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Contents

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Abstract. Based on our earlier results in decision theory, we demonstrate how decision trees can be integrated into a general framework for analysing decision situations with respect to different criteria, and suggest an evaluation rule taking into account all strategies, criteria, probabilities and utilities involved in the situations under consideration. A significant property of the framework is that it admits the representation of imprecise information at all stages. This information is modelled in sets of measures constrained by interval estimates. The strategies are then evaluated relative to different decision rules, e.g., a set of generalisations of the principle of admissibility. Decision situations are evaluated using fast algorithms developed particularly for solving these kinds of problems. The presented framework has been developed and used within a large-scale evaluation project at the Swedish National Rail Administration.

Keywords: Multiple Attribute Utility Theory, Decision Analysis, Decision Theory, Utility Theory

1 Introduction

Aggregation of utility functions under a variety of criteria is investigated in the area of Multi Attribute Utility Theory (MAUT) [12-14]. A number of techniques used in MAUT has been implemented as computer programs such as SMART [5] and EXPERT CHOICE, the latter which is based on the widely used AHP [23-25]. AHP has been criticised in a variety of respects [2, 29, 30] and models using geometric mean value techniques has been suggested instead [1, 15]. Techniques based on the geometric

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mean value has, for instance, been implemented by Lootsma and Rog in REMBRANDT [18].

All these approaches have their advantages, but the requirement to provide numerically precise information sometimes seems to be unrealistic in real-life decisions situations, and a number of models with representations allowing imprecise statements have been suggested. For instance, [27] extends the AHP-method in this respect and also make use of structural information when the alternatives are evaluated into overlapping intervals. The system ARIADNE [26] also allows the decision maker to use imprecise estimates, but does not discriminate between alternatives when these are evaluated into overlapping intervals. Fuzzy set theory is a more widespread approach to relaxing the requirement of numerically precise data by providing a more realistic model of the vagueness in subjective estimates of probabilities, weights, and values [3, 16]. These approaches allow, among other features, the decision maker to model and evaluate a decision situation in vague linguistic terms.

The methods we propose herein originate from earlier work on handling probabilistic decision problems involving a number of alternatives and consequences when the background information is vague or numerically imprecise [4, 8, 20]. The aim of this paper is to generalise the work into the realm of multiple criteria decision aids, but still conform to classical statistical theory rather than to fuzzy set theory. By doing so, we try to avoid problems emanating from difficulties in providing set membership functions and in defining set operators having a satisfying intuitive correspondence. Parts of the framework presented in this paper has also been implemented in the DELTA tool which at present is used in a large-scale evaluation at the Swedish National Rail Administration.

The next section describes how imprecise sentences are modelled and how the model subsequently can be evaluated. Section 3 extends the results from Section 2 and describes how consequence analyses can be incorporated into the method. Section 4 concludes the paper.

2 Modelling Decision Situations

As was mentioned above, a significant feature of the framework is that it allows for decision situations where numerically imprecise or comparative sentences occur. These sentences are represented in a numerical format and with respect to this the strategies can be evaluated using a variety of decision rules. The further discriminating analyses try to show which parts of the given information are the most critical and must be given extra careful consideration.

2.1. Information Frames

The decision maker's importance (weight) estimates are represented by linear constraints and we treat three classes of weight sentences: vague sentences, interval sentences, and comparative sentences (cf. [6]).

Typical *vague sentences* include: "The criterion K_i is the most important" or "The criterion K_j is of some importance". They may be represented by suitable intervals according to the decision maker. Suppose that a decision maker stipulates that for K_i to be called 'important', the weight must be greater than 0.5 but less than 0.9. In this case, the translation will be $w_i \in [0.5,0.9]$, represented by the two linear inequalities

$w_i \geq 0.5$ and $0.9 \geq w_i$. Similar translations apply when representing other vague sentences. *Interval sentences* are of the form: "The importance of K_i lies between the numbers a_i and b_i" and are translated into $w_i \in [a_i,b_i]$. Finally, *comparative sentences* are of the form: "The importance of K_i is greater than the importance of K_j". Such a sentence is translated into an inequality $w_i \geq w_j$. Each statement is thus represented by one or more constraints. We call the conjunction of constraints of the types above, together with the normalisation constraint $\Sigma_{i \leq n} w_i = 1$, the *criteria base* (K).

