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MULTIATTRIBUTE UTILITY ANALYSIS: A BRIEF SURVEY

Ralph L. Keeney

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Abstract

The role of multiattribute utility theory is first placed in the overall context of decision analysis. Then an approach that has proven useful in adapting the theory to be a practical tool is illustrated. Several cases where multiattribute utility has been used are briefly discussed. These include both operational and strategic problems involving, for example, siting of large-scale facilities (airports, power plants), medical treatment, the structuring corporate objectives, environmental management, and personal investment strategy.

1. Introduction

This paper has two purposes. The first is to briefly describe a general approach which has proven in practice to be useful in assessing multiattribute utility functions in a variety of contexts. The second is to illustrate the broad range of problems for which this approach may be helpful. We do this using short descriptions of a number of actual cases where preferences have been formalized by a utility function. Because of length considerations, there is no attempt to be complete in either describing the approach or surveying applications. Our goal is to be illustrative, not definitive.

Many complex decision problems have the characteristic of being multiple objective in nature. Inevitably, these multiple objectives are conflicting in the sense that, once dominated alternatives have been eliminated, further achievement in terms of one objective can occur at the expense of

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*International Institute for Applied Systems Analysis, 2361 Laxenburg, Austria.

some achievement of another objective. Thus, in evaluating alternatives, the decision maker must consider his preference tradeoffs between various degrees of achievement of one objective and degrees of achievement of others. The real problems are even more complicated because uncertainty is usually present. One cannot predict with certainty what the consequences of each of the alternatives under consideration will be.

In evaluating alternatives, it is very difficult to logically and consistently consider the above complexities informally in one's mind. Hence there is a need for formal analysis. Decision analysis is an approach which does explicitly address the multiple objective and uncertainty issues. The theoretical basis for this is well established, (see von Neumann and Morgenstern [27]). However, an important practical problem concerns quantifying the decision maker's preference structure for multiple objectives. Without this mathematical representation--called a utility function--of the decision maker's preferences, one cannot formally evaluate the alternatives.

1.1 Decision Analysis

By briefly outlining the decision analysis approach, we hope to motivate the work described here and place it properly in a broader context. Raiffa [24] discusses the philosophy and techniques of decision analysis in detail. For our purposes, let us categorize it with four steps:

1) structuring the problem,
2) quantifying the uncertainties involved,
3) quantifying the decision maker's preferences, and
4) evaluating the alternatives.

Structuring includes problem specification and identification of the decision maker. The decision maker must articulate his objectives and attributes (i.e. measure of effectiveness) for each objective. An attribute is a measurement scale used to indicate the degree to which the corresponding objective is achieved. The alternatives must also be specified. Let us designate our set of attributes as $X_1, X_2, \ldots, X_n$ and use x_i to indicate a specific amount of attribute X_i. For instance, X_1 may designate profit in 1975 measured in thousands of dollars and x_1 may be 188. With this convention, the consequence of any alternative is $\underline{x} \equiv (x_1, x_2, \ldots, x_n)$.

Quantifying uncertainties involves describing the uncertainty about the possible consequences of each alternative. For each alternative A_j, a probability distribution $p_j(\underline{x})$ indicating which consequences might occur and their likelihood is required. The p_j may be specified using any combination of analytical models, simulation models, subjective assessments, and data that are available and appropriate.

Quantifying preferences means assessing the decision maker's utility function $u(\underline{x}) \equiv u(x_1, x_2, \ldots, x_n)$, which is called a multiattribute utility function since the argument of the utility function is a vector indicating levels of the several attributes. The multiattribute utility function, which will be referred to by the mnemonic MUF, has two properties which make it useful in addressing the issues of uncertainty and tradeoffs between objectives. These properties are:

a) $u(\underline{x}') > u(\underline{x}'')$ if and only if $\underline{x}'$ is preferred to $\underline{x}''$, and
b) in situations with uncertainty, the expected value of u is the appropriate guide to make decisions; i.e., the alternative with the highest expected value is the most preferred.

