Working Paper

Aspiration Level Approach to Interactive Multi-objective Programming and its Applications

Hirotaka Nakayama

WP-94-112
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Foreword

For a long time, it has been recognized that most decisions are taken with respect to several criteria. The main difficulty, however, has been to find an approach for integrating the multi-criterion feature into the decision making methodology in such a way that it is at the same time practically acceptable and mathematically feasible. In the development of such approaches, IIASA has played a central role. Particularly, the aspiration level approach has been developed largely at IIASA. The author of the present Working Paper, Professor Hirotaka Nakayama of Konan University in Kobe (Japan), has cooperated with the MDA-project for many years on the topic of multi-criteria analysis. Professor Nakayama specializes in refining and adapting the aspiration level approach in order to increase its practical usability without losing the mathematical tractability. In the present Working Paper, he presents his approach and summarizes his experiences with this adapted approach.

Abstract

Several kinds of techniques for multiple criteria decision making have been developed for the last few decades. Above all, the aspiration level approach to multi-objective programming problems is widely recognized to be effective in many practical fields. As a variant of the aspiration level approach, the author developed the satisficing trade-off method. In addition, he has been applying the method to several kinds of practical problems for these ten years. Some of them were already performed in real life. Typical examples such as feed formulation for live stock, erection management of a cable stayed bridge and bond portfolio selection will be included in this paper.

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*Hirotaka Nakayama**

1 Introduction

Multi-objective programming problems are formulated as follows:

$$\text{(MOP)} \qquad \text{Minimize} \quad f(x) \equiv (f_1(x),\ f_2(x), \ldots,\ f_r(x)) \quad \text{over} \quad x \in X.$$

The constraint set X may be given by

$$g_j(x) \leq 0, \qquad j = 1, \ldots, m,$$

and/or a subset of R^n itself. For the problem (MOP), we define Pareto solutions as follows:

Definition 1.1 A solution $\hat{x}$ is said *Pareto optimal*, if there is no better solution $x \in X$ other than $\hat{x}$, namely, if

$$f(x) \not\leq f(\hat{x}) \qquad \text{for any} \quad x \neq \hat{x} \in X.$$

In general, there may be many Pareto solutions. The final decision is made among them taking the total balance over all criteria into account. This is a problem of value judgment of decision maker (in abbreviation, DM). The totally balancing over criteria is usually called *trade-off*. It should be noted that there are very many criteria, say, over one hundred in some practical problems such as erection management of cable stayed bridge, and camera lens design. Therefore, it is very important to develop effective methods for helping DM to trade-off easily even in problems with very many criteria.

Interactive multi-objective programming search a solution in an interactive way with DM while eliciting information on his/her value judgment. Along this line, several methods have been developed remarkably for the last about fifteen years: Among them, the aspiration level approach is now recognized very effective in practice, because

(i) it does not require any consistency of DM's judgment,

(ii) aspiration levels reflect the wish of DM very well,

and

(iii) aspiration levels play the role of probe better than the weight for objective functions.

In the following, we will discuss the difficulty in weighting method which is commonly used in the traditional goal programming.

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2 Why is the Weighting Method Ineffective?

In multi-objective programming problems, the final decision is made on the basis of the value judgment of DM. Hence it is important how we elicit the value judgment of DM. In many practical cases, the vector objective function is scalarized in such a manner that the value judgment of DM can be incorporated.

The most well known scalarization technique is the linearly weighted sum:

$$\sum_{i=1}^{r} w_i f_i(x). \tag{2.1}$$

The value judgment of DM is reflected by the weight. Although this type of scalarization is widely used in many practical problems, there is serious drawbacks in it. Namely, it can not provide a solution among sunken parts of Pareto surface due to "*duality gap*" for nonconvex cases. Even for convex cases, for example, in linear cases, even if we want to get a point in the middle of line segment between two vertices, we merely get a vertex of Pareto surface, as long as the well known simplex method is used. This implies that depending on the structure of problem, the linearly weighted sum can not necessarily provide a solution as DM desires.

In the traditional goal programming (Charnes-Cooper, 1961), some kind of metric function from the goal f^* is used as the one representing the preference of DM. For example, the following is well known:

$$\left(\sum_{i=1}^{r} w_i |f_i(x) - f_i^*|^p\right)^{1/p} \tag{2.2}$$

The preference of DM is reflected by the weight w_i, the value of p, and the value of the goal f_i^*. If the value of p is chosen appropriately, a Pareto solution among a sunken part of Pareto surface can be obtained by minimizing the function (2.2). However, it is usually difficult to pre-determine appropriate values of them. Moreover, the solution minimizing (2.2) can not be better than the goal $\boldsymbol{f}^*$, even though the goal is underestimated.

In addition, one of the most serious drawbacks in the goal programming is that people tend to misunderstand that a desirable solution can be obtained by adjusting the weight. It should be noted that there is no positive correlation between the weight w_i and the value $f(\hat{x})$ corresponding to the resulting solution $\hat{x}$ as will be seen in the following example.

Example 2.1 Let $y_1 = f_1(x)$, $y_2 = f_2(x)$ and $y_3 = f_3(x)$, and let the feasible region in the objective space be given by

$$\{(y_1, y_2, y_3) | \ (y_1 - 1))^2 + (y_2 - 1)^2 + (y_3 - 1)^2 \leq 1\}.$$

Suppose that the goal is $(y_1^*,\ y_2^*,\ y_3^*) = (0,\ 0,\ 0)$.The solution minimizing the metric function (2.2) with $p = 1$ and $w_1 = w_2 = w_3 = 1$ is $(y_1,\ y_2,\ y_3) = (1 - 1/\sqrt{3},\ 1 - 1/\sqrt{3},\ 1 - 1/\sqrt{3})$. Now suppose that DM wants to decrease the value of f_1 a lot more and that of f_2 a little more, and hence modify the weight into $w_1' = 10,\ w_2' = 2,\ w_3' = 1$. The solution associated with the new weight is $(1 - 10/\sqrt{105},\ 1 - 2/\sqrt{105},\ 1 - 1/\sqrt{105})$. Note that the value of f_2 is worse than before despite that DM wants to improve it and hence increase the weight of f_2 up to twice. Someone might think that this is due to the normalization of weight. Therefore, we normalize the weight by $w_1 + w_2 + w_3 = 1$. The original weight normalized in this way is $w_1 = w_2 = w_3 = 1/3$ and the renewed weight by the same normalization is $w_1' = 10/13,\ w_2' = 2/13,\ w_3' = 1/13$. We can observe that w_2'

is less than w_2. Now increase the normalized weight w_2 to be greater than 1/3. To this end, set the unnormalized weight $w_1 = 10$, $w_2 = 7$ and $w_3 = 1$. With this new weight, we have a solution $(1 - 10/\sqrt{150},\ 1 - 7/\sqrt{150},\ 1 - 1/\sqrt{150})$. Despite that the normalized weight $w_2'' = 7/18$ is greater than the original one ($= 1/3$), the obtained solution is still worse than the previous one.

As is readily seen in the above example, it is usually very difficult to adjust the weight in order to obtain a solution as DM wants. Therefore, it seems much better to take the aspiration level of DM rather than the weight as the probe. Interactive multi-objective programming techniques based on aspiration levels have been developed so that the drawbacks of the traditional goal programming may be overcome. In the following section, we shall discuss the satisficing trade-off method developed by the author (Nakayama 1984) as one of them.