The *strategy base* (S) consists of similar translations of vague and numerically imprecise utility estimates.[2] A strategy base with n criteria and m strategies is expressed in strategy variables $\{u_{11},...,u_{1n},...,u_{m1},...,u_{mn}\}$ stating the utility of the strategies according to the different criteria. The term u_{ij} denotes the utility of strategy S_i with respect to criterion K_j. The collection of weight and utility statements constitutes the *information frame*. It is assumed that the variables' respective ranges are real numbers in the interval [0,1]. Below, we will refer to an information frame as a structure $\langle S, K \rangle$.

Example: A decision maker gives assessments concerning the strategies for a risk policy of a company. The objective of the investigation is to decide how to allocate resources for preventing potential losses of the company. The available strategies are to prevent disruption of productions and services, to prevent obstruction of research and development, or to distribute the resources over both these objectives. These strategies are labelled S_1, S_2, and S_3 below. Assume that the decision is supposed to be evaluated with respect to a short-term financial perspective as well as credibility in the long run. These criteria are denoted K_1 and K_2 below. The utilities involved could, for example, be monetary values. In that case, they are linearly transformed to real values in the interval [0,1].

For instance, the assessments with respect to criteria K_1 could be the following:

- The utility of strategy S_1 is between 0.20 and 0.50
- The utility of strategy S_2 is between 0.20 and 0.60
- The utility of strategy S_3 is between 0.40 and 0.60
- The utility of strategy S_2 is at least 0.10 better than that of S_1

Similar utility assessments can be asserted with respect to K_2.[3]

Moreover, the decision maker may estimate the importance of K_1 and K_2 as numbers in the interval [0,1]. The number 0 denotes the lowest importance and 1 the highest. Thus, the assessments about the criteria could be:

- Criteria K_2 is at least as important as K_1
- The importance of criteria K_1 is between 0.30 and 0.70

One further reason for allowing interval as well as comparative assessments is that the background information may have different sources. For instance, intervals naturally occur from aggregated quantitative information while qualitative analyses

[2]The values can be cost values, utility values, or values on any other appropriate scale, cf. [28].

[3]Note that we only discuss the representation of the situation from a global point of view. The individual criteria assessors may have used different kinds of risk evaluation methods to determine their utilities (cf. [7, 10]).

often result in comparisons. Since the sources may be different, the assessments are not necessarily consistent with each other.

The utility estimates with respect to K_1 are translated into the following expressions.

$\mathbf{u_{11}} \in [0.20, 0.50]$	$\mathbf{u_{21}} \in [0.20, 0.60]$
$\mathbf{u_{31}} \in [0.40, 0.60]$	$\mathbf{u_{21}} \geq \mathbf{u_{11}} + 0.10$

The importance of K_1 and K_2 are also represented as numbers in the interval [0,1], and the translation of the assessments above results in the following expressions.

$\mathbf{w_2} \geq \mathbf{w_1}$ $\qquad$ $\mathbf{w_1} \in [0.30, 0.70]$ ■

2.2. Aggregations

In the following, we will assume that the bases are consistent, i.e. that there is at least one solution vector to each system of inequalities.[4]

One candidate for an aggregation principle could be based on a weighted sum of the utilities and the following notation will be used to define this with respect to an information frame representing n criteria and m strategies:

Definition: Given an information frame $\langle S, K \rangle$, the *global utility $G(S_i)$* of a strategy S_i is $G(S_i) = \Sigma_{k \leq n} w_k \cdot u_{ik}$, where w_k and u_{ik} are variables in K and S, respectively.

Definition: Given an information frame $\langle S, K \rangle$, the *difference in global utility* δ_{ij} between two strategies S_i and S_j are $\delta_{ij} = G(S_i) - G(S_j) = \Sigma_{k \leq n} w_k \cdot (u_{ik} - u_{jk})$, where w_k, u_{ik}, and u_{jk} are variables in *S* and *K*, respectively. ■

Definition: Given an information frame $\langle S, K \rangle$, let a and d be two vectors of real numbers $(a_1,...,a_n)$ and $(b_{11},...,b_{mn})$. ${}^{ab}G(S_i) = \Sigma_{k \leq n} a_k \cdot b_{ik}$, where a_k and b_{ik} are numbers substituted for w_k and u_{ik} in $G(S_i)$). Similarly, ${}^{abd}\delta_{ij} = {}^{ab}G(S_i) - {}^{ad}G(S_j)$.

