Change in Economic Mechanism: Model of Evolutionary Transition from Budgets Regulation to Competitive Market

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Summary. In the framework of dynamic equilibrium theory we propose a model of evolutionary transition from Economy with centralized budgets regulation to Market Economy (with self-financing). It is assumed that information about possible change of economic mechanism affects essentially on behavior of agents. Duration of transition period is regarded as a random variable. We study conditions when such transition allows firms to adapt their plans to future market and guarantees an existence of equilibrium paths. It is also discussed the case of Shock (instantaneous transition) which may bring to bankruptcy, jump of prices and deficit.

1 Introduction

The paper deals with modelling changes of economic mechanism. We consider an economic system with finite number of goods and agents. Unlike the most of dynamic equilibrium models (see e.g. [1–3]), both producers and consumers in our model act under budget restrictions. It seems that financial constraints may have an essential role for firms in economics with poor financial system. This approach allows to associate different principles of budgets forming with different economic mechanisms and to state a problem of transition from one economic mechanism to another.

We focus on the following two economy functioning mechanisms.

The first one assumes the presence of a certain Central Planning Board ("the State") and may be identify with centralized (state-controlled) economy. At every time t the State sets up prices and derives a total budget (income) as a total cost of goods produced at this moment. The total budget is distributed over the agents according some priorities prescribed by the State (Budgets Regulation). Consumers solve maximization utility problem (under the corresponding budgets). Producers choose their plans (i.e. pairs "input–output", where we associate "input" with beginning-of-period t, and "output" – with end-of period $t+1$) such that buying resources by current prices at time t subjected to budget constraints to maximize their incomes by prices of next time $t+1$.

Under the second economic mechanism role of the State is eliminated and his distributive functions are moved to the Market. An income of producer at time t is resulted as

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receipts from the sale of his own production by current prices. This income is divided into two parts. The first one is distributed among consumers accordingly their limited liability shares in firms. The second is directed for reproduction at next period and producer tries to maximize his income to the moment $t + 1$ under this budget constraint. Consumer wishes to maximize his current utility subjected to his budget resulted as total dividends on liability shares in firms' incomes.

Suppose that economic system at moments $t < 0$ is developed under Budget Regulation and at $t = 0$ a policymaker make a decision to change economic mechanism and go to Market. We do not discuss here any motivations of this decision and, in particular, a problem "which mechanism is better". Our aim is to construct a Transition process with some "desirable" properties.

Let present three possible variants of such transition. The first is a Shock (i.e. instantaneous unexpected transition), when all agents being at time t under Budget Regulation find yourself under Market at next moment $t + 1$. Since at time t agents count on getting budgets from the State, then "own" incomes of some firms can become zero (or, near zero) that leads to bankruptcy and decline of production. The second variant is declared instantaneous transition, when it becomes known precisely that at time, say, $t + 1$ system turns out under Market economy. But this case can not guarantee the balance of supply and demand at time t because of an emergence of rush demands and needs in goods which probably did not produce before. So, both variants of instantaneous transition can be accompanied by "undesirable" phenomena. We discuss these problems in example at section 7.

Proceed to gradual transition as a main object of our paper. We can describe this variant in a following way. The State at some moment, say $t = 0$, declares on future change in economic mechanism, but does not fix precisely the moment of this change. All agents consider this moment as a random variable and at time t they have their own beliefs (subjective probabilities) about change in mechanism at time $t+1$. Therefore, firms choosing their plans at time t have to take into account "state" prices as well as "market" prices at next time $t + 1$. Our hypothesis on producer's behavior is a maximization of future income in "weighted" prices with conditional subjective probabilities of change as weights (some arguments in the favour of this hypothesis discussed in section 6). The proposed transition process is free from drawbacks of instantaneous transition mentioned above and guarantees (under appropriate conditions) an existence of equilibrium parts with positive budgets for all agents.

The structure of the paper is as follows. In section 3 and 4 we present two equilibrium dynamic models. Unlike traditional general equilibrium theory possibilities of firms in this paper are restricted not only by technologies but budget constraints as well as consumers. Mechanism of firms' budgets forming is different in these two models. Then in section 5 we present a model of gradual transition from one economic mechanism to another and prove an existence of equilibrium transition process. Optimal properties of equilibria in proposed models are discussed in section 6. At last, in section 7 we demonstrate some phenomena which can emerge under various types of transition (including Shock).

2 Description of the System

We consider an economic system with l goods and finite set of agents. Each agent lives for an finite or infinite number of periods $t = 0, 1, \dots, T$ $\quad (T \leq \infty$ is a planning horizon). We share all agents into N producers and M consumers (note that an agent may be viewed as producer and consumer simultaneously).

Productive possibilities of ***producer i*** at period $(t, t+1)$ (in sequel we shall say, briefly, period t) is represented as a set of all feasible "input-output" pairs $(x, y) \in Q_t^i \subseteq R_+^l \times R_+^l$ (technology). Assume as usual that Q_t^i are convex closed sets with $(0,0) \in Q_t^i$. We, also, suppose that these sets are unbounded in general (at all coordinates), but local bounded, i.e. sets $\{y : (x, y) \in Q_t^i, \quad x \in A\}$ are bounded for any bounded A.

Preferences of *consumer j* at period t is induced by utility function $u_t^j(c)$ which is defined on commodity set $C_t^j \subseteq R_+^l$. Assume that u_t^j are nonnegative quasi-concave continuous functions, and C_t^j are convex closed sets with $0 \in C_t^j$.

Denote by I and J set of producers and consumers, (resp. $\#I = N, \#J = M$). Moreover, for vectors $x^k = (x_1^k, \ldots, x_l^k) \quad (k = 1, 2)$ inequality $x^1 \geq x^2$ means $x_i^1 \geq x_i^2$ for all i, $\quad x^1 > x^2$ means $x^1 \geq x^2$ and $x^1 \neq x^2$, and $x^1 \gg x^2$ means $x_i^1 > x_i^2$ for all i.

3 Budget Regulation Economy Model

Now we describe behavior of agents in economic system with centralized distribution of budgets. It means that at the beginning of each period the State sets up agents' budgets according to the activity of all agents at previous period. Then producers and consumers operates for unit period in framework of their budget constraints. We shall refer to this model as Budget Regulation, or Centralized, Economy Model (BRE model).

Let p_t be a nonnegative price vector in the system at the beginning of period t. In this moment producers and consumers are provided with budgets ρ_t^i and π_t^j respectively $(i \in I, j \in J)$. The agent's problem is to maximize the end-of-period income (for producer) or current utility of consumption (for consumer) subject to their budget constraints, i.e.

$$p_{t+1}y \to \max \tag{1}$$
$$(x,y) \in Q_t^i, \quad p_t x \le \rho_t^i \qquad (i \in I),$$

$$u_t^j(c) \to \max \tag{2}$$
$$c \in C_t^j, \qquad p_t c \le \pi_t^j \qquad (j \in J).$$

(We assume that for finite T p_{T+1} are given final prices).

Distribution Budget Rule in this model at time t is the following: $\rho_t^i = \alpha_t^i K_t$, $\pi_t^j = \beta_t^j K_t$ where coefficients α_t^i, β_t^j are associated with agents' priorities (in view of the State) and K_t is total income in the system at time t.

