

# Working Paper

## An Interactive Reservoir Management System For Lake Kariba

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September 1995



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# **AN INTERACTIVE RESERVOIR MANAGEMENT SYSTEM FOR LAKE KARIBA**

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## Foreword

Reservoir management might be characterised as one of the oldest topics for decision support. On one hand this is due to the fact that it is very clear what has to be decided. On the other hand this is due to the important consequences of the decisions. In the first years of reservoir management, the emphasis was on optimization. Later, other aspects were included like the use of multiple criteria and forecasts for future inflow. The current paper presents a state-of-the-art approach to the reservoir management of Lake Kariba by integrating virtually all essential aspects with the expert knowledge of the reservoir manager in an interactive system based on up-to-date information technology. The present paper again demonstrates how fruitful it is to combine within IIASA the expertise on water problems with the expertise on decision support methodology.

# AN INTERACTIVE RESERVOIR MANAGEMENT SYSTEM FOR LAKE KARIBA

## ABSTRACT

This paper presents a user-interactive decision support system (DSS) for the management of the Lake Kariba reservoir. Built in the fourth-generation computer language IFPS, the system takes into account relevant reservoir characteristics and parameters, such as the amount of hydropower generated, reservoir storage throughout the year, and the amount of water released for down-stream usage. The system blends water release rules determined previously using optimization and simulation-based scenario analyses with expert input from an experienced reservoir manager, yielding an intuitive and realistic DSS with which the reservoir manager may easily identify. The DSS also includes a Box-Jenkins time series model that forecasts future inflows. Each month, the system provides the manager with a proposed release schedule, which the manager then uses to explore and evaluate the consequences in terms of the decision criteria, over an extended period of time. The types of information provided to and sought from the manager correspond closely with actual reservoir management practice. An important characteristic of the system is that the manager can quickly explore various different potential release decisions *a priori*, for a variety of potential inflow scenarios, including predicted inflows for average hydrological years, as well as inflows reflecting extreme events such as drought and flood periods. The manager can compare the results of the release decisions made in the scenario analysis, both with the release strategy proposed by the system and with historical release decisions, thus aiding the manager in establishing effective reservoir management policies in practice. Thus, rather than a mechanical value, our DSS offers the manager a flexible problem analysis with suggested courses of action. We illustrate the system using example sessions with an experienced reservoir manager. While the system is designed specifically to support the management of Lake Kariba, its extension to a more general class of reservoir management problems is straightforward.

**Key words:** Decision Support Systems, Water Management, Reservoir Management.

# AN INTERACTIVE RESERVOIR MANAGEMENT SYSTEM FOR LAKE KARIBA

## 1. INTRODUCTION

The literature abounds with reservoir management models, covering a wide range of both theoretical methods and applications. Reservoir management involves two fundamentally different decision problems, reservoir planning and reservoir operation. At the planning stage, the reservoir design, location and size are determined, while day-to-day reservoir management involves establishing and implementing decision rules that guide the amount of water to be released from the reservoir at any point of time. This set of decision rules is also called the reservoir operating policy or control strategy. Even though the reservoir design process requires the formulation of meaningful operating policies, the search for acceptable reservoir operation policies may be viewed as a crucial component of reservoir management at all stages.

The reservoir management problem can be analyzed using various different quantitative techniques. In a review paper, Klemes (1981) shows among others simple Ripple diagram method, developed at the beginning of the 20<sup>th</sup> century. More sophisticated modeling frameworks include simulation-based (Ford 1990; Sigvaldason 1976), optimal control (Georgakakos 1993) and optimization-based models. The latter can be divided into single objective (Rabinowitz, Mehrez and Oron 1988) and multiple objective models (Can and Houck 1984; Goulter and Castensson 1988; Haines, Hall and Freedman 1975; Yang, Burn and Lence 1992). Giles and Wunderlich (1981) use dynamic programming to solve an operational model with five criteria. Reznicek and Cheng (1991) discuss the implementation of stochastic methods for reservoir operation. Tatano *et al.* (1994) propose a chance constrained model for determining optimal operating policies in the presence of extreme conditions such as droughts. Yeh (1985) gives a comprehensive review of state-of-the-art reservoir management models.

Each particular reservoir management problem has its unique aspects, and no single universally applicable problem formulation exists (Yeh 1985). While providing in-depth insights in basic principles of reservoir management, general problem formulations are based on a number of simplifying assumptions that may fail to represent a comprehensive framework for a particular application. Therefore, one needs to be careful in generalizing the applicability of approaches developed for specific problem situations.

The task of formulating acceptable reservoir operating policies is complicated by the uncertainty of reservoir inflows, the existence of multiple, conflicting objectives, and impreciseness in the problem objectives and constraints. Moreover, the optimal operating policies derived through

quantitative methods may not be implemented at the reservoir site, because when making the actual decisions, reservoir managers depend in part on their own expertise and use additional information that is not included in the formal models. Thus, rather than striving for an optimal operating policy, it may be more realistic to use manager-interactive methods to determine an acceptable and feasible operating policy that leads to a satisfactory achievement of the objectives, taking into account formal parameters as well as the reservoir manager's personal intuition and expertise.

In this paper, we describe a decision support system (DSS) for managing the Lake Kariba reservoir. The system has several important characteristics. We develop a tool of analysis that is easy to use and easily accepted by managers who have little training in quantitative analysis, and that corresponds as closely as possible to the way in which water release decisions were made historically. Of course, the overriding goal is to design decision rules that significantly improve on previously practiced rules, especially in the event of extreme hydrological conditions such as flooding or drought.

The manager can easily use the DSS on a daily, weekly or monthly basis, to quickly analyze the impact of several different alternative courses of action (such as water release schemes) on the relevant objectives and other quantities of interest (such as electricity generation, reservoir storage, number of flood gates opened). The system is built in the fourth generation software Interactive Financial Planning System (IFPS 1988), which combines a powerful spreadsheet-like format and various scenario generation and report generation capabilities with an English-like user interface. This system has been used successfully as the model-base of many real DSSs, especially in applications involving scenario analysis over time, so that the IFPS software is particularly suitable for our purposes.

The remainder of our paper is organized as follows. In the next section, we briefly discuss general characteristics of the Lake Kariba reservoir, followed in Section 3 by an overview of the inflow forecasting model and the operating policy guidelines and used in our system. Section 3 also reviews the mathematical formulation of our model. We introduce our DSS framework and the nature of the model-user interaction in Section 4. In Section 5, we present an illustration of the use of our DSS, followed by a discussion of possible extensions in Section 6 and concluding remarks in Section 7.

## 2. THE LAKE KARIBA RESERVOIR

Located in Africa along the border of Zambia and Zimbabwe, Lake Kariba is the fourth-largest man-made lake in the world. At its maximum retention level, the lake covers an area of over 5,600 km<sup>2</sup> and has an active storage capacity that exceeds 70 km<sup>3</sup>. More details on the Zambezi River Basin can be found in Pinay (1988). Hydropower plants installed at the northern (Zambian) and Southern (Zimbabwean) banks of the reservoir dam, and a smaller hydropower facility located on the Kafue river,

the northern tributary to the Zambezi river, jointly supply over 70 percent of the energy produced in these two countries combined (ZESA 1986). Since their completion in 1977, the hydropower facilities have supplied an average monthly energy of about 600 gigawatthour (GWh), with an almost constant distribution throughout the year. Zambia and Zimbabwe operate the facility jointly, and share the electricity generated on a fifty-fifty basis. The geographic location of the Zambezi River Basin is shown in Figure 1.

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Figure 1 About Here

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Gandolfi and Salewicz (1990, 1991) discuss extensively the hydrological conditions and operational objectives associated with managing of Lake Kariba, and conclude that the Lake Kariba reservoir management problem revolves essentially around the balancing of two conflicting objectives: (1) to maintain a fixed and as high as possible level of energy production; and (2) to maintain a flood reserve at the beginning of the rainy season, in order to avoid high discharges through flood gates during peak flow periods.

The first objective derives from the fact that the Kariba hydropower scheme operates in the base load, and must supply the electrical network with as high and reliable an energy output as possible. The second objective reflects that the opening of flood gates can have undesirable and potentially dangerous consequences. For instance, the release of large amounts of water through the flood gates causes vibrations in the dam, which may compromise the dam's structural integrity. Since the inflows into Lake Kariba fluctuate wildly and are difficult to predict, it is impossible to avoid using the flood gates altogether, but their use should be limited as much as possible. Another consideration is that high discharges from the reservoir may endanger the population living downstream and create operational problems at the downstream Cabora Bassa Reservoir in Mozambique. There also exist other objectives, related to human activities and wildlife protection in the areas downstream of the reservoir. However, these objectives are difficult to quantify, and appear to be less important for the management of the Lake Kariba (Gandolfi and Salewicz 1990, 1991).

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Figure 2 About Here

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The catchment area at the dam site upstream of Kariba gorge covers approximately 664,000 km<sup>2</sup>. Figure 2, which shows the reservoir inflows for three hydrological years from October 1967 until September 1970, indicates that the rainfall pattern is strongly seasonal, with a typical rainy season from November to March, and a dry period for the remainder of the year. The inflow into Lake Kariba lags several months behind the rainfall upstream. Consequently, the period of high inflows typically starts in February, with peak flows in April and May, after which the inflows decrease substantially for the remainder of the year. Almost 60 percent of the  $9 \times 10^9$  m<sup>3</sup> average annual flow from the lower catchment occurs between January and March (Santa Clara 1988). In addition to the seasonality of the inflow pattern, there are also large variations in inflow quantities across different years, complicating the task of accurately forecasting lake inflows.

### 3. KARIBA RESERVOIR MANAGEMENT MODEL

We next provide an overview of the most important components of our model, the inflow forecasting model, the operating policies, and an overview of the major equations of our model. A summary of the variables and model equations is given in the appendix.

#### 3.1. Inflow Forecasting

The uncertainty of reservoir inflows can be dealt with in several different ways. One way is to explicitly use probabilistic methods and techniques in formulating the model. Another is a stochastic optimization approach, taking into account either the probabilities or expected values of critical parameters associated with the reservoir inflow and storage (Tatano *et al.* 1994). However, both these approaches require strong, often overly simplifying and unrealistic assumptions about the system, for instance stationarity of the inflow process. One can also account for the stochastic nature implicitly, by combining a deterministic formulation of the reservoir operation problem with a forecasting model to predict the inflows, estimated using statistical time series techniques. Gandolfi and Salewicz (1991) report that naive forecasts and a Markovian model did not provide accurate forecasts of the inflows into Lake Kariba, and did not significantly improve the system's performance. Ríos Insua and Salewicz (1993) use a complex Bayesian dynamic linear forecasting model to predict the inflows. Although this Bayesian model can provide accurate forecasts, its formulation is very complex and may not be transparent to the user, while the calculations require specialized software and are computationally intensive.