With respect to these definitions, we can, for instance, express the concept of admissibility in the sense of [17].

Definition: Given an information frame $\langle S,K \rangle$, S_i is *at least as good* as S_j iff ${}^{abd}\delta_{ij} \geq 0$, for all a, b, d, where $\{w_1 = a_1\}$ & ... & $\{w_n = a_n\}$ is consistent with K and $\{u_{i1} = b_{i1}\}$ & ... & $\{u_{in} = b_{in}\}$ & $\{u_{j1} = d_{j1}\}$ & ... & $\{u_{jn} = d_{in}\}$ is consistent with S.

S_i is *better* than S_j iff S_i is at least as good as S_j and ${}^{abd}\delta_{ij} > 0$, for some a, b, d, that are consistent with K and S as above.

S_i is *admissible* iff no other S_j is better. ■

[4]Recall that a list of numbers $[n_1,...,n_s]$ is a *solution vector* to a set of inequalities S containing variables $x_1,...,x_s$, if the substitution of n_i for x_i in S, for all $1 \leq i \leq s$, does not yield a contradiction. The set of solution vectors to S constitutes the *solution set* for S. If there is a non-empty solution set for S, it is *consistent*. Otherwise S is *inconsistent*. Given two sets of inequalities S and T, if $S \cup T$ is consistent we will sometimes say that S is consistent with T or vice versa. Needless to say, the solution sets to the bases can be determined by ordinary linear programming (LP) methods.

The concept of admissibility is computationally meaningful in our framework as demonstrated in [4]. However, the admissibility often seems to be too weak to form a decision rule by itself, and in [4, 9] we introduce further discriminating principles in the case of decisions under risk. These are readily adapted to the multi-criteria case. We first introduce some notations that will be used in the sequel.

Definition: Given a base Y and a function f into the set of real numbers, ${}^{Y}max(f(y))$ is sup(a | f(y) > a is consistent with Y). Similarly, ${}^{Y}\min(f(y))$ is inf(a | f(y) < a is consistent with Y). Likewise, given an information frame ⟨S,K⟩, ${}^{SK}max(G(S_i))$ is sup(d | ${}^{ab}G(S_i) > d$ for all vectors a and b such that $\{w_1 = a_1\}$ & ... & $\{w_n = a_n\}$ is consistent with S and $\{u_{i1} = b_{i1}\}$ & ... & $\{u_{in} = b_{in}\}$ is consistent with K). ■

Next, the problem of finding optima in the bases is addressed from an interactive point of view. Determining admissibility are computationally fairly demanding tasks in the general case, using quadratic programming (QP), and the main issue in the following section is to provide a procedure to reduce problems of this kind to linear systems, solvable with linear programming (LP) methods.

2.3. Bilinear Optimisation

Our purpose now is to evaluate expressions such as δ_{ij} and δ_{ji} for all pairwise combinations of alternatives under consideration. This leads to quadratic problems with certain structural properties. Each comparison of two alternatives results in exactly one bilinear objective function together with many linear constraint equations, a bilinear programming (BP) problem. Since the objective function is quadratic and all the constraint equations are linear, the optimising problem could be solved with QP methods. However, QP algorithms are in general too demanding from an interactive point of view. [20] suggests a bilinear elimination (BE) algorithm for solving the BP problem by generating a large number of systems to solve. At the time of writing, solving these systems will not admit fast response and thus BE is not well suited for an interactive tool. The same problems occur when determining the strengths of the strategies. Therefore, an LP based method for use in an interactive environment is necessary. The algorithm described is the bilinear optimisation (^{K}B-Opt). In describing this algorithm we will make use of the following concepts.

Definition C: Given an information frame ⟨S,K⟩.
Then ${}^{S}G_i^{max}$ is $w_1 \cdot a_{i1} + ... + w_n \cdot a_{in}$, where a_{ik}, $1 \le k \le n$, is sup(b | $\{b \le u_{ik}\}$ & $\{a_{i(k-1)} = u_{i(k-1)}\}$ & ... & $\{a_{i1} = u_{i1}\}$ is consistent with S).

