SPATIAL PROCESS MODELLING FOR AIR POLLUTION STANDARDS: A Problem Statement

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Spatial Process Modelling for Air Pollution Standards: A Problem Statement

Spatial process models have application to problems in several disciplines. The problem presented here treats monitoring and control of air pollution, but the methodological base seems similar to several other problems, and the hope in outlining this problem is to perhaps generate interest in others working on similar problems, or towards work on this problem itself.

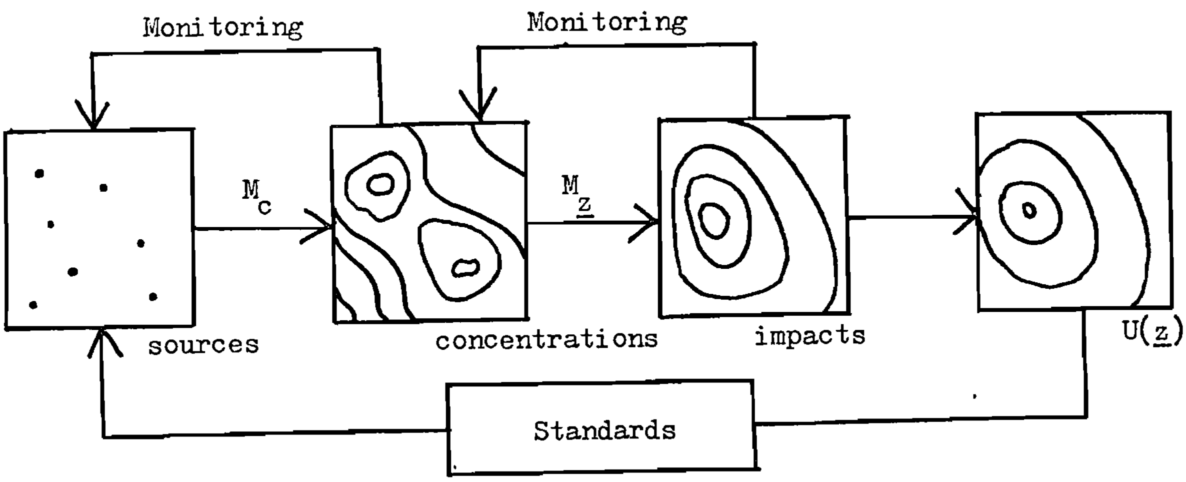

Figure 1.

Consider some geographic region within which there are pollution sources of point and distributed types whose locations we know, and monitoring stations whose locations are at our discretion. Assume that we have some analytical model, M_C, with which to make air pollution concentration predictions and that this model uses the spatial distribution of sources and their respective discharges as input from which it produces probabilistic predictions of the spatial distribution of air pollution concentration, C(X,Y). This might be a probabilistic analogue to the Oak Ridge Laboratories ADDL model.

The spatial distribution of pollutant concentration causes impacts against certain societal, environmental, and economic objectives which are generally held to be important (e.g., it increases the morbidity and mortality rates, causes destruction of agricultural production, etc.) Let these impacts be defined as performance against some predetermined set of objectives (e.g., "minimize mortality rate"), and let this performance be measured on a vector of indices, $\underline{z}$; the value of this vector at some spatial location (x,y) will be $\underline{z}(x,y)$. Clearly, these impacts depend on the spatial distributions of exogenous properties, $\underline{\theta}(x,y)$ such as population composition and size, and land-use type and intensity as well. Further assume that we have some second analytical model $M_{\underline{z}}$ which, given the distributions of pollutant concentration and exoginous variables predicts a probability density function of $\underline{z}(x,y)$. Finally, let there be some multiattributed utility function, $U(\underline{z})$, defined over the vector space of all $\underline{z}$'s and that the objective in decision making with respect to regional air pollution is to maximize the expected value of $U(\underline{z})$.

If we assume that some regional administrator or agency is responsible for air pollution control, there are two types of decisions which are allowed them: first, setting standards for source emissions; and second, designing the monitoring network. Clearly, the ultimate aim is to set standards such that the expected value of $U(\underline{X})$ is maximized.

We will consider two decision problems: one is the static case in which standards are set for long periods of time (e.g., annually up-dated); the other is the dynamic case in which standards are used as continually reviewed dynamic control variables.

