# Working Paper

## Social Justice and Individual Choice

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WP-94-25
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## Abstract

Consider a divisible resource or cost that is to be fairly distributed among a group, and suppose that various members of the group have different opinions about what a fair distribution might be. We exhibit a class of mechanisms that aggregate individual opinions into a group opinion, and which have the property that no one can manipulate the size of his own share. There is a unique such mechanism that satisfies a variant of Arrow's conditions for social choice functions. We illustrate its application to distributing dues among the member countries of IIASA. Other potential applications include distributing shares in formerly state-owned enterprises, and in allocating voting power among different states or regions in a federal system of government.

## Social Justice and Individual Choice

### 1. The eye of the beholder

Fairness is an everyday concern in economic and political life. Scarcely any discussion of public policy occurs without the term being used by some or all parties to the debate. Yet for many people fairness remains a disturbingly vague and inchoate concept. Some think that fairness is a mere fig-leaf that disguises arguments based on naked self-interest. Others, while not going quite so far, hold that fairness is largely a matter of opinion and therefore nothing interesting can be said about it. Yet neither of these positions is very credible. The first is not logically coherent, because a word that is used *only* as a cover ceases to be one; it becomes transparent and therefore ineffectual. The second argument -- that fairness is merely a matter of *opinion* -- cuts little mustard because it applies with equal force to the other tools we use to analyze individual choice. One might just as well dismiss attitudes toward risk, marginal rates of substitution, and the propensity to save as merely matters of opinion and therefore of no intrinsic interest. Yet they form the backbone of economic theory. Why, then, do opinions about fairness not have equal standing?

Part of the reason is that we do not yet have an accepted empirical framework for assessing attitudes toward fairness with the same confidence that we measure, say, attitudes toward risk. And part of the reason is that debates about fairness have traditionally been the province of philosophers, for whom fairness is anything but a matter of opinion, but a question of high moral principle. If philosophers had produced a convincing recipe for fairness we would need to go no further, but instead they offer a menu of competing definitions that have radically different policy implications. Egalitarians maintain that goods should be divided equally, Aristotelians that they should be allotted in proportion to each person's contribution, Rawlsians that they should go to the least well-off, utilitarians that they should be given to those who would benefit most at the margin, anti-invidians that they should be distributed so that no one envies another, and so forth. Clearly this presents a dilemma for anyone who wants to incorporate distributive considerations into questions of public policy.

It is of course true that *in certain cases* one of these principles may be particularly compelling. If a group of people invest in a joint enterprise, for example, it is natural to follow the Aristotelian principle and divide the profits in proportion to the investors' contributions. If a company of soldiers receives an allotment of food they will probably adopt the egalitarian principle. If there is only enough flu vaccine for one shot it would probably be given to the person who is most at risk.[1]

There are many situations, however, in which it is unclear what principle applies. Take the current economic reform in Russia and Eastern Europe. A key step in the transition to market economies is to privatize state enterprises and to distribute ownership rights among individuals. But exactly who is entitled to these rights and what is the extent of their entitlement? The Aristotelian theory suggests that the shares should be allotted in proportion to individual contributions. The question then becomes how to measure a person's contribution. Is it his contribution to a particular enterprise? If so, one might divide the shares among the enterprise's current (and former) employees according to the length of their employment, or perhaps to their total earnings during employment. Alternatively, one could argue that *everyone* contributed to the enterprise through state subsidies that were funded over the years by taxes. On this view an individual's share would be larger the more taxes he paid. Similar, if not more perplexing, difficulties arise in applying the Rawlsian and utilitarian principles. Even the egalitarian principle is ambiguous. Should equality hold for all residents or only citizens? For all citizens or only adult citizens?

Difficulties like these lead us back to the proposition that fairness is, at least in some cases, mainly a matter of opinion. It does *not* follow, however, that these opinions do not matter. When common property or a collective burden is to be distributed among a group, the stakeholders will often have strongly-held views about the justice or injustice of different distributions. By *just* I do not necessarily mean ethical or moral, but that which the parties believe to be *appropriate* under a given set of circumstances. Sometimes opinions about "appropriateness" flow from a sense of moral imperative; more often they are simply opinions based on intuition and experience, but this does not make them any the less interesting or important.

