Working Paper

Uncertainties and Decision Making

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1 Introduction

With these notes we want to outline some practical and important issues concerning decision making under uncertainty and risks.

Decision making under uncertainty is as old as mankind. Cäsar crossing the Rubicon said "Alea iacta est" (the dice are cast) implying that our decisions are affected by chance.

Uncertainty is an essential feature for any study directed towards the future. The tendency in decision making involving uncertainty is to postpone decisions until uncertainties are resolved. Unfortunately, uncertainties are inherent in virtually all systems related to economics, meteorology, demography, ecology, etc., and will never be resolved. In order to encourage a decisionmaker to act we need appropriate tools to explicitly treat the uncertainties involved. When effects of decisions become evident only after a considerable length of time, the purely adaptive "trial and errors" market mechanisms become inefficient and need to be combined with anticipative mechanisms based on models, predictions, regulations and verifications.

2 Models, Verbal Analysis

We cannot predict or optimize everything unless we make some assumptions on patterns of the phenomena: create a model. Sometime the assumptions are made in an explicit form that leads to mathematical models and a rigorous analysis. Sometime they are made implicitly and that leads to a verbal analysis, often providing more questions than answers. It is impossible to rely on such statements as "the market brings to the equilibrium demands and prices" or "adaptation and incremental improvements will find an optimal decision", since it is true only when underlying processes have certain properties, for instance, smoothness, concavity, product homogeneity and divisibility, unchanging "environment", full information (certainty), lack of strategic behaviour, absence of externalities and increasing return to scale. Otherwise the "convergence" cannot be proved even on paper (mathematically) and such mechanisms without the help of a regulatory body may create unpredictable behavior and dangerous "locked-in" structures even by small disturbances at the initial stage[1] as may be the case in the former Soviet Union.

Uncertainty enters various steps of a model building process, starting from the model structure: purely verbal or with some precise details; which variables are driving?; levels of aggregation and decomposition (region, country, the universe) coinciding with uncertainty of available information; relations between variables, for instance, diffusion equations or only simple "predictors" such as the transfer coefficients are needed in the pollution

[1]For more information see W.B. Arthur, Yu.M. Ermoliev and Yu.M. Kaniovski *Path-dependent processes and the emergence of macro-structure*, European Journal of Operations Research, 30 (1987), p.294-310.

management if we take into account uncertainties in the weather conditions, "doses–effects" relations, current and projected emission patterns, etc.

Modeling usually starts with an attempt to create a model which makes sense of observable data. From the formal point of view it can be interpreted as an approximation of available data by some functions such as polynomial, or other functions which may also be given implicitly as solutions of differential equations, or Petry and neural nets with unknown parameters to be identified. Since there may be errors of measurements, the "fitness" must be understood also in a certain way that often leads to non unique solutions of the identifications procedures and additional uncertainties of predictions.

3 Decisions, Uncertainties—Searching for Improvements

The existence of decision variables creates additional and essential sources of uncertainties in contrast to classical natural science models. The aim of a decision making model is not only to make sense of a limited set of observable data, but also the change of the current practice and structure for a better state which may be unlike anything we experienced in the past. The experiments or "trial and errors" mechanisms may be dangerous, time consuming or simply impossible and we have to rely only on models equipped with tools to treat arising uncertainties explicitly.

Goals of a decision making process and constraints create more uncertainties and cases when good intentions may lead to bad results and public distrust. Constraints induce feedback, for instance, if carbon dioxide of the water C_w and carbon dioxide of the atmosphere C_a satisfy a symbolic constraint (in a time interval)

$$\alpha C_w + \beta C_a = const$$

with some positive coefficients α, β, then the positive feedback occurs: the warming diminishes C_w and thus increases C_a which leads to more warming, diminishing C_w and so on.

Uncertainties in constraints such as in values α, β influence a balance of positive and negative multiple feedback and may lead to opposite conclusions, for instance, on warming or on the ozone layer.

It is not true that any other, e.g. verbal, analyses avoids all above mentioned difficulties: it is only possible by ignoring them.

