RM-75-11

AN OPTIMIZATION TECHNIQUE FOR THE BUDWORM FOREST-PEST MODEL

Carlos Winkler

March 1975

An Optimization Technique for the Budworm Forest-Pest Model

Carlos Winkler*

1. Introduction

As part of a multidisciplinary approach to studying the budworm-forest system at IIASA, the methodology group investigated the possibilities of development of an optimization model. In this report we describe our work and the results obtained so far. Frequently, as the investigation proceeded, new constraints were fed back to us; this required a change in direction of our efforts, or a postponement of some approach to a later time, when we could count on a vital piece of software. Here we will loosely trace the evolution of our efforts in order to motivate each step and to point out what further work might be done on the loose ends.

Also, as requested by the ecology group, we will give an exposition on the dynamic programming technique used and its interaction with the ecological model.

2. Stander's Simulation Model

The Ecology Group brought to IIASA Stander's Simulation Model for the budworm-forest system which they believed gives a reasonable picture of the real world, and was therefore suitable for our optimization purposes. As the Stander's model is described in detail elsewhere [8], we will emphasize here only a few points that are important for our work.

---

*International Institute for Applied Systems Analysis, Laxenburg, Austria.

We cite from Stander's paper: "The model is of the discrete, state-transition type. Its time step is one year, and events are assumed to transpire within the year in a sequence determined by the model organization. The model is specifically of New Brunswick. To account for spatial heterogeneity, the province was divided into 265 six-by-nine mile rectangles, which are called 'sites' in this report. The long axis of each site is in N - S direction.

Within each site the computer keeps track of the population of budworms (adults, eggs, larvae, survival rates, etc.). The critical process of adult budworm dispersal is assumed to occur only among the 265 sites, since each site is treated as being homogeneous in itself; i.e. trees and budworms are treated as being uniformly distributed within sites.

The various budworm control policies which can be manifested are all effective at the site level; that is, spraying for larvae of adults may be done in sites with high levels of budworm hazard, or only at sites where there are high larval concentrations; and host management can take the form of clear-cutting high hazard sites or select-cutting older host species trees."

At the beginning of a one-year time interval the state of the system is determined by the values of egg densities, forest composition, and stress index, which condenses the past history of budworm attacks on the forest.

These values, together with some control criteria, make it possible to calculate, for each site and each stochastic weather outcome, the values of the state variables for the next year without accounting for contamination from other sites and contamination from each site to the others. Next, by means of a dispersion model, the contamination distribution can then be calculated and used to obtain the real values of the egg densities for the next year. Thus, for each possible control criterion and weather pattern, we are able to generate a picture of the evolution of the system. Through comparison of these "future pictures," decisions as to which controls are preferred can be taken. Unfortunately, the number of possible controls is exceedingly high, and only a few can be analyzed and compared in this way; the difficulties are particularly compounded by a stochastic weather pattern.

In the following, we will give a schematic representation of the mechanics of Stander's model as modified by D. Jones (see [7]). We will not follow the organization of his computer model, but will otherwise use the same relationships between variables.

The following notation will be used for a given site:

x_i = acres covered by trees in i^{th} age group;

E = eggs/acre in site;

F_T = Foliage level (0 - 3.8) at site (replaces Stander's stress index);

N = number of age groups;

X = $(X_1, X_2, \ldots, X_N)$ forest composition vector;

P_F = proportion of site covered by balsam fir.

A superscript t is used to denote the variables on the t^{th} time period. In Stander's model there are 25 three-year-age classes. Other intermediate variables will be defined as they appear.

With values E^{t-1}, F_T^{t-1}, and X^{t-1} at the beginning of a period, the following relationships allow us to sequentially compute the values for E^t, F_T^t, X^t at the beginning of the next period.

2.1 Mean age of trees:

$$MA^t = \sum_{i=1}^{N} MA_i * X_i^{t-1} / \sum_{i=1}^{N} X_i^{t-1} \quad , \qquad (2-1)$$

where MA_i is the mean age in the ith age class.

2.2 Surface area [10 sq.ft./acre]:

$$A_s^t = g_1(F_T^{t-1}) * .0356 * (MA^t)^{1.5} * (1136 - 3.0224 * MA^t) * EXP(-MA^t/30) \quad , \qquad (2-2)$$

where

$$g_1(F_T) = \begin{cases} 1 \quad , & \text{if } F_T > 2.5 \\ 1.25 * (2.5 - F_T)/(3 - F_T) \quad , & \text{otherwise} \end{cases} \qquad (2-3)$$

2.3 Egg densities [eggs/10 sq. ft.]:

$$E_d^t = E^{t-1}/A_s^t \quad . \tag{2-4}$$

2.4 Tree mortality fraction:

$$d_i^t = \hat{d}_i + f_i * g_2(F_T^{t-1}) \quad , \quad i = 1,\ldots,N \quad , \tag{2-5}$$

where

$$g_2(F_T) = \begin{cases} 0 \quad , & \text{if } F_T \geq 2 \\ 1.25 * (2 - F_T)/(2.5 - F_T) \quad , & 0 \leq F_T < 2 \quad . \end{cases} \tag{2-6}$$

$\hat{d}_i = 0$ for $i = 1,\ldots,N-1$, $\hat{d}_N$ is the natural mortality rate for old trees, and f_i is given in Figure 1 as a function of age.

2.5 Surviving trees:

$$Y_i^t = (1 - d_i^t - p_i^t)X_i^{t-1} \quad , \quad i = 1,\ldots,N \quad , \tag{2-7}$$

$$Y_o^t = \sum_{i=1}^{N} (d_i^t + p_i^t)X_i^{t-1} \quad ,$$

where $p_i^t \leq 1 - d_i^t$ is the fraction of acres covered with trees in the i^{th} age class that are logged in year t.

2.6 Forest composition at end of period:

$$X_i^t = (1 - a_i) * Y_i^t + a_{i-1}Y_{i-1}^t \quad , \quad i = 1,\ldots,N \, , \tag{2-8}$$

where a_i is the fraction of trees in age class i that ages to class i + 1 ($a_i = 1/3$, $i = 1,\ldots,N-1$ when N = 25 and $a_0 = 1$).

2.7 Foliage level during period:

$$\hat{F}_T^t = [1 - (3.8 - F_T^{t-1}) * (.95 - F_T^{t-1}) * .129]F_T^{t-1} \quad . \tag{2-9}$$

2.8 New foliage during period:

$$F_1^t = F_T^{t-1}/3.8 \quad . \tag{2-10}$$

2.9 Third instar larvae density before spraying:

$$TL_d^t = 2.11 * 10^{-5} * A_s^t * F_1^t * (2 - F_1^t) * E_d^t \quad . \tag{2-11}$$

2.10 Third instar larvae density after spraying:

$$\widehat{TL}_d^t = TL_d^t * \text{EXP } (-.4 * S_\ell^t) \quad . \tag{2-12}$$

2.11 Adult density:

$$A_d^t = f_2(S_A^t) * g_3(TL_d^t, W) * \hat{TL}_d^t \quad , \qquad (2\text{-}13)$$

where $S_A^t = 0$ and $f_2(0) = 1$, if adults are not sprayed, $S_A^t = 1$ and $f_2(1) = .1$ if adults are sprayed, and

$$g_3(TL_d^t, W) = \max \{ [g_4(TL_d^t, W) * g_5(TL_d, W)], g_6(W) \} \quad , \qquad (2\text{-}14)$$

where g_4 is given in Figure 2 and[1]

$$g_5(TL_d, W, F_1) = \begin{cases} 1 \quad , \quad \text{if } TL_d \leq 135 \cdot F_1 \\ 1 - 3.33 * 10^{-3} * TL_d, \text{for } TL_d > 135 * F_1 \text{ and } W = 3 \\ 1 - 4 * 10^{-3} * TL_d, \text{ otherwise} \quad ; \end{cases} \qquad (2\text{-}15)$$

$g_6(W) = .20$ for $W = 1$, $g_6(W) = .16$, $W = 2$, and $g_6(W) = .10$ for $W = 3$.

2.12 Fecundity per adult:

$$FEC^t = \max \{20, (90 - .566 * TL_d^t / \hat{F}_T^t)\} \quad . \qquad (2\text{-}16)$$

2.13 Total number of potential eggs:

$$E_T^t = A_s^t * FEC^t * A_d^t \quad . \qquad (2\text{-}17)$$

2.14 Fraction of eggs laid in site:

$$SL1^t = \max \{0, .01375 * FEC^t - .4875\} \quad . \qquad (2\text{-}18)$$

2.15 Eggs to other sites:

$$E_0^t = (1 - SL1^t) * E_T^t \quad . \qquad (2\text{-}19)$$

2.16 Eggs next year

$$E^t = \frac{\hat{F}_T^t}{3.8} * \left(2 - \frac{\hat{F}_T^t}{3.8}\right) * \left[SL1^t * E_T^t + P_f * E_{IN}^t\right] \qquad (2\text{-}20)$$

where E_{IN}^t is the number of eggs coming from other sites.

2.17 Foliage level at the end of period:

$$F_T^t = \max \{0 \, , \, \hat{F}_T - .0037 * (TL_d^t + \hat{TL}_d^t)\} \quad . \qquad (2\text{-}21)$$

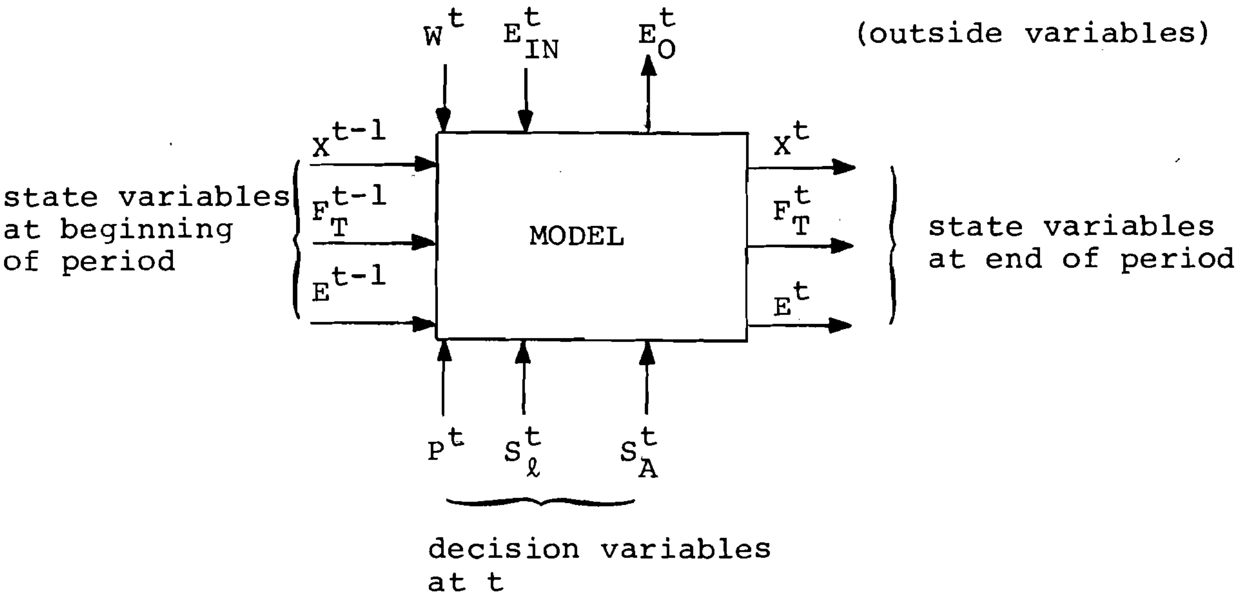
---

[1]W represents the stochastic weather outcome. W = 1 stands for 30, W = 2 for 20, and W = 3 for 10 days of good weather, respectively.

Relations (2-1) through (2-21) represent the ecological model. Letting $P = (p_1, \ldots, p_N)$ be the vectors of acres logged, we can express the relationships of the model as

$$\begin{aligned} X^t &= G_1(X^{t-1}, F_T^{t-1}, P^t) \\ F_T^t &= G_2(X^{t-1}, F_T^{t-1}, E^{t-1}, P^t, S_\ell^t) \qquad (2\text{-}22) \\ E^t &= G_3(X^{t-1}, F_T^{t-1}, E^{t-1}, P^t, S_\ell^t, S_A^t, W^t, E_{IN}^t) \quad , \end{aligned}$$

or by



The model allows us to compute the value of the variables shown with arrows pointing outward for each set of values of the variables shown with arrows pointing inward.

Using a more compact notation, let

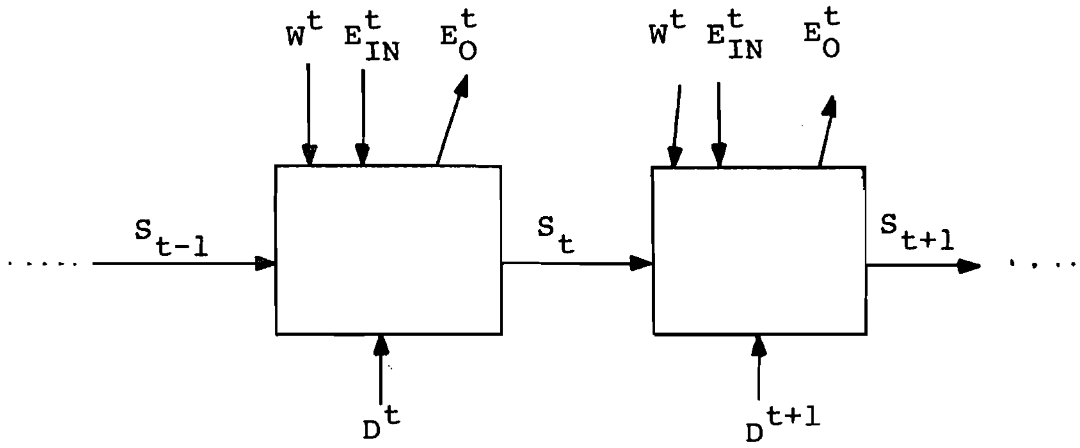
$S^t = (X^t, F_T^t, E^t)$ denote the state variables at time t, and

$D^t = (P^t, S_\ell^t, S_A^t)$ denote the decision variables at time t.