In this paper we propose a method for assessing opinions about distributive justice that is in the pluralistic spirit that animates the modern theory of social choice. We accept from the outset that people may have different views about what a fair distribution ought to be. These views are taken as given, and our task is to find a *process* that aggregates them into a group opinion. Of course, this pushes the fairness issue onto a higher plane -- the design of fair process. We shall argue, however, that it is often easier to achieve consensus on fair procedural principles than to bargain over particular distributions.

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[1] These cases illustrate three general distributive principles -- proportionality, parity, and priority -- that figure in almost all discussions of distributive justice (Young, 1994).

Specifically, we propose to construct an aggregation procedure with the following four properties: i) everyone's opinion counts equally; ii) individuals have no incentive to misreport their opinions in order to manipulate their own shares; iii) the size of a person's share depends only on data that are relevant to that person; iv) if there is unanimous support for a particular ratio between two individuals, then this ratio is implemented. These conditions are deliberately modelled on standard axioms in the social choice literature (including Arrow's axioms) in order to illustrate that fair division can be approached with the familiar tools of social choice theory. Unlike many such axiom schemes, however, they yield positive results. We shall show, in fact, that there exists a ***unique*** aggregation method with these four properties, and then illustrate its application to a situation with real data.

## 2. The model

Consider a group of n individuals $i = 1, 2, \ldots, n$ who are to divide a common asset, which we assume is perfectly divisible. The individuals may have various distinguishing characteristics -- need, income, age, prior contribution, etc. -- that entitle them to more or less of the good. They may also have different views about how these characteristics translate into specific entitlements to the good. Whether these differences result from distinct philosophical positions about fairness is unimportant for our purposes. In other words, we shall not inquire into the *reasons* that they hold the opinions they do any more than we ask for the reasons why they are risk averse. We do recognize, however, that a person's opinion about the proper size of his *own* share may be dictated largely by self-interest. For this reason we shall be concerned mainly with how much people think the *others* should get, not how much they *themselves* should get.[2]

Formally, the *opinion* of individual i is a vector of nonnegative numbers $r^*_{ij}$, $1 \leq j \leq n$, that sum to one. These are the relative amounts that i believes everyone should receive. The opinion is *nondegenerate* if i believes the others are entitled to something, that is, $r^*_{ij} > 0$ for some $j \neq i$. We shall assume throughout that all opinions are nondegenerate. A *partial report* by individual i is a vector $\mathbf{r}_i = (r_{ij})_{j \neq i}$ that is nonnegative and not identically zero. These are the relative shares that i *says* the others should receive. The partial report $\mathbf{r}_i$ is *true* if there is a factor $\lambda_i > 0$ such that $r_{ij} = \lambda_i r^*_{ij}$ for every $j \neq i$. We can always normalize a partial report so that its sum is unity, and we shall assume this has been done for every individual.

[2] There is a large body of empirical work assessing intuitions of distributive justice by experimental methods that differ from those suggested here. See, for example, Selten (1978), Yaari and Bar-Hillel (1982), Hoffman and Spitzer (1985), Roth ed. (1987), Mathieu and Zajac (1991), and Frohlich and Oppenheimer (1992).

Fix the number of individuals n and let $\mathbf{r} = (\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n)$ be a set of normalized partial reports. Assume for the moment that the reports are true. We would like to find a ***division*** -- a set of n shares that sum to one -- that balances the revealed differences of opinion among the members of the group. An aggregation ***method*** is a function $F(\mathbf{r})$ defined for every n-vector of normalized partial reports $\mathbf{r} = (\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n)$, such that $F(\mathbf{r}) = (x_1, x_2, \ldots, x_n) \geq 0$ and $\Sigma x_i = 1$.

