Working Paper

Multiple Criteria Analysis for Regional Water Quality Management: the Nitra River Case

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Foreword

The research described in this Working Paper is a continuation of collaborative work between the Methodology of Decision Analysis (MDA) and the Water Resources (WAT) Projects that began during the Young Scientists Summer Program (YSSP) in 1993. The research is aimed at an application of multicriteria decision analysis techniques and tools for water quality management in a river basin. These techniques and tools have been applied to the Nitra River Basin in Slovakia, which has been the subject of collaborative research between the Water Resources Project of IIASA, the Water Research Institute (Bratislava) and the Vah River Basin Authority.

The first goal of this Working Paper is to illustrate the capabilities of the applied methodology and the developed tools. The second goal is to document the formulation of the underlying mathematical programming model and the data used, since this is essential not only for using the reported results at IIASA but also for possible future applications.

Abstract

This Working Paper documents the implementation of an element of a Decision Support System (DSS) for regional water quality management, applied in cooperation with the Water Research Institute (VUVH, Bratislava) and the Vah River Basin Authority to the Nitra River case study in Slovakia. Several re-usable, modular software tools have been developed and implemented – a problem-specific generator to produce the core part of the mathematical programming model, tools for the generation and interactive modification of multicriteria problems, and a solver for the resulting mixed-integer optimization problem.

Provided in the paper are the following: a complete formulation of the mathematical model (including the applied well-known dissolved oxygen model), a detailed discussion of the data used, documentation of the developed software, an overview of results which might be of interest, and recommendations for future work. Emphasis is placed on the advantages of multicriteria analysis for the regional water quality management problem.

Key Words: decision support, regional water quality management, multi-criteria programming, aspiration-reservation-led decision support, reference point, mixed-integer linear programming.

Contents

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1 Introduction

Surface water quality in many Central and Eastern European countries is generally poor, and the cost of cleaning up the rivers in this region is estimated to be enormous due to the low level of municipal and industrial wastewater treatment (and associated infrastructures), the significant role of uncontrolled non-point source pollution, as well as sediment and soil contamination. In the light of these features and existing severe financial constraints in these countries, imposing effluent water quality standards on the basis of the "best available technology" typical of Western Europe and North America may not feasible in the near future (cf [Som93]). For this reason, there is a need to set strategies which are realistic in the short-term and also consistent with long-term planning goals (when implementation of higher quality standards will be feasible due to improved economic conditions). Thus, decision-makers need to evaluate the trade-offs among a broad range of possible policies based on, among other things, effluent and/or ambient water quality standards and goals, capital investment and annual operating costs, and the principles of equity, uniformity, and efficiency.

The scope of our problem is a river basin or a larger region composed of several basins where untreated or inadequately treated municipal and industrial wastewater emissions should be reduced in order to improve ambient water quality. At each discharge, one technology to be selected out of a set of possible technologies can be implemented in order to meet the desired water quality goals in the region. This technology selection, or strategy development, can be performed in many different ways (depending on underlying principles and methodologies employed), among which three are outlined subsequently.

The traditional approach (as used in Western developed countries) is based on the selection of generally uniform effluent standards which are often based on given technologies (this is the well-known policy of "best available technology"). Under such an approach, both ambient water quality standards and budget requirements are considered indirectly. The assumption is twofold:

- if effluent standards are defined stringently enough, ambient quality will be "good" enough, and

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- enough money or willingness to pay is available to achieve "safe" environmental conditions (without raising the questions of how "safe" they are and how much should be paid for them).

As noted before, such a robust and uniform policy may not be affordable for countries in the Central and Eastern European region for the coming decade or so (order of magnitude estimates suggest the per capita cost requirement of reaching surface water quality close to western norms to be approximately equal to the per capita GDP – a few thousend USD). Thus, another approach is to specify ambient water quality goals and to look for a regional least-cost policy [SoP92]. As shown [Som93, SMPK94], such a non-uniform strategy can lead to significant savings, though the implementation is less straightforward than for a policy based on effluent standards. From a methodological point of view, the regional approach generally leads to a single-criterion optimization task [KuS94]. In practice, water quality criteria used as constraints are changed systematically by the analyst, thus learning the multi-objective aspects of the problem (see Section 3.3 for details on handling multiple criteria problems within the framework of single-criterion optimization). This approach has been implemented in the decision support system DESERT, incorporating rather generic hydraulic and water quality simulation models, parameter estimation and uncertainty analysis methods, dynamic programming etc., which was developed by IIASA's Water Resources Project (cf. [IMK+95]). Detailed results for the Nitra River basin can be found in [SMPK94].

A third approach – though still belonging to the same genre as the previous one – is possible by using multi-criteria optimization methods. Here, the concept of policy development is the same as in DESERT, but the methodology is different: as contrasted to the sequential analysis of a set of single-criterion optimization solutions, the problem is handled in one step (under ideal conditions) with the interactive incorporation of decision makers' preferences. The objective of the present paper is to apply such an approach and to develop a related prototype Regional Water Quality Management Decision-Support System (RWQM DSS). The first steps towards the development of RWQM are documented in [BMW93].

As already noted, the RWQM differs from traditional single-criterion optimization approaches, and also from more recent expert system/AI approaches, in that "hard" constraints are limited as much as possible through the use of a multicriteria model. Attractive traits of multicriteria decision aid (MCDA) are simplicity and flexibility, which allow the user to learn about the decision situation during the process of decision-making. One possible disadvantage of the implemented approach is that simple, linear models are used rather than more physically-based (i.e., non-linear) ones. However, the usefulness of complex models for management is often limited by uncertainty in system identification and a lack of high quality field data. Thus, the models applied in the Nitra River Case Study are relatively simple, yet they provide adequate information for making real decisions.

In comparison to DESERT (which includes various models and tools, optimization, a simplified GIS, interfaces, and a detailed database characterizing "physical" and other properties of the watershed considered), RWQM is narrower in scope since it considers only the strategy development issue. Certainly, compatibility between the corresponding elements of the two systems is crucial in order to make comparisons. This was not the case for the prototype of RWQM, for which data was assembled from different sources, but this shortcoming has been basically eliminated by using data (listed in Section 5) computed by the simulation models of DESERT.

The broad objective of the present paper was defined earlier. Detailed goals can now

be summarized as follows:

- To implement one of several possible mathematical programming models and techniques in order to provide a tool for supporting decision making.
- To illustrate the capabilities of multicriteria model analysis in the field of water quality management.
- To test, using a real-life problem, several modular and re-usable software tools aimed at facilitating the implementation of Decision Support Systems.

The structure of the paper is as follows. In Section 2, a mathematical formulation of the problem is detailed, along with model assumptions. This formulation was done in a way most suitable for checking the correctness of the model. However, this form is different from the standard formulation of optimization problems, so a reformulation which corresponds to an equivalent MPS standard form is provided in Appendix C.

A discussion of multicriteria model analysis is given in Section 3, followed in Section 4 by a description of the organization of software and data. Section 5 contains a description of the data used, and the results of the applications are discussed in Section 6. Conclusions and recommendations for future work are given in Section 7.

2 Core model formulation

Although there are many ways of formulating a water quality management problem (cf [LSH81, SMPK94]), they can be grouped into two basic approaches:

- to use scenario analysis (i.e., simulating the results of assumed decisions), or
- to optimize with respect to a selected criterion while meeting prescribed constraints (which in contrast to scenario analysis often incorporates only a simplified, linear simulation model or the use of linear transfer coefficients[1]). For instance, one can minimize costs subject to water quality constraints, maximize water quality subject to a budget constraint, or find a Pareto efficient solution using a multiple objective programming method for different combinations of criteria (performance indices) and values of reference points. There are also a number of variations for each of these three alternatives, such as the minimization of investment and annual costs or the maximization of the improvement in several water quality criteria.

However, both approaches (scenario analysis and optimization) require the definition of a *core model* which relates waste water emissions, treatment decisions, and the resulting ambient water quality. Specification of the core model (complex or simple) and the data used for it are obviously of critical importance for any model-based decision support system. Therefore, to document the research and provide a basis for possible future work, we specify in detail both the mathematical formulation and the data used in this model.

2.1 Assumptions

The adopted assumptions and the resulting model formulation slightly differ from the assumptions and formulation reported in [BMW93]. As mentioned previously, these differences are not only the result of examining available data and alternative model formulations, but also of modifications necessary for achieving results consistent with the models described in [SMPK94]. Therefore, the following assumptions have been adopted:

[1] A transfer coefficient expresses the linear impact of an emission at a given location on the water quality at another, downstream location, or, in a non-dimensional form, the ratio of corresponding concentrations at two locations considered. See later.

1. Our water quality simulation model is simple and uses the widely used concept of transfer coefficients. Regarding the hydraulics of the river, we employ a steady-state formulation, using a "critical design flow" from the Nitra study (cf [SMPK94]). We assume - as usually done in practice - complete mixing downstream of each emission and tributary confluence, and we assume uniform flow along the river between these points. Since the primary water quality problem of the Nitra River is related to dissolved oxygen (DO), we use the well-known (linear) extended Streeter-Phelps model incorporating nitrogenous oxygen demand (see e.g. Thomann and Mueller [ThM87]). In order to analyze the joint impact of carbon (or organic material), nitrogen, and total phosphorus removal, we can consider phosphorus in a simplified manner by assuming a first-order decay due to apparent settling (i.e., the impact of P on algae biomass and DO is neglected). However, this model capability has not yet been tested.
2. We consider six water quality constituents (the following values of subscript l are used for the respective constituents):
   0. DO, dissolved oxygen
   1. CBOD, carbonaceous oxygen demand
   2. NBOD, nitrogenous oxygen demand
   3. NH4, ammonia
   4. P, total phosphorous
   5. SOD, sediment oxygen demand.

   We note that NH4 is directly obtained from NBOD, P is not coupled to any of the other variables (and actually is not a criterion in the present effort), and SOD is considered as a model parameter to be calibrated (see later).
3. For a given river system a set of locations or points should be defined (given implicitly by the provided data), each of which is at least one of the following:
   - Emission point: waste water is discharged at this point. The amount of discharged pollutants (e.g. BOD) depends on the treatment technology chosen in the decision process. These include municipal and industrial discharges.
   - Abstraction point: water is withdrawn from the river. At these points one can consider a "negative" emission, whereby the constituent loads are reduced proportionally to the reduction in river flow.
   - Monitoring point: water quality is compared to given standards at this point.
   - Confluence point: junction point of two rivers. Constituent loads are the sum of loads from both rivers.
   - Weir point: DO is added to the river due to the increase in turbulence downstream of a weir or small dam.
   - Other points: points for which hydraulic and hydrologic data exist and therefore new travel time and transfer coefficients are calculated. The load of waste does not change at these points.

   Each of these points is called a *node*, denoted by the subscript j. At every node the equations that define water quality are given. This enables us to employ a formulation for the mass balances of constituents at each of these points on the river.
4. For every emission point, one emission source is assumed[2]. At each emission node there exist a number of technology options, denoted by the subscript k. Included in

[2]This has been done to simplify the description. Actual implementation can easily be modified to accommodate any number of sources in a single point.

each of the k treatment technologies are the option of no treatment (with raw waste concentrations and no cost), as well as the option of maintaining the existing technology (with the operating cost but no investment cost).

5. For monitoring points, standards for constituents 0, 1, 2, 3, and 4 (above listed) may be set. For each monitoring node, a variable which corresponds to a relative violation of the standard is defined. A matrix of such variables (rows corresponding to nodes, columns to water quality constituents) may then be used for various analyses of the resulting water quality.

2.2 Decision variables

The decision variables are the treatment technologies to be implemented at the j-th node where waste-water emissions occur. Let these be denoted by x_{jk}, where j is the index of an emission node and k is the technology choice. Since only one technology can be implemented at each point, we impose the following constraint:

$$\sum_{k \in K(j)} x_{jk} = 1 \qquad x_{jk} \in \{0,1\}, \qquad j \in E \tag{1}$$

where $K(j)$ is a set of technologies considered for the emission node j, and E is a set of nodes where emissions occur.

2.3 Auxiliary variables

Auxiliary variables are the model quantities whose values depend on the values of decision variables. Other model quantities (called parameters and defined in Section 5) are assumed to be given or to be calculated from the provided data. This distinction is necessary since parameters are actually computed before scenario analysis or optimization starts.

Auxiliary variables are defined to ease both the problem formulation and the interpretation of results. Not all of the auxiliary variables will be used in every possible formulation of the examined model (cf Section 3), but for consistency all variables that might be used are defined below. They are divided into two groups: variables related to water quality and variables related to costs.

2.3.1 Water quality variables

- The following three indices of water quality have been defined:

$$DOmin = \min_{j \in M}(aq_{j0}) \tag{2}$$

$$BODmax = \max_{j \in M}(aq_{j1}) \tag{3}$$

$$NH4max = \max_{j \in M}(aq_{j3}) \tag{4}$$

  where aq_{jl} (defined by (9) or (10)) is the ambient concentration of the l-th constituent at node j, and set M contains indices of monitoring nodes.
- At each monitoring point a vector wq_j of relative water quality indices is defined as:

$$wq_{j0} = (aqs_{j0} - aq_{j0})/aqs_{j0} \qquad j \in M \tag{5}$$

$$wq_{jl} = (aq_{jl} - aqs_{jl})/aqs_{jl} \qquad l \in [1,4] \qquad j \in M \tag{6}$$

Note that the water quality index for DO (dissolved oxygen) is defined by eq. (5) in a different way than the indices defined by eq. (6). Since DO should be maximized while other ambient concentrations should be minimized, such an approach allows for the minimization of all water quality indices, thus simplifying various model formulations.