We use Box-Jenkins seasonal ARIMA time series modeling (Box and Jenkins 1976) to estimate the average monthly inflows into the reservoir, using inflow data from October 1929 until September 1984, for a total of 432 observations. After estimating several alternative models, the one with the best

overall fit was a model with first and second order nonseasonal autoregressive components, reflecting that the current month's inflow depends on the inflows of the previous two months, and a seasonal autoregressive component that relates current inflows with those of 6, 12, 18 and 24 months prior. The estimated ARIMA model is given in (1),

$$(1-0.8441B+0.2791B^2)(1+0.1272B^6-0.2464B^{12}+0.1315B^{18}-0.1003B^{24})i_t = 1659.5+a_t, \quad (1)$$

where  $i_t$  is the inflow into the reservoir during month  $t$ , in  $10^6 \text{ m}^3$  per month,  $B$  is the backshift operator such that  $Bi_t = i_{t-1}$ , 1659.5 is the mean value of the time series, and  $a_t$  is a normally distributed random error with mean 0 and standard deviation  $\sigma_a$ . Multiplying out (1), yielding  $i_t = 1659.5 + \sum_{j=1}^{26} \phi_j i_{t-j} + a_t$ , we derive the  $k$ -step ahead forecast  $\hat{i}_{t+k}$  ( $k \geq 0$ ) in (2),

$$\hat{i}_{t+k} = \sum_{1 \leq m \leq k-1} \phi_m \hat{i}_{t+k-m} + \sum_{k \leq r \leq 26} \phi_r i_{t+k-r}. \quad (2)$$

For instance, the one-step ahead forecast  $\hat{i}_{t+1} = \sum_{r=1}^{26} \phi_r i_{t+1-r} = 0.8441i_t - 0.2791i_{t-1} - 0.1272i_{t-5} + 0.1074i_{t-6} - 0.0355i_{t-7} + 0.2464i_{t-11} - 0.2080i_{t-12} + 0.0688i_{t-13} - 0.1315i_{t-17} + 0.1110i_{t-18} - 0.0367i_{t-19} + 0.1003i_{t-23} - 0.0847i_{t-24} + 0.0280i_{t-25}$ . Note that in this formula several of the  $\phi_j$  equal 0, because the forecasting model is seasonal.

The model in (1) passed all Box-Jenkins diagnostic checks (Box and Jenkins 1976). In a further validation of the model, comparing the one-step ahead forecasts of (2) over the 432 months from October 1929 until September 1984 with the forecasts of Ríos Insua and Salewicz's (1993) Bayesian model over the same period, we found the difference between both methods in mean squared error to be statistically insignificant at  $\alpha = 0.05$ . Although the model proposed by Ríos Insua has been shown to yield accurate forecasts, it is complex, requires special purpose software tools and is computationally expensive, whereas (1) and (2) are simple, easily interpreted and can be embedded directly in an IFPS planning model. Therefore, the Box-Jenkins model is better suited for our purposes.

Comparing the one-step inflow forecasts from (2) for the period from October 1977 to September 1980 with the historical inflows during this period in Figure 3, we see that our model predicts the next month's inflow with good accuracy, except during the unusually wet period from March to May 1978, when the true inflow exceeded the predicted inflow. Figure 3 shows that for this period the forecasting model reacts to the changes in the inflows with a one month delay.

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Figure 3 About Here  
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### 3.2. Operating Rules

The operating rules implemented in our IFPS model are based on the rules estimated and tested by Gandolfi and Salewicz (1991), through simulation and optimization, in their study of the Lake Kariba Reservoir. Figure 4 shows that in the rules of Gandolfi and Salewicz (1991), the release from the reservoir  $r_t$  depends, among others, on active reservoir storage  $s_t$ , the time of the year  $t$ , and the maximum reservoir storage. Within the framework of our model, these operating rules are calibrated by expert managers at the Lake Kariba site.

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 Figures 4 and 5 About Here  
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Figure 5 depicts the physical maximum active storage of the lake,  $s_{\max} = 70,970 \times 10^6 \text{ m}^3$ , the operating rules estimated and tested by Gandolfi and Salewicz (1991) through simulation and optimization in their study of the Lake Kariba Reservoir labeled “upper bound” and “lower bound,” and the historical rule curve that was actually utilized by reservoir management labeled “rule curve,” throughout one hydrological year. Within the feasible range of 0 to  $s_{\max}$ , the active storage of the reservoir is divided into 4 zones. The boundaries of these zones vary by the season.

In the case of Lake Kariba, the quantity of water to be released is determined largely by energy production targets and the necessity to maintain the reservoir storage within the feasible range. We denote the release amounts necessary to achieve the lower and upper energy production targets by  $R_L$  and  $R_U$ , respectively. In the first storage zone defined by releases between 0 and  $R_L$ , one can release only as much water as is available in the active storage of the reservoir. Storage values in this range are indicative of extreme drought conditions. In the second zone, up to a storage of  $S_{Lt}$ , the release is constant ( $r_t = R_L$ ), at a level which is just enough to achieve the lower energy production target. Thus, this zone can be described as the reduced energy output zone. In the third zone, for storage values of  $S_{Lt} < s_t \leq S_{Ut}$ , release first rises linearly from  $R_L$  to  $R_U$ , the release necessary to meet the upper energy production target, and then remains at this level. The third zone is the normal operating zone for the reservoir. Finally, the fourth zone, where  $S_{Ut} < s_t \leq s_{\max}$ , reflects flood conditions, and release grows with a slope  $\alpha$ . Obviously, storage values above  $s_{\max}$  are impossible, as the reservoir would overflow.

In their analysis, Gandolfi and Salewicz (1991) determine the parameters of the operating rule shown in Figure 4, in particular the boundaries between the storage zones  $S_{Lt}$  and  $S_{Ut}$ , and the slope  $\alpha$ . The upper and lower bound rule curves of Gandolfi and Salewicz (1991) in Figure 5 reflect the

values of  $S_{Lt}$  and  $S_{Ut}$ . Since the rules estimated by Gandolfi and Salewicz (1991) form the basis of our modeling framework, as explained below, in the remainder of this paper we will refer to these rules as the proposed release rules, and to the corresponding storages as the proposed storage values.

The actual rule curve in the historical operation of the reservoir is very similar to Gandolfi and Salewicz's (1991) proposed rule curve, which clearly shows that management applied a similar concept of dividing the range of reservoir storage values into different storage zones was used. In fact, this rule curve approximates an "ideal" average operating condition for the reservoir (Loucks and Sigvaldasson 1982). In our DSS, the actual rule curve in a sense constitutes the reference trajectory for reservoir management, and was used as an additional decision aid during the simulation.

Of course, due to the stochastic nature of the actual reservoir inflows it is impossible to attain the proposed storage at all times, and in reality reservoir management will seek to approximate the proposed trajectory as closely as possible. Nevertheless, the proposed trajectory provides a useful benchmark for the actual release decision process. Actual (or predicted) reservoir storage values that exceed the upper bound ( $S_{Ut}$ ) or fall below the lower bound ( $S_{Lt}$ ) indicate that the process is in danger of moving out of control. If the current storage is too high, then there exists a potential danger of future flooding, requiring an increased release, while low storage values imply that the release in upcoming periods will likely be insufficient to satisfy energy production and water supply downstream, unless the amount of water currently released is reduced. Thus, if the reservoir storage reaches beyond the upper or lower bound, a change in the water release policy is required to avoid or mitigate a disastrous situation in the future.

Generally, even the actual storage values that are within the bounds, but have forecasted storage values that deviate substantially from the proposed trajectory, serve as an indication that reservoir management may be heading for problems down the road, unless immediate corrective action is taken. In most formal reservoir management models, reaching a critical reservoir storage value implies a prescribed change in water release policy, leaving reservoir management merely to implement the revised policy. In contrast, our DSS model presents the manager with a proposed revised release schedule based on the curves in Figures 4 and 5, which the manager can use to explore the implications for various different inflow scenarios. Thus, as we discuss in more detail in Sections 4 and 5, rather than a mechanical value, our DSS offers the manager a flexible problem analysis with suggested courses of action, and predicted consequences associated with various different release decisions.

### 3.3. Model Description

We next describe the mathematical model formulation. As this is an important issue within our interactive modeling framework, at this point we distinguish between *proposed* release strategies and active storage ( $r_t$  and  $s_t$ , respectively, for month  $t$ ), and *actual* release strategies and active storage values ( $u_t$  and  $y_t$ , respectively, for month  $t$ ). The proposed values are obtained by applying Gandolfi and Salewicz's (1991) release rule curve, and represent the values that are initially presented to the decision maker for evaluation. The actual values represent the release and storage values that are finally selected by the decision maker, after interactively exploring various tradeoffs and scenarios within the DSS model. At the start of the interactive process, the proposed and actual release and storage values are assumed to be the same, but of course this may not be the case later in the process. We also remark that throughout this paper the term reservoir storage should be interpreted as active reservoir storage, which excludes the reservoir volume below the lowest level of the outlet tunnels.

We will denote the predicted value of a variable  $x$  by  $\hat{x}$ . Such values are usually predicted within our DSS modeling framework, but can be adjusted by the decision maker as part of the interactive scenario analysis. In generic terms, the reservoir mass balance equation, *i.e.*, the definitional equation relating the actual reservoir storage at the beginning of month  $t+1$  ( $s_{t+1}$ ) to the storage at the beginning of month  $t$ , taking into consideration inflows ( $i_t$ ), evaporation losses ( $v_t$ ) and actual releases ( $u_t$ ) during month  $t$ , is given in (3),

$$s_{t+1} = s_t + i_t - u_t - v_t \quad (3)$$

For the purpose of managing the reservoir, in (3) we want to predict  $s_{t+1}$  at the beginning of period  $t$ , based on estimates for  $i_t$  and  $v_t$ , and the decision variable  $u_t$ . Thus, within the context of our models, we replace (3) by  $\hat{s}_{t+1} = s_t + \hat{i}_t - u_t - \hat{v}_t$ .

