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Spatial Modeling of Resource Allocation and Agriculture Production under Environmental Risk and Uncertainty

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Abstract

Within the limits of the physical production potential, effective land performance is largely determined by anthropogenic factors, e.g., the availability of infra-structure, market access and a complex interaction of behavioral, socio-economic, cultural and technological conditions. Any discussion of interactions with the environment must come to grips with uncertainties and resulting risks. The paper outlines a possible approach to modeling agricultural production in a spatial setting under environmental risk and uncertainty. The methodology proposes the distinction of 'compartments', i.e., geo-referenced collections of homogenous land units which are coherent in terms of natural resources and economic production conditions. The compartments interact through commodity markets and 'trade' of mobile resources, compete for allocation of limited public resources, and are jointly affected by government policies, regulations and other regional constraints.

The proposed model incorporates two types of decisions: *strategic* ex-ante decisions and flexible *adaptive* ex-post decisions. The notion of strategic decisions is especially relevant when dealing with the risks of virtually irreversible impacts, such as placing a dam, clearing rainforests for agricultural purposes, or diverting agriculture land to urban and industrial uses. Adaptive ex-post decisions allow to correct the strategic decisions relying on the ability to learn from experience and to adapt to observed situations.

From a formal point of view, this process can be interpreted as a stochastic decomposition procedure, integrating individual agents through market clearing conditions and allocation of public resources in order to find strategic decisions that are robust and most effective in an uncertain setting.

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1. Introduction

"How people or nations use their land depends on complex, interrelated factors which include the characteristics of the land itself, economic factors, social, legal, and political constraints, and the needs and objectives of the land users. In order to make rational decisions it is necessary to:

- collect the right information about physical, social, and economic aspects of the land area in question; and
- assess the land's relative suitability for different uses in the light of the needs and objectives of the land user and the community" (FAO, 1990a).

With these words the Food and Agriculture Organization of the United Nations (FAO) points out the need for comprehensive new approaches in land-use and development planning. Basic principles to be observed in sound land evaluation were first published in 1976 (FAO, 1976; FAO 1984; FAO 1985; FAO, 1990b) and applied to assessing the capability of land in the developing world (FAO/IIASA/UNFPA, 1982). While this assessment was severely limited by the availability and quality of data and the capacity to store and process spatial data sets by computer, the importance of such work for development was recognized by the 1983 FAO Conference. Consequently, the approach was further developed in a case study (FAO/IIASA, 1991) concerned with the development and implementation of a national level methodology for the determination of land use potentials in Kenya, as a tool for policy formulation and development planning. The specific role of this policy tool may be defined as assisting in the planning of sectors and regions bridging the gap between conventional macro-planning and specific project planning.

Decision making in agriculture, at the local level by farmers and by policy makers at the regional/national level, starts from an assessment of production options. Assessment of agricultural production traditionally has been classified along two lines. One approach is followed in biological and physical sciences and is concerned with the production potential of land, usually without too much concern regarding the socio-economic constraints and dynamic aspects of realizing this potential. A second approach is supported by social and economic sciences and concentrates on behavioral aspects of farmers or authorities. Resources like land or water are only generally described and with only a very elementary idea of their physical production potential. Both approaches have their merits, but each gives only partial explanations of current agricultural practices or policy implications.

There is urgency in linking these two disciplines as evidence suggests that increasing pressure on land has resulted in the implementation of agricultural production systems which go beyond the land capacity. Recommendations of alternative sustainable land uses which imply changes in actual agricultural production systems should prevent land degradation or environmental damage and also be an improvement in terms of the land user's economic viability and performance.

It is well recognized that 'sustainability' is a politically powerful concept (IIASA, 1992) with various, sometimes controversial, definitions put forward. One such definition, published by the National Research Council (N.R.C., 1991), postulates that sustainable development "should include management of the use of a resource so it can meet human demands of the present generation without decreasing opportunities for future generations". Although intuitively clear in its meaning, the application of such a definition in the selection of concrete development strategies is difficult because of the complexity and numerous interactions, in space and time, in managed and unmanaged ecosystems.

As stated by FAO (FAO, 1990b), principles to be adopted for promoting sustainable land use include:

- managing the land in order to maintain or improve its productive capacity: this means protection against erosion, maintaining the nutrient status by adequate use of manure or fertilizer, safely disposing of toxic wastes;
- assessing and preparing for predictable hazards: natural hazards such as drought, floods and landslides cannot be completely avoided but the risks can be reduced, for example by water supply and flood protection schemes and by avoiding risky sites;
- minimizing the loss of productive land: prime farmland and land of unique value for any desirable land use should be conserved.

Change, be it for socio-economic or environmental reasons, creates conflicts between competing uses of resources, in particular land, and between the interests of individual land users and the common good. For instance, development of new farmland competes with the preservation of natural biotopes or forestry; urbanization and industrialization may irreversibly divert valuable agricultural land.

Environmental changes, such as erosion, physical soil destruction, salinization, acidification, toxification, radioactive contamination, or the increasing concentrations of greenhouse gases, affect different regions in various ways. One of them is a degradation of soils leading to a deterioration of the agricultural production potential.

A natural step in the assessment of the environmental changes in a region is the classification and evaluation of the land according to bio-chemical or physical properties such as content of organic matter, base saturation (pH level), soil depth and nutrient status, availability of chemical elements essential for grazing animals or human food, and presence of contaminants, e.g. toxic chemicals in ground and surface water, radionuclides, etc.

But soil properties alone do not determine land production potential. Climate characteristics such as variability of rainfall, drought hazard, occurrence of floods, storms, and occurrence of frost during the growing season, all affect potential land productivity. Site location, e.g. distance to markets or vicinity to nuclear sites and radioactive fallout areas, may also be a critical variable in land evaluation. For instance, radionuclides intercepted by crops or pasture may very soon enter the food chain as food or feed. Therefore, public health protection may demand some serious constraints upon agricultural and farm practices, harvest and food management.

Within the limits of physical production potential, effective land performance is largely determined by anthropogenic factors, i.e. the availability of infra-structure, market access and a complex interaction of behavioral, socio-economic, cultural and technological factors. In a realistic assessment, technological considerations must include aspects such as availability and viability of agro-technologies, scope for irrigation, soil improvements and clean-up of contamination. Socio-economic factors include land tenure system, farming objectives, crop composition and structure of livestock. Therefore, land evaluation must refer to well defined conditions and assumptions, and the assessment of land production potential must be carried out in a spatial and dynamic context.

A succession of droughts may flip a region out of a 'stable' performance. Of course, the land productivity can be improved by applying appropriate adaptive measures, such as agro-chemicals and supplementary irrigation, subject to economic limitations, budget constraints, as well as physical limitations and the ability of land to recover from degradation and contamination. Radioactively contaminated areas can, for instance, be used for crops that take up less radioactivity or accumulate the radioactivity in parts which are not used for consumption, or crops which discharge radioactivity during food processing.

Various aspects of land evaluation and spatial modeling of agricultural production are discussed by Keyzer (Keyzer, 1992), in particular a possible model representation in terms of a (large-scale) optimization problem. The aim of this paper is to develop ideas towards an operational approach enabling to evaluate the land potential by simulating various interacting activities in a region driving to achieve an optimal performance within limited resources, local and mobile, and limited possibilities to improve or recover its production potential (using fertilizers, pest control, machinery or decontamination measures, etc.) against uncertainties of weather, market situation or other risks such as. the contamination of crops, pastures or livestock.

The proposed simulation procedures require the development of certain decomposition schemes making use of non-differentiable and stochastic optimization techniques. Special attention is paid to incorporating uncertainties and risk. We also aim to incorporate in a formal way dynamic aspects.

The approach we adopt is based on subdividing the region into 'compartments'. Depending on scale, a compartment may simply correspond to a farm or a collection of farms. They are defined as to reflect structured entities (sub-systems) of the region under consideration and their economic (and other) interactions. The notion of a compartment, as used her, does not exclude internal heterogeneity of certain characteristics, such as soil or landform; a compartment may itself be subdivided into smaller homogenous units to form the basis for the bio-physical evaluation[1]. For instance, a valley in a mountainous region that economically depends on, say forestry, dairy production and tourism, may become a compartment even though there is likely to be a large heterogeneity of resources within that compartment, e.g. in terms of steepness of slopes, soil type and even climate zones. Land units within the compartment should refer to relevant combinations of such heterogeneous attributes.

In practice, compartments will often be defined by superimposing maps of different aspects of the land and then drawing boundaries that best reflect the most important distinctions in the separate maps. Geographical information systems (GIS) provide powerful assistance in storing and manipulating geo-referenced data. The details of defining and characterizing compartments will vary with the intensity of a study and the scale of the study area, and may be constrained by availability of data. The description must refer to relevant

[1] In land evaluation as carried out by FAO and others, such basic land units have been termed agro-ecological cells. Often these cells cannot be geo-coded but are known (in a statistical sense) to exist in the wider context of a geo-referenced map unit, e.g. FAO/UNESCO soil map unit.

endowments, applicable economic and physical balance equations, with identification of 'permanent' resources of each compartment and 'mobile' resources which can be redistributed or 'traded' among them.

The iterative decomposition procedure simulating the behavior of interacting compartments takes place with respect to allocating mobile resources and meeting other 'regional' constraints such as targeted production levels, tolerable environmental impacts and market clearing conditions.

Each compartment is represented by a stochastic and dynamic model. The stochastic features of the model reflect risks to production and economic returns, effectiveness and efficiency of investments and other measures applied to improve characteristics of soils and the capability of land. The dynamics trace future stochastic trends of long-term strategic, possibly irreversible, decisions. The time step reflects the seasonal regenerative process of agricultural and livestock activities. Within the regenerative cycle (one season) the model has static nature.

Uncertainties within a time step are represented by a number of 'scenarios', e.g. dry, wet or normal weather conditions. In many cases the 'weather factor' can be considered as the driving variable attributing to various risks of agricultural production, including yield failure and market situation. Other kinds of uncertainties, such as accidents, droughts, flooding, etc., can also be represented by stochastic variables with a finite or infinite number of attainable values.