- One can consider a maximum violation of a standard for a particular constituent, namely:

$$g_l = \max_{j \in M}(wq_{jl}) \qquad l \in [0,4] \tag{7}$$

- Additionally, one may wish to consider an aggregate index of regional water quality which can be defined as

$$g_{all} = \max_{l \in [0,4]}(g_l) \tag{8}$$

Indices defined by (7) and (8) – if positive – show a maximum relative violation of water quality standards for the l-th constituent or for all constituents, respectively. A negative value of g_l or g_{all} indicates that water quality standards are observed at every monitoring location, and the corresponding absolute value in such a case represents the relative "margin of safety" of water quality at the worst monitoring location.

- The ambient concentration of DO (denoted for the j-th node by aq_{j0}) is affected by several constituents, as well as by the saturated dissolved oxygen concentration (see [ThM87] for details). DO is given by the extended Streeter-Phelps model, analytically integrated stretch by stretch

$$\begin{aligned} aq_{j0} \;=\; & [1./(Q_j + W_j)] * \Big(\sum_{i \in I(j)} \Big(b_{i0} + Q_i * (DOsat_j - TC_{i0}(DOsat_i - aqy_{i0}) \\ & - \sum_{l \in \{1,2,5\}} TCp_{il} aq_{il} \Big) \Big) + ioxy_j \Big) \end{aligned} \tag{9}$$

where the set $I(j)$ contains indices of nodes located immediately up-stream of the j-th node (this set contains two elements for confluence nodes and one element otherwise), aqy_{i0} is equal to max(aq_{i0},0), aq_{il} is defined by eq. (10), and the remaining right hand side quantities are given (or computed from given data - cf Section 5): $DOsat_j$ is DO saturation level at j-th node, TC_{il} are transfer coefficients for water quality constituents, TCp_{il} are transfer coefficients for the oxygen-demanding constituents (applied to the DO balance), Q_j is the river flow at (or just below) node j, W_j is the withdrawal occurring at node j, b_{i0} is the background level of DO mass entering the river upstream of node j, and $ioxy_j$ is the DO "loading" from an emission at node j, as defined in the data section. The second set of transfer coefficients (TCp_{il}) applies to the DO balance, i.e. the effect of CBOD, NBOD, and SOD on the DO level. Note that the first set of transfer coefficients (TC_{il}) applies only to the decay of each constituent. Thus, the summation term represents the DO mass coming from upstream, which consists of oxygen transfer from the upstream node(s) as well as "background" oxygen from groundwater infiltration flow (for simplicity, we assume that background loads of other constituents do not affect DO until the next reach downstream). This upstream mass is then mixed with the DO load from the wastewater emission, $ioxy_j$, hence the division by the total flow $Q_j + W_j$. Note that indices 1, 2, and 5 correspond to CBOD, NBOD and SOD, respectively.

- Ambient concentrations of other constituents (denoted by aq_{jl}) are defined by:

$$aq_{jl} = \Big(\sum_{i \in I(j)} (b_{il} + TC_{il} aq_{il} Q_i) + e_{jl} \Big) / (Q_j + W_j) \qquad l \in [1,4] \tag{10}$$

As in eq. (9), the first term in this equation represents the background load of constituent l which accounts for non-point or non-controllable source pollution, the second term represents the load of the constituent l arriving from the upstream reach(es), and the third term represents the emission load of constituent l at node j, as discussed below. Thus, Q_i and Q_j are the flows at points i and j, W_j is the amount of water withdrawn from the river at point j, and TC_{il} is a dimensionless transfer coefficient for constituent l in a segment from node i to the nearest node downstream, j. Note that the eq. (10) is formulated with the assumption that Q_j accounts for the waste flow and the withdrawal, that is, the waste flow already has been added to and the withdrawal subtracted from Q_j. If this assumption[3] were not true then the equation (10) would be replaced by:

$$aq_{jl} = \Big(\sum_{i \in I(j)} (TC_{il} aq_{il}(Q_i - W_i) + b_{il}) + e_{jl} \Big) / (Q_j + q_j) \qquad l \in [1,4] \tag{11}$$

where q_j is the given waste flow rate [m^3/day] at node j.

- Along with the waste flow rate, q_j, there are water quality constituent concentrations resulting from the implementation of the k-th technology at the j-th emission node, em_{jkl} [mg/l]. The emission load of the l-th constituent at the j-th node is denoted by e_{jl} and is defined by:

$$e_{jl} = q_j \sum_{k \in K(j)} x_{jk} em_{jlk} \qquad l \in [1,4] \qquad j \in E \tag{12}$$

Note that – due to the eq. (1) – for each j exactly one out of $K(j)$ binary variables, x_{jk}, will be equal to one while the others will be equal to zero.

2.3.2 Cost variables

Corresponding to the k-th treatment technology implemented at the j-th node are an investment cost IC_{jk} and an operating and maintenance cost OMC_{jk}. As noted earlier, included in the technologies are the option of no treatment (with raw waste concentrations and no cost) and the option of maintaining the existing technology (with O&M cost but no investment cost).

- The investment costs Inv_j for the j-th emission point are defined by

$$Inv_j = \sum_{k \in K(j)} x_{jk} IC_{jk} \qquad j \in E \tag{13}$$

- The O&M costs OM_j are given by:

$$OM_j = \sum_{k \in K(j)} x_{jk} OMC_{jk} \qquad j \in E \tag{14}$$

- The total annual cost (TAC) of each technology is defined by using the combining the two previous cost components as

$$TAC_j = [r(r+1)^n / ((r+1)^n - 1)] Inv_j + OM_j \qquad j \in E \tag{15}$$

where r is a given discount rate, n is a given capital recovery period, and the multiplier of the first term is the so-called capital recovery factor.

[3]This assumption has been adopted for the data currently used.

• Finally, one may want to consider the sums of respective costs for the whole region:

$$Tot\_Inv = \sum_{j \in E} Inv_j \tag{16}$$

$$Tot\_OM = \sum_{j \in E} OM_j \tag{17}$$

$$Tot\_TAC = \sum_{j \in E} TAC_j \tag{18}$$

For details of costs of various discharges and treatment alternatives in the Nitra river basin, the reader is referred to [SMPK94].

2.4 Performance indices

Performance indices serve for comparing the solutions obtained as a result of solving an optimization problem. For a single criterion optimization, one such index has to be selected as an objective function, while constraints are usually set for some of the other indices (cf Section 3.3). For multicriteria optimization, a number of objectives is selected from a set of possible performance indices, and setting proper aspiration and reservation values may be equivalent[4] to specifying constraints for respective objectives.

The *core model* specification currently allows the following selection of performance indices (this list can easily be extended if the need for other objectives arises):

• The three indices of water quality, $DOmin, BODmax, NH4max$ (eqs. 2 - 4).
• The relative violation of standards set for each water quality constituent among the set of monitoring points; this is equivalent to minimizing $wq_{jl}, j \in M$ (eq. 7) for the l-th constituent,
• The regional water quality index g_{all} (eq. 8)
• The total annual cost for the whole region Tot_TAC (eq. 18).
• The total investment cost for the whole region Tot_Inv (eq. 16).
• The total operating, maintenance, and replacement cost for the whole region Tot_OM (eq. 17).

In order to facilitate both the formulation and analysis of the model, all indices except $DOmin$ have been defined in such a way that a smaller value is preferred to a larger one.

3 Model analysis for decision support

3.1 Model-based decision support

The term *Decision Support System* (DSS) is widely used in both research and in practice, but there is little consensus as to its meaning. In order to avoid possible misunderstandings, it is necessary to present the basic characteristics and features of the class of DSSs with which we will be dealing. Let us start with a brief discussion of the environment in which a DSS may be used. The key person in this environment is an individual who uses a DSS. By convention such a person is called a *Decision Maker* (DM). By this term we mean both a person who makes real decisions (whether a manager or an engineer or an operator) or an expert who may be his/her advisor. Decisions are made within a

[4]This is a much more flexible approach for the typical case in which constraints set to some goals are too tight and cause the problem to be infeasible. In such a case, the multiobjective approach provides a nearest feasible solution while the single objective formulation only reports a problem to be infeasible.

Decision Making Process (DMP), which, in situations that justify the use of a DSS, is a relatively complex and unstructured task. The purpose of a DSS is not to automate decision-making, but to help a DM understand the consequences of different decisions, and thereby make a better decision. In other words, a DSS can be considered as a tool which, under full control of a DM, performs the cumbersome tasks of data management and analysis and provides relevant information that enables a DM to concentrate on the part of the DMP which can not be formalized and automated.

A model-based DSS requires the development of a mathematical programming model which can adequately represent the real-world situation in which a decision is needed. To represent this *decision situation*, the model must be capable of being used for predicting and evaluating the consequences of decisions. Such a model is typically composed of the following elements (cf e.g. [WiM92]):

- Decision variables, which represent the actual decisions (choices, options) to be made. In RWQM the decision variables are selections of waste water treatment technologies[5] at each of the controllable emission points.
- Potential objectives (goals, performance indices), which can be used for evaluating the consequences of implementing the computed or chosen decisions. In RWQM such objectives include various costs (total annualized, investment, operational) and ambient water quality indicators (concentration of different constituents, violations of water quality standards), both for selected monitoring points and for the entire region.
- Various intermediate and parametric variables (balance and/or state variables, resources, external[6] decisions). In RWQM these variables include all potential objectives and some auxiliary variables which facilitate the understanding of the model formulation and the interpretation of results.
- Constraining relations (inequalities, equations, etc.) between variables that indirectly determine the set of admissible (feasible) decisions. Many of the constraints represent physical processes (such as mass balance) which can not be violated. Clearly, this is the most important part of any model. For comments about its implementation in RWQM, see Section 2.
- Outcome relations, which define goals as functions of variables. In RWQM the intermediate variables have been selected in such a way that the definition of such relations is not needed.

For more details about the formulation of the core model used in RWQM, see Section 2.

3.2 Optimization in decision support

Every DSS should be used in two basic modes, simulation and optimization, which can be briefly characterized as follows:

- In simulation, decision variables are inputs and goals are outcomes. Therefore, this technique is good for exploring the intuition of a DM and for verification of the model. It is also good for providing a DM with information about consequences of applying certain decisions. One can thus consider simulation as an alternative-focused method of analysis, in which the user examines the effects of implementing prespecified alternatives.
- In contrast, optimization can be considered as a goal-oriented (value-focused) approach which is directed towards creating alternatives. Optimization is driven by a desire to

[5]This also includes the *do nothing* option.

[6]Those not directly controlled by a DM.

reach a set of goals expressed in terms of values of the objective(s). Therefore, goals are a driving force, and the values of decision variables are outcomes.

Interchangeable use of both simulation and optimization has obvious advantages, especially in the learning phase of using a DSS.

Simulation capabilities of RWQM have been implemented in the prototype documented in [BMW93] by an object oriented prototyping tool ORVAN (cf [Pla90]). However, this activity has been discontinued because other tools (cf [SMPK94]) provide the same functions with much more detailed models. Therefore, only the optimization capabilities of RWQM will be discussed further in this paper.

Using optimization, a DM may want to consider different types of related costs and standards for water quality. However, he/she knows that specifying particular water quality criteria (ambient or effluent) may lead to solutions which are too expensive (even if least-cost solutions are considered). On the other hand, assuming constraints for costs (with water quality standards being goals) could result in unacceptable quality of water. Therefore, in the most general model, one should treat both costs and water quality standards as goals (objectives). Doing so provides the flexibility of examining trade-offs between costs and water quality.

Nonetheless, single-criterion optimization for water quality management is still common and useful for enhancing the understanding of major features of a problem. This usually entails one of three basic formulations:

1. cost is minimized subject to water quality constraints,
2. water quality improvement is maximized subject to a cost constraint, or
3. cost is minimized with the costs of water quality violations included in the form of penalty functions (cf e.g.[LSH81])

With respect to the first two formulations, the constraints may be considered "soft" in that the decision-makers must also decide how much funding to allocate for waste-water treatment and at what levels to set water quality standards. In analyzing the problem, decision-makers might wish to run many simulations, varying the cost/water quality constraints each time. Regarding the third formulation, placing monetary values on environmental quality is controversial and requires much case-specific research (cf e.g. [SCK89]).

Multiple criteria decision analysis (MCDA) can be a valuable way to evaluate the trade-offs among costs and water quality: rather than imposing constraints (such as water quality or cost constraints) which may not be identifiable in the real world, the decision-maker can choose ranges of values for a set of objectives, and thus view the problem in a more flexible and realistic manner.

3.3 Single criterion optimization

For single criterion optimization one of the performance indices must be chosen as a goal function. However, practical problems have usually several criteria, so it is necessary to introduce constraints for other criteria. For the sake of illustrating this approach, we outline two of the well known approaches, namely:

1. Maximization of the environmental quality under given financial resources. For our model this is equivalent to the minimization of the regional water quality index g_{all} (where g_{all} is equal to the maximum violation of the given standards) under a given constraint on total annual cost, which takes the following form:

$$\min g_{all}$$

$$\text{subject to:} \qquad Tot\_TAC \leq \overline{TAC} \tag{19}$$

where g_{all} and Tot_TAC are defined by equations (8) and (18), respectively.

2. Minimization of costs for achieving a given water quality standard. Such a standard might be represented by g_{all} so that the problem has the following form:

$$\min Tot\_TAC$$

$$\text{subject to:} \qquad \min g_{all} \leq \overline{g_{all}} \tag{20}$$

Note that $g_{all} \leq 0$ implies that water quality standards are held at every node. Therefore, selecting $\overline{g_{all}} = 0$ is equivalent to setting hard constraints for the water quality standards. Solution of the problem (20) for such hard constraints could result in costs which are not acceptable. Therefore, one usually has to consider a series a problems with different values of constraints for the water quality standards, which can be achieved by trying different values of $\overline{g_{all}}$. In practice it is usually necessary to repeat this analysis for several such constraints (in our case representing the different water quality constituents).

