Working Paper

Decision Analysis under Extreme Uncertainities - Evaluating Unusual Weathers Caused by Global Warming

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Foreword

A typical feature of many decision problems is the uncertainty about possible consequences. By uncertainty we mean here that there are several possible consequences and that, moreover, there is not much information about their relative likeliness of occurrence. The present paper is devoted to situations with extreme uncertainities, which occur frequently with environmental policy problems. The authors revise and refine an existing approach in order to make it more realistic. They demonstrate the capabilities of the approach by applying it to policy evaluation for decreasing the emission of carbon dioxide. The reported research is part of IIASA's project Methodology of Decision Analysis.

Abstract

In this paper we propose a new methodology of decision analysis under extreme uncertainties which could analyze the preference of various type of persons from pessimistic ones to optimistic ones. For this purpose we revise a previous axiom of dominance for constructing a measurable value function (utility function) under uncertainty based on Dempster-Shafer theory of probability . The previous axiom of dominance has dealt with only the best and the worst results in the set element. Here, we propose a new axiom of dominance after defining the value of the set element taking into account the average of the value of all the results included in the set element. It is shown that we can construct a measurable value function under uncertainty for a pessimistic, an ordinary or an optimistic person, based on this new axiom of dominance. An example of evaluating the alternative policies to decrease the emission of carbon dioxide for avoiding global warming is included.

Keywords: Decision analysis; Extreme uncertainty; Dempster-Shafer theory; Dominance; Environmental engineering; Global warming

Decision Analysis under Extreme Uncertainties

- Evaluating Unusual Weathers Caused by Global Warming -

Hiroyuki TAMURA#, Hiroyasu KITAMURA*, Itsuo HATONO**
*and Motohide UMANO**

1. Introduction

Progress of science and technology and rapid economic development have increased welfare of humankind enormously. Rapid economic development, however, has introduced large-scale energy consumption and the rapid increase of greenhouse gas, especially carbon dioxide emission. Environmental risk of global warming has thus increased as the result of greehouse gas effect. We need to develop systems methodology to predict such negative effect under extreme uncertainties and to make decision on selecting an appropriate countermeasure.

Expected utility theory of von Neumann and Morgenstern (1947) has been a powerful tool for decision making under risk, where probability of

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been a powerful tool for decision making under risk, where probability of getting each consequence is given. In real situations there exist many cases where probability of getting each consequence is unknown. For these cases, we need a methodology for decision making under uncertainty.

Dempster-Shafer theory of probability (Shafer 1976) is a powerful tool to cope with such uncertainties. For constructing a value function under uncertainty based on Dempster-Shafer theory of probability, only the best consequence and the worst consequence in the set element have been used in the previous axiom of dominance (Jaffray 1988). When there exist more than or equal to three elements in a set element, the values of the elements except the best one and the worst one are disregarded when we evaluate the set element.

In this paper a new axiom of dominance is proposed in which all the elements in the set element will be used to judge the value of the set element. It will be shown how this new axiom of dominance is used to construct a value function under uncertainty. It is assumed that the probability of occurring each element is unknown, instead, the basic probability for each set element is given.

Using the methodology described in this paper we will try to evaluate alternative policies to decrease the emission of carbon dioxide which is closely related with global warming.

2. Value Function under Uncertainty Based on Dempster-Shafer Theory of Probability

Let the value function under uncertainty based on the basic probability of Dempster-Shafer be

$$f^*(B,\mu) = w'(\mu)v^*(B \mid \mu) \tag{1}$$

where B denotes a set element, μ denotes basic probability, w' denotes a weighting function for basic probability, and v^* denotes a value function with respect to a set element. Set element B is a subset of $\Lambda=2^{\Theta}$, where Θ contains all the elements.

Equation (1) is an extended version of value function based on based on Bayes probability theory can be found in Tamura et al.(1987).

Axiom of Dominance 1: In the set element B let the worst consequence be m_B and the best consequence be M_B. For any B_1, B_2 $\in \Lambda = 2^{\Theta}$ if

$$m_{B_1} \precsim m_{B_2},\ M_{B_1} \precsim M_{B_2} \text{ then } B_1 \precsim B_2$$

where $a \precsim b$ denotes "b is preferred to or indifferent to a."