2. Risk, Scenarios, Anticipative and Adaptive Decisions

Uncertainties and risks are inherent in agricultural practices. Land performance has always been affected by weather and climate, and sophisticated adaptive strategies to cope with climatic vulnerability have evolved in many societies. Floods, droughts, storms and frost may decisively affect the agricultural potential of a region.

With the commercialization of agriculture new kinds of risks occurred, such as the reliability of access to markets, to purchase commercial farm inputs and to sell surplus production, and level of prices in local and international markets.

Anthropogenic additions of contaminants to soils, in particular through accident or inappropriate waste management in industry or military sites, 'chemical time bombs', may also be vital to soils for continued agriculture, or likewise to the suitability of water for fisheries. Impacts of 'Chernobyl' have been observed upon European agriculture. Contamination of vital natural resources and accumulation of toxic elements in the food chain exerts stochastic effects on human health and biosphere and requires changes in agricultural practice and food production to 'acceptable' levels.

Traditional approaches to land evaluation often ignore possible variability in production conditions, simply by averaging them, which is equivalent to dealing with only one scenario of possible developments. In general, however, there may be an infinite number of scenarios and the challenge is to find strategies robust against all eventualities. Averaging deprives us of the diversity which is necessary to meet risks and may lead to wrong conclusions. Let us illustrate this by the following simple examples:

Example 1:

Suppose there are two farms with the same crop structure and average yields, but with different variances of yield, e.g. due to weather conditions. It is clear that the farm with a larger variance in yields is more vulnerable and may be less profitable but it is impossible to distinguish them on the basis of averaged data.

Example 2:

Suppose there is only one type of soil and two crops, A and B. Crop A performs better in dry seasons and crop B outperforms crop A in wet seasons. On average, the weather condition may only be dry or wet, implying a monocropping structure as an optimal solution, i.e. cultivation of only crop A or only crop B. By taking into account probabilities for both weather conditions, dry and wet, the structure of the optimal solution is changed to a multicropping structure: crop A and crop B must be included in the optimal solution in proportions related to frequencies of wet and dry seasons, prices on the market, etc.

In the short term, the sustainable supporting capacity of a region, limited by conditions in 'bad' years, essentially depends on the region's ability to adapt to changing conditions, e.g. through additional land management measures, food storage, or finance of additional imports and infrastructure for distribution. In the longer term, strategic decisions are important to keep pace with changing requirements.

In the model representation these aspects are taken care of by incorporating two types of mechanisms:

(a) long-term (ex-ante) strategic decisions such as investment in machinery, irrigation schemes, storage facilities, major land improvements, structure of livestock and cropping patterns, and

(b) short-term adaptive (ex-post) decisions such as use of irrigation water, level of fertilizer application, allocation of manual labor and machinery in a given situation, change of planting dates, replanting of crops, etc.

Yields in agriculture, of crops and livestock, are a result of human activity and management skills, both ex-ante and ex-post decisions, and natural endowment and characteristics, some of which are uncertain, like weather conditions or occurrence of extreme events such as floods, droughts or accidents.

Assume that each of these uncertain factors can be characterized by a finite number of scenarios with weights describing associated frequency distributions. For example, weather conditions could be classified by three situations (scenarios): (1) dry season with frequency P_1, (2) wet season with frequency P_2, and (3) 'normal' season with frequency P_3. Of course, more details could be incorporated, like characterizing weather conditions as 'dry and cold', 'dry and hot', 'wet and cold', etc.

Therefore, generally speaking, each scenario can be described by a vector s of integer numbers,

$$s = (s_1,...,s_N)$$

where the components $s_1,...,s_N$ are random variables assuming a finite number of integer values with given frequencies (probabilities). For example, $s_1 \in \{s_{11}, s_{12}, ...\}$ may represent scenarios of weather, $s_2 \in \{s_{21}, s_{22}, ...\}$ describes scenarios of flood hazard, $s_3 \in \{s_{31}, s_{32}, ...\}$ models scenarios of possible accidents and contamination, etc.

Note that although the number of possible situations (scenarios) $s=1,...,S$ is a finite number S, it may easily become very large so that special stochastic optimization techniques are required to efficiently deal with the associated decision making problems.

Let us denote the vector of strategic decisions by x^k and the vector of adaptive decisions in situation s by $a^k(s)$, where k denotes a compartment, $k=1,...,K$. We also assume, without loss of generality, that the set of scenarios is the same for all compartments, e.g. by taking the union of all possible combinations of stochastic variables and applying zero probability weights in specific compartments as appropriate. Therefore we can write $s=1,...,S$, with compartment specific weights $P_1^k,...,P_S^k$.

Then, for a given scenario s, ex-ante decisions x^k and adaptive decisions $a^k(s)$ in a land compartment k, yields $y^k(s)$ of crops and livestock can be represented[2] by a function f^k,

$$y^k(s) = f^k(s,x^k,a^k(s)),\ k=1,...,K$$

and the use of production inputs and resources in scenario s can be characterized by a vector function r^k,

$$r^k(s) = \varphi^k(s,x^k,a^k(s))$$

For a given scenario s in compartment k, and ex-ante and ex-post decisions x^k and $a^k(s)$, respectively, we define an optimality measure, a valuation of welfare $u^k(s,x^k,a^k(s))$. Then the short-term agricultural potential U^k of farm compartment k is obtained by selecting adaptive decisions $a^k(s)$ so as to maximize welfare (for given s and x^k),

$$U^k(s,x^k) = \max_{a\in\mathcal{A}^k(s,x^k)} u^k(s,x^k,a) \tag{1}$$

where the set of feasible ex-post decisions $\mathcal{A}^k$ generally depends on the particular situation s and ex-ante decisions x^k, $\mathcal{A}^k=\mathcal{A}^k(s,x^k)$

Let us note that relationship (1) implicitly defines optimal adaptive decisions a^* as a vector function $a^*(s,x^k)\in\mathcal{A}^k(s,x^k)$, such that

$$U^k(s,x^k) = u^k(s,x^k,a^*)$$

Decision x^k is constrained to lie in a feasible set X^k and it has to be chosen in an optimal way against all possible scenarios $s=1,...,S$. For each decision x^k in compartment k we can think of ranking values of optimal welfare $U^k(s,x^k) = u^k(s,x,a^*(x^k))$ obtainable under all the different scenarios:

$$U^k(s_{\nu_1},x) \le U^k(s_{\nu_2},x) \le \ldots \le U^k(s_{\nu_S},x)$$

where $(\nu_1,\nu_2,...,\nu_S)$ represents a permutation of the vector $(1,2,...,S)$ formed from the scenario indexes $s=1,...,S$. In other words, a decision x^k can be characterized by the distribution of the scenario specific outcomes $U^k(s,x^k)$, $s=1,...,S$. The main idea then is to find an

[2] Function f^k constitutes a yield model that, based on relevant natural conditions in compartment k (climate, soils, landform), available technologies and feasible farm management measures, calculates a vector of crop and animal yields. In practical applications, the derivation of such a function may be a non-trivial task.

optimal solution $x^* \in X^k$ contained in the set X^k of feasible solutions, that leads to a 'best' choice of the distribution (in some yet to be defined sense). For example, maximizing expected welfare $W^k(x)$,

$$W^k(x) = E\,U^k(s,x) = \sum_s P_s U^k(s,x) \tag{2}$$

or maximizing some other objective, such as the probability $P\{U^k(s,x) \geq u_0\}$ to reach a certain welfare level u_0, or a criterion taking into account not only expected welfare $EU^k(s,x)$ but also its variance, using an appropriate weight γ:

$$EU^k(s,x) + \gamma E[U^k(s,x)-EU^k(s,x)]^2,$$

or

$$EU^k(s,x) + \gamma E[\max(0,U^k(s,x)-EU^k(s,x))]$$

Let us note, that all these criteria can formally be treated in the same way as mathematical expectation of some 'welfare' function $g^k(s,x)$. For example,

$$P\{U^k(s,x) \geq u_0\} = E[g^k(s,x)]$$

where $g^k(s,x) = 1$ if $U^k(s,x) \geq u_0$, and $g^k(s,x)=0$ otherwise.

Therefore, we can often restrict our general discussion to the case of expected welfare, as specified in (2) above. Specific features of a problem may be important for developing efficient solution algorithms.

Before proceeding further, let us illustrate some possible features of the maximization problem stated in (1) and (2) above, where the maximum value of expected welfare (2) is considered a quantification of limits for the agricultural potential of a region.

Consider a simple situation: Suppose that in a particular farm area, say geographical compartment k, n_s soil types, $j=1,\dots,n_s$, have been distinguished. Let $x_{ij} \geq 0$ denote the acreage of soil type j allocated to producing crop i, $i=1,\dots,n_c$. If A_j is the total extent of soil type j in the compartment, then the ex-ante decisions x_{ij} are constrained by land availability,

$$\sum_{i=1}^{n_c} x_{ij} \leq A_j \qquad j=1,\dots,n_s \tag{3}$$

Suppose that $y_{ij}(s)$ denotes the yield attainable for crop i in situation s on soil type j. Then production $q_i(s)$ of crop i under (stochastic) situation s is

$$q_i(s) = \sum_{j=1}^{n_s} y_{ij}(s) x_{ij} \qquad i=1,\dots,n_c \tag{4}$$

In a particular situation, for instance in the case of a dry season, various parts of the area x_{ij} can be treated specifically through ex-post adaptive measures $a_{ijl}(s)$, $l=1,\dots,n_a$. For example, a portion $a_{ij1}(s)$ of x_{ij} may be replanted with another crop, another part $a_{ij2}(s)$ may get additional irrigation, etc. In general, there may be several adaptive decisions to treat x_{ij}, for an observed situation s:

$$x_{ij} = \sum_{l=1}^{n_a} a_{ijl}(s) \qquad i=1,\dots,n_c;\ j=1,\dots,n_s$$

There may also be areas x_{ij}, $(i,j)\in\bar{I}$, where it is impossible to improve the productivity in a given situation s by any specific adaptive actions. Therefore, constraints (3) can be written in the form of joint constraints on ex-ante decisions x and ex-post adaptive decisions a(s):

$$\sum_{ij\in\bar{I}} x_{ij} + \sum_{ij\in I}\sum_{l} a_{ijl}(s) \le A_j,$$

where I is the set of ex-ante land allocation decisions x_{ij} which may be adaptively adjusted in an observed situation s, and $\bar{I}$ represents the set of fixed non-adaptive decisions. Together, ex-ante decisions x and ex-post decisions a(s) define the feasible set $\mathcal{A}^k(s,x)$ in equation (1). The set of feasible adaptive actions may involve additional activities and costs, such as purchasing of fertilizers, seeds, hiring of labor or machinery, etc., which need to be appropriately accounted for in the objective function.