Note that the above formulations are simplifications of more realistic formulations which include a number of constraints for the other criteria.

There are several techniques to deal with de facto multiple criteria problems within the framework of single criterion optimization. For example, Haimes proposed in [HaH74] the ϵ *constraint approach*, in which (n-1) objectives are placed into constraints with given *tolerable levels* (which can be interpreted as aspirations for the criteria that have to be achieved). This hard requirement can be relaxed by representing requirements for the values of criteria as *soft* constraints. This approach is discussed in more detail, and a number of extensions of traditional single-objective optimization are summarized, in [Mak94a]. One can also treat soft constraints (cf. [Mak94c] for details) as a special case of multiple criteria optimization. Since a single criterion approach has frequently no advantages over a multicriteria approach (cf Section 3.7 for reasons), we will focus only on multicriteria optimization.

3.4 Multiple criteria model analysis

3.4.1 General remarks

An adequate discussion of different approaches to Multiple Criteria Decision Analysis (MCDA) is beyond the scope of this paper, so we will present only a short summary of the approach that we have implemented, namely the *reference point* (RFP) approach. A reader interested in the methodological foundations and details related to different MCDA approaches is advised to consult one of many publications which provide a bibliography of this subject (cf e.g. [Mak94a] for an overview and bibliography).

Recently, multi-criteria optimization has been applied to a number of water resources problems. For instance, multiple objective decision making techniques have been used to derive reservoir operating rules (cf e.g. [LaS92] and e.g. [IkK92]), to design groundwater remediation plans (cf [SDM92]), and to assist in water resources conflict negotiations ([ThL92]). A comprehensive overview of different methodologies is given in [Hip92]. To our knowledge, however, this represents the first application of the RFP method to a water resources problem.

From the user's point of view, the critical step of MCDA is generating a part of the Pareto-optimal solution set[7]. Generating the entire Pareto-set is practically impossible

[7] Efficient, or Pareto-optimal, solutions are those for which an improvement in the value of one criterion cannot be attained without worsening the value of at least one other criterion.

and – even if done – would result in a vast amount of useless information. Therefore, most MCDA methods generate a very limited number of Pareto-solutions and then provide a tool for the analysis of these solutions and for generating another set of Pareto-optimal solutions based on these results. One danger of these methods is that they could be used as "black boxes." We believe that for a water quality DSS, the best method would be an interactive one which enhances learning during the decision process. Learning means understanding the correspondence between the aspirations (represented by desired values of criteria) of a user and the attainability of such aspirations. Since aspirations are usually not attainable, a user has to learn (using the mechanisms of the RFP method) how to adjust aspirations in order to find a feasible solution which best meets his/her expectations.

We do not discuss here approaches based on the idea of converting a multi-criteria problem into a single-criterion one by summing up weighted criteria. The arguments for avoiding this approach are discussed in detail in [Mak94c].

3.4.2 Reference point method

The RFP method is based on the concept of *satisficing behavior* (also called *bounded rationality*), in which the decision maker attempts first to improve the criterion which shows the worst performance (cf e.g. [MaS58]). This method has a number of noteworthy advantages over other MCDA methods, as discussed in detail together with a more formal presentation of the RFP technique in [LeW89, Mak94c]. Here we summarize only the RFP method in the form of the following stages:

1. The decision maker (DM) specifies a number of criteria (objectives). In typical applications there are 2–7 criteria. For an LP problem a criterion is often a linear combination of variables, but criteria may have other forms for specific applications (cf e.g. [Mak94c]).
2. The DM specifies an aspiration level $\bar{q} = \{\bar{q}_1, \ldots, \bar{q}_n\}$, where $\bar{q}_i$ are the desired values for each criterion and n is a number of criteria. Additionally, the DM specifies a reservation level $\underline{q}$, which is composed of the worst values of criteria that a DM would like to consider.
3. The underlying formulation of the problem is the minimization of an (piece-wise linear) *achievement scalarizing function*, which can be interpreted as an ad-hoc non-stationary approximation of the DM's value function dependent on the currently selected aspiration and reservation levels. Then, the problem is transformed by the DSS into an auxiliary parametric single-objective problem, the solution of which gives a Pareto-optimal point[8]. If a specified aspiration level $\bar{q}$ is not attainable, then the Pareto-optimal point is the nearest (in the sense of a Chebyshev weighted norm) to the aspiration level. If the aspiration level is attainable, then the Pareto-optimal point is uniformly better than $\bar{q}$. Therefore, this approach may be considered an extension of goal programming (see [OgL92]). Properties of the Pareto-optimal point depend on the localization of the reference point (aspiration and reservation levels) associated with the criteria. In order to correctly handle criteria that may have different magnitudes of values a proper (automatic) scaling in the criteria space is implemented (cf [Mak94c] for details).
4. The DM explores various Pareto-optimal points by changing the aspiration level $\bar{q}$ and reservation level $\underline{q}$ for each criterion. Additionally, a DM may stabilize a criterion

[8]For the sake of brevity we will refer to properly Pareto-optimal solutions as Pareto solutions (unless otherwise mentioned). A Pareto-optimal point is composed of values of all criteria for a corresponding Pareto-optimal solution.

(i.e. specify a desired value instead of minimizing or maximizing the value of this criterion) or temporarily remove a criterion from the analysis. This results in the computation of a Pareto optimal point with respect to the remaining "active" criteria, but values of criteria that are not active are still available for review.

5. The procedure described in points 2, 3 and 4 is repeated until a set of satisfactory solutions is found.

Thus, multiple criteria optimization with the reference point method can be thought of as inverse simulation: rather than repeatedly adjusting the decision variables to determine acceptable states (expressed as constraints in the classical approach to optimization), the user chooses desired states (in terms of ranges of values of objectives) and determines the resulting values of the decision variables. This provides a useful complement to scenario analysis.

3.5 Formulation of multicriteria problem

Multicriteria problem formulations are composed of the several objectives and of a core model. In our case the core model is composed of equations (1) through (10), and equations (12) through (18). Note that these equations involve only the constraints related to water quality constituents and to the definitions of variables. Therefore, none of the decision variables is constrained by a quantity which is actually an exogenous decision variable (such as a maximum available budget or an acceptable constituent concentration). Due to the nature of the RFP method, one can examine various Pareto-optimal solutions which represent compromises between costs and water quality. The basic advantage of this approach is that it provides a natural way to examine a number of Pareto-efficient solutions without facing the risk of infeasibility.

For the multicriteria analysis we have selected (out of the set of possible criteria defined in Section 2) the following six criteria:

- TAC, the total annualized cost (eq. 18).
- INV, the total investment cost (eq. 16).
- OMRC, the total operations, maintenance, and replacement cost (eq. 17).
- DOmin, the minimum DO concentration at any monitoring point (eq. 2)
- BODmax, the maximum CBOD concentration at any monitoring point (eq. 3).
- NH4max, the maximum NH4 concentration at any monitoring point (eq. 4).

This set of criteria can easily be modified (cf [Mak94c] for details).

3.6 Interactive multicriteria analysis

The first step in the analysis is to determine the utopia point and an approximation of the nadir point. These are determined by finding the "selfish solution" for each criterion. For instance, the utopia point value for DOMIN can be found by solving the single criterion problem of maximizing DOMIN without considering other criteria. The results of this step provide the so-called payoff table, which is a useful guide for selecting reference points and evaluating trade-offs. Note that one should not expect values of criteria that are better than the respective utopia point values or worse than the corresponding nadir values. Also, it is important to note that a nadir point value is typically much better than the "worst possible" value for a criterion (cf the discussion of results in Section 6 for an example).

A typical starting point for multicriteria analysis is to choose the utopia point as the first aspiration point and the nadir point as the first reservation point. The corresponding

Pareto-optimal solution represents a compromise solution for all criteria. This computation concludes the preparatory phase of the analysis, which is done automatically by LP-MULTI (cf [Mak94c] for details). At this point, the control is passed to a user who specifies aspiration and reservation levels in an interactive manner – new aspiration and reservation levels are selected upon the analysis of Pareto-optimal solutions obtained for previously specified reference points.

In the current software implementation (cf Section 4 for more details) the specification of aspiration and reservation levels is done with the help of the FT tool (cf [GrM95] for the description). FT is an interactive, graphical tool that supports specification of aspiration and reservation levels, allows for modification of the criteria status (a criterion can be active, stabilized, or temporarily disregarded) and the display of previous solutions. FT is linked with LP-MULTI, which generates a corresponding auxiliary mixed-integer programming problem and calls the MOMIP solver (cf [OgZ94]). A Pareto-optimal solution for given aspiration and reservation levels is found by solving an auxiliary single-criterion mixed-integer programming problem. A solution of the auxiliary problem is a properly Pareto-optimal solution that lies on a line defined (in the criteria space) by the aspiration and reservation levels. The definition and generation of the auxiliary problems is presented in detail in [Mak94c].

FT provides additional possibilities for more advanced users, such as specifying preferences in terms of fuzzy sets using the extended-value membership function (cf [GrW94] for details).

3.7 Multi-criteria vs. single-criterion model analysis

In practice, any decision problem is in fact a multicriteria problem. Single criterion optimization is used mainly for historical reasons, since for many years this was the only known optimization approach. Therefore, users were forced to select only one criterion as a goal function for optimization and to treat other goals as constraints. Hence, the practical use of any single criterion problem formulation would seldom result in one acceptable solution. On the contrary, it would require the generation and solution of many problems for different values of constraints (in our example, for $\overline{TAC}$ and $\overline{g_{all}}$, respectively). Many of these problems will not have feasible solutions and, without a good *a priori* understanding of the solution characteristics, it is generally not easy to find a set of acceptable solutions.

Multicriteria optimization techniques provide much better functionality without a substantial increase of computational complexity (in our case, the numbers of additional rows and columns were smaller then 1% of the respective dimensions of the *core* model). The applications of multicriteria optimization have been limited mainly by lack of modular tools that facilitate multicriteria model analysis.

One should point out that multicriteria model analysis based on the RFP approach provides an easy way for generating also solutions which are typically generated by a single-criterion optimization. However, both scenario generation and sensitivity analysis are much easier with the multicriteria optimization. Finally, it is possible to use a multicriteria tool for a single-criterion optimization (by selecting only one active criterion), but this is practically never required.

The RFP approach also provides an equivalent of the so-called *soft* constraints often needed in the single-criterion optimization. Namely, one can replace a soft constraint (or group of constraints) by an objective, and then set the aspiration level equal to the desired value of the constraint and the reservation level to the worst acceptable value. Hence, violations of *soft* constraints can be treated as goals (to be minimized) in the multicriteria

approach.

4 Organization of software and data

RWQM is being implemented with the principle of re-usability in mind. Therefore, reusable modular tools are being developed parallel to its implementation. We briefly characterize the software tools which can be applied in development of other DSS:

FT – Fuzzy Tool is a prototype implementation of the methodology outlined in Section 3 with an optional extension for interactive specification of user preferences in terms of fuzzy sets (cf [GrM95] for details). Currently this tool is operational only under MOTIF running on Solaris 2.3 and on MS Windows.

LP-DIT – Data Interchange Tool for Linear Programming Problems (cf [Mak94b] for details) is a prototype implementation for handling data that define a MIP or LP problem. LP-DIT provides an easy and efficient way for the definition and modification of MIP problems, as well as the interchange of data between a problem generator, a solver, and software modules which serve for problem modification and solution analysis.

LP-MULTI – Modular tool for multiple criteria problems (cf [Mak94c] for details) is a prototype implementation of a tool for generation and interactive modification of a multiple criteria problem. It currently uses LP-DIT for data handling and FT for interaction with a user.

MOMIP – Modular Optimizer for Mixed Integer Programming (cf [OgZ94] for details). It also uses LP-DIT for data handling.

This approach has several important advantages which, for the sake of brevity, will not be discussed fully here. Instead, we summarize only the functional structure of the software.

Data handling: The data used in the model (cf Section 5 for details) has been output from the simulation model documented in [SMPK94] and has been combined in one free–format ASCII file. The data file is composed of several segments containing groups of related data and a description of data items. The organization of the data file is flexible and provides adequate documentation so that its organization is easy to modify.

Problem generation: A problem-specific model generator (subsequently referred to as *the generator*) has been implemented. The generator (which uses LP-DIT) generates a *core* model, described in Section 2, in the form suitable for a mathematical programming problem (cf Appendix C for details).

Multicriteria problem analysis: The core model is used by LP-MULTI for the generation of a multicriteria problem. The currently used set of criteria (cf Section 3) can be easily modified. First, the utopia and nadir points are automatically computed. After this stage is completed, the interactive phase is started. In this phase the FT TOOL allows for an interactive analysis of solutions and the selection of new aspiration and reservation levels. A user can also change the status of a criteria and specify preferences in terms of fuzzy sets. The solutions are stored, and a summary

of solutions is logged, so that it is easy to continue analysis during another session and to produce a report based on a set of selected solutions.

Solution of multi-criteria problem: LP-MULTI converts the multicriteria problem and generates a corresponding MIP problem in the LP-DIT format. Then it calls the MOMIP solver. The currently examined model has (after conversion of a multiple criteria problem into a single-criterion problem by using the achievement scalarizing function) about 800 rows and 800 variables (including 90 binary variables), and it typically takes less than one minute to solve it on the Sun Workstation.