#### *Proposed Release and Storage*

Equation (4) indicates that, as already discussed above, the proposed amount of water released during month  $t$  according to the rule curve estimated by Gandolfi and Salewicz (1991),  $r_t$ , depends on the storage at the beginning of  $t$ ,  $s_t$ , the physical maximum ( $s_{\max}$ ), the intermediate storage amounts  $s_{Lt}$  and  $s_{Ut}$ , and the time of the year (season). The values of  $s_{Lt}$  and  $s_{Ut}$  are seasonally dependent, with lower values just prior to and during the rainy season and higher values during the dry season.

$$r_t = r_t(s_t, s_{Lt}, s_{Ut}, s_{\max}, t) \quad (4)$$

The target release level  $r_L$  in Figure 4 corresponds with an energy production of 600 GWh per month, while  $r_U$  yields 700 GWh per month. From (3), we know that the mass balance equation predicting the storage  $y_{t+1}$  based on the release rule  $r_t$  proposed by Gandolfi and Salewicz (1991) is given by (5),

$$\hat{y}_{t+1} = s_t + \hat{i}_t - r_t - \hat{v}_t, \quad (5)$$

where  $\hat{v}_t$  is defined in (6),

$$\hat{v}_t = l_t[a(s_t + \hat{y}_{t+1})/2 + b]. \quad (6)$$

In (6),  $l_t$  is the seasonally dependent evaporation rate in month  $t$ , whereas  $a$  and  $b$  are scalar coefficients (Gandolfi and Salewicz 1991). By substitution of (6) into (5), the mass balance equation can be simplified to (7),

$$\hat{y}_{t+1} = c_{1t}s_t + c_{2t}(\hat{i}_t - r_t) + c_{3t} \quad (7)$$

As  $\hat{v}_t$  is seasonal, the coefficients  $c_{1t}$ ,  $c_{2t}$  and  $c_{3t}$  in (7) are seasonal as well. Our DSS model uses the values estimated by Gandolfi and Salewicz (1991).

#### *Actual Release and Storage*

The actual release during month  $t$ ,  $u_t$ , is determined by the reservoir manager, and may or may not be identical to  $r_t$ . Defining the amount of water actually released according to its use, we have (8),

$$u_t = u_{t1} + u_{t2} \quad (8)$$

where  $u_{t1}$  is the amount released for energy production, and  $u_{t2}$  the amount released in order to control the reservoir level, but not used for energy production, during month  $t$ .  $u_{t2}$  is also called the spill. The maximum flow through the turbines,  $\bar{u}_{\text{turb}}$ , determines the upper bound of  $u_{t1}$ , *i.e.*,

$$u_{t1} \leq \bar{u}_{\text{turb}}. \quad (9)$$

From (3), we determine the actual predicted storage  $\hat{s}_{t+1}$ , after simplification by substituting  $\hat{v}_t = l_t[a(s_t + \hat{s}_{t+1})/2 + b]$ , as (10),

$$\hat{s}_{t+1} = c_{1t}s_t + c_{2t}(\hat{i}_t - u_t) + c_{3t}. \quad (10)$$

### Remaining Equations

Following Gandolfi and Salewicz (1991), the tailrace level of the dam,  $g_t$ , is estimated as a function of release, as in (11),

$$g_t = \alpha u_t^\beta + \gamma, \quad (11)$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are scalars. The head value  $h_t$  at the beginning of month  $t$  is given by (12),

$$h_t = L_t - g_t, \quad (12)$$

where  $L_t$  is the reservoir level. Due to the characteristics of Lake Kariba, the reservoir level can be interpolated accurately as a piecewise linear function of storage, as expressed in (13),

$$L_t = L(s_t) = \lambda_j s_t + \mu_j, \text{ if } s_t \in \mathcal{F}_j, \quad (13)$$

where  $\mathcal{F}_j$  is the  $j^{\text{th}}$  segment of the piecewise linear function, and  $\lambda_j$  and  $\mu_j$  are the slope and constant term of the  $j^{\text{th}}$  segment, respectively. Finally, the total amount of energy produced during month  $t$  depends on the release  $u_{1t}$  and the estimated average reservoir level during month  $t$ , and thus, through (12), as a function of the estimated average head level during month  $t$ , is given in (14),

$$\hat{e}_t = \eta \epsilon u_{1t} (h_t + \hat{h}_{t+1}) / 2 \quad (14)$$

where  $\eta$  is the efficiency coefficient of the energy generating facilities,  $\hat{h}_{t+1}$  is the predicted head level at the beginning of month  $t+1$ , and  $\epsilon$  a scalar coefficient.

The number of flood gates that need to be open during month  $t$ ,  $f_t$ , depends on the amount of water spilled, *i.e.*, on  $u_{t2}$ . The capacity of each flood gate is  $4,178.5 \times 10^6 \text{ m}^3$  per month, and there are 6 flood gates.

## 4. AN INTERACTIVE RESERVOIR MANAGEMENT SYSTEM

### 4.1. DSS Framework

We selected the fourth generation IFPS computer package (Gray 1988) to implement our interactive DSS. This software is particularly well-suited for our particular application, because it uses a spreadsheet-like user interface, allows for a model implementation using English-like statements, is designed specifically for temporal analysis, with a built-in capability for not really defining variables over time, and has specialized functions that are relevant for scenario analysis, such as goal seeking and what-if analysis. Moreover, IFPS is a well-tested commercial package that has been used successfully

as the model base in many DSS applications.

The interactive nature of our DSS, combined with its user-friendly interface, enables the reservoir manager to conduct an extensive scenario analysis within minutes. The structure of the model, including the inflow forecasting model, are straightforward and transparent to the manager. Even though we did not include this option in our application, it is possible to extend the model to include a formal multicriteria optimization analysis, if the mainframe version of IFPS is used. The PC version of IFPS does not include the optimization facility.

#### 4.2. The Interactive Process

For each month, the scenario analysis in our DSS is decomposed into three stages. Each stage differs with respect to the type of information used during the analysis, the type of model(s) applied, the way in which decisions are made, and the type of decisions made. The schematic structure of the three-stage process is shown in Figure 6.

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 Figure 6 About Here  
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As noted above, the two major objectives of the reservoir operation are to meet a predefined energy production target and to minimize reservoir discharge during the peak flood period. These objectives are clearly conflicting, as the energy production objective implies that one should keep as much water in the reservoir as possible once the energy production target is met, to secure meeting the energy production in future months, whereas the flood protection objective implies that the reservoir should be kept at as low a level as possible, so that the reservoir can absorb the inflows during flood periods. Thus, any reasonable operating policy must thus seek a compromise between energy production on the one hand, and a reservoir discharge level that maintains "controllability" of the reservoir, if possible under any hydrological and operating conditions, on the other hand. In this context, controllability means the ability to smooth the natural inflow trends, by releasing additional water for energy production during low inflow periods, and storing part of the peak inflows during flood periods.

In practice, the overriding consideration is to secure the controllability of the reservoir, and every operator of the multi purpose reservoir system attempts to meet this objective by searching for various compromise solutions, which in turn allows for the achievement of other objectives, such as hydropower generation, water supply and flood control. To our knowledge, such a user-interactive formulation of hierarchically structured objectives has not been applied in the field of reservoir

management. This may stem from a lack of theoretical and empirical tools and methodological considerations that would allow one to address formally the notion of “controllability” of the reservoir and related issues.

Stages 1 and 2 of the process take place at the beginning of each month  $t$ , when the release decision is to be made. Stage 3 involves updating the model at the end of the month, when the actual inflow, evaporation, *etc.*, during month  $t$  are known. Thus, Stage 3 in month  $t$  prepares the model analysis of Stages 1 and 2 at the beginning of month  $t+1$ . Our three-stage procedure is also termed a rolling horizon procedure. We next describe the three stages in detail.

### *Stage 1*

As shown in the flowchart of Figure 6, Stage 1 of the process contains two models, the forecasting model and the operating policy model. At month  $t$ , the forecasting model uses (2) to generate predicted inflows to the reservoir  $\hat{i}_{t+k}$  for the next  $k = 12$  months. The operating policy model uses the inflow forecasts  $\hat{i}_{t+k}$  ( $k = 1, \dots, 12$ ) and the current reservoir storage  $s_t$  to determine a sequence of proposed releases  $r_{t+k}$  ( $k = 1, \dots, 12$ ), according to the decision rule developed by Gandolfi and Salewicz (1991). The sequence of proposed releases  $r_{t+k}$ , predicted inflows  $\hat{i}_{t+k}$ , and predicted values of other variables, such as the storage and reservoir levels, the reservoir head, the amount of spill and energy production output, serve as input into Stage 2 of the process.

### *Stage 2*

In Stage 2, the manager makes the control decisions. The reservoir manager interactively analyzes and evaluates the impact of both the proposed release decisions and alternative release strategies on the hydropower scheme and flood control, in terms of the above-mentioned variables of interest, over a twelve month planning horizon. As we will discuss further in our illustration below, in analyzing alternative scenarios the manager is not limited to evaluating the consequences of various different release strategies, but can also explore the impact on the reservoir system of various deviations from the inflow forecasts. Throughout the interactive analysis, the manager can compare the scenarios with the historical hydrological events and the historical reservoir operation.

A typical analysis of a particular release strategy involves an evaluation of how the state of the reservoir evolves over the next twelve months, in terms of storage and level, and in particular the state of the reservoir in twelve months; the extent to which the projected storage trajectory deviates from the historical rule curve used by operators of the Lake Kariba reservoir in the past; the position of the storage trajectory relative to the upper and lower storage limits; the rate of filling or depleting the reservoir; the rate at which the projected trajectory approaches or departs from the proposed rule curve

(Gandolfi and Salewicz 1991) and the historical rule curve; the evolution of the projected storage in February when storage should attain its lowest value in the annual cycle, in order to anticipate the extent to which the reservoir can absorb the anticipated inflows during the wet period; a comparison of this value with that projected for February one month prior; the evolution of the projected storage in June and July at the end of the period of high inflows and at the beginning of the low inflow season; a comparison of these values with the projections made one month prior; differences between forecasted and observed inflows; the extent of the energy production deficit, if any; the distribution over time of projected future energy production deficits; the expected amount of spill, its distribution over time (*e.g.*, it is better to spill  $5,000 \times 10^6 \text{ m}^3$  during two consecutive months than to spill  $10,000 \times 10^6 \text{ m}^3$  in one month); and the need for opening or closing additional flood gates.

Of course, this list is the bare minimum of what the manager will want to consider in deciding on the appropriate release strategy. Whenever relevant, the manager can access other types of information as well. The final decision about how much to release during the next month not only involves the series of analyses and comparisons in Stage 2, but also takes into account the intuition and non-quantifiable reasoning of the human decision maker. This judgmental model component is very important, and distinguishes our approach from many previous reservoir management models.

