GEOGRAPHICAL DIFFUSION PROCESSES: A Working Paper on Alternative Methodological Approaches of an Operational Type

Ross D. MacKinnon

March 1975 WP-75-29

Preface

I was requested to write this paper when I attended the Congress of Brazilian Geography at Belem last summer. As a member of the Institute of Pan American Geography and History's working group on the spatial diffusion of innovations, I was asked to identify the range of operational models which might be relevant to the problem. Partly because the problem is rather vaguely specified and partly because of commitments to do other things the paper is probably not as detailed or comprehensive as it should be. It may be revised in late spring, after members of the working group have had an opportunity to comment.

Fortuitously, there is a strong connection between spatial diffusion problems and the urban project's national settlement systems research. The empirical work in Brazil, undertaken largely by the Institute of Brazilian Geography and Statistics (IBGE), will be focusing on regional economic development processes.

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Ross D. MacKinnon

International Institute for Applied Systems Analysis
and
Department of Geography, University of Toronto

1. Introduction

The purpose of this working paper is to present alternative technical frameworks in which the study of spatial diffusion can be modelled--in detail sufficient to allow for a determination to be made on the relevance or feasibility of such approaches to particular problems, but the level of detail will not be sufficient for the use of the technique without referring to other sources. Illustrative sources will be identified but the list of references should by no means be regarded as complete. The paper then is to be thought of as a menu with the general nature of each item indicated but in no way is it the cook book from which the dishes can be prepared.

The paper is composed of two main parts. Although essentially methodological in nature a brief discussion of some subtantive issues was thought to be desirable and is presented in the following section. Alternative modelling approaches are then discussed in section three. It should be emphasized that this report is a working paper intended for distribution within the Institute of Pan American Geography and History working group on diffusion. It is a rather rough document and certainly incomplete in that it represents the personal perspectives of someone who has not been a very active participant in this area of study.

2. Substantive Issues

Spatial diffusion can be defined narrowly to include those processes which move through space and time and in which the system increases its spatial extent by means of very small changes such that differential equations can be used to model the process. Although gradual change characterizes most diffusion studies in geography only a few use differential equations explicitly. Moreover, there is a tendency to use a rather broader definition of spatial diffusion such that the spatial growth of virtually any type even quite discontinuous, phenomena is included.

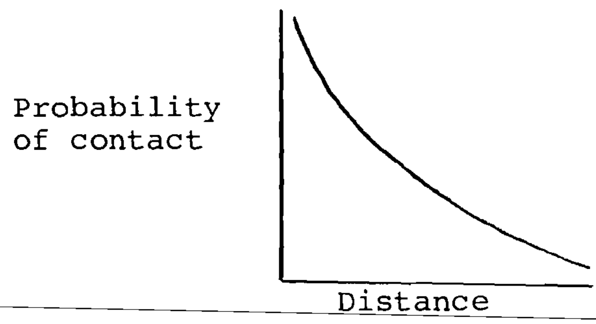
Implicit in the narrow definition is that the mechanism of diffusion is several atomic agents communicating with each other rather than a few large agents directing the spread of the phenomemon. The focus of this discussion will be on processes of gradual change i.e. towards the narrow end of the definitional spectrum--but certainly not emphasizing differential equations *per se*. In fact, we will be concerned with operational models and therefore generally in the discrete time domain. From time to time discontinuous change will be introduced either because these phenomena are of interest in their own right or because they act as agents for the subsequent diffusion processes of a more conventional type.

Geographers have studied diffusion or spread of many phenomena--new ideas (innovations in business or administrative practices or new products; rumours or information of other sorts; diseases (epidemiology); people animals and plants (migration).

Most of these models are propelled by a communication or interaction process of some sort whereby some or all of the knowers (infectives) act as spread or diffusion agents in subsequent periods of the process. Assumptions about "forgetting" or "resistance" may alter the speed of the diffusion process, but the spatial nature of the process is more often than not dominantly influenced by nature of the postulated interaction or communication process unless, of course, resistance factors have a strong spatial pattern.

What are the general sorts of categories of assumptions that have been made concerning the spatial diffusion interaction mechanism? The most frequently used is that people are more likely to interact if they are close together rather than far apart. This assumption is incorporated by what is usually called a distance decay function. Figure 1.