## 3. Statistical solutions

We begin by treating the problem as one of statistical inference. Suppose we knew the true opinion $r^*_i$ of each member of the group. Let us normalize it so that it sums to unity. A statistician would find it natural to average the opinions and let $x^*_j = \Sigma_i r^*_{ij}/n$ for every j. These shares $\mathbf{x}^* = (x^*_1, \ldots, x^*_n)$ minimize the squared deviations from the true opinions.

Now suppose that we do not know the true opinions, but we have in hand a set of normalized partial reports $(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n)$. Suppose further that these reports are true. Then there exist numbers $\lambda_1, \lambda_2, \ldots, \lambda_n$ in the interval [0, 1) such that $r^*_{ii} = \lambda_i$, $r^*_{ij} = (1 - \lambda_i)\, r_{ij}$, $j \neq i$. Although we do not know the values of the $\lambda_i$, we can assume a prior distribution over them. Let us assume, for example, that the numbers $\lambda_i$ are independently and uniformly distributed over the interval [0, 1). Then the relative shares that minimize the *expected* squared deviations from the true opinions are given by

$$x = \operatorname{argmin} \sum_{j=1}^{n} \int\int \ldots \int \Big[(x_j - \lambda_j)^2 + \sum_{i:\, i \neq j} (x_j - (1 - \lambda_i) r_{ij})^2\Big]\, d\lambda_1 d\lambda_2 \ldots d\lambda_n,$$

which has the solution

$$x_j = (1/2n)\,\Big(1 + \sum_{i:\, i \neq j} r_{ij}\Big). \tag{1}$$

Equivalently, $\mathbf{x}$ is the normalized sum of the rows of the matrix:

$$\begin{matrix} 1 & r_{12} & \cdots & r_{1n} \\ r_{21} & 1 & \cdots & r_{2n} \\ \vdots & & & \vdots \\ r_{n1} & & \cdots & 1 \end{matrix} \tag{2}$$

Note that each person's share is independent of his own report, so that no one has an incentive to manipulate his report in order to get more for himself.

Consider the following hypothetical application. There are three claimants for shares in a newly privatized enterprise. Individual 1 is a manager, 2 is a laborer, and 3 is a citizen who is neither a manager nor a laborer. The manager thinks that the laborer should get twice as much as the citizen. The laborer thinks that the manager and the citizen should be treated alike. The citizen thinks that the laborer should get nothing. Thus they write down the following partial reports, where each is normalized to sum to 1:

| | mgr | lab | cit |
|---|---|---|---|
| mgr | -- | 2/3 | 1/3 |
| lab | 1/2 | -- | 1/2 |
| cit | 1 | 0 | -- |

The solution is found by placing a "1" in each blank, summing the entries in each column, and normalizing so that the total is unity. The answer is $\mathbf{x} = (15/36, 10/36, 11/36)$.

Although this method is plausible in some ways, it has several drawbacks. In the first place its motivation is weak -- we have no good reason to assume a uniform prior, and we have even less reason to assume that the true opinions are independently distributed. (Indeed, there may be considerable agreement among members of the group about what the appropriate standard of justice is.) Even if we accept these assumptions, however, the outcome of the method is unsatisfactory because it is *skewed* toward equal division: no matter what the true opinions are, everyone gets at least 1/2n of the pie.

Let us therefore consider a second statistical estimation approach. Instead of assuming a uniform prior, let us estimate the distribution of shares that comes *closest* to the partial reports, given that each report is only meaningful up to a choice of scale, and that all reports are to count equally. In other words, we wish to find a vector $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ and a single positive factor $\lambda$ such that

$$\mathbf{x}, \lambda \text{ minimizes } \sum_j \Big(\sum_{i: i \neq j} (x_j - \lambda r_{ij})^2\Big) \text{ subject to } \sum_j x_j = 1.$$

The unique solution is

$$\lambda = 1 - 1/n$$
$$x_j = (1/n)\sum_{i:i\neq j} r_{ij} \text{ for all } j. \qquad (3)$$

This is equivalent to saying that **x** is the normalized sum of the rows of the matrix

$$\begin{matrix} 0 & r_{12} & \cdots & r_{1n} \\ r_{21} & 0 & \cdots & r_{2n} \\ \vdots & & & \vdots \\ r_{n1} & & \cdots & 0 \end{matrix} \qquad (4)$$

This process has the following natural interpretation: each individual is given 1/n of the total "pie" to divide among the *others*, and the sum of these divisions is the required allocation.