Of course, the building of a decision making model (conceptual or more formal, including details such as concrete values α, β) is not only a scientific task but also an art. The main purpose is not to take a picture of the situation, but to make a sketch—a laboratory world to examine possible concepts and alternatives. In the presence of essential uncertainties the most important task of the modeling seems to be a search for better solutions, comparative studies or optimization rather then prediction and assessment. It is impossible to explore all details of the environment, biochemical, hydro-meteorological, genetic, etc. differences involving large numbers of variables. Therefore assessments and predictions will always yield poor absolute values. Despite this, the preference structure among decisions might be stable, which is similar to the difference between measuring exact weights of parcels and only guessing which is heavier.

4 Robust optimal decisions and Scenario Analysis

The study of interactions between uncertainties, decisions and outcome is a methodologically challenging task. Relations among variables may be changed considerably with the change of decisions aggravating various risks. It is possible to speak about reliability of a decision: the best decision combined with "bad luck" may lead to negative effects, and a wrong decision with "good luck" to positive effects—at least for a while, as it was in the case of Chernobyl.

How can we choose decisions which are optimal and still robust against all eventualities? For instance, decisions in the case of greenhouse effects which might fundamentally endanger modern civilization, but on the other hand might be beneficial or perhaps even not big enough to matter[2].

As uncertainty is a broad concept, it is useful to approach it in many different ways[3].

The most commonly used technique, scenario analysis, to deal with long term planning under uncertainty is seriously flawed. Although it can identify "optimal" solutions for each scenario, it does not provide any idea as to how these "optimal" solutions should be combined to produce merely a reasonable decision. A suggestion to mix the best solutions with weights assigned to corresponding scenarios may lead to wrong decisions as it can be seen from simplest situations. There is also a suggestion to use mixed strategies derived from an appropriate "pay-off" (decision/scenarios) matrix. In this case an optimal decision appears as a result of random choice among decisions which are optimal only to one scenario (e.g., best crop for "dry", "normal" or "wet" season).[4] Of course, such a solution lacks diversity (multiple crops, different energy sources, various products, etc.) needed for its robustness against all possible scenarios.

5 Set-Valued Estimates. Guaranteed Strategies

One clear and easy way to characterize the uncertainty is by ranges or even sets of possible values (without identifying their likelihood): set-valued estimates. Since such a depiction of uncertainty does not provide any idea of more reasonable values, the choice of optimal decisions is usually based on the calculation of upper and lower bounds of outcomes with respect to all possible uncertainties. In particular, it is suggested to make decisions from the worst case situation—so called guaranteed decisions (strategies).

In the case of dynamic systems modeled by differential equations guaranteed strategies often lead to instabilities and irregular behavior of corresponding trajectories. The absence of optimal trajectories in the commonly used sense requires generalization, and it is often suggested to describe the state of the system also as a set-valued estimate. Since nonlinear transformations of a dynamic system may map a simple set into a rather complicated domain, the set-valued estimates of the state are sometimes searched among a rather simple approximation, for instance, ellipsoids.

The characterization of uncertainties by ranges and worst case analysis may provide

[2]Clark, W.C., 1986, *"Policy Procedures on the Carbon Dioxide Question: Risk Uncertainties and Extreme Events"*, CP-86-26, International Institute for Applied Systems Analysis, Laxenburg, Austria, pp. 287-303.

[3]See, for example, *Decision analysis and behavioral research*, D. von Winderfeld, W. Edwards, Cambridge University Press, 1986.

[4]Yuri Ermoliev, Günther Fischer, 1993 *Spatial Modeling of Resource Allocation and Agriculture Production under Environmental Risk and Uncertainty*, IIASA, WP-93-11.

useful insights. On the other hand, "because[5] a range may be derived through a process of ruling out impossible values rather then through critical analysis of the relative likelihood of more reasonable values this depiction sometimes arouses scepticism and can appear non-scientific."

The following is a typical illustration:[6]

> Giving only the mean annual income....or only the median and bounds would not reveal that a substantial proportion of the total national income accrues to a relatively small number of very wealthy persons.