### *Stage 3*

Once the final release decision for month  $t$  has been made and implemented in Stage 2, the model parameters and variables are updated at the end of the month in Stage 3, in preparation of the Stage 1 analysis at the beginning of month  $t+1$ . For instance, at the end of month  $t$ , the actual release decision, the true values of hydrological variables, such as evaporation and inflow, and the corresponding values of energy production and spill are calculated for month  $t$ , as well as the initial storage  $s_{t+1}$  for month  $t+1$ . Once the relevant information has been updated, the rolling horizon procedure is repeated for month  $t+1$ .

## 5. ILLUSTRATION

To test the effectiveness and accuracy of our DSS, we conducted a simulated analysis of the reservoir operation over the period from October 1962 (soon after commissioning the reservoir) to September 1984, using a decision maker who was very familiar with the Lake Kariba reservoir management problem. This period of 22 hydrological years is rich in events and covers very different hydrological situations, including floods, lasting droughts and average hydrological conditions.

All simulations were performed on a monthly basis. Each month of the simulation, the decision maker went through the three-stage decision process shown in Figure 6. Throughout the analysis, the decision maker was provided with exactly the same information as he would have had available if he would really have been managing the reservoir. For instance, in our simulations the decision maker relied on the forecasted inflows provided by (2), and never had access to the next month's true inflow, until the update in Stage 3 of the decision process at the end of the month.

Consistent with our modeling framework, during the simulation process the decision maker considered two main objectives of plant operation: (1) to achieve an energy production target of 732 GWh per month, which exceeds the current energy production target (used at the dam site) of 600 GWh per month by over 20 percent. As the current target of 600 GWh per month appears conservative, the decision maker set a quite ambitious energy production target, thus testing the ability of the system to deliver more energy, without seriously compromising the minimum sustainable storage and water supply requirements under drought conditions; and (2) to minimize release through the flood gates, which translates into two sub objectives: to reduce the total number of months that the flood gates are open, and to reduce the maximum flood discharges, at the cost of eventual prolongation of the time that the flood gates remain open.

One can analyze the results of the simulation and the comparison with the historical operation scheme from many different points of view, and it is possible to use several different indicators to describe the state of the reservoir at a given time. In our simulation, the decision maker focused on reservoir storage, as the trajectory of this quantity summarizes the dam operation, and forms the most suitable basis for characterizing the reservoir operation over a long planning horizon.

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Figures 7 and 8 About Here  
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Figure 7 shows both the real reservoir storage trajectory, obtained from its historical operation, and the simulated storage trajectory that reflects the release decisions made by the decision maker. Both trajectories start from the same initial state in October 1962, shortly after the commissioning of the dam, and run until September 1984, in the midst of a severe drought which occurred in the Zambezi basin in the mid to late eighties. Interestingly, we observe that the actual and proposed storage patterns are quite similar, suggesting that the Lake Kariba reservoir has been operated by quite experienced managers, who tend to make release decisions that correspond closely to the proposed rule curve, at least on an average basis. While similar for the most part, the two curves differ significantly during several periods. The first large discrepancy occurs between January 1963 and July 1964, which

marked the time just after one of the largest floods in the history of the Zambezi river basin. Apparently, at this time reservoir management of the Kariba dam feared further flooding, and decided to draw down the level of the reservoir, in preparation for the next flood. However, this flood never occurred, and as Figure 8 shows clearly, too much water was released during this time, depleting the reservoir. In contrast, the forecasts yielded by (2) predicted moderate future inflows, rather than floods, so that in our simulation the decision maker was able to balance the discharge policy with the risk of overflowing the reservoir. The result was a release policy that avoided the danger of filling the reservoir, while allowing a much lower maximum release quantity than what happened in reality. Figure 8 shows that for the simulated operation of the reservoir the highest release during this period was about  $7,000 \times 10^6 \text{ m}^3$  per month, whereas the actual operators of the scheme released over  $12,000 \times 10^6 \text{ m}^3$  per month.

The emphasis in the decision process on reducing flood releases is reflected by the fact that from October 1962 to September 1984 (22 years, 264 months) the flood gates were open for only 48 months, while in reality the reservoir operators discharged water through the flood gates during 77 months. Moreover, the maximum release through the flood gates during the historic operation of the dam amounted to  $17,137 \times 10^6 \text{ m}^3$  per month (4 gates), while during the simulation process, the largest release was  $10,493 \times 10^6 \text{ m}^3$  per month (3 gates).

The period from October 1968 to September 1970 serves as a good illustration of the differences between the release policy implemented by the operators of the dam and that derived with the aid of our DSS. This period included two consecutive wet years. The reservoir was already filling up quickly in January – February 1969, and there was a risk that the storage would not only exceed the proposed storage trajectory, but eventually reach the maximum value and overflow the reservoir. During the historical operation, the reservoir operators were changing releases dramatically from month to month, depending upon observed fluctuations of the inflows, reflecting a narrow control strategy. The result was that in December 1969 approximately  $1,900 \times 10^6 \text{ m}^3$  was released, and in the next month,  $7,500 \times 10^6 \text{ m}^3$  (see Figure 9).

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 Figure 9 About Here  
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During the same time period, the release policy derived by the decision maker using the DSS avoided sharp and unnecessary release variations, remained relatively stable, and consequently resulted in a reservoir storage that was  $20,000 \times 10^6 \text{ m}^3$  higher than during the historical operation at the beginning of the dry period that immediately followed in 1971–1973. This, in turn, enabled the

decision maker to maintain a much higher level of energy production and water availability downstream during 1971–1973.

We next illustrate how the decision maker used the three-stage framework of Figure 6 in making the month-by-month release decisions with the assistance from DSS. First, the decision maker carefully studied the state of the reservoir, the inflow forecasts from (2), and the impact of the proposed release schedule. As Figure 10 shows, for the period from October 1968 until September 1970, the proposed releases based on Gandolfi and Salewicz's (1991) operating rule and the *current* state only were constant, and determined solely by energy production considerations. However, the inflow *forecasts* over the next 12 months derived with (2) signaled the possibility of future flooding, and the projected storage trajectory based on these forecasts far exceeded the proposed rule curve, crossing over the maximum reservoir storage  $s_{\max}$ . Therefore, the decision maker opted to spill water in the period of December 1968 to April 1969, although the operating rule at that time (based on the current state of the reservoir) did not require this action. In doing so, a buffer storage was prepared for the incoming flood predicted for May and June of 1969.

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 Figure 10 About Here  
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Moreover, although reservoir storage was still in the flood zone in July and August of 1969, the decision maker decided to release less water than proposed by the operating rule, again based on the inflow forecasts for the next 12 months. Given the inflow forecasts in September 1969, the reduced inflow during the previous month and a reluctance to spill too much water, the decision maker decided to release as much water as was needed for maximum energy production (*i.e.*,  $u_{t1} = \bar{u}_{\text{turb}}$  and  $u_{t2} = 0$ ) in September. However, one month later it became evident that the expected decrease of inflows could not be confirmed, and the inflows as well as predicted inflows for the next 12 months were above average. This trend alarmed the decision maker, who subsequently gave a higher priority to the flood protection objective, by releasing more water than proposed by the operating rule, allowing him to gain some slack in the reservoir, and avoid high releases from February to May 1970. Figure 11 combines Figures 9 and 10, and shows the historical (real) releases, the releases proposed by applying Gandolfi and Salewicz's (1991) rules, and the simulated releases determined by the decision maker within the framework of our DSS.

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 Figure 11 About Here  
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Figure 11 demonstrates that, during October 1968 to September 1970, the release rule developed by Gandolfi and Salewicz (1991) determined release volume based on the current reservoir storage only, and did not contain any mechanism that would take into account information available on changes in the inflow pattern. In contrast, the DSS is capable of incorporating various sorts of information (even informal) into the decision making process. The feedback nature of the interactive process, allowing for an active anticipation of extreme events, results in more flexible release policies.

It should be noted that, although the analytical process leading to the final release decision was very fast, our simulation was performed under laboratory conditions, and was free of many factors (*e.g.*, political and economic) which influence real decision processes associated with the operation of hydropower dams. However, the simulation and analytical tool IFPS used during our experiment proved to be so flexible and versatile, that it is possible that requests by political and economic authorities for additional analysis and explanation can be satisfied without destroying the structure of our DSS model.

## 6. EXTENSIONS

One way to extend the current model is to include an optimization model that estimates appropriate values of  $S_{Lt}$  and  $S_{Ut}$ , based explicitly on current and forecasted hydrological conditions. These estimates can then be presented to the reservoir manager in Stage 1 at the beginning of the interactive decision process. It is important that the results from such an optimization are interpreted only as proposed release decisions, so that the manager's decision making flexibility is not restricted.

In our current model, we use the formulation by Gandolfi and Salewicz (1991), who linearize several key relationships in the model, such as that between active reservoir storage and reservoir level, which is approximated by a piecewise linear function, and that between the head and tailrace level. The model can be extended by treating these relationships as nonlinear.

During its construction, the Lake Kariba dam had to be redesigned, because the reservoir inflows were underestimated due to a lack of reliable hydrological data. In order to handle the unanticipated large reservoir inflow, the designers decided to increase the number of flood gates. Due to the vibrations, it is never desirable to open and close flood gates frequently. However, in the case of the Lake Kariba dam, the issue of operating stability is even more pressing, due to the structural characteristics and design flaws of the dam. Hence, management needs to be extremely careful with the simultaneous opening of multiple flood gates, and it is of interest to consider the number of times the flood gates are opened and closed, as well as the number of months that the flood gates are open. This extension is straightforward within our modeling framework.

Given the user-interactive nature of our DSS, it is of interest to implement the system using state-of-the-art graphical tools to display the different scenarios during the interactive decision process. The recently released PC-based IFPS/Visual software package (IFPS 1995), which also includes improved simulation capabilities, is an excellent candidate for providing such an extension.

## 7. SUMMARY AND CONCLUSIONS

We introduce a user-friendly IFPS-based DSS for reservoir management, which was shown to yield accurate operating rules for the Lake Kariba Reservoir, forecasts future inflows accurately, integrates the various model components effectively, and provides a reliable and user-friendly decision aid.