3 Satisficing Trade-off Method

In the aspiration level approach, the aspiration level at the k-th iteration $\overline{f}^k$ is modified as follows:

$$\overline{f}^{k+1} = T \circ P(\overline{f}^k) \tag{3.1}$$

Here, the operator P selects the Pareto solution nearest in some sense to the given aspiration level $\overline{f}^k$. The operator T is the trade-off operator which changes the k-th aspiration level $\overline{f}^k$ if DM does not compromise with the shown solution $P(\overline{f}^k)$. Of course, since $P(\overline{f}^k)$ is a Pareto solution, there exists no feasible solution which makes all criteria better than $P(\overline{f}^k)$, and thus DM has to trade-off among criteria if he wants to improve some of criteria. Based on this trade-off, a new aspiration level is decided as $T \circ P(\overline{f}^k)$. Similar process is continued until DM obtains an agreeable solution. This idea is implemented in DIDASS (Wierzbicki 1981 and Grauer *et al.* 1984) and the satisficing trade-off method (Nakayama 1984). In particular, the satisficing trade-off method provides several devices which make the trade-off analysis easier by using the sensitivity analysis and parametric optimization techniques in traditional mathematical programming.

3.1 On The Operation P

The operation which gives a Pareto solution $P(\overline{f}^k)$ nearest to $\overline{f}^k$ is performed by some auxiliary scalar optimization. It has been shown in Sawaragi-Nakayama-Tanino (1985) and Wierzbicki (1986) that the only one scalarization technique, which provides any Pareto solution regardless of the structure of problem, is of the Tchebyshev norm type. However, the scalarization function of Tchebyshev norm type yields not only a Pareto solution but also a weak Pareto solution. Since weak Pareto solutions have a possibility that there may be another solution which improves a critera while others being fixed, they are not necessarily "*efficient*" as a solution in decision making. In order to exclude weak Pareto solutions, the following scalarization function of the augmented Tchebyshev type can be used:

$$\max_{1 \leq i \leq r} w_i(f_i(x) - \overline{f}_i) + \alpha \sum_{i=1}^{r} w_i f_i(x). \tag{3.2}$$

where α is usually set a sufficiently small positive number, say 10^{-6}.

Theorem 3.1 (Nakayama-Tanino 1994) For arbitrary $w \geq 0$ and $\alpha > 0$, $\hat{x} \in X$ minimizing (3.2) is a properly Pareto optimal solution to (MOP). Conversely, if $\hat{x}$ is a properly Pareto optimal solution to (MOP), then there exist $w > 0$, $\alpha > 0$ and $\overline{f}$ such that $\hat{x}$ minimizes (3.2) over X.

The weight w_i is usually given as follows: Let f_i^* be an ideal value which is usually given in such a way that $f_i^* < \mathrm{Min}\{f_i(x) | \; x \in X\}$, and let f_{*i} be a nadir value which is usually given by

$$f_{*i} = \max_{1 \leq j \leq r} f_i(x_j^*) \tag{3.3}$$

where

$$x_j^* = arg \min_{x \in X} f_j(x). \tag{3.4}$$

For this circumstance, we set

$$w_i^k = \frac{1}{\overline{f}_i^k - f_i^*} \tag{3.5}$$

or

$$w_i^{k'} = \frac{1}{f_{*i} - f_i^*}. \tag{3.6}$$

The minimization of (3.2) with (3.5) or (3.6) is usually performed by solving the following equivalent optimization problem, because the original one is not smooth:

$$\text{(Q)} \qquad \text{Minimize} \quad z + \alpha \textstyle\sum_{i=1}^r w_i f_i(x)$$
$$\text{subject to}$$

$$w_i^k(f_i(x) - \overline{f}_i^k) \;\leq\; z \tag{3.7}$$

$$x \in X.$$

Remark 3.1 Note the weight (3.5) depends on the k-th aspiration level, while the one by (3.6) is independent of aspiration levels. The difference between solutions to (Q) for these two kinds of weight can be illustrated in Fig. 1. In the auxiliary min-max problem (Q) with the weight by (3.5), $\overline{f}_i^k$ in the constraint (3.7) may be replaced with f_i^* without any change in the solution. For we have

$$\frac{f_i(x) - f_i^*}{\overline{f}_i^k - f_i^*} = \frac{f_i(x) - \overline{f}_i^k}{\overline{f}_i^k - f_i^*} + 1.$$

3.2 On The Operation T

In cases DM is not satisfied with the solution for $P(\overline{\boldsymbol{f}}^k)$, he/she is requested to answer his/her new aspiration level $\overline{\boldsymbol{f}}^{k+1}$. Let x^k denote the Pareto solution obtained by projection $P(\overline{\boldsymbol{f}}^k)$, and classify the objective functions into the following three groups:

(i) the class of criteria which are to be improved more,
(ii) the class of criteria which may be relaxed,
(iii) the class of criteria which are acceptable as they are.

Let the index set of each class be denoted by I_I^k, I_R^k, I_A^k, respectively. Clearly, $\overline{f}_i^{k+1} < f_i(x^k)$ for all $i \in I_I^k$. Usually, for $i \in I_A^k$, we set $\overline{f}_i^{k+1} = f_i(x^k)$. For $i \in I_R^k$, DM has to agree to increase the value of $\overline{f}_i^{k+1}$. It should be noted that an appropriate sacrifice of f_j for $j \in I_R^k$ is needed for attaining the improvement of f_i for $i \in I_I^k$.

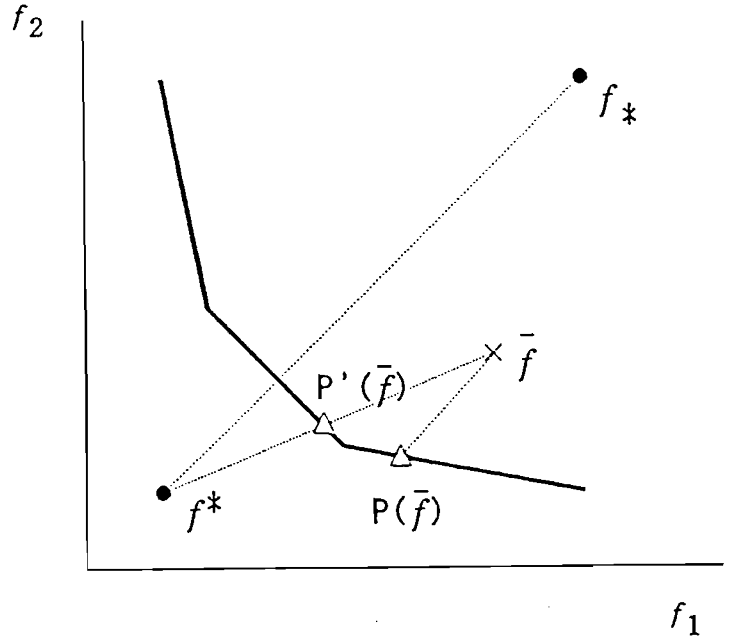

Figure 1: The difference between solutions associated with two kinds of weight.

Example 3.1 Consider the same problem as in Example 2.1: Let $y_1 = f_1(x)$, $y_2 = f_2(x)$ and $y_3 = f_3(x)$, and let the feasible region in the objective space be given by

$$\{(y_1, y_2, y_3)|\ (y_1 - 1))^2 + (y_2 - 1)^2 + (y_3 - 1)^2 \leq 1\}.$$

Suppose that the ideal point is $(y_1^*,\ y_2^*,\ y_3^*) = (0, 0, 0)$, and the nadir point is $(y_{*1},\ y_{*2},\ y_{*3}) = (1, 1, 1)$. Therefore, using (3.6) we have $w_1 = w_2 = w_3 = 1.0$. Let the first aspiration level be $(\overline{y}_1^1,\ \overline{y}_2^1,\ \overline{y}_3^1) = (0.4,\ 0.4,\ 0.4)$. Then the solution to (Q) is $(y_1^1,\ y_2^1,\ y_3^1) = (0.423,\ 0.423,\ 0.423)$. Now suppose that DM wants to decrease the value of f_1 a lot more and that of f_2 a little more, and hence modify the aspiration level into $\overline{y}_1^2 = 0.35$ and $\overline{y}_2^2 = 0.4$. Since the present solution $(y_1^1,\ y_2^1,\ y_3^1) = (0.423,\ 0.423,\ 0.423)$ is already Pareto optimal, it is impossible to improve all of criteria. Therefore, suppose that DM agrees to relax f_3, and with its new aspiration level of $\overline{y}_3^2 = 0.5$. With this new aspiration level, the solution to (Q) is $(y_1^2,\ y_2^2,\ y_3^2) = (0.359,\ 0.409,\ 0.509)$. Although the obtained solution does not attain the aspiration level of f_1 and f_2 a little bit, it should be noted that the solution is improved more than the previous one. The reason why the improvement of f_1 and f_2 does not attain the wish of DM is that the amount of relaxation of f_3 is not much enough to compensate for the improvement of f_1 and f_2. In the satisficing trade-off method, DM can find a satisfactory solution easily by making the trade-off analysis deliberately. To this end, it is also possible to use the sensitivity analysis in mathematical programming. (Refer to the following automatic trade-off or exact trade-off). We have observed in the previous section that it is difficult to adjust weights for criteria so that we may get a desirable solution in the goal programming. However, the aspiration level can lead DM to his/her desirable solution easily in many practical problems.