Using this Axiom of Dominance 1, we will restrict a set element B to

$$\Omega = \{(m, M) \in \Theta \times \Theta : m \prec M\}$$

where m and M denote the worst and the best consequence in the set element B, respectively. Then, eqn.(1) is reduced to

$$f^*(\Omega, \mu) = w'(\mu) v^*(\Omega \mid \mu). \qquad (2)$$

Suppose we look at an index of pessimism $\alpha(m,M)$ such that the following two alternatives are indifferent:

Alternative 1: One can receive m for the worst case and M for the best case. There exists no other information.

Alternative 2: One receives m with probability $\alpha(m,M)$ and receives M with probability $1-\alpha(m,M)$, where $0<\alpha(m,M)<1$.

If one is quite pessimistic, $\alpha(m,M)$ becomes nearly equal to 1, and if he is quite optimistic $\alpha(m,M)$ becomes nearly equal to zero. If we incorporate pessimism index $\alpha(m,M)$ in eqn.(2), value function is obtained as

$$\begin{aligned} v^*(\Omega \mid \mu) &= v^*((m, M) \mid \mu) \\ &= \alpha(m, M) v'(m \mid \mu) + \{1 - \alpha(m, M)\} v'(M \mid \mu). \end{aligned} \qquad (3)$$

3. Deficiency in the Previous Models

The value function based on Dempster-Shafer theory of probability can handle a set element in which multiple elements are included. However, we could easily raise contradicting examples when we follow Axiom of Dominance 1.

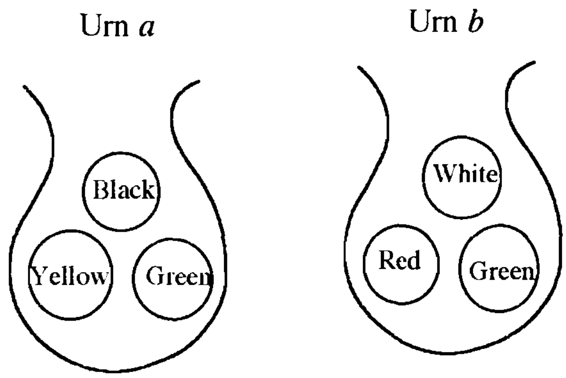

Figure 1. Urns in Example 1

Example 1: Suppose there exist two urns, a and $b$as shown in **Fig. 1**. Urn a contains 30 balls of black, yellow and green, however, the number of balls with each color is unknown. Urn b contains 30 balls of white, red and green. The number of balls with each color is again unknown. Suppose a decision maker (DM) selects urn a or urn b and he will pick up a ball from the urn he selected. He will receive the following amount of money in Japanese yen depending upon the color of ball he picks up.

Black: 1000, Yellow: 5000, Green: 6000,
White: 2000, Red: 3000

Let a_1, a_2, a_3 be the events of picking up black ball, yellow ball and green ball, respectively, from urn a, and let a set element B_1 be

$$B_1 = \{a_1, a_2, a_3\} \tag{4}$$

Let b_1, b_2, b_3 be the events of picking up white ball, red ball and green ball, respectively, from urn b, and let a set element B_2 be

$$B_2 = \{b_1, b_2, b_3\}. \tag{5}$$

Let $v(a_i)$ and $v(b_i)$ be monetary value of each element for i=1,2,3. If we follow Axiom of Dominance 1

$$a_1 \prec b_1 \quad (b_1 \text{ is preferred to } a_1) \tag{6a}$$

$$a_3 \sim b_3 \quad (a_3 \text{ is indifferent to } b_3) \tag{6b}$$

Then, we will get

$$B_1 \prec B_2 \tag{7}$$

However, the monetary value of B_1 and B_2 are

$$B_1 = \sum_{i=1}^{3} v(a_i) = 12{,}000 \tag{8a}$$

$$B_2 = \sum_{i=1}^{3} v(b_i) = 11{,}000 \tag{8b}$$

This leads to $B_1 \succ B_2$, which is contradicting with eqn.(7).