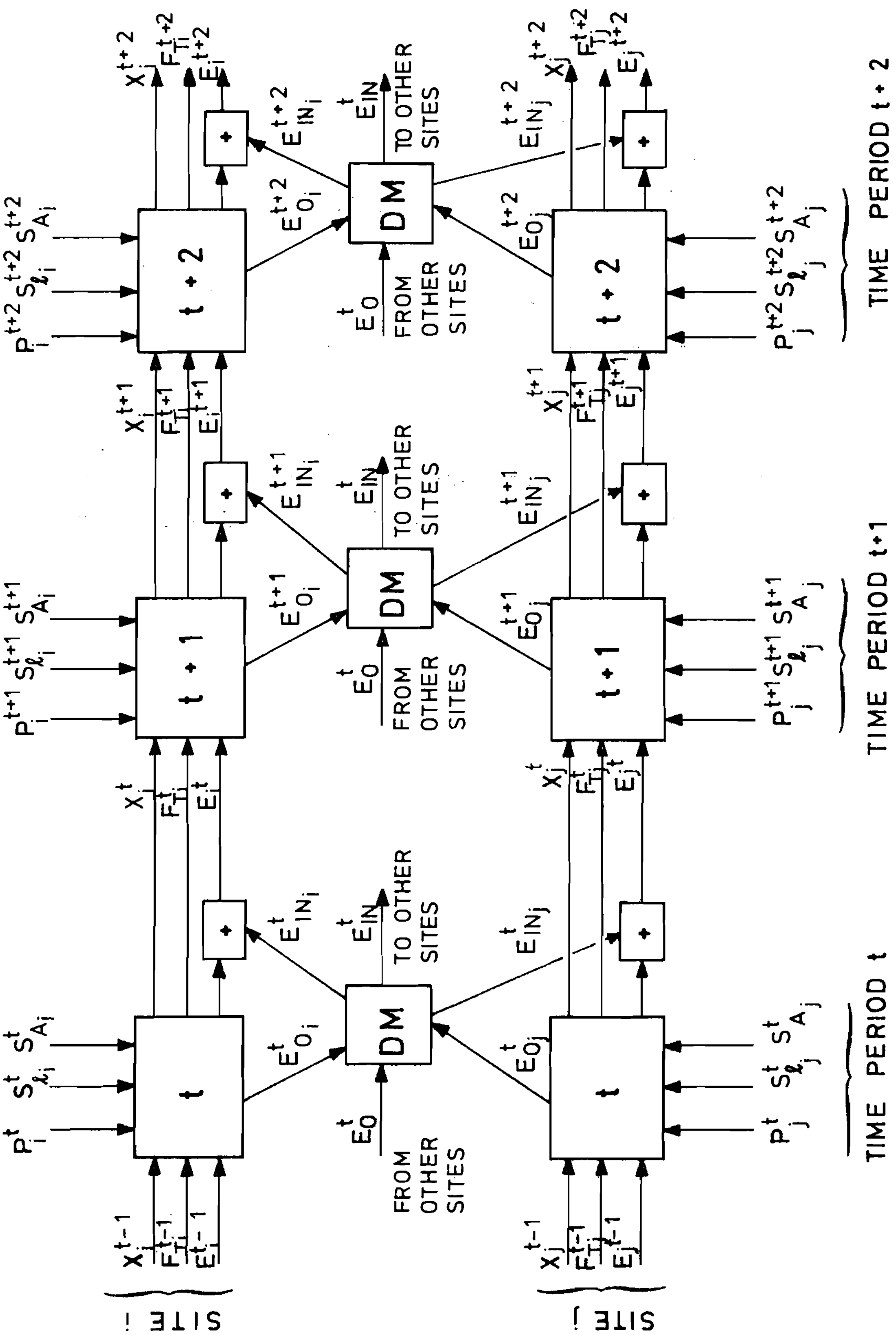
Then the model defines

$$S^t = F(S^{t-1}, D^t, W^t, E_{IN}^t) \quad , \qquad (2\text{-}23)$$

that is, knowing W^t and E_{IN}^t, we can use the model to obtain, for each state at the beginning of period t and for each possible decision during period t, the state of the system at the beginning of period t + 1. This can be shown schematically by:

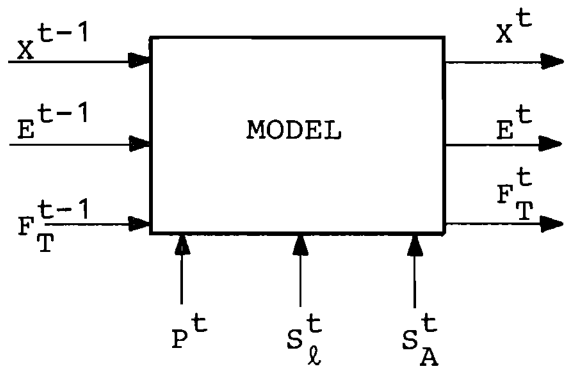


It is now possible to represent the information flow through the whole regional model by the following diagram.


SITE i
SITE j
DM
FROM OTHER SITES
TO OTHER SITES
TIME PERIOD t
TIME PERIOD t+1
TIME PERIOD t+2

3. The Dynamic Programming Approach

Consider the following simplified model for one time period:



This corresponds to assuming known weather patterns and contamination values from other sites. Assume that we can evaluate or assign a value $V_t(X^t,E^t,F_T^t)$ to any possible state (X^t,E^t,F_T^t) at the end of period t, and that we derive a benefit $C(X^{t-1},E^{t-1},F_T^{t-1},P^t,S_\ell^t,S_A^t)$ when starting in state $(X^{t-1},E^{t-1},F_T^{t-1})$ at the beginning of year t, and by taking actions (P^t,S_ℓ^t,S_A^t). Assuming further that we are using a discount factor λ, our decision problem at the beginning of period t can be stated as

$$\max_{P^t,S_\ell^t,S_A^t} \{C(X^{t-1},E^{t-1},F_T^{t-1},P^t,S_A^t,S_\ell^t) + \lambda V_t(X^t,E^t,F_T^t)\} \qquad (3\text{-}1)$$

or, using the more compact notation defined earlier,

$$\max_{D^t} \{C(S^{t-1},D^t) + \lambda V_t(F(S^{t-1},D^t))\} \quad , \qquad (3\text{-}2)$$

where $S^t = F(S^{t-1},D^t)$ are the relations represented by our model.

If for some reason we do not know in which state we are at the beginning of period t, we can use the above to evaluate each possible initial state through

$$V_{t-1}(S^{t-1}) = \max_{D^t} \{C(S^{t-1},D^t) + \lambda V_t(F(S^{t-1},D^t))\} \quad . \tag{3-3}$$

This is the dynamic programming recurrence relationship (see [2]). In the case of a stochastic weather pattern, the basic recurrence relation is the same except that we work with expected values where appropriate. That is,

$$V_{t-1}(S^{t-1}) = \max_{D^t} \{C(S^{t-1},D^t) + \lambda E[V_t(F(S^{t-1},D^t,W))]\} \quad . \tag{3-4}$$

From the recurrence relationships it becomes apparent that if we know $V_t(S^t)$ for some $t = T$, then we can evaluate recursively the payoff functions for $t = T - 1, T - 2,\ldots,1$. Also for each state S^{t-1} we then obtain an optimum action D_o^t (at least one) for which the optimum is attained. We can formalize this relationship as

$$D_o^t = g_t(S^{t-1}) \ , \qquad t = 1,\ldots,T \quad . \tag{3-5}$$

After having completed the backward evaluation for $T = T - 1, T - 2,\ldots,1$ we can reconstruct the sequence of optimal policy:

<u>Deterministic Case</u>: Set $t = 0$, initial state S^o. Then for $t = 1,\ldots,T - 1$ we obtain

$$D_o^t = g_t(S^{t-1})$$

$$S^t = F(S^{t-1},D_o^t) = F(S^{t-1},g_t(S^{t-1})) = F_t(S^{t-1}) \ ; \tag{3-6}$$

and in this way we can determine the optimal sequence of policies and the optimal path.

<u>Stochastic Case</u>: Set $t = 0$, initial state S^o. For $t = 1,\ldots,T - 1$ we now have

$$D_o^t = g_t(S^{t-1}) \quad ;$$

then we observe the stochastic outcome W^t, and the state of the next year is given by

$$S^t = f(S^{t-1}, D_o^t, W^t) \qquad (3\text{-}7)$$

$$S^t = f(S^{t-1}, g_t(S^{t-1}), W^t) = F(S^{t-1}, W^t) \quad .$$

Observe that it is not possible to determine beforehand the optimal path of the system. At each period we observe the state of the system and take the optimal action; then we must wait for the weather outcome before we can observe the state of the next year. Only then can we determine its optimal decision.

3.1 "Curse of Dimensionality" and Other Complications

So far we have purposely skipped over some of the difficulties of dynamic programming in order to present a clearer picture to those who are not acquainted with the technique. Now we will touch on some of the difficult points.

3.2 Computational Aspects of Dynamic Programming

For some simple comparisons let us assume that at each time period we can be in one of K possible states and take one of L possible decisions. Then the total problem can be viewed as follows: for each time period, take one decision (i.e. a value for the triple P, S_ℓ, S_A) so as to optimize the behavior of the system during the time span considered. Thus, the total number of combinations of decisions is L^T, where T is the number of time periods. We must, therefore, pick the best sequence of decisions out of L^T possible sequences; if we chose the optimum by evaluating and comparing each sequence, the work would be proportional to L^T.

On the other hand, with dynamic programming we would, for a given time period, have to do L evaluations for each state, i.e. KL evaluations per time period, and

$$T * L * K \ , \quad \text{in total} \quad . \qquad (3\text{-}8)$$

From this simplified analysis both the advantages and limitations of dynamic programming become apparant. The advantage is that the complexity increases linearly with the number of time periods, instead of exponentially. The disadvantage is that the complexity also depends on the number of possible states, while in the direct evaluation case it does not.

Again, if we have N_s state variables and each one can take on M discrete values, then

$$K = M^{N_s} , \tag{3-9}$$

i.e. the complexity increases exponentially with the number of state variables (for fixed M).

3.3 Continuous State Variables

From our previous analysis it is clear that it would be impossible to carry out a numerical optimization for all possible states in the case of continuous state variables, because we would have an uncountable number of such states. Therefore, we must restrict ourselves to a discrete set of points in the range of each variable, and the payoff function is evaluated only on these grid points. Of course, the grid chosen can be as close as desired, but from the expression $K = M^{N_s}$ we see that too fine a grid (i.e. large M) excessively increases the computational requirements. On the other hand, a denser grid gives us more values of the payoff and hence better approximations.

3.4 Approximation to the Payoff Function

In between grid points we need to interpolate the values of the payoff function, or alternatively we can approximate the payoff function by a polynomial, and use the evaluations on the grid points to obtain the coefficients of the approximating polynomial by regression. With this approach there are many possible tradeoffs: higher order polynomials give better fits but require more work in the regression; or is it better to fit a high order polynomial over the whole grid, or several lower order polynomials on subregions of the grid? These questions cannot be answered beforehand and experimentation is necessary.

Error Propagation. Due to the use of an approximated payoff function for period t, it is likely that we are introducing some error into the evaluations for period t-1 during the backward evaluation phase; this error is likely to increase from stage to stage. Consequently, starting from the initial state, forward evaluation techniques have been proposed and implemented to improve on the optimal policy obtained during backward transformation, in case it had been affected by the error propagation. As these techniques are discussed elsewhere [3], we will not describe them here.

4. Dynamic Programming Models for the Budworm Problem

The huge number of variables in the regional budworm problem (due to the 265 sites and the 100 time periods, i.e. 26500 x number of variables per site per time period) precludes the use of a general non-linear programming technique.

On the other hand, due to the time dimension, this problem has an inherent dynamic structure and thus can be considered as a large dynamic programming problem. The number of state variables would be 265 x number of state variables of a site. Thus, if the forest variables could be aggregated to be represented by two, we would have 265 x 4 = 1060 state variables, which is beyond present possibilities.

The weak interactions between sites during a given time period (see Figure 1)--only E^{t+1} is affected through the dispersion--can be used to reduce the dimensionality of the problem.

4.1 The Method of Successive Approximations

The method of successive approximations (see [2]) can be used to reduce the number of state variables in dynamic programming problems. In the present context, suppose we have a feasible solution for the whole region over the entire time period of interest. The method works by fixing the value of the state variables in all sites except one. A reduced dynamic programming problem, with the state variables of only one site but with the decision variables of all sites, is solved. Of course, the restriction that the state variables are fixed puts some constraints on the decision variables of all other sites. They can be viewed as dependent variables that are adjusted to keep the state variables constant in response to changes in the egg contamination from the site being currently optimized.