3.3 Automatic Trade-off

It is of course possible for DM to answer new aspiration levels of all objective functions. In practical problem, however, we often encounter cases with very many objective functions, for which DM tends to get tired with answering new aspiration levels for all objective functions. Therefore, it is more practical in problems with very many objective functions for DM to answer only his/her improvement rather than both improvement and relaxation. At this stage, we can use the assignment of sacrifice for f_j $(j \in I_R)$ which is automatically set in the equal proportion to $(\lambda_i + \alpha)w_i$, namely, by

$$\Delta f_j = \frac{-1}{N(\lambda_j + \alpha)w_j} \sum_{i \in I_I} (\lambda_i + \alpha)w_i \Delta f_i, \qquad j \in I_R \tag{3.8}$$

where N is the number of elements of the set I_R, and λ is the Lagrange multiplier associated with the constraints in Problem (Q). The reason why (3.8) is available is given by the following:

Theorem 3.2 (Nakayama 1991-b, Nakayama-Tanino 1994) Let $\tilde{x}$ be a solution to (Q) with $X = \{x|\ g_j(x) \leq 0,\ j = 1, \ldots, m\}$ and $\tilde{f} = f(\tilde{x})$. Suppose that the second order sufficient condition holds; namely,

$$u^T \nabla^2 L(\tilde{x}; \lambda, \mu) u > 0$$

for any $u \neq 0$ such that

$$\nabla f_i(\tilde{x}) u = 0, \quad i = 1, \ldots, r-1$$

and

$$\nabla g_j(\tilde{x}) u = 0, \quad j \in J = \{j|\ g_j(\tilde{x}) = 0\}.$$

Suppose also that the vectors $(\nabla f_1(\tilde{x}), -1), \ldots, (\nabla f_r(\tilde{x}), -1)$, $(\nabla g_{k_1}(\tilde{x}), 0)$, $\ldots, (\nabla g_{k_s}(\tilde{x}), 0)$ are linearly independent, where $(k_1, \ldots, k_s)$ is the index set of active constraints of (Q). In addition, suppose that the following strict complementary slackness condition holds:

$$\lambda_i > 0 \quad \text{for any} \quad i \in 1, \ldots, r$$

$$\mu_j > 0 \quad \text{for any} \quad j \in J.$$

Then, with Δf_i $(i = 1, \ldots, r)$ such that $(\tilde{f}_1 + \Delta f_1, \ldots, \tilde{f}_r + \Delta f_r)$ is on the Pareto surface, we have

$$0 = \sum_{i=1}^{r} \lambda_i \Delta f_i + o(\| \Delta f \|)$$

Therefore, under some appropriate condition, $((\lambda_1+\alpha)w_1, \ldots, (\lambda_r+\alpha)w_r)$ is the normal vector of the tangent hyperplane of the Pareto surface. In particular, in multi-objective linear programming problems, the simplex multipliers corresponding to a nondegenerated solution is the feasible trade-off vector of Pareto surface (Nakayama 1992).

By using the above automatic trade-off method, the burden of DM can be decreased so much in cases that there are a large number of criteria. Of course, if DM does not agree with this quota Δf_j laid down automatically, he/she can modify it in a manual way.

Example 3.2 Consider the same problem as in Example 3.1. Suppose that DM has the solution $(y_1^1,\ y_2^1,\ y_3^1) = (0.423,\ 0.423,\ 0.423)$ associated with his first aspiration level $(\overline{y}_1^1,\ \overline{y}_2^1,\ \overline{y}_3^1) = (0.4,\ 0.4,\ 0.4)$. The Lagrange multipliers at this solution is $\lambda_1 = \lambda_2 = \lambda_3 = 0.333$. Now suppose that DM modifies the aspiration level into $\overline{y}_1^2 = 0.35$ and $\overline{y}_2^2 = 0.4$. For the amount of improvement of $|\Delta f_1| = 0.073$ and $|\Delta f_2| = 0.023$, the amount of relaxation of f_3 on the basis of automatic trade-off is $|\Delta f_3| = 0.095$. In other words, the new aspiration level of f_3 should be 0.52. If DM agrees with this trade-off, he/she will have the new Pareto solution $(y_1^2,\ y_2^2,\ y_3^2) = (0.354,\ 0.404,\ 0.524)$ to the problem (Q) corresponding to the new aspiration level $(\overline{y}_1^2,\ \overline{y}_2^2,\ \overline{y}_3^2) = (0.35,\ 0.4,\ 0.52)$. It should be noted that the obtained solution is much closer to DM's wish rather than the one in Example 3.1.

Example 3.3 Consider the following multiple objective linear programming problem:

$$\begin{aligned} f_1 &= -2x_1 - x_2 + 25 \rightarrow Min \\ f_2 &= x_1 - 2x_2 + 18 \rightarrow Min \end{aligned}$$

subject to

$$\begin{aligned} -x_1 + 3x_2 &\leq 21 \\ x_1 + 3x_2 &\leq 27 \\ 4x_1 + 3x_2 &\leq 45 \\ 3x_1 + x_2 &\leq 30 \\ x_1,\ x_2 &\geq 0 \end{aligned}$$

Suppose that the ideal point $\boldsymbol{f}^* = (4,\ 4)$ and the nadir point $\boldsymbol{f}_* = (18,\ 21)$ by using the pay-off matrix based on minimization of each objective function separately. Lettiing the first aspiration level be $\overline{\boldsymbol{f}}^1 = (15,\ 9)$, we have the first Pareto solution (11.632, 4.910) by solving the auxiliary min-max problem (Q). This Pareto point in the objective function space is the intercept of the line parallel to the line passing through $\boldsymbol{f}^*$ and $\boldsymbol{f}_*$ with the Pareto surface (curve, in this case). Now we shall consider the following three cases:

<Case 1> Suppose that DM is not satisfied with the obtained Pareto solution, and he/she wants to improve the value of f_2. Let the new aspiration level of f_2 be 4.5. Instead, suppose that DM agrees with some sacrifice of f_1. The new aspiration level of f_1 based on the automatic trade-off is 14.5. Since the automatic trade-off is based on the linear approximation of Pareto surface at the present point, the new aspiration level obtained by the automatic trade-off is itself Pareto optimal in this case as shown in Fig. 2. **<Case 2>** Suppose that DM wants to improve f_1 rather than f_2 at the moment when the first Pareto solution (11.632, 4.910) is obtained. Let the new asipration level of f_1 that DM desires be 9.0. Then the new aspiration level by the automatic trade-off is $\overline{\boldsymbol{f}}^2 = (9.0,\ 5.286)$, and is not Pareto optimal. Solving the auxiliary min-max problem (Q) with the new aspiration level, in this case, we have the new Pareto solution (9.774, 6.226). Since the improvement which DM desires is not so large after solving the min-max problem (Q) with an aspiration level in many practical cases, the new aspiration level produced by automatic trade-off based on the linear approximation of Pareto surface is close to the Pareto surface. Therefore, the satisficing trade-off method using the automatic trade-off yields the desirable solution usually only in a few iterations.