An Equilibrium in Centralized Economy Model with given initial states $(y_0^i; i \in I)$ is a bundle $\left((\hat{x}_t^i, \hat{y}_{t+1}^i), \hat{c}_t^j, \hat{p}_t, \quad 0 \le t \le T;\ i \in I, j \in J\right)$ where $(\hat{x}_t^i, \hat{y}_{t+1}^i) \in Q_t^i$, $\hat{c}_t^j \in C_t^j$, $\hat{p}_t \in R_+^l \setminus \{0\}$, satisfying for any $0 \le t \le T, i \in I, j \in J$ the following conditions:

$$\hat{p}_{t+1}\hat{y}_{t+1}^i \ge \hat{p}_{t+1}y \quad \text{for any } (x,y) \in Q_t^i:\ \hat{p}_t x \le \hat{\rho}_t^i, \tag{3}$$
$$(\hat{p}_{T+1} = p_{T+1} \quad \text{for finite } T)$$
$$u_t^j(\hat{c}_t^j) \ge u_t^j(c) \quad \text{for any } c \in C_t^j : \hat{p}_t c \le \hat{\pi}_t^j; \tag{4}$$

$$\hat{\rho}_t^i = \alpha_t^i \hat{K}_t, \quad \hat{\pi}_t^j = \beta_t^j \hat{K}_t, \quad \hat{K}_t = \sum_i \hat{p}_t \hat{y}_t^i, \quad \hat{y}_0^i = y_0^i; \tag{5}$$

$$\sum_j \hat{c}_t^j + \sum_i \hat{x}_t^i = \sum_i \hat{y}_t^i. \tag{6}$$

Relation (3) ((4)) means that $(\hat{x}_t^i, \hat{y}_{t+1}^i)$ (resp. $\hat{c}_t^j$) is solution of producer (consumer) problem under price system $(\hat{p}_t)$ and budgets $\hat{\rho}_t^i$ and $\hat{\pi}_t^j$. (5) shows that agents' budgets are formed as a distribution from total cost of goods produced at the end of previous period. Finally, (6) is usual resource balance condition.

To prove an existence of such equilibrium we need some additional (but nonrestrictive enough) assumptions holding for any $0 \le t \le T$:

(P1) there exist technological processes $(\check{x}_t^i, \check{y}_{t+1}^i) \in Q_t^i$ such that $\sum_i \check{y}_{t+1}^i \gg 0$ *(at any time any good can be produced)*;

(P2) for any $i \in I$ and $(x,y) \in Q_t^i$ there is process $(x',y') \in Q_t^i$ such that $y' > y$ *(non-satiation of producers)*;

(C1) for any $1 \le s \le l$ there is $j \in J$ such that function $u_t^j(c_1, \dots, c_l)$ is strictly monotone at c_s *(non-satiation of consumption for any good)*;

(C2) for any $j \in J$ and $c \in C_t^j$ there is $c' \in C_t^j$ such that $u_t^j(c') > u_t^j(c)$ *(non-satiation of consumers in general)*;

(D) α_t^i, β_t^j are positive and $\sum_i \alpha_t^i + \sum_j \beta_t^j = 1$.

Theorem 1. *Let above assumptions (P1),(P2),(C1),(C2), and (D) hold. Then for every initial states $(y_0^i; i \in I)$ such that $\sum_i y_0^i \gg 0$ there exist an equilibrium in BRE model with strictly positive prices $(\hat{p}_t \gg 0)$.*

In order to prove this and similar results in further sections we shall use the following generalization of known Gale's Lemma.

Let $\sigma = \{x = (x_1, \ldots, x_l) \in R^l_+ : x_1 + \ldots + x_l = 1\}$ be a standard simplex in R^l_+, $\Delta = rint\ \sigma = \{x \in \sigma : x \gg 0\}$, $P_m = \Delta^m$ – Cartesian product of m open simplexes σ.

Theorem A. *Let $F : P_m \to 2^{R^{lm}}$ be upper semi-continuous, convex-valued mapping, which maps compacts in compacts and satisfy the following conditions:*

i) for any $p^n_t \in \Delta$, $p^n_t \to p_t$ $(n \to \infty)$ $(1 \le t \le m)$, where $p_r \in \sigma \setminus \Delta$ for some r, and $x^n = (x^n_1, \ldots, x^n_m) \in F(p^n_1, \ldots, p^n_m)$ there exists $1 \le t \le m$ and $1 \le s \le l$ such that $p_{t,s} = 0$ and $\limsup_{n\to\infty} x^n_{t,s} > 0$;

ii) $p_t x_t = 0$ for any $p = (p_1, \ldots, p_m) \in P_m$, $(x_1, \ldots, x_m) \in F(p)$, $1 \le t \le m$.

Then $0 \in F(P_m)$, i.e. $0 \in F(\hat{p})$ for some $\hat{p} \in P_m$.

(We took this formulation from [4], close result can be found in [5].)

Proof of Theorem 1. At first note that without loss of generality we may consider bounded sets Q^i_t and C^j_t. Indeed, let $(\hat{p}_t, (\hat{x}^i_t, \hat{y}^i_{t+1}), \hat{c}^j_t;\ \ i \in I, j \in J, 0 \le t \le T)$ be an equilibrium with initial states y^i_0. Then using (6) we have

$$\hat{x}^i_0 \le \sum_i y^i_0, \qquad \hat{c}^j_0 \le \sum_i y^i_0 \qquad (i \in I, j \in J),$$

$$\hat{y}^i_1 \le b_1, \qquad \hat{x}^i_1 \le N b_1, \qquad \hat{c}^j_1 \le N b_1,$$

$$\cdots\cdots\cdots\cdots$$

$$\hat{y}^i_t \le b_t, \qquad \hat{x}^i_t \le N b_t, \qquad \hat{c}^j_t \le N b_t,$$

for some strictly positive vectors b_t, which exist by local boundness of technologies Q^i_t. Hence, we may consider an equilibrium only at bounded sets $\hat{Q}^i_t = \{(x, y) \in Q^i_t : x \le \bar{x},\ y \le \bar{y}\}$, and $\hat{C}^j_t = \{c \in C^j_t : c \le \bar{c}\}$, where $\bar{x}, \bar{y}$ are sufficiently large (note, that its do not depend on planning horizon T), and we can choose $\bar{c} \gg N\bar{y}$ (N stands for the total number of producers).

Next proceed to specify excess demand correspondence (e.d.c.). Problems 1 and 2 imply that we may consider only prices $p_t \in \sigma$.

Case 1 ($T < \infty$). Put $P_t = \Delta^{T+1}$ be Cartesian product of $T+1$ simplexes Δ, $p = (p_0, \ldots, p_T) \in P$, $p^t = (p_0, \ldots, p_t)$ and define e.d.c. step by step.

Let $K_0 = \sum_i p_0 y^i_0$, $\rho^i_0(p^0) = \alpha^i_0 K_0$, $\pi^j_0(p^0) = \beta^j_0 K_0$ and define $\psi^i_0(p^1)$ as a set of solutions $(\tilde{x}^i_0, \tilde{y}^i_1)$ for producer problem (1) at time $t = 0$ under prices p^1. Similarly, φ^j_0 will be a set of optimal consumptions for consumer problem (2) at time $t = 0$ under prices p^0.