Figure 1



Another way of representing this mechanism is by Hägerstrand's Mean Information Field (MIF). See Figure 2)

Figure 2

.05	.1	.05
.1	.4	.1
.05	.1	.05

Each person is assumed to make a contact with people in each cell according to the indicated probabilities. Beginning with a single cluster of innovations and in the absence of other information, it is clear such a process will have a very simple expected distribution over time--growing outwards in a symmetrical concentric wave-like pattern. However, it should be emphasized that with a probablistic formulation, a large number of patterns could result, some of them less likely than others but all of them possible. It is the identification of ranges and variances of distributions which is one of the most rewarding in the study of spatial diffusion processes--recognizing explicitly that a great deal of uncertainty and randomness underlies the spread of many phenomena and the communication or interaction processes on which that diffusion is based.

Peculiarities in the communications process may necessitate the adjustments to the contact probabilities. These peculiarities may be of a rather general type (e.g. arising from biases in movement towards the central business district or towards some other attractive activity centre). The information field in cross-section may therefore look like Figure 3(a) rather than Figure 3(b)

Figure 3(a)

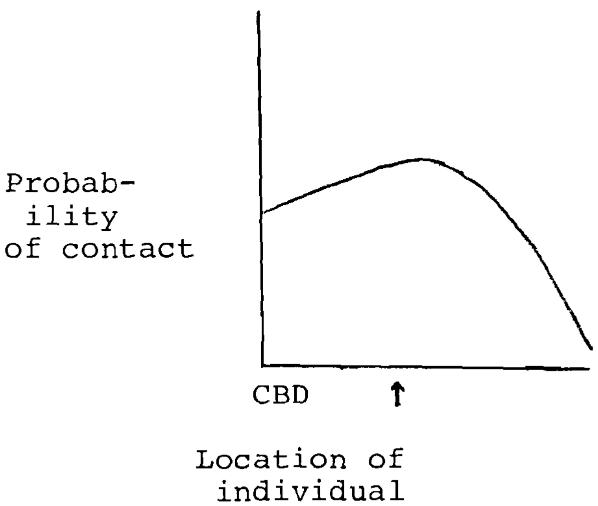
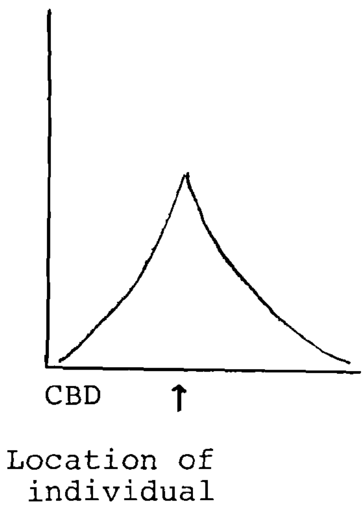


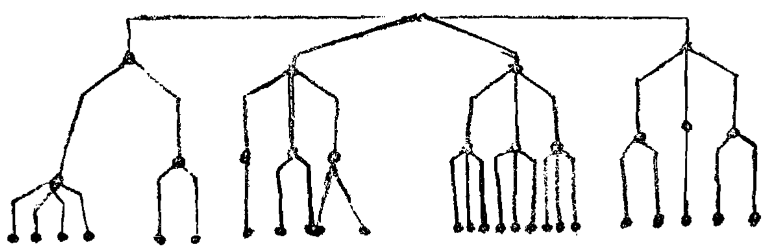
Figure 3(b)



"Irregularities" in the communication process may be of a more <u>ad hoc</u> type reflecting physical (rivers, mountains) or social barriers (different cultural groups) to communication. These may be accommodated by either adjusting the MIF probabilities or making it difficult or impossible for people to interact directly with people in certain locations.

Either superimposed on the approximately smooth distance decay function or perhaps supplementing it completely can be a hierarchical communication-diffusion process. According to central place theory shopping behaviour and other activities are hierarchically structured. Within a diffusion context this is usually interpreted as meaning communication and innovations move from high order centres to centres of lower order. A new business practice originating in the largest city in the nation (city of rank k) would next appear in the cities of next lowest rank (k-1), etc.