<Case 3> Suppose that DM wants to make f_1 less than 9.0 absolutely at the moment when the first Pareto solution (11.632, 4.910) is obtained. In this case, we have to treat f_1 as the constraint

$$f_1(x) \leq 9.0$$

As will be shown in the subsection 3.5 below, the interchange between objectives and constraints can be made so easily in the formulation of auxiliary min-max problem. (We can treat the criteria as DM wishes between objectives and constraints by adjusting one parameter β in the min-max problem.)

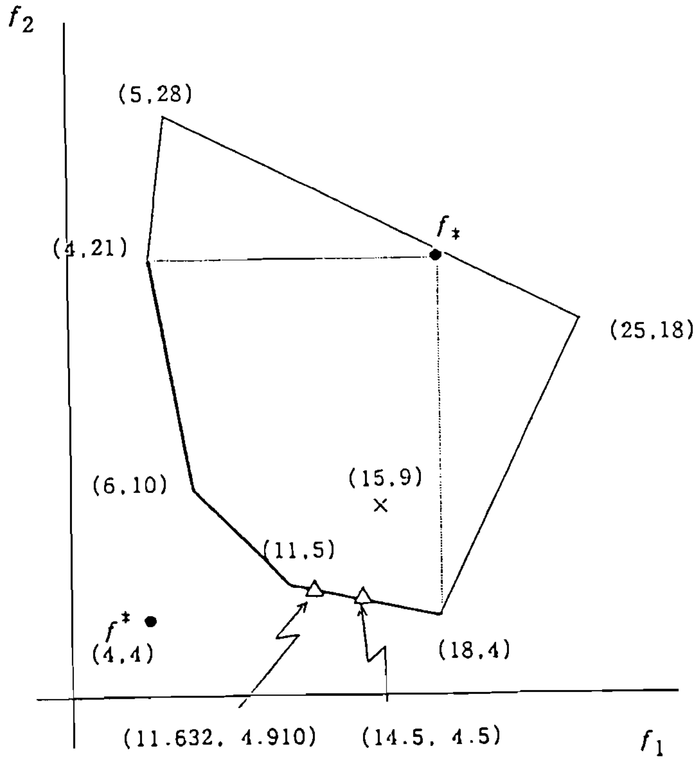

Figure 2: Automatic trade-off (case 1)

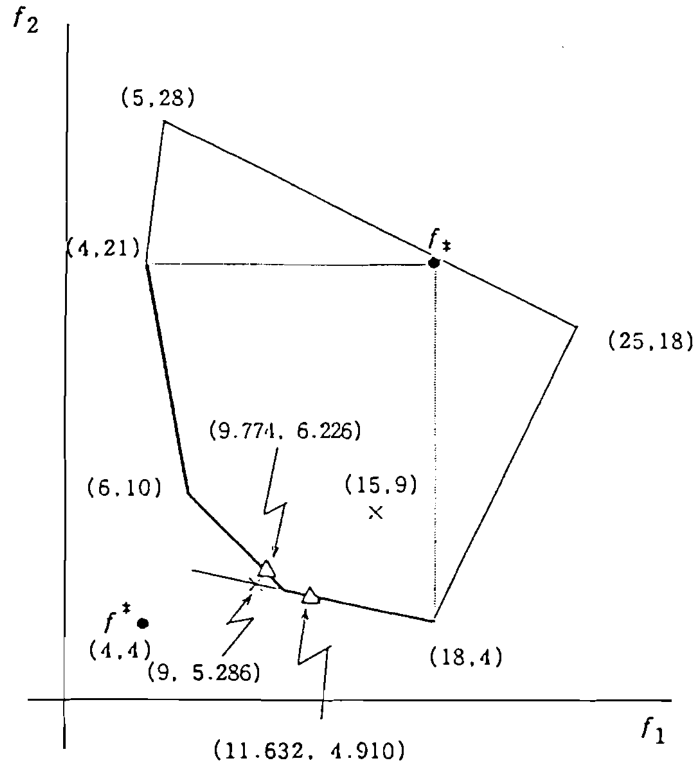

Figure 3: Automatic trade-off (case 2)

3.4 Exact Trade-off

In linear or quadratic cases, we can evaluate the exact trade-off in an extended form of the automatic trade-off stated above. This implies that we can calculate exactly the amount of relaxation such that the new aspiration level is on the Pareto surface (Nakayama, 1992). The main idea in it is that the parametric optimization technique is used in stead of the simple sensitivity analysis. Using this technique, a new Pareto solution can be obtained without solving the auxiliary sclarized optimization problem again. This implies that we can obtain the new solution very quickly. Therefore, using some graphic presentation as computer outputs, DM can see the trade-off among criteria in a dynamic way, e.g. as an animation. This makes DM's judgement easier.

3.5 Interchange between Objectives and Constraints

In the formulation of the auxiliary scalarized optimization prolem (Q), change the right hand side of the equation (3.7) into $\beta_i z$, namely

$$w_i(f_i(x) - \overline{f}_i) \leq \beta_i z. \tag{3.9}$$

As is readily seen, if $\beta_i = 1$, then the function f_i is considered to be an objective function, for which the aspiration level $\overline{f}_i$ is not necessarily attained, but the level of f_i should be better as much as possible. On the other hand, if $\beta_i = 0$, then f_i is considered to be a constraint function, for which the aspiration level $\overline{f}_i$ should be guaranteed. In many practical problems, there is almost no cases in which we consider the role of objective and constraint fixed from the beginning, but usually we want to interchange them depending on the situation. Using the formula (3.9), this can be done very easily (Korhonen 1987). In addition, if the value of β_i is set in the middle of 0 and 1, f_i can play a role in the middle of objective and constraint which is neither a complete objective nor a complete constraint (Kamenoi *et al.* 1992). This is also very effective in many practical problems.

3.6 Remarks on Trade-off for Objective Functions with 0-Sensitivity

Since α is sufficiently small like 10^{-6} and $\lambda_1 + \ldots \lambda_r = 1$, we can consider in many cases

$$(\lambda_i + \alpha)w_i \simeq \lambda_i w_i.$$

When $\lambda_j = 0$ and f_j is not to be improved, we set $\Delta f_j = 0$ in the automatic trade-off. Therefore, unless we select at least one f_j with $\lambda_j \neq 0$ as an objective function to be relaxed, we can not attain the improvement that DM wishes.

Since $(\lambda_i + \alpha)w_i$ (or approximately, $\lambda_i w_i$) can be regarded to provide the sensitivity information in the trade-off, $\lambda_i = 0$ means that the objective function f_i does not contribute to the trade-off among the objective functions. In other words, since the trade-off is the negative correlation among objective functions, $\lambda_i = 0$ means that f_i has the positive correlation with some other objective functions. Therefore, if all objective functions to be relaxed, $f_j \ (j \in I_R)$, have $\lambda_j = 0 \ (j \in I_R)$, then they can not compensate for the improvement which DM wishes, because they are affected positively by some of objective functions to be improved.