Next put $K_1 = \sum_i p_1 \tilde{y}^i_1$, where $(\tilde{x}^i_0, \tilde{y}^i_1) \in \psi^i_0(p^1)$, $\rho^i_1(p^1) = \alpha^i_1 K_1$, $\pi^j_1(p^1) = \beta^j_1 K_1$. Define $\psi^i_1(p^2)$ and $\varphi^j_1(p^1)$ like the first step. Further we can specify sequentially total income $K_t = \sum_i p_t \tilde{y}^i_t$, where $(\tilde{x}^i_{t-1}, \tilde{y}^i_t) \in \psi^i_{t-1}(p^t)$, budgets $\rho^i_t(p^t)$ and $\pi^j_t(p^t)$ and so on $(t \le T)$.

Now we can define e.d.c. as mapping $\chi(p) = (\chi_0(p), \ldots, \chi_T(p))$ where

$$\chi_t = \sum_j \tilde{c}^j_t + \sum_i \tilde{x}^i_t - \sum_i \tilde{y}^i_t, \qquad \tilde{c}^j_t \in \varphi^j_t(p^t),\ (\tilde{x}^i_t, \tilde{y}^i_{t+1}) \in \psi^i_t(p^{t+1}).$$

Show that $\chi(p)$ satisfies all conditions of Theorem A. Convexity of values of χ follows immediately from convexity of sets Q^i_t and C^j_t. In order to prove that χ is upper semi-continuous (u.s.c.) mapping we shall use the following:

Lemma. *Let X and Y be finite-dimensional sets, Y be closed and convex, F and g be continuous functions, defined on $X \times Y$, and $g(x,y)$ is convex on y. Then, if for any $x \in X$ set $D(x) = \{y \in Y : g(x,y) \leq 0\}$ is bounded and $g(x,y') < 0$ for some $y' \in Y$, then $\Phi(x) = \max_{y \in D(x)} F(x,y)$ is continuous function and $G(x) = \{y^* \in D(x) : F(x,y^*) = \Phi(x)\}$ is u.s.c. mapping.*

To prove this lemma with known "Theorem of the Maximum" (see [6]) we need only in continuity of mapping $D(x)$. Continuity of g immediately implies that $D(x)$ is u.s.c. mapping. Let $x_n \to x$, $y \in D(x)$, i.e. $g(x,y) \leq 0$. Put $y_n = \theta_n y + (1-\theta_n)y'$, where $g(x,y') = -\epsilon < 0$, $0 < \theta_n \leq 1$. Then $y_n \in Y$ and $g(x_n,y_n) \leq \theta_n g(x_n,y) + (1-\theta_n)g(x_n,y') \leq \theta_n\delta_{1n} + (1-\theta_n)(-\epsilon+\delta_{2n})$, $\delta_{1n}, \delta_{2n} \to 0$. If we take $\theta_n = (\epsilon-\delta_{1n})/(\epsilon+\delta_{1n}-\delta_{2n})$ then $y_n \to y$ and $g(x_n,y_n) \leq 0$, i.e. $y_n \in D(x_n)$. Therefore, $D(x)$ is lower semi-continuous, and lemma is proved.

Return to proof of Theorem 1. Using positivity of budgets $\rho_0^j(p^0)$, $\pi_0^j(p^0)$ and the fact that sets Q_t^i, C_t^j contains 0, Lemma implies that mappings $\psi_0^i(p^0)$, $\varphi_0^j(p^0)$ are u.s.c., and functions $F_1^i(p^1) = p_1\tilde{y}_1^i$, where $(\tilde{x}_0^i, \tilde{y}_1^i) \in \psi_0^i(p^1)$ are continuous. Proceeding this process, one can prove that mappings $\varphi_t^j(p^t), \psi_t^i(p^{t+1})$, $0 \leq t \leq T$ are u.s.c. Hence $\chi(p)$ is u.s.c also. Then, image of compact is closed (by upper semi-continuity) and bounded (by above note to consider bounded sets $\hat{Q}_t^i$ and $\hat{C}_t^j$).

To examine condition i) let $(p_0(n), \ldots, p_T(n)) \to (p_0, \ldots, p_T)$, where $p_t \in \sigma \setminus \Delta$ for some t. Put $r = \min\{t : p_t \in \sigma \setminus \Delta\}$ and distinguish two cases.

1) $r = 0$, i.e. $p_{0,s} = 0$ for some s and $\eta_0(n) = \sum_j \tilde{c}_0^j + \sum_i \tilde{x}_0^i - \sum_i y_0^i$, where $\tilde{c}_0^j \in \varphi_0^j(p^0(n)), (\tilde{x}_0^i, \tilde{y}_1^i) \in \psi_0^i(p^1(n))$. By (C1) there exists $j \in J$ such that $u_0^j(c_1, \ldots, c_l)$ is strictly monotone at c_s. Then

$$u_0^j(\tilde{c}_0^j) = \max\left\{u_0^j(c) : c \in \hat{C}_0^j,\ p_0(n)c = \sum_{k \neq s} p_{0,k}(n)c_k + p_{0,s}(n)c_s \leq \pi_0^j\right\}.$$

Since $\sum_i y_0^i \gg 0$ and $p_0(n) \to p_0 > 0$ then $\lim_n \pi_0^j(n) > 0$ and by strict monotonicity of u_0^j it is easy to see that $\tilde{c}_{0,s}^j \to \bar{c}_s$. Therefore, $\limsup_n \eta_{0,s}(n) \geq \bar{c}_s - \sum_i y_0^i > 0$.

2) Let $r \geq 1$ and $p_{r,s} = 0$, $\eta_t(n) = \sum_j \tilde{c}_t^j + \sum_i \tilde{x}_t^i - \sum_i \tilde{y}_t^i$, where $\tilde{c}_t^j \in \varphi_t^j(p^t(n)), (\tilde{x}_t^i, \tilde{y}_{t+1}^i) \in \psi_t^i(p^{t+1}(n))$. Put $j \in J$ such that $u_r^j(c)$ is strictly monotone at c_s and processes $(\breve{x}_t^i, \breve{y}_{t+1}^i)$ $(t = 0, \ldots, r-1)$ from assumption (P1). Since $p_t \gg 0$ for $t \leq r-1$, then for some $0 < \theta_t < 1$ and sufficiently large n we have $\theta_t p_t(n)\breve{x}_t^i \leq \alpha_t^i \sum_i p_t(n)\breve{y}_t^i$ $(i \in I, 0 \leq t \leq r-1, \breve{y}_0^i = y_0^i)$. Hence $\theta_0 p_0(n)\breve{x}_0^i \leq \rho_0^i(n)$ and $p_1(n)\tilde{y}_1^i \geq \theta_0 p_1(n)\breve{y}_1^i$, i.e. $\theta_0\theta_1 p_1(n)\breve{x}_1^i \leq \rho_1^i(n)$. Continuing this process one can obtain $\theta_0 \cdots \theta_{r-1} p_{r-1}(n)\breve{x}_{r-1}^i \leq \rho_{r-1}^i(n)$. This inequality implies $\liminf_n \pi_r^j(n) > 0$ and as in case 1) $\limsup_n \eta_{r,s}(n) \geq \bar{c}_s - \bar{y}_s > 0$. So , condition i) is valid.