The major contribution of our flexible DSS modeling framework is that it enables a scenario analysis that captures the decision maker's judgments, facilitating a flexible decision process. Our scheme of decision making process does not limit the decision maker to a narrow set of models, tools and information to be used in the control process. Instead, it offers a framework for almost unlimited possibilities of search for a good policy, by using different forecasting and scenario generation methods. Depending on the decision situation and expertise available, it is possible to use simple simulation methods, as we did in our application, or sophisticated optimization methods. The participation in the decision process is not limited to reservoir operators, and many parties – such as administrators and politicians – can be involved in the decision making process as well.

In our simulation experiment, comparing our approach with the historical performance at the Lake Kariba dam, the effectiveness of our DSS illustrated by an increased energy output of up to 22 percent (from 600 to 732 GWh per month), a reduction in the number of months that at least one flood gate is open from 77 to 48, and a reduced maximum discharge from  $17,137 \times 10^6 \text{ m}^3$  to  $10,493 \times 10^6 \text{ m}^3$ . Given the potential structural problems with the Lake Kariba dam, the importance of the latter two statistics is underscored by the dangers associated with excessive vibrations in the dam.

Although due to the location of Lake Kariba in our application non-energy demands such as agriculture and recreation do not play a major role, our modeling approach can easily be modified to incorporate these considerations. Hence, our basic DSS framework can be generalized to a wider class of reservoir management problems.

## REFERENCES

- Box, G. E. P. and Jenkins, G. M. (1976), *Time Series Analysis: Forecasting and Control*," Revised Edition, Holden-Day, Oakland, CA.
- Can, E. K. and Houck, M. H. (1984), "Real Time Reservoir Operations by Goal Programming," *ASCE Journal of Water Resources Planning and Management*, **110** (3), 297–309.
- Cohon J. L. and Marks, D. H. (1973), "Multiobjective Screening Models and Water Resource Investment," *Water Resources Research*, **9** (4), 826–836.
- Ford, D. T. (1990), "Reservoir Storage Reallocation Analysis With PC," *Journal of Water Resources Planning and Management*, **116** (3), 402–416.
- Gandolfi, C. and Salewicz, K. A. (1990), "Multiobjective Operation of the Zambezi River Reservoirs," Working Paper WP-90-31, International Institute for Applied Systems Analysis, Laxenburg, Austria.
- Gandolfi, C. and Salewicz, K. A. (1991), "Water Resources Management in the Zambezi Valley: Analysis of the Lake Kariba Operation," in *Hydrology for the Management of Large Rivers*, F. H. M. Van de Ven, D. Gutknecht and K. A. Salewicz (Eds.), IAHS Publication No. 201, 13–25.
- Georgakakos, A. P. (1993), "Operational Trade-Offs in Reservoir Control," *Water Resources Research*, **29** (11), 3801–3819.
- Giles, J. E. and Wunderlich, W. O. (1981), "Weekly Multipurpose Planning Model for TVA Reservoir System," *ASCE Journal of Water Resources Planning and Management*, **107** (2), 495–511.
- Goulter, I. C. and Castensson, R. (1988), "Multi-Objective Analysis of Boating and Fishlife in Lake Sommen, Sweden," *Water Resources Development*, **4** (3), 191–198.
- Gray, P. (1988), *Guide to IFPS/Personal*, McGraw-Hill, New York, NY.
- Haimes, Y. Y., Hall, W. A. and Freedman, W. A. (1975), *Multiobjective Optimization in Water Resources Systems*, Elsevier, Amsterdam, The Netherlands.
- IFPS, *User Guide, IFPS/Visual for the PC*, Comshare, Chicago, IL, 1995.
- Kindler, J., Salewicz, K. A., Slota, H. and Terlikowski, T. (1980), "Operation of Multiple Reservoir Systems: A Case Study of the Upper Vistula System," in *The Operation of Multiple Reservoir Systems*, Z. Kaczmarek and J. Kindler (Eds.), Collaborative Paper CP-82-S02, International Institute for Applied Systems Analysis, Laxenburg, Austria, 259–294.
- Klemes, V. (1981), "Applied Stochastic Theory of Storage in Evolution," *Advances in Hydrosciences*, **12**, 79–141.
- Loucks, D. P. and Sigvaldasson, O. T. (1982), "Multiple Reservoir Operation in North America," in *The Operation of Multiple Reservoir Systems*, Z. Kaczmarek and J. Kindler (Eds.), Collaborative Paper CP-82-S02, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1–104.
- Pinay, G. (1988), "Hydrobiological Assessment of the Zambezi River System: A Review," Working Paper WP-88-089, International Institute for Applied Systems Analysis, Laxenburg, Austria.

- Rabinowitz, G., Mehrez, A. and Oron, G. (1988), "A Nonlinear Optimization Model of Water Allocation for Hydrologic Energy Production and Irrigation," *Management Science*, **34**, 973–990.
- Reznicek, K. and Cheng, T. (1991), "Stochastic Modelling of Reservoir Operations," *European Journal of Operational Research*, **50**, 235–248.
- Ríos Insua, D. and Salewicz, K. A., "The Operation of Lake Kariba: A Bayesian Analysis," *Journal of Multi-Criteria Decision Analysis*, Forthcoming, 1995.
- Ríos Insua, D. and Salewicz, K. A. (1995), "The Coupling of Forecasting and Optimization: Development of the Operating Policy for the Lake Kariba Hydropower Scheme," unpublished manuscript.
- Ryan, B. F., Joiner, B. L. and Ryan, T. A., Jr. (1985), *Minitab Handbook*, Second Edition, Duxbury Press, Boston, MA.
- Salewicz, K. A. and Terlikowski, T. (1981), "Case Study in Hierarchical Control – The Upper Vistula Multireservoir System," Working Paper WP–81–44, International Institute for Applied Systems Analysis, Laxenburg, Austria.
- Santa Clara, J. M. A. (1988), "The Hydrological Operation of the Kariba Hydroelectric Scheme: Past, Present and Future," paper presented at *The 16th Congress on Big Dams*, San Francisco, CA.
- Sigvaldason, O. T. (1976), "A Simulation Model for Operating a Multi-Purpose Reservoir System," *Water Resources Research*, **12** (2), 263–278.
- Tatano, H., Okada, N., Yoshikawa, K. and Kawari, H. (1994), "A Frequency and Duration Constrained Model for the Optimization of a Single Reservoir Operation," in *Stochastic and Statistical Methods for Hydrology and Environmental Engineering*, K. W. Hipel and L. Fang (Eds.), **4**, 375–388.
- Yang, Y., Burn, D. H. and Lence, B. J. (1992), "Development of a Framework for the Selection of a Reservoir Operating Policy," *Canadian Journal of Civil Engineering*, **19**, 865–874.
- Yeh, W. W.-G. (1985), "Reservoir Management and Operations Models: A State of the Art Review," *Water Resources Research*, **21** (12), 1797–1818.
- ZESA, *Annual Report and Accounts*, Zimbabwe Electricity Supply Authority, Harare, Zimbabwe, 1986.

**APPENDIX: Summary of Notation and Model Equations**

***Notation***

$a, b$	Scalar coefficients of the function relating the predicted evaporation losses $v_t$ to the predicted average storage volume during month $t$ .
$a_t$	Random error term in ARIMA forecasting model for $i_t$ ; $a_t \sim N(0, \sigma_a)$ .
$c_{it}$	Scalar coefficients of the reservoir mass-balance equation ( $i = 1, 2, 3$ ).
$e_t$	Amount of energy produced during month $t$ , GWh.
$g_t$	Tailrace level at the beginning of month $t$ , meter.
$h_t$	Head of the reservoir at the beginning of month $t$ , meter.
$i_t$	Inflow into Lake Kariba during month $t$ , $10^6 \text{ m}^3$ .
$l_t$	Evaporation rate, month $t$ , $10^6 \text{ m}^3$ per month.
$L_t$	Reservoir level at the beginning of month $t$ , meter.
$r_t$	Proposed total amount of water to be released during month $t$ , $10^6 \text{ m}^3$ .
$R_L$	Release level associated with meeting the lower energy production target, $10^6 \text{ m}^3$ .
$R_U$	Release level associated with meeting the upper energy production target, $10^6 \text{ m}^3$ .
$s_t$	Actual active storage, beginning of month $t$ , $10^6 \text{ m}^3$ .
$s_{\max}$	Maximum active reservoir storage, $10^6 \text{ m}^3$ .
$s_{tL}$	Lower storage bound of the proposed release rule, beginning of month $t$ , $10^6 \text{ m}^3$ .
$s_{tU}$	Upper storage bound of the proposed release rule, beginning of month $t$ , $10^6 \text{ m}^3$ .
$u_{1t}$	Actual amount of water released for energy production during month $t$ , $10^6 \text{ m}^3$ .
$u_{2t}$	Actual amount of water released in order to control the reservoir storage, not used for energy production (spill) during month $t$ , $10^6 \text{ m}^3$ .
$u_t$	Actual total amount of water released during month $t$ , $10^6 \text{ m}^3$ .
$\bar{u}_{\text{turb}}$	Maximum flow through the turbines, $10^6 \text{ m}^3$ .
$v_t$	Evaporation losses during month $t$ , $10^6 \text{ m}^3$ .
$y_t$	Proposed amount of water stored, beginning of month $t$ , $10^6 \text{ m}^3$ .
$\alpha, \beta, \gamma$	Scalar coefficients of the function relating the tailrace level $g_t$ to storage $s_t$ .
$\delta, \epsilon$	Scalar coefficients of the function used to relate storage volume $s_t$ and the head.
$\eta$	Efficiency coefficient of the energy generating facilities.
$\lambda, \mu$	Scalar coefficients of the function relating the reservoir level $L_t$ to storage $s_t$ .
$\phi_j$	The autoregressive coefficients in the ARIMA forecasting equations ( $j = 1, \dots, 24$ ).