Example 3.4 Consider the following problem:

$$\text{Minimize} \quad (f_1,\ f_2,\ f_3) \ = \ (x_1,\ x_2,\ x_3)$$

subject to

$$\begin{aligned} x_1 + x_2 + x_3 &\geq 1 \\ x_1 \qquad - x_3 &\geq 0 \\ x_1,\ x_2,\ x_3 &\geq 0 \end{aligned}$$

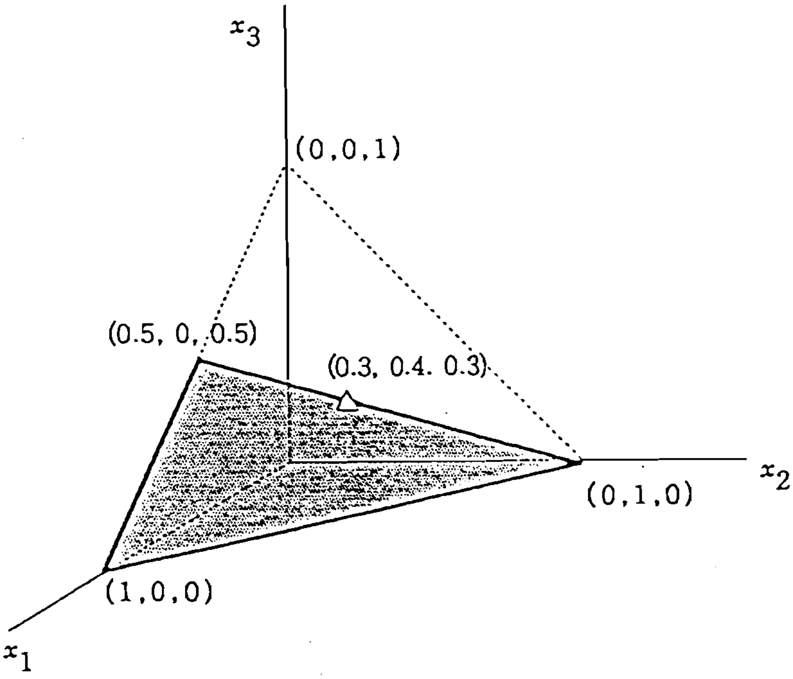

Figure 4: A Case with 0-Sensitivity

The ideal point is (0, 0, 0) and the nadir point is (1.0, 1.0, 0.5). For the first aspiration level (0.2, 0.2, 0.4), we have a Pareto value (0.333, 0.333, 0.333) and the corresponding simplex multiplier $(\lambda_1,\ \lambda_2,\ \lambda_3) = (2/3,\ 1/3,\ 0)$. Suppose that DM wants to improve f_1 and f_2, and sets their new aspiration levels 0.2 and 0.3, respectively. Since the relaxation $\Delta f_3 = 0$ by the automatic trade-off, the new aspiration level becomes (0.2, 0.3, 0.333). Associated with the new aspiration level, we have the Pareto value (0.3, 0.4, 0.3), in which neither f_2 is improved nor f_3 is relaxed. This is because that the objective functions f_1 and f_3 has a positive correlation along the edge of Pareto surface at the point (0.333, 0.333, 0.333), while f_1 and f_2 have trade-off relation with each other there. As a result, though f_3 was considered to be relaxed, it was affected strongly by f_1 and hence improved. On the other hand, despite that f_3 was considered to be improved, it was relaxed finally. This is due to the fact that the objective function to be relaxed is only f_3 despite that λ_3 is 0, and the fact that we did not consider that f_3 has positive correlation with f_1. This example suggests that we should make the trade-off analysis deliberately seeing the value of simplex multiplier (Lagrange multiplier, in nonlinear cases). Like this, the satisficing trade-off method makes the DM's trade-off analysis easier by utilizing the information of sensitivity.

3.7 Relationship to Fuzzy Mathematical Programming

In the aspiration level approach to multi-objective programming such as the satisficing trade-off method, the wish of DM is attained by adjusting his/her aspiration level. In other words, this means that the aspiration level approach can deals with the fuzziness of right hand side value in traditional mathematical programming as well as the total balance among the criteria. There is another method, namely fuzzy mathematical programming, which treat the fuzziness of right hand side value of constraint in traditional mathematical programming. In the following, we shall discuss the relationship between the satisficing trade-off method and the fuzzy mathematical programming.

For simplicity, consider the following problem:

(F) Maximixe $f_0(x)$

subject to $f_1(x) = \overline{f}_1$

Suppose that the right hand side value $\overline{f}_1$ is not needed to meet so strictly, but that it is fuzzy. The membership function for the criterion f_1 is usually given as in Fig. 5. Since our aim is to maximize this membership function, we can adopt the following function without change in the solution:

$$m_1(x) = \min\ \{(\overline{f}_1 - f_1(x))/\epsilon + 1,\quad -(\overline{f}_1 - f_1(x))/\epsilon + 1\},$$

where ϵ is a parameter representing the admissible error for the target $\overline{f}_1$.

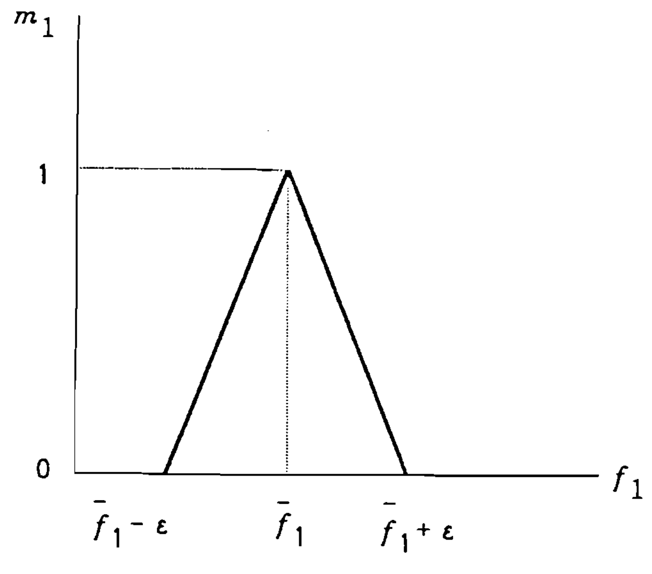

Figure 5: Membership function for f_1 in (F)

Now, the problem (F) is reduced to a kind of multi-objective optimization in which f_0 and m_1 should be maximized. Then a membership function for maximization of f_0 is usually defined with its aspiration level $\overline{f}_0$. For example,

$$m_0'(x) = \min\{-(\overline{f}_0 - f_0(x))/(f_0^* - f_{0*}) + 1,\quad 1\},$$

However, if we maximize the above m_0' as it is, the solution will be merely the one to the satisficing problem for which f_0 is to be just greater than $\overline{f}_0$. As was stated in the previous section, we shall use the following function in stead of m_0' in order to assure the Pareto optimality of the solution:

$$m_0(x) = -(\overline{f}_0 - f_0(x))/(f_0^* - f_{0*}) + 1.$$

Finally, our problem is to maximize both m_0 and m_1, which is usually reduced to the following problem:

Minimize $\quad z$

subject to

$$\begin{aligned}(\overline{f}_0 - f_0(x))/(f_0^* - f_{0*}) - 1 &\leq z\\ (\overline{f}_1 - f_1(x))/\epsilon - 1 &\leq z\\ -(\overline{f}_1 - f_1(x))/\epsilon - 1 &\leq z\end{aligned}$$

Now, one may see some similarity between the above formulation and the one in the satisficing trade-off method. In the satisficing trade-off method, the objective function with target such as $f_1 \to \overline{f}_1$ is usually treated as two objective functions, $f_1 \to Max$ and $f_1 \to Min$. Under this circumstance, suppose that for $f_1 \to Max$ we set the ideal value $f_1^* = \overline{f}_1$, the nadir value $f_{1*} = \overline{f}_1 - \epsilon$ and the aspiration level $\overline{f}_1 - \epsilon$; for $f_1 \to Min$ we set the ideal value $f_1^* = \overline{f}_1$, the nadir value $f_{1*} = \overline{f}_1 + \epsilon$ and the aspiration level $\overline{f}_1 + \epsilon$. Then the treatment of f_1 is the same between the above formulation and the satisficing trade-off method.