This second property follows directly from the axioms of utility theory postulated first in von Neumann and Morgenstern [27].

Evaluating alternatives involves calculating the expected utility of each of the alternatives and conducting sensitivity analysis. Given p_j for each A_j and u from the previous steps, the expected utilities for the alternatives can be evaluated. To gain additional confidence and insight into which alternative should be chosen and why, various parameters in both the probability distributions and the utility function can be varied to see how these affect the expected utility of the alternatives.

1.2 Statement of the Problem

The weakest link of the four above steps in rendering decision analysis operational for multiple objective problems is quantifying the decision maker's preferences. Defining the problem is common to all attempts to systematize the decision making process. Quantifying uncertainties has also been widely addressed in modelling efforts. The outputs of many simulation models include probability distributions over the relevant attributes for each of the alternatives under consideration. However, the decision maker is usually required to review these outputs--informally combining them with his preferences--to select an alternative. Because multiattribute utility theory was only recently developed [7,8,16,20,23,25] and because the operational procedures to put it into practice are not well developed, the third and fourth steps are informally carried out simultaneously. The critical step is actually the quantification of preferences because, as indicated above, evaluation of alternatives is fairly straightforward once probabilities and preferences are quantified.

Much of multiattribute utility theory is developed as follows. Assumptions about the decision maker's preferences are postulated, and the restrictions these assumptions place on the functional form of the utility function are derived. Then, for any specific problem, the appropriateness of the assumptions for a particular MUF should be verified with the decision maker and parameters for the utility function assessed and checked for internal consistency. Ideally, the functional form of the MUF would have the following properties:

1) be general enough to allow application to many real problems,
2) require a minimal number of assessment questions to be asked of the decision maker,
3) require assessments which are reasonable for a decision maker to consider, and
4) be easy to use in evaluating alternatives and conducting sensitivity analyses.

The next section describes two convenient functional forms for the MUF which measure up well in terms of these properties.

1.3 Organization of the Paper

Section 2 illustrates the theoretical approach to generate functional forms of MUF's. Section 3 surveys many recent applications of multiattribute utility analysis. The last section offers a brief comment and perspective on its possible use.

2. The Additive and Multiplicative Utility Functions

Conditions which imply that a MUF is either additive or multiplicative are very similar. None of the conditions require the decision maker to consider preference tradeoffs among more than two attributes simultaneously or to consider lotteries (specifying various levels of $\underline{x}$ and the probabilities of receiving them) with the level of more than one attribute being varied. Furthermore, the assessments needed to specify an n-attribute utility function are n one-attribute utility functions and n scaling constants.

2.1 The Basic Assumptions

The two basic assumptions which we use for both additive and multiplicative utility functions are referred to as preferential independence and utility independence. These are defined as follows:

Preferential Independence: The pair of attributes $\{X_1,X_2\}$ is preferentially independent of the other attributes $\{X_3,\ldots,X_n\}$ if preferences among $\{X_1,X_2\}$ pairs, given that $\{X_3,\ldots,X_n\}$ are held fixed, do not depend on the level where $\{X_3,\ldots,X_n\}$ are fixed.

Preferential independence implies that the tradeoffs between attributes X_1 and X_2 do not depend on $X_3,\ldots,X_n$.

Utility Independence: The attribute X_1 is utility independent of the other attributes $\{X_2,\ldots,X_n\}$ if preferences among lotteries over X_1, (i.e. lotteries with uncertainty about the level of X_1 only) given $X_2,\ldots,X_n$ are fixed, do not depend on the level where those attributes are fixed.

A main result can now be stated.