Further, note that if $\tilde{c}_t^j \in \varphi_t^j(p^t)$, $(\tilde{x}_t^i, \tilde{y}_{t+1}^i) \in \psi_t^i(p^{t+1})$, then non-satiation conditions (P2) and (C2) implies $p_t\tilde{x}_t^i = \rho_t^i(p^t), p_t\tilde{c}_t^j = \pi_t^j(p^t)$. Indeed, let $p_t\tilde{x}_t^i < \rho_t^i(p^t)$. By (P2) there exists $(x,y) \in Q_t^i$ such that $y > \tilde{y}_{t+1}^i$. Then, the process $(x_\theta, y_\theta) = (\theta x + (1-\theta)\tilde{x}_t^i, \theta y + (1-\theta)\tilde{y}_{t+1}^i)$ (where $0 < \theta < 1$) belongs to Q_t^i and $p_t\tilde{x}_\theta \leq \rho_t^i(p^t)$ for sufficiently small θ. On the other hand, $p_{t+1}y_\theta > p_{t+1}\tilde{y}_{t+1}^i$ that contradicts to optimality of $\tilde{y}_{t+1}^i$. Similarly, using condition (C2) one can obtain an equality for consumers' budgets.

At last, if $\eta = (\eta_0, \ldots, \eta_T) \in \chi(p)$ then we have $p_t\eta_t = \sum_j p_t\tilde{c}_t^j + \sum_i p_t\tilde{x}_t^i - \sum_i p_t\tilde{y}_t^i = \sum_j \pi_t^j(p^t) + \sum_i \rho_t^i(p^t) - \sum_i p_t\tilde{y}_t^i = 0$ by assumption (D).

Hence, statement of Theorem 1 follows immediately from Theorem A.

Case 2 ($T = \infty$). By previous case, for any $\tilde{T} = 1, 2, \ldots$ there exists an equilibrium $e(\tilde{T}) = (e_t(\tilde{T}),\ 0 \le t \le \tilde{T})$ with planning horizon $\tilde{T}$, where $e_t(\tilde{T}) = ((\hat{x}_t^i(\tilde{T}), \hat{y}_{t+1}^i(\tilde{T})), \hat{c}_t^j(\tilde{T}), \hat{p}_t(\tilde{T});\ i \in I, j \in J)$. As we see above sequence $\{e_t(\tilde{T}),\ \tilde{T} = 1, 2, \ldots\}$ is bounded for any t. Therefore, $e_1(\tilde{T}') \to e_1$ for some sequence $\{\tilde{T}'\}$, $e_2(\tilde{T}'') \to e_2$ for some subsequence $\{\tilde{T}''\} \subset \{\tilde{T}'\}$ and so on, where $e_t = ((\hat{x}_t^i, \hat{y}_{t+1}^i), \hat{c}_t^j, \hat{p}_t;\ i \in I, j \in J)$. It is easy to see that e_t ($t \ge 0$) satisfies relations (3)-(6). To complete proof we have to show that $\hat{p}_t \gg 0$ for any t. If it is not so, i.e. $p_t \in \sigma \setminus \Delta$ for some $t \ge 0$, then using arguments similar to those in case 1 one can obtain that $\eta_r = \sum_j \hat{c}_r^j + \sum_i \hat{x}_r^i - \sum_i \hat{y}_r^i > 0$ for some r. The last inequality contradicts to relation (6).

Now, Theorem 1 proved completely. ◇

4 Competitive Market Economy Model

In this section we shall consider another model of economy with many agents. This model is very like to Centralized Economy model but differs essentially in mechanism of budgets' forming, namely, producer forms his budget on the base of his own income. We shall refer to this model as Competitive Market Economy Model (CME model).

More precisely, behavior of producer and consumer is described by problems (1) and (2) (respectively) with budgets ρ_t^i, π_t^j determined by the following formulas

$$\rho_t^i = (1 - \gamma_t^i) p_t y_t^i, \quad (0 \le t \le T);$$

$$\pi_0^j = p_0 \omega_0^j + \sum_i \alpha_0^{ij} \gamma_0^i p_0 y_0^i;$$

$$\pi_t^j = \sum_i \alpha_t^{ij} \gamma_t^i p_t y_t^i \quad (1 \le t \le T),$$

where ω_0^j is an initial endowment of consumer j, $0 < \gamma_t^i < 1$ represents share of producer i's income at time t directed for consumption, and non-negative $\alpha_t^{ij}, \sum_j \alpha_t^{ij} = 1$, are consumer j's share in producer i's income (for consumption).

We can define *an equilibrium* in CME model like previous model as a bundle $((\hat{x}_t^i, \hat{y}_{t+1}^i), \hat{c}_t^j, \hat{p}_t)$ satisfying relations (3), (4) with budgets derived by above formulas, and, also, resource balance (6) for $t \ge 1$ and, resource balance $\sum_j \hat{c}_0^j + \sum_i \hat{x}_0^i = \sum_i y_0^i + \sum_j \omega_0^j$ at time $t = 0$.

Denote $I_t(j) = \{i \in I :\ \alpha_t^{ij} > 0\}$ - set of producers with positive share of income for consumer j ("own" producers for consumer j). Suppose that for any $j \in J$, $0 \le t \le T$ sets $I_t(j)$ are non-empty, and the following assumptions hold:

(P3) there exist $(\check{x}_{t-1}^i, \check{y}_t^i) \in Q_{t-1}^i$ such that $\sum_{i \in I_t(j)} \check{y}_t^i \gg 0$ *(any good can be produced by "own" producers of any consumer)*;

(E) $\omega_0^j \gg 0$ for all $j \in J$.

The last assumption (E) seems to be very strong and really can be weaken for positive (in some sense) initial states y_0^i. We even can put $\omega_0^j = 0$ for all $j \in J$ whenever $\sum_i y_0^i \gg 0$. But we specially don't require any additional conditions on initial states that allows us in further use this model as a stage in transition from centralization to market.

Theorem 2. *Let assumptions (P2),(P3),(C1),(C2), and (E) hold. Then for every non-zero initial states ($y_0^i; i \in I$) there exist an equilibrium in CME model with strictly positive prices ($\hat{p}_t \gg 0$).*

Proof of Theorem 2 follows the line of the Proof of Theorem 1. After remarks on possible replacement of sets Q^it and C_t^j to bounded $\hat{Q}_t^i$ and $\hat{C}_t^j$, choose $\bar{c}$ such that $\bar{c} \gg N\bar{y} + \sum_j \omega_0^j$, and begin from the case of finite T.

The construction of e.d.c. is almost the same as above. The only distinction appears in checking condition i) of Theorem A. Let $(p_0(n), \dots, p_T(n)) \to (p_0, \dots, p_T)$, where $p_t \in \sigma \setminus \Delta$ for some t, and $r = \min\{t : \; p_t \in \sigma \setminus \Delta\} \geq 1$ (the case $r = 0$ examines with the same arguments as in Theorem 1).