However, usually in the satisfcing trade-off method, we set f_1^* and f_{1*} based on the pay-off matrix (i.e., different from $\overline{f}_1$ and $\overline{f}_1 \pm \epsilon$, respectively). Hence, we do not contain ϵ in the denominator of constraints in the min-max problem, because we make the trade-off analysis by adjusting ϵ rather than the target $\overline{f}_1$; for example, using (3.6) the constraint for f_1 in the min-max problem is given by

$$\begin{aligned}(\overline{f}_1 - \epsilon - f_1(x))/(f_1^* - f_{1*}) &\leq z\\ -(\overline{f}_1 + \epsilon - f_1(x))/(f_1^* - f_{1*}) &\leq z\end{aligned}$$

With this formulation, even if DM wants $\epsilon = 0$ and if there is no solution to $f_1(x) = \overline{f}_1$, we can get a solution approximate to $f_1(x) = \overline{f}_1$ as much as possible. In the fuzzy mathematical programming, however, if $\epsilon = 0$, then we have a crisp constraint $f_1(x) = \overline{f}_1$, and we sometimes have no feasible solution to it.

Finally as a result, we can see that the satisficing trade-off method deals with the fuzziness of right hand side value of constraint automatically and can effectively treat problems for which fuzzy mathematical programming provides no solution. Due to this reason, we can conclude that it is better to formulate the given problem as a multi-objective optimization from the beginning and to solve it by the aspiration level approach such as the satisficing trade-off method.

4 Applications

Interactive multi-objective programming methods have been applied to a wide range of practical problems. Good examples in engineering applications can be seen in Eshenauer *et al.* (1990). The author himself also has applied to several real problems:

1. blending
    - (a) feed formulation for live stock (Nakayama *et al.* 1992)
    - (b) plastic materials (Nakayama *et al.* 1986)
    - (c) cement production (Nakayama 1991-a)

   (d) portfolio (Nakayama 1989)

2. design

   (a) camera lens

   (b) erection management of cable-stayed bridge (Furukawa *et al.* 1986)

3. route search

   (a) car navigation system (Nakayama *et al.* submitted)

4. planning

   (a) scheduling of string selection in steel manufacturing (Ueno *et al.* 1990)

   (b) long term planning of atomic power plants

In the following, some of examples are introduced briefly.

4.1 Feed Formulation for Live Stock

Stock farms in Japan are modernized recently. Above all, the feeding system in some farms is fully controled by computer: Each cow has its own place to eat which has a locked gate. And each cow has a key on her neck, which can open the corresponding gate only. Every day, on the basis of ingredient analysis of milk and/or of the growth situation of cow, the appropriate blending ratio of materials from several viewpoints should be made.

There are about 20-30 kinds of raw materials for feed in cow farms such as corn, cereals, fish meal, etc.

About 10 criteria are usually taken into account for feed formulation of cow:

- cost
- neutrition
  - protein
  - TDN
  - cellulose
  - calcium
  - magnesium
  - etc.
- stock amount of materials
- etc.

This feeding problem is well known as the diet problem from the beginning of the history of mathematical programming, which can be formulated as the traditional linear programming. In the traditional mathematical programming, the solution often occurs on the boundary of constraints. In many cases, however, the right hand side value of constraint such as neutrition needs not to be satisfied rigidly. Rather, it seems to be natural to consider that a criterion such as neutrition is an objective function whose target has some allowable range. As was seen in the previous section, the satisficing trade-off method deals well with the fuzziness of target of such an objective function. The author and others have developed a software for feed formulation using the satisficing trade-off method, called F-STOM (Feed formulation by Satisficing Trade-Off Method) (Nakayama *et al.* 1993). This software is being distributed to live stock farmers and feed companies in Japan through an association of live stock systems.

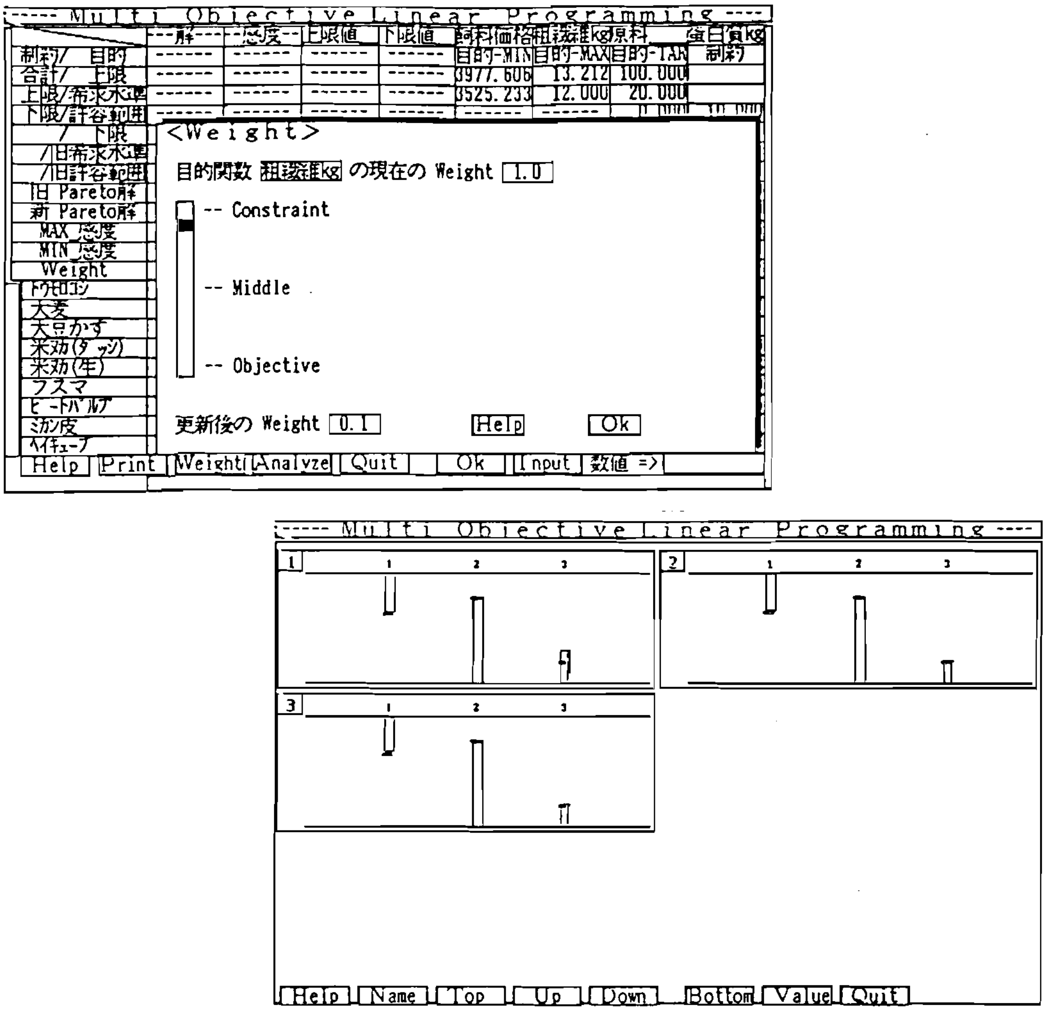

Figure 6: A Phase of F-STOM

4.2 Erection Management of Cable Stayed Bridge

In erection of cable stayed bridge, the following criteria are considered for accuracy control (Furukawa *et al.* 1986):

i. residual error in each cable tension,

ii. residual error in camber at each node,

iii. amount of shim adjustment for each cable,

iv. number of cables to be adjusted.