Reporting: Tools for examining complete results are currently very simple. One can obviously examine complete solutions (i.e. values of all variables listed in Sections 2.2 and 2.3). Additionally, a simple tool has been developed for plotting the resulting ambient concentrations at each node and for each constituent. Figure 1 (see Section 6.1) is an example of such a plot, which can be examined on a graphical terminal and stored in the Postscript format.

5 Data

All of the primary data (listed in Section 5.1) used in the research reported here have been provided from the simulation model documented in [SMPK94]. The data collection methods, selection of a design scenario, calibration of a water quality model, and analysis of parameter uncertainty are well documented in [SMPK94] and are beyond the scope of this paper. This section simply provides a list the data used and corresponding model parameters, followed by brief discussions of data conversions and calculated parameters.

5.1 List of Data

The following list is provided to summarize the data used and the corresponding model parameters. The data listed in this subsection is output from the simulation model documented in [SMPK94] and is stored in a free format ASCII file (cf Section 4 for more details). Therefore it can be easily modified.

KM_j – distance of each node from the mouth of the river on which it is located, km
Q_j – river flow at each node, m^3/s
W_j – withdrawal from river at each node, m^3/s
IF_j – infiltration flow accumulated at each node, $m^3/s/km$
q_j – waste flow from source at node j, m^3/s
T_j – temperature of the river at each node, C
$CBOD5_j$ – 5-day CBOD concentration at each node, mg/l
$NH4_j$ – ammonia concentration at each node, mg/l
P_j – dissolved phosphorous concentration at each node, mg/l
HW_j – height of weir at node j, m
TR_j – travel time to each node from the first surveyed point upstream, $hours$
v_j – velocity at each node, m/s
h_j – depth of water at each node, m
ad_{jl} – background concentrations of water quality constituents in infiltration flow, mg/l
K_0 – average reaeration rate coefficient for the entire Nitra River, $/day$
K_1 – average CBOD removal rate for the entire river, $/day$
K_2 – average NBOD removal rate for the entire river, $/day$

em_{j0} – effluent concentration of DO, considered a constant for each source, mg/l
em_{jl} – effluent concentration of other constituents resulting from each technology, mg/l
IC_{jk} – investment cost of each treatment technology at each emission point, (in 10^6 US$)
OMC_{jk} – operating cost of each treatment technology at each emission point, (in 10^6 US$)

5.2 Implemented data conversion

Several data items are provided in a form that requires simple conversions before using them in the model. Such conversions are listed below for the sake of documentation.

- The water quality data includes values of 5-day BOD (CBOD5) and ammonia (NH4), from which we can calculate ultimate CBOD emissions em_{j1}:

$$em_{j1} = CBOD5_j/(1 - exp(-5 * K_{1j})) \qquad j \in E \tag{21}$$

where K_{1j} is the CBOD removal rate [/day] at emission point j (cf eq. (28)), and NBOD emissions em_{j2}:

$$em_{j2} = \alpha * em_{j3} \qquad j \in E \tag{22}$$

where the coefficient α is estimated from the stoichiometry of the nitrogen-oxygen cycle and is equal to 4.57 if it is assumed that all of the ammonia consumes oxygen.

- Travel times TR_j, in hours, are provided for each node from the farthest upstream measurement point. For each river and for all but the last nodes, a difference of those quantities is computed, converted into days and stored as TR_j. The last node on each tributary is located immediately upstream of the confluence, so that zero travel time is considered to the confluence.
- We assume that the sediment oxygen demand [g/m^2-day] is negligible along the river

$$SOD_j = 0 \tag{23}$$

5.3 Calculated parameters

Several parameters used in the model formulation are calculated in advance from the provided data. These calculations are as follows:

- The saturation DO concentration, $DOsat_j$, is empirically defined as

$$DOsat_j = 14.652 - 0.41022T_j + 0.007991T_j^2 - 0.000077774T_j^3 \tag{24}$$

where T_j is the temperature (degrees Centigrade) at the j-th node.

- Assuming first-order decay, the dimensionless transfer coefficients in equations (9, 10, 11) are defined as follows:

$$TC_{jl} = exp(-K_{lj}TR_j) \qquad l \in [1,4] \tag{25}$$

where K_{lj} is a temperature-dependent decay rate [/day], and TR_j is the travel time (in days) in river segment that starts at node j. Since DO is also affected by one or more weirs along the river, the corresponding transfer coefficient takes the form

$$TC_{jl} = (1/r)exp(-K_{lj}TR_j) \qquad l = 0 \tag{26}$$

where r is the ratio of the DO deficits above and below the weir and is given by (cf [Gam57])

$$r = 1 + 0.38[c_1 * c_2 * HW_j(1 - 0.11HW_j)(1 + 0.046T_j)] \tag{27}$$

where HW_j is the height of the weir, and c_1 and c_2 are coefficients dependent on the type of weir and upstream DO deficit. Since we do not know the upstream DO deficit ahead of time, we have selected a conservative value for c_2. Note that this formulation considers the weir to be at the beginning of reach j, and that $r = 1$ when $HW_j = 0$ (i.e., where no weir exists).

- As already mentioned, we treat ammonia (NH4) and NBOD synonymously. Their concentrations differ by a factor of α (cf eq (22)) and their decay rates are the same ($K_{2j} = K_{3j}$).
- Each of the decay rates in the transfer coefficients are temperature-dependent. The rate coefficients (for 20°C) and temperature corrections are given as follows:

$$K_{ij} = K_{ij}^{20}\beta_i^{(T_j-20)} \qquad i = 1,2 \tag{28}$$

where $\beta_1 = 1.04$ and $\beta_2 = 1.08$. The reaeration rate (K_0) is also dependent on the hydraulic of the river so that (cf [SMPK94]):

$$K_{0j} = K_{0j}^{20}(1.024)^{(T_j-20)}\sqrt{v_j/h_j^3} \tag{29}$$

where v_j is the average velocity [m/s], h_j is the depth [m], and T_j is the temperature [C]. The values of coefficients K_{0j}^{20}, K_{1j}^{20}, and K_{2j}^{20} provided in the data file (cf Section 5.1) are for 20°C.

- Not only must the decay of CBOD, NBOD, and SOD along the river be modeled, but so must their effect on the DO balance. Therefore, the effects of the oxygen-demanding constituents on the DO concentration (CBOD, NBOD, and SOD) are represented by different transfer coefficients than calculated in (25). We denote these by TCp_{jl} and define them as follows:

$$TCp_{j1} = [K_{1j}/(K_{0j} - K_{1j})][exp(-K_{1j}TRj) - exp(-K_{0j}TR_j)] \tag{30}$$

$$TCp_{j2} = [K_{2j}/(K_{0j} - K_{2j})][exp(-K_{2j}TR_j) - exp(-K_{0j}TR_j)] \tag{31}$$

$$TCp_{j5} = (K_{0j}/h_j)[1 - exp(-K_{0j}TR_j)] \tag{32}$$

where h_j is the depth [m] of the river.

- The additional influx of pollutants due to infiltration flow (cf eqs. (9) and (10)) is given by

$$b_{jl} = ad_{jl}IF_jL_j \tag{33}$$

where ad_{jl} is the background concentration of constituent l, IF_j is the volume of infiltration flow entering the river upstream of node j, and L_j is the length of the reach upstream of node j.

- The additional input of DO due to emissions (cf eq. (9) is defined as

$$ioxy_j = q_jem_{j0} \tag{34}$$

for all nodes j which are source nodes, where q_j is the waste flow and em_{j0} is the DO concentration in the wastewater. This convention is also used for tributaries.

- Since deoxygenation in the river may occur below large emissions, the relation $aqy_{i0} = max(aq_{i0}, 0)$ is needed in eq. (9) to compute the correct DO and maintain a linear model. To implement this relation, it is possible to add a dummy variable y_i as follows:

$$aqy_{i0} = aq_{i0} + y_i \tag{35}$$

$$aqy_{i0} \geq 0 \tag{36}$$

$$y_i \geq 0 \tag{37}$$

A penalty term corresponding to the dummy variables should be then added to the objective function in order to make y_i as close to zero as possible.

For the data currently used, however, this situation occurs only on one of the tributaries (the Handlovka River, see [SMPK94]). Since this case is not an acceptable solution, we can simply prevent the computation of a negative DO value by setting the constraints:

$$aq_{i0} \geq 0 \tag{38}$$

This prevents the generation of about 150 additional variables, which would be required in the implementation of eq. (35) However, an option for the generation of eq. (35) has been implemented in the problem generator and can easily be used, if needed.

6 Discussion of results

In this section we present some results from the multicriteria model analysis described in Section 3. First, we compare the results from RWQM and the simulation model used within DESERT (see [SMPK94] for a detailed description). Second, we present general results from the six-criteria formulation. Finally, we highlight results which show some of the advantages of the multicriteria analysis over traditional single-criterion analysis. In no sense are these results "complete", nor should they be used for policy recommendations. Instead, the results were selected to demonstrate the capabilities of multi-criteria optimization for decision support and to provide some interesting insights to the case study investigated.

6.1 Comparison of models

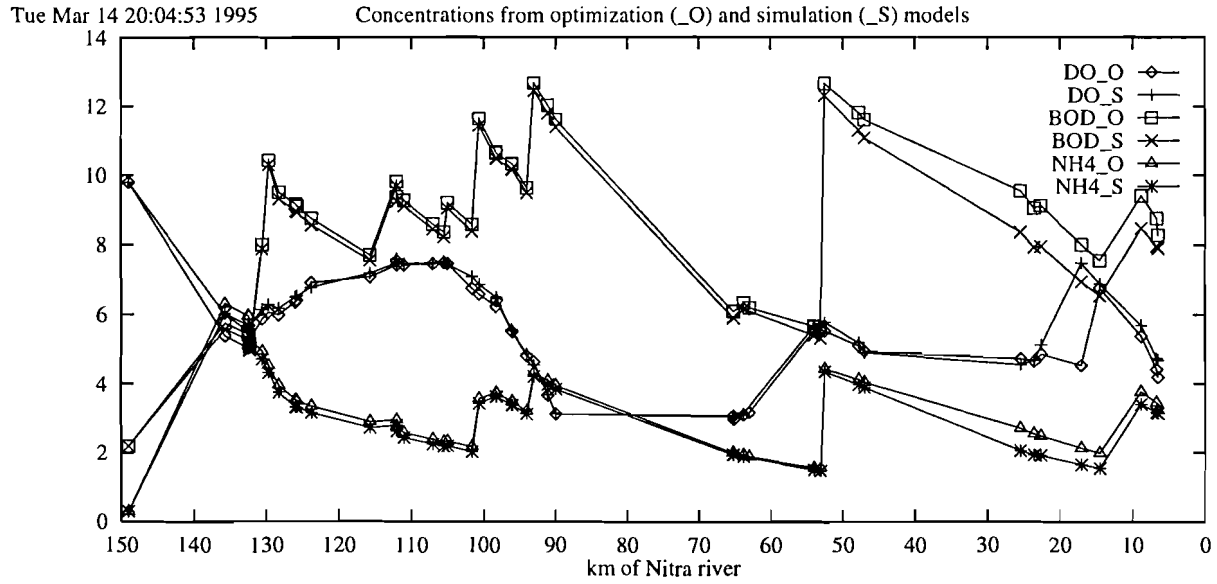

Figure 1: DO, BOD and NH4 profiles from RWQM and simulation models.

First, we compare the results of a "base case" to those obtained from the simulation model documented in [SMPK94]. Shown in Figure 1 are the resulting concentrations of

DO, BOD, and NH4 for the case in which the currently used treatment technologies are implemented in each model. Overall concentrations match very closely along the entire river, reflecting the success of harmonizing the two approaches. Smaller deviations (e.g. around 135 Rkm and 17 Rkm) are due result from discrepancies in modeling conventions. The maximum[9] deviation in DO is less than 0.5 mg/l. Similarly, BOD and NH4 concentrations are quite similar, with the critical differences in BODmax and NH4max (at kilometers 53 and 107, respectively) smaller than 0.1 mg/l.

A more detailed discussion of the identified technical reasons for these discrepancies is presented in Appendix B. However, there are two important differences between the sets of solutions obtained from both models due to the optimization techniques used which can lead to different results. The first difference is due to the fact that in single-criterion optimization only one criterion is specified (usually a cost related criterion), and water quality indices (represented by concentrations) are treated as constraints. In such analyses, for some (a priori unknown) constraint values, one can expect large changes in the value of the selected objective to result from relatively small changes in constraint values. For example, a significantly lower cost might be obtained with the minimum DO value set to 4.90 mg/l instead of a round value like 5.0 mg/l (which is a common constraint value selected with national water quality goals and classification systems in mind). This is a typical observation for mixed integer programming problems and the results presented in Table 1 also illustrate this point. Since multi-criteria analysis facilitates the generation of solutions, we show in Table 4 a number of solutions with similar values of DO and quite different costs. The second difference between model results is due to the dynamic programming technique used for the single-criterion optimization. Due the discretization of the state space (needed to alleviate the "curse of dimensionality"), this technique provides suboptimal solutions. The difference between the suboptimal values and the true optimum may strongly depend on the selected constraint values. From our analysis, it seems that the single-criterion solutions from the dynamic programming model are indeed close to optimal, but probably not always[10] optimal.

These considerations are illustrated in Table 1, in which several multicriteria solutions (numbered by 1 through 5) are given, along with the dynamic programming result (# 6) for the minimization of investment costs with the constraint DO $>$ 5 mg/l (cf p. 163 in [SMPK94]). It is apparent from the table that small differences in the water quality goals used as constraints are associated with relatively large differences in investment costs. This is due to the fact that differences in treatment technologies at the various locations, as illustrated in the right side of the table (see [SMPK94] for details), leads to discontinuous changes in the investment cost (the impact is nearly insignificant on the OMRC cost), particularly with the DO constraint value close to 5 mg/l. At this level, investments show diminishing returns with respect to DO due to "non-controllable" background loads (e.g. nonpoint pollution and lots of small industrial emissions considered non-controllable here), which prevent the DO level from being improved above 5.4 mg/l (see Sec 6.2) at a reasonable cost.