Theorem 1. For $n \geq 3$, if for some X_i, $\{X_i,X_j\}$ is preferentially independent of the other attributes for all $j \neq i$ and X_i is utility independent of all the other attributes, then either

$$u(\underline{x}) = \sum_{i=1}^{n} k_i u_i(x_i) \quad , \qquad \text{if } \sum k_i = 1 \quad , \qquad (1)$$

or

$$1 + ku(\underline{x}) = \prod_{i=1}^{n} [1 + kk_i u_i(x_i)] \quad , \quad \text{if } \sum k_i \neq 1 \quad , \qquad (2)$$

where u and u_i are utility functions scaled from zero to one, the k_i's are scaling constants with $0 < k_i < 1$, and $k > -1$ is a non-zero scaling constant satisfying the equation

$$1 + \prod_{i=1}^{n} (1 + kk_i) \quad . \qquad (3)$$

The proof of this result is found in Keeney [15]. Alternative sets of assumptions leading to either form (1) or (2) are found in Fishburn [7], Meyer [20], and Pollak [23]. The functional form (1) is referred to as the additive utility function and (2) is the multiplicative utility function. For the case of two attributes, the following is proved in Keeney [16].

Theorem 2. For $n = 2$, if X_1 is utility independent of X_2 and X_2 is utility independent of X_1, then the utility function $u(x_1,x_2)$ is either additive or multiplicative.

Using either (1) or (2), if $\sum_{i=1}^{n} k_i = 1$, the utility function is additive, and if $\sum_{i=1}^{n} k_i \neq 1$, it is multiplicative. When $\sum_{i=1}^{n} k_i > 1$, then $-1 < k < 0$, and when $\sum_{i=1}^{n} k_i < 1$, then $0 < k < \infty$. To use either the additive or multiplicative form, we need to obtain exactly the same information, the n single-attribute utility functions $u_i(x_i)$ and the n scaling constants k_i. How this information is obtained and used is considered in detail in Keeney and Raiffa [17].

In terms of the required assessments and general robustness, the additive and multiplicative utility functions appear to be the practical ones for say $n \geq 4$. Even when the requisite assumptions do not precisely hold over the domains of all the attributes, it may be a good approximation to assume they do, or it may be reasonable to integrate different additive and multiplicative utility functions over separate regions of these attributes. More general functional forms, requiring more assessments, have been developed using a similar approach for cases requiring additional flexibility in the preference structure. See, for example, Bell [2], Farquhar [5], Fishburn [6], Keeney [16], Kirkwood [18], and Oksman [22].

3. Applications

This section surveys a number of applications which have explicitly used results such as Theorems 1 and 2 to quantify a decision maker's preferences over more than one attribute. Original references are included for the reader interested in more depth. Here one can at most just get a feeling for the range of problems being addressed using multiattribute utility.

The Safety of Landing Aircraft. The safety of landing an airplane depends on many factors: wind, visibility, ceiling, other aircraft in the vicinity, etc. Yntema and Klem [28] attempted to quantify the relative safety of various situations which differed in terms of ceiling, visibility, and amount of fuel that would remain at touchdown given a normal landing. Other relevant factors were fixed at a standard level.

The decision makers for this study were twenty US Air Force pilots, each of whom had a good deal of experience in landing aircraft in a variety of situations. The three attribute utility function chosen was a generalization of the multiplicative form (2) referred to as the multilinear form. This required assessing three single attribute utility functions plus eight scaling constants. In attribute space, the ceiling ranged from 100 to 5,000 feet, visibility from 0.25 to five miles, and remaining fuel from fifteen to 250 gallons. To examine the implications of each decision maker's utility function, each decision maker was presented with forty pairs of consequences and asked to select the preferable one from each pair. These were compared with the choices implied by the utility function. Yntema and Klem concluded the results were satisfactory.

Blood Bank Inventory Control. Jennings [10] developed a detailed model of a whole-blood inventory system for a blood bank in a hospital and examined policy options in such a system. Alternative policies were evaluated in terms of the percent of blood shortage and the percent of blood outdating. Shortage is the blood requested by a doctor not available in the hospital inventory. Outdated blood is blood not used during its legal lifetime, which is currently twenty-one days in most hospitals in the United States.

As a part of my doctoral dissertation (see [13]), I tried to assess the preferences of the nurse who was in charge of ordering blood for the blood bank of the Cambridge Hospital in Cambridge, Massachusetts. For the two attributes, shortage and outdating, utility independence properties were verified, and Theorem 2 was used directly. A utility function, which appeared to be a resonably good representation of the nurses preferences, was developed.