4. A Revised Axiom of Dominance

Axiom of Dominance 2: Let m_1 and M_1 be the worst and the best consequences, respectively, in the set element B_1, and m_2 and M_2 be the worst and the best consequences, respectively, in the set element B_2. Let g_1 and g_2 be hypothetical elements such that

$$v(g_1) = \sum_{i=1}^{n_1} v(a_i)/n_1$$

$$v(g_2) = \sum_{i=1}^{n_2} v(b_i)/n_2$$

where

n_1: number of elements in B_1

n_2: number of elements in B_2.

If

$$m_1 \precsim m_2,\ M_1 \precsim M_2,\ g_1 \precsim g_2$$

then

$$B_1 \precsim B_2\ .$$

Using Axiom of Dominance 2, we could restrict a set element B to

$$\Omega = \{(m, g, M) \in \Theta \times \Theta \times \Theta : m \prec g \prec M\}.$$

Then, eqn.(1) is reduced to eqn.(2), and furthermore

$$f^*(\Omega, \mu) = f^*((m, g, M), \mu)$$

$$= \beta_1 f(m, \mu) + \beta_2 f(g, \mu) + \beta_3 f(M, \mu) \quad (9)$$

$$\beta_1 + \beta_2 + \beta_3 = 1$$

whereβ_1, β_2, β_3 denote weighting coefficients for the worst element, for the hypothetical element such that its value is the average of all the elements, and for the best element, respectively.

If we follow Axiom of Dominance 2, B_1 and B_2 in Example 1 cannot be compared, since

$$m_1 \prec m_2, \;\; M_1 \sim M_2 \text{ and } g_1 \succ g_2.$$

Example 2: Suppose there exist two urns *a* and *b* as shown in **Fig. 2**. Urn *a* contains 30 balls of black, yellow and green, however, the number of balls with each color is unknown. Urn *b* contains 30 balls of white, blue and green. The number of balls with each color is again unknown. Suppose a DM selects urn *a* or urn *b* and will pick up a ball from the urn he selected. Depending upon the color of ball he picks up, he will receive the following amount of money:

Black: 1000, Yellow: 5000, Green: 6000,

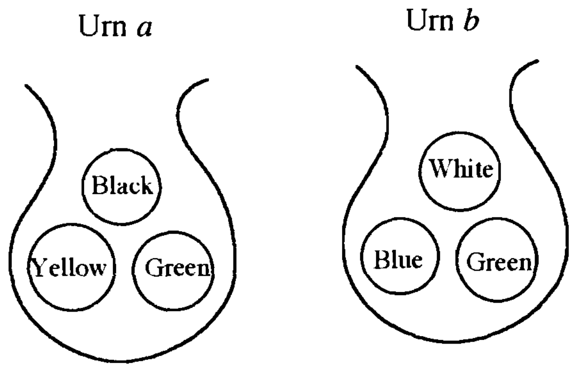

Figure 2. Urns in Example 2

White: 2000, Blue: 4000

As shown like in Example 1, let

$$B_1 = \{a_1(\text{black}), a_2(\text{yellow}), a_3(\text{green})\} \tag{10a}$$

$$B_2 = \{b_1(\text{white}), b_2(\text{blue}), b_3(\text{green})\} \tag{10b}$$

An ordinary conservative person prefers B_2 to B_1. A value function for such person should be convex, e.g.

$$v(x) = \sqrt{\frac{x-1000}{5000}}\,. \tag{11}$$

In this case

$$v(a_1) = 0,\ v(a_2) = 0.89,\ v(a_3) = 1 \tag{12a}$$

$$v(b_1) = 0.45,\ v(b_2) = 0.77,\ v(b_3) = 1 \tag{12b}$$

which leads to

$$\sum_{i=1}^{3} v(a_i)/3 = 0.63, \qquad \sum_{i=1}^{3} v(b_i)/3 = 0.74 \tag{13}$$

and

$$m_1(=a_1) \prec m_2(=b_1)$$

$$g_1 \prec g_2$$

$$M_1(=a_3) \sim M_2(=b_3)$$

This example shows that Axiom of Dominance 2 can describe the preference of ordinary conservative person.