The technique is iterative. At each iteration the state variables in N - 1 sites are kept constant and a dynamic program is solved to change the values of the other site. After one cycle, all state variables of all sites have been allowed, in turn, to be modified and the procedure is repeated. Convergence is monotonic [2].

This is essentially a hill-climbing technique, and hence if there are local optima we cannot guarantee global optimality. On the other hand, starting from any feasible solution allows us to find a better one, even if we stop before reaching optimality.

In terms of the present application we must consider that:

a) The method requires a feasible path for the entire time span, i.e. it can be applied to deterministic problems. For the stochastic budworm problem we would have to generate a weather sequence and then work with the resulting deterministic problem. For each possible weather sequence we would have a different deterministic problem;

b) The huge number of variables and data, and the large number of dynamic programs that must be solved iteratively, require a sophisticated dynamic programming and data management computer system. Even then it could turn out that the convergence characteristics are poor.

For these reasons the method of successive approximation was not pursued further.

4.2 The Site Model

Observe in Figure 1 that if contamination effects were not important, i.e. $E_O^t \approx 0$, $E_{IN}^t \approx 0$, or $E_O^t \approx E_{IN}^t$, we could treat each site independently of the others. Solving site problems independently has the following advantages:

a) The reduction in dimensionality makes it much easier to obtain a computational solution.

b) At present, sites are actually managed independently of each other, so that the solving of each site problem would give the right answers to site manager's problems.

In addition, if in an optimal policy there are no outbreaks and the net contamination effect is negligible, the optimal solutions to the site model may provide the optimal solution to the global problem. This possibility is worth examining. The optimal policies obtained on the site optimization can be used on the simulation model to check a posteriori if the assumptions on negligible contamination effects were warranted.

Two alternative formulations were considered which are described in Section 5.

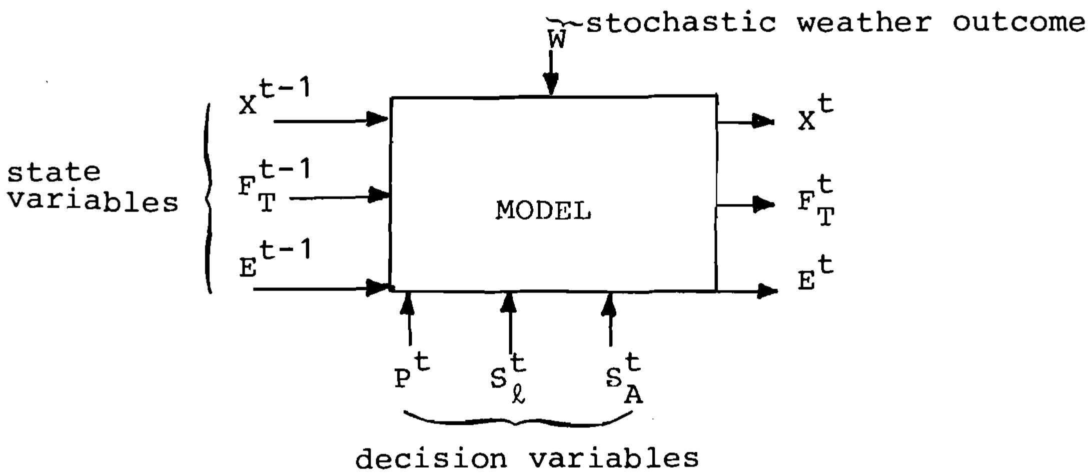
5. Site Model Formulations

5.1 A General Site Model

We will refer to the model described in Section 2 (equations (2-1) through (2-21), with the additional relation

$$E_{IN}^t = E_O^t \qquad (5-1)$$

as the general site model. Thus we can represent one time period by



Recall that $X^t = (X_1^t, X_2^t, \ldots, X_N^t)$, where N is the number of age classes in the forest. Thus the number of state variables is N + 2. By assuming that the area covered by balsam fir is constant over time on any given site, we have

$$\sum_{i=1}^{N} X_i^t = K \quad , \tag{5-2}$$

and

$$X_1^t = K - \sum_{i=2}^{N} X_i^t \quad , \tag{5-3}$$

where X_1^t is a dependent variable, and we reduce the number of state variables to N + 1. Since the computational complexity increases exponentially with the number of state variables it is desirable to aggregate the forest variables to obtain a number of age classes as small as possible.

The ecologists at IIASA felt that the principal features of the forest response could be captured by a three-age group model (see [6]) where

Age group 1: 0 - 9 year old trees

Age group 2: 10 - 29 year old trees

Age group 3: 30 and over.

Thus a dynamic programming problem with four continuous state variables must be solved.

Furthermore, at a workshop in January with representatives from the Canadian Forest Service, it was established that only trees of age group three can be logged, so that $P^t = P_3^t$ is a scalar. So, for this case we have

$$S^t = (X_2^t, X_3^t, F_T^t, E^t) \quad , \tag{5-4}$$

$$D^t = (P_3^t, S_A^t, S_\ell^t) \quad , \tag{5-5}$$

and

$$S^t = F(S^{t-1}, D^t, W) \tag{5-6}$$

is given by the model (equations (2-1)-(2-21) together with (5-1) and N = 3.

5.1-1 Objective Function

Consider the dynamic programming recursion relation

$$V_{t-1}(S^{t-1}) = \max_{D^t} \{C(S^{t-1}, D^t) + \lambda V_t(S^t)\} \quad . \tag{5-7}$$

In order to carry out the dynamic programming optimization we need the final value function $V_T(S_T)$ for the final period and the function $C(S^{t-1}, D^t)$ for the benefits (or costs) obtained in period t.

At the January workshop (see [6]), the following functions were selected:

a) Final Value Function: Since discounting is used and the final time period is far in the future ($\sim$100 years), it was felt that the impact of the final value function on present-day policies would not be too great. Thus for simplicity, a final value function reflecting the lumber value at that moment was selected, i.e.

$$V_T(S_T) = a_1 X_1^T + a_2 X_2^T + a_3 X_3^T [1. - g_2(F_T^T)] \tag{5-8}$$

where

a_i = value in [$/acre] of one acre of trees of age group i and

$g_2(F_T)$ = the function defined in relation (2-6).

b) Benefit Function: It was decided that the benefits consist of revenues from logging and recreational benefits, and that the costs are these incurred for spraying. In [$/acre] these are the following:

b.1 Logging benefits

Benefits of 100 [$/acre] were assumed for trees in age group 3, i.e.

$$LB = 100 \cdot P_3^t \text{ [\$/acre]} \quad . \tag{5-10}$$

b.2 Wildlife/recreation benefits

$$= 58 \cdot \min\left(\frac{X_1^t}{K}, .15\right) \cdot \min\left(\frac{X_2^t}{K}, .30\right) \cdot \frac{X_3^t}{K} \text{ [\$/acres]} \tag{5-9}$$

where K is the total area in the site covered by balsam fir.

b.3 Spraying larvae

There is a fixed cost of .20 [$/acre] for spraying, plus a variable cost of .078 [$/ounce]. Hence, if the dosage is S_ℓ^t ounces/acre,

$$C_\ell(S_\ell^t) = \begin{cases} 0 \ , & \text{if } S_\ell^t = 0 \\ .20 + .078 \cdot S_\ell^t \ , & S_\ell^t > 0 \ . \end{cases} \tag{5-11}$$

b.4 Spraying Adults

$$C_A(S_A^t) = \begin{cases} 0 \ , & \text{if no spraying} \\ .30 \ , & \text{[\$/acre] otherwise} \ . \end{cases} \tag{5-12}$$

From this the following expression is obtained for the benefit function

$$C(S^{t-1}, D^t) = 100 \cdot P_3^t + 58 \cdot \min\left(\frac{X_1^t}{K}, .15\right) \cdot \min\left(\frac{X_2^t}{K}, .30\right) \cdot \frac{X_3^t}{K} - C_\ell(S_\ell^t) - C_A(S_A^t) \text{ [\$/acre]} \quad . \tag{5-13}$$

5.1-2 Other Considerations

The site model in 5.1, together with the objective function developed in 5.1-1, determines a dynamic program. The dimensionality has been reduced to four continuous state variables through aggregation of the forest model. Ecologists are interested in the behavior of the system over a long period of time, in this case at least 100 years. Because of the complexity of the model, a successful implementation requires an efficient dynamic programming package which can deal effectively with continuous state variables. This implies having some sort of polynomial approximation or interpolation scheme over a grid. Flexibility in the choice of the grid points is important. In addition, error propagation estimates and forward evaluation techniques are required. Writing such a code is a major undertaking and it was not attempted here, though negotiations were started to obtain one from commercial sources.[2]

In the meantime an alternative approach suggested by G.B. Dantzig (see [5]) was pursued, which lead to a considerably simplified site dynamic program.

5.2 A Simplified Site Dynamic Program

As mentioned before, when the state variables are continuous it is necessary to discretize them by imposing a set of grid points upon them. In the backward evaluation the payoff function is evaluated only for these grid points, and some sort of approximation is used for values between grid points. The payoff function can be approximated by a polynomial, and the values on the grid points can be used to evaluate the constants of the polynomial. Depending on the functional form of the polynomial the effort to evaluate it will be greater or smaller. Hence it would be nice to find a functional form that i) can reasonably be expected, and ii) is easy

---

[2] As of this writing such a code, DYGAM, is available in IIASA (see [4]).

to evaluate. G.B. Dantzig (see [5]) suggested the following assumption to obtain a functional form having these nice properties.

<u>Assumption</u>: The value of the forest is the sum of the value of its parts:

$$V(X^t,E^t,I^t) = \sum_{i=1}^{N} X_i^t \hat{V}_i(E^t,I^t) \quad , \tag{5-14}$$

where $V_i(E^t,I^t)$ is the value of one acre of i-year-old trees when the egg density is E^t and the foliage index is I^t. The foliage index is defined as

$$I = 3.8 - F_T \tag{5-15}$$

so that a value of 0 is associated with no damage, and a value of 3.8 with complete destruction of the foliage. Without loss of generality we assume

$$\hat{V}_i(E^t,I^t) = a_i f_i(E^t,I^t) \quad , \tag{5-16}$$

where a_i is the value of one acre of i-year-old trees in the absence of any budworm contamination, and $f_i(E^t,I^t)$ is the reduction in value when in state (E^t,I^t). Thus

$$f_i(0,0) = 1 \; , \qquad \forall\, i \quad , \tag{5-17}$$

and we assume $f_i(E^t,I^t)$ to be non-increasing in E^t and I^t, i.e. the value decreases as the level of contamination increases. By substituting (5-16) in (5-14), we have

$$V(X^t,E^t,I^t) = \sum_{i=1}^{N} X_i^t a_i f_i(E^t,I^t) \quad . \tag{5-18}$$

Implicit in this treatment are the assumptions that prices and discount rates are constant over time so that all value functions are time-invariant. Furthermore, recreational benefits having cross-terms as in equation (5-9) are inconsistent with (5-14) and hence are dropped. Essentially in the simplified approach we are maximizing the long-range value of the forest to the lumber industry. This is not as serious a drawback as it appears at first, because all users considered, i.e. lumber industry, ecologists, recreational interests, etc., are concerned with a healthy and green forest, though for different purposes. Thus, when optimizing from the point of view of one of them, we are likely to obtain a policy that will preserve a healthy forest, of value to all of them.

5.2-1 Evaluation of the Terms in the Payoff Function

Using (5-18) we can rewrite the recurrence relation (3-4) as

$$\sum_{i=1}^{N} X_i^{t-1} a_i f_i (E^{t-1}, I^{t-1})$$

$$= \max_{D^t} \left\{ C(S^{t-1}, D^t) + \lambda E \left(\sum_{i=1}^{N} X_i^t a_i f_i (E^t, I^t) \right) \right\} \qquad (5-19)$$

where $X^t = (X_1^t, \ldots, X_N^t), E^t, I^t$ are obtained from X^{t-1}, E^{t-1}, I^{t-1} and D^t through the model. Now for some state with $X_j^{t-1} = 0 \quad \forall_j \neq i$, (5-19) reduces considerably because an i-year-old tree will, in the following year, be i+1 years old if it survives, or 1 year old if it dies or is cut and then replaced; i.e.

$$X_{i+1}^t = (1 - \Theta_i - d_i(I^{t-1}))X_i^{t-1}$$

$$X_1^t = (\Theta_i + d_i(I^{t-1})) X_i^{t-1}$$

$$X_j^t = 0 \quad j = 2, \ldots, N \quad , \quad j \neq i+1 \quad ,$$

where Θ_i is the fraction of the i-year-old trees logged and $d_i(I^{t-1})$ is the fraction that dies because of budworm damage. The benefit function also simplifies to

$$C(S^{t-1}, D^t) = -C_\ell(S_\ell^t)X_i^{t-1} - C_A(S_A^t)X_i^{t-1} + C_i \ \Theta \ X_i^{t-1} \quad ,$$

where $C_\ell(S_\ell^t)$ and $C_A(S_A^t)$ are given by (5-11) and (5-12), respectively, and the C_i's are the benefits of logging one acre of i-year-old trees. By substituting these quantities in (5-19), and dividing by $X_i^{t-1} > 0$ we obtain

$$a_i f_i(E^{t-1},I^{t-1}) = \max_{D^t} \Big\{ - C_\ell(S_\ell^t) + C_A(S_A^t) + C_i\Theta_i$$
$$+ \lambda[(1 - \Theta_i - d_i(I^{t-1}))a_{i+1}\; E\; f_{i+1}(E^t,I^t)$$
$$+ (\Theta_i + d_i(I^{t-1}))a_1\; E\; f_1(E^t,I^t)]\Big\} \quad . \quad (5\text{-}20)$$

Observe from (5-20) that if the terms

$$\hat{V}_j(E,I) = a_j f_j(E,I)$$

are known for j = 1 and j = i + 1, it is possible to recursively calculate them for all 1 < j < i + 1. Notice that this reduces a problem with N state variables to N one-stage problems with two state variables.