Other kinds of constraints, e.g. minimum production target levels $Q_i(s)$ of crop i in a subsistence farming situation, can be expressed as:

$$\sum_{j=1}^{n_s} y_{ij}(s)x_{ij} \ge Q_i(s)$$

For a given ex-ante decision x, consisting of crop and soil type specific allocations x_{ij}, there will in general be two types of situations s:

$$\sum_{j=1}^{n_s} y_{ij}(s)x_{ij} < Q_i(s), \qquad \text{i.e. a shortage of crop i in relation to target } Q_i(s)\text{, and}$$

$$\sum_{j=1}^{n_s} y_{ij}(s)x_{ij} \ge Q_i(s), \qquad \text{i.e. a surplus of crop i in relation to target } Q_i(s).$$

To mitigate the risk of a shortfall in commodity supply, we stipulate the existence of commodity markets where crops can be sold and purchased. This is modeled by introducing additional ex-post decision variables, purchases $a^+(s)\ge 0$, and sales $a^-(s)\ge 0$, such that

$$\sum_{j=1}^{n_s} y_{ij}(s)x_{ij} + a_i^+(s) = Q_i(s), \quad \text{in case of a shortage of crop i, and}$$

$$\sum_{j=1}^{n_s} y_{ij}(s)x_{ij} - a_i^-(s) = Q_i(s), \quad \text{in case of a surplus of crop i.}$$

Both these conditions can be combined into one constraint,

$$\sum_{j=1}^{n_i} y_{ij}(s)x_{ij} + a_i^+(s) - a_i^-(s) = Q_i(s)$$

by introducing into the objective function (purchasing) costs $-c_i^+ a_i^+(s)$ and (selling) benefits $c_i^- a_i^-(s)$. Clearly, if $c^+>0$ and $c^->0$, then in the optimal solution the complementarity condition $a^+(s)\cdot a^-(s) = 0$ will be fulfilled.

In a similar way, we can introduce constraints regarding the availability, use and 'trade' of fixed and mobile resources, as well as on the level of tolerable environmental impacts, e.g. tolerable soil loss due to erosion or tolerable emission of pollutants such as nitrates or pesticides into the groundwater, or methane and other trace gases to the atmosphere.

Therefore, the feasible set of decisions $\mathcal{A}^k(s,x,a(s))$ can in general be described by a set of linear inequalities:

$$\sum_{\nu} \alpha_{l\nu}^k x_\nu^k + \sum_{\mu} \beta_{l\mu}^k a_\mu^k(s) \le R_l^k(s) \qquad l=1,\ldots,n_R \qquad (5)$$

where the farming systems and management decisions available for consideration in compartment k are described by the set of ex-ante decision $x^k=\{x_\nu^k, \nu=1,2,\ldots\}$ and ex-post adaptive decisions $a^k(s)=\{a_\mu^k(s), \mu=1,2,\ldots\}$, technological parameters $\alpha_{l\nu}^k(s)$ and $\beta_{l\mu}^k(s)$, and a vector of generalized resources $R_l^k(s)$, including items such as land of different types and qualities, labor, capital, external production inputs, intermediate consumption items, environmental impacts, etc.

Many refinements to this general framework are conceivable as required for specific studies. For example, in studies analyzing possible utilization of contaminated areas the yield function used in (4) will require further differentiation to model sources, sinks and levels of different contaminants. Soil types, in addition to the usual considerations on fertility, workability and toxicity, will have to be distinguished by bio-chemical and physical characteristics, as well as levels and types of contamination. Also, the notion of crop type needs to be extended to include representation of different levels of contamination. For instance, vegetable oils from 'clean' crops could be used for food processing while oil from crops grown on contaminated soils could be allocated to industrial uses. Relevant constraints derived from safety regulations could thus be included in the optimization framework.

Another important question in development projects relates to the dynamic aspects of the system and timing of decisions. The ex-ante decision variables x can be thought of as trajectories,

$$x = (x(1),\ldots,x(T))$$

of strategic decisions over periods t=1,2,...,T. The notion of ex-ante time dependent decisions is particularly important for the choice of irreversible decisions (see discussion in section 3).

Adaptive actions, such as 'trading' activities $a^+(s)$ and $a^-(s)$, may also depend on t: $a^+(s(t),t)$, $a^-(s(t),t)$ where s(t) is the given situation in time period t. A scenario s then itself comprises of a trajectory,

$$s = (s(1),...,s(T))$$

where s(1),...,s(T) are random vectors. For instance, each s(t) may assume values $s_1,s_2,...,s_S$ with probabilities $P_1,P_2,...,P_S$ (which may themselves depend on time). Such types of models, even when the number of possible situations S for a given time period t is small, may require special stochastic optimization techniques, as the number of all possible combinations for t=1,...,T is equal to S^T, which may be an astronomical number. Hence, it may prove impossible or impractical to list and evaluate all possible combinations of situations, as would be required to solve the optimal decision problem (2) by conventional optimization tools.

Before we formulate the general modeling framework, let us now illustrate the concepts by another example.

3. Strategic and Irreversible Decisions

The two-stage approach described above, ex-ante strategic and ex-post adaptive decisions, is an important technique of stochastic optimization. It should be noted that the two stage decisions do not necessarily correspond to two time periods. Such a concept can incorporate both conventional risk averse and risk seeking decisions. It is also possible to speak of two stage N-period dynamic models. The idea of a two-stage approach has also become important in the discussion of irreversible decisions. Let us briefly study the connection between the concepts of strategic (ex-ante) decisions used here and the mechanisms proposed by Arrow and Fisher (Arrow and Fisher(1974)) for the acceptance of irreversible decisions.

Strategic decisions are defined as those decisions which cannot be altered according to an observed situation. In order to ensure flexibility of a system under such decisions, they are supplemented by a set of corrective (ex-post) decisions. Some strategic decisions, for instance, such as placing a dam, the use of nuclear energy, deforestation of tropical rainforests, etc., may involve irreversible transformations of the environment, or at least may be characterized as extremely costly in terms of options to reverse their impact. Society would be 'locked in' to such decisions for a long time, often with considerable uncertainties at the initial state, the time when a decision must be taken, as to future costs and benefits associated with the irreversible decision.

In order to reduce the risks involved in irreversible decisions, Arrow and Fisher suggested a two stage approach in the decision making process: in the presence of uncertainty decisions are only partially accepted and applied in the initial stage, and can then be corrected by learning from experience. For example, in this case under-investments can be remedied before the second period; uneconomic over-investments, although impossible to be corrected, would not lead to as costly losses. The challenge is in how to extract the robust elements of irreversible decisions to adopt during the initial stage. Let us show that this question leads to conceptual models similar to what was discussed above.

Consider the same simple example as in Arrow and Fisher (1974). Suppose that a decision maker faces the question to what extent, if at all, to proceed with some form of commercial development of an area, i.e. to convert it irreversibly from its natural state through investment to a specific economic use. The area is also capable of yielding benefits in its preserved state. Let us denote per hectare benefits from preservation and development by b_p and b_d, respectively. If the cost of development per hectare is c_d, then in the absence or unawareness of uncertainty this leads to a simple 'yes or no' decision: if uncertain costs and benefits b_p, b_d and c_d are replaced by averaged values $\overline{b}_p, \overline{b}_d$ and $\overline{c}_d$, then the entire area is developed when $\overline{b}_d - \overline{c}_d - \overline{b}_p > 0$, and it should be preserved otherwise.

In the presence of risks it is less likely that the entire area be developed. The optimal extent of development, say x, may be evaluated through costs and benefits as incurred at the stage of implementation. As suggested by Arrow and Fisher, let us split the decision on the development of the area into two stages. During the first stage the development of x hectares, $0 \le x \le d$, is accepted. In the second stage, decision x is revised by recalculating expected costs and benefits according to observed values b_p, b_d and c_d, depending on scenarios s=1,2,... of possible developments.

Let us denote the expected benefits in the second stage obtainable per hectare from preservation and development by β_p and β_d, respectively, and the cost of development in the second stage by γ_d. Then, the total benefit from the area for a given scenario s and (stage one) decision x is:

$$W(s,x,a) = b_p(d-x) + b_d x - c_d x + \beta_p(d-x-a) + \beta_d(x+a) - \gamma_d a$$

where a is the extent of land developed in the second stage. Suppose, as was assumed in Arrows and Fisher, that the decision a depends on the observed scenario s=1,2,... and costs and benefits $b_p(s)$, $b_d(s)$, $c_d(s)$, $\beta_p(s)$, $\beta_d(s)$ and $\gamma_d(s)$. The second stage adaptive decision must satisfy the constraints

$$x + a(s) \le d, \; a(s) \ge 0$$

and can be chosen as to maximize total benefit W(s,x,a) with respect to a for given s and x. Since total benefit can be rewritten as

$$W(s,x,a) = b_p d + (b_d - b_p - c_d)x + \beta_p(d-x) + \beta_d x + (\beta_d - \beta_p - \gamma_d)a$$

the optimal second stage adaptive decision becomes:

$$a(s,x) = \begin{cases} d-x, & \text{if } \beta_d(s) - \beta_p(s) - \gamma_d(s) > 0 \\ 0 & \text{otherwise} \end{cases}$$

Then it is easy to verify that the expected total welfare, $\overline{W}(x)$, is obtained as follows:

$$\overline{W}(x) = E[(b_d - b_p - c_d + \min(\gamma_d, \beta_d - \beta_p))x + b_p d + \max(\beta_d - \gamma_d, \beta_p)]$$

$\overline{W}(x)$ is a linear function, and therefore the optimal decision will be a 'corner' solution, x=d or x=0, like in the case of averaged cost and benefit coefficients $\overline{b}_p, \overline{b}_d$ and $\overline{c}_d$. Note that the introduction of a second stage with the ability to observe may postpone the decision on the development, from the fist stage (x=d) to the second stage (x=0, a(s)=d).