So there exist $1 \leq s \leq l$ and $j \in J$ such that $p_{t,s} = 0$ and $u_r^j(c)$ is strictly monotone with respect to c_s. By assumption *(P3)* $p_r \bar{y}_r^i > 0$ for some $i \in I_r(j)$. Put

$$\eta_0(n) = \sum_j \tilde{c}_0^j + \sum_i \tilde{x}_0^i - \sum_i y_0^i - \sum_j \omega_0^j, \quad \eta_t(n) = \sum_j \tilde{c}_t^j + \sum_i \tilde{x}_t^i - \sum_i \tilde{y}^i t \quad (t \geq 1)$$

where $\tilde{c}_t^j \in \varphi_t^j(p^t(n))$, $(\tilde{x}_t^i, \tilde{y}_{t+1}^i) \in \psi_t^i(p^{t+1}(n))$.

Since $p_{r-1} \gg 0$ there exist $0 < \theta < 1$ such that $\theta p_{r-1}(n)\check{x}_{r-1}^i \leq p_{r-1}(n)\tilde{y}_{r-1}^i$ and, therefore, $\theta p_r(n)\check{y}_r^i \leq p_r(n)\tilde{y}_r^i$. Hence, $\liminf_n \pi_r^j(n) \geq \liminf_n \alpha_r^{ij}\gamma_r^i p_r(n)\tilde{y}_r^i \geq \theta\alpha_r^{ij}\gamma_r^i p_r \check{y}_r^i$, and as above, $\limsup \eta_{r,s}(n) \geq \bar{c}_s - N\bar{y}_s \gg 0$.

So, an application of Theorem A completes the proof.

The case of infinite horizon is similar to those in proof of Theorem 1. ◇

5 Model of Transition from Budget Regulation Economy to Competitive Market Economy

In previous sections we presented two models associated, roughly speaking, with centralized and market economies. As it was mentioned in Introduction at time $t = 0$ we make a decision about transition from Centralized Economy to Market Economy. And the main problem is raised: How can we adapt to future market economy staying in framework of Centralized Economy.

We propose to introduce a *Transition Period* in a course of which agents subjecting to BRE will change their behavior using information on prices in future ME and uncertainty of its emergence moment.

Duration of Transition Period ϑ can be regarded as a random variable with given distribution $\Pr\{\vartheta = t\}$ $(t = 1, 2, \dots, T)$.

Description of agents' behavior in Transition Model will be more complicated than in previous models. Denote $\tau_0 = \max\{t : \; \Pr\{\vartheta \geq t\} = 1\}$, $\tau_1 = \min\{t : \; \Pr\{\vartheta \leq t\} = 1\}$. In other words, segment $[\tau_0, \tau_1]$ be a support of random variable ϑ, i.e. minimal segment containing ϑ with probability one.

We consider a situation when producers can have no complete information about the distribution of "jump moment" ϑ and are forced to choose their behavior on the base of subjective probabilities $\Pr_i\{\vartheta = t\}$, $i \in I$. It means that the State only declares its intention to change economic mechanism and specify the possible interval of this event, and agents have to use their own notions about this fact. For the simplicity we assume that supports of distributions $\Pr_i$ are the same as for Pr.

Moreover, a set of consumers in market economy can, in general, differs from those under centralized economy (for example, the State have to eliminate as consumer, whereas some new consumers may appear). We shall denote the set of "market" consumers as $\tilde{J}$.

Let for some τ we have a collection of prices $\{p_t \quad (t \leq \tau);\ p_t(\tau) \quad (\tau \leq t \leq T)\}$. If τ is interpreted as a moment of change in economic mechanism, then p_t be a price vector in the system at time t provided that system is developed in the framework of BRE model, and $p_t(\tau)$ be prices at time t provided that change in economic mechanism (i.e. emergence of CME) was at moment τ. It is easy to see that prices p_t are well defined for $0 \leq t \leq \tau_1 - 1$, and prices $p_t(\tau)$ — for $\tau_0 \leq \tau \leq \tau_1, \tau \leq t \leq T$. Budgets $\rho_t^i, \pi_t^j, \rho_t^i(\tau), \pi_t^j(\tau)$ will have an analogous sense. So we have to consider two different types of problems for producers and consumers at time t in dependence of current economic mechanism

$$\bar{p}_{t+1}^i y \to \max \tag{7}$$

$$(x,y) \in Q_t^i, \quad p_t x \leq \rho_t^i \qquad (i \in I),$$

where $\bar{p}_{t+1}^i = p_{t+1} q_{t+1}^i + p_{t+1}(t+1)(1 - q_{t+1}^i), \quad q_{t+1}^i = \Pr_i\{\vartheta > t+1 \mid \vartheta \geq t+1\}$;

$$u_t^j(c) \to \max \tag{8}$$

$$c \in C_t^j, \quad p_t c \leq \pi_t^j \qquad (j \in J);$$

$$p_{t+1}(\tau) y \to \max \tag{9}$$

$$(x,y) \in Q_t^i, \quad p_t(\tau) x \leq \rho_t^i(\tau) \qquad (i \in I);$$

$$u_t^j(c) \to \max \tag{10}$$

$$c \in C_t^j, \quad p_t(\tau) c \leq \pi_t^j(\tau) \qquad (j \in \tilde{J}).$$

(for finite horizon at time $T+1$ all prices are given) (7) is the basic problem for producers at Transition Period. The main difference from similar problem (1) in BRE model is that the firm i should evaluate his output cost at the end of period t in expected (forecasting) prices $\bar{p}_{t+1}^i$ rather then in prices p_{t+1}. At the beginning of period t when producer chooses his plan it is not known whether economic mechanism will change at time $t+1$. Subjective probability of change at time $t+1$ for producer i provided it did not occur before t is equal to $1 - q_{t+1}^i$ and corresponding prices will be $p_{t+1}(t+1)$. Similarly, prices at time $t+1$ provided BRE model remained at this time equals p_{t+1} and probability of such event equals q_{t+1}^i. Thus for producer i $\bar{p}_{t+1}^i$ represents an average weighted price at time $t+1$ with conditional probabilities of preserving or change of economic mechanism at time $t+1$ as weights.

Other problems are standard agents' problems in corresponding models (BRE for (8) and ME for (9)-(10)).