Static Case

Given the abstraction of the problem, one can readily formalize an expression for the optimal standard level as,

$$(1) \qquad \max E[u] = \max_{\underline{s}} \int_x \int_y \int_{c(x,y)} u[\underline{z}(x,y)] \; \mathrm{pdf}[\underline{z}(x,y)/c(x,y)\underline{\theta}(x,y)] \; \mathrm{pdf}[c(x,y)/\underline{s}] \; dc(x,y) \; dxdy$$

in which pdf stands for "probability density function", $c(x,y)$ is the pollutant concentration at point (x,y), and $\underline{s}$ is the set of standards. The function $\mathrm{pdf}[c(x,y)/\underline{s}]$ is the prediction of model M_1; the function $\mathrm{pdf}[\underline{z}(x,y)/c(x,y), \underline{\theta}(x,y)]$ is the prediction of model M_2. From the criterion of optimality, the set of standards, $\underline{s}$, which maximizes equation 1 is the optimal standard. In this case monitoring is of use only in validating the models M_c and $M_{\underline{z}}$, and for record keeping. Optimization of the monitoring network is without meaning in the standard setting decision, and would of course be accomplished by minimizing the error of spatial estimates or some weighted spatial estimate accounting for exogenous variables.

Dynamic Case

The second problem in standard setting is to use standards as a dynamic control variable. This would apply in the case where whenever pollution concentrations as measured through some monitoring network became too high, restrictive standards would be enforced. When concentrations fell, less restrictive standards would be reinstituted. A typical example is "smog alert days" in some major American cities during which private auto transport is discouraged and certain industries are forced to slow production. This problem is somewhat more interesting than the static case because both standard decisions and monitoring networks may be optimized within the same decision frame-work.

Using the abstraction presented in Figure 1 the structure of decision becomes,

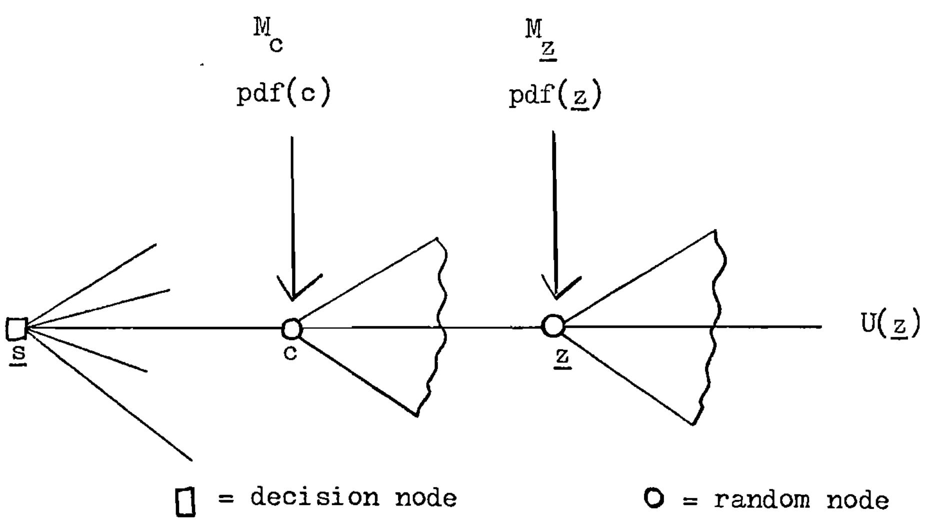

Figure 2.

and the optimization is,

$$\max_{\underline{s}} \int_{y}\int_{x}\int_{\underline{z}_t}\int_{c_t}\int_{c_{t-1}} u[\underline{z}_t]\ \mathrm{pdf}[\underline{z}_t | c_t, c_{t-1}, \underline{s}_t, \underline{\theta}]$$

$$\mathrm{pdf}[c_t | c_{t-1}, \underline{s}_t]$$

$$\mathrm{pdf}[c_{t-1}]\ dc_{t-1} dc_t d\underline{z}_t dx dy$$

in which $\underline{z}_t$, c_t, c_{t-1}, and $\underline{\theta}$ are functions of location (x,y), and in which the model $M_{\underline{z}}$, M_c, and the error resulting from the monitoring network correspond to the respective probability density functions,

$$M_{\underline{z}}:\quad \mathrm{pdf}[\underline{z}_t | c_t,\ c_{t-1},\ \underline{s}_t,\ \underline{\theta}]$$

$$M_c:\quad \mathrm{pdf}[c_t | c_{t-1},\ \underline{s}_t]$$

$$\text{Monitoring error:}\quad \mathrm{pdf}[c_{t-1}]$$

where,

c_t = the pollution concentration at (x,y) during time t,

c_{t-1} = the pollution concentration at (x,y) during time t-1,

$\underline{s}_t$ = the set of standards during time t,

and the distribution of exoginous variables $\underline{\theta}$ is assumed known. This is an optimization problem which Gros and Ostrom[1] suggest can be solved by methods of dynamic control theory.