The essential drawback of the suggested rule how to choose the second stage decision is its dependence on observed costs and benefits. By 'bad luck' the observed scenario may imply to develop the entire area, when $\beta_d - \beta_p - \gamma_d > 0$, even though the probability of such an event is small. The challenge posed by the irreversibility of a decision demands more rigorous analysis of the involved random variables and better decision rules that should not necessarily lead to 'corner' solutions: at each stage optimal decisions may require to develop only a fraction of the remaining area. For example, this happens already with a slight modification of the above problem. Suppose that the total negative benefit (i.e. net cost) at any stage must not exceed a given budget R. Then, it leads to a different second stage decision rule:

$$a(s,x) = \begin{cases} d-x, & \text{if } \beta_d(s) - \beta_p(s) - \gamma_d(s) > 0 \text{ and } W(s,x,d-x) > -R \\ 0 & \text{otherwise} \end{cases}$$

Under such a decision rule, expected total welfare becomes a nonlinear function of x, and the optimal decision x^* is not necessarily a corner solution, $x^*=0$ or $x^*=d$. The notion of strategic, two stage time-dependent decisions provides a possible framework to formalize such policies.

4. Land Use and Stochastic Resources - An Example

In this section we consider examples of simple situations which illustrate that without consideration of (at least) a two-stage decision process an appropriate formulation of the model is impossible and degenerates to simplistic conclusions. As discussed in the introduction, the productivity of land at a given geographical location is an indicator of the local economic conditions and environmental properties subject to stochastic variation and dynamic development. This indicator is affected, for instance, by variations of water resources which may be due to seasonal or interannual fluctuations (stochastic) or a changing climate, a dynamic change manifesting itself in a gradual change of stochastic properties, e.g. occurrence of droughts or floods. The significance of such changes shall be evaluated in relation to other factors such as quality and capacity of storage facilities, infrastructure, pricing and distribution network of irrigation water, pollution levels and dump sites, etc.

Although we consider in the following only a simple example, it shows that the land value of a region is a rather complicated function of different factors. The sensitivity analysis of this function to potential climate changes, through variations of water resources, may show their relevance to the welfare of the region is negligible in contrast to, for instance, mismanagement of regional capital resources or accidental pollution.

Consider a region with only two compartments. The agricultural performance of the first compartment can be improved by irrigation, whereas in the second compartment additional water resources are not available. Furthermore, the maximum level and efficiency of irrigation depends on the water level of the river.

If the water level is characterized by its average value, the decision to use irrigation is trivial and depends, for instance, on whether the net revenue per hectare of irrigated area in the first compartment, value c_1^1, is greater than the profit c_2^1 from a hectare without use of irrigation. The coefficient c_1^1 includes costs of measures necessary for the use of irrigation such as leveling, construction of water distribution network, pumping stations, etc.

The stochastic variation of the river water level creates essential difficulties. In situations of low water levels the land prepared in advance can only be partially supplemented with additional water, resulting in a profit $c_3^1 - c_2^1$ per hectare in the remainder of such land. Besides these physical limitations the situation may also be affected by variations in water prices, and it is easy to imagine a scenario when in a dry season the use of irrigation water may become unprofitable although irrigation is beneficial at average conditions. Let us formulate a model allowing to analyze similar situations.

Suppose $Q_1^1, Q_2^1, ..., Q_S^1$ are levels of river water in scenario s=1,...,S with frequencies $P_1^1, P_2^1, ..., P_S^1$. This may include, for example, frequencies of droughts and floods. Let us denote by $x^1 \leq A^1$ the area which must be prepared in advanced for irrigation, where A^1 is the total irrigable acreage in the first compartment. Suppose that q^1 is the amount of water required for irrigation of a hectare. There may be two types of risks: In situations s when $Q_s^1 < x^1 q^1$ there is the risk to forego a profit $c_2^1 - c_3^1$ per hectare of land that was prepared for irrigated cultivation but cannot be irrigated, extent $x^1 - Q_s^1/q^1$. In the case $Q_s^1 > x^1 q^1$ there is the risk to forego profit $c_1^1 - c_2^1$ per hectare of land not prepared in advance for irrigation, extent $Q_s^1/q^1 - x^1$. The extent of irrigated acreage, value x^1, is a strategic ex-ante decision.

In each situation s there may also be an ex-post adaptive decision a_s^1: the area of prepared land in the compartment that should receive irrigation water (amount q^1 per hectare). We have constraints $a_s^1 \leq x^1$ and $a_s^1 \leq Q_s^1/q^1$, for each s=1,...,S. Since x^1 is given, and with $c_1^1 > c_2^1$, the optimal ex-post decision is $a_s^1(x^1) = \min(x^1, Q_s^1/q^1)$. Let us note that in cases when the price of water is subject to variation, $\pi_1, ..., \pi_S$, revenues per hectare, coefficients c_1^1, c_2^1, c_3^1, depend on scenarios: $c_1^1(s), c_2^1(s), c_3^1(s)$.

Then the welfare of the first compartment in scenario s is defined as

$$W^1(s, x^1, a_s^1(x^1)) = \begin{cases} c_1^1(s) Q_s^1/q^1 + (x^1 - Q_s^1/q^1) c_3^1(s) + (A^1 - x^1) c_2^1(s), \text{ if } Q_s^1 < x^1 q^1 \\ x^1 c_1^1(s) + (A^1 - x^1) c_2^1(s), \text{ if } Q_s^1 \geq x^1 q^1 \end{cases}$$

The expected welfare of the first compartment is defined as:

$$W^1(x^1) = \sum_{\{s: Q_s^1 < x^1 q^1\}} [c_1^1(s) \frac{Q_s^1}{q^1} + c_3^1(s)(x^1 - \frac{Q_s^1}{q^1}) + c_2^1(s)(A^1 - x^1)] P_s^1 +$$
$$\sum_{\{s: Q_s^1 \geq x^1 q^1\}} [c_s^1(s) x^1 + c_s^2(s)(A^1 - x^1)] P_s^1$$

It is easy to see that for each scenario s the welfare function $W^1(s, x^1, a_s^1(x^1))$ is a convex but non-differentiable function, with discontinuities of derivatives. Such structure of welfare functions is typical for management under risk and uncertainty, since the presence of risk results in different profits depending on whether the agent in the ex-ante decision 'hits' or 'misses' the uncertainties, like $x^1 q^1 \leq Q_s^1$ or $x^1 q^1 > Q_s^1$.

Let us notice that the evaluation of the land potential, as discussed above in a simple example, cannot be formulated correctly without introducing ex-ante decisions x and adaptive ex-post decisions $a_s(x)$. The formulated model incorporates both types of decisions and allows to find such a strategic decision x^* which is optimal against all possible scenarios. It leads to decisions which in general will be quite different from the degenerate solutions of a deterministic approach: all acreage in the compartment is irrigated, or none.

Of course, the land potential in the compartment will also depend on resources other than water leading to constraints of the type: $\alpha_1^1 x^1 + \alpha_2^1 (A^1 - x^1) \le V^1$. This may include constraints on the availability of inputs, such as labor or machinery, fertilizers and pesticides, and budget and investment constraints. A larger variety of constraints will also call for extending the list of strategic decisions, e.g. related to investment and resource allocation, and including additional adaptive actions, for instance import in 'bad' years.

So far we have only considered one compartment. The interests of the region may also require improvements in the productivity of the second compartment. In general, the different compartments will compete for mobile regional resources, e.g. government investment. The evaluation of the regional welfare becomes an optimization problem with joint resource constraints.

Suppose $Q_1^2, Q_2^2, \dots, Q_S^2$ are possible scenarios of crop production for feeding livestock in the second compartment. The productivity per unit of livestock is expressed as a function $\delta(a)$ of a feeding intensity, a, which satisfies constraints $\underline{a} \le a \le \bar{a}$ that relate to limits of livestock nutrition. In a particular situation s, s=1,...,S, the feed intensity per livestock unit is given by $a_s^2 \le Q_s^2 / x^2$, where x^2 is the number of livestock units kept in the compartment. Let c_1^2 denote the net revenue per unit of livestock at a given feeding intensity a. Also, c_2^2 is the profit that would accrue from selling crops, and c_3^2 is the price for purchasing additional feed from outside the compartment. (All of these may have stochastic components which we ignore here in the example for ease of notation). We can assume that $\delta(a)c_1^2 > ac_2^2$. Then the optimal ex-post decision on feeding intensity is:

$$a_s^2(x) = \max(\underline{a}, \min(\bar{a}, Q_s^2 / x^2))$$

The welfare of the second compartment under scenario s becomes:

$$W^2(s, x^2, a_s^2(x^2)) = \begin{cases} \delta(\underline{a}) x^2 c_1^2 - (\underline{a}x^2 - Q_s^2) c_3^2, \text{ if } 0 \le Q_s^2 < \underline{a}x^2 \\ \delta(Q_s^2 / x^2) x^2 c_1^2, \text{ if } \underline{a}x^2 \le Q_s^2 < \bar{a}x^2 \\ \delta(\bar{a}) x^2 c_1^2 + (Q_s^2 - \bar{a}x^2), \text{ if } \bar{a}x^2 \le Q_s^2 \end{cases}$$

and the expected welfare is:

$$W^2(x^2) = \sum_{\{s: Q_s^2 < \underline{a}x^2\}} P_s^2 [\delta(\underline{a}) x^2 c_1^2 - (\underline{a}x^2 - Q_s^2) c_3^2] + \sum_{\{s: \underline{a}x^2 \le Q_s^2 < \bar{a}x^2\}} P_s^2 \delta(Q_s^2 / x^2) x^2 c_1^2 +$$
$$\sum_{\{s: \bar{a}x^2 \le Q_s^2\}} P_s^2 [\delta(\bar{a}) x^2 c_1^2 + (Q_s^2 - \bar{a}x^2) c_2^2]$$

Again, the welfare function $W^2(s, x^2, a_s^2, (x^2))$ is non-differentiable with discontinuous marginal values. There are also additional constraints on ex-ante decisions: $\alpha^2 x^2 \le V^2$.