Figure 4 - A Hierarchical Structure of Communication



4th order centre

3rd order centres

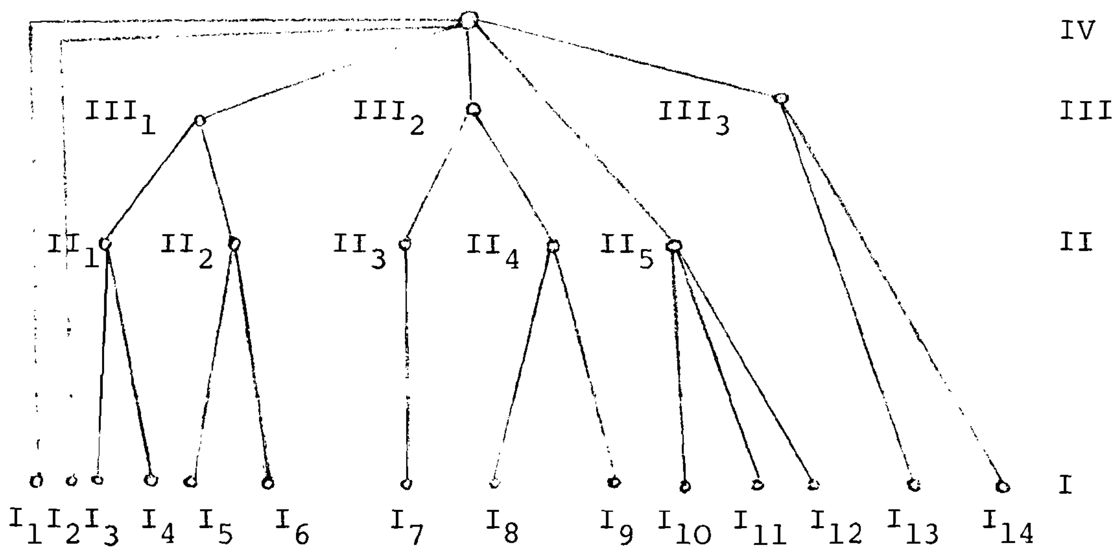
2nd order centres

1st order centres

Although innovations are generally assumed to proceed from larger to smaller centres, a more complex pattern is possible--they may originate in intermediate even smaller centres and then diffuse both upwards and downwards through the hierarchy. Even in such cases the larger cities are critical points of diffusion in that they are so well connected to centres throughout the national settlement system. Only when the largest centre has adopted the innovation, it is argued, will the innovation have a good chance of being widely adopted. Thus large centres are critical in the diffusion of innovations either because of their assumed importance in the generation of innovation or because of their role as strategic focal points in the communications network.

Of course, when the hierarchical structure is combined with a neighbourhood effect, the diffusion network can be quite different from that shown in Figure 4. In the immediate hinterland of largest centre of centres of several sizes, and large to intermediate centes also contain in their sphere of influence centres of many sizes. Thus very small centres within 100 kilometers of Rio may adopt the innovation long before centres of much larger size in Amazonia.

Figure 5 - A Hierarchical Structure with a Neighbourhood Effect



City I_1 for example, may receive information about the innovation before city II_1 and simultaneously with III_1.

Of course, communications networks may not reflect only hierarchical and neighbourhood effects. Pairs of cities may be closely connected at the same level in the hierarchy or at quite distant levels in a hierarchy due to a close functional relationship (a complementary economic structure for example--an iron ore town and a steel making city; or two cities with strong social ties arising from past migration tendencies).