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[1]Gros, J. and T. Ostrom (1975), "A decision analytic approach to river basis pollution control", Research Memorandum (in preparation) IIASA.

Qualitatively, consider the time series of pollution concentration at one point in space (Figure 3). The decision

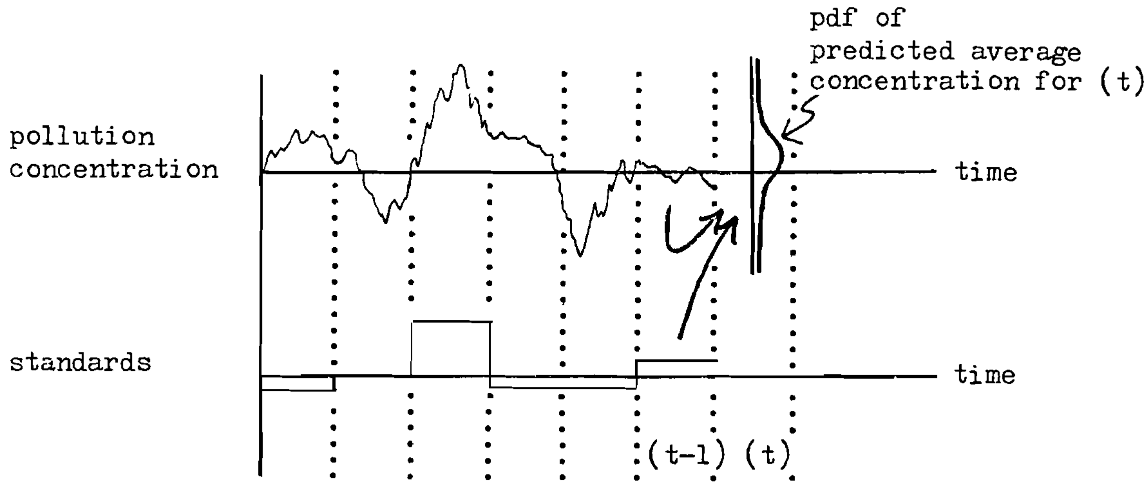

Figure 3.

model just proposed assumes that pollution concentration at some time t is dependent only on the concentration at t-1 and the standard selected for t: It assumes a discrete time series. Further, it considers maximizing utility only in the forthcoming increment, and not in the entire future (which of course would be affected by a decision). Nevertheless, this is only a first statement of the problem which might be further refined.

In application, the standard setting decision may not be over a continuous variable, but rather over a discrete variable which may only take on two or three possible levels. For example, in the air pollution case a regional authority might have only three possible courses of action: leave the standards at their normal level, adopt short duration strict standards, or totally stop certain types of pollution activities. In this case the mathematics of equation 2 become somewhat easier as the optimization reduces to comparing three values of expected utility rather than optimizing a continuous function.

Optimization of the monitoring network enters this problem through error in the estimation of c_{t-1}. The greater the error in measurement, the broader the pdf of c_{t-1} and thus the more dispersion in the derived pdf of $\underline{z}(x,y)$. The problem of investment in improving the monitoring network thus depends on whether or not an improvement will lead to a net increase in expected utility, given the economic cost of improvement. The geometry of optimization for a given investment depends on (1) how information changes the pdf of c_{t-1}, and (2) on the set of exogenous variables, $\underline{\theta}(x.y)$, which are also spatially distributed. These things are considered in a very simple way in Darby, et al. (1974)[3]. Although the optimization seems at first observation to be almost intractable, upon closer examination this may not be the case. If, however, the optimization is not possible, the second line of approach would be to develop some spatially weighted error function (i.e., weighted on the basis of where pollutant concentration is most damaging with respect to the set of societal

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[3]Darby, W.P., Ossenbruggen, P.J., and Gregory, C.J. (1974). "Optimization of Urban Air Monitoring Networks", Jour. of the A.S.C.E., EE 3 : 577-591

objectives) with which to compare alternative network designs. But this doesn't allow easy access to the question of how much to invest in monitoring.

A periferal problem here, which will only be mentioned, is decisions for investment and allocation in monitoring the effects of pollution against the index set $\underline{z}(x,y)$. This would be for verification of model $M_{\underline{z}}$ as in the static case, and not directly part of the dynamic control problem.