Since the change of cable rigidity is small enough to be neglected with respect to shim adjustment, both the residual error in each cable tension and that in each camber are linear functions of amount of shim adjustment. Let us define n as the number of cable in use, ΔT_i $(i = 1, \ldots, n)$ as the difference between the designed tension values and the measured ones, and x_{ik} as the tension change of i-th cable caused from the change of the k-th cable length by a unit. The residual error in cable tension caused by the shim adjustment is given by

$$p_i = |\Delta T_i - \sum_{k=1}^{n} x_{ik} \Delta l_k| \qquad (i = 1, \ldots, n)$$

Let m be the number of nodes, Δz_j $(j = 1, \ldots, m)$ the difference between the designed camber values and the measured ones, and y_{jk} the camber change at j-th node caused from the change of the k-th cable length by a unit. Then the residual error in the camber caused by the shim adjustments of $\Delta l_1, \ldots, \Delta l_n$ is given by

$$q_j = |\Delta Z_j - \sum_{k=1}^{n} y_{jk} \Delta l_k| \qquad (j = 1, \ldots, m)$$

In addition, the amount of shim adjustment can be treated as objective functions of

$$r_i = |\Delta l_i| \qquad (i = 1, \ldots, n)$$

And the upper and lower bounds of shim adjustment inherent in the structure of the cable anchorage are as follows;

$$\Delta l_{Li} \leq \Delta l_i \leq \Delta l_{Ui} \qquad (i = 1, \ldots, n).$$

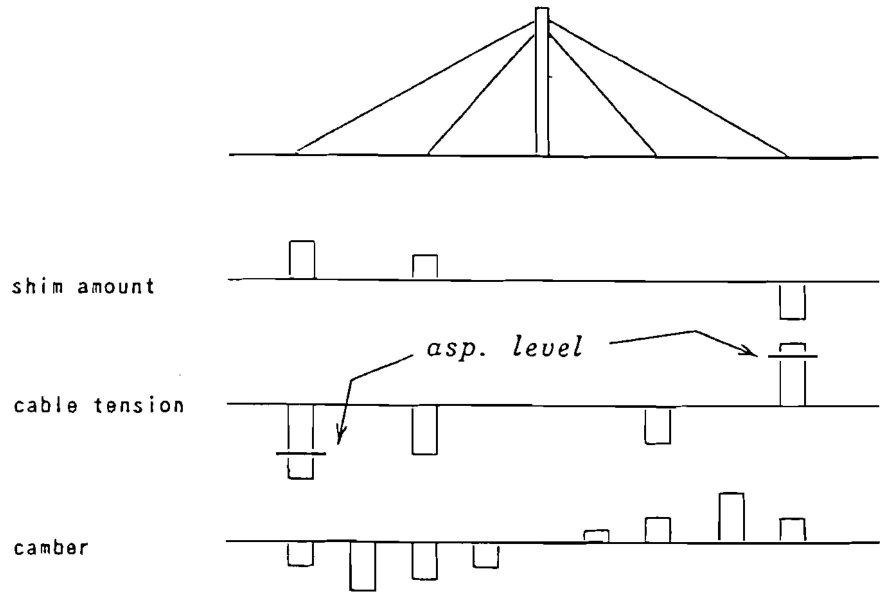

Figure 7: A phase of the proposed erection management system

Fig. 7 shows one phase of erection management system of cable stayed bridge using the satisficing trade-off method. The residual error of each criterion and the amount of shim adjustment are represented by graphs. The aspiration level is inputted by a mouse on the graph. After solving the auxiliary min-max problem, the Pareto solution according to the aspiration level is represented by a graph in a similar fashion. This procedure is continued until the designer can obtain a desirable shim-adjustment. This operation is very easy for the designer, and the visual information on trade-off among criteria is user-friendly. The software was used for real bridge construction, say, Tokiwa Great Bridge (Ube City) and Karasuo Harp Bridge (Kita-Kyusyu City) in 1992.

4.3 An Interactive Support System for Bond Trading

In portfolio selection problems, many companies are now widely trying to use mathematical analysis for bond trading. In this subsection, some bond trading problem is formulated as a kind of multi-objective fractional problem. It will be seen in the following that the satisficing trade-off method can be effectively applied to such a porfolio problem.

Bond traders are facing almost every day a problem which bonds and what amount they should sell and/or buy in order to attain their customers' desires. The economic environment is changing day by day, and sometimes gives us a drastic change. Bond traders have to take into account many factors, and make their decisions very rapidly and flexibly according to these changes. The number of bonds to be considered is usually more than 500, and that of criteria is about ten as will be shown later. The amount of trading is usually more than 100 million yen, and hence even a slight difference of portfolio combination might cause a big difference to profit or loss. This situation requires some effective method which helps bond traders following faithfully their value-judgment on a real time basis not only mathematically but also in such a way that their intuition fostered by their experiences can be taken in.

Bond portfolio problems are a kind of blending problems. Therefore, mathemetical programming approach can be used very effectively. However, the traditional mathematical programming approach with a single objective function can not take in the value-judgment and intuition of bond traders so easily in a flexible manner for the changes of environment. We shall show that the satisficing trade-off method fits to this purpose.

4.3.1 Mathematical Formulation for Bond Portfolio Problems

We shall follow the mathematical model given by Konno and Innori (1987). Assume that an investor holds u_j units of bonds B_j, $j = 1, \dots, N$. Associated with B_j, we have the following indices:

c_j: coupon to be paid at a fixed rate (yen/bond/year)

f_j: principal value to be refunded at maturity (yen/bond)

p_j: present price in the market (yen/bond)

t_j: maturity (number of years until its principal value is refunded)

Returns from bonds are the income from coupon and the capital gain due to price increase. Bond portfolio problems are to determine which bonds and what amount the investor should sell and/or buy taking into account many factors, say, expected returns and risk, the time needs money for another investment, and so on. Therefore, we have the following criteria for each bond B_j:

[I] returns

(i) direct yield (short term index of return)

$$\gamma_j = \frac{c_j}{p_j}$$

(ii) effective yield

$$\nu_j = \frac{[c_j\{(1+\alpha)^{t_j}-1\}/\alpha + f_j]^{1/t_j}}{p_j} - 1$$

where α is the interest rate.

[II] risk

(iii) price variation

$$\pi_j = \frac{t_j}{1+\mu_j t_j}, \quad j = 1, \dots, N$$

Let x_j $(j = 1, \dots, n_1)$ and X_k $(k = 1, \dots, n_2)$ denote the number of bonds to be sold and to be purchased, respectively. Then S_0 and S_1 represent respectively the total quantity of bonds and the total value of bonds after the transaction. Namely,

$$\begin{aligned} S_0 &= \sum_{j=1}^{N} u_j - \sum_{j=1}^{n_1} x_j + \sum_{k=1}^{n_2} X_k \\ S_1 &= \sum_{j=1}^{N} p_j u_j - \sum_{j=1}^{n_1} p_j x_j + \sum_{k=1}^{n_2} P_k X_k. \end{aligned}$$

In addition, we set

$$S_2 = \sum_{j=1}^{N} p_j t_j u_j - \sum_{j=1}^{n_1} p_j t_j x_j + \sum_{k=1}^{n_2} P_k T_k X_k.$$

Then the average for each index is taken as an objective function:

(i′) average direct yield

$$F_1 = \left(\sum_{j=1}^{N} \gamma_j p_j u_j - \sum_{j=1}^{n_1} \gamma_j p_j x_j + \sum_{k=1}^{n_2} \gamma_k P_k X_k \right) / S_1$$

(ii′) average effective yield

$$F_2 = \left(\sum_{j=1}^{N} \nu_j p_j t_j u_j - \sum_{j=1}^{n_1} \nu_j p_j t_j x_j + \sum_{k=1}^{n_2} \nu_k P_k T_k X_k \right) / S_2$$

(iii′) average price variation

$$F_3 = \left(\sum_{j=1}^{N} \pi_j u_j - \sum_{j=1}^{n_1} \pi_j x_j + \sum_{k=1}^{n_2} \pi_k X_k \right) / S_0$$

Our constraints are divided into soft constraints and hard constraints:

(iv) average unit price

$$F_4 = \left(\sum_{j=1}^{N} p_j u_j - \sum_{j=1}^{n_1} p_j x_j + \sum_{k=1}^{n_2} P_k X_k \right) / S_0$$

(v) average maturity

$$F_5 = \left(\sum_{j=1}^{N} t_j u_j - \sum_{j=1}^{n_1} t_j x_j + \sum_{k=1}^{n_2} T_k X_k\right) / S_0$$

(vi) specification of time of coupon to be paid

$$F_6 = \sum_{i \in I_1} x_i / S_0$$
$$F_7 = \sum_{i \in I_2} x_i / S_0$$

where I_m is the set of indices of bonds whose coupon are paid at the time t_m.