Table 1 also shows similarities among the treatment configurations. For example, at the largest plants such as Topolcany (To), Nitra (Ni), and Nove Zamky (No), there is little

[9]Not counting the difference shown at the weir (Rkm 17) caused by different conventions applied by the two models: the simulation model output the DO concentration just below the weir, while the optimization model reports the concentrations just above the weir.

[10]We can not evaluate suboptimality in an exact way due to the model discrepancies listed above. However, this easily could be done by running the simulation model with the technologies set to those corresponding to the optimal multicriteria solution.

#		INV	OM	DO	BOD	NH4	Ha	Le	Pr	Pa	Ba	To	Ni	Zl	Vr	Su	No
1	MC	11.3	6.8	4.93	10.6	3.29	1	3	2	1	3	6	1	2	3	2	1
2	MC	12.6	6.9	4.98	10.6	3.26	2	1	2	1	3	6	1	3	2	2	1
3	MC	14.5	6.6	5.05	10.6	3.26	2	1	2	1	3	6	1	2	0	0	2
4	MC	14.8	6.7	5.08	10.6	3.26	3	1	2	1	3	6	1	2	0	0	2
5	MC	13.1	6.9	5.10	10.2	3.1	2	1	2	1	3	6	2	1	2	1	1
6	DP	15.0	6.9	5.20	11.4	3.2	1	1	2	1	3	6	2	1	1	1	2

Table 1: Solutions from multicriteria (MC) and dynamic programming (DP) optimizations. Aspiration level and constraint, respectively, for DO set to 5.0 mg/l

deviation among the different solutions. However, for different selections of aspiration and reservation levels in MCDM one can easily find a number of solutions that differ slightly (but perhaps significantly) from solution # 6, thus providing arguments for a more detailed consideration of trade-offs between the criteria. In particular, solutions # 2,3 and 4 are slightly cheaper, in terms of investment costs, than solution #5 (all having lower, but still acceptable, DO levels); and solutions #1 and 2 are substantially cheaper, but are considered by a single criterion model (with the constraint for DO set to 5 mg/l) as infeasible. Additionally, solutions #2-4 show the trade-off between investment and OMRC costs. All solutions presented in Table 1 illustrate the high sensitivity of the single-criterion problem.

6.2 Results and their discussion

As noted, our primary purpose is to test the methodology developed. Other results and their practical implications can be found in [SMPK94]. Thus, we did not prescribe a minimum treatment level nor consider it "compulsory" to operate all the existing facilities, as would be typically done in practice. Also, we should note that the multicriteria analysis is aimed at interactive model examination, and any *ex post* summary of results can not replace the interactive analysis. Therefore, the selected results presented in this section can be considered only as an illustration of the methodology.

The present case study can not demonstrate all the possible advantages of the MCDM. Namely, we face a so-called multiple pollutant issue in which the water quality indicators and costs used as criteria are correlated. For instance, the oxidation of both organic material (BOD) and NH4-N reduces the DO level, and thus a single DO criterion will automatically lead to changes in BOD and NH4-N as well. Specifying BOD and NH4-N as additional environmental criteria may not significantly change the solution. This is also true in an indirect way: setting even a moderate aspiration level for the NH4-N would also result in significant changes in DO and BOD, since the coupling of the pollutants takes place not only in the river through biochemical processes but also at treatment plants. For example, the control of NH4-N (i.e., if water supply is the major use) requires nitrification at the plant which can be done only after carbon (or BOD) removal is done, and both have a positive impact on the DO level in the river. Similarly on the cost side, it is obvious that TAC is uniquely determined by INV and OMR, if the discount rate and the project life time are known (in a broader management setting these could be also incorporated into the MCDM). Thus, trade-offs between the three economic and the three environmental criteria exist, but they are weaker than in many other decision problems.

Before the multicriteria analysis starts, the LP-MULTI computes a "pay-off table".

Criterion minimized	Criteria value					
	TAC	IC	OMRC	DOmin	BODmax	NH4max
TAC	1.55	0.0	1.55	0.14	25.8	4.71
IC	6.06	0.0	6.06	3.60	11.6	3.84
OMRC	1.55	0.0	1.55	0.14	25.8	4.71
DOmin	14.4	34.3	8.45	5.38	9.81	1.70
BODmax	14.4	32.7	8.45	5.38	9.81	1.70
NH4max	14.4	29.9	8.45	5.38	9.81	1.70
Utopia	1.55	0.0	1.55	5.38	9.81	1.70
Nadir	14.4	50.3	8.69	0.14	25.8	4.71
Nadir*	14.4	34.3	8.45	0.14	25.8	4.71

Table 2: The pay-off table for the 6 criteria problem.

The results of such computations for the six criteria (minimization of TAC, IC, OMRC, BODmax, NH4max and maximization of DOmin)[11] are presented in Table 2. The row labeled "Utopia" summarizes the criteria values obtained from selfish solutions, for which each criterion is optimized in successive single-criterion optimization runs. It is commonly known that the Nadir point cannot be computed by selfish optimization (cf [Mak94c] for details). Therefore, the row labeled "Nadir" summarizes the worst values of criteria which were obtained in any part of the analysis. This is a better approximation of the Nadir point than the values provided in the row labeled "Nadir*", which is composed of the worst values obtained during the selfish optimizations only.

The pay-off table is useful as a guide for the evaluation of trade-offs among the criteria, providing good initial information about the ranges of criteria values that can be expected from any rational decision. A typical starting point is to choose criteria values at the utopia point as aspiration levels and values at the nadir point as reservation levels. The resulting "compromise" solution is shown in the Table 3 as solution # 1. The compromise solution is obtained using automatically calculated trade-offs between the criteria based on the Utopia point and on the current approximation of the Nadir point. Therefore, this solution has a low value of *DOmin* (i.e. the minimum concentration of DO over the set of points where standards are checked) but a relatively good (low) NH4max value. This is due to the very low Nadir value for DO. Since the DO level is a much more important indicator than the NH4 level, this compromise solution is far from acceptable. However, one can easily find better solutions (e.g. solutions #18 and 19, see Table 3) with similar costs, much better values of DO, and only slightly worse values of NH4.

At this point an interactive multicriteria analysis may start. The analysis is typically composed of several sessions, during which a number of solutions are generated and analyzed. A cycle composed of analysis of previous solutions, selection of new aspiration and reservation levels, and optimization is conventionally called an iteration (one iteration for this problem takes about 2-4 min.). Usually several iterations are needed to explore one region of the Pareto optimal solution space. We have examined a total of 6 criteria, but at different stages some criteria were considered more closely than others. For example, at the initial stage we were motivated by the results of the compromise solution to examine more closely the relation between DOmin and INV. Therefore, the aspiration and reservation values for those criteria were changed more carefully, while those values

[11]The subscripts *max* and *min* in the criteria names indicate that the corresponding value is a maximum or minimum, respectively, of values over the set of monitoring points.

#'s	TAC	INV	OM	DO	BOD	NH4	Ha	Le	Pr	Pa	Ba	To	Ni	Zl	Vr	Su	No	Sum
1	6.6	13.5	5.1	2.8	17.5	3.2	2	0	2	1	3	0	5	1	0	2	0	16
6	7.5	21.5	5.0	3.4	16.8	3.1	1	1	2	0	3	3	5	2	0	1	0	18
20	8.7	28.5	5.3	4.8	17.3	3.0	1	0	2	1	3	4	4	2	0	0	0	17
32	8.7	22.5	6.0	5.0	16.5	3.0	2	0	2	1	3	6	5	2	1	0	0	22
40	10.0	23.1	7.3	5.1	10.2	2.4	2	2	2	1	3	6	5	2	1	2	1	27
45	10.9	26.3	7.9	5.2	10.1	2.2	3	1	2	1	4	6	5	2	1	1	3	29
46	10.3	28.0	7.0	5.2	10.1	1.8	2	1	3	1	3	6	5	0	0	0	2	23
2-5	3.6-4.2	1-2.8	3.5-4.1	2.9-3.1	19.5-21.8	4.0-4.3	1	0-2	2	0	3	1-2	0-1	1-3	0-1	0-2	0	9-14
8-11	5.4-5.8	1.2-2.6	5.3-5.5	3.7-4.0	17.7-17.8	3.7-3.8	1	0-1	2	1	3	1	1	1-3	1-3	1-2	0	13-17
12-14	6.2-6.6	1.0-2.3	6.1-6.4	4.0-4.2	11.2-11.5	3.7	1	1-2	2	1	3	1-2	1	1	1	0-1	1	13-16
15-17	7.7-7.9	6.7-8.0	6.9-7.0	4.5	11.1	3.6	2-3	1-2	3	1	3-4	2	1	1	1-2	1	1	18-20
18-19	7.7-7.9	10.0-12.0	6.5	4.7	10.4-10.6	3.2-3.3	1	0-1	2	1	3	6	1-2	1	1	0-1	1	18-19
21	7.5	17.0	5.5	4.8	20.2	3.8	1	0	2	1	3	4	0	2	1	2	2	18
A	7.9-8.2	10.5-12.5	6.6-6.8	4.8-4.9	10.2-10.6	3.1-3.3	1	1-3	2	1	3	6	1-2	2	1-3	0-2	1	20-25
24	8.1	16.0	6.2	4.9	10.7	3.3	1	1	2	1	3	4	1	1	0	0	2	16
B	8.2-9.6	12.6-17.6	6.5-7.5	4.9-5.2	10.1-10.6	2.9-3.3	1-3	0-3	2-3	1	3-4	6	1-2	0-3	0-3	0-2	1-3	19-28
50,51	10.8-11.8	29.9-32.9	7.3-8.0	5.3	9.9-10.1	1.7	3	2	3-4	1	4	6	5	0-2	0	0-2	2	26-31
52,53	14.1-14.4	50.3	8.2-8.5	5.4	9.8	1.7	4	2	4	1	5	8	5	0-1	0	0	2	31-32

Table 3: Clustered selected solutions from multicriteria optimization. Solutions marked by A contain solutions # 22, 23, 25 and 26. Solutions marked by B contain solutions # 27 through 49, excluding solutions # 32, 40, 45 and 46.

for the other four criteria were set relatively loosely (i.e. the reservation level was close to the nadir point, and the aspiration level was near the corresponding value of a last solution). Then we stabilized the values of INV and DO (by setting relatively narrow ranges between the respective aspiration and reservation levels) and began narrowing the range between aspiration and reservation levels for the other criteria. This is why we have obtained several sets of solutions which have rather similar values of criteria (those solutions are presented in Table 3). However, we include in Appendix A all solutions in order to provide data for a more detailed analysis.

Selected and grouped solutions from two interactive sessions are presented in Table 3. The solutions presented in Table 3 have been grouped in order to illustrate the main characteristics of the decision problem (discussed below). All solutions have been sorted according to increasing values of DO and are labeled by their sequence number (to be subsequently referred to as "solution #"). DO was selected as the metric since the Nitra River suffers from low dissolved oxygen levels (and their consequences), and the treatment of the DO balance of a receiving water is the prerequisite for handling the other pollution issues.

Subsequently, we evaluate the results with a special focus on the trade-offs between investment cost and operating cost, as well as those among the three water quality constituents:

1. Considering all of the solutions, Figure 2 shows the "exponential" increase of TAC as a function of the DO level due to the diminishing returns discussed earlier. The relation between INV and DO, shown in Figure 2, shows that this increase is much faster for the investment costs. The patterns of the solutions are similar to those obtained from a single-criterion analysis (see [SMPK94]), except that the multiple-criteria analysis results in a much larger number of solutions and some scatter in the points. The shapes of the TAC(DO) and INV(DO) "functions" further justify that DO is the most important water quality indicator to be used for management purposes. However, it is evident that the aspiration levels for the other criteria also have a significant role. In particular, one can observe five outlying solutions (marked by shadowed circles) in Figure 2. Those solutions (# 1 and 6 in Table 3) have DO levels similar to other solutions which have substantially higher TAC and INV values. This is because these solutions include capital-intensive technologies for the reduction of NH4.
2. Figure 3 illustrates in more detail the relation between DO and INV for investment costs less than 16 mln US$. We have generated a number of non-dominated solutions for the most interesting range of DO, namely between 3.7 and 5.2 mg/l (cf. solutions #8 through 49 in Table 4). The DO level of 4.2 mg/l (solution # 14) can be reached with a very low investment cost, whereas improving the DO above 4.5 mg/l requires substantial investments.
3. Figure 4 shows in the (TAC, NH4, DO) coordinate system clusters of "DO driven" solutions, along with some solutions with different aspiration levels for NH4. The appearance of three groups of solutions (for DO levels below 3, around 4, and around 5 mg/l, respectively) clearly demonstrates the trade off between the two water quality criteria (DO and NH4) and the cost consequences. Figure 5 shows a similar relationship in the (INV, NH4, DO) coordinate system. Both figures also illustrate the strong correlation (discussed in the first part of this Section) between improvement of the DO and NH4 levels.
4. In many cases TAC is used as a measure to compare different project alternatives. However, if a budget for investments is limited or absent (which is the current situation in the Central and Eastern European countries), INV could be a preferred criterion, and