With diffusion of a hierarchical type, two causal mechanisms may be at work and which one is predominant may be difficult if not impossible to identify. Strictly speaking, hierarchal diffusion in its pure form arises from the hierarchical nature of the communications process. Another mechanism which may be at work is threshold profitability levels which are strongly correlated with market size. As an innovation is introduced larger centres are often able to insure a large profit margin--even if the innovation proves to be unsuccessful in the long run, the large market provides a hedge against such risks that smaller centres cannot. With successful innovations, the market threshold usually diminishes over time. Initial attempts to introduce the innovation can be risky and be characterized by costly experimental trial-and-error methods. As the diffusion process proceeds, the characteristics of the innovation become clearer, in many cases mass production is possible resulting in lower risks and costs, allowing centres with smaller markets to adopt the innovation. We can note that profitability effects can also be associated with an apparently contiguous diffusion process. In an agricultural context, for example, wavelike patterns of innovation may arise because there are regional patterns in land capability and agricultural practices. Thus, an innovation may move from areas of high to low capability, either because of the gradient in profitability conditions, the nature of the underlying communications process, or some mixture of these two reasons. The extent to which the pattern of diffusion is a result of information flows on the one hand or profitability thresholds on the other is often difficult to determine. Careful sampling of the data may be desirable in an attempt to hold one set of factors constant while allowing the others to vary.

Of course identical patterns of diffusion may have completely different policy implications depending on which of the two mechanisms underlies the pattern of spread. If the information factor predominates, then an increase in the effectiveness of the dissemination of information in both qualitative and quantitative terms by means of advertising, demonstration projects or other promotional means may be implied. Also, the existing channels of communication and transportation may be improved--daily delivery of magazines and newspapers, radio and television frequent bus, train and air services to outlying area. That is, with respect to the communications process, policy can be implemented to take specific information to the potential adopters or to increase the likelihood that they will come into contact with people or institutions which are already using the innovation. The first method can be far more selective in its effects and limited in the resources necessary to implement it. The second can have far reaching and perhaps unanticipated consequences and in that it involves structural change may necessitate expenditures of considerably greater magnitude.

If, on the other hand, profitability conditions are predominant, the policy implications may be quite different. Certainly a system of subsidies and penalities could be implemented in order to control the variations in profitability and therefore the pattern of diffusion. Changes in the infrastructure can also change the pattern of profitability. For example, the potential market for an innovation may be sufficient but ineffective because it is spread too thinly over an area which is poorly served by transportation. Improved transportation, perhaps with an explicit focus on a regional centre could have the effect of making this demand effective by making profit levels attractive so that these and other services are provided even in peripheral areas. The decisions regarding the location of other public facilities can also affect profitability conditions.

Another way to deal with diffusion problems which are plagued by insufficient markets, or equivalently, demands that are spread too thinly is to encourage a settlement pattern which has a better defined hierarchy--more centres at an intermediate level perhaps. This area is where diffusion modelling interfaces closely with growth pole theories (See Berry, 1971).

A quite different way to deal with the same problem is to attempt to encourage the development and adoption of innovations with lower profitability thresholds. While examples of such strategies are difficult to identify, the principle should be clear. Agricultural innovations suitable in a capital intensive, large farm economy may not be suitable in a small farm labour intensive context. Enormous and inefficient subsidies, both direct and indirect, may be necessary to force the diffusion of certain innovations. In such cases, perhaps some attempt should be made to investigate the feasibility of other innovations which serve similar purposes but are more appropriate to the country at hand.

Before turning to more technical issues, it should be noted that if a diffusion study is to be relevant to policy in an economic development context, and important research area which should not be overlooked is the way in which the innovation interacts with, causes and is caused by, economic and social development. Which innovations are symptoms and which are real forces in development? What are the multiplier effects--magnitude and spatial extent--of various innovations? A special case of this category of problem is the way one or more innovations may follow another perhaps in a dominant industry. This multi-level diffusion process where one innovation in a catalytic way stimulates other innovations is certainly an under-researched problem but a potentially very rewarding one within an economic development context. As the "initial" diffusion patterns can strongly bias the pattern of subsequent innovations. Another example of multiple dynamic processes is the linkage of diffusion and migration processes. Migrants may behave according to one set of relationships. Some of these migrants are innovators, however. Thus the pattern of diffusion is the result of the joint migration and communication processes. Carrying the argument one step further one could hypothesize that migration itself may be influenced by the pattern of innovation adoptions and the benefits which flow therefrom. One can see that the dynamics of diffusion when linked to other dynamic processes may become very complicated indeed.

This concludes the brief summary of substantive issues relating to spatial diffusion studies. More thorough reviews are available at various levels (Gould 1969, Brown 1968, and Hudson, 1972). A recent discussion paper series, "Studies in the Diffusion of Innovation" edited by Larry Brown at Ohio State University includes reviews and recent theoretical and empirical research in this field.