Then we define *an equilibrium* in Transition Model with given initial states $(y_0^i; i \in I)$ and endowments $(\omega_t^j;\ j \in \tilde{J})$ as a bundle $\{((\hat{x}_t^i, \hat{y}_{t+1}^i), \hat{c}_t^j, \hat{p}_t;\ 0 \leq t \leq \tau_1 - 1),\ ((\hat{x}_t^i(\tau), \hat{y}_{t+1}^i(\tau)), \hat{c}_t^j(\tau), \hat{p}_t(\tau);\ \tau_0 \leq \tau \leq \tau_1, \tau \leq t \leq T,\ j \in \tilde{J}),\ i \in I\}$, satisfying the following relations:

$(\hat{x}_t^i, \hat{y}_{t+1}^i)$ solves problem (7) under prices $\hat{p}_t, \hat{p}_{t+1}, \hat{p}_{t+1}(t+1)$ and budget $\hat{\rho}_t^i$;

$\hat{c}_t^j$ solves (8) under prices $\hat{p}_t$ and budget $\hat{\pi}_t^j$;

$\hat{\rho}_t^i = \alpha_t^i \hat{K}_t, \quad \hat{\pi}_t^j = \beta_t^j \hat{K}_t, \quad \hat{K}_t = \sum_i \hat{p}_t \hat{y}_t^i, \quad \hat{y}_0^i = y_0^i$;

$(\hat{x}_t^i(\tau), \hat{y}_{t+1}^i(\tau))$ solves problem (9) under prices $\hat{p}_t(\tau)$ and budget $\hat{\rho}_t^i(\tau)$;

$\hat{c}_t^j(\tau)$ solves (10) under prices $\hat{p}_t(\tau)$ and budget $\hat{\pi}_t^j(\tau)$;

$\hat{\rho}_t^i(\tau) = (1-\gamma_t^i)\hat{p}_t(\tau)\hat{y}_t^i(\tau), \quad \hat{\pi}_t^j(t) = \hat{p}_t(t)\omega_t^j + \sum_i \alpha_t^{ij}\gamma_t^i\hat{p}_t(t)\hat{y}_t^i(t);$

$\hat{\pi}_t^j(\tau) = \sum_i \alpha_t^{ij}\gamma_t^i\hat{p}_t(\tau)\hat{y}_t^i(\tau) \ (\tau+1 \le t), \quad \hat{y}_t^i(t) = \hat{y}_t^i;$

$\sum_j \hat{c}_t^j + \sum_i \hat{x}_t^i = \sum_i \hat{y}_t^i; \quad \sum_j \hat{c}_t^j(t) + \sum_i \hat{x}_t^i(t) = \sum_i \hat{y}_t^i + \sum_j \omega_t^j;$

$\sum_j \hat{c}_t^j(\tau) + \sum_i \hat{x}_t^i(\tau) = \sum_i \hat{y}_t^i(\tau) \qquad (\tau+1 \le t \le T).$

Theorem 3. *Let assumptions (P1)-(P3),(C1),(C2) and (D) hold. Then for every initial states $(y_0^i; i \in I)$ such that $\sum_i y_0^i \gg 0$, and positive endowments $(\omega_t^j; j \in \tilde{J})$, there exist an equilibrium in Transition Model (with positive prices $\hat{p}_t$ and $\hat{p}_t(\tau)$).*

Proof of this Theorem combines methods used in proving Theorems 1 and 2 above.

For finite T put $P = \Delta^{(\tau_1-1)}\prod_{\tau=\tau_0}^{\tau_1}\Delta^{(T-\tau+1)}$.

Then for $p = ((p_t,\ 0 \le t \le \tau_1-1),\ (p_t(\tau),\ \tau_0 \le \tau \le \tau_1, \tau \le t \le T)) \in P$ one can define step by step demand and supply correspondences $\varphi_t^j, \varphi_t^j(\tau), \psi_t^i, \psi_t^i(\tau)$ and e.d.c. as $\chi = ((\chi_t,\ 0 \le t \le \tau_1-1),\ (\chi_t(\tau),\ \tau_0 \le \tau \le \tau_1, \tau \le t \le T))$, where

$$\chi_t = \sum_j \tilde{c}_t^j + \sum_i \tilde{x}_t^i - \sum_i \tilde{y}_t^i, \quad (\tilde{x}_t^i, \tilde{y}_{t+1}^i) \in \psi_t^i, \tilde{c}_t^j \in \varphi_t^j,$$

$$\chi_t(t) = \sum_j \tilde{c}_t^j(t) + \sum_i \tilde{x}_t^i(t) - \sum_i \tilde{y}_t^i - \sum_j \omega_t^j,$$

$$\chi_t(\tau) = \sum_j \tilde{c}_t^j(\tau) + \sum_i \tilde{x}_t^i(\tau) - \sum_i \tilde{y}_t^i(\tau), \quad (\tilde{x}_t^i(\tau), \tilde{y}_{t+1}^i(\tau)) \in \psi_t^i(\tau), \tilde{c}_t^j(\tau) \in \varphi_t^j(\tau).$$

Then using similar considerations as in proofs of Theorems 1 and 2 it can be obtained that χ satisfies all assumptions of Theorem A. Thus application of Theorem A completes the proof for the case $T < \infty$.

Another case is considered as above. $\diamond$

6 Optimal Properties of Equilibria

An equilibrium in Walrasian type models is connected usually with Pareto–optimality. However, in models with budgets' restrictions for all agents (in particular, in BRE and CME models) an equilibrium is not Pareto-optimal in general. But for finite horizon T it can be represented as a solution of optimization dynamic model of the following type:

$$\sum_{t=0}^{T}(U_t(c_t) + F_t(z_t)) \to \max \tag{11}$$

over all paths $c_t \in C_t = \sum_j C_t^j,\ z_t = (x_t, y_{t+1}) \in Q_t = \sum_i Q_t^i:\ c_t + x_t \le y_t$ with given initial state $y_0 = \sum_i y_0^i$.

Recall that prices $(p_t, 0 \le t \le T < \infty)$ are said to be *supporting* prices for the path $(\hat{c}_t, \hat{z}_t)$ if for any $0 \le t \le T$:

i) $U_t(\hat c_t) - p_t \hat c_t = \max_{c\in C_t} (U_t(c) - p_t c)$;

ii) $F_t(\hat z_t) + p_{t+1}\hat y_{t+1} - p_t \hat x_t = \max_{z\in Q_t} (F_t(z) + p_{t+1} y - p_t x) \quad (p_{T+1} = 0)$;

iii) $p_t(\hat y_t - \hat x_t - \hat c_t) = 0$.

It is known that path $(\hat c_t, \hat z_t)$ which is supported by some prices is optimal for the problem (11).

A bundle $\left((x_t^i, y_{t+1}^i), c_t^j, \quad 0 \le t \le T;\ i \in I, j \in J\right)$ where $(x_t^i, y_{t+1}^i) \in Q_t^i$, $c_t^j \in C_t^j$, is called an *allocation* if $\sum_j c_t^j + \sum_i x_t^i \le \sum_i y_t^i \quad (0 \le t \le T)$.

Theorem 4. *Let assumptions (P2),(C2) hold and* $(\hat p_t, (\hat x_t^i, \hat y_{t+1}^i), \hat c_t^j; i \in I, j \in J, 0 \le t \le T)$ *be an equilibrium in BRE model with initial states* $(y_0^i; i \in I)$ *and positive budgets* $\hat\rho_t^i$ *and* $\hat\pi_t^j$. *Then there exist positive* μ_t^j *and* $\psi_t^i > -1 \quad (i \in I, j \in J, 0 \le t \le T)$ *such that* $((\hat x_t^i, \hat y_{t+1}^i), \hat c_t^j)$ *is optimal solution for the problem*

$$\sum_{t=0}^{T} \left[\sum_j \mu_t^j u_t^j(c_t^j) + \sum_i \psi_t^i \hat p_{t+1} y_{t+1}^i \right] \to \max \tag{12}$$

over all allocations $((x_t^i, y_{t+1}^i), c_t^j)$ *with initial states* $(y_0^i; i \in I)$ *and* $\hat p_t$ *are supporting prices for the path* $\left(\sum_i (\hat x_t^i, \hat y_{t+1}^i), \sum_j \hat c_t^j, \ 0 \le t \le T\right)$ *in problem (11) with* $U_t(c_t) = \max\left\{\sum_j \mu_t^j u_t^j(c_t^j) :\ c_t^j \in C_t^j, \sum_j c_t^j = c_t\right\}$, $\hat p_{t+1} y_{t+1}^i :\ (x_t^i, y_{t+1}^i) \in Q_t^i, \sum_i (x_t^i, y_{t+1}^i) = z_t\big\}$.