5. Relaxation of Dominance

Axiom of Dominance 2 is more restricted than Axiom of Dominance 1. Axiom of Dominance 2 can handle a conservative or pessimistic preference, but not suitable for the other type of preference, e.g. risk prone or optimistic preference. In Example 2, it depends on the type of DM's preference whether he would prefer urn a or urn b. Pessimistic DM may prefer urn b to urn a, and optimistic DM may prefer urn a to urn b. We need a new Axiom of Dominance which could represent such preference correctly.

Definition 1: Let a set B contain n elements as

$$B = \{a_1, a_2, \dots, a_n\} \tag{14a}$$

$$a_i \prec a_{i+1}, \qquad i = 1, 2, \dots, n-1 \tag{14b}$$

and let $v(a_i)$ be value of each element. Let

$$v(g) = \sum_{i=1}^{n} v(a_i)/n \tag{15}$$

and let $\alpha(m,M)$ be the index of pessimism which has been defined in Axiom of Dominance 1. Suppose h is a value function of a set element B such that

$$h(B \mid \alpha) = a + be^{-c\alpha(m,M)}, \quad \text{if } v(g) \neq \{v(M) + v(m)\}/2 \tag{16a}$$

$$h(B \mid \alpha) = a + b\alpha(m,M), \quad \text{if } v(g) = \{v(M) + v(m)\}/2 \tag{16b}$$

where parameters a, b, c are determined by

$$h(B \mid 0) = v(M), \quad h(B \mid 0.5) = v(g),$$
$$h(B \mid 1) = v(m)$$

Axiom of Dominance 3: If

$$h(B_1 \mid \alpha) \leq h(B_2 \mid \alpha)$$

then

$$B_1 \precsim B_2.$$

Using Axiom of Dominance 3, eqn.(1) is reduced to

$$f^*(h(B \mid \alpha), \mu) = w'(\mu)h(B \mid \alpha). \tag{17}$$

Using Axiom of Dominance 3, let us look at Example 2 again in the following Example 3.

Example 3: For pessimistic person (say α=0.8) if we use eqn.(11), we get eqns.(12) and (13). We then obtain

$$h(B_1 \mid \alpha) = 1.54 - 0.54 \times 2.86^{0.8} = 0.29 \tag{18a}$$

$$h(B_2 \mid \alpha) = 3.25 - 2.25 \times 1.24^{0.8} = 0.58. \tag{18b}$$

This implies that such pessimistic person prefers B_2 to B_1.

For optimistic person (say α=0.2) if we use a value function

$$v(x) = \left(\frac{x-1000}{5000}\right)^2 \tag{19}$$

we obtain

$$v(a_1) = 0, \quad v(a_2) = 0.64, \; v(a_3) = 1 \tag{20a}$$

$$v(b_1) = 0.04, \; v(b_2) = 0.36, \; v(b_3) = 1 \tag{20b}$$

which lead to

$$\sum_{i=1}^{3} v(a_i)/3 = 0.55\,, \; \sum_{i=1}^{3} v(b_i)/3 = 0.47\,. \tag{21}$$

We then obtain

$$h(B_1 \mid \alpha) = 3.02 - 2.02 \times 1.50^{0.2} = 0.83 \tag{22a}$$

$$h(B_2 \mid \alpha) = -1.81 + 2.81 \times 0.66^{0.2} = 0.78\,. \tag{22b}$$

This implies that such optimistic person prefers B_1 to B_2.

As shown in this Example 3, Axiom of Dominance 3 proposed in this paper could represent the preference of both pessimistic person and optimistic person, properly.

6. Global Warming and Evaluation of Various Alternative Policies to Decrease Carbon Dioxide Emission

Recent increase of carbon dioxide concentration around the globe is serious and it is said that the resulting global warming may cause serious damages in our life. Therefore, we need to restrict the emission of carbon dioxide somehow.

Suppose we look at three alternatives as follows:

Alternative *a*: Amount of carbon dioxide emission is reduced to 1990 level by the year 2000.

Alternative *b*: Amount of carbon dioxide emission is not increased any more.

Alternative *c*: No restriction is imposed on carbon dioxide emission.