Note that this method is reliable in the sense that *each agent's share is independent of his own report.* Moreover it is clear how to construct a family of such procedures. Fix a nonnegative number c, and for every set of n normalized partial reports $\mathbf{r} = (\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n)$, define the rule $F^c$ such that

$$F_j^c(\mathbf{r}) = [1/(nc + n)][c + \sum_{i:i\neq j} r_{ij}]. \qquad (5)$$

The first rule that we defined is $F^1$ and the second is $F^0$. Letting c go to infinity we also obtain the constant method $F(\mathbf{r}) = (1/n, 1/n, \ldots, 1/n)$.

These methods can obviously be extended to allow each agent to submit a full report about himself as well as the others (one just ignores what each says about himself). Define a *report* by individual i to be a vector of n numbers $\mathbf{r}_i = (\mathbf{r}_{i1}, \mathbf{r}_{i2}, \ldots, \mathbf{r}_{in}) \geq \mathbf{0}$. It is *nondegenerate* if $r_{ij} > 0$ for at least one $j \neq i$. In this setting a *method* is a function $F(\mathbf{r})$ that associates a unique distribution $\mathbf{x} \geq \mathbf{0}$, $x_j = 1$, with each set of n nondegenerate, normalized reports $\mathbf{r} = (\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n)$. From now on we shall treat methods in this extended sense, since they form a less restricted class than those that ignore agents' reports about themselves.

## 4. A solution from first principles

Fix the number of individuals n and an extended method F, and consider the following procedural fairness conditions.

I. ANONYMITY. For every permutation $\pi$ of the indices 1, 2, . . ., n, and every full report $\mathbf{r}_i$, consider the report $\mathbf{r}^{\pi}_i = (r_{\pi(i)\pi(1)}, \ldots, r_{\pi(i)\pi(n)})$. F is *anonymous* if for every $\pi$ and every $\mathbf{r}$,

$$F_{\pi(i)}(\mathbf{r}^{\pi}_1, \mathbf{r}^{\pi}_2, \ldots, \mathbf{r}^{\pi}_n) = F_i(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n).$$

This condition says that the method should not be biased in favor of any individual. In particular, although the individuals may get different shares, they have equal voice in determining the shares.

II. RELIABILITY. For all i and $\mathbf{r}$, $F_i(\mathbf{r})$ does not depend on i's report.

III. RELEVANCE. For all i and $\mathbf{r}$, $F_i(\mathbf{r})$ does not depend on reports about individuals different from i, that is, $F_i(\mathbf{r})$ is independent of $r_{jk}$ for all $j \neq i$ and all $k \neq i$.

Note that this condition is similar to Arrow's independence of irrelevant alternatives condition. Strictly speaking, the analog of Arrow's condition would be that any two shares $x_i$, $x_j$ depend only on data concerning i and j. Since we are dealing here with cardinal rather than ordinal comparisons, however, we may go one step further and require that the share of any individual depend only on data about that individual.

IV. UNANIMITY. For all i, j, and $\mathbf{r}$, if there is a nonnegative number c such that $r_{ij}/r_{ji} = r_{kj}/r_{ki} = c$ for all $k \neq i, j$, then $F_j/F_i = c$.

This condition states that if, for some pair i and j, i's reported share for j is c times as large as j's reported share for i, and if everyone else concurs on this ratio, then i should receive c times as much as j under F. This means, in particular, that if $r_{kj} = 0$ for all $k \neq j$ and $r_{ki} > 0$ for all $k \neq i$, then $F_j/F_i = 0$.