Further, ${}^{S}G_i^{min}$ is $w_1 \cdot a_{i1} + ... + w_n \cdot a_{in}$, where a_{ik}, $1 \le k \le n$, is inf(b | $\{b \ge u_{ik}\}$ & $\{a_{i(k-1)} = u_{i(k-1)}\}$ & ... & $\{a_{i1} = u_{i1}\}$ is consistent with S). ■

By using the above definitions, the strategies can be evaluated with respect to a variety of decision rules using simple LP methods only. The evaluation of admissibility is quite straightforward, but also other decision rules can be formed. We demonstrate this by forming the relative strength.

Definition: Given an information frame ⟨S,K⟩, the *relative strength* Δ_{ij} of S_i compared to S_j is $({}^{SK}\max(\delta_{ij}) - {}^{SK}\max(\delta_{ji}))/2$. ■

Using the definition C above the following expression can be formed.

Definition: Given an information frame ⟨S,K⟩, ${}^{S}\delta_{ij}$ is ${}^{S}G_i{}^{max} - {}^{S}G_j{}^{min}$ and ${}^{K}\Delta_{ij}$ is $({}^{K}max({}^{S}\delta_{ij}) - {}^{K}max({}^{S}\delta_{ji}))/2$. ■

We will now demonstrate that ${}^{K}\Delta_{ij}$ is equal to Δ_{ij}. under specific circumstances. This means that the relative strength can be determined by using LP methods only. The idea behind ${}^{K}\Delta_{ij}$ is to transform a bilinear expression into a linear expression with the property of having the same extremal value under specific conditions. Thus, the evaluation of the relative strength Δ_{ij} involves the evaluation of ${}^{SK}max(\delta_{ij})$. To avoid the non-linearity inherent in the δ_{ij} formula, an LP procedure is employed for calculating δ_{ij}. The following proposition follows immediately from a similar proposition that is proved in [4].

Proposition: Given an information frame ⟨S,K⟩, assume that none of the comparative statements in S involve variables from different S_i's. Then $\Delta_{ij} = {}^{K}\Delta_{ij}$ for any pair S_i and S_j. ■

2.4. Contractions

Furthermore, in non-trivial decision situations, when an information frame contains numerically imprecise information, the principles suggested above are sometimes too weak to yield a conclusive result. A way to refine the analysis is to investigate how much the different intervals can be contracted before an expression such as $\delta_{ij} > 0$ ceases to be consistent. This contraction avoids the complexity inherent in combinatorial analyses, but it is still possible to study the stability of a result by gaining a better understanding of how important the interval boundary points are. By co-varying the contractions of an arbitrary set of intervals, it is possible to gain much better insight into the influence of the structure of the information frame on the solutions.[5] Contrary to volume estimates, contractions are not measures of the sizes of the solution sets but rather of the strength of statements when the original solution sets are modified in controlled ways. Both the set of intervals under investigation and the scale of individual contractions can be controlled. Consequently, a contraction can be regarded as a focus parameter that zooms in on central sub-intervals of the full statement intervals.

Definition: X is a base with the variables $x_1,...,x_n$, $\pi \in [0,1]$ is a real number, and $\{\pi_i \in [0,1] : i = 1,...,n\}$ is a set of real numbers. $[a_i, b_i]$ is the interval corresponding to the variable x_i in the solution set of the base, and $\mathbf{k} = (k_1,...,k_n)$ is a consistent point in X. A *π-contraction* of X is to add the interval statements $\{x_i \in [a_i+\pi\cdot\pi_i\cdot(k_i-a_i), b_i-\pi\cdot\pi_i\cdot(b_i-k_i)] : i = 1,...,n\}$ to the base X. **k** is called the *contraction point.* ■

By varying π from 0 to 1, the intervals are decreased proportionally using the gain factors in the π_i-set, thereby facilitating the study of co-variation among the variables.

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[5] For a 100% contraction, the volume of each base is reduced to a single point. For this special case, the results from the algorithms for comparing alternatives coincide with the ordinary expected value.