Under such constraints the optimal values of expected welfare in each compartment, $W^1(V^1)$ and $W^2(V^2)$, depend on the availability of resources in the region, say investment. The problem then is to allocate available investment so as to maximize expected regional welfare,

$$W^* = \max_{V^1,V^2}[W^1(V^1) + W^2(V^2)]$$

under joint resource constraints

$$V^1 + V^2 \leq R$$

In a realistic application, there will be many more joint regional constraints that link the individual compartments, such as commodity market clearing conditions, availability of mobile resources, or joint limits on environmental impacts.

Several strategies are conceivable to apply as solution techniques: decomposition schemes, distinguishing two or more hierarchical layers, or direct (conventional) large scale optimization:

(i) To deal in the low dimensional space of variables V^1, V^2.

(ii) To deal in the space of variables $(x,V)=(x^1,x^2,V^1,V^2)$

(iii) To deal in the large dimensional space of variables (x,V,a(s),s=1,...,S).

The selection of the most appropriate solution method will depend on the specific properties of a problem: the number of strategic and adaptive variables, the number of linking constraints, the number of independent stochastic variables in the model, and the total number S of scenarios to be considered. In the case of dynamic models the number of time steps will also enter the considerations.

The first approach above, a stochastic decomposition technique, distinguishes an 'outer' problem formulated in terms of the linking variables, V, and requires the solution of 'inner' sub-problems in the space of variables (x, a(s), s=1,...,S). This method is only practical if the solutions of the stochastic sub-problems can be obtained relatively fast. Usually this will exclude cases with a large number of independent stochastic variables.

Obviously, the same concerns hold for the third approach above, application of large scale optimization, that makes no attempts to exploit the specific structure and features of the problem.

The most promising method for the solution of problems with implicitly given sets of scenarios will often be the second approach above: a stochastic decomposition technique that formulates an 'outer' problem in terms of the strategic ex-ante decisions and the linking variables (x,V), and 'inner' problems that solve for the scenario specific optimal levels of adaptive ex-post decisions a(s).

In the next sections we will formulate a resource allocation and decision model, as introduced in the previous example, in more general terms and will outline appropriate solution techniques.

5. A Conceptual Large Scale Linear Programming Model

Let us illustrate the approach by a simple (yet important) case: a linear objective function and linear constraint functions. Therefore, suppose that the yield relationship is defined for any compartment k and scenario s by a linear vector function $f^k(s,x,a(s))$ of decision variables (x,a(s)). Also, we assume that the set of feasible farm activities $\mathcal{A}^k(s,x)$ can be described by a system of linear inequalities as introduced in (5), and that the measure of welfare $W^k(s,x,a(s))$ is linear with respect to decisions (x,a(s)), for any compartment k=1,...,K and scenario s=1,...,S. Then, in the short term, the optimal farm resource allocation model (1) for compartment k can be specified so as to maximize a linear function W^k,

$$\max_{a(s)\in\mathcal{A}^k(s,x)} W^k(x,a(s)) = \max_{a(s)\in\mathcal{A}^k(s,x)} \left[\sum_{j=1}^{nx} c_j^k(s)x_j + \sum_{l=1}^{na} d_l^k(s)a_l^k(s)\right] \tag{6}$$

with respect to the vector of adaptive decisions $a^k(s) = (a_1^k(s),\dots,a_{na}^k(s)) \geq 0$ satisfying a set of linear inequalities

$$\sum_{j=1}^{nx} \alpha_{ij}^k x_j^k + \sum_{l=1}^{na} \beta_{il}^k a_l^k(s) \leq R_i^k(s) \qquad i=1,\dots,n_R \tag{7}$$

for a given scenario s and strategic decisions $x^k=(x_1,\dots,x_{n_X})$. The coefficients of the welfare function (6), $c_j(s)$ and $d_l(s)$, are to be taken negative or positive, depending whether the corresponding activities result in costs or benefits. Let us denote optimal adaptive decisions or corrections of x under a specific situation s as $a^k(s,x)$.

Without loss of generality, we can assume that for any feasible ex-ante decision $x^k \in X^k$ there exists a feasible adaptive decision $a^k(s)$ for any possible scenario s=1,...,S. Otherwise feasibility can be achieved by introducing auxiliary artificial activities, e.g. variables that represent a measure of non-fulfillment of a constraint, and reformulating problem (1) accordingly so as to minimize the scaled sum of infeasibilities in 'bad' situations. Whenever a feasible solution of problem (1) exists, the auxiliary artificial variables would be kept at zero activity level and the solution of the modified problem would be identical to the solution of the original problem (1). The strategic optimization problem then requires to find a decision vector x^k that maximizes the expected welfare:

$$\max_{x\in X^k} W^{*k}(x^k) = \max_{x\in X^k} E[U^k(s,x^k)] = \max_{x\in X^k} \left\{\sum_{j=1}^{nx} \bar{c}_j^k x_j^k + \sum_{s=1}^{S} P_s\left[\sum_{l=1}^{na} d_l^k(s)a_l^k(s,x^k)\right]\right\} \tag{8}$$

with $a_l^k(s) = a_l^k(s,x)$ and expected values $\bar{c}$ of coefficients c(s),

$$\bar{c}_j^k = \sum_s P_s c_j^k(s)$$

and subject to some linear constraints,

$$\sum_{j=1}^{nx} \delta_{hj}^{k} x_j^{k} \leq Q_h^{k} \qquad h=1,\ldots,n_Q, \qquad (9)$$

defining the feasible set X^k, and where $a_l^k(s) = a_l^k(s,x^k)$ is an ex-post adaptive decision maximizing (6) subject to constraints (7). It is easy to see that maximization of (8), interpreted as a non-linear function of x^k subject to constraints (9), is equivalent to large-scale maximization of the linear function (8) with respect to the variables $(x^k,a^k(1),\ldots,a^k(S))$ subject to linear constraints (7) and (9).

6. Regional Exchange and Resource Allocation

In the discussion so far we have mainly dealt with strategic decisions and adaptive measures to maximize welfare of a particular geographical compartment k with fixed resources and in response to environmental risks and uncertainty. In matrix form this compartment specific optimization problem can be rewritten as to find vectors x^k, $a^k(1),\dots,a^k(S)$ maximizing expected welfare of the compartment,

$$\max_{x,a(s)} W^k(x^k,a^k(s)) = \max_{x,a(s)}[(\bar{c}^k)' \cdot x^k + \sum_{s=1}^{S} P_s (d^k(s))' \cdot a^k(s)], \tag{10}$$

subject to constraints

$$A^k(s)x^k + B^k(s)a^k(s) \leq R^k(s) \qquad s=1,\dots,S \tag{11}$$

$$D^k x^k \leq Q^k \tag{12}$$

$$x^k \geq 0,\ a^k(s) \geq 0, \qquad s=1,\dots,S \tag{13}$$

where $A^k(s)$ denotes the matrix of coefficients $A^k(s)=\{\alpha^k_{ij}(s)\}$ related to generalized resource constraints (11) involving ex-ante decisions x^k, and matrix $B^k(s)=\{\beta^k_{il}(s)\}$ refers to constraints in (11) involving adaptive actions $a^k(s)$, respectively, together defining the limits of the adaptive capacity of agricultural systems in compartment k. Matrix $D^k=\{\delta^k_{hj}\}$ in (12) defines the feasible set of ex-ante strategic decisions X^k. The dimensions of vectors x, a, R and Q are n_X, n_A, n_R and n_Q, respectively[3].

The problem (10)-(13) is formulated in the space of all variables $(x^k,a^k(s),s=1,\dots,S)$. This problem can also be formulated as maximization of a (nondifferentiable) welfare function $W^{*k}(x)$ (see equation (8)) in the space of variables x^k and subject to constraints (9).

In reality, the compartments k=1,...,K of a region interact with each other in various ways. Economically they interact through commodity markets, and there may be joint constraints on natural resources, e.g. water for irrigation, or mobile resources such as farm labor and capital, or environmental standards like water quality, emission permits, etc.

Consider the case when part of the resources denoted by the right hand side of constraints (11) and (12) can be exchanged or redistributed in order to improve overall regional performance. We partition vectors $R^k(s)$ and Q^k into components that are fixed with regard to compartment k and mobile, respectively, such as:

$$R^k(s) = R^k_0(s) + R^k_Z(s)$$

$$Q^k = Q^k_0 + Q^k_Z$$

[3] usually we think of vectors as columns, although often use rows to denote their components.

where $R_0^k(s)$ denotes the immobile portion of the generalized resource vector R^k, and $R_Z^k(s)$ relates to the mobile component of resource vector R^k. Similarly, we partition the right hand side Q^k in (12) into an immobile component Q_0^k and a component related to exchange Q_Z^k.

Let us denote the outflow (or export) of resources from compartment k to another compartment h under scenario s by $r_{kh}(s)\geq 0$ and inflow (import) of mobile resources from compartment h into compartment k by $r_{hk}(s)\geq 0$. Then total net migration of resources into compartment k is:

$$\begin{aligned} &Z_R^k(s) = \sum_{h=1}^{K} r_{hk}(s) - \sum_{h=1}^{K} r_{kh}(s), \\ &R_Z^k(s) + Z_R^k(s) \geq 0 \\ &0 \leq r_{kh}(s) \leq \bar{r}_{kh}(s) \leq R_Z^k(s) \qquad s=1,\dots,S \end{aligned} \tag{14}$$

and reasoning similarly for exchange of resources related to vector Q,

$$\begin{aligned} &Z_Q^k = \sum_{h=1}^{K} q_{hk} - \sum_{h=1}^{K} q_{kh}, \\ &Q_Z^k + Z_Q^k \geq 0 \\ &0 \leq q_{kh} \leq \bar{q}_{kh} \leq Q_Z^k \end{aligned} \tag{15}$$

Variables $r_{kh}(s)$ model adaptive ex-post migration of resources, e.g. such as labor. Variables q_{kh} describe ex-ante strategic exchange of resources, related to constraints (12) that do not depend on adaptive decisions.