3. Alternative Methodologies

Diffusion processes can be studied using a wide variety of technical frameworks at many levels of formality. Often the subjective and intuitive approach can give insights into the spread of innovations which more formal methods cannot because of the rich experience of the interpreter and the lack of adequate data or because of the inherent limitations (computational or theoretical) of the formal approaches themselves. In site of the frequent superiority of the informal methods, this review limits itself to the formal ones in part because my activities tend to be of that type and because these approaches in spite of any limitations can offer strong quantitative and qualitative insights. Most of the time they should be used in conjunction with the more subjective methods. The hypotheses to be tested, the formal method to be used, and the interpretation of the results are all more or less subjective and judgmental decisions.

Differential equation models adopted from the physical sciences represent the pinnacle of formal elegance in studies of diffusion processes. They attempt to give insights of a qualitative type in that even more than most models, they abstract from the complexities of real processes and the ultimate "testing" of the model is often either weak or non-existent. Examples of this type of spatial diffusion model are found in Beckmann (1970) and Hudson (1972). These approaches are without doubt interesting and challenging but lie outside the scope of this review as it is concerned with operational or empirical models.

The operational analogue to differential equation models are the statistical difference equation models the simplest of which are models of the autoregressive type. In such models the value of a variable at one time is a function of values of that same variable in previous time periods. For example, a linear model of this type is:

$$X(t) = \alpha_0 + \sum_{k=1}^{r} \alpha_k X(t-k) + \varphi(t)$$

where r is the number of time periods assumed to influence the current value of X (in the simplest case r = 1)

α_i (k=1, ...r) are parameters estimated from the data which indicate the strength of the influence of previous values of X on the current value.

$\varepsilon(t)$ is a random error term.

In a geographical model, the location of X in space as well as time is taken into account. The state variable in this case X(i,t) is for example the number of adopters at location i at time t. It is assumed to be dependent on previous values of that variable at that location <u>and</u> at other locations. In the simplest case, X(i,t) may have both a spatial and temporal lag of one. That is, only values of the variable in the previous time periods and in immediately adjacent neighbouring (contiguous) locations are assumed to influence the variable in question.

$$X(i,t) = \alpha_0 + \alpha X(i,t-1) + \beta\, X(i+\delta,t-1) + \varepsilon_i(t)$$

where α_0, α, β are parameters as before and $\bar{X}$ $(i+\delta,t-1)$ is some index of the number of adopters at time t-1 in contiguous or nearby locations (this could be simply the total number of adopters in all contiguous locations or the average number or the maximum number of adopters in a contiguous location.

Of course, the locations to be identified as "nearby" can be determined a priori by the modeller. The modeller may input any arbitrary contiguity matrix A such that $a_{ij} = 1$ if the j^{th} location is to be allowed to influence the i^{th} location and $a_{ij} = 0$ if such influence is not to be allowed. Thus, a hierarchical or structural diffusion network could be incorporated as well as one of a simple contiguity type. The primary source of this type of methodology is Cliff and Ord (1973). An empirical example within a diffusion context can be found in MacKinnon (1974).

It should be noted that autoregressive models represent a means to predict in cases either where other variables are not known or where the causal structure is not well understood. Information on past behaviour is substituted for theoretical knowledge or information on a wide number of other variables. The use of autoregressive and related models does not imply that models with a strong causal structure would not be useful, only that they are either infeasible or impractical at the current stage of development. For models which use this implicit theoretical structure to be effective, a fairly long time series on the single variable must be available. Moreoever, they are typically used for rather short term projections.

The basic autoregressive model can be supplemented with information on other variables. Thus in the simple linear first order case,

$$X(i,t) = \alpha_0 + \alpha X(i,t-1) + \beta \overline{X}(i+\delta,t-1) + \sum_j \gamma_j Y_j(i,t-1) + \varepsilon_i(t)$$

where Y_j is the j^{th} independent variable temporally and perhaps spatially lagged.

Thus, the number of adopters could be hypothesized to be a function of income and education levels as well as the number of previous adopters in the same and "nearby" locations.