*Theorem. For every number of claimants n, $F^0$ is the unique fairness procedure that is anonymous, reliable, relevant, and unanimous.*

*Proof.* It is straightforward to check that $F^0$ satisfies all four conditions. Conversely, let F be a method that satisfies these conditions on some set of n respondents. Relevance implies that, for every j, $F_j(\mathbf{r})$ is a function solely of the numbers $r_{1j}, r_{2j}, \ldots, r_{nj}$. Moreover, reliability implies that $F_j(\mathbf{r})$ does not depend on $r_{jj}$. In effect, then, $F_j(\mathbf{r})$ is a function from $[0, 1]^{n-1} \to [0, 1]$. Call this function g. From anonymity it follows that g is a symmetric function of its arguments, and that g represents $F_i(\mathbf{r})$ for all i.

To prove the theorem, it suffices to show that, for all $\mathbf{y} \in [0, 1]^{n-1}$, $g(y_1, \ldots, y_{n-1}) = \Sigma y_k/n$. First consider any $\mathbf{y} \geq \mathbf{0}$ such that $\Sigma y_k = 1$. Define the reports $\mathbf{r}_i$ as follows:

$$\mathbf{r}_1 = 0, \ y_1, \ldots, y_{n-1}$$
$$\mathbf{r}_2 = y_{n-1}, 0, \ldots, y_{n-2}$$
$$\vdots$$
$$\mathbf{r}_n = y_1, \ y_2, \ \ldots, \ 0.$$

Since r is fixed by the cyclic permutation $1 \to 2 \to \ldots \to n \to 1$, condition I implies that $F_i(\mathbf{r}) = F_j(\mathbf{r})$ for all i, j, and hence $F(\mathbf{r}) = (1/n, 1/n, \ldots, 1/n)$. From this and condition III it follows that $g(\mathbf{y}) = \Sigma y_k/n$ whenever $\Sigma y_k = 1$.

Now choose $\mathbf{y} \geq \mathbf{0}$ such that $\Sigma y_k = a$, where $0 < a < 1$. Without loss of generality we may assume that $y_1$ is the largest of the values $y_i$. Construct a matrix of reports $\mathbf{r}$ such that the first *column* of $\mathbf{r}$ is $(0, y_1, y_2, \ldots, y_{n-1})^T$ and the second column is $(1/a)(y_1, 0, y_2, \ldots, y_{n-1})^T$. (Here T denotes "transpose".) For each $i \geq 2$ we have $y_1 + y_i \leq a < 1$. Since $y_i \leq y_1$ it follows that $y_i < 1/2$ for all $i \geq 2$. From this we obtain

$$y_i + y_i/a \leq y_i + y_i/(y_1 + y_i) \leq y_i + 1/2 < 1/2 + 1/2 = 1.$$

Thus the remaining columns of $\mathbf{r}$ may be filled in such that the sum of each row equals 1.

Let $F(\mathbf{r}) = \mathbf{x}$. Pairwise unanimity asserts that $x_1/x_2 = a$. Since the sum of the second column is unity, the preceding argument shows that $x_2 = 1/n$. Thus $x_1 = a/n$. This establishes that $g(\mathbf{y}) = \Sigma\, y_k/n$ whenever $0 < \Sigma y_k < 1$. If $\Sigma\, y_k = 0$, then $\mathbf{y}$ is identically 0. In this case we can construct reports so that the first column is identically zero and the second is positive and sums to 1. Pairwise unanimity then implies that $x_1/x_2 = 0$, which shows that $x_1 = 0$. Thus $g(\mathbf{0}) = 0$.