In general, the right hand sides of constraints (11) and (12) may also include ex-ante distribution (investment) of regional (public) resources, variables V_R^k and V_Q^k. Constraints (11) and (12), when permitting distribution and exchange of resources, therefore become

$$R^k(s) = R_0^k(s) + R_Z^k(s) + Z_R^k(s) + V_R^k \tag{16}$$

and

$$Q^k = Q_0^k + Q_Z^k + Z_Q^k + V_Q^k, \tag{17}$$

respectively.

We note that migration of resources must satisfy regional clearing and resource availability conditions:

$$\sum_{k=1}^{K} Z_R^k(s) = Z_R^T(s), \tag{18}$$

$$\sum_{k=1}^{K} V_R^k \leq V_R^T,$$
$$R_Z^k(s) + Z_Z^k(s) \geq 0, \;\; V_R^k \geq 0, \;\; k = 1,\dots,K$$

$$\sum_{k=1}^{K} Z_Q^k = Z_Q^T, \tag{19}$$
$$\sum_{k=1}^{K} V_Q^k \leq V_Q^T,$$
$$Q_Z^k + Z_Q^k \geq 0, \;\; V_Q^k \geq 0, \;\; k = 1,\dots,K$$

where $Z_R^T(s)$ and Z_Q^T denote the net inflow of resources R and Q, a negative value in the case of a net outflow, into the region of consideration, and K is the total number of compartments belonging to the region[4]. Vectors V_R^T and V_Q^T denote total availability of public resources and

$$R^T(s) = \sum_{k=1}^{K} R^k(s)$$

and

$$Q^T = \sum_{k=1}^{K} Q^k$$

represent limits of regional resources.

It may be noted that the proposed methodology allows for a hierarchical systems approach, spanning from geographical compartments, to regions within countries, nations and the globe.

The migration of mobile resources extends the set $\mathcal{A}^k(s,a^k(s))$ of adaptive decisions to a set $\mathcal{A}^k(s,a^k(s),r^k(s))$, including the vector of ex-post mutual resource exchange decisions $r^k(s) = \{r_{kh}(s)\}$ as used in (14). We specify transfer and distribution costs (e.g. including transportation costs), $\rho^k(s) = \{\rho_{kh}(s)\}$ and $\xi^k = \{\xi_{kh}\}$ relating to the exchange of resources $r^k(s) = \{r_{kh}(s)\}$ and $q^k = \{q_{kh}\}$, respectively; furthermore, let $V^k = (V_R^k, V_Q^k)$ and $\gamma^k = (\gamma_R^k, \gamma_Q^k)$ denote distribution of public resource and corresponding costs. We extend the objective function of the 'local' optimal decision problem (10) to include transfer of resources:

$$W^k = (\bar{c}^k)' \cdot x^k - (\xi^k)' \cdot q^k - (\gamma^k)' \cdot V^k - (\sigma_Q^k)' \cdot Z_Q^k + \sum_{s=1}^{S} P_s[(d^k(s))' \cdot a^k(s) - (\rho^k(s))' \cdot r^k(s) - (\sigma_R^k(s))' \cdot Z_R^k(s)] \tag{20}$$

[4] without loss of generality and for convenience of notation we may assume that $Z_Q^T = 0, \; Z_R^T(s) = 0$.

We note that in equation (20) variables x^k and $a^k(s)$ represent 'local' decisions in a given compartment k, vectors q^k and $r^k(s)$ denote resource migration, and net resource exchange $Z_Q = \{Z_Q^1,\ldots,Z_Q^K\}$, $Z_R(s) = \{Z_R^1(s),\ldots,Z_R^K(s)\}$ and public resources V=$\{V_R^1,\ldots,V_R^K,V_Q^1,\ldots,V_Q^K\}$ connect and integrate the compartments of a region. By jointly solving for these variables subject to compartment constraints (11) - (13), with $R^k(s)$ and Q^k as defined in (16) and (17), and satisfying regional constraints (14), (15), (18) and (19), the evaluation of land potential at the regional level is formulated as a large scale optimization problem, maximizing a function $W(x,a(s),r(s),q,Z_R(s),Z_Q,V)$,

$$W(x,a(s),r(s),q,Z_R(s),Z_Q,V) = \sum_{k=1}^{K} \varpi_k W^k(x^k,a^k(s),r^k(s),q^k,Z_R^k(s),Z_Q^k,V^k) \quad (21)$$

Since any real world problem will involve a large number K of compartments as well as possibly a large number of scenarios S, it may be impractical and difficult to solve optimization problem (21) directly. All the more, it is hardly conceivable that any real world problem formulation will lend itself to analytical solution. We therefore propose an iterative decomposition method that takes advantage of the hierarchical structure of the resource allocation and adaptive decision problem and yields an optimal solution of problem (21).

7. Decomposition of the Spatial Resource Allocation Problem

Welfare of a region, W, is assumed to comprise of the weighted sum of welfare achieved in each compartment constituting the region[5]. Recognizing that for any ex-ante decisions $x=\{x^1,...,x^K\}$, $Z_Q=\{Z_Q^1,...,Z_Q^K\}$ and $V=\{V^1,...,V^K\}$ the local problems may often be solved easily, we can view regional welfare as a function of x, Z_Q and V:

$$W^* = \max_{x,Z_Q,V} \sum_{k=1}^{K} \varpi_k W^{*k}(x^k, Z_Q^k, V^k) \tag{22}$$

where W^{*k} here denotes the maximum welfare value of the 'local' problem (see equation (20) above) of finding optimal adaptive decisions, i.e. decisions depending on scenario s, s=1,...,S, in compartment k, given ex-ante decisions x^k, Z_Q^k and V^k.

We observe that for given vectors x, Z_Q, and V, the solution of the 'local' decision problems may be derived from (10) with constraints (11) to (13) modified to take into account migration of resources, as defined in (14) to (17), where the local components of the right hand sides, $R^k(s)$ and Q^k, are adjusted by flows of net exchange and public distribution, that is:

$$R^k(s) = R_0^k(s) + R_Z^k(s) + Z_R^k(s) + V_R^k \tag{23}$$

$$Q^k = Q_0^k + Q_Z^k + Z_Q^k + V_Q^k, \tag{24}$$

with

$$Z_R^k(s) = \sum_{h=1}^{K} r_{hk}(s) - \sum_{h=1}^{K} r_{kh}(s), \qquad R_Z^k(s) + Z_R^k(s) \geq 0 \qquad s=1,...,S \tag{25}$$

$$Z_Q^k = \sum_{h=1}^{K} q_{hk} - \sum_{h=1}^{K} q_{kh}, \qquad Q_Z^k + Z_Q^k \geq 0 \tag{26}$$

and variables defined as in section 6.

Regional welfare as represented by equation (22), $W(x,Z_Q,V)$, is a convex but in general non-differentiable function due to the nature of the local decision problems. The subgradient set of this function has a simple structure which can be derived from the solution of the dual problems of the 'local' decision problems in (22).

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[5] in the present discussion of the decomposition algorithm we assume fixed (non-negative) compartment weights ϖ_k, k=1,...,K, scaled to sum up to unity, i.e. $\Sigma\varpi_k=1$. Other formulations derived from welfare economics may require variable weights to meet additional constraints on dual variables. This could be achieved with additional 'outer' iteration levels over a set of variables ϖ.

Let us denote the optimal dual variables related to (11), (12) with (23), (24) by $\lambda_R^k(s)$ and λ_Q^k, respectively. A separate sub-problem is defined by the flow balance equations (26); the objective function to be minimized is:

$$\min_q \sum_{k=1}^{K} (\xi^k)' q^k \tag{27}$$

Let us denote the optimal dual variables related to equation (26) by μ_Q^k. We specify components of subgradients of the welfare function $W(x,Z_Q,V)$ related to ex-ante variables $x=(x^1,\ldots,x^K)$, $Z_Q = (Z_Q^1,\ldots,Z_Q^K)$ and $V = (V_R^1,\ldots,V_R^K,V_Q^1,\ldots,V_Q^K)$ by W_x, W_{Z_Q} and W_V, respectively:

$$W_x = \{\varpi_k \bar{c}^k + (\lambda_R^k,\lambda_Q^k)' \begin{pmatrix} A^k \\ D^k \end{pmatrix}, k = 1,\ldots,K\}$$

$$W_{Z_Q} = \{-(\lambda_Q^k + \mu_Q^k + \sigma_Q^k), k = 1,\ldots,K\}$$

$$W_V = \{-(\bar{\lambda}_R^k + \gamma_R^k), k = 1,\ldots,K, -(\lambda_Q^k + \gamma_Q^k), k = 1,\ldots,K\}$$

where we use the notation

$$\bar{\lambda}_R^k = \sum_{s=1}^{S} P_s \lambda_R^k(s)$$

$$\lambda_R^k = \begin{pmatrix} \lambda_R^k(1) \\ \ldots \\ \lambda_R^k(S) \end{pmatrix}$$

$$A^k = \begin{pmatrix} A^k(1) \\ \ldots \\ A^k(S) \end{pmatrix}$$

The collection $W_{x,Z_Q,V}=\{W_x, W_{Z_Q}, W_V\}$ is a subgradient of the function $W(x,Z_Q,V)$ with respect to $(x^1,\ldots,x^K,Z_Q^1,\ldots,Z_Q^K,V_R^1,\ldots,V_R^K,V_Q^1,\ldots,V_Q^K)$. We note that for fixed x, Z_Q, V the 'local' maximization problems in (22) usually have a simple structure and the solution can be achieved rapidly provided the number of scenarios S is not too big. We can therefore derive an iterative decomposition scheme to solve (22) as follows:

1. Start from an initial vector of ex-ante decisions $x^{(0)}=(x^{1(0)},\ldots,x^{K(0)})$ and resource exchange and distribution decisions $Z_Q^{(0)} = (Z_Q^{1(0)},\ldots,Z_Q^{K(0)})$ and $V^{(0)}=(V^{1(0)},\ldots,V^{K(0)})$, respectively, which meet (18) and (19). Let each farm compartment k, k=1,...,K, find a

collection of shadow prices (λ,μ) for this allocation of resources: $\lambda_R^{k(0)}(s)$,.s=1,...S, $\lambda_Q^{k(0)}$ and $\mu^{k(0)}$.