Obviously much experimentation with the functional form and the variables to be included could be undertaken within this statistical difference equation methodology. More broadly the large field of time series analysis offers a number of opportunities for the empirical study of diffusion processes. (Box and Jenkins, 1970). The trends seasonal, cyclical and other components of fluctuations in time series of adopters can be identified for each location separately and then locations which tend to vary together or perhaps with a lag can be identified by factor analytic methods, cross-spectral analysis, trend surface analysis, or other methods. See Casetti, King and Jefferies ( ), Haggett and Bassett ( ) and Rayner (1971) for examples of such approaches in a geographical context.

There is a large battery of multivariate statistical methods which can be brought to bear on diffusion problems. Which one or which group will be used will depend on data availability, the level of knowledge of process and the personal preferences of the research team. Some are for exploratory purposes, i.e."fishing expeditions;" others are to test a highly structured set of hypotheses or a theory. Regression models although used for both purposes are more satisfactory in the latter role. Some methods such as principal components analysis are good for taxonomical purposes--to group together locations that behave similarly with respect to the diffusion process. Others such as discriminant analysis start with groups (delimited by principal components methods or perhaps more simply as early, medium and late adopting groups of regions) and attempt to identify variables which distinguish between the groups. Are early adopting locations, for example, consistently characterized by high income and accessibility measures?

As an alternative to statistical methods, one can adopt a simulation approach in which a number of postulated relationships are integrated into a single model, some of these may be verified before running the model using statistical approaches; others may not be tested and may in fact not even be testable. Well-known examples of simulation approaches to diffusion are Hägerstrand (1967) and Morrill (1965). These are models where the underlying communications process is assumed to be probabilistic that is where the same people can behave quite differently even under identical circumstances. Thus to get a true picture of the possible development of the diffusion process, one must run the model several times; only in this way can the range or the variance structure be identified. Monte Carlo simulation methods of this kind use random number generators and, depending on the complexity of the model, may require substantial amounts of computer time.

Of course simulation models may be of a deterministic type where the spread of the process is postulated to be related to a number of different factors where feedback effects may play an important role. Simulation models have the advantage of being able to cope with a large number of relationships simultaneously and not being limited to artitrarily well-behaved relationships. Many _ad hoc_ decision rules and constraints may quite easily be incorporated into such models. One should bear in mind, however, that the inclusion of many _ad hoc_ rules may be counter productive in scientific terms in spite of the fact that a closer replication of actual patterns of diffusion may be possible. A goal of research is generally to identify general and pervasive factors underlying diffusion processes. The undisciplined use of the flexibility provided by simulation approaches may result in highly place-specific and spurious "explanations".

Very similar in nature to the autoregressive approach is one which utilizes the theory of Markov processes (See Kemeny and Snell (1960), Bartholomew (1973) and Curry and MacKinnon (1974) for introduction to Markov processes). In a diffusion context, the state vector is the number of adopters at each location. In the simplest case, the expected distribution of knowers can be represented as:

$$Y(t+1) = Y(t) \left[I+Q\right]$$

assuming each innovation tells no more than one person in a time period according to the probability distributions given by the matrix Q. The parameters q_{ij} indicate the probability of a knower in i telling someone in j about the innovation over the specified time period. ($\Sigma q_{ij} \leqslant 1.0$, $1 - \sum_j q_{ij}$ being the probability that the person in i tells someone outside the regional system or tells nobody at all.) Note that the Mean Information Field methodology can still be retained and incorporated in the rows of the matrix Q. This formulation is deficient in that someone may be told who has already heard about and adopted the innovation. This defect can be ameliorated by multiplying Q by the diagonal matrix $R(t) = r_{ij}(t) = (1 - \frac{Y_j(t)}{X_j})$, the probability that a person contacted in j has not yet heard about the innovation. X_j is of course the total relevant population in j. The equation

$$Y(t+1) = Y(t) \left[I + QR(t)\right]$$

provides a consistent first approximation to a diffusion simulation model of a Markovian type. Modifications to this model could be made as guided by policy interests and data availability.