5.2-2 Evaluation of the a_i Terms

In the absence of budworm contamination (i.e. E = 0, I = 0) there is no need for spraying and no budworm mortality, i.e. $d_i(0) = 0$ for all i. Also by (5-17), $f_i(0,0) = 1$, and (5-20) reduces to

$$a_i = \max(\lambda a_{i+1}, C_i + a_0) \qquad (5\text{-}22)$$

with $a_0 = \lambda a_1$. Define

$$V_i = \lambda^i C_i + \lambda^{2i} C_i + \lambda^{3i} C_i \ldots = \sum_{j=1}^{\infty} (\lambda^i)^j C_i \quad ,$$

i.e.

$$V_i = \frac{\lambda^i}{1-\lambda^i} C_i \quad . \qquad (5\text{-}23)$$

Then V_i is the present value of all future incomes from the forest if the stationary policy "let it grow i years and then cut it" is used indefinitely. Choose i* such that

$$V_{i*} = \max_{i=1,\ldots,N} V_i \quad . \qquad (5\text{-}24)$$

It can be verified easily that the myopic policy "let the trees grow if i < i* and cut them if i $\geq$ i*" is optimal for (5-22). That is, by letting

$$a_0 = V_{i*} \quad , \qquad (5\text{-}25)$$

we have

$$a_i = C_i + a_0 \quad , \quad i = i^*,\ldots,N \quad ,$$

$$a_i = \lambda a_{i+1} = \lambda^{(i^*-i)} a_{i^*} \quad , \quad i=1,\ldots,i^* - 1 \quad . \qquad (5\text{-}26)$$

We will call i* the optimal cutting age.

5.2-3 Evaluation of the f_i Functions

The assumption that for one-year-old trees $f_i(E,I) = 1$ for all E and I considerably simplifies the evaluation of the f_i functions through the recursion (5-20). This assumption is based on the observation that larvae do not survive on young trees and there is no budworm mortality for them (see Jones [7]) which, together with the effects of discounting, make it reasonable to expect. This assumption can be verified a posteriori by calculating values for $f_1(E,I)$ at the end of optimization; and if necessary the procedure can be repeated using these values. Making use of this assumption, (5-20) simplifies to

$$f_i(E^{t-1},I^{t-1}) = \frac{1}{a_i} \max_{D^t} \Big\{ -C_\ell(S_\ell^t) - C_A(S_A^t) + C_i\Theta$$

$$+ (\Theta_i + d_i(I^{t-1}))a_0 + \lambda(1 - \Theta_i$$

$$- d_i(I^{t-1}))a_{i+1} E\, f_{i+1}(E^t,I^t)\Big\}, \qquad (5\text{-}27)$$

with $a_0 = \lambda a_1$.

We further assume that in the sequence of events in one time period, tree death occurs at the beginning of the period, before they can be logged, and that logging has the effect of removing parts of the forest without affecting the densities and relationships in the remaining parts. Thus for any value of S_ℓ^t or S_A^t, (5-27) is linear in Θ_i, and it would be optimum (for a given age class) to cut either all ($\Theta_i = 1 - d_i(I^{t-1})$) or nothing ($\Theta_i = 0$). Hence letting g denote the maximum in the right hand side of (5-27) when $\Theta_i = 0$, and h when $\Theta_i = 1 - d_i(I^{t-1})$, we have

$$a_i f_i(E^{t-1},I^{t-1}) = \max\,(g,h) \qquad (5\text{-}28)$$

with

$$g = \max_{S_A^t, S_\ell^t} \Big\{ -C_\ell(S_\ell^t) - C_A(S_A^t) + a_0 d_i(I^{t-1}) \qquad (5\text{-}29)$$

$$+ \lambda(1 - d_i(I^{t-1}))a_{i+1} E\, f_{i+1}(E^t, I^t) \Big\} \quad ,$$

and

$$h = \max_{S_A^t, S_\ell^t} \Big\{ -C_\ell(S_\ell^t) - C_A(S_A^t) + C_i(1 - d_i(I^{t-1})) + a_0 \Big\} \quad . \qquad (5\text{-}30)$$

Clearly the maximum of (5-30) is achieved when there is no spraying, so that

$$h = C_i(1 - d_i(I^{t-1})) + a_0 \quad . \qquad (5\text{-}31)$$

Furthermore, since $f_{i+1}(E^t, I^t) \leq 1$, from (5-29) then

$$g \leq \max_{S_A^t, S_\ell^t} \Big\{ -C_\ell(S_\ell^t) - C_A(S_A^t) + d_i(I^{t-1})a_0 + \lambda(1 - d_i(I^{t-1}))a_{i+1} \Big\}$$

$$\leq d_i(I^{t-1})a_0 + \lambda(1 - d_i(I^{t-1}))a_{i+1}$$

$$= \hat{g} \quad . \qquad (5\text{-}32)$$

Thus, whenever $g \leq \hat{g} \leq h$, we have, by (5-28),

$$a_i f_i(E^{t-1}, I^{t-1}) = h = C_i(1 - d_i(I^{t-1})) + a_0 \quad , \qquad (5\text{-}33)$$

and the optimal policy is to log. In particular,

$$h - \hat{g} = C_i(1 - d_i(I^{t-1})) + a_0 - d_i(I^{t-1})a_0$$

$$- \lambda(1 - d_i(I^{t-1}))a_{i+1}$$

$$= (1 - d_i(I^{t-1}))(C_i + a_0 - \lambda a_{i+1}) \geq 0 \quad ,$$

for all $i \geq i^*$ by (5-22) and (5-26). Hence for all $i \geq i^*$ the optimal policy is to log always, and the f_i function is computed from (5-33). For $i < i^*$ we can now use Equation (5-27) to recursively evaluate the f_i functions.

5.2-4 Computational Aspects

As previously mentioned, in order to proceed with the numerical evaluations of the f_i expressions using (5-27), it is necessary to discretize the continuous variables E and I, and to evaluate the functions at this discrete set of points using some sort of approximation for values between grid points. For the numerical implementation on the computer of the approach outlined in 5.2-2 and 5.2-3, a by-linear interpolation scheme was used. That is, if we let I_i, $i = 1,\ldots,N_I$, and E_j, $j = 1,\ldots,N_E$, be the grid points for I and E respectively (both increasing with index), and if for arbitrary E and I (within their range) we define the indices ℓ and m by $E_\ell \leq E \leq E_{\ell+1}$, $I_m \leq I \leq I_{m+1}$,

$$\alpha = \frac{E_{\ell+1} - E}{E_{\ell+1} - E} \quad , \qquad \beta = \frac{I_{m+1} - I}{I_{m+1} - I} \quad , \qquad (5\text{-}34)$$

then $f_i(E,I)$ is approximated by

$$f_i(E,I) = \alpha\beta f_i(E_\ell,I_m) + \alpha(1-\beta)\, f_i(E_\ell,I_{m+1}) + (1-\alpha)\,\beta f_i(E_{\ell+1},I_m) + (1-\alpha)\,(1-\beta) \cdot f_i(E_{\ell+1},I_{m+1}) \quad . \qquad (5\text{-}35)$$

For the logging benefits the following expression is used:

$$C_i = L_i * (P_R - C_{ki} - C_T) \quad , \qquad (5\text{-}36)$$

where

- L_i [cunits/acre) is the yield in cubic units of lumber of one acre of i-year-old trees. Figure 3 gives L_i as a function of i;
- P_R [\$/cunit] is the price at the mill of one cubic unit of lumber;
- C_{ki} [\$/cunit] is the cost to harvest one cunit of lumber from an i-year-old stand. C_{ki} is given in Figure 4 as a function of i;

C_T [$/cunit] is an average transportation cost from the stand to the mill. A value of $C_T = 7$ was used for calculations.

5.2-5 Interpretation as a Markov Chain

As was pointed out by Dantzig (see [5]), the simplified Site Dynamic Program can be reinterpreted as a Markov chain stochastic control problem. For this we consider a unit of the forest (one acre for instance, or one tree) to be in a state characterized by a triple (i,j,k), where

$i = 1,\ldots,N$ - age of the unit;

$j = 1,\ldots,N_E$ - indices of the discrete set of egg densities E_j;

$k = 1,\ldots,N_k$ - indices of the discrete set of foliage indices I_k.

The unit of the forest is considered homogeneous, i.e. having age i, with egg density E_j and foliage index I_k. In order to obtain the transition probabilities the fraction $(1 - d_i^t - p_i^t)$ in (2-7) is reinterpreted as the probability that an i-year-old unit in period t will survive to be i + 1 years old in period t + 1, and $(d_i^t + p_i^t)$ is reinterpreted accordingly as the probability that it will be replaced by a one-year-old tree in period t + 1.

Thus, starting from a state $(i^{t-1},j^{t-1},k^{t-1})$ at time t - 1, the unit survives with probability $p_s = (1 - d_i^t - p_i^t)$ to a state $i^t = i^{t-1} + 1$, and through the model, values E_n^t and I^t are obtained for each stochastic weather outcome n (n = 1, 2 or 3 with probability p_n). (From (2-1) to (2-21), notice that $I^t(F_T^t)$ does not depend on the weather.) By means of (5-34), indices ℓ_n and m are determined and the α_n and β are interpreted as the probabilities

$$\alpha_n = P\{j^t = \ell_n \mid (i^{t-1},j^{t-1},k^{t-1}),\ i^t = i^{t-1} + 1,\ n\}\ ,$$

$$1 - \alpha_n = P\{j^t = \ell_n + 1 \mid (i^{t-1},j^{t-1},k^{t-1}),\ i^t = i^{t-1} + 1,\ n\}$$

$$\beta = P\{k^t = m \mid (i^{t-1}, j^{t-1}, k^{t-1}),\ i^t = i^{t-1} + 1,\ n\} \quad ,$$

$$1 - \beta = P\{k^t = m + 1 \mid (i^{t-1}, j^{t-1}, k^{t-1}),$$
$$i^t = i^{t-1} + 1,\ n\} \quad .$$

Thus, if the unit survives it can end up in states

$(i + 1, \ell_n, m)$,	with probability	$p_s p_n \alpha_n \beta$
$(i + 1, \ell_n + 1, m)$,	with probability	$p_s p_n (1 - \alpha_n)\ \beta$
$(i + 1, \ell_n, m + 1)$,	with probability	$p_s p_n \alpha_n (1 - \beta)$
$(i + 1, \ell_n + 1,\ m + 1)$,	with probability	$p_s p_n (1 - \alpha_n)\ (1 - \beta)$

for $n = 1,2,3$. If it is replaced some assumption has to be made as to the probability distribution of the egg density and foliage index of the replacement.

Though p_s, ℓ_n, m, α_n and β depend on the decisions taken, once the optimal decisions have been obtained, they are fixed and we can calculate a matrix of transition probabilities for the optimal policies. With this transition probability matrix it is then possible to calculate several probabilities of interest such as:

a) probability of a unit being harvested;
b) probability of a unit being harvested at optimal age, given that it is harvested;
c) probability of a unit dying as a consequence of budworm damage.

6. Results of the Optimization

A FORTRAN program was written to solve the Simplified Site Dynamic Programming Problem. Several runs were made using different values for the price P_R of a cubic unit (cunit) of wood and for the interest rate ρ. Table 1 describes the runs and contains some of the quantities computed in the optimization.

For each run the results of the optimization give the optimal cutting age and the optimal policies, as well as

Table 1. Description and results of runs.

Run	Parameters used		Results of Optimization			
No.	P_R	ρ	i^*	P(H)	$P(H_i^* \mid H)$	P(M)
1	65.	5	50	1.	1.	0.
2	60	5	53	1.	0.98	0.
3	57.5	5	56	1.	0.99	0.
4	55.	5	59	1.	0.99	0.
5	52.5	5	60	1.	1.	0.
6	50.	5	60	1.	1.	0.
7	47.5	5	66	1.	0.97	0.
8	45.	5	70	0.97	0.99	0.03
9	42.5	5	74	0.51	0.96	0.49
10	40.	5	75	0.14	1.	0.86
11	45.	1	75	1.	1.	0.
12	45.	8	65	0.50	0.98	0.50
13	45.	10	62	0.33	0.99	0.67
14	55.	1	70	1.	1.	0.
15	55.	8	51	1.	1.	0.
16	55.	10	50	1.	1.	0.
17	45.	2	75	1.	1.	0.
18	45.	3	73	1.	0.96	0.
19	45.	4	70	1.	1.	0.
20	45.	6	69	0.80	0.97	0.20

P(H) = probability of a unit being harvested;

$P(H_i^* | H)$ = conditional probability of a unit being harvested at optimal age (given that it is harvested);

P(M) = probability of death of a unit as a consequence of budworm damage.

some probabilities. As the output of each run is quite large, it is not feasible to include it all here; only those parts that serve an illustrative purpose, or are of interest to our subsequent analysis, will be given here.