Finally, consider the case where $\Sigma y_k > 1$. Then we can construct a matrix of reports **r** such that the first column is $(0, \mathbf{y})^T$, each row sums to 1, and each of the remaining columns sums to at most 1. From the preceding argument it follows that $x_j = \Sigma_i r_{ij}/n$ for every $j \geq 2$. Since $\Sigma x_i = 1$ and $\Sigma_{i,j} r_{ij}/n = 1$, it follows that $x_1 = \Sigma_i r_{i1}/n$ as was to be shown. Thus $g(\mathbf{y}) = \Sigma y_k/n$ in all cases, which completes the proof.

## 5. An application

The International Institute for Applied Systems Analysis (IIASA) is an interdisciplinary research organization to promote scientific cooperation between Eastern and Western countries. As of mid-1993, IIASA had the fourteen member countries shown in Table 1 below. Each country contributes annual dues to support the operating budget of the institute according to a graduated dues schedule. Initially there were just two categories of countries: Russia and the United States in category I and all other countries in category II. The dues ratio of the former to the latter was 6 to 1. As of June, 1993 when this study was undertaken there were three categories of members with the dues ratio 6 : 2 : 1. The recent changes in Eastern Europe and Russia have led some to question whether this scheme is still appropriate.

This situation provides a natural environment in which to apply the ideas sketched above. A simple experiment was conducted in which a questionnaire was distributed to fourteen researchers at IIASA, one drawn at random from each of the member countries. The questionnaire presented the information in Table 1, and invited the respondents to state their opinion about what they think the *fairest ratio of payments* would be for each of the thirteen countries *other* than their own. The results are presented in Table 2. The last line of the table gives the shares (in percent) according to the method $F^0$.

| Country | Current Dues Ratio | Pop ($10^3$) | GDP Per Cap ($) |
|---|---|---|---|
| Austria | 2 | 7,618 | 17,250 |
| Bulgaria | 1 | 9,004 | 5,690 |
| Canada | 2 | 26,219 | 20,370 |
| Czechoslovakia | 1 | 15,639 | 7,900 |
| Finland | 2 | 4,962 | 22,110 |
| Germany | 2 | 78,620 | 18,200 |
| Hungary | 1 | 10,576 | 6,090 |
| Italy | 2 | 57,517 | 15,180 |
| Japan | 2 | 123,116 | 24,240 |
| Netherlands | 1 | 14,835 | 16,030 |
| Poland | 1 | 37,854 | 4,560 |
| Russia | 6 | 285,861 | 9,230 |
| Sweden | 2 | 8,493 | 21,620 |
| USA | 6 | 249,250 | 20,850 |

Table 1. The fourteen member countries of IIASA as of July, 1993. Per capita GDP estimates for Eastern Europe and Russia (including the Ukraine) are from Comecon Data 1990 (Vienna Institute for Comparative Economic Studies, p. 48). The figures for Germany combine separate estimates for East and West Germany.

| | Au | Bu | Ca | Cz | Fi | Ge | Hu | It | Ja | Ne | Po | Ru | Sw | US |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | -- | 1 | 4 | 1 | 4 | 4 | 1 | 2 | 4 | 2 | 1 | 4 | 4 | 6 |
| | 1 | -- | 2 | 1 | 2 | 2 | 1 | 2 | 3 | 2 | 1 | 1 | 2 | 3 |
| | 3 | 1 | -- | 1 | 3 | 3 | 1 | 3 | 3 | 3 | 1 | 3 | 3 | 3 |
| | 2 | 1 | 3 | -- | 2 | 4 | 1 | 3 | 4 | 2 | 1 | 6 | 2 | 6 |
| | 2 | 1 | 4 | 1 | -- | 4 | 1 | 2 | 4 | 2 | 1 | 4 | 2 | 6 |
| | 2 | 1 | 2 | 1 | 2 | -- | 1 | 2 | 2 | 2 | 1 | 2 | 2 | 6 |
| | 2 | 1 | 3 | 1 | 2 | 3 | -- | 2 | 4 | 2 | 1 | 3 | 3 | 6 |
| | 1 | 1 | 2 | 1 | 1 | 2 | 1 | -- | 6 | 1 | 1 | 5 | 1 | 6 |
| | 2 | 1 | 2 | 1 | 2 | 2 | 1 | 2 | -- | 1 | 1 | 2 | 2 | 6 |
| | 2 | 1 | 2 | 1 | 2 | 2 | 1 | 2 | 2 | -- | 1 | 2 | 2 | 6 |
| | 1 | 1 | 2 | 1 | 1 | 6 | 1 | 2 | 6 | 2 | -- | 6 | 1 | 6 |
| | 2 | 1 | 2 | 1 | 2 | 2 | 1 | 2 | 2 | 2 | 1 | -- | 2 | 6 |
| | 1.9 | 0.7 | 2.4 | 1.0 | 2.4 | 2.7 | 0.8 | 2.2 | 3.7 | 1.9 | 0.8 | 3.6 | -- | 4.5 |
| | 2 | 1 | 3 | 1 | 2 | 2 | 1 | 2 | 3 | 1 | 1 | 5 | 2 | -- |
| $F^0$: | 5.9 | 3.0 | 7.9 | 3.2 | 6.6 | 8.9 | 3.2 | 6.8 | 11.1 | 5.7 | 3.2 | 10.8 | 6.7 | 16.8 |