One modification could be to link the diffusion model to a migration model. In other words both the agents of diffusion and the population of which they are a subgroup are relocating their place of business or their residence with certain probabilities. The simplest pure migration model of this sort of process is

$$X(t+1) = X(t)\ P$$

where X(t) is a vector indicating the population distribution (both knowers and non-knowers) at time t. The change in the distribution of the knower subgroup is a function of both P, migration propensities, and Q, the pattern of telling. Thus,

$$Y(t+1) = Y(t)\ \left[P + QR(t)\right]$$

where $r_{ij}(t) = (1 - \frac{Y_j(t)}{X_j(t)})$

Although $r_{ij}(t)$ has the same interpretation as in the "pure" diffusion model, note that both the numerator and denominator of the second term vary with time.

Another modification of some interest is to introduce variables over which government planners have some control and some sort of goal set which the planning agency uses to guide its actions. A simple example of such an extension could be achieved by assuming the government can introduce (at a cost) information inputs into the system (trade fairs, agricultural agents, advertisements, etc.).

The distribution of knowers in the system is equal to past distribution, plus those who are told by these people, plus those who hear about the innovation as a direct result of government actions. Thus,

$$Y(t+1) = Y(t)\left[I + QR(t)\right] + BU(t)$$

where $U_j(t)$ is some measure of government propagandizing in region j at time t; b_{ij} is the effectiveness of government propagandizing in j upon the increase in knowers in i--in other words, the increase in the number of knowers in i resulting from a unit effort in j. (In the simplest case B could be the identity matrix and $U_j(t)$ interpreted simply as the number of people in j who are newly told about the innovation by the government agent at time t.

In addition to this equation describing the dynamics of the process, we must include something about the goals of the planning agency. A desired distribution of knowers may be specified or some desirable characteristics of such distribution. The easiest goals of this sort to incorporate are those state desirable trajectories or time paths of knowers. For example,

$$Y_j^{L}(t) \leqslant Y_j(t) \leqslant Y_j^{u}(t) \qquad \begin{array}{l} t=1, \;\ldots\ldots\; T \\ j=1, \;\ldots\ldots\; r \end{array}$$

where $Y_j^{L}(t)$ and $Y_j^{u}(t)$ are lower and upper bounds for the numbers of knowers. Thus one could see the implications of specifying an equitable (or more equitable) distribution of knowledge in the system.

One of the implications is the costs associated with the decision variable U. These costs can be taken into account by specifying an objective function Z which is to be minimized: For example,

$$\text{Minimize } Z = \sum_t \sum_j C_j(t) U_j(t)$$

where $C_j(t)$ is the present value of the per unit cost of the unit of government advertising or promotional effort in region j at time t.

One formulation of the diffusion control problem then is the linear programming problem outlined above--find the decision variables (information inputs, government effort) which achieve certain goals and use the system's rules of behaviour and use no more resources than are necessary.

The problem could be formulated in different ways using the same linear programming methodology; for example, minimize the deviation between goals and outcomes without exceeding a specified budget; minimize the time to attain certain objectives without exceeding a budget. Alternatively, the more general and analytical methods of optimal control theory could be used to solve problems of lower dimensionality.

No matter what normative technique is utilized, it should be emphasized that these models seldom if ever provide definitive solutions--they can provide insights into both the specific and general nature of the types of controls which may prove beneficial in guiding the pattern of innovations (and thus perhaps economic development) along desired paths.

By making explicit the implications of a variety of goals the goals themselves may be revised. In summary then the object of these models is not to make decisions, but rather to provide decision-makers with relevant information so that they can make better decisions.

4. Closure

This concludes a rather cursory survey of some of the methodological alternatives in spatial diffusion studies. More detail and specific references can be provided on request.

Although purely operational approaches have been emphasized, it should be recognized that these approaches can possibly be enriched by the study of diffusion from a more abstract, analytical viewpoint (See for example Itô and McKean (1974) and Erdős and Rényi (1960

It is clear that the choice of approach will be partly progmatic (depending on the goals of the research and the resources and data available to undertake the research) and partly subjective (depending on the style and capabilities of the researcher). However, in a broadly based empirical study all of the operational approaches outlined in this paper--statistical, simulation and normative models--deserve careful consideration.

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