The same applies for concentrations of pollutants or populations, toxicants, energy demand for the next year, etc. For instance, a daily concentration of a toxicant may be within the normal level, but for five minutes it may vitally exceed the survival level. A simplified depiction of the uncertainty by decision makers may easily create a syndrome of "public concern" as the following example illustrates.

Consider a situation where two types of accidents might occur to a group of ten people. The first type of accident will result in the death of all ten persons in one out of ten cases. A second type of accident will result in the death of each of the ten persons (independently) also in one out of ten cases. The range of possible death (set-valued estimate) and the averaged value are the same in both cases, that is 10 and $(1/10)\ 10 = 1$. But the chance of ten deaths in the second type of accident is only $10^{-10} = 0.0000000001$ in contrast to 0.1 of the first type.

6 Weights, Fuzzy-Sets

A more general idea to depict uncertainties is to assign weights to possible values of uncertainties (parameters, events), such as frequencies in the case of repetitive events, or confidence measures in the case of non-repetitive events (an accident of a particular plant). Such weights are often interpreted as a probabilistic measure (possibly of subjective nature). There may also be other versions, for instance when the support of the weight-function is interpreted as the "Fuzzy set".

The difficulty of such an approach is that although the weights of initial data are known, their propagation through the system creates enormous computational difficulties for the analytical evaluation of outcome weights. The decision variables as we can see further may dramatically affect these weights and create a higher order source of uncertainties.

The interpretation of weights as a probabilistic measure has essential advantages to any other concepts since the study of the propagation in this case can be based on the Monte Carlo simulation techniques. In other words, it is possible to incorporate a Monte Carlo simulation model into an optimization process. In addition the probabilistic approach enables us to formalize ideas of learning from observations, ex ante (anticipative) and ex post (adaptive) control strategies. Unfortunately it is impossible in other approaches such as the Fuzzy-set theory. Besides, in such approaches there is no well established empirical method to quantify fuzziness (or vagueness, plausibility) similar to frequencies analysis

[5]See Adam M. Finkel "Confronting Uncertainty in Risk Management", Center for Risk Management, Resources for the Future, Washington, January 1990, p.28.

[6]See Adam M. Finkel, p.XIII.

of real observations, experiments, results of questionnaires or expert judgements of the probability theory. This produces various inconsistencies.[7]

How can we design an optimal strategy by utilizing only weights of the initial data?

7 Stochastic Optimization. Anticipation and Adaptation

The task of the Adaptation and Optimization Project (1982-1985) at IIASA was to study the answers to this question. Extended discussions of motivations, developed tools with its possible applications and implementations has been published in the volume Yu. Ermoliev, R. Wets (Eds.), 1988, *"Numerical Techniques for Stochastic Optimization"*, Springer-Verlag, Computational Mathematics, 10.

Let us note that the search procedure cannot be based on straightforward Monte Carlo simulations since even the evaluation of the best decision among two alternatives in such a case is equivalent to well-known problems of hypotheses testing. Since results of initial data propagation can be studied (in general) only by a sampling procedure, the above question is equivalent to the following:

> How can we find an optimal decision among an infinite number of feasible decisions without calculation of exact values of "objectives" and "constraints" and with a large number of decision variables and uncertainties?

The answer to this question leads us to stochastic optimization tools conceivable with only partially known distribution functions (and incomplete observations of unknown parameters), which have been successfully applied to a wide variety of problems.

There are differences between the typical formulation of the optimization problems that come from statistics and those from decision making under uncertainty. Stochastic optimization models are mostly motivated by problems arising in so-called "here–and–now", or ex ante situations, when decisions must be made on the basis of existing or assumed a priori information about uncertainties. The situation is typical for problems of long-term planning (strategic behavior) possibly involving irreversibility that arise in systems analysis. In mathematical statistics we are mostly dealing with "wait–and–see", or ex post situations when decisions are made on the basis of observations "during" the decision making process. Such a situation is encountered in short-term planning or in driving a car.