The damage considered here is just the damage caused by unusual weather. Let us consider a two-attribute problem of cost to realize each alternative and the damage caused by unusual weather.

Table 1: Alternatives, cost, and damage caused by unusual weather

Alternative	Cost	Damage Caused by Unusual Weather	
a	x_1	$\{d_2\}$ $\{d_3\}$	p_1 $1\text{-}p_1$
b	x_2	$\{d_1, d_2\}$ $\{d_1, d_2, d_3\}$	μ_2 $1\text{-}\mu_2$
c	x_3	$\{d_1, d_2\}$ $\{d_1, d_2, d_3\}$	μ_3 $1\text{-}\mu_3$

Suppose cost to realize alternative a, b, c are x_1, x_2, x_3 ($x_1>x_2>x_3=0$), respectively, and probability of getting damage caused by unusual weather are p_1, μ_2, μ_3 ($p_1<\mu_2<\mu_3$) where p_1 denotes a Bayesian probability, μ_2 and μ_3 denote basic probability used in Dempster-Shafer theory.

Let us define three events as follows:

d_1: Get damage caused by unusual weather due to global warming.

d_2: Get damage caused by unusual weather which is not related with global warming.

d_3: No damage.

Three alternatives are summarized in **Table 1**. In Table 1, for alternative a, p_1 denotes probability of getting damage caused by unusual weather based on

the meteorological data obtained up to 1990, and 1- p_1 denotes probability of not getting any damage. For alternative b, probability of getting damage caused by unusual weather will be increased, but it will be fuzzy. Basic probability μ_2 is assigned to the set element of $\{d_1, d_2\}$. Actually, when we get damage caused by unusual weather, we do not know whether it is due to global warming or not. Basic probability 1- μ_2 is assigned to the set element composed by all the elements $\{d_1, d_2, d_3\}$. For alternative c, basic probability μ_3 and 1- μ_3 are assigned similarly.

By using value function under uncertainty, we evaluate three alternatives a, b, c. Since our problem is with two attributes, value functions can be described as

$$F^*(a) = f^*(x_1, \{d_2\}, p_1) + f^*(x_1, \{d_3\}, 1 - p_1) \tag{23a}$$

$$F^*(b) = f^*(x_2, \{d_1, d_2\}, \mu_2) + f^*(x_2, \{d_1, d_2, d_3\}, 1 - \mu_2) \tag{23b}$$

$$F^*(c) = f^*(x_3, \{d_1, d_2\}, \mu_3) + f^*(x_3, \{d_1, d_2, d_3\}, 1 - \mu_3). \tag{23c}$$

Let $v_1(x_i)$ be value function for cost, $v_2(d_i)$ be value function for getting damage caused by unusual weather, $h(B|\alpha)$ be value function for set element, w(*) be value function (weighting function) for probability of getting damage, k_1 and k_2 be scaling coefficients for cost and for getting damage caused by unusual weather, respectively. Then eqn.(23) is reduced to

$$F^*(a) = k_1 w(p_1) v_1(x_1) + k_2 w(p_1) v_2(d_2) + k_1 w(1 - p_1) v_1(x_1) + k_2 w(1 - p_1) v_2(d_3) \tag{24a}$$

$$F^*(b) = k_1 w(\mu_2) v_1(x_2) + k_2 w(\mu_2) h(\{d_1, d_2\} \mid \alpha) + k_1 w(1 - \mu_2) v_1(x_2) + k_2 w(1 - \mu_2) h(\{d_1, d_2, d_3\} \mid \alpha) \tag{24b}$$

$$F^*(c) = k_1 w(\mu_3) v_1(x_3) + k_2 w(\mu_3) h(\{d_1, d_2\} \mid \alpha) + k_1 w(1 - \mu_3) v_1(x_3) + k_2 w(1 - \mu_3) h(\{d_1, d_2, d_3\} \mid \alpha) \tag{24c}$$

Let

$$v_1(x_1) = 1, \quad v_1(x_2) = 0.3, \quad v_1(x_3) = 0$$

$$w(p_1) = 0, \quad w(\mu_2) = 0.3, \quad w(\mu_3) = 1, \quad w(p) = 1 \text{ for } p \geq \mu_3$$

$$v_2(d_1) = 1, \quad v_2(d_3) = 0$$

where each value shows disutility, that is, higher value implies high disutility. The interpretation for these values is as follows:

Concerned with cost, the value for the least cost is set to 0, the value for the highest cost is set to 1, and the value for keeping the present status is set to 0.3. Concerned with probability, the value for the best case is set to 0, the value for keeping the present status is set to 0.3, and the value for probability higher than or equal to μ_3 is set to 1.