[III] hard constraints

(vii) budget constraints:

$$\sum_{j=1}^{n_1} -p_j x_j + \sum_{k=1}^{n_2} P_k X_k \leq C$$
$$\sum_{k=1}^{n_2} P_k X_k \geq C$$

(viii) specification of bond

$$l_j \leq x_j \leq u_j, \quad j = 1, ..., n_1$$
$$L_k \leq X_k \leq U_k, \quad k = 1, ..., n_2$$

For this kind of problem, the satisficing trade-off method can be effectively applied. Then we have to solve a linear fractional min-max problem. In the following subsection, we shall give a brief review of the method for solving it.

4.3.2 An Algorithm for Linear Fractional Min-Max Problems

Let each objective function in our bond trading problem be of the form:

$$F_i(x) = p_i(x)/q_i(x) \quad (i = 1, \ldots, r)$$

where p_i and q_i are linear in x. Then since

$$F_i^* - F_i(x) = \frac{F_i^* q_i(x) - p_i(x)}{q_i(x)} := \frac{f_i(x)}{g_i(x)}$$

the auxiliary min-max problem (Q) becomes a kind of linear fractional Min-Max problem. For this kind of problem, several methods have been developed: Here we shall use a Dinkelbach type algorithm (Borde-Crouzeix 1987, Ferland-Potvin 1985) as is stated in the following:

Step 1: Let $x^0 \in X$. Set $\theta^0 = \max_{1 \leq i \leq r} f_i(x^0)/g_i(x^0)$ and $k = 0$.

Step 2: Solve the problem

$$(P_k) \qquad T_k(\theta^k) = \min_{x \in X} \max_{1 \leq i \leq r} (f_i(x) - \theta^k g_i(x))/g_i(x^k)$$

Let x^{k+1} be a solution to (P_k).
Step 3: If $T_k(\theta^k) = 0$ then stop: θ^k is the optimal value of the given min-max Problem, and x^{k+1} is the optimal solution.
Step 4: If not, take $\theta^{k+1} = \max_{1 \le i \le r} f_i(x^{k+1})/g_i(x^{k+1})$. Replace k by $k+1$ and go to Step 2.

Note that the problem (P_k) is the usual linear min-max problem. Therrefore, we can obtain its solution by solving the following equivalent problem in a usual manner:

(Q_k) Minimize z
subject to

$$\frac{f_i(x) - \theta^k g_i(x)}{g_i(x^k)} \le z, \quad i = 1, ..., r$$

4.3.3 An Experimental Result

A result of our experiments is shown below: The problem is to decide a bond portfolio among 37 bonds selected from the market. The holding bonds are $x(1) = 5000$, $x(9) = 1000$, $x(13) = 2500$, $x(17) = 4500$, $x(19) = 5500$, $x(21) = 6000$, $x(23) = 5200$, $x(25) = 4200$, $x(27) = 3200$ and $x(37) = 3800$. The experiment was performed by a worker of a security company in Japan who has a carreer of acting as a bond trader.

		Pareto sol.	Asp.Level (Target Range)	Lowest	Highest	Sensitivity
	F1 (max)	5.8335	5.8000	5.4788	5.9655	0.0133
	F2 (max)	6.8482	6.8000	6.7165	6.8698	0.0000
	F3 (min)	0.1292	0.1300	0.1262	0.1325	1.0000
*	F4	102.7157	$F4 \le 103.00$			
*	F5	4.0000	$4.00 \le F5 \le 5.00$			
*	F6	0.2000	$0.20 \le F6$			
*	F7	0.2000	$0.20 \le F7$			

x(1)=	1042.0643	x(2)=	400.0000
x(3)=	400.0000	x(4)=	200.0000
x(5)=	0.0000	x(6)=	400.0000
x(7)=	0.0000	x(8)=	0.0000
x(9)=	547.3548	x(10)=	0.0000
x(11)=	0.0000	x(12)=	0.0000
x(13)=	2500.0000	x(14)=	200.0000
x(15)=	0.0000	x(16)=	200.0000
x(17)=	4500.0000	x(18)=	0.0000
x(19)=	5321.9573	x(20)=	0.0000
x(21)=	6000.0000	x(22)=	0.0000
x(23)=	5200.0000	x(24)=	0.0000
x(25)=	4200.0000	x(26)=	0.0000
x(27)=	3200.0000	x(28)=	274.6025
x(29)=	400.0000	x(30)=	200.0000
x(31)=	0.0000	x(32)=	200.0000
x(33)=	400.0000	x(34)=	400.0000
x(35)=	0.0000	x(36)=	0.0000
x(37)=	3800.0000		

The asterisk of F4 to F7 implies soft constraints. In our system, we can change objective function into soft constraints and vice versa. Here, the bond trader changed F2 (effective yield) into a soft constraint, and F4 (unit price) into an objective function. Then under the modified aspiration level by trade-off, the obtained result is as follows:

		Pareto sol.	Asp.Level (Target Range)	Lowest	Highest	Sensitivity
	F1 (max)	5.8629	5.9000	5.8193	5.9608	0.0000
*	F2 (max)	6.8482	$6.8482 \leq$ F2			
	F3 (min)	0.1302	0.1292	0.1291	0.1322	1.0000
	F4 (min)	102.3555	102.1000	102.0676	102.7228	0.0043
*	F5	4.0000	$4.00 \leq$ F5 $\leq$ 5.00			
*	F6	0.2000	$0.20 \leq$ F6			
*	F7	0.2000	$0.20 \leq$ F7			

x(1)=	0.0000	x(2)=	400.0000
x(3)=	400.0000	x(4)=	200.0000
x(5)=	0.0000	x(6)=	400.0000
x(7)=	0.0000	x(8)=	0.0000
x(9)=	139.4913	x(10)=	0.0000
x(11)=	0.0000	x(12)=	0.0000
x(13)=	2500.0000	x(14)=	381.5260
x(15)=	0.0000	x(16)=	200.0000
x(17)=	4500.0000	x(18)=	0.0000
x(19)=	5277.1577	x(20)=	0.0000
x(21)=	6000.0000	x(22)=	0.0000
x(23)=	5200.0000	x(24)=	0.0000
x(25)=	4200.0000	x(26)=	0.0000
x(27)=	3200.0000	x(28)=	14.8920
x(29)=	400.0000	x(30)=	200.0000
x(31)=	0.0000	x(32)=	200.0000
x(33)=	400.0000	x(34)=	400.0000
x(35)=	400.0000	x(36)=	222.7743
x(37)=	3800.0000		

The result was satisfactory to the testee. He recognized that this method (software) is very easy to use and flexible for variation of the desire of investers. Transaction cost should be taken into account in the future.

5 Concluding Remarks

Recently, much attention has been being paid to the intelligent information process by computers. Such intelligent information process as diagnosis with less value-judgment will be treated in an effective way by artificial intelligence or knowledge engineering. However, among intelligent information process, the value-judgment will remain hard to treat to the last. In many practical problems such as portfolio problems, it is very important to get a solution reflecting faithfully the value-judgment of customers. In order to make decisions in a flexible manner for the multiplicity of value-judgment and complex changes of environment of decision making, the cooperative system of man and computers are very attractive: above all, interactive multi-objective programming methods seem promising.

Among several interactive multi-objective programming techniques, the aspiration level approach has been applied to several kinds of real problems, because

i. it does not require any consistency of judgment of DM,

ii. it reflects the value of DM very well,

iii. it is easy to implement.

In particular, the point (i) is very important, because DM tends to change his attitude even during the decision making process. This implies that the aspiration level approach such as the satisficing trade-off method can work well not only with the multiplicity of value judgment of DMs but also the dynamics of value judgment of them.

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