Two-stage and more general multi-stage models of the stochastic optimization attempt to develop an operational framework for the concept of "now–and–then" decisions incorporating "here–and–now" strategies with "wait–and–see" decisions. When effects of actions are uncertain and the actions cannot be reversed, the timing of decisions becomes important. The choice of anticipative ex-ante decision must be subjected to how future information and learning could change the level of uncertainties and developments. Thus multi-stage approaches of the stochastic optimization incorporate classical risk aversion and risk seaking concepts. The notion of stage refers to various pre-posterior information levels rather then time intervals: each state may consist of some time intervals and we can also speak, for example, of two-stage dynamic models.

Generally speaking, in the case of uncertainties, non-stationarities or disequilibrium there are two major mechanisms to facilitate our response to uncertainty and changing

[7] D.V. Lindley "The Probability Approach to the Treatment of Uncertainty in Artificial Intelligence and Expert Systems", *Statistical Science* 2, 17-24 (1987).

conditions: the short-term adaptive adjustments (defensive driving, marketing, inventory control, emergency service, etc.) and the long-term anticipative actions (engineering design, policy setting, investment strategies, insurance, pollution reduction strategies, land development, even keeping an umbrella at the office, etc.). The anticipative, long-term perspectives are important for the environmental problems, since purely adaptive market mechanisms are inefficient in such cases.

The major challenge to the systems analyst is to develop a "now-and-then" approach that combines both mechanisms (adaptive and anticipative) in the presence of a large number of uncertainties, and this in such a way that it is computationally tractable.

8 Chaotic Behavior

Dynamics and nonlinearities bring uncertainties of chaotic behavior similar to probabilistic and well-known from the pseudo-random numbers generators theory. Again, often the famous question arises whether the good plays dice or the nature is certain and can be predicted.[8] If this is the case, then one may become a very wealthy person by finding a deterministic equation describing fluctuations of prices on a stock exchange. How shall we characterize uncertainties of the chaotic nature, for instance, sequences of pseudo-random numbers—by ranges, trajectories, generating equations or remarkably stable frequences? Of course, it depends on the problem at hand.

9 Externalities, Negotiations

Decisions create more uncertainties through externalities existing in economics and environment, when results of a participant (firm, region, country) are uncertain until other participants reveal their decisions. Such uncertainties require not only exchange of information, but a concept of mutual interests and joint constraints which are not always understood and may often emerge only through a process of successive negotiations. There exists the game theory: theoretical framework with various concepts of possible equilibriums in the presence of different interests of participants (of cooperative or non-cooperative nature).

The treatment of the environment as this generation's "public good" brings uncertainties in the evaluation of real costs, benefits or efficiencies. Of course, the round table discussion will never produce values of damages and the negotiations must be supplemented by unified approaches to treat involved uncertainties. It is possible to imagine a model accepted by participants to serve the purpose of a witness or victim during a negotiation process.

10 Irreversibility, Dynamic Approaches, "Dialogue" with Models

Accumulative effects and irreversibility of environmental damages when values may be lost forever accentuate on the long-term and anticipative nature of decisions and require to establish "contacts" between the current and the future generations. What are "taxes" of the current generation? What are "social" costs and benefits? The answer often calls for a

[8]Contradicting even with W. Heisenberg's "uncertainty principle".

regulatory body (or number of competing authorities) to establish environmental quality in certain "norms" with appropriate monitoring regulations, and verification procedures. It requires a dynamic approach to the decision making under uncertainty to enable the tracing back to the sources of observable episodes.

A decision making process can easily be directed towards desired results by manipulations with goals, constraints, parameters, relations between variables, etc. (with model), that may lead to contradictory conclusions and public distrust. It is important to understand that the structure of a decision making model is not as well defined as the structure of natural science models. The formulation of such a model always runs into difficulties in identifying objectives and constraints. Their understanding and quantification is usually achieved through a "dialogue" with the model when each run of the model provides new ideas of possible variables, constraints and goals. Such process proceeds until a compromised decision emerges. Therefore, the model doesn't make decisions—they are made by the decision maker (participant) and the most important is not "**what** he decides" but "**how** he decides".