Proof. Since $\hat\pi_t^j > 0$ then by Kuhn-Tucker Theorem there exist $\lambda_t^j \ge 0$ such that for any $c \in C_t^j$

$$u_t^j(c) - \lambda_t^j \hat p_t c \le u_t^j(\hat c_t) - \lambda_t^j \hat\pi_t^j \le u_t^j(\hat c_t) - \lambda_t^j \hat p_t \hat c_t.$$

Non-satiation condition *(C2)* implies $\lambda_t^j > 0$. Thus for any $c_t^j \in C_t^j$

$$U_t(c_t^j; j \in J) - \sum_j \hat p_t c_t^j \le U_t(\hat c_t^j; j \in J) - \sum_j \hat p_t \hat c_t^j \tag{13}$$

where $U_t(c_t^j; j \in J) = \sum_j \mu_t^j u_t^j(c_t^j)$, $\quad \mu_t^j = 1/\lambda_t^j$.

Similarly one can obtain that there exist such $\psi_t^i \ge 0$ that for any $z_t^i = (x_t^i, y_{t+1}^i) \in Q_t^i \quad \hat p_{t+1} y_{t+1}^i - \psi_t^i \hat p_t x_t^i \le \hat p_{t+1} \hat y_{t+1}^i - \psi_t^i \hat p_t \hat x_t^i$, and therefore

$$F_t(z_t^i; i \in I) + \sum_i (\hat p_{t+1} y_{t+1}^i - \hat p_t x_t^i) \le F_t(\hat z_t^i; i \in I) + \sum_i (\hat p_{t+1} \hat y_{t+1}^i - \hat p_t \hat x_t^i) \tag{14}$$

where $F_t(z_t^i; i \in I) = \sum_i (1 - \psi_t^i) \hat p_{t+1} y_{t+1}^i$. Moreover, (6) implies

$$\hat p_t \left(\sum_i \hat y_t^i - \sum_i \hat x_t^i - \sum_j \hat c_t^j \right) = 0, \quad 0 \le t \le T. \tag{15}$$

At last, from relations (13)-(15) it follows that $(\hat p_t)$ are supporting prices for path $((\hat x_t^i, \hat y_{t+1}^i), \hat c_t^j)$. Hence, this path is optimal for problem (12) over all allocations with given initial state. ◇

As for optimality properties of an equilibrium in CME model one can see that proving Theorem 4 we did not use the concrete structure of budgets ρ_t^i and π_t^j. It means that a complete analogue of this theorem is valid for CME model.

Using the representation of equilibrium models as optimization problems we can explain a structure of prices $\bar{p}_t^i$ in Transition model. Let BRE model is associated with problem (11) with some functions $U_t^1(c)$ and $F_t^1(z)$, and CME model — with corresponding functions $U_t^2(c)$ and $F_t^2(z)$. If ϑ is a moment of a change in economic mechanism, then Transition model is naturally associated with maximization of the functional

$$E_\vartheta \left[\sum_{t=0}^{\vartheta-1} F_t^1(c_t^1, z_t^1) + \sum_{t=\vartheta}^{T} F_t^2(c_t^2, z_t^2) \right], \tag{16}$$

where $F_t^k(c,z) = U_t^k(c) + F_t^k(z) \quad (k=1,2)$, and expectation E_ϑ is evaluated with respect to distribution of ϑ. As it known, the supporting prices in (16) have the same "weighted" structure as prices $\bar{p}_t^i$ in problem (7) when all subjective distributions Pr_i are the same as "true" distribution of ϑ (for more general optimization problems with jumps this result was obtained, e.g. in [7–8]). This fact allows us to say that prices in proposed Transition model, at least for the case of "complete information" (when distribution of jump moment is known exactly for all agents) have an "optimal" (in some sense) structure, and agents behave in "optimal" way.

7 An Example of Transition Process

In this section we give an example of economy in which the transition from one economic mechanism to another (in the sense of above considerations) implies essential change in production plans. Though we can not formally apply theorems proved above to this example (because, prices with zero components are admitted), we think it will be useful to show phenomena arising under various strategies of transition from one economic mechanism to another.

Let economy consists of four goods (x_1, x_2, x_3, x_4) and two producers with following technologies:

$$Q_1 = \{((x_1,x_2,x_3,x_4),(y_1,0,y_3,y_4)) \text{ where } y_1 \le f_1(x_2'),$$

$$y_3 \le f_3(x_2'', x_4),\ y_4 \le f_4(x_2'''),\ x_2' + x_2'' + x_2''' = x_2\}$$

$$Q_2 = \{((x_1,x_2,x_3,x_4),(0,y_2,0,0)) \text{ where } y_2 \le f_2(x_2,x_3)\}$$

where $x_j, y_j \ge 0$ $(j = 1,2,3,4)$. It is easy to see that if production functions f_j are concave then Q_i $(i=1,2)$ are convex sets. Moreover, there is one consumer of the good x_1 with utility function $u(c_1)$.

In order to describe agents' problems let p_j^t be the price of good j, $(j=1,2,3,4)$ and ρ_i^t, $(i=1,2), \pi^t$ be budgets of producer i and consumer at the time t. At the beginning of period t agents wish to maximize their incomes (utility) at the end of period t subject to budget constraints, i.e.

$$\begin{aligned} p_1^{t+1} f_1(x_2') + p_3^{t+1} f_3(x_2'', x_4) + p_4^{t+1} f_4(x_2''') \to \max, \\ p_2^t(x_2' + x_2'' + x_2''') + p_4^t x_4 \le \rho_1^t; \end{aligned} \tag{17}$$

$$f_2(x_2, x_3) \to \max, \tag{18}$$

$$p_2^t x_2 + p_3^t x_3 \le \rho_2^t;$$

$$u(c_1) \to \max,$$
$$p_1^t c_1 \le \pi^t. \qquad (19)$$

With this economy we can associate BRE model - and CME model in dependence of the way of budgets forming.