In this case eqn.(24) is reduced to

$$F^*(a) = k_1 \times 0 \times 1 + k_2 \times 0 \times v_2(d_2) + k_1 \times 1 \times 1 + k_2 \times 1 \times 0 = k_1 \tag{25a}$$

$$F^*(b) = k_1 \times 0.3 \times 0.3 + k_2 \times 0.3 \times h(\{d_1, d_2\} \mid \alpha)$$

$$+k_1 \times 1 \times 0.3 + k_2 \times 1 \times h(\{d_1, d_2, d_3\} \mid \alpha)$$

$$= 0.39k_1 + k_2\{0.3h(\{d_1, d_2\} \mid \alpha) + h(\{d_1, d_2, d_3\} \mid \alpha)\} \quad (25b)$$

$$F^*(c) = k_1 \times 1 \times 0 + k_2 \times 1 \times h(\{d_1, d_2\} \mid \alpha)$$

$$+k_1 \times 1 \times 0 + k_2 \times 1 \times h(\{d_1, d_2, d_3\} \mid \alpha)$$

$$= k_2\{h(\{d_1, d_2\} \mid \alpha) + h(\{d_1, d_2, d_3\} \mid \alpha)\} \quad (25c)$$

Let

$$v_2(d_2) = 0.1.$$

Since disutility of getting damage by unusual weather which is not related with global warming, is not so high.

Using pessimism index, disutility for set element is obtained as

$$h(\{d_1, d_2\} \mid \alpha) = \alpha(d_1, d_2)v_2(d_1) + (1 - \alpha(d_1, d_2))v_2(d_2)$$

$$= 0.9\alpha(d_1, d_2) + 0.1 \quad (26a)$$

$$h(\{d_1, d_2, d_3\} \mid \alpha) = -0.53 + 0.53 \times 2.89^{\alpha} \quad (26b)$$

Using eqn.(26), eqn.(25) is reduced to

$$F^*(a) = k_1 \quad (27a)$$

$$F^*(b) = 0.39k_1 + k_2\{0.27\alpha(d_1, d_2) + 0.03 + h(\{d_1, d_2, d_3\} \mid \alpha)\} \quad (27b)$$

$$F^*(c) = k_2\{0.9\alpha(d_1, d_2) + 0.1 + h(\{d_1, d_2, d_3\} \mid \alpha)\} \quad (27c)$$

It will be shown that eqn.(27) can represent the preference of pessimistic person, ordinary person and optimistic person, where each type of person is described as follows:

Pessimistic Person: Global warming is quite serious, progress of global warming is irreversible, and once warming has started, it will never be recovered. (α =0.8)

Ordinary person: Global warming is serious, the increase of temperature might be few degrees centigrade by the middle of 21st century. (α =0.5)

Optimistic Person: Global warming is within meteorological change, or it could be solved by the progress of science and technology. (α =0.2)