The optimal policies can be presented conveniently in the form of policy tables (see Figures 5 and 6). For every run there is one policy table for each age group which gives the optimal policy for an area covered by trees of age i as a function of the foliage level F_T, and the logarithm (base 10) of the egg density E [egg/acres]. Thus for an area covered with 60-year-old trees (see Figure 5), according to the values of F_T and log E, the policy table tells us: a) None (i.e. do nothing this year), b) Log, or c) Spray. In this last case the computer table also specifies the dosage, and whether larvae or adults or both should be sprayed. Only a few of these tables are included in this report for illustrative purposes, but they are all available to a decision maker in the computer outputs.

Optimal cutting ages have been plotted in Figure 7 as a function of the value of a cubic unit of wood and, in Figure 8, as a function of the interest rate. Similarly, the probabilities P(H) of a unit being harvested have been plotted in Figure 9 as a function of the price and, in Figure 10, as a function of the interest rate.

In Figures 11 and 12 the probabilities of survival of a unit to age i, $P(S_i)$, and of spraying, given that it has survived to age i, $P(S_p|S_i)$, are plotted as a function of the age i for two different runs. Figure 11 corresponds to run 19 with P_R = 45 ($/cunit) and ρ = 4(%), which was chosen as an example where there is no tree mortality due to budworm damage; Figure 12 corresponds to run 12 with P_R = 45 [$/cunit] and ρ = 8(%), chosen as an example where there is budworm-caused tree mortality.

All the probabilities in Figures 9 through 12 are based on the arbitrary assumption that there is an egg density E_d = 5.68 (eggs/10 sq.ft.) at age 20.

7. Observations

Recall that the motivation for the site model was the observation that if in an optimal policy there are no outbreaks, then the net contamination effect is negligible; and that by solving the site problem with $E^t_{IN} = E^t_0$, the optimal solutions to the site models provide the optimal solution to the global problem. It was mentioned that this assumption could be tested a posteriori through use of the optimal site policies in the overall simulation model. Though not a substitute for validation through simulation, analysis of Figures 9 and 10 provides additional evidence that the above assumption holds for a wide range of values of the economic parameters. Observe in Figure 9 that for an interest rate of 5% and prices over 45 [$/cunit] there is no budworm-caused tree mortality (i.e. no outbreaks), and that for a value of 45 [$/cunit] this mortality is small. Moreover, it follows from Figure 10 that for an interest rate of 4% and a price of 45 ($/cunit] there is no budworm-caused tree mortality. Under all these economic conditions, where it is optimal at the site level to save the trees, we can expect that the above assumption on negligible contamination effects will hold. On the other hand, when the economic conditions are such that the optimal site policies allow for outbreaks to occur, the assumption will not hold; and in the site optimization we are not accounting for the cost of damage to other sites through contamination. If we could account at the site level for these costs, policies would be shifted into the direction of saving trees. By these heuristics, the policies for $P_R = 45$ [%/cunit], $\rho = 4(\%)$ probably would still be good for $\rho = 5(\%)$ when these contamination costs are assessed.

Figures 11 and 12 provide an explanation for the occurence of budworm-caused tree mortality at higher interest rates for $P_R = 45$[$/cunit]. Thus, from Figure 11 for $\rho = 4(\%)$, observe that spraying starts at age 21 and no outbreak is allowed to occur. On the other hand, observe from Figure 12 for $\rho = 8(\%)$ that no spraying is done before age 30, and that there is a positive probability of .50 of budworm-caused tree mortality due to outbreaks between ages 22 and 36. That is, the higher interest rate discounts the future benefits to such an extent that it is not worthwhile to spray between ages 20 and 30 in order to prevent a possible outbreak; if the right weather sequences occur, the unit will die before reaching age 34. However, if the weather sequence is such that the unit has survived to age 30, then the potential benefits from logging are not so distant and it becomes worthwhile to save the unit through spraying. Notice that no tree mortality is allowed to occur after age 35.

Some preliminary runs done in Vancouver, using the optimal policies from the Simplified Site Dynamic Program in the regional simulation program, seem to justify our simplifying assumptions. Considerable improvement over management policies currently in use was obtained, (see Figure 13) which gives the fraction of "bad" recreational sites as a function of time for both policies over the next hundred years as predicted by the simulation model. (For a definition of "bad" recreational site see Bell [1].)

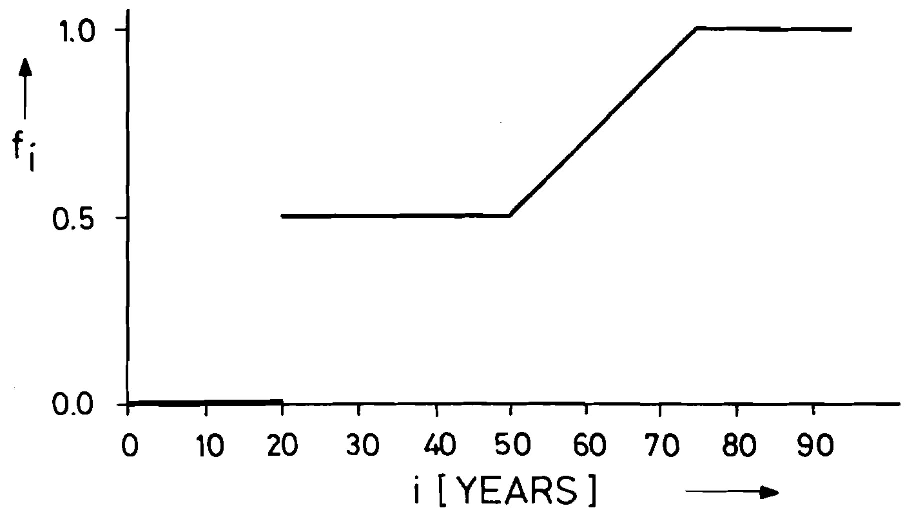


FIGURE 1. AGE SPECIFIC MORTALITY FACTOR (f_i) VS. TREE AGE (i)

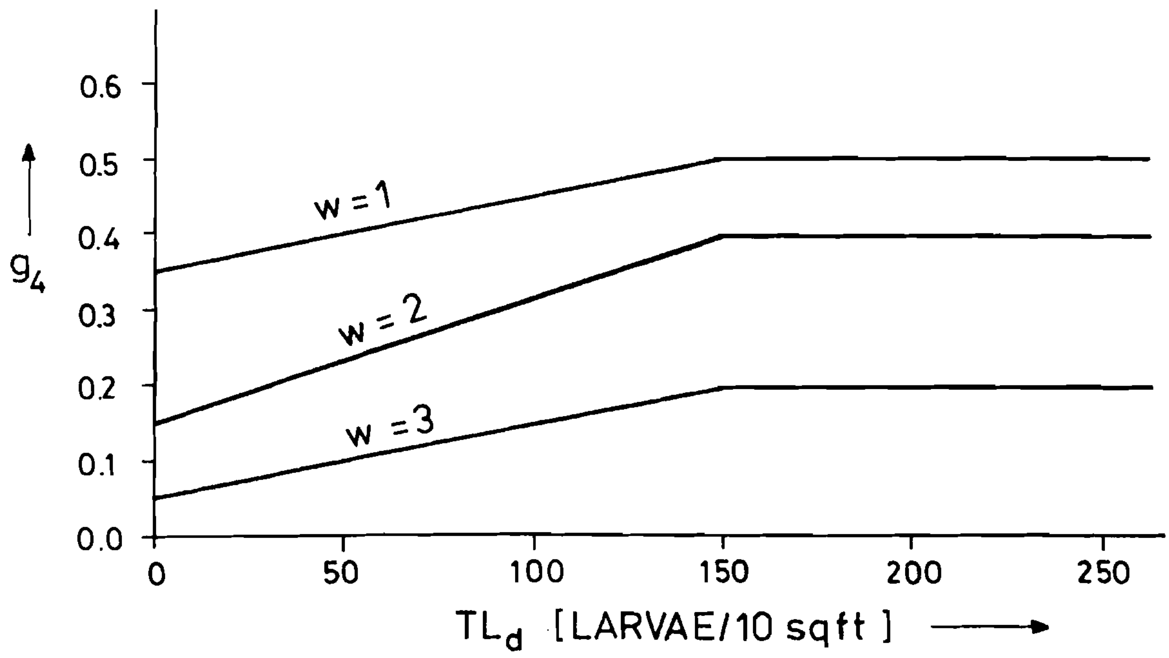


FIGURE 2. SURVIVAL FACTOR FOR LARVAE (g_4) VS. THIRD INSTAR DENSITY

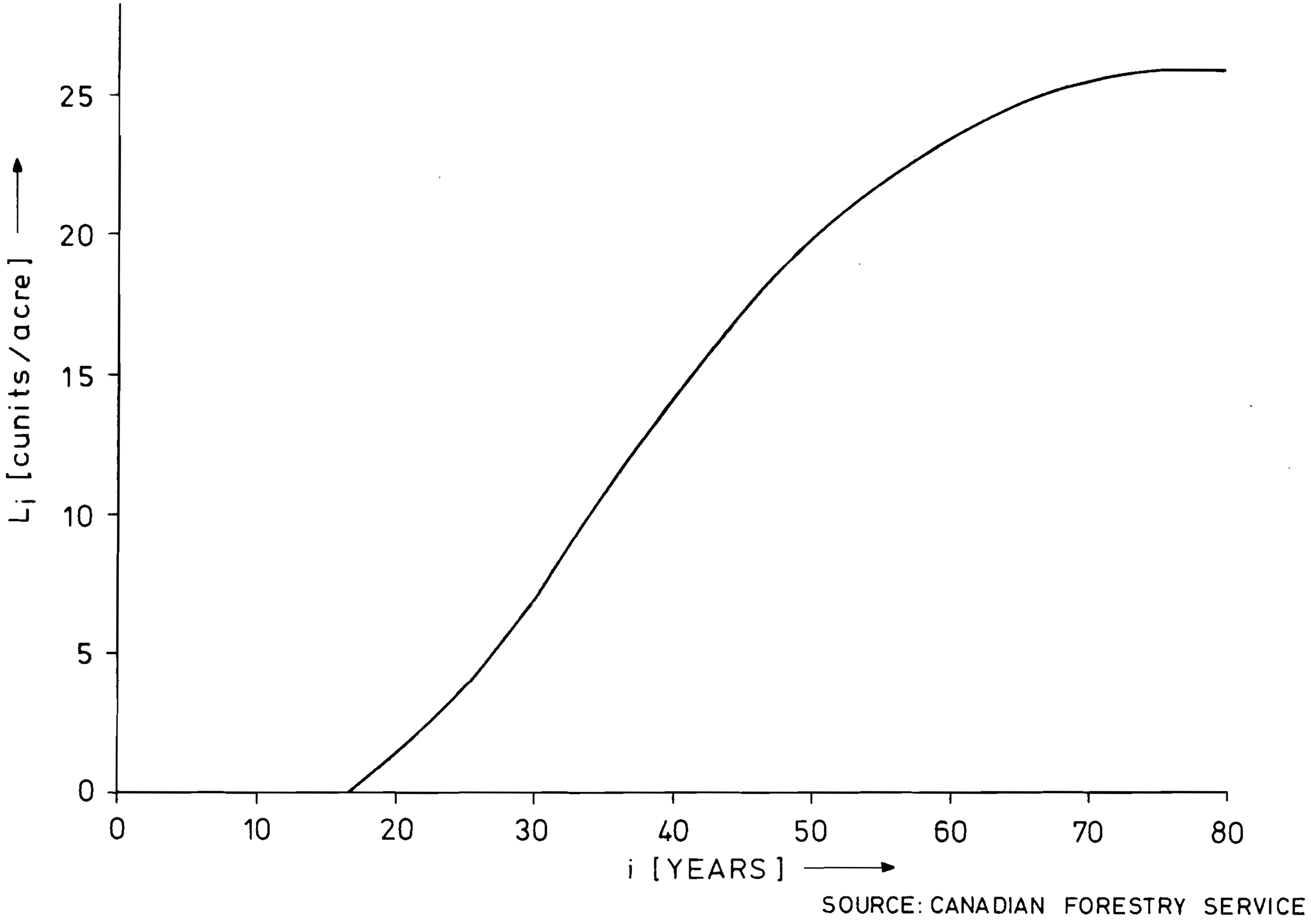


FIGURE 3. MERCHANTABLE PULP WOOD (L_i) VS. AGE OF STAND (i)