We can say that in the decision making process nothing can be claimed to be the true decision; it varies with changes in goals, alternatives, constraints, information. The whole process must be viewed as a process of successive adjustments rather than a comprehensive choice. The study of such non-stationary decision making processes is rather challenging. Important "adjustments" may not necessarily lead to incremental improvements even in the case of concave (or convex) but rapidly changing (non-differentiable) objective (goal) function.

The complexity of the decision making process itself creates additional uncertainties and constraints. We can also study the propagation of decisions through the "decision events tree", where an event may be "public rejection".

11 Nonlinearities. Discontinuity of Risk Functions

The nature of uncertainty avoids certainty so as even the best solution, as mentioned above, may have (with bad luck) a negative result. It requires transparent representation of scientific understanding of inherent uncertainty, which is often quite different from the public and decision maker perception.

The uncertainty creates easily nonlinearity and even discontinuity, which in turn creates new uncertainty.

In the case of guaranteed strategies, essential uncertainties are created by "inner" subproblems which are often not completely solved, and thus the guaranteed results are unknown. Sometime this difficulty can be avoided by using nondifferentiable optimization techniques that have been a focal point of IIASA's research since the mid 70s.

Stochastic approaches to the decision making under uncertainty aim to model situations of a risk, when each given decision may have both positive or negative results or externalities (win or loose, hit or miss, cost or profit, over or under-estimating, etc.). The possibility of positive and negative externality for the same solution results in nonlinearities and even discontinuities of corresponding risk indicators.

Risk-based environmental management provides a great deal of such examples. We can think of a typical "hit or miss" situation, that of reducing accumulative pollutants such as greenhouse gases and stratospheric ozone-destroying gases. Such problems are characterized by uncertain thresholds, which, if exceeded, may result in drastic losses.

In this example the discontinuity occurs due to irreversible environmental impacts.

The presence of risks creates also a discontinuity in a at first glance rather "smooth" situation.

To illustrate this fact by verbal discussion is difficult. Therefore, let us consider a simple pollution control model, given by a linear inequality ("safety constraint"):

$$hx \leq 2 ,$$

where x is a level of the emission; h is a "predictor", which computes the average deposition level at a receptor point from the unit of the emission. The norm of the "daily" depositions equal 2. Such safety constraints are important as a "surrogate indicator" in the case, when the evaluation of real damages (costs) is impossible.

If h is known, then a permitted level of the emission x is defined easily from the inequality. Suppose now that h is a random variable which takes only two values 0.5 and 1 with probability 0.5. Then for any $x \geq 0$ there may be two possibilities

$$hx \leq 2 \text{ or } hx > 2$$

and the simplest indicator to characterize risk to violate the safety constraint is (risk-function)

$$F(x) = Pr[hx > 2] ,$$

The graph of this step function is shown in Figure 1. Accumulative effect (increasing level of pollution x) results in discontinuous changes which is similar to so-called "chemical time bomb" phenomena.[9] They are unexpected and uncontrolled unless the rate of change is characterized. For instance, in a pollution control problem we might be interested in minimizing risk by a process of incremental improvements. Since the marginal value (derivatives) of $F(x)$ at any x is 0, the search process cannot be based on evaluations of these values. How can one characterize the increasing rate of two functions, shown in Figure 2 in order to utilize it in the search of an optimal decision.

[9] See Stigliani, W., P. Doelman, W. Salomons, R. Schulin, G. Smidt and S. van der Zee (1991). Chemical Time Bombs: Predicting the Unpredictable, Environment 33:4-9, 26-30.