In this situation, the decision maker (the nurse) had a degree in liberal arts and no formal scientific training. On the other hand, she seemed interested and enthusiastic about the procedures. I felt the assessment procedure went very smoothly. Similar experiences more recently have led me to suspect that willingness to think hard about the consequences is more important for obtaining a good utility function--one which captures the decision makers preferences reasonably well--than a formal quantitative education.

Instructional Programs. Roche [26] examined the problem of a decision maker who must choose among alternate budget allocations in a small school district. In particular, he concentrated on the allocation of funds for four junior-high programs: English/language arts, science, mathematics, and social studies. For measurement purposes, Roche and members of the school system chose "the percentage of students achieving at or above grade level on the standardized achievement tests" for each of the four programs.

The preferences of nine individuals were assessed, namely, the principal and assistant principal of the junior high school, the superintendent and assistant superintendent, and all five committee members of the town's school committee. First it was verified that each pair of attributes was preferential independent of the other attributes. This allowed one to construct an additive value function (see [25]), which assigns a number to each possible set of four-tuples of the achievement indices such that higher numbers are preferred. It is not necessarily appropriate to use this function on decisions involving uncertainty although the value function helps considerably in obtaining a multiattribute utility function.

<u>Fire Department Response Time</u>. A classical question facing fire departments is "how much is a minute of response time worth." Specifically, in New York City, the standard response to an alarm is to send three engines and two ladder trucks to the alarm location. Thus the quality of response might be measured by a utility function with five arguments: the response times of the three engines and the two ladders. Given this utility function, various operational policies could be examined for their overall impact on response time.

In conjunction with the joint work of the New York City Fire Department and the New York City Rand Institute, the utility function of one of the deputy fire chiefs was assessed. The procedure and results are found in Keeney [12].

<u>Mexico City Airport</u>. In 1971, the Ministry of Public Works in Mexico had the responsibility to recommend a strategy to the President for developing the airport facilities for Mexico City over the remainder of this century. Together with the ministry, R. deNeufville, H. Raiffa, and myself conducted a

decision analysis for evaluating possible developmental strategies. This first model examined the impact of operating various functional types of aircraft (e.g. domestic, international, military) at different airports in the Mexico City area on six variables: capacity, cost, safety, social disruption, noise, and access time. A formal utility function was assessed over these for the Director of Airports of Mexico. Details are found in [11].

A second model was constructed to examine strategies for the Mexican Government to proceed with the project. This model explicitly included less tangible factors, such as, political effects, overall effectiveness, flexibility to adapt the policy when needed, prestige, etc. This is described in deNeufville and Keeney [4]. This work did have a major impact on the Ministry of Public Works' recommendations to the President of Mexico. It is not clear to what degree it has had an impact on the final decision process.

Treatment for Cleft Lip and Palate. The second most common congenital defect in the US is cleft lip and cleft palate. There are alternative approaches for treatment, each having a multitude of effects, but none of these involve life-and-death type decisions. Nevertheless, the problem is obviously very important; thus, the treatment strategy is important. Krischer [19] evaluated various treatment strategies using multiattribute utility analysis. The four variables which he explicitly used in the evaluation were cost of treatment, impact on speech intelligibility, impact on hearing, and the cosmetic effects.

Utility functions for eighty-nine clinicians and thirty parents of children with cleft lip and palate were assessed using a questionnaire. Overall little differences in preferences were found among doctors or between doctors and parents. Using a representative utility function, alternative treatment strategies were examined.

Nuclear Power Siting. Two studies have been conducted of nuclear power plant siting using multiattribute utility. In a doctoral dissertation, Gros [9] studied the problem of deploying 1,000-Mw base-load units on the New England coast. Specifically, he examined the benefits and costs accruing to four separate groups: environmentalists, the utility companies, regulatory agencies, and local groups. Using individuals with a knowledge of each groups interests, utility functions for each of these were assessed over four proxy attributes: monetary costs, population within fifteen miles of the site, temperature of water released after cooling, and capacity of the site measured in number of 1,000-Mw units.