**Major Model Equations**

$$(1-0.8441B+0.2791B^2)(1+0.1272B^6-0.2464B^{12}+0.1315B^{18}-0.1003B^{24})i_t = 1659.5+a_t, \quad (1)$$

$$\widehat{i}_{t+k} = \sum_{1 \leq m \leq k-1} \phi_m \widehat{i}_{t+k-m} + \sum_{k \leq r \leq 26} \phi_r i_{t+k-r} \quad (2)$$

$$s_{t+1} = s_t + i_t - u_t - v_t \quad (3)$$

$$r_t = r_t(s_t, S_{Lt}, S_{Ut}, s_{\max}, t) \quad (4)$$

$$\widehat{y}_{t+1} = s_t + \widehat{i}_t - r_t - \widehat{v}_t, \quad (5)$$

$$\widehat{v}_t = l_t[a(s_t + \widehat{y}_{t+1})/2 + b]. \quad (6)$$

$$\widehat{y}_{t+1} = c_{1t}s_t + c_{2t}(\widehat{i}_t - r_t) + c_{3t} \quad (7)$$

$$u_t = u_{t1} + u_{t2} \quad (8)$$

$$u_{t1} \leq \bar{u}_{\text{turb}}. \quad (9)$$

$$\widehat{s}_{t+1} = c_{1t}s_t + c_{2t}(\widehat{i}_t - u_t) + c_{3t}. \quad (10)$$

$$g_t = \alpha u_t^\beta + \gamma, \quad (11)$$

$$h_t = L_t - g_t, \quad (12)$$

$$L_t = \lambda s_t + \mu, \quad (13)$$

$$e_t = \eta u_{1t}(\delta(h_t + h_{t+1})/2 + \epsilon) \quad (14)$$

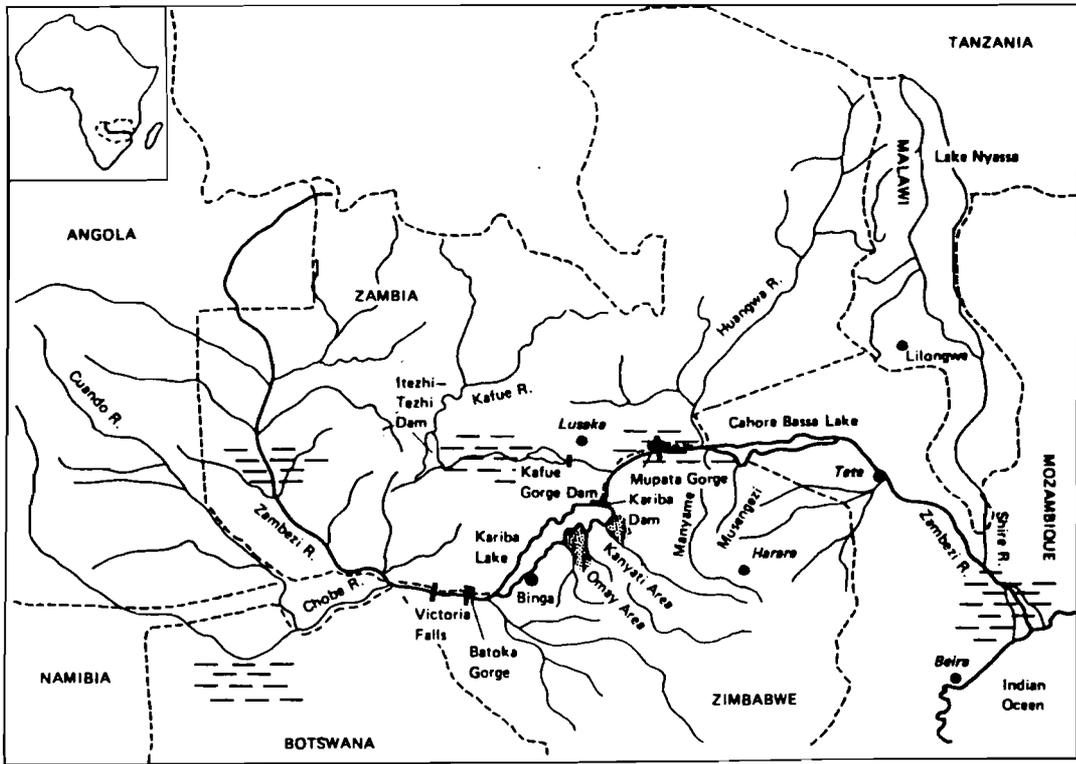


FIGURE 1: Geographic Location of the Zambezi River Basin

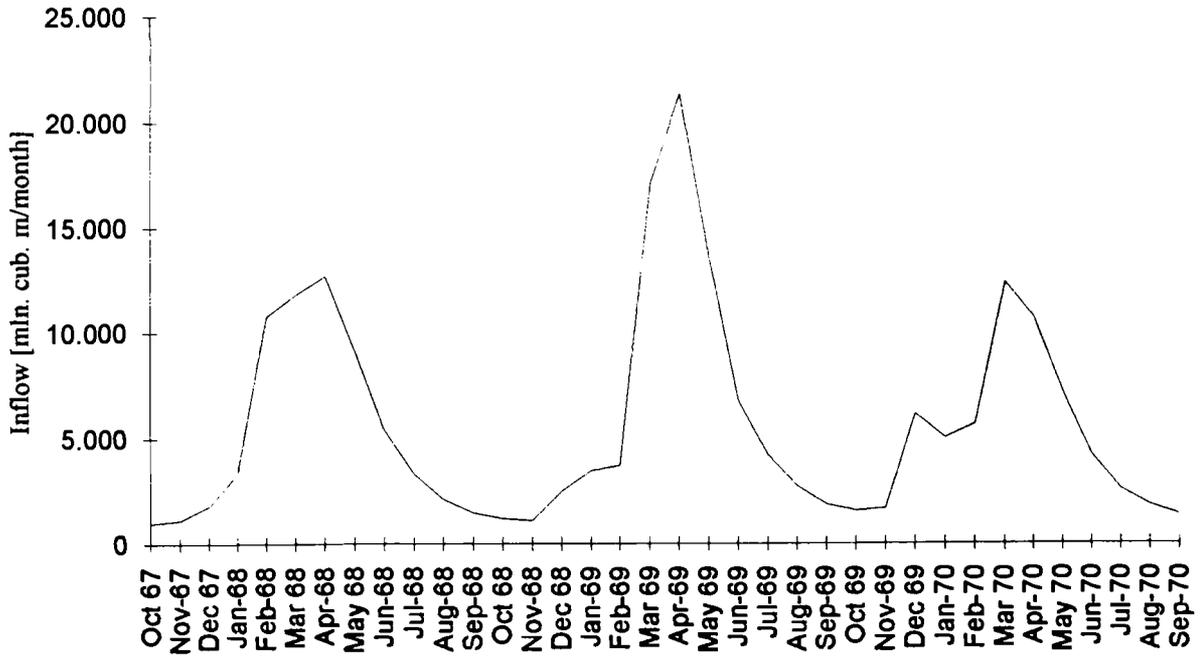


FIGURE 2: Historical Inflows of Lake Kariba, October 1967–September 1970

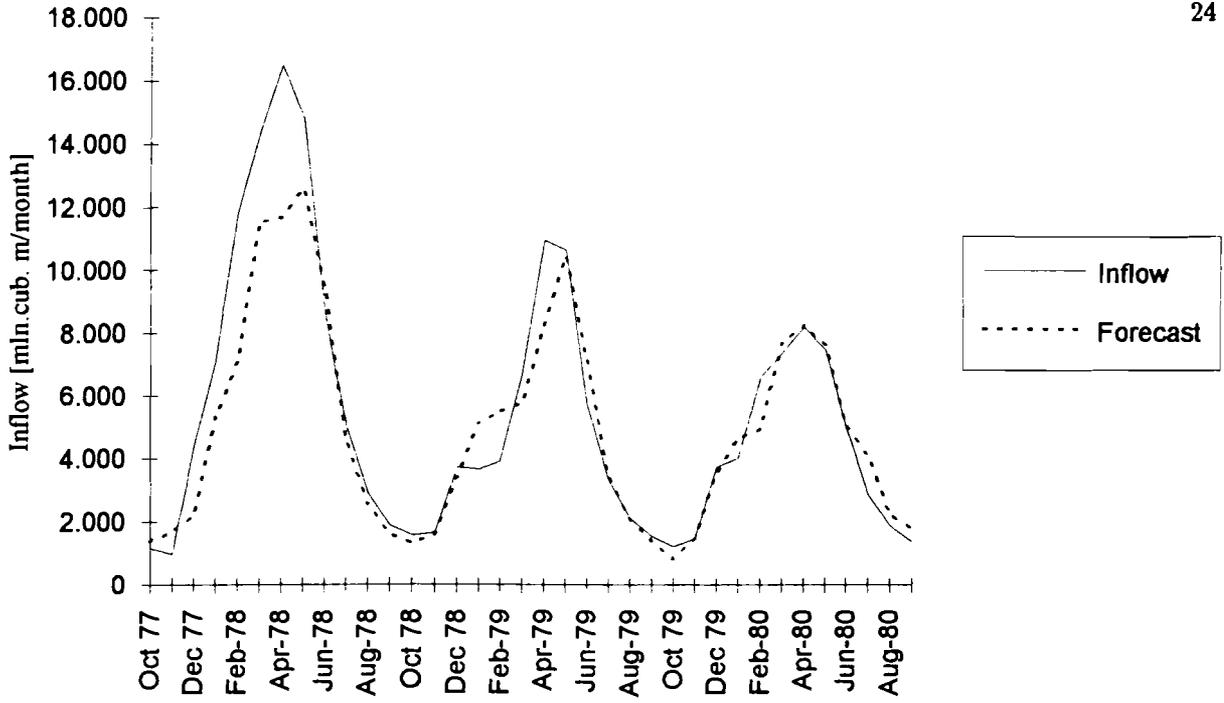


FIGURE 3: Historical and Forecasted Inflows into Lake Kariba, October 1977–September 1980