3 Multi-Level Decision Trees

In Section 2 decision problems were modelled without taking into account how a decision maker arrived at his preferences and there were no requirements on the methods he used in this process. By extending the concept of strategy, and using techniques similar to those proposed in that section, more general decision models can be handled. Consider a decision situation under risk as in Fig. 1 (cf., e.g., [22]).

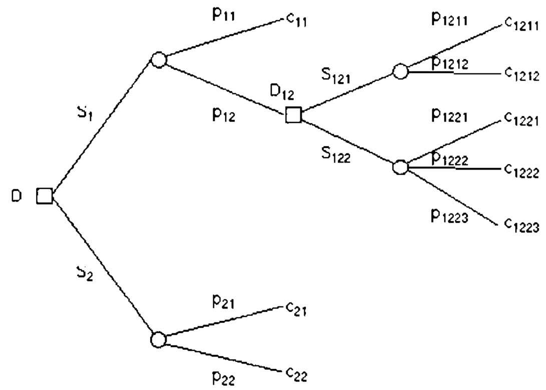

Fig. 1 A Multi-Level Tree

The directed edges (labelled S) in the figure denote alternatives, and the c's different consequences. The squares are decision nodes, i.e., where a decision has to be made by a decision maker. The circles denote chance nodes, from which edges lead to leaves or to new decision nodes. Finally, the leaves correspond to ultimate consequences. A directed edge (labelled p) denotes the probability of the node where the edge terminates, given that the strategy (leading to the chance node where p begins) is chosen. The preferences among the consequences are supposed to be expressed by some kind of value function, for instance a utility function. If such a function exists, the value of consequence c_{ij} can be mapped onto a value u_{ij}, and the situation can be evaluated with respect to different evaluation rules (cf. [9, 11, 19, 21]).

This model could be extended in a way similar to Section 2 by allowing for imprecise assessments. To simplify the presentation in the sequel, it is assumed that, to each chance node, there is at most one directed edge leading to a decision node. The general case is very similar.

Definition: Given a decision tree, a set $\{c_{i1},...,c_{is_i},D_{i(s_i+1)}\}$ is an *alternative associated with a chance node C_i* if the elements of the set are exhaustive and pairwise disjoint with respect to C_i. (This notation will be used even if an alternative does not contain an element $D_{i(s_i+1)}$). ■

Informally, this means that exactly one of $c_{i1},...,c_{is_i},D_{i(s_i+1)}$ will occur given that the alternative, represented by the directed edge to C_i is chosen.

Definition: Given a decision tree, a sequence of edges $[S_1,...,S_r]$ is a *strategy*, if for all elements in the set, S_{i-1} is a directed edge from a decision node to a chance node C_{i-1}, and there is a directed edge from C_{i-1} to a decision node from which S_i is a directed edge. ■

Definition: Given a criterion K, a decision tree associated with K, and a strategy $[S_1,...,S_r]$, where each S_i is an alternative $\{c_{i1},...,c_{is_i},D_{i(s_i+1)}\}$ associated with a chance node C_i. The *expected utility of $[S_1,...,S_r]$ with respect to criterion K*, $E^K(S_1,...,S_r)$, is defined by the following:

(i) $E^K(S_i) = \Sigma_{k \leq s_i} p_{ik} \cdot u_{ik}$, when S_i is an alternative $\{c_{i1},...,c_{is_i}\}$,

(ii) $E^K(S_i,...,S_r) = \Sigma_{k \leq s_i} p_{ik} \cdot u_{ik} + (p_{i(s_i+1)} \cdot E^K(S_{i+1},...,S_r))$, when S_i is an alternative $\{c_{i1},...,c_{is_i},D_{i(s_i+1)}\}$, u_{ij} denotes the utility of the consequence c_{ij}, and p_{ij} denotes the probability of the consequence c_{ij} (or D_{ij}), under criterion K. ■