Assume that all functions $f_1(x_2), f_2(x_2, x_3), f_3(x_2, x_4)$ and $f_4(x_2)$ are concave, strictly increasing (with respect to their arguments) and, besides, $f_1(0) = f_3(0, x_4) = f_3(x_2, 0) = f_4(0) = 0$ for any positive x_2, x_4. Utility function $u(c_1)$ strictly increases on c_1, also. Let consider an equilibrium in BRE model with zero prices at good 3 for any moments, i.e. $\hat{p}_3^t = 0\ \forall t$. For this case it can be shown that amount of goods 3 and 4 will be zero at any moment except, may be, initial moment. Indeed, assume that $y_1^t, y_2^t, y_3^t, y_4^t$ be some amounts of goods in the system at time t, and $\tilde{x}_2^{\prime\prime t}, \tilde{x}_2^{\prime\prime\prime t}, \tilde{x}_2^{\prime\prime\prime\prime t}, \tilde{x}_4^t,$ be optimal solution in problem (17) (with $p_3^{t+1} = 0$). Obviously, $\tilde{x}_2^{\prime\prime t} = 0$ and, therefore, $y_3^{t+1} = f_3(\tilde{x}_2^{\prime\prime t}, \tilde{x}_4^t) = 0$. Moreover, $p_4^t x_4^t = 0$ and, hence, on equilibrium $\hat{p}_4^t y_4^t = \hat{p}_4^t \tilde{x}_4^t = 0$ for any moments t. Furthermore, if $\tilde{x}_2^{\prime\prime\prime t} > 0$ then $y_4^{t+1} = f_4(\tilde{x}_2^{\prime\prime\prime t}) > 0$ and therefore $\hat{p}_4^{t+1} = 0$. This implies that $\tilde{x}_2^{\prime\prime\prime t} = 0$ (see (17)). This contradiction shows that $y_4^{t+1} = 0$. So, in BRE model an equilibrium with $p_3^t = 0\ \ \forall t$, implies that $y_3^t = y_4^t = 0$ for any t and it maximizes the following problems:

$$f_1(x_2) \to \max,$$
$$p_2^t x_2 \le \rho_1^t;$$

$$f_2(x_2, 0) \to \max,$$
$$p_2^t x_2 \le \rho_2^t;$$

and (19), where $\rho_i^t = \alpha_i K^t$, $\pi^t = \beta K^t$, $\alpha_1 + \alpha_2 + \beta = 1$, $K^t = p_1^t y_1^t + p_2^t y_2^t$ be a total budget at time t. It is easy to see that

$$\hat{p}_1^t = \frac{\beta y_2^t}{y_1^t + \beta(y_2^t - y_1^t)}, \qquad \hat{p}_2^t = \frac{(1-\beta) y_1^t}{y_1^t + \beta(y_2^t - y_1^t)},$$

be an equilibrium prices on goods 1 and 2.

Now proceed to CME model. Suppose that consumer of good 1 vanishes in CME and, therefore, demand of good 1 is zero at any moments. Show that this good will not produced in the system. Let y_1^t be an amount of good 1 at moment t, and $\tilde{x}_2^{\prime t}$ be optimal in problem (17) with $p_1^{t+1} = 0$. At the equilibrium we have $p_1^t y_1^t = 0$ (because of zero demand). If $\tilde{x}_2^{\prime t} > 0$ then $y_1^{t+1} = f_1(\tilde{x}_2^{\prime t}) > 0$ and , therefore, $p_1^{t+1} = 0$ (as above) . This implies $\tilde{x}_2^{\prime t} = 0$, i.e. we get contradiction. Hence, $y_1^{t+1} = 0$. So, for the case under discussion the system is reduced to the model with three goods (x_2, x_3, x_4) and two producers with the following problems:

$$p_3^{t+1} f_3(x_2^{\prime\prime}, x_4) + p_4^{t+1} f_4(x_2^{\prime\prime\prime}) \to \max,$$

$$p_2^t (x_2^{\prime\prime} + x_2^{\prime\prime\prime}) + p_4^t x_4 \le \rho_1^t = p_3^t y_3^t + p_4^t y_4^t;$$

and (18) with $\rho_2^t = p_2^t y_2^t$, where (y_2^t, y_3^t, y_4^t) be amount of goods at time t. Although we can't use directly Theorem 2 (by formal reasons), best applying similar arguments one can prove that for any positive initial state (y_2^0, y_3^0, y_4^0) there exist an equilibrium with positive prices $\bar{p}_2^t, \bar{p}_3^t$ and $\bar{p}_4^t$. If we assume additionally that $\frac{\partial f_3(0,x_4)}{\partial x_2} = f_4'(0) = +\infty$ for any positive x_4, then amounts of goods 3 and 4 at the equilibrium will always be positive. So, we see that behavior of the first producer will be essentially different in different models: in BRE model it produces only good 1, whereas in CME model- only goods 3 and 4. Consider variants of Shock transition in this system (i.e. without any transition period). The first one is simply a case when at some moment consumer of good 1 eliminates and, therefore, an income of the first producer becomes zero that leads to its bankruptcy. In second variant we declare at moment t that at $t+1$ will be the change. Then the first producer has to solve the problem

$$\bar{p}_1^{t+1} f_1(x_2') + \bar{p}_3^{t+1} f_3(x_2'', x_4) + \bar{p}_4^{t+1} f_4(x_2''') \rightarrow \max,$$

$$\hat{p}_2^t(x_2' + x_2'' + x_2''') + \hat{p}_4^t x_4 \leq \rho_1^t;$$

where $\bar{p}_j^{t+1}$ be equilibrium prices in CME, and $\hat{p}_j^t$ be equilibrium prices in BRE (j=1,2,3,4). Obviously, $y_3^{t+1} = 0$, and budget at time $t+1$ will be $\rho_1^{t+1} = \bar{p}_4^{t+1} y_4^{t+1}$. Then, at time $t+1$ the agent solves the problem

$$\bar{p}_3^{t+2} f_3(x_2'', x_4) + \bar{p}_4^{t+2} f_4(x_2''') \rightarrow \max,$$

$$\bar{p}_2^{t+1}(x_2'' + x_2''') + \bar{p}_4^{t+1} x_4 \ \leq \ \bar{p}_4^{t+1} y_4^{t+1}; \tag{20}$$

Since at equilibrium $\tilde{x}_4^{t+1} = y_4^{t+1}$, then constraint (20) implies $\bar{p}_2^{t+1}(x_2'' + x_2''') \leq 0$, i.e. $x_2'' = x_2''' = 0$, and, hence, $y_3^{t+2} = y_4^{t+2} = 0$. So this agent ceases its activity due to bankruptcy at moment $t+2$. So any version of instantaneous transition (shock) in this example gives unsatisfactory results . However such undesirable effect vanishes in framework of Transition Model from section 5. Indeed, the problem (7) for our example is transformed to the following:

$$p_1^{*,t+1} f_1(x_2') + p_3^{*,t+1} f_3(x_2'', x_4) + p_4^{*,t+1} f_4(x_2''') \rightarrow \max,$$

$$p_2^t(x_2' + x_2'' + x_2''') + p_4^t x_4 \ \leq \ \rho_1^t;$$

Where $p_i^{*,t+1}$ be "weighted" prices (note that $p_3^{*,t+1}$ and $p_4^{*,t+1}$ are positive because of positivity of corresponding "market" prices). If we assume that

$$f_1'(0) = f_4'(0) = \frac{\partial f_3(0, x_4)}{\partial x_2} = +\infty \quad \text{for any } x_4 > 0$$

then amount of all goods in the system will be positive for two and more moments from the beginning of transition. If we apply again arguments similar to those in proof of theorem 3 then we can obtain an existence of equilibrium transition process with positive amounts of all goods. Thus for this example the gradual transition process essentially change the technological mode of agent and force him to produce a good which is unprofitable at present but will be profitable in future market economy.

8 References

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