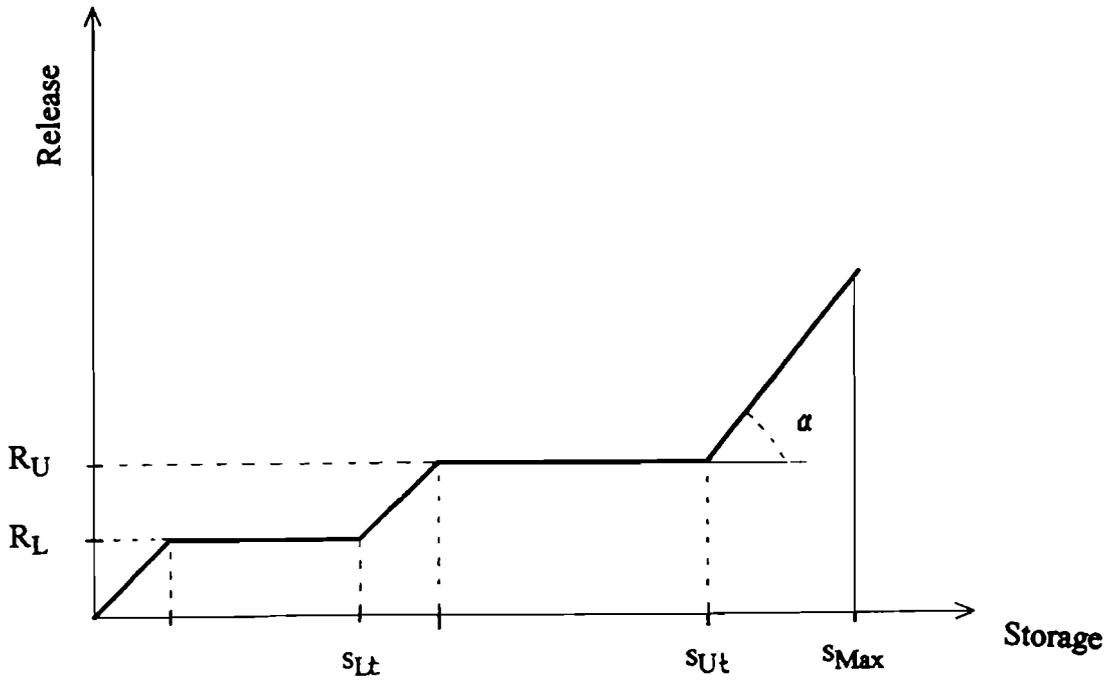


FIGURE 4: Proposed Release Policies for Lake Kariba (Adapted from Gandolfi and Salewicz, 1991)

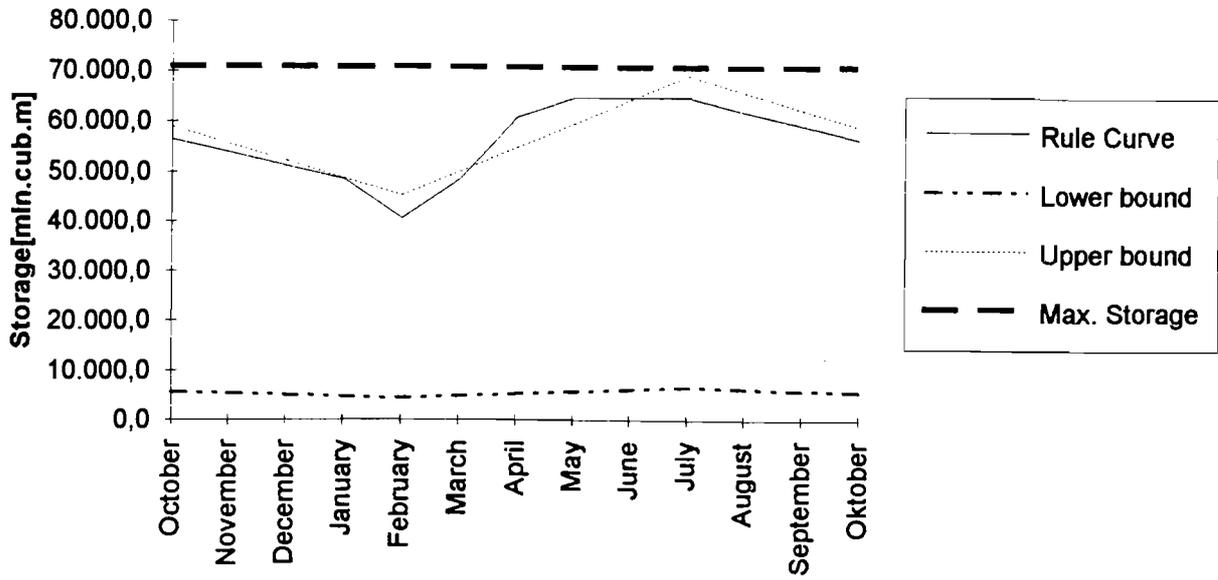


FIGURE 5: Proposed Storage Rule Curve (Gandolfi and Salewicz 1991) and Storage Limits for Lake Kariba