Given a decision tree T, a decision node D in T can be considered a set $\{S_1,...,S_q\}$ of strategies, i.e. all directed edges from D. Two bases may be associated to D, one containing the probability variables of the edges from each S_i, and one containing the utility variables corresponding to possible leaves emanating from each S_i. Using such a structure, vague and numerically imprecise assessments can be represented and evaluated in a way similar to Section 2. The inequalities containing utility variables are included in the utility base *V(D)*, and inequalities containing probability variables are included in the probability base *P(D)*. These bases comprise the *local decision frame corresponding to D and criterion K* $\langle P^K(D),v^K(D)\rangle$. This framework for evaluating the expected utility of a strategy can be combined with the framework described in Section 2 and the total decision situation can be evaluated with respect to all criteria, strategies, probabilities and utilities involved in the decision situation under consideration. The decision maker may assert probability and utility assessments with respect to the tree. In this sense the probability and utility bases are local to each criteria. What remains is to substitute the utilities of strategies in Section 2 with the expected utility of a strategy as defined in this section.

Definition: Given a set of criteria $\{K_1,...,K_n\}$, n decision trees – each associated with exactly one criterion, and a strategy $[S_1,...,S_r]$, the *global expected utility of $[S_1,...,S_r]$*, $G(S_1,...,S_r)$, is defined as:

$G(S_1,...,S_r) = \Sigma_{k \leq n} E^{K_k}(S_1,...,S_r) \cdot w_k$, where w_k is a variable denoting the weight of criterion K_k as in the corresponding definition in Section 2. ■

Note that the definition does not presume that the decision trees for the different criteria are identical. For some domains the tree could be the same for all criteria and only the probability and utility assessments may differ. In other domains the decision maker may have constructed different decision trees involving the strategies under consideration. Similar to Section 2, the strategies are evaluated with respect to the information in the criteria base. The difference here is that the strategy base is replaced by a set of probability and utility bases.

Consider the prerequisites in the definition above. Each S_i in the strategy $[S_1,...,S_r]$ is an alternative on the form $\{c_{i1},...,c_{is_i},D_{i(s_i+1)}\}$, for each criterion K. Each S_i is associated with a chance node C_i. Assume that the directed edge leading to C_i emanates from the decision node D_i, to which a local decision frame $\langle P^K(D_i),V^K(D_i)\rangle$ corresponds. Such a frame contains constraints representing the probability and utility assessments of criterion K. Consequently, $G^K(S_1,...,S_r)$ is associated with the set $\{\langle P^K(D_i),V^K(D_i)\rangle\}^j$, j = 1,...,r, in the same way as the strategy variables used in Section 2 are associated with the strategy base.

Definition: Given a criterion K, a decision tree T, and a strategy $[S_1,...,S_r]$ in T, where each S_i is an alternative $\{c_{i1},...,c_{is_i},D_{i(s_i+1)}\}$ associated with a chance node C_i. Let $a_1,...,a_r$, $b_1,...,b_r$ be vectors of real numbers $\{(a_{i1},...,a_{i(s_i+1)})\}i=1,...r$, $\{(b_{i1},...,b_{is_i})\}i=1,...r$. Now, the expected utility of $[S_1,...,S_r]$ according to criterion K is defined by the following:

(i) $^{a_ib_i}E^K(S_i) = \Sigma_{k\leq s_i} a_{ik}\cdot b_{ik}$, when S_i is an alternative $\{c_{i1},...,c_{is_i}\}$,

(ii) $^{a_ib_i...a_rb_r}E^K(S_i,...,S_r) = \Sigma_{k\leq s_i} a_{ik}\cdot b_{ik} +(a_{i(s_i+1)}\cdot E^K(S_{i+1},...,S_r))$, when S_i is an alternative $\{c_{i1},...,c_{is_i},D_{i(s_i+1)}\}$. ■

This may now be combined with the notation for instantiations of the global expected utility of an strategy in Section 2 into the following:

Definition: Given a set of criteria $\{K_1,...,K_n\}$, n decision trees – each associated with exactly one criterion, and a strategy $[S_1,...,S_r]$. Let $a_1,...,a_n$, $b_1,...,b_n$ be vectors of vectors $\{(^ja_1,...,^ja_r), (^jb_1,...,^jb_r)\}$, j=1,...n. The latter are vectors of real numbers $\{(^ja_{i1},...,^ja_{i(s_i+1)})\}$, i=1,...,r, $\{(^jb_{i1},...,^jb_{i(s_i)})\}$, i=1,...,r. Also let d be a vector of real numbers $(d_1,d_2,...,d_n)$. Now,

$$^{a_ib_i...a_nb_nd}G(S_1,...,S_r) = \Sigma_{k\leq n}\ {}^{{}^ka_i{}^kb_i...{}^ka_n{}^kb_n}E^{K_k}(S_1,...,S_r)\cdot d_k. ■$$