Pessimistic Person: Let

$$\alpha(d_1, d_2) = \alpha(d_1, d_3) = 0.8$$

then

$$h(\{d_1, d_2, d_3\} \mid 0.8) = 0.71.$$

Equation (27) is then reduced to

$$F^*(a) = k_1$$

$$F^*(b) = 0.39k_1 + 0.96k_2$$

$$F^*(c) = 1.53k_2.$$

Ordinary person: Let

$$\alpha(d_1, d_2) = \alpha(d_1, d_3) = 0.5$$

then

$$h(\{d_1, d_2, d_3\} \mid 0.5) = 0.37$$

Equation (27) is then reduced to

$$F^*(a) = k_1$$

$$F^*(b) = 0.39k_1 + 0.54k_2$$

$$F^*(c) = 0.92k_2$$

Optimistic Person: Let

$$\alpha(d_1, d_2) = \alpha(d_1, d_3) = 0.2$$

then

$$h(\{d_1, d_2, d_3\} \mid 0.2) = 0.13.$$

Equation (27) is then reduced to

$$F^*(a) = k_1$$

$$F^*(b) = 0.39k_1 + 0.21k_2$$

$$F^*(c) = 0.41k_2.$$

Table 2 shows the value of disutility obtained for various values of scaling coefficients k_1 and k_2. In this table bold number shows the value for the best alternative among alternatives *a*, *b*, *c*. For the same value of scaling coefficient, pessimistic person chooses alternative *a* which is the most strict alternative for carbon dioxide emission. On the contrary optimistic person tends to choose alternative *c* which is the most loose alternative. That is, the value function under uncertainty proposed in this paper can properly represent the preference of pessimistic person, ordinary person, and optimistic person.

Table 2: Evaluated Values of Alternatives for Various Scaling Coefficients.

k_1	k_2	α =0.8			α =0.5			α =0.2		
		a	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*
0.0	1.0	**0.00**	0.96	1.53	**0.00**	0.54	0.92	**0.00**	0.21	0.41
0.1	0.9	**0.10**	0.90	1.38	**0.10**	0.53	0.83	**0.10**	0.23	0.37
0.2	0.8	**0.20**	0.85	1.22	**0.20**	0.51	0.74	**0.20**	0.25	0.33
0.3	0.7	**0.30**	0.79	1.07	**0.30**	0.50	0.64	0.30	**0.26**	0.29
0.4	0.6	**0.40**	0.73	0.92	**0.40**	0.48	0.55	0.40	0.28	**0.25**
0.5	0.5	**0.50**	0.67	0.76	0.50	**0.46**	**0.46**	0.50	0.30	**0.21**
0.6	0.4	**0.60**	0.62	0.61	0.60	0.45	**0.37**	0.60	0.32	**0.16**
0.7	0.3	0.70	0.56	**0.46**	0.70	0.44	**0.28**	0.70	0.34	**0.12**
0.8	0.2	0.80	0.50	**0.31**	0.80	0.42	**0.18**	0.80	0.35	**0.08**
0.9	0.1	0.90	0.45	**0.15**	0.90	0.41	**0.09**	0.90	0.37	**0.04**
1.0	0.0	1.00	0.39	**0.00**	1.00	0.39	**0.00**	1.00	0.39	**0.00**

7. Conclusion

In this paper a new axiom of dominance for constructing a value function under uncertainty based on Dempster-Shafer theory is proposed. Since all the elements in a set element are used to evaluate the set element, we obtained the mean value of the values of all the elements in the set element. Then, it is postulated that an ordinary person evaluates the value of a set element by this mean value, a pessimistic person evaluates the value of a set element lower than the mean value, and an optimistic person evaluates the value of a set element higher than the mean value.

Using this hypothesis a new axiom of dominance is proposed. It could enable us to evaluate a set element by using the best element, the worst element, pessimism index and the mean value of the values of all the elements in the set element. Based on the value of a set element evaluated through this axiom of dominance, a value function under uncertainty is constructed to evaluate various alternatives.

Using the methodology proposed in this paper various alternatives to restrict the emission of carbon dioxide are evaluated. In general, when we come across unusual weather, we do not know whether it is due to the increase of carbon dioxide or not. Using Dempster-Shafer theory we assigned basic probability to a set element which contains multiple elements such as "unusual weather is due to the increase of carbon dioxide," and "unusual weather is not due to the increase of carbon dioxide." The value function constructed based on this problem formulation could properly evaluate the preference of various type of persons, ordinary, pessimistic or optimistic ones.

Since the value function under uncertainty could be reduced to the prospect theory (Kahneman and Tversky 1979, Jackson and Sarin 1989) or expected utility theory (von Neumann and Morgenstern 1947) under specified conditions, it is possible to incorporate Dempster-Shafer theory to these theories.

For further research

1) we need to investigate a method to find and assign basic probability to each set element,

2) instead of using arithmetic mean we need to try to use geometric mean or other averaging operator for evaluating the value of a set element.

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