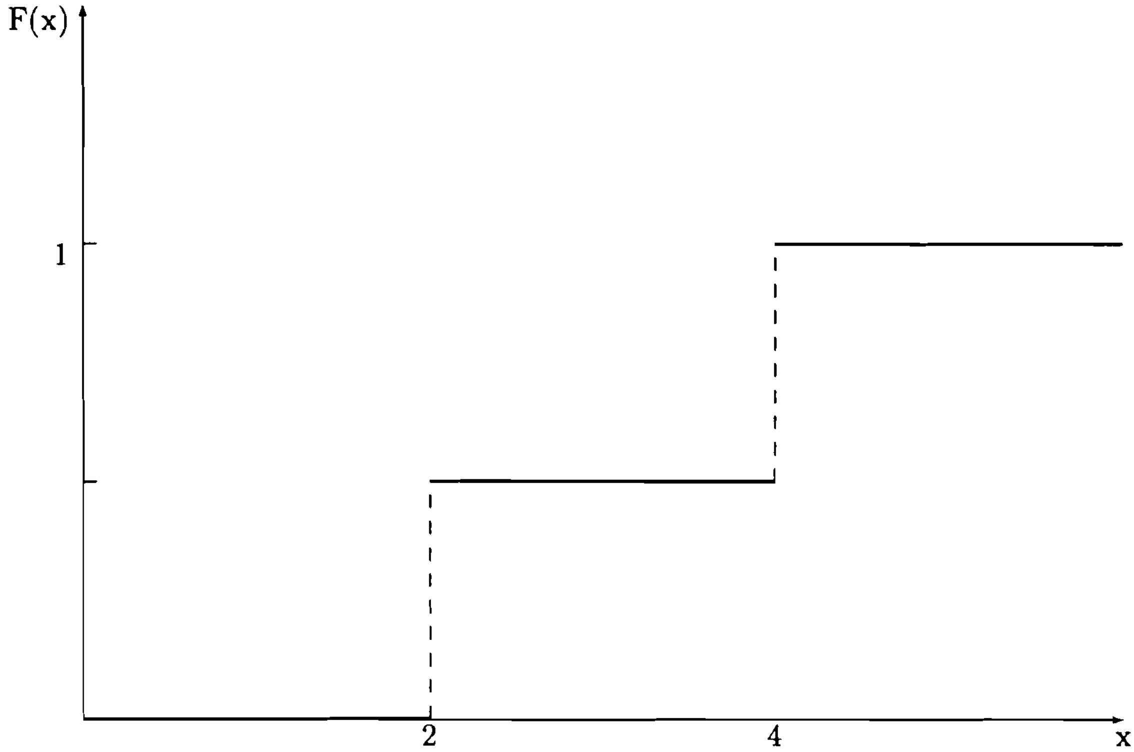

Figure 1. Discontinuity of the risk function. The marginal value of the function is 0 but the risk increases with increasing x. Accumulative effect results in rapid and sudden changes.