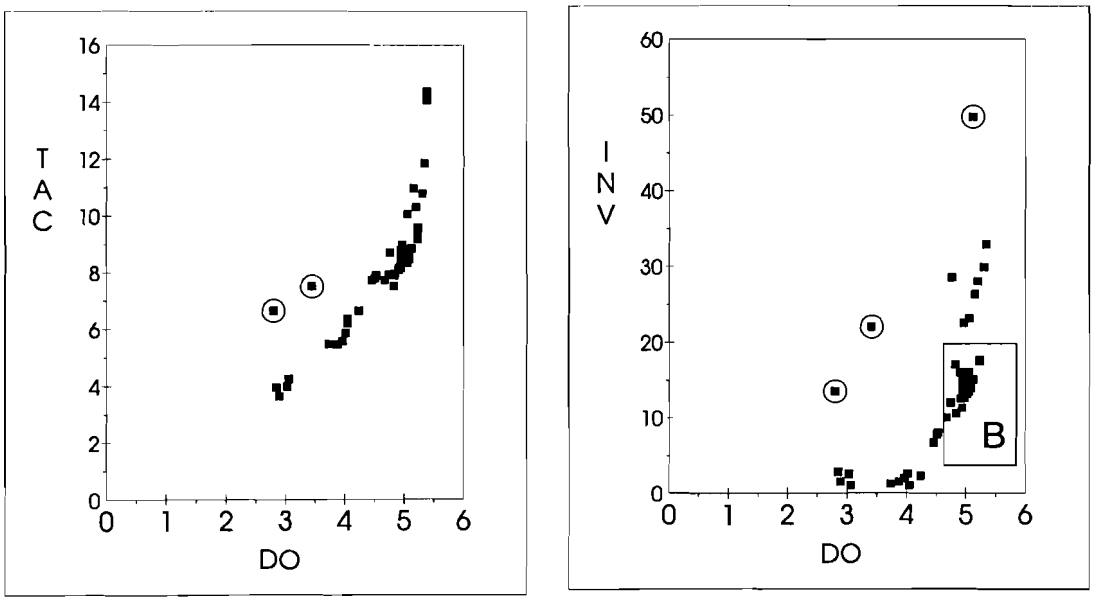

Figure 2: TAC and INV (both in mln US$) vs DO (mg/l)

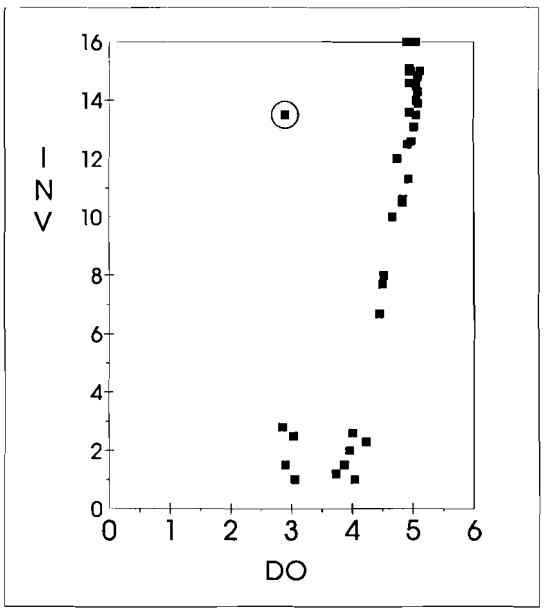

Figure 3: INV vs. DO (for INV < 16 mln US$)

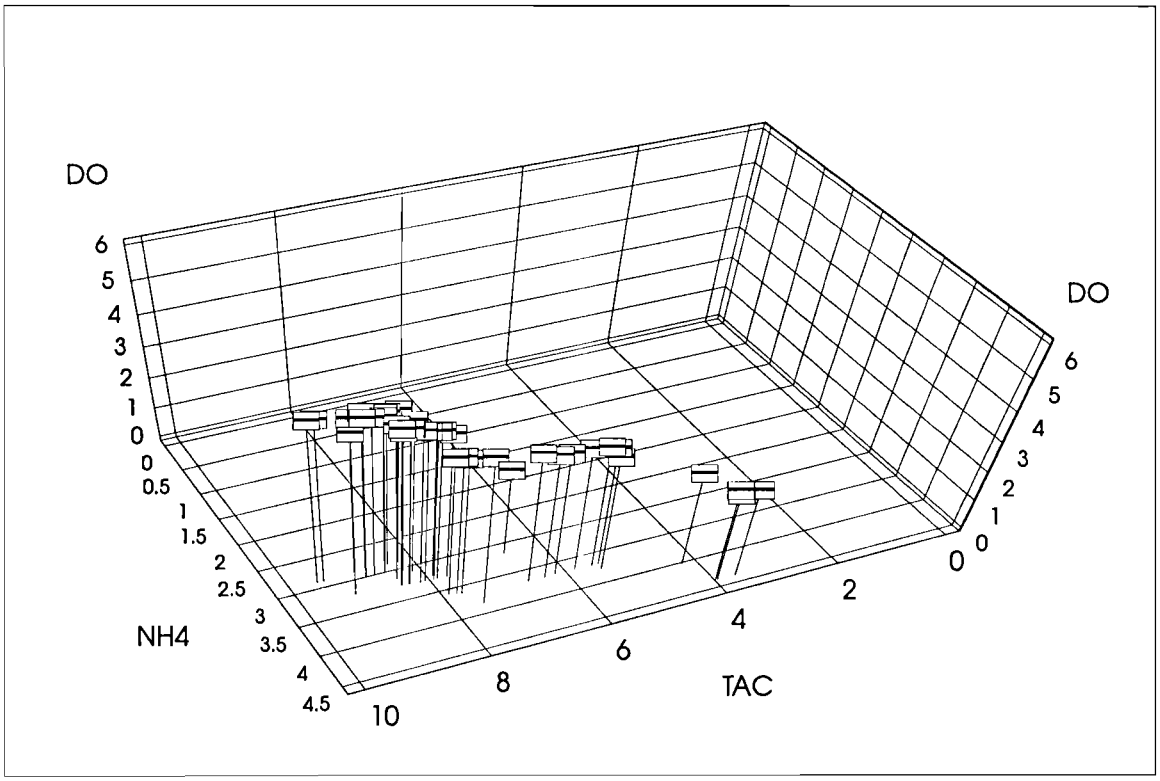

Figure 4: TAC vs. NH4 and DO

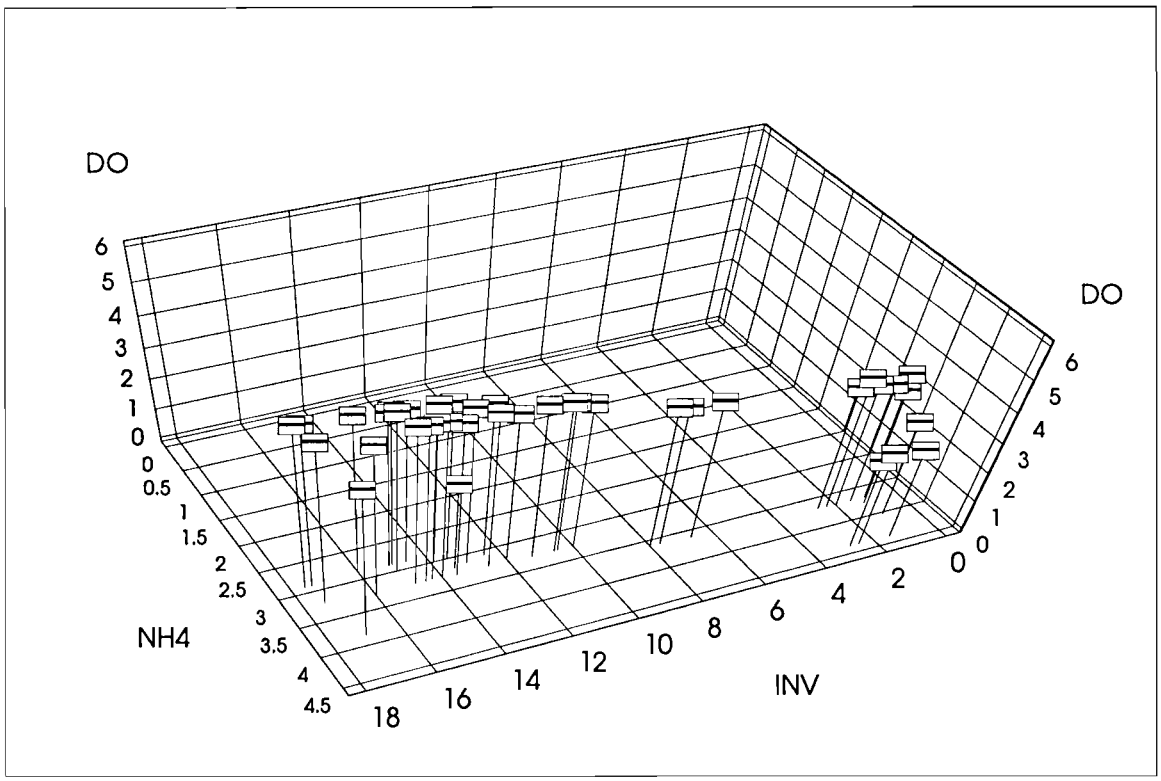

Figure 5: INV vs. NH4 and DO

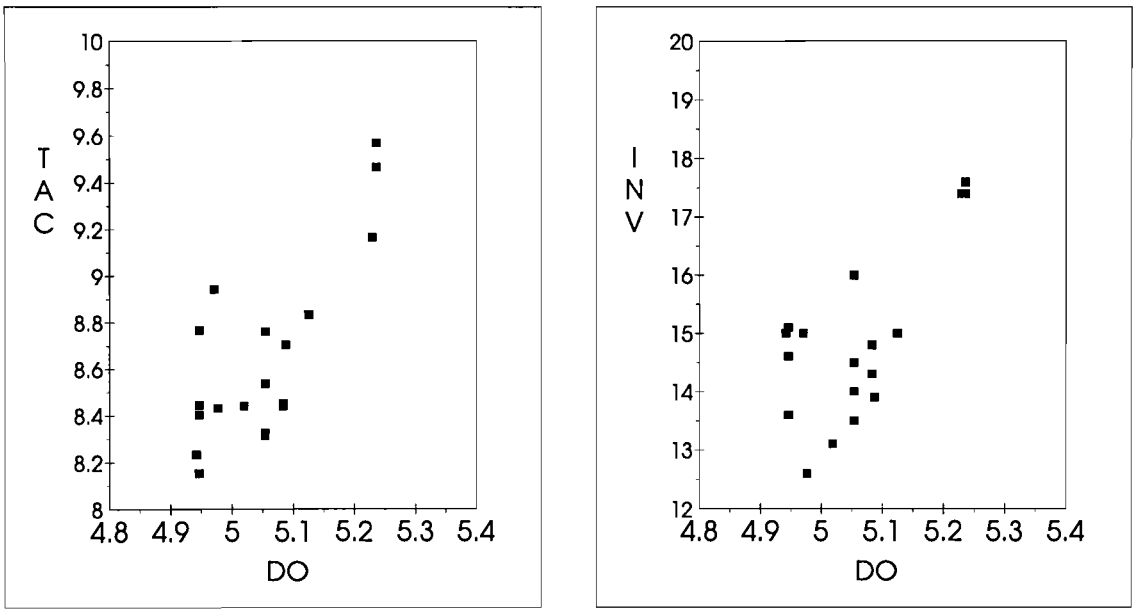

Figure 6: TAC and INV vs. DO for the cluster of solutions marked by B in Table 3.

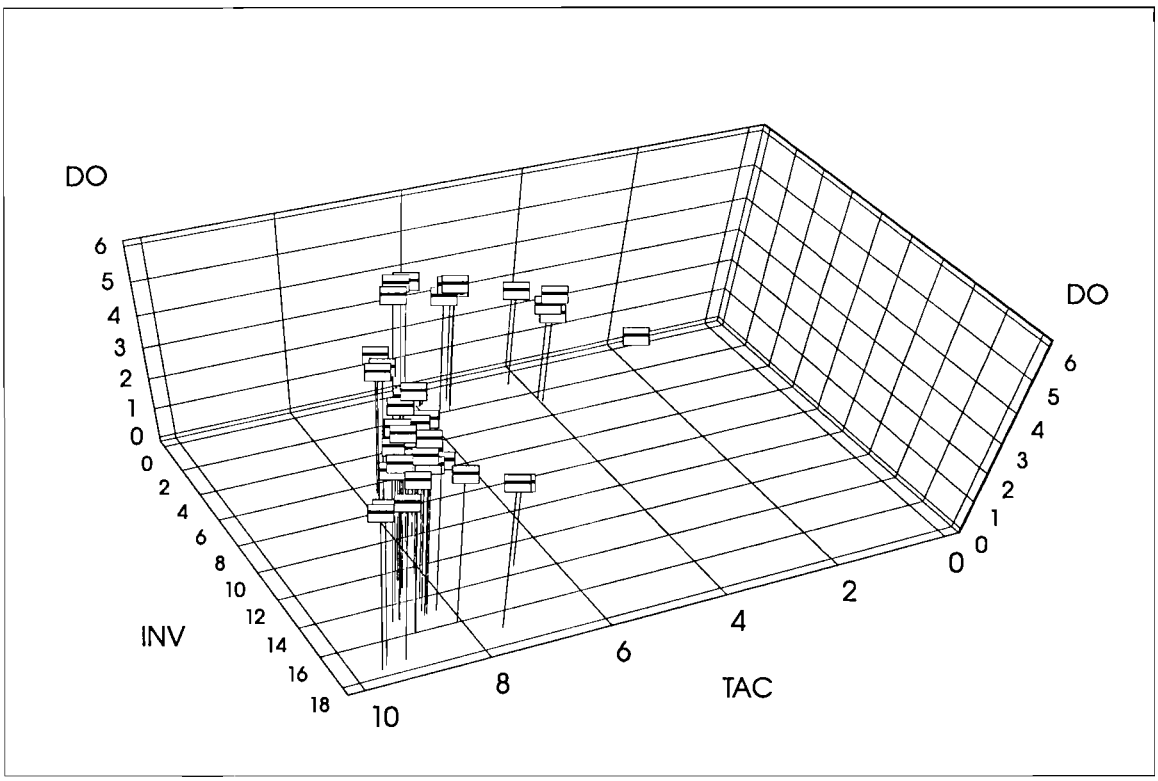

Figure 7: TAC vs INV vs. DO

there is a trade off between the two. This is illustrated by solutions #22-26, in which investments between 10.5 and 16 million USD result in practically the same ambient water quality (see Table 3 and Figure 3). Obviously, OMR and therefore TAC costs are also different, but this is not so significant.