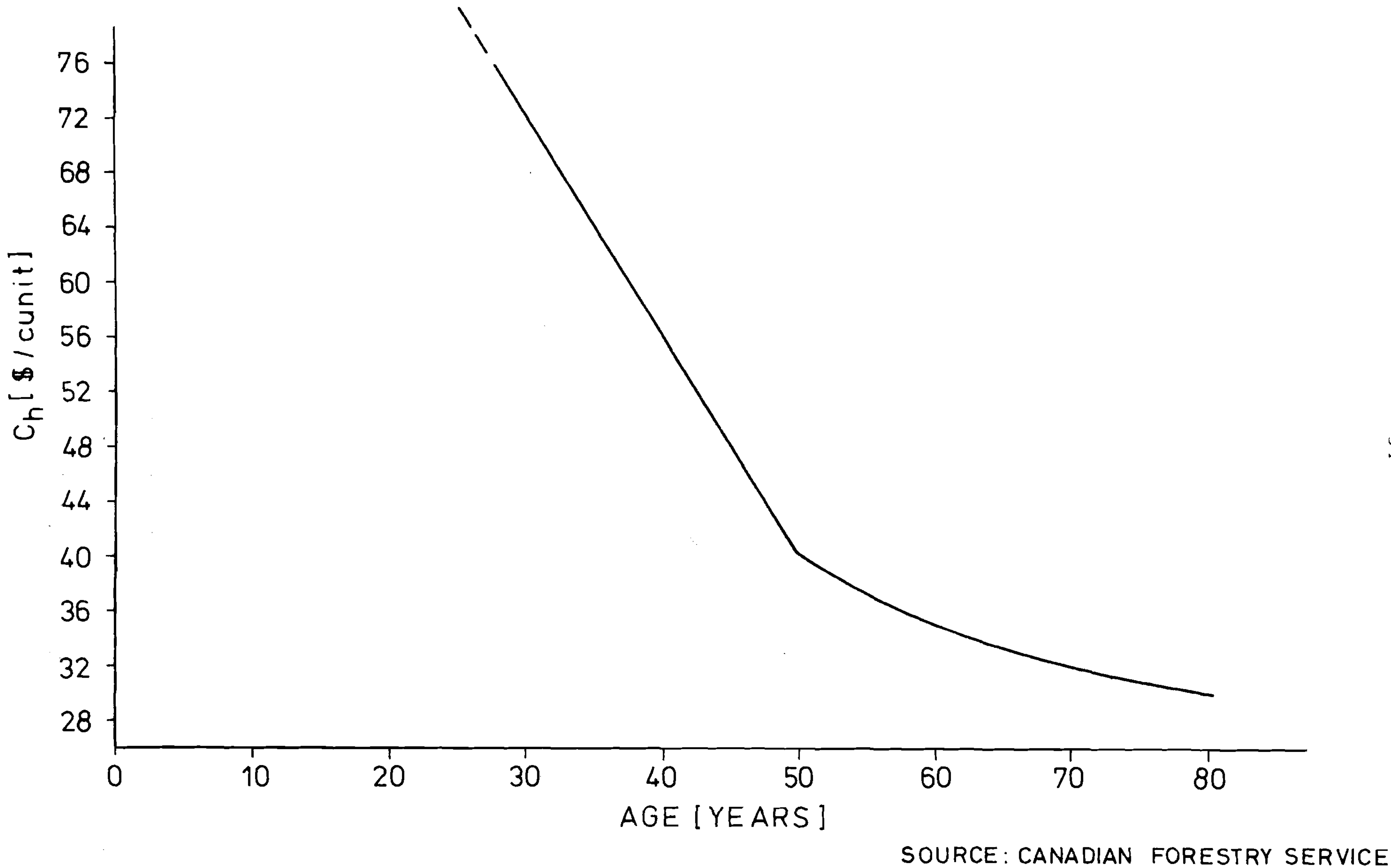


FIGURE 4. COST OF HARVESTING A CUBIC UNIT OF WOOD (C_h) VS. AGE OF STAND

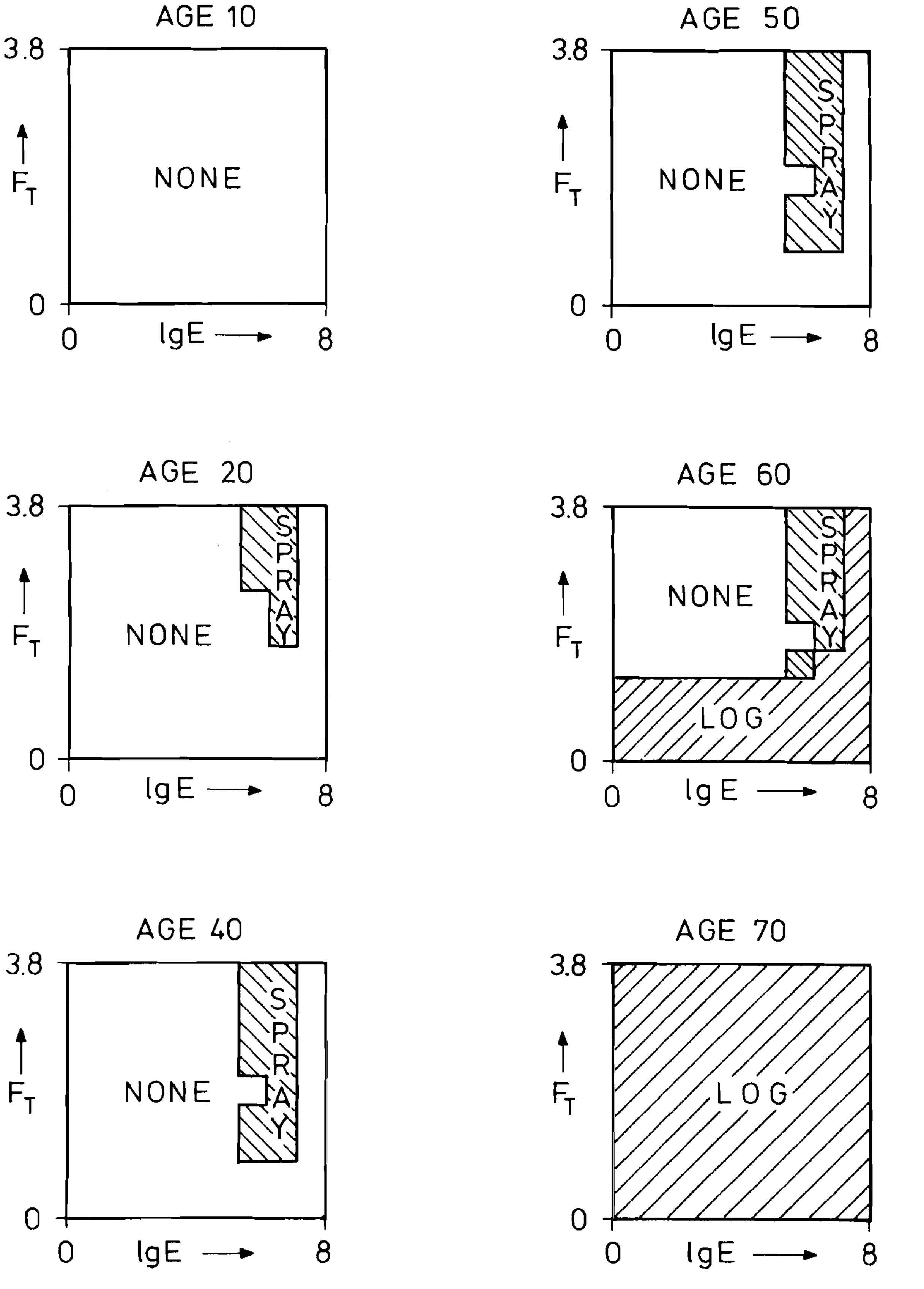


FIGURE 5. POLICY TABLES FOR REPRESENTATIVE AGES
(PRICE = 45 [$ / cunit], ρ = 4[%])

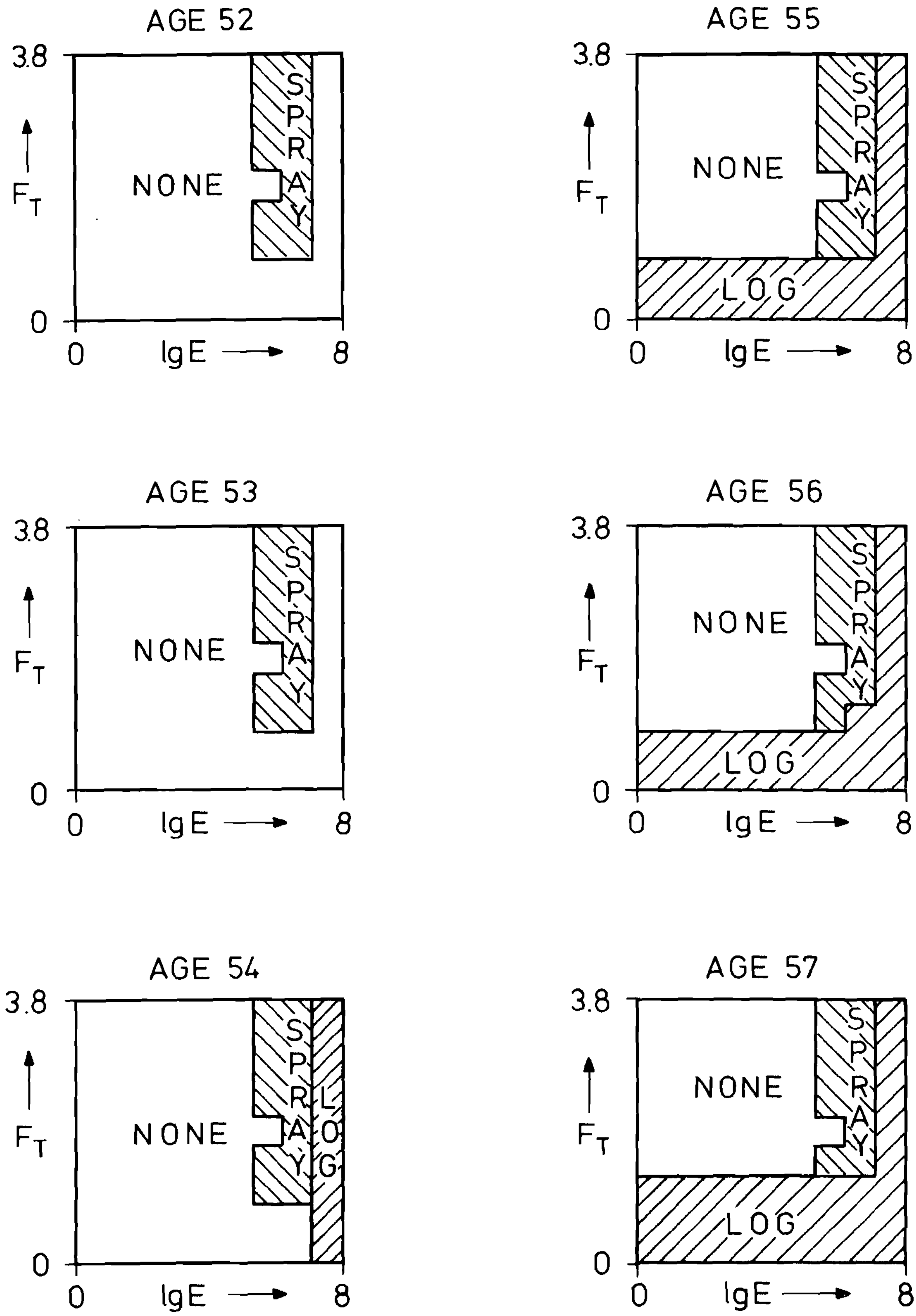


FIGURE 6. POLICY TABLES FOR AGES 52-57
(PRICE = 45 [$ / cunit], ρ = 4 [%])