Using the overall approach described in Nair et al. [21], Woodward-Clyde Consultants of San Francisco has been using multiattribute decision analysis in their professional practice while consulting for utility companies. The first study was recently completed. First candidate sites were selected considering several factors such as earthquake potential, faults, water availability, etc. Evaluation of these sites involved assessing a utility function including the environmental impact on fish, waterfowl, and rare and endangered species; the socio-economic impact on the communities near the site due to the boom-bust cycle; the safety of the population due to radiation and possible accidents; costs; and system reliability. This work will be reported soon.

Corporate Objectives. For many operational problems in firms producing products--as opposed to service oriented firms--it may be appropriate to use a single monetary attribute in evaluating alternatives. However, in decisions concerning strategic policy of a firm, the board of directors weighs the alternatives in terms of several different criteria. Recently, Woodward-Clyde Consultants has specified its overall objectives in a hierarchy and utility functions of several board members were assessed. The final attributes concerned such diverse aspects as retained earnings; increase in salaries; incentative compensation; growth in the retirement plan; US and foreign coverage; depth, breadth, and balance of professional personnel; and the increase in professional capability. For details of the assessments, see Keeney [14].

Personal Consumption and Investment Strategy. How should an individual allocate his or her income and accumulated wealth on consumption over the years? Should one spend more on consumption during the ages thirty to forty and have less for retirement at sixty or vice versa? Richard Meyer of the Harvard Business School has conducted an applications oriented research program on this general topic over the last six years. His attributes concern the consumption in each year until death. Death itself is treated as an unknown. Utility functions have been assessed for several individuals and strategies evaluated with the aid of an interactive computer program. Some results are found in a doctoral dissertation by Oksman [22].

Forest Pest Management. The ecology project at the International Institute for Applied Systems Analysis (IIASA) in Laxenburg, Austria, developed a model for evaluating control strategies for the spruce budworm. This pest periodically destroys much of the forests of New Brunswick, Canada. Major variables of interest impacted by the various strategies are the lumber industry profits in New Brunswick, the unemployment in the area (since the lumber industry is very significant), and the recreational value of the forest. The problem is complicated because the impact of any strategy effects periods of several years. David Bell has made a considerable effort to assess a utility function over these three attributes over time for purposes of evaluating treatment alternatives. Although the work is not yet complete, several aspects of the program are reported in Bell [1,3].

4. Comments

The cases discussed above are neither exhaustive nor representative of all the work being done under the title multiattribute utility. Several other cases, conducted mainly by other individuals, are described in a book on the topic [17]. It is a fact that most complex problems do involve multiple objectives. Hence if one considers analysis worthwhile, the question is whether to formally or informally include this aspect. The alternative of forgetting the multiple objectives altogether seems unreasonable if one hopes to have any subsequent impact on the decision.

In many complicated problems, several man-years may be spent developing a model (e.g. simulation) to relate the multiple input, output, and decision variables. The final result of all this is a report to "the decision maker" who, after perhaps a week of thinking and consulting with his "advisors," makes a decision. If the problem requires so much modelling, it seems that in some cases it may be very difficult to sort out the overall preferences in one's head in a week. Assessing a multiattribute utility function offers the decision maker an option to an in-the-head analysis to both a) get his preferences straight, and b) evaluate policies using his preferences and the model output in a logically consistent manner. It would appear that for some problems, the shifting of a few man-months effort from the modelling aspects to the preference aspects would prove to be worthwhile.

Another class of problems where one may find multiattribute utility useful are those where the value of preference issues are critical. An example discussed earlier involved the corporate objectives. Here the purpose is not to make a decision, but rather to assist in 1) articulating substantiative issues, 2) sensitizing different individuals to the issues involved, 3) generating creative alternatives, 4) communicating, 5) isolating differences in judgment and preferences, and 6) resolving those differences. The usefulness of such analyses is only beginning to be recognized.

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