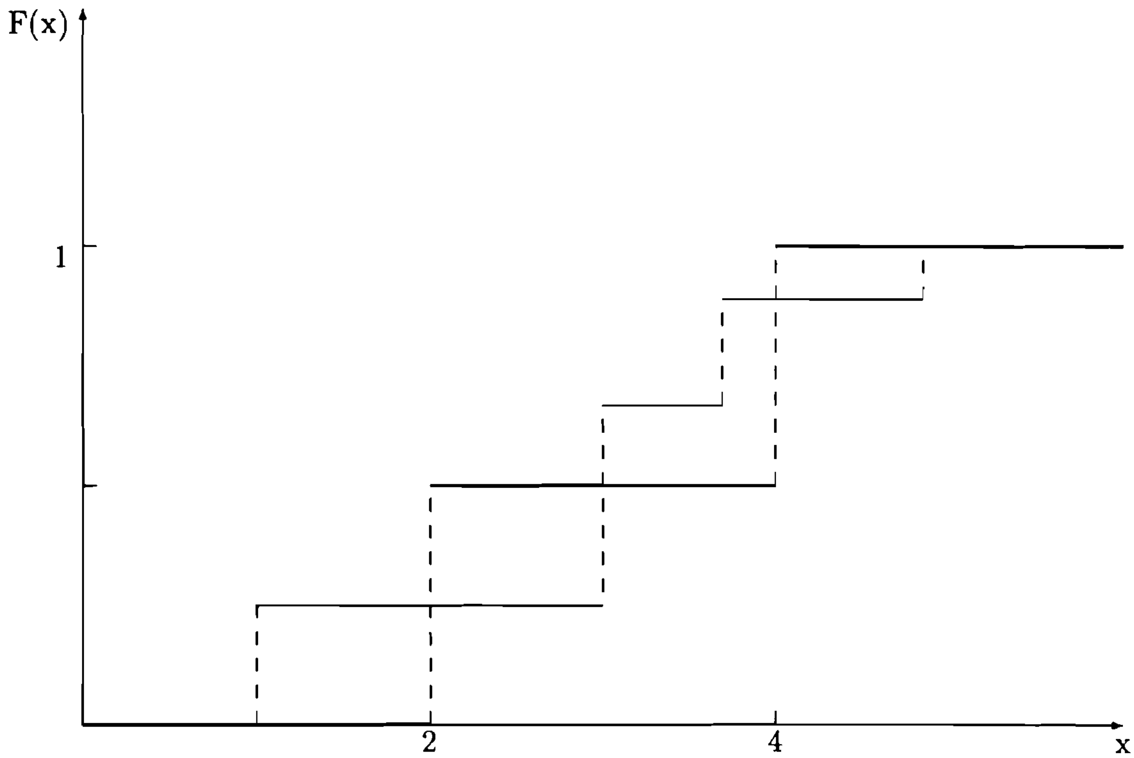

Figure 2. What is the rate of increase of two functions.

We can imagine now possible nonlinearities and discontinuities of a similar risk function defined for the case n emittors:

$$F(x) = Pr\left[h_1x_1 + h_2x_2 + \ldots + h_nx_n > q\right],$$

where $h_1, \ldots, h_n$ are random predictors (for each emittor $i = 1, \ldots, n$); $x_1, \ldots, x_n$–levels of emission; q is the norm (critical load).

The discontinuity of $F(x)$ creates the uncertainty of the change or in the indication of improvements. In addition there are critical uncertainties in the evaluation of the risk indicator $F(x)$ as a function of x. The choice of a decision x may dramatically affect the value $F(x)$: let us compare decisions $x = (1, 0, \ldots, 0)$ and $x = (1, 1, \ldots, 1)$. Although uncertainties in the initial data (predictors) $h_1, \ldots, h_n$ are characterized by probabilistic measures, the probability of an outcome

$$h_1x_1 + \ldots + h_nx_n > q$$

as a function x is evaluated exactly only in exceptional cases (especially for more general risk indicators involving "damages").

The fundamental problem in the design of an optimal strategy is to bypass the above mentioned discontinuities, uncertainties of the change and the evaluation of exact values of functions similar to $F(x)$, in general involving also costs, benefits, damages, etc.

Within the stochastic optimization framework such search technique is developing for problems with large numbers of decisions variables and uncertainties, practically arbitrary "distributions" and rather general "objectives" and "constraints". The technique is essentially based on using only random observations of risk functions through Monte Carlo simulations or actual measurements.

12 Enforcement and Education

A more rigorous and explicit formulation of assumptions may be a great advantage for decision making, even if uncertainty exists in the model. But policy formation is a complicated process that involves political pressure, self-interest and the interest of various groups. The reluctance to reach firm[10] conclusions enables a policy maker to use research as to support or justify a pre-determined decision. Background and educational training is also important. Decision makers usually have difficulties assessing variable results and often only simple estimates and indicators are encouraged, which are often sources of public concern and distrust. For instance, such indicators as "life expectancy" and "collective dose" or "concentration level" alone are not able to dipict variations of effects within a population or country. What are enforcements for using more adequate data in order to choose "fair" decisions? Of course, such a question can only be answered on a case by case basis.

[10]See C. G. Miller "Cases in the applications of air quality models in policy making", pp. 23-24 in the Proceedings of an October 1979 IIASA Workshop on "Mathematical models for planning and controlling air quality", Pergamon Press.