5. Figure 6 illustrates TAC(DO) and INV(DO) relations for the cluster of solutions marked by B in Table 3 and in Figure 2. These solutions also have a relatively small range in DO, BOD, NH4-N values used as water quality criteria. However, a rather broad range of combinations of these similar water quality values can be obtained with significantly different combinations of INV and TAC, and of INV and OMRC. For example, DO values around 4.9 mg/l can be obtained with an INV of 10.6 and an OMRC of 6.69, or with an INV of 16 and an OMRC of 6.20. This trade-off is also illustrated in Figure 7.

These examples clearly indicate the attractiveness of MCDA for the analysis of the trade-offs between economic and environmental criteria, particularly in finding the range of economic criteria values that would result in a similar quality of the environment.

6.3 Benefits of multicriteria analysis

Before presenting the benefits from multicriteria analysis we present the following observation that illustrates the necessity of a careful analysis of optimal solutions (regardless of whether they are obtained from SCDA or MCDA). Within a given cluster, the treatment configuration obtained (see Table 3) shows certain variability. This appears primarily for smaller emissions, while the technologies for larger ones such as To or Ni remain rather robust. If we consider the significantly different solutions of Table 3, the large number of potential policies is evident. However, a significant portion of them may not be feasible in practice for reasons external to the model. For example, in practice it is crucial to introduce reliable and easily implementable strategies, notions of which play an important role in formulating legislation. As was shown in [SMPK94], "cheap" alternatives in Table 3 may be too vulnerable (if in practice the assumed "design scenario" is not realized). Thus, a decision maker would most likely select a solution with DO about 5 mg/l, requiring INV=13-25 million USD. The direct incorporation of a vulnerability criterion into the MCDA would be a logical desire of a DM. Though such a criterion is difficult to implement explicitly, it should be kept in mind in the ensuing model analysis.

Multiple-Criteria Decision Analysis (MCDA) is, from a purely methodological point of view, superior to a single-criterion analysis (SCDA) or a sequence of SCDAs. This is because SCDA can be considered a particular case of MCDA (used for one criterion only), and full functionality of SCDA should be supported by MCDA at a small additional cost (since the computational complexities of both approaches are similar). Our present experiences show a number of advantages of MCDA over SCDA, but also a number of limitations or disadvantages.

The main advantages of MCDA are the following:

- The MCDA developed is interactive and very fast, so that the development of a few dozen alternatives does not require more than perhaps two hours for an experienced user with a good understanding of the problem. As shown, a user can easily find a number of Pareto-optimal solutions which are most interesting to him/her.
- MCDA provides a natural way for dealing with multiple criteria problems. Application of SCDA to a multiple-criteria problem requires the conversion of all but one criteria into constraints so that the analysis is driven by the one criterion selected as the goal function.
- MCDA can provide a number of solutions which are not available from the SCDA, even

if SCDA uses as a goal function a weighted sum of criteria (see [Mak94c] for details).

- MCDA provides a much better approach to analysis of solutions which *nearly* meet the constraints imposed in the single-criterion sensitivity analysis. Sensitivity analysis in SCDA has a number of both theoretical and practical limitations and is very time-consuming and tedious for a user. For instance, shown in Table 1 are some efficient solutions obtained through MCDA compared with one corresponding solution obtained by the sensitivity analysis performed on the DP model. In practice, it is not too likely that a DM would obtain such solutions from a sensitivity analysis of the SCDA model.

The main advantages of SCDA and limitations of MCDA are the following:

- SCDA is simpler, especially for inexperienced users, because it does not require knowledge of the methodological background of the relatively new MCDA.
- SCDA is more trusted, because it is a classical, well known, and established methodology. Confidence in the methodology plays a key role in real applications.
- SCDA provides, at least theoretically, one solution for each set of constraint values without the need for interaction. MCDA requires the active participation of a user in specifying preferences in the form of aspiration and reservation levels. Although this methodology is easy and fits well with the way in which decisions are evaluated, it still requires some basic knowledge of the methodology of MCDA.
- MCDA makes it easy to generate a large number of solutions. However, the evaluation of these solutions can be more problematic and much more time-consuming. For instance, solutions which are very close to each other except for values of one of the criteria can appear strange at first, perhaps confusing the DM. Since developers of a DSS often overestimate the technical background of the DM, as well as the time available for the problem analysis, the potantial benefits of MCDA may not be realized. In such a case SCDA, which allows the user to specify a well defined criterion and set of constraints, could prove to be more fruitful.
- Many users have difficulty evaluating six criteria visually and quickly. Although there exist special MCDA techniques (see e.g. [Mak94c]) that allow the consideration of a much larger number of criteria, our experience shows that it is easier to consider only 2 or 3 criteria at the time, and to either stabilize or give a broad range of aspiration/reservation levels to the other criteria. Moreover, the specification of values for stabilized criteria is difficult in a case like ours, because groups of criteria are highly correlated (see Section 6.2).

Summing up the benefits from multicriteria model analysis, the main advantage is due to the flexibility provided in model examination. In solving practical problems, a user is interested in finding and comparing Pareto-efficient alternatives. Using the RFP method in MCDA, one can easily analyze different "regions" of Pareto-efficient solutions. Selections of these regions depend on the preferences of the user expressed in terms of objective values, and can be easily changed during the model analysis upon learning about possible efficient solutions. The constraints imposed on objectives in the single-criterion optimization are replaced by the selection of aspiration and reservation levels, which allows the examination of trade-offs between different objectives without the risk of passing over a number of interesting solutions, without the risk of infeasibility, and without all of the problems related to sensitivity analysis (see [Mak94c] for more details). On the other hand, one should note that the use of MCDA requires additional methodological background, some experience in interacting with a computer, and more time for the detailed analysis of a larger set of solutions worthy of examination.

Finally, it should be also stressed that none of the SCDA and MCDA methodologies will lead directly to actual decisions. Instead, they are tools which – if properly used – can

tremendeously help the DM to understand the issue and identify rational decisions. SCDA is indeed a more well-known and accepted methodology, but MCDA (once understood) can provide more flexibility in a decision support process.

7 Concluding remarks

This paper is not aimed at providing policy recommendations for the Nitra River Basin. The reader interested in such recommendations is advised to refer to [SMPK94]. The research documented in this paper illustrates some capabilities of multiple-criteria model analysis (MCMA) applied to regional water quality management. A substantial improvement in the results (in comparison with those documented in [BMW93]) was possible by refining the model formulation and using the hydraulic data computed by the simulation model [SMPK94].

The discussion presented in Section 6.3 shows known, but often forgotten, limitations of any optimization technique applied to a real problem. Namely, the various optimization techniques provide only tools which help to understand the problem at hand and to select a number of solutions for further, more detailed analysis. For example, an attempt to apply a so-called optimal solution, without a deep knowledge of the consequences, could easily result in the implementation of a vulnerable solution. Therefore, in practical applications, optimization techniques should often be combined with simulation techniques, which allow for a more detailed examination of the consequences of decisions.

The criteria of reliability (or the inverse, vulnerability) is critical for the actual implementation of a decision. The reliability of a solution (perhaps obtained from optimization) is often assessed through simulation, typically using a more detailed model. An alternative, which in some ways combines simulation and optimization, is to develop a stochastic optimization model (cf. e.g. [ErW80]) which allows the DM the ability to control the reliability of the solution. The framework for such a model can easily be laid out for this situation. In short, it would involve the consideration of a number of other "design" scenarios (including different flow rates, values of the reaeration rate coefficient, etc.) which are deemed likely or at least possible. The optimization model would still seek one "robust" solution (in our case, one set of values for the binary variables representing the technology alternatives). However, the number of intermediate variables and constraining relations would grow by a factor of the number of scenarios. Thus, the formulation and solution of such a stochastic optimization model is not a trivial task. Difficulties include the selection of design scenarios, the estimation of their probability of occurrence, and the solution of the resulting large-scale optimization problem. Nonetheless, such a model could prove to be an extremely valuable addition to the decision support tools already developed. Implementations of such techniques (see e.g. [SoW88, RoR94]) illustrate well applicability of this methodology.

The methodology and software tools developed by the two collaborating projects now make it easier to apply advanced decision support methods to other case studies on regional water quality management. However, a number of methodological questions (mainly related to the specification and calibration of a water quality model, to the consideration of reliability, and to the design of the user interface for MCDA) is open and provides challenging tasks for further research.

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A Summary of all solutions

#	TAC	INV	OM	DO	BOD	NH4	Ha	Le	Pr	Pa	Ba	To	Ni	Zl	Vr	Su	No	Sum
0	1.550	0.0	1.55	0.139	25.752	4.709	1	0	2	0	0	0	0	0	0	0	0	3
1	6.646	13.5	5.06	2.801	17.517	3.169	2	0	2	1	3	0	5	1	0	2	0	16
2	3.949	2.8	3.62	2.851	21.458	4.384	1	2	2	0	3	2	0	1	1	2	0	14
3	3.646	1.5	3.47	2.894	21.864	4.396	1	0	2	0	3	1	0	2	1	1	0	11
4	3.984	2.5	3.69	3.030	21.813	4.384	1	1	2	0	3	1	0	3	1	2	0	14
5	4.247	1.0	4.13	3.059	19.534	4.016	1	0	2	0	3	1	1	1	0	0	0	9
6	7.495	21.5	4.97	3.447	16.777	3.140	1	1	2	0	3	3	5	2	0	1	0	18
8	5.461	1.2	5.32	3.733	17.711	3.776	1	1	2	1	3	1	1	1	3	1	0	15
9	5.446	1.5	5.27	3.874	17.757	3.692	1	0	2	1	3	1	1	2	1	1	0	13
10	5.555	2.0	5.32	3.962	17.757	3.692	1	0	2	1	3	1	1	2	1	2	0	14
11	5.845	2.6	5.54	4.015	17.684	3.681	1	1	2	1	3	1	1	3	2	2	0	17
12	6.187	1.0	6.07	4.045	11.523	3.681	1	1	2	1	3	1	1	1	1	0	1	13
13	6.357	1.0	6.24	4.045	11.523	3.681	1	1	2	1	3	1	1	1	1	1	1	14
14	6.640	2.3	6.37	4.235	11.162	3.680	1	2	2	1	3	2	1	1	1	1	1	16
15	7.707	6.7	6.92	4.454	11.133	3.610	3	2	3	1	3	2	1	1	2	1	1	20
16	7.774	7.7	6.87	4.506	11.128	3.591	2	1	3	1	4	2	1	1	1	1	1	18
17	7.900	8.0	6.96	4.528	11.123	3.584	3	1	3	1	4	2	1	1	1	1	1	19
18	7.715	10.0	6.54	4.670	10.605	3.292	1	1	2	1	3	6	1	1	1	1	1	19
19	7.910	12.0	6.50	4.745	10.386	3.234	1	0	2	1	3	6	2	1	1	0	1	18
20	8.678	28.5	5.33	4.767	17.251	2.999	1	0	2	1	3	4	4	2	0	0	0	17
21	7.517	17.0	5.52	4.821	20.248	3.846	1	0	2	1	3	4	0	2	1	2	2	18
22	7.873	10.5	6.64	4.834	10.605	3.292	1	1	2	1	3	6	1	2	1	1	1	20
23	7.935	10.6	6.69	4.838	10.605	3.292	1	1	2	1	3	6	1	2	2	1	1	21
24	8.079	16.0	6.20	4.905	10.656	3.292	1	1	2	1	3	4	1	1	0	0	2	16
25	8.138	12.5	6.67	4.916	10.204	3.088	1	1	2	1	3	6	2	2	1	0	1	20
26	8.153	11.3	6.82	4.937	10.604	3.291	1	3	2	1	3	6	1	2	3	2	1	25
27	8.232	15.0	6.47	4.942	10.204	2.885	1	1	2	1	3	6	2	0	1	0	2	19
28	8.153	13.6	6.55	4.946	10.604	3.291	1	3	2	1	3	6	1	2	0	0	2	21
29	8.404	15.1	6.63	4.946	10.200	2.884	1	2	2	1	3	6	2	1	0	0	2	20
30	8.444	15.1	6.67	4.946	10.200	2.884	1	2	2	1	3	6	2	0	1	1	2	21
31	8.765	14.6	7.05	4.946	10.200	2.884	1	2	2	1	3	6	2	1	0	2	3	23
32	8.653	22.5	6.01	4.971	16.479	2.950	2	0	2	1	3	6	5	2	1	0	0	22
33	8.942	15.0	7.18	4.971	10.367	2.868	2	0	2	1	3	6	2	1	1	1	3	22
34	8.430	12.6	6.95	4.977	10.600	3.264	2	1	2	1	3	6	1	3	2	2	1	24
35	8.439	13.1	6.90	5.019	10.185	3.141	2	1	2	1	3	6	2	1	2	1	1	22
36	8.313	14.5	6.61	5.054	10.600	3.264	2	1	2	1	3	6	1	2	0	0	2	20
37	8.324	14.0	6.68	5.054	10.600	3.264	2	1	2	1	3	6	1	1	0	1	2	20
38	8.536	13.5	6.95	5.054	10.185	2.997	2	1	2	1	3	6	2	2	1	1	1	22
39	8.759	16.0	6.88	5.054	10.185	2.856	2	1	2	1	3	6	2	1	1	0	2	21
40	10.043	23.1	7.33	5.057	10.182	2.384	2	2	2	1	3	6	5	2	1	2	1	27
41	8.438	14.8	6.70	5.083	10.595	3.257	3	1	2	1	3	6	1	2	0	0	2	21
42	8.450	14.3	6.77	5.083	10.595	3.257	3	1	2	1	3	6	1	1	1	0	2	21
43	8.703	13.9	7.07	5.087	10.164	2.995	3	2	2	1	3	6	2	2	1	1	1	24
44	8.832	15.0	7.07	5.125	10.146	2.990	2	1	2	1	4	6	2	2	1	1	1	23
45	10.949	26.3	7.86	5.154	10.127	2.151	3	1	2	1	4	6	5	2	1	1	3	29
46	10.299	28.0	7.01	5.203	10.118	1.822	2	1	3	1	3	6	5	0	0	0	2	23
47	9.164	17.4	7.12	5.229	10.576	3.221	3	1	3	1	3	6	1	1	2	0	2	23
48	9.464	17.4	7.42	5.236	10.097	2.906	3	2	3	1	3	6	2	2	1	2	1	26
49	9.567	17.6	7.50	5.236	10.097	2.899	3	2	3	1	3	6	2	2	3	2	1	28
50	10.762	29.9	7.25	5.307	10.057	1.695	3	2	3	1	4	6	5	0	0	0	2	26
51	11.834	32.9	7.97	5.343	9.891	1.695	3	2	4	1	4	6	5	2	0	2	2	31
52	14.058	50.3	8.15	5.379	9.814	1.695	4	2	4	1	5	8	5	0	0	0	2	31
53	14.358	50.3	8.45	5.379	9.814	1.695	4	2	4	1	5	8	5	1	0	0	2	32