Table 2. **Reported fair shares by respondents from each of the fourteen member countries, and the computed fair shares (in percent) using method $F^0$.** Each row is a report by a respondent from the country that is indicated by a dash.

Several features of these responses are worth noting. First, there was a considerable amount of agreement among the respondents even though no two opinions were the same. For example, everyone except the Swedish respondent said that Bulgaria, Czechoslovakia, Hungary, and Poland should pay the same amount (or they reported no opinion). Similarly, ten of the respondents thought that the US should pay six times the amount paid by the smallest dues-payer (as it does now). Moreover, the Swede's estimate, which was based on a formula weighting population and GDP, yielded almost the same ratio (4.5/.7 = 6.4).[3]

Second, it appears that precedent (i.e., the current dues ratios) weighed fairly heavily in the respondents' opinions. But while most respondents thought that the current shares were about right, none of them thought they were exactly right. For example, no one thought that Russia should continue to pay three times the amount that Japan does. (In fact all but two people who evaluated both countries thought that Japan should pay as much or more than Russia.) Similarly, of those who evaluated both Russia and the U.S., only three thought that they should continue to pay equal amounts. And no one thought that Italy should continue to pay at the same rate as Poland, Hungary and Bulgaria.

A third interesting feature of the data is that all but one respondent (the Swede) gave answers in small whole numbers. In effect everyone except the Swede divided the countries into between two and five equivalence classes of dues-payers. This is not to say that the respondents ignored differences in population and GDP; the written comments accompanying their answers show that they did. Most, however, preferred to round the results instead of drawing fine distinctions. Of course, the existing dues structure encouraged this approach. It appears, however, that most respondents felt uncomfortable with a weighting scheme that *appears* to be very precise, even though in fact it is based on estimates of GDP and population that are subject to considerable error, and that in any event are proxies for more elusive concepts like ability to pay.

## 6. Class constraints

The grouping of stakeholders into equivalence classes is quite common in problems of this type. Indeed for practical reasons this may be necessary when the number of stakeholders is large. In distributing shares from state enterprises, for example, it seems natural to declare in advance that all laborers are to be treated alike, all managers alike, and all citizens

[3] The Swedish respondent proposed that the ratios be in proportion to the following criterion: (population/maximum population) + (GDP per capita/maximum GDP per capita).