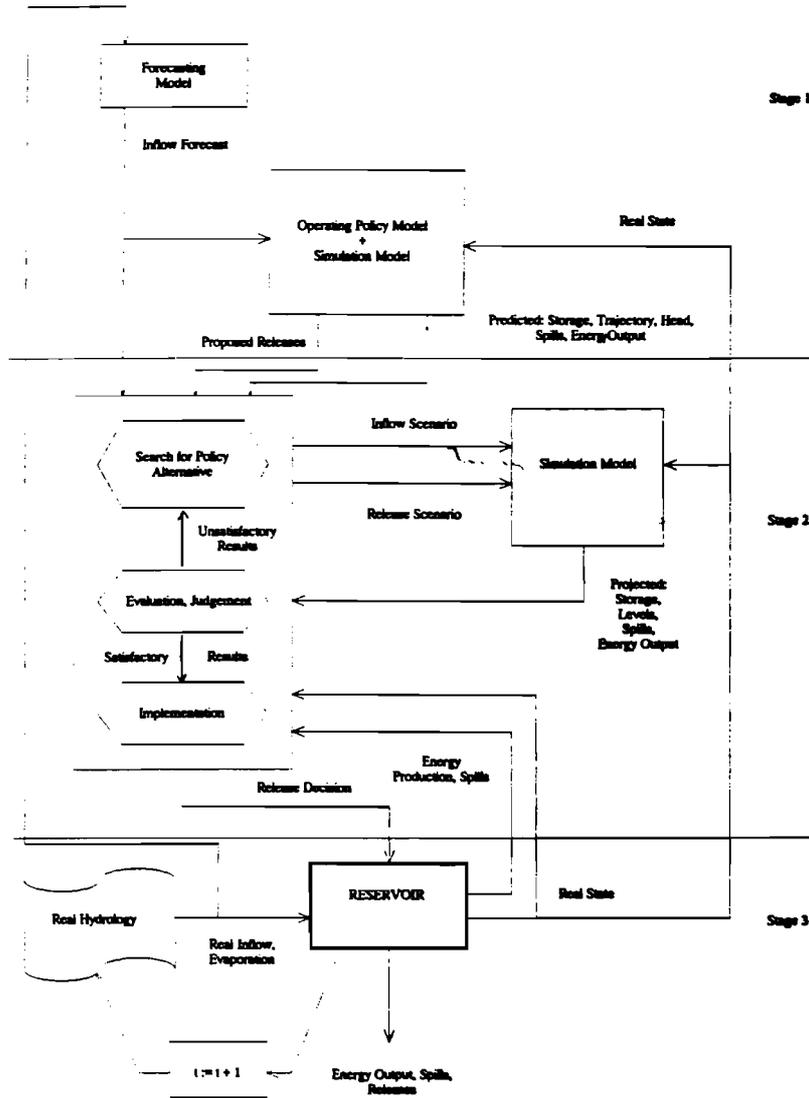


FIGURE 6: Flowchart of Iterative Three-Stage Decision Process

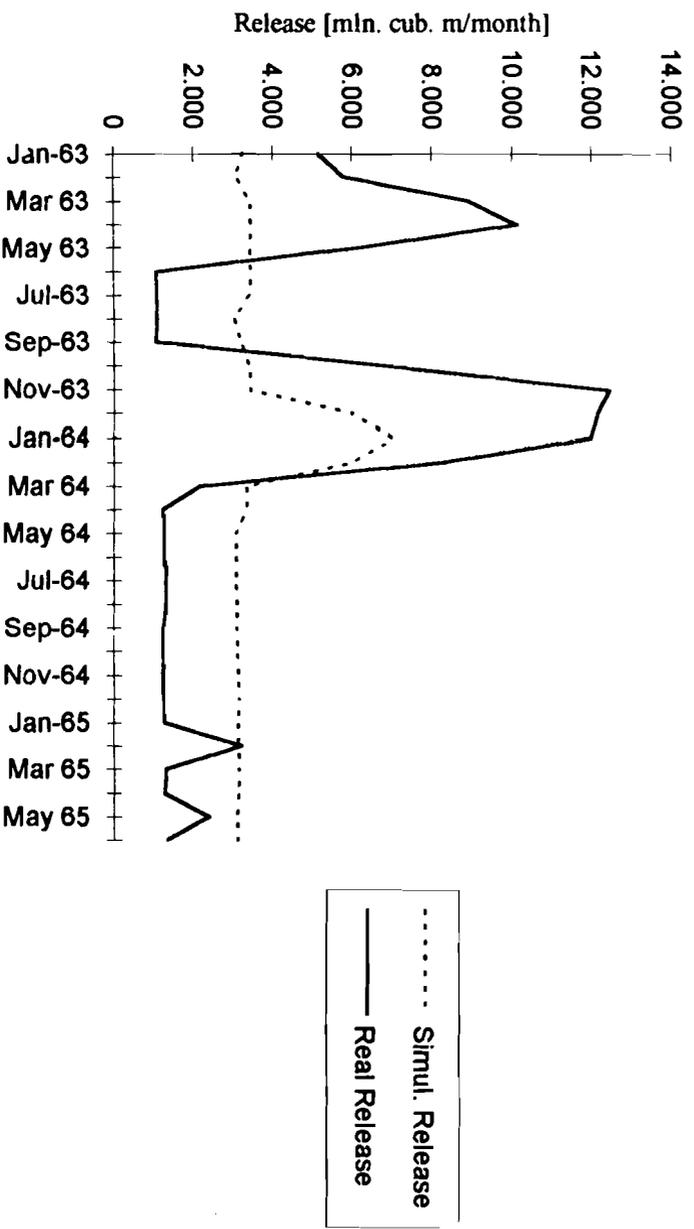


FIGURE 8: Historical and Simulated Releases, January 1963—May 1965

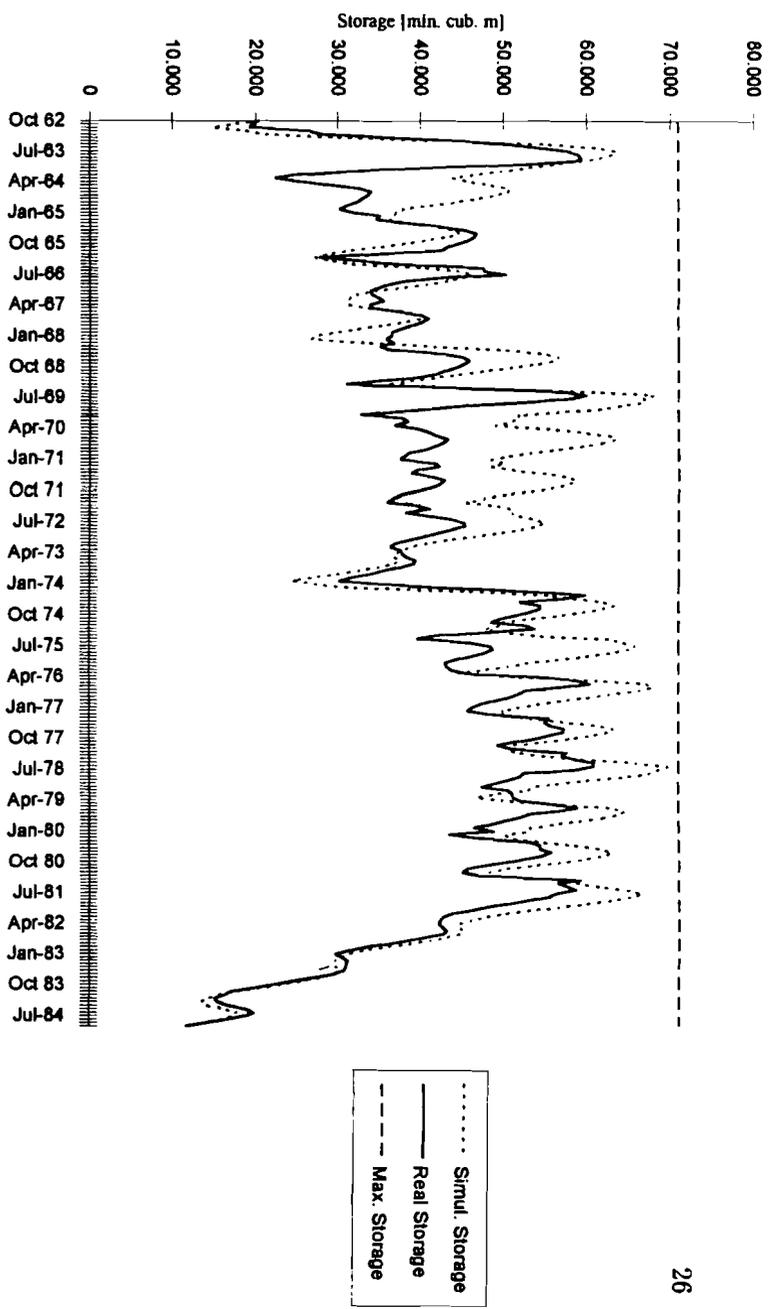


FIGURE 7: Historical and Simulated Reservoir Storage Levels, October 1962—September 1984

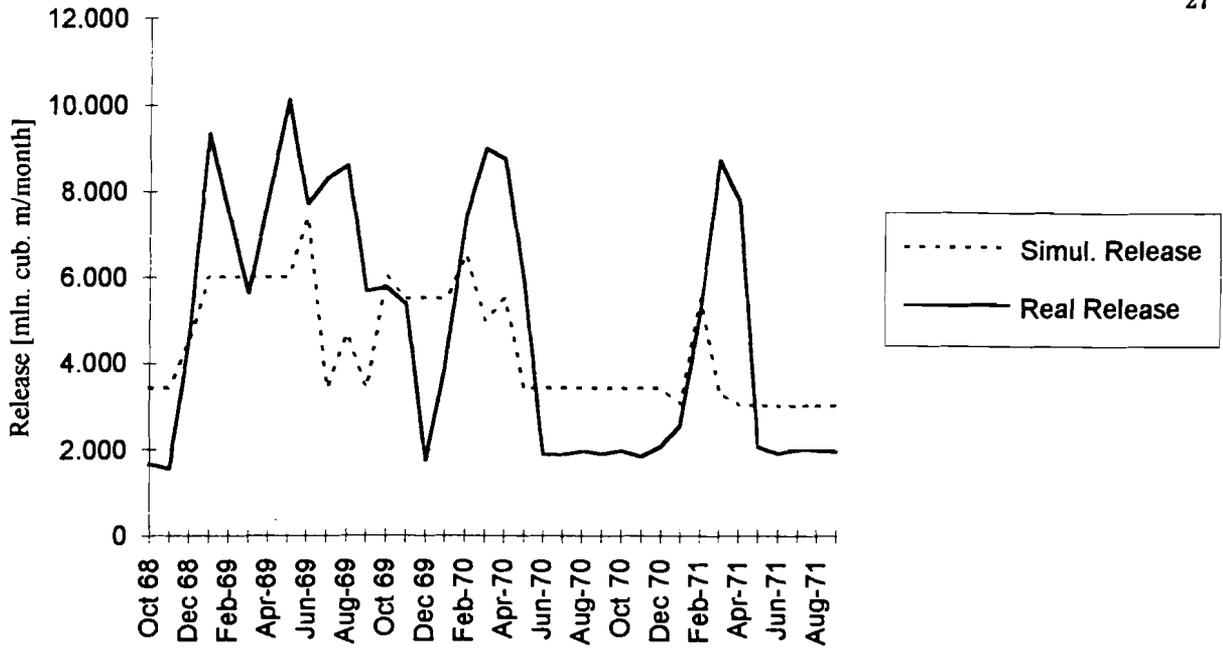


FIGURE 9: Historical and Simulated Releases, October 1968–September 1970

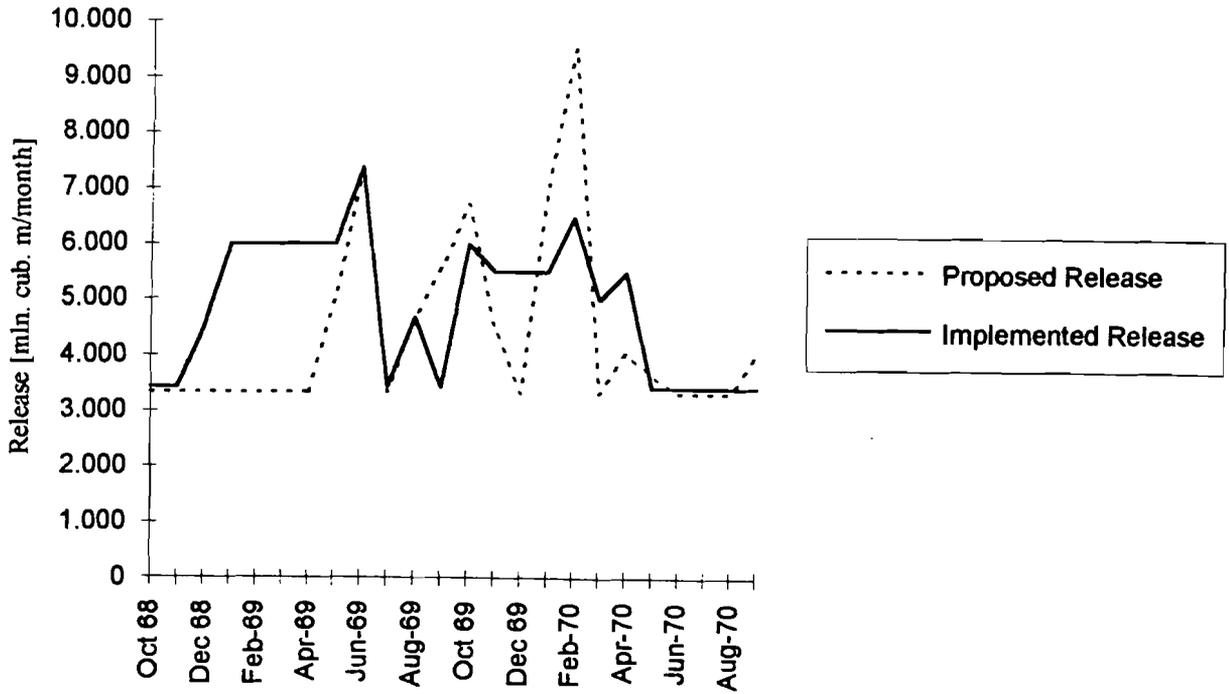


FIGURE 10: Historical and Proposed Releases, October 1968–September 1970

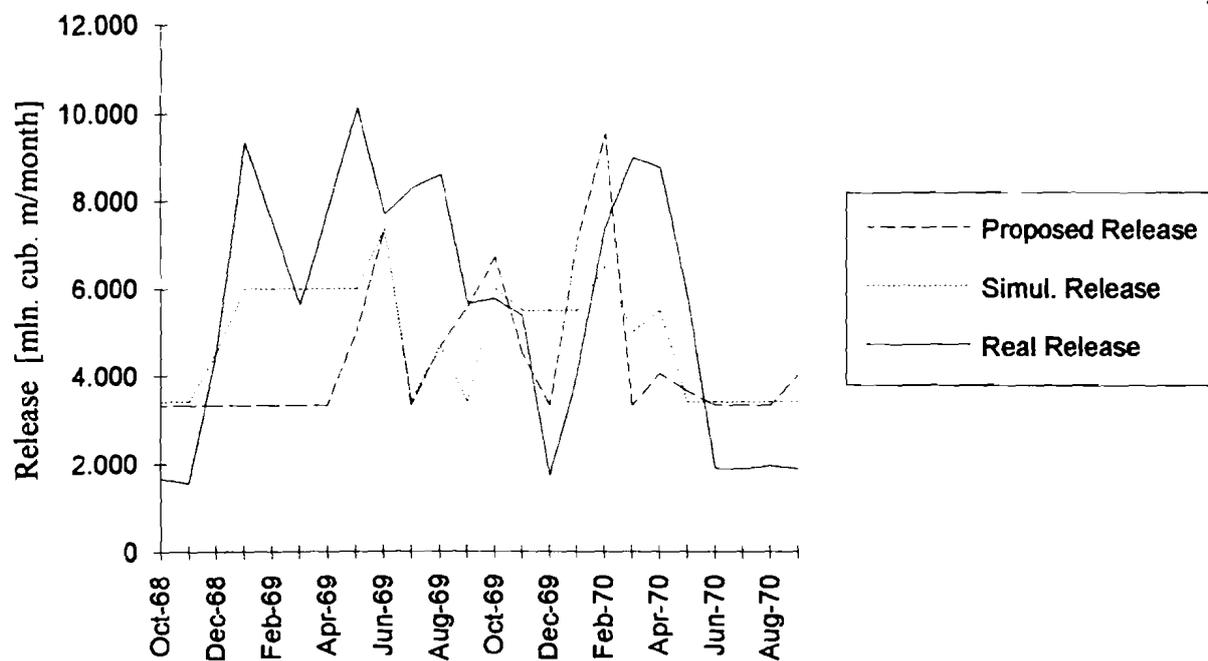


FIGURE 11: Historical, Proposed and Simulated Releases, October 1968–September 1970