2. Suppose that after M 'outer' iterations of problem (22), the regional resources are allocated according to decisions $x^{(M)} = (x^{1(M)},...,x^{K(M)})$, $Z_Q^{(M)} = (Z_Q^{1(M)},...,Z_Q^{K(M)})$ and $V^{(M)} = (V^{1(M)},...,V^{K(M)})$. Compute the shadow prices in farm compartment k derived for this distribution as $\lambda_R^{k(M)}(s)$,.s=1,...S, $\lambda_Q^{k(M)}$ and $\mu^{k(M)}$.

3. At iteration M, derive a new ex-ante decision vector $x^{(M+1)}$ and resource exchange and distribution vectors $Z_Q^{(M+1)}$ and $V^{(M+1)}$ according to:

$$x^{k(M+1)} = \max\{0, x^{k(M)} + \varphi_M[\varpi_k \bar{c}^k + (\lambda_R^{k(M)}, \lambda_Q^{k(M)})\begin{pmatrix} A^k \\ D^k \end{pmatrix}]\}, \tag{28}$$

$$Z_Q^{(M+1)} = \Pr j[Z_Q^{(M)} - \varphi_M(\lambda_Q^{(M)} + \mu_Q^{(M)} + \sigma_Q)], \tag{29}$$

$$V_R^{(M+1)} = \Pr j[V_R^{(M)} - \varphi_M(\bar{\lambda}_R^{(M)} + \gamma_R)], \tag{30}$$

$$V_Q^{(M+1)} = \Pr j[V_Q^{(M)} - \varphi_M(\lambda_Q^{(M)} + \gamma_Q)], \tag{31}$$

where φ_M is a non-negative stepsize, M=0,1,...; we use the general vector notation $b^{(M)}$, like in $Z_Q^{(M)}, \lambda_Q^{(M)}, \lambda_R^{(M)}, V_Q^{(M)}$ and $V_R^{(M)}$, to denote vectors with components $(b^{1(M)},...,b^{K(M)})$. The diagonal matrix ϖ in (29) - (31) contains compartment weights ϖ_k, k=1,...,K, as introduced in (22).

Projection operations, denoted by symbol Prj, as used in (29) - (31) take place onto the sets defined by conditions (18) and (19):

In (29) onto the set defined by

$$\sum_{k=1}^{K} Z_Q^k = Z_Q^T,\ Q_Z^k + Z_Q^k \geq 0,\ k = 1,...,K,$$

in (30) onto the set

$$\sum_{k=1}^{K} V_R^k \leq V_R^T,\ V_R^k \geq 0,\ k = 1,...,K,$$

and in (31) onto the set

$$\sum_{k=1}^{K} V_Q^k \leq V_Q^T,\ V_Q^k \geq 0,\ k = 1,...,K,$$

We note that such projection operations can easily be implemented.

4. Such a process converges to the optimal allocation when the adjustment stepsize is chosen, for instance, according to φ_M = const/M.

The iterative procedure introduced in (28) - (31) requires the collection of shadow prices for all scenarios s=1,...,S. This may be a rather tedious task and practically impossible when the scenarios are generated from a distribution rather than being a small finite number given in advance. In such cases, instead of procedure (28) - (31), a stochastic decomposition technique may be implemented, as follows:

1. As in the previous procedure, start from an initial vector of ex-ante decisions $x^{(0)}=(x^{1(0)},...,x^{K(0)})$ and resource exchange and distribution decisions $Z_Q^{(0)} = (Z_Q^{1(0)},...,Z_Q^{K(0)})$ and $V^{(0)} = (V^{1(0)},...,V^{K(0)})$, respectively, which meet (18) and (19). Randomly observe a scenario s_0 and find a collection of shadow prices only for this scenario:

$$(\lambda_R^{(0)}(s_0),\lambda_Q^{(0)},\mu_Q^{(0)}) = \{\lambda_R^{1(0)}(s_0),...,\lambda_R^{K(0)}(s_0),\lambda_Q^{1(0)},...,\lambda_Q^{K(0)},\mu_Q^{1(0)},...,\mu_Q^{K(0)}\}$$

2. Suppose that after M 'outer' iterations of problem (22), the regional resources are allocated according to decisions $x^{(M)} = (x^{1(M)},...,x^{K(M)})$, $Z_Q^{(M)} = (Z_Q^{1(M)},...,Z_Q^{K(M)})$ and $V^{(M)} = (V^{1(M)},...,V^{K(M)})$. Observe, at random, a new scenario s_M and compute the collection of corresponding shadow prices in farm compartment k, k=1,...,K, derived for this distribution and scenario as $\lambda_R^{k(M)}(s_M)$, $\lambda_Q^{k(M)}$ and $\mu^{k(M)}$.

3. At iteration M, derive a new resource allocation decision according to the following scheme (which is similar to (28) - (31)):

$$x^{k(M+1)} = \max\{0, x^{k(M)} + \varphi_M[\varpi_k \bar{c}^k + (\lambda_R^{k(M)}(s_M),\lambda_Q^{k(M)})\begin{pmatrix} A^k(s_M) \\ D^k \end{pmatrix}]\}, \tag{32}$$

$$Z_Q^{(M+1)} = \Pr j[Z_Q^{(M)} - \varphi_M(\lambda_Q^{(M)} + \mu_Q^{(M)} + \sigma_Q)], \tag{33}$$

$$V_R^{(M+1)} = \Pr j[V_R^{(M)} - \varphi_M(\lambda_R^{(M)}(s_M) + \gamma_R)], \tag{34}$$

$$V_Q^{(M+1)} = \Pr j[V_Q^{(M)} - \varphi_M(\lambda_Q^{(M)} + \gamma_Q)], \tag{35}$$

where M=0,1,....

The convergence with probability 1 of a procedure as outlined above can be derived from general stochastic optimization techniques under very general assumptions (for details see Ermoliev and Wets (1988)).

Let us note that the iterative procedures (28) - (31) and (32) - (35) simulate an adaptation of the resource allocation, Z_Q and V, and strategic decisions x, to the set of all possible scenarios in order to find strategies that are robust and optimal against uncertainties. The process results in optimal solutions $x^{*k}, a^{*k}(s), Z_R^{*k}(s), Z_Q^{*k}, r_{kh}^{*}(s), q_{kh}^{*}$ and dual variables ('prices') $\lambda_R^{*k}(s), \lambda_Q^{*k}$, under rather general assumptions. An actual implementation, of course, will depend on the specification and properties of the problem at hand.

It may be worth noting that the presence of uncertainties results in specific optimality conditions. It is easy to see, for instance, that for ex-ante decisions $x^{*k} = (x_1^{*k},\ldots,x_{n_x}^{*k})$, we obtain for all non-zero components $x_j^{*k} > 0$:

$$E[\sum_{i=1}^{n_R} \alpha_{ij}^k(s)\lambda_{iR}^{*k}(s) + \sum_{l=1}^{n_Q} \delta_{lj}^k \lambda_{lQ}^{*k}] = \overline{c}_j^k$$

Therefore, the efficiency of ex-ante decisions x^{*k} cannot be judged from individual seasonal observations. By 'bad' or 'good' luck, imputed costs associated with a decision x_j^{*k} in a particular season s,

$$\kappa_j^k(s) = \sum_{i=1}^{n_R} \alpha_{ij}^k(s)\lambda_{iR}^{*k}(s) + \sum_{l=1}^{n_Q} \delta_{lj}^k \lambda_{lQ}^{*k},$$

may exceed or fall below $\overline{c}_j^k$. On the other hand, the efficiency of ex-post decisions, such as $a^{*k}(s)$, may be established on a seasonal basis. If a particular $a_h^{*k}(s) > 0$, then optimality implies:

$$\sum_{i=1}^{n_R} \beta_{ih}^k(s)\lambda_{iR}^{*k}(s) = d_h^k(s)$$

Similar 'clearing' conditions, as illustrated above for ex-ante and ex-post variables, also hold for the other decision variables, including migration flows.

Iteration procedure (32) - (35) can easily be extended from linear welfare functions W^k, as in (20), and linear resource constraints (11), (12), to the more general case of nonlinear functions:

$$W^k(x^k,a^k(s)) = \sum_{s=1}^{S} P_s[f^{k0}(s,x^k,a^k(s)) - (\rho^k(s))' r^k(s) - (\sigma_R^k(s))' Z_R^k(s)] - \\ (\xi^k)' q^k - (\gamma^k)' V^k - (\sigma_Q^k)' Z_Q^k \quad (36)$$

$$f^{ki}(s,x^k,a^k(s)) \le R^{ki}(s),\ i = 1,\ldots,n_R^k, \quad (37)$$

$$g^{kh}(x^k) \le Q^{kh},\ h = 1,\ldots,n_Q^k, \quad (38)$$

$$x^k \ge 0,\ a^k(s) \ge 0,\ s = 1,\ldots,S.$$

Then iteration scheme (32) changes to the following procedure:

$$x^{k(M+1)} = \max\{0, x^{k(M)} + \varphi_M \varpi_k [f_x^{k0}(s_M, x^{k(M)}, a^{k(M)}(s_M)) + \\ \sum_{i=1}^{n_R^k} \lambda_{Ri}^{k(M)}(s_M) f_x^{ki}(s_M, x^{k(M)}, a^{k(M)}(s_M)) + \sum_{h=1}^{n_Q^k} \lambda_{Qh}^{k(M)} g_x^{kh}(x^{k(M)})]\} \quad (39)$$

where $\lambda_{Ri}^{k(M)}(s_M), \lambda_{Qh}^{k(M)}$ are vectors of dual variables related to constraints (37), (38) specified according to (23) - (26). Symbols f_x, g_x denote gradients with respect to variables x. Adjustments of resource exchange and distribution $Z_Q^{(M)}$, $V_R^{(M)}$, and $V_Q^{(M)}$ follow relationships (33) - (35).