alike. The issue then becomes what the appropriate ratio should be between a representative laborer, a representative citizen, and a representative manager. The scheme developed above is easily adapted to this case. Let there be m equivalence classes $C_1$, $C_2$, . . ., $C_m$, and let $n_j$ be the number of individuals in the jth class. Let $n = \Sigma n_j$ denote the total number of individuals. It is given that all members of the same class must receive equal shares. The method $F^0$ may then be modified as follows. First, each individual states the relative shares that should hold between representative individuals in the *other* classes. Thus a *partial report* of individual i is a vector $\mathbf{r}_i$ of m - 1 numbers $r_{ij}$, where j ranges over all classes that do not contain i. The numbers $r_{ij} : r_{ik}$ represent the proportions that i believes should hold between each member of class j and each member of class k. A *solution* is an m-vector $\mathbf{x} \geq \mathbf{0}$ such that $x_j$ is the share of each individual in class j and $\Sigma n_j x_j = 1$. Given n partial reports $\mathbf{r}_i$ (i = 1, 2, . . ., n) the process $F^0$ chooses the unique $\mathbf{x} \in R^m$ such that

$$x_j = (1/n) \sum_{i:i \neq j} (r_{ij}/\sum_{k:k \neq i} r_{ik} n_k).$$

Concretely, we give 1/n of the pie to each individual, who then divides it among the n - 1 remaining individuals *subject to* the constraint that all members of the same class must receive equal shares in his division.

To illustrate, consider the following variant of the first example. There is one manager, two laborers, and three citizens. Every citizen thinks the proper ratio for a manager and a laborer is 1 : 0, every laborer thinks the proper ratio for a manager and a citizen is 1 : 1, and the manager thinks the proper ratio for a laborer and a citizen is 2 : 1.[4]

[4] In general, individuals in a given class will have different opinions, but this presents no real complications.

Normalizing so that each report sums to one, we obtain the matrix:

| | mgr | lab1 | lab2 | cit1 | cit2 | cit3 |
|---|---|---|---|---|---|---|
| mgr | * | 2/7 | 2/7 | 1/7 | 1/7 | 1/7 |
| lab1 | 1/4 | * | * | 3/4 | 3/4 | 3/4 |
| lab2 | 1/4 | * | * | 3/4 | 3/4 | 3/4 |
| cit1 | 1 | 0 | 0 | * | * | * |
| cit2 | 1 | 0 | 0 | * | * | * |
| cit3 | 1 | 0 | 0 | * | * | * |

The row average yields the relative shares $F^0(\mathbf{r}) = (49/84, 4/84, 9/84)$ for manager, laborer, and citizen respectively.

## 7. Conclusion

We conclude by suggesting several other applications of this approach. Suppose, for example, that a person dies without leaving a will. Ordinarily the probate court would distribute the estate among the surviving spouse and children according to ratios specified in the law. If for some reason there were no applicable law, however, then the heirs might determine a fair solution using the method $F^0$ described above. Or they might try to negotiate a solution, knowing that if they failed to agree they would resort to this procedure.

As a second example, consider a village that receives a shipment of grain to ward off famine. Different classes of villagers -- children, the elderly, pregnant women, etc. -- might be thought to deserve different amounts. The traditional way of solving such problems is to have an authority figure such as the chief of the tribe divide it, and in a traditional society this might well be considered fair. A more democratic approach, however, would be to employ a method like that described above, using the expressed opinions of the villagers as inputs.

A third potential application is to the design of constitutional conventions. Suppose that a group of n states plans to form a federation with a single legislature. The states differ in population and perhaps in other ways that call for giving them different numbers of representatives in the new confederation. The precise division of seats will be one of the issues decided in the constitutional convention. This raises the perplexing question of how many delegates each state gets to send. If each sends the same number, and they vote by majority, the outcome will probably be biased in favor of equal division. This might be regarded as unfair if the states are very different in size.

The preceding analysis shows how this difficulty may be surmounted. Let each state send an equal number of delegates, and let each delegate specify a fair ratio of representation for the other states. Thus the pie is the number of seats in the legislature, and the delegates give their opinions about the relative number of seats that each of the other states should receive. The outcome using the method $F^0$ is a ratio of representation between the various states. The actual distribution of seats is determined by a choosing a total number of seats, and allocating them in proportion to the shares according to some standard apportionment formula like Webster's method (Balinski and Young, 1982). In this way one avoids the circularity problem that results when the distribution of seats in the convention influences the distribution of seats in the legislature itself.

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