Definition: A *general decision frame* is a structure $\langle \mathcal{T}, \mathcal{S}, \mathcal{L}, \mathcal{K}\rangle$. $\mathcal{T}$ is a set of T_j's – decision trees associated with the criteria K_j, j=1,...,n. $\mathcal{S}$ is the set of possible strategies modelled in the trees. $\mathcal{L}$ is a set of local decision frames $\langle P^{K_j}(D_i),V^{K_j}(D_i)\rangle$ corresponding to D_i and criterion K_j,where D_i is a node in the tree T_j. $\mathcal{K}$ is the criteria base as in Section 2. ■

The different strategies can then be evaluated, for instance with respect to admissibility as in Section 2.

Definition: Given a general decision frame $\mathcal{F}$ and a real number t in the interval [0,1]. The strategy $[S_{i_1},...,S_{i_r}]$ is at least as *good* as the strategy $[S_{j_1},...,S_{j_q}]$ iff $^{a_1b_1...a_nb_nd}G(S_{i_1},...,S_{i_r}) - {}^{f_1g_1...f_ng_ne}G(S_{j_1},...,S_{j_q}) \geq 0$, for all d, e where d and e are solution vectors to $\mathcal{K}$. Furthermore, each ^ja_i in a_i, and each ^jf_i in f_i are solution vectors to $P^{K_j}(D_i)$, and each ^jb_i in b_i, and each ^jg_i in g_i are solution vectors to $V^{K_j}(D_i)$.

The strategy $[S_{i_1},\ldots,S_{i_r}]$ is *better* than the strategy $[S_{j_1},\ldots,S_{j_q}]$ iff $[S_{i_1},\ldots,S_{i_r}]$ is at least as good as $[S_{j_1},\ldots,S_{j_q}]$ and ${}^{a_1b_1\ldots a_nb_nd}G(S_{i_1},\ldots,S_{i_r}) - {}^{f_1g_1\ldots f_ng_ne}G(S_{j_1},\ldots,S_{j_q}) > 0$ for some d, e where d and e are solution vectors to $\mathcal{K}$, and for every a_i, b_i, f_i, g_i, $i=1,\ldots,k$, a_i in a_i, and ${}^jf_i$ in f_i are solution vectors to $P^{Kj}(D_i)$, and ${}^jb_i$ in b_i, and ${}^jg_i$ in g_i are solution vectors to $V^{Kj}(D_i)$.

The strategy $[S_{i_1},\ldots,S_{i_r}]$ is *admissible* iff no other strategy in $\mathcal{J}$ is better. ■

If the set of admissible strategies is too large, contraction methods similar to those suggested in Section 2 can be used for investigating the stability of the result.

4 Concluding Remarks

We have shown how a set of vague and numerically imprecise statements can be evaluated with respect to a set of criteria and how to determine which strategies are reasonable to choose among. The approach considers a decision problem with respect to the different criteria as well as the consequence analysis of the different strategies involved. These aspects are modelled into information frames consisting of systems of linear expressions stating inequalities and interval assessments. The strategies may be evaluated relative to a variety of principles, for example generalisations of the principle of maximising the expected utility. We also demonstrate how decision trees can be integrated into the framework and suggest an evaluation rule taking into account all strategies, criteria, probabilities and utilities involved in the framework.

Contractions are introduced as an automated sensitivity analysis. This concept allows us to investigate critical variables and the stability of the results. An important feature is the investigation into effects of decreasing the different intervals, since without such an option the set of admissible alternatives is often relatively large. In this paper, we have proposed a contraction principle that seems to be reasonable. However, a number of modifications are possible, such as decreasing the intervals from either side as far as possible in steps of different lengths in order to approximate a set of reliability criteria. Some suggestions for decision rules are described in the paper, but we have also noted that these are not the only possible ones and the framework could use other decision rules as well.

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