Table 4: Summary of selected solutions from multicriteria optimization

B Discrepancies between simulation and MC optimization models

The following reasons for the discrepancies (see Section 6.1) between the results from the simulation [SMPK94] and the multiple-criteria optimization models have been identified:

- The simulation model uses fixed values of DO, BOD, and NH4 at headwater points and the ends of tributaries. Using fixed values in this manner is not appropriate for mathematical programming formulations. Therefore, RWQM assumes that DO equals the saturation level at headwater points, and travel times estimated from the simulation model ([BrS94]).
- In RWQM a non-negativity constraint for DO is added, rather than using a conditional statement as in the dynamic programming model. (cf Section 5.3 for details). The implemented approach results in differences in results only for scenarios in which technology 1 at the Handlova WWTP and technology 0 or 1 at the Prievidza WWTP are selected by the dynamic programming model. This is because more costly technologies are required by RWQM to meet the non-negativity constraint on DO. Note that this discrepancy is irrelevant if the water quality standards are also checked on the Handlovka river (cf [SMPK94] for details about locations where water quality standards are currently checked).
- Transfer coefficients in the last reaches of tributaries differ slightly between the two models. For these reaches, RWQM uses the convention that data for a segment is stored in the up-stream end of the segment (since the down-stream node is "shared" by two reaches which join at the confluence), whereas the simulation model generates an additional node at the end of each reach and is able to store the corresponding data there.
- The simulation model allows for conditional constructs, such as *"If DO concentration is greater than a, then weir coefficient equals b."* which are not easily implemented in linear programming. Thus, we assume a conservative coefficient value for the weir at km 17.0 on the Nitra river.
- For this weir, the DO concentration provided by the simulation model is the concentration just below the weir, while the RWQM output is the DO concentration just up-stream of the weir (cf eq. (26)). This does not change the results of either model since the location of the weir is not at a critical water quality point, though plots will show a large difference in the DO concentrations, as in Figure 1.

It is likely that most of the differences in analysis results can be attributed to these model discrepancies. The *core model* documented in Section 2 provides results very close to the results from the simulation model in DESERT [SMPK94]. The following extensions and modifications of both models might be considered in further development in order to improve results and decrease the existing discrepancies:

- In the calculation of the reaeration rate, velocity, depth, and temperature data are used from the downstream end of a reach (as opposed to using the upstream or an average value). Trial and error analyses have shown that the choice of data to use can have a significant impact on the resulting transfer coefficients. Therefore, one should also consider using average values.
- As in DESERT, improved results could be obtained with better knowledge of the flow rates and travel times for tributaries, as well as the so-called uncontrollable wastewater emissions. As mentioned, an adequate representation of the physical system is essential to the beneficial use of any decision support system.

- Currently, only one monitoring point is critical for each constituent. Hence, the water quality at most of monitoring points is not controlled (by criteria in MCDA or by constraints in SCDA). Introducing different standards for different parts of the rivers (or acceptable violations of standards at some locations) could result in a substantial improvement of water quality at other locations.

C Mathematical Programming Problem

This Appendix contains a formulation of the mathematical programming problem equivalent to the formulation in Section 2. Section C.1 contains equations converted to the form corresponding to the standard formulation of an LP (Linear Programming) problem. Section C.2 contains definitions of names used for rows and columns in the LP problem formulation. Finally, Section C.3 contains definitions of goal functions used for different optimization problems.

C.1 Conversion of equations

The equations defined so far have been defined in a way that is easy to interpret. However, most of them have had to be converted to a form that is accepted for the formulation of an optimization problem. In order to document those conversions, all of the equations used in the definition of optimization problems are listed below in this new form.

To remind the reader, the following definition of sets is being used in the equations:
M - set of indices of all monitoring nodes
E - set of indices of all emissions nodes
J - set of indices of all nodes
L - set of indices of all water quality constituents
$K(j)$ - set of indices of all technologies considered at the emission node j
$I(j)$ - set of indices of all nodes located immediately upstream of node j

The indices of water quality, l, are as follows:
0. DO, dissolved oxygen
1. CBOD, carbonaceous biological oxygen demand
2. NBOD, nitrogenous biological oxygen demand
3. NH4, ammonia
4. P, dissolved phosphorous
5. SOD, sediment oxygen demand.

The only equation without changes is eq (1):

$$\sum_{k \in K(j)} x_{jk} = 1 \qquad x_{jk} \in \{0,1\}, \qquad j \in E$$

Equation (5), $wq_{j0} = (aqs_{j0} - aq_{j0})/aqs_{j0}$ is converted to:

$$wq_{j0} + aq_{j0}/aqs_{j0} = 1. \qquad j \in M \tag{39}$$

Equation (6), $wq_{jl} = (aq_{jl} - aqs_{jl})/aqs_{jl}$ is converted to:

$$wq_{jl} - aq_{jl}/aqs_{jl} = -1. \qquad l \in [1,4] \qquad j \in M \tag{40}$$

Equation (7), $g_l = \max_{j \in M}(wq_{jl})$ is converted to:

$$wq_{jl} - g_l \leq 0. \qquad l \in [0,4] \qquad j \in M \tag{41}$$

and equation (8) $g_{all} = \max_{l\in[0,4]}(g_l)$ is converted to:

$$g_l - g_{all} \leq 0. \qquad l \in [0,4] \tag{42}$$

Consult Section C.3 for additional information about the last two conversions.

Equation (9):

$$\begin{aligned} aq_{j0} \;=\; & 1./(Q_j + W_j) * \Big(\sum_{i\in I(j)} \Big(b_{i0} + Q_i * (DOsat_j - TC_{i0}(DOsat_i - aqy_{i0}) \\ & - \sum_{l\in\{1,2,5\}} TCp_{il} aq_{il}) \Big) + ioxy_j \Big) \end{aligned} \tag{43}$$

now has the form:

$$aq_{j0} + 1./(Q_j + W_j) * \sum_{i\in I(j)} Q_i * (-TC_{i0} aqy_{i0} + \sum_{l\in\{1,2\}} TCp_{il} aq_{il}) = RHS_j \qquad j \in J \tag{44}$$

where RHS_j is given by:

$$\begin{aligned} RHS_j \;=\; & 1./(Q_j + W_j) * \Big(\sum_{i\in I(j)} \Big(b_{i0} + Q_i * (DOsat_j - TC_{i0} DOsat_i \\ & - TCp_{i5} SOD_i) \Big) + ioxy_j \Big) \end{aligned} \tag{45}$$

and SOD_i is a given SOD at the i-th node.

Equation (10), $aq_{jl} = \left(\sum_{i\in I(j)}(TC_{il} aq_{il} Q_i + b_{il}) + e_{jl}\right)/(Q_j + W_j)$ is converted to:

$$aq_{jl} - (e_{jl} + \sum_{i\in I(j)} TC_{il} aq_{il} Q_i)/(Q_j + W_j) = 1./(Q_j + W_j) * \sum_{i\in I(j)} b_{il} \qquad l \in [1,4] \qquad j \in J \tag{46}$$

Equation (12), $e_{jl} = q_j \sum_{k\in K(j)} x_{jk} em_{jlk}$ is converted to:

$$e_{jl} - q_j \sum_{k\in K(j)} x_{jk} em_{jlk} = 0. \qquad l \in [1,4] \qquad j \in E \tag{47}$$

Equation (13), $Inv_j = \sum_{k\in K(j)} x_{jk} IC_{jk}$ takes the form:

$$Inv_j - \sum_{k\in K(j)} x_{jk} IC_{jk} = 0. \qquad j \in E \tag{48}$$

Equation (14), $OM_j = \sum_{k\in K(j)} x_{jk} OMC_{jk}$ is now:

$$OM_j - \sum_{k\in K(j)} x_{jk} OMC_{jk} = 0. \qquad j \in E \tag{49}$$

Equation (15), $TAC_j = [r(r+1)^n/((r+1)^n - 1)]Inv_j + OM_j$ is now:

$$TAC_j - [r(r+1)^n/((r+1)^n - 1)]Inv_j - OM_j = 0. \qquad j \in E \tag{50}$$

Equation (16), $Tot\_Inv = \sum_{j \in E} Inv_j$ is converted to:

$$Tot\_Inv - \sum_{j \in E} Inv_j = 0. \tag{51}$$

Equation (17), $Tot\_OM = \sum_{j \in E} OM_j$ is now:

$$Tot\_OM - \sum_{j \in E} OM_j = 0. \tag{52}$$

Equation (18), $Tot\_TAC = \sum_{j \in E} TAC_j$ becomes:

$$Tot\_TAC - \sum_{j \in E} TAC_j = 0. \tag{53}$$

C.2 Names in the MPS formulation

C.2.1 General remarks

All names are generated automatically by the problem generator, and their compositions are currently pre-specified according to the following rules:

- Names are composed of an id followed by a node number (j) and a constituent number (l). Whenever any of these numbers is not relevant, it is skipped.
- Numbers are counted from 0.
- Name's id and numbers are separated by _ (an underscore).
- Every name is exactly 8 characters long, so dots are appended to any name which is shorter than 8 characters.

Additionally a number of rows and columns is generated during the conversion of multicriteria problem into an equivalent single criterion using the achievement scalarizing function (cf [Mak94c] for the description of the applied conversion method).

C.2.2 Rows

The following id's are used for the rows, which are generated in the sequence given below. The variables *n_waste*, *n_nodes*, *n_mon*, and *n_emm* denote the number of the water quality constituents, all nodes, monitoring, and emission nodes, respectively.

goal - one goal function row,
x - n_emm rows for eq. (1),
g_all - n_waste rows for eq. (42),
g - n_mon * n_waste rows for eq. (41),
wq - n_mon * n_waste rows for eq. (40),
aq - n_nodes * n_waste rows for eq. (43) and (46),
e - n_emm * n_waste rows for eq. (47),
inv - n_emm rows for eq. (48),
om - n_emm rows for eq. (49),
tac - n_emm rows for eq. (50),
tot_inv - n_emm rows for eq. (51),
tot_om - n_emm rows for eq. (52),
tot_tac - n_emm rows for eq. (53).

C.2.3 Columns

The following id's are used for the columns, which are generated in the sequence given below. In addition to the numbers defined in the previous section, *n_tech* denotes the number of all technologies considered at all emission points.
x – n_emm * n_waste columns for decision variables
g_all – one columns for the regional water quality index g_{all}
g – n_waste columns for water quality indices (for each type of waste),
wq – n_mon * n_waste columns for water quality indices,
aq – n_nodes * n_waste columns for ambient water quality concentrations,
e – n_emm * (n_waste - 1) columns[12] for amounts of discharged constituents,
inv – n_emm columns for investment costs,
om – n_emm columns for O&M costs,
tac – n_emm columns for TAC,
tot_inv – one column for total investment costs,
tot_om – one column for total O&M costs,
tot_tac – one column for total TAC.

C.3 Auxiliary goal function

The goal function for a multicriteria problem is defined by the LP-MULTI package (consult [Mak94c] for details of implementation). The definition of the goal function is done in such a way that proper conversion of equations (7) and (8) into equations (41) and (42), respectively, is made. Specifically, the goal function contains the following term:

$$\epsilon * (g_{all} + \sum_{i=0}^{n\_waste} g_i) \tag{54}$$

where ϵ is a predefined coefficient (currently set to 0.001). However, if the respective variable enters the goal function because of the definition of the achievement scalarizing function, then the ϵ coefficient for this variable is overwritten by a coefficient generated by LP-MULTI.

[12] No columns are generated for DO.