The decomposition procedures discussed here assume that for fixed ex-ante decision variables x, Z_Q and V, the resulting subproblems can easily be solved with respect to 'fast' seasonal ex-post decisions a(s) and migration flows r(s), q. Examples which have briefly been analyzed suggest that this may often be the case in practical applications: the short-term nature of ex-post decisions allows to assume a simple, even linear, structure of the objective and constraints functions with respect to these variables.

8. Pricing Mechanisms

The model formulation discussed in section 6 primarily deals with supply response of a region subject to technological and behavioral conditions, resource constraints and environmental risks and uncertainty. This seems appropriate for assessing the economic potential of a region.

In reality, activity levels and resource migration flows depend, among others, on prices, say a vector π, entering the objective function and constraints set. There are various possible approaches to systematically analyze the sensitivity of regional supply (and demand) with respect to the choice of π.

Sensitivity analysis may, for instance, be concerned with estimates of lower and upper bounds on prices and resulting regional supply. Price variability with possible shocks can also be included, e.g. by means of stochastic price signals in the scenario formulation. This would increase the complexity of the analysis and add to the number of scenarios. However, since prices would essentially be exogenously determined, the solution techniques introduced in the previous section would still be applicable. For instance, the formulation of a model with stochastic supply-demand response is illustrated in section 2.

Endogenizing prices could be achieved in a spatial general equilibrium approach. Such an economic model can be formulated in a straightforward way; however, the computational efforts involved and the availability of appropriate tools may limit its practical value for policy analysis.

Application of fixed point algorithms requires the calculation of demand and supply; these may be given implicitly only as solutions of corresponding optimization subproblems, e.g. derived from profit and utility maximization. In the presence of uncertainties, solution of a stochastic optimization sub-problem itself may pose a challenging task. The traditional equilibrium approach assumes that economic agents reveal information on their preferences. Representing 'externalities', such as public goods and environmental quality, requires integrated approaches incorporating markets, conventional measures of government intervention, e.g. taxes, subsidies and quota, and other regulating incentives, such as emission permits.

From a formal point of view possible computational schemes can be interpreted as distributed optimization processes. The procedures proposed here incorporate ideas of a scheme by Kornai and Liptak with nondifferentiable and stochastic optimization techniques. Decomposition techniques allow to incorporate price dependent constraints; this in turn may affect the properties and computational speed for global convergence.

Let us briefly sketch a possible procedure analogous to the decomposition schemes developed in section 7:

1. Start from an initial vector of ex-ante decisions $x^{(0)}=(x^{1(0)},...,x^{K(0)})$ and resource distribution decisions $V^{(0)}=(V_R^{(0)},V_Q^{(0)})=(V_R^{1(0)},...,V_R^{K(0)},V_Q^{1(0)},...,V_Q^{K(0)})$ which meet conditions (18) and (19), and prices $\pi_R^{(0)}(s), s=1,...,S, \pi_Q^{(0)}$ related to resources and commodities as defined in (16) and (17). Let each farm compartment k, k=1,...,K, find a collection of shadow prices $\lambda_R^{k(0)}(s)$,.s=1,...S, and $\lambda_Q^{k(0)}$ with regard to optimal solutions maximizing subproblems equivalent to (20), subject to:

$$A^k(s)x^k + B^k(s)a^k(s) \le R_0^k(s) + R_Z^k(s) + \sum_{h=1}^{K} r_{kh}(s) - \sum_{h=1}^{K} r_{hk}(s) + V_R^k \qquad (40)$$

where $r_{kh}(s) \ge 0$ and with fixed $x^k = x^{k(0)}$ and $V_R^k = V_R^{k(0)}$, k=1,...,M, and consequently minimizing transportation cost (27) subject to

$$D^k x^k \le Q_0^k + Q_Z^k + \sum_{h=1}^{K} q_{kh} - \sum_{h=1}^{K} q_{hk} + V_Q^k \qquad (41)$$

where $q_{kh} \ge 0$, $x^k = x^{k(0)}$ and $V_Q^k = V_Q^{k(0)}$

2. Suppose that after M 'outer' iterations there are ex-ante decisions $x^{(M)}, V_R^{(M)}, V_Q^{(M)}$ and prices $\pi_R^{(M)}(s), \pi_Q^{(M)}$. Find a collection of shadow prices $\lambda_R^{k(M)}(s)$,.s=1,...S, $\lambda_Q^{k(M)}$ related to maximizing problem (20) subject to constraints (40), where $r_{kh}(s) \ge 0$, $x^k = x^{k(M)}$ and $V_R^k = V_R^{k(M)}$; and by minimizing (27) subject to constraints (41), where $q_{kh} \ge 0$, $x^k = x^{k(M)}$ and $V_Q^k = V_Q^{k(M)}$

3. Define

$$Z_R^{k(M+1)}(s) = \sum_{h=1}^{K} r_{kh}^{(M)}(s) - \sum_{h=1}^{K} r_{hk}^{(M)}(s),$$

$$Z_Q^{k(M+1)} = \sum_{h=1}^{K} q_{kh}^{(M)} - \sum_{h=1}^{K} q_{hk}^{(M)},$$

and compute, by explicit or implicit methods, corresponding prices $\pi_R^{k(M+1)}(s), \pi_Q^{k(M+1)}$.

4. Adjust ex-ante decisions $x^{(M+1)}, V_R^{(M+1)}$ and $V_Q^{(M+1)}$ according to scheme (28) - (31).

There may be other versions of decomposition schemes of the type (28) - (31), for instance, in cases when the welfare of a region can be specified as a function of public allocation decisions V and market prices π_Q and $\pi_R(s)$, s=1,..,S, subject to market clearing and resource availability conditions similar to (18) and (19).

In other words, when it is possible to specify relationships $h_Q^k(\pi_Q)$ and $h_R^k(\pi_R(s))$ such that ex-ante resource exchange decisions Z_Q^k and adaptive exchange $Z_R^k(s)$, as used in (14) to (19), can be obtained from:

$$Z_Q^k = h_Q^k(\pi_Q)$$
$$Z_R^k(s) = h_R^k(\pi_R(s))$$

subject to market clearing conditions:

$$\sum_{k=1}^{K} Z_R^k(s, \pi_R(s)) = Z_R^T(s), \sum_{k=1}^{K} Z_Q^k(\pi_Q) = Z_Q^T \tag{42}$$

When it is possible, for given vectors $\pi=(\pi_Q,\pi_R(s))$ and V, to efficiently compute an optimal compartment solution $W^{*k}(\pi,V)$,

$$W^{*k}(\pi, V) = \max_{x,a(s)} W^k(x, a(s), \pi, V) \tag{43}$$

then an iterative procedure in the spirit of (28) - (31) or (32) - (35), may similarly be taken with respect to a sequence of ex-ante decisions $x^{(M)}$, market prices $\pi^{(M)}$ and public resource allocation decisions $V^{(M)}$. Depending on the assumptions made about mechanisms to bring about commodity balances and market clearing, additional constraints on shadow prices, for instance, derived from welfare economics (e.g. see Fischer et al., 1988), may have to be added to such a scheme.

9. Concluding Remarks

The value and use of land as well as the quality of other resources, like water or forest resources, plays a critical role in the discussion of viable and sustainable economic development, environmental change and pollution control strategies. The production potential of agricultural resources is often degraded by physical and chemical destruction, toxification and contamination, or at risk of negative impacts from climate change. On the other hand, agriculture itself may generate pollution affecting, for example, water quality or climate. Environmental protection and improvement can be achieved through various countermeasures, such as land-use regulation, purification technologies, clean-up and emission control strategies, for example, through emission permits. In the overall evaluation of the land potential such policies should be included in an integrated manner in comparison to other policy alternatives, such as investment in infrastructure of a region or investments in land reclamation and improvement.

Any discussion of interactions with the environment must come to grips with uncertainties and resulting risks. In this paper we discuss an approach to the study of agricultural development in the presence of uncertainties. The proposed model incorporates two types of decisions: strategic ex-ante decisions and flexible adaptive ex-post decisions.

The notion of strategic decisions is especially relevant when dealing with the risks of virtually irreversible impacts, such as placing a dam, clearing rainforests for agricultural purposes, or diverting agriculture land to urban and industrial uses. Regions would be 'locked in' to such decisions for a long time with essential uncertainties as to future costs and benefits.

The adaptive ex-post decisions then allow to correct the strategic decisions relying on the ability to learn from experience and to adapt to observed situations. This is a most important idea of stochastic optimization when dealing with policy formulation in the presence of risks. To put these ideas into an operational framework requires the development of appropriate tools. Since the spatial aspects are essential, the resulting models have large dimensionality which is further multiplied by possible combinations of uncertainties.

In this paper we have outlined a possible approach to modeling agricultural production in a spatial setting under environmental risk and uncertainty. The methodology proposes the distinction of 'compartments', i.e. geo-referenced collections of homogenous land units which are coherent in terms of natural resources and economic production conditions. The compartments interact through commodity markets and 'trade' of mobile resources, compete for allocation of limited public resources, and are jointly affected by government policies, regulations and other regional constraints.

From a formal point of view, this process can be interpreted as a stochastic decomposition procedure, integrating individual agents through market clearing conditions and allocation of public resources in order to find strategic decisions that are robust and most effective in an uncertain setting. There may be various modifications of the proposed scheme, depending on the 'market mechanisms' and 'environmental externalities' to be studied.

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