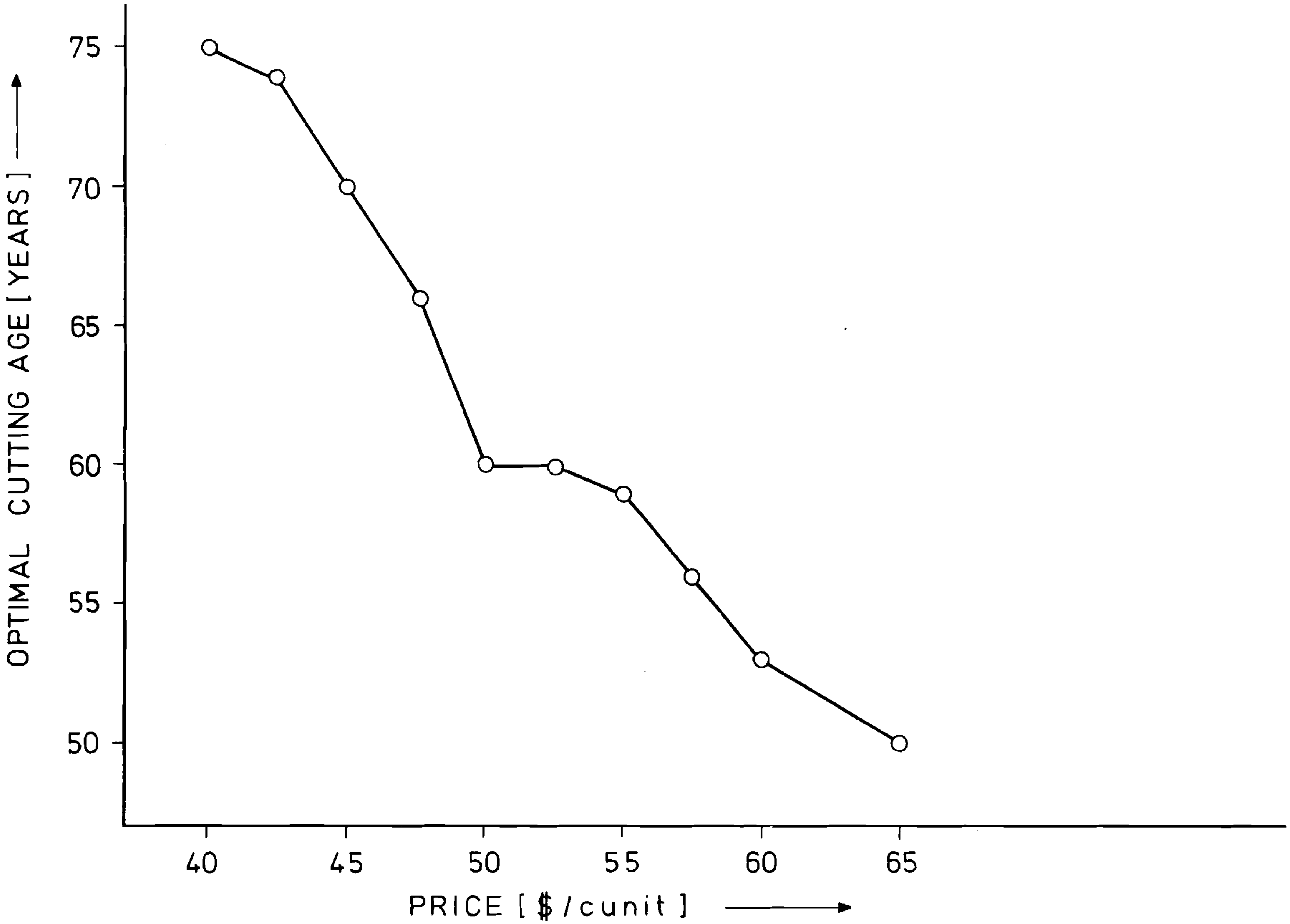


FIGURE 7. OPTIMAL CUTTING AGE VS. PRICE OF CUBIC UNIT OF WOOD FOR $\rho = 5\%$

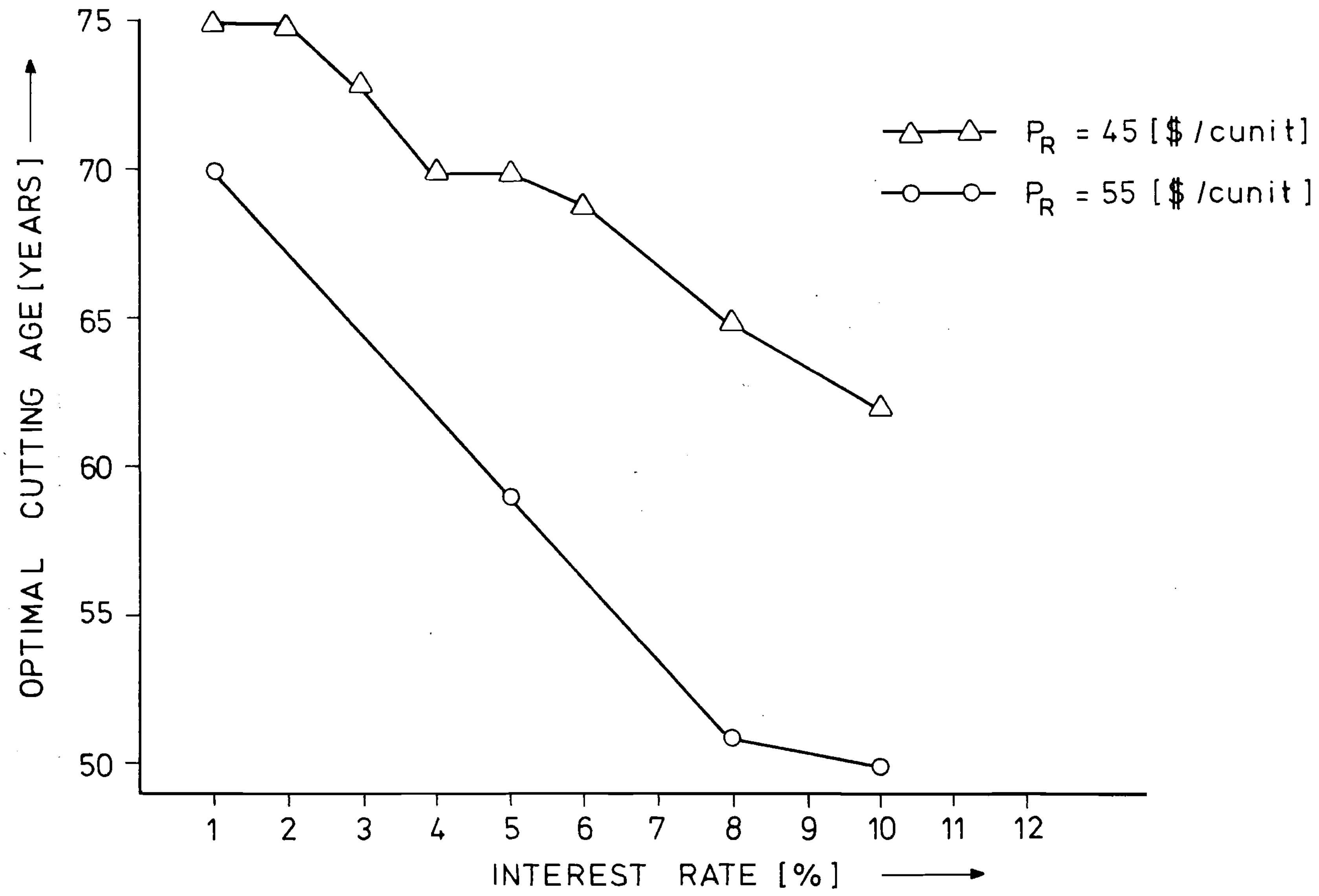


FIGURE 8. OPTIMAL CUTTING AGE VS. INTEREST RATE

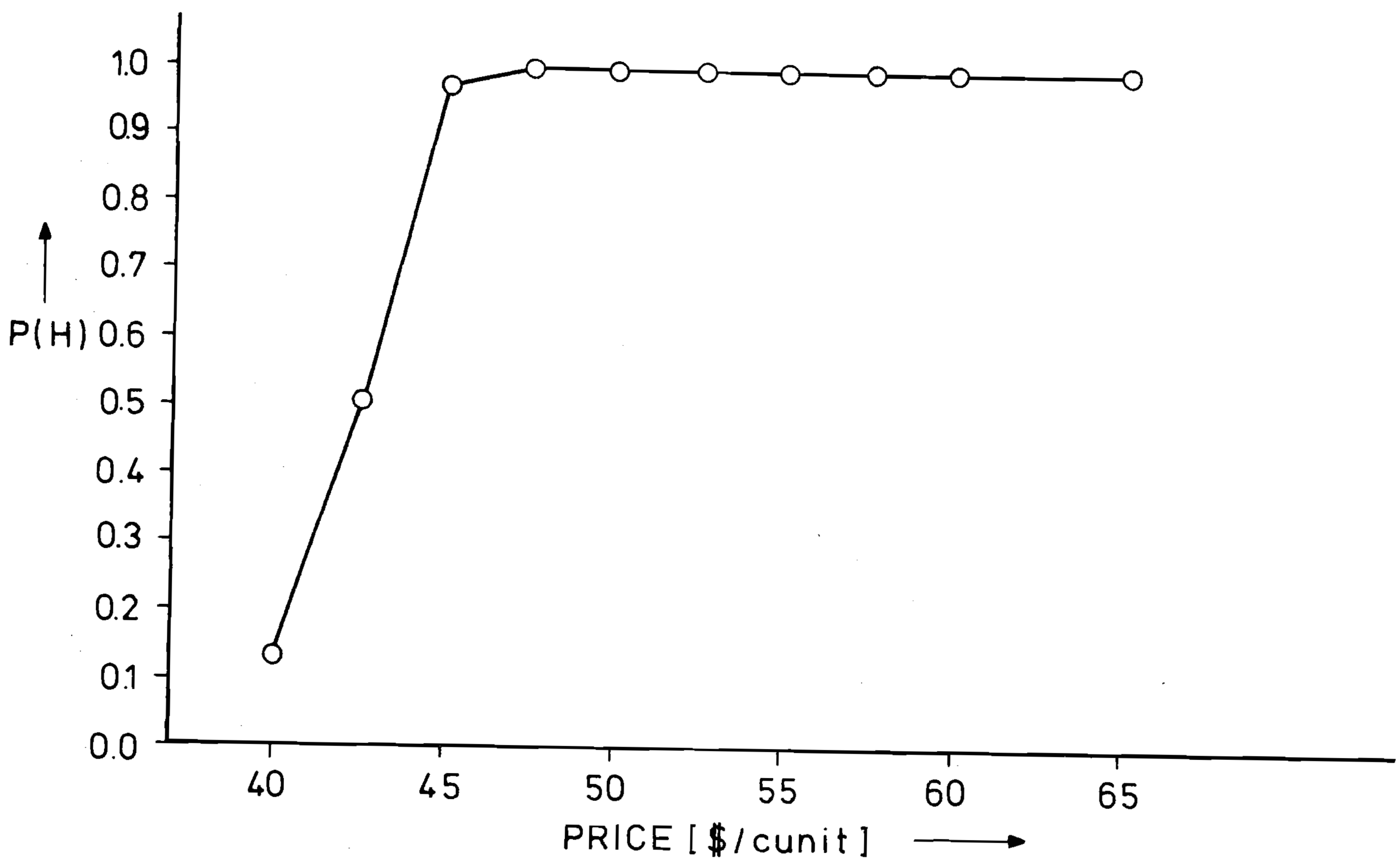


FIGURE 9. PROBABILITY OF HARVESTING A UNIT, P (H), VS. PRICE FOR ρ = 5 %

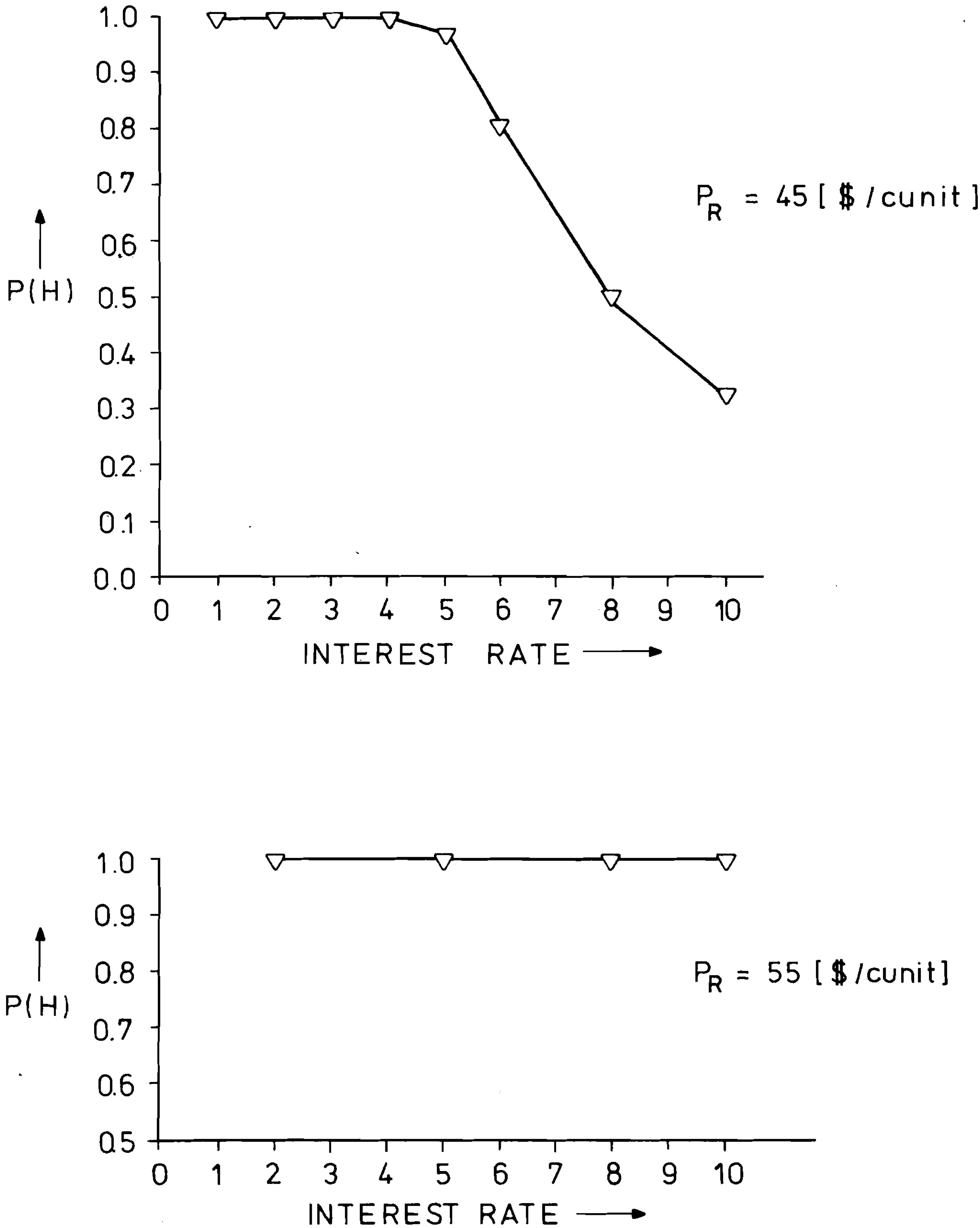


FIGURE 10. PROBABILITY OF HARVESTING A UNIT, P(H), VS. INTEREST RATE

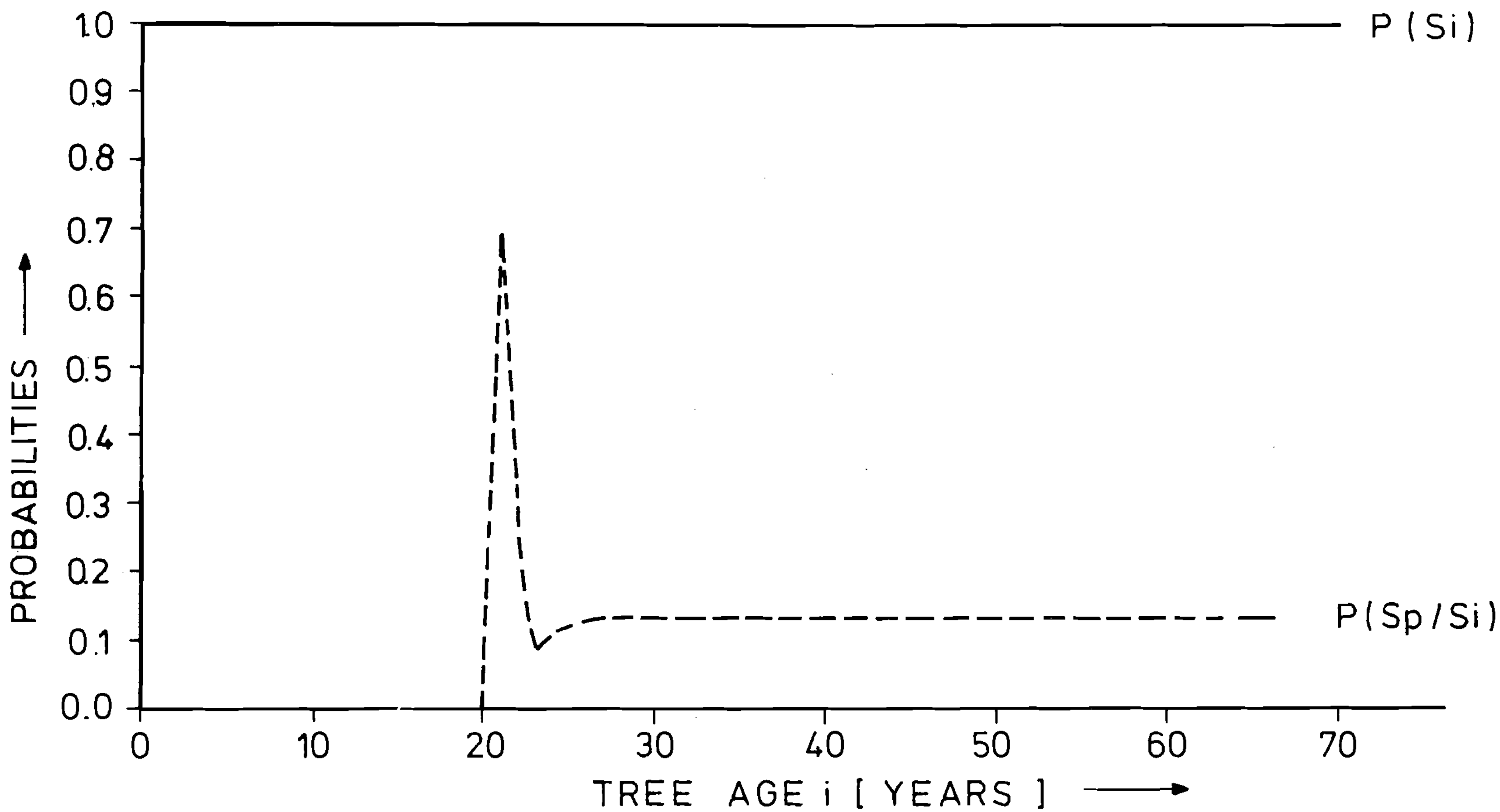


FIGURE 11. PROBABILITIES OF SURVIVING TO AGE i, P(Si), AND OF SPRAYING AT AGE i, P(Sp / Si), VS. TREE AGE (PRICE = 45 [\$/cunit] , ρ = 4 [%])

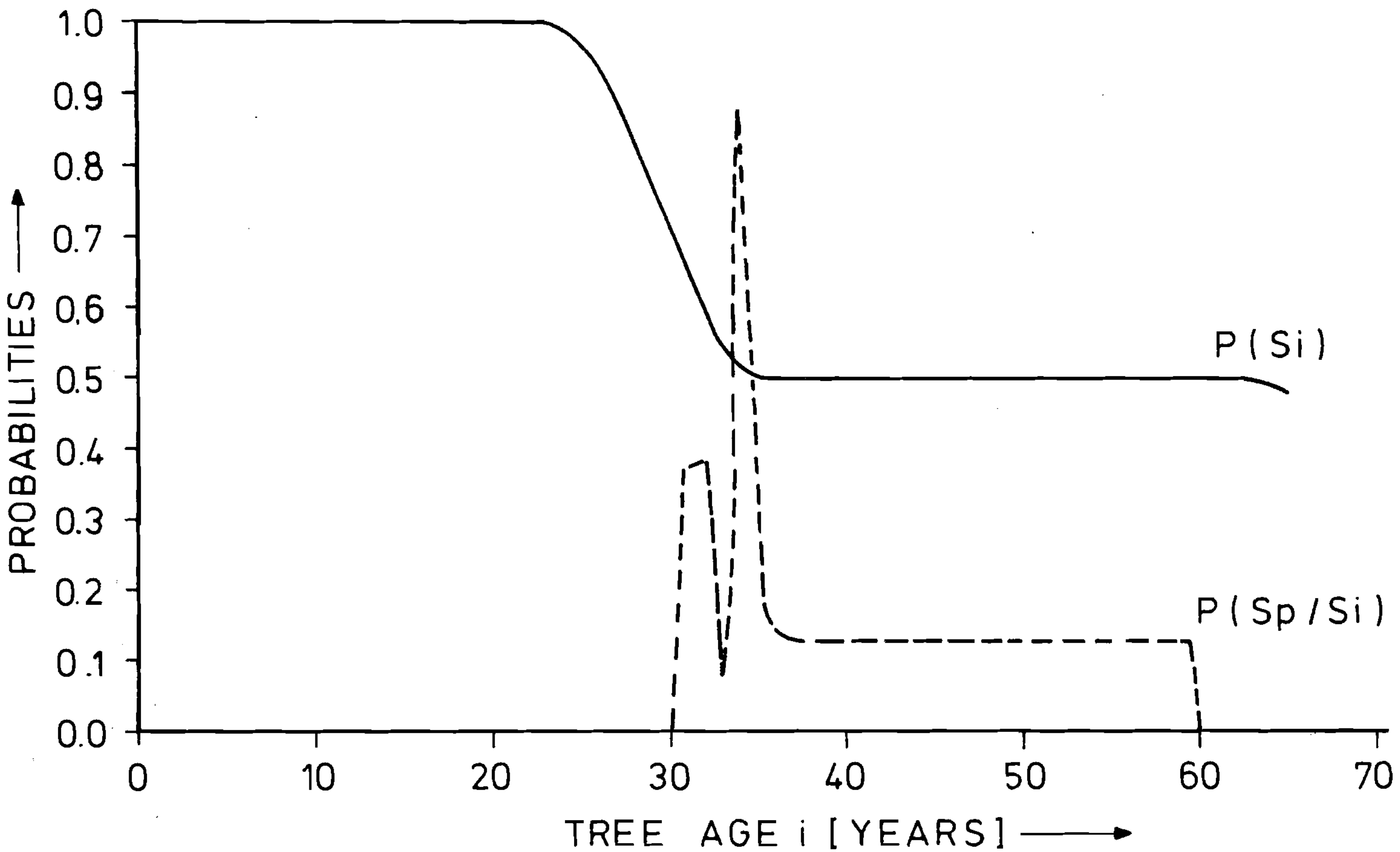


FIGURE 12. PROBABILITIES OF SURVIVING TO AGE i, P(Si), AND OF SPRAYING AT AGE i, P(Sp / Si) VS. TREE AGE (PRICE = 45 [\$ / cunit], ρ = 8 [%])

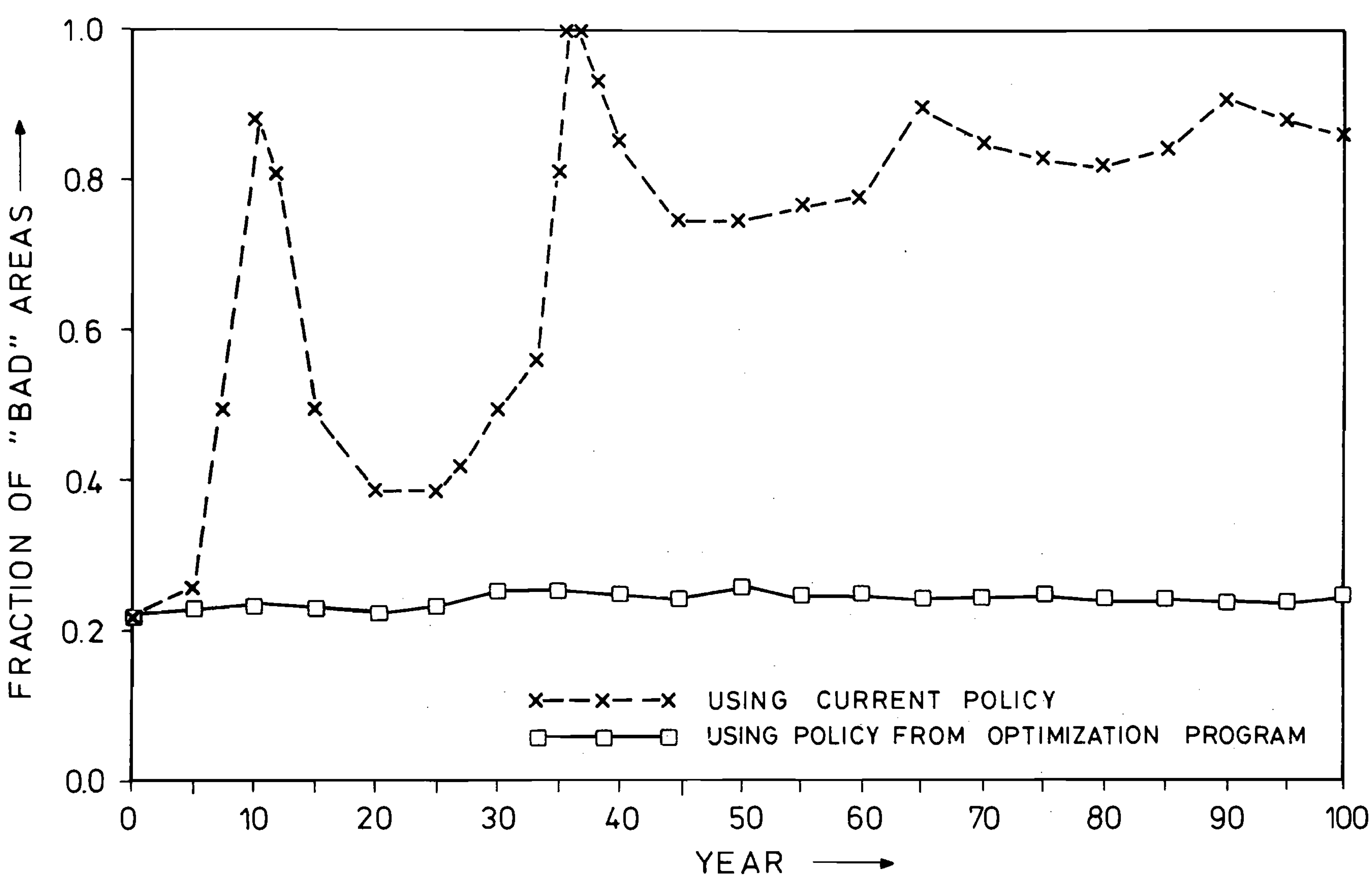


FIGURE 13. FRACTION OF "BAD" RECREATIONAL SITES VS. YEAR OF SIMULATION

References

[1] Bell, D.E. "Defining and Quantifying Goals for Measuring the State of an Ecosystem," IIASA, RR-74-26, Laxenburg, Austria, 1974.

[2] Bellman, R.E. and S.E. Dreyfus. "Applied Dynamic Programming," The Rand Corporation, Santa Monica, California, USA, 1962.

[3] Control Analysis Corporation. "Development of an Algorithm to Solve Multi-Stage Games," Palo Alto, California, USA, 1973.

[4] Control Analysis Corporation. "DYGAM - A Computer System for the Solution of Dynamic Programs," Palo Alto, California, USA, 1974.

[5] Dantzig, G.B. "Determining Optimal Policies for Ecosystems," Technical Report 74-11, Department of Operations Research, Stanford University, Stanford, California, USA, August 1974.

[6] Holling, C.S. "Summary of Budworm Workshop," IIASA, Personal Communication, Laxenburg, Austria, 1974.

[7] Jones, D.D. "Biology of the Budworm Model," IIASA, RM-74-3, Laxenburg, Austria, 1974.

[8] Stander, J.M. "A Simulation Model of the Spruce Budworm and the Forest in New Brunswick," MS, Institute of Resource Ecology, University of British Columbia, Canada, 1973.