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INTERIM REPORT IR-98-092/October

Formulation and spatial aggregation of agricultural production relationships within the Land Use Change (LUC) model.

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Contents

Abstract

The Land Use Change (LUC) project at IIASA in which the Centre for World Food Studies participates, is currently engaged in the development of a case study for China (Fischer et al., 1996). Its focus on land use and land cover change makes it necessary to ensure that geographic detail and statistical information on conditions prevailing within these regions be preserved to the extent possible. This raises two issues that are addressed in this paper. First, while it is not practical to formulate a regional optimization model with a very large number of sub-regions, over two thousand in the case of China, one would like to avoid the loss of information associated with aggregation. For a generalized version of the Mitscherlich-Baule yield function that is commonly used in agronomy the paper describes a consistent aggregation procedure over sub-regions, which leads to simple aggregate functions at regional level, but has the special property that, once the regional model has been solved, all results can be recovered fully at sub-regional level. Secondly, agronomy studies commonly use yield functions in which the per hectare yield of a particular crop depends on inputs per hectare. Unfortunately, in the case of China, as for most countries, input use is not available by crop and only recorded for a particular geographical unit. The paper proposes a more crude formulation whose parameters, however, can be estimated by cross-section on the basis of the available data.

Acknowledgments

The methodology proposed in this paper takes a major practical step towards embedding spatial characteristics into economic analysis.

The Land Use Change (LUC) project at IIASA gratefully acknowledges this important methodological contribution of Professor Michiel Keyzer in developing the welfare approach being implemented for China by the IIASA LUC group.

We would also like to thank the Centre for World Food Studies for devoting precious time and resources to jointly implementing with LUC staff the theoretical concepts proposed in this report.

About the Author

Professor Michiel Keyzer is an economist, professor of mathematical economics and Director of the Centre for World Food Studies of the Vrije Universiteit, Amsterdam. Professor Keyzer's main activities are in research and research co-ordination in the areas of mathematical economics, and economic model building applied to SOW-VU projects.

Professor Keyzer has been involved in an European Union study on reform of the Common Agricultural Policy and in a World Bank study on farm restructuring and land tenure in reforming socialist economies

Professor Keyzer is co-author (with Victor Ginsburgh) of a major book on *The Structure of Applied General Equilibrium Models* published by MIT Press in 1997.

The Centre for World Food Studies (Dutch acronym SOW-VU) is a research institute related to the Department of Economics and Econometrics of the Vrije Universiteit Amsterdam. It was established in 1977 and engages in quantitative analyses to support national and international policy formulation in the areas of food, agriculture and development cooperation.

SOW-VU's research is directed towards the theoretical and empirical assessment of the mechanisms which determine food production, food consumption and nutritional status. Its main activities concern the design and application of regional and national models which put special emphasis on the food and agricultural sector. An analysis of the behaviour and options of socio-economic groups, including their response to price and investment policies and to externally induced changes, can contribute to the evaluation of alternative development strategies.

SOW-VU emphasizes the need to collaborate with local researchers and policy makers and to increase their planning capacity.

SOW-VU's research record consists of a series of staff working papers (for mainly internal use), research memoranda (refereed) and research reports (refereed, prepared through team work).

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Formulation and spatial aggregation of agricultural production relationships within the Land Use Change (LUC) model.

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1. Introduction

The Land Use Change (LUC) project at IIASA is currently engaged, among others, in the development of a model for China (Fischer et al. (1996)). This model maximizes intertemporal social welfare for a national economy consisting of a small number of economic regions. Its focus on landuse and land cover change makes it necessary to ensure that geographic detail on conditions prevailing within these regions be preserved to the maximum. This raises the issue of consistent aggregation over farm models within each region. This paper discusses the estimation of crop (and livestock) production functions at sub-regional level as part of a static revenue maximizing decision by the farmer at the sub-regional level, typically with a large number of sub-regions, to be aggregated to economic regions. Every sub-region distinguishes three consumer groups: crop farmers, livestock farmers and nonagriculturalists who jointly constitute one dynasty and face a common intertemporal budget constraint. We will restrict our attention to the formulation and analysis of a static revenue maximization problem for a crop producing farm in a particular sub-region, using a generalized version of the Mitscherlich-Baule yield function (Section 2) and the possibility of determining the parameters of this model by cross sectional estimation over sub-regions (Ssection 3), and, finally, of aggregating the relationships over sub-regions, so as to serve as technical relationships within the regional model, from which the results at sub-regional level can be recovered, once the optimal solution has been found (Section 4).

2. The static decision problem of a crop producing farm

2.1 The decision problem

Agronomic studies commonly use yield functions in which the per hectare yield of a particular crop depends on inputs per hectare. To estimate such relations statistically it is necessary to avail of data on these variables. However, input use is rarely measured by crop and often only available for a particular geographical or social unit. This makes it necessary to specify a more crude formulation, and for this we propose to use a transformation function. The revenue maximizing problem of a given sub-region is then written:

$$\max_{V \geq 0,\ Y \geq 0} p^y Y - p^v V \tag{1}$$

subject to

$T(Y,-V,-A,x) \leq 0$, where the choice variables are

$$V = (V_1,...,V_K) \quad \text{for inputs,}$$
$$Y = (Y_1,...,Y_C) \quad \text{for outputs,}$$

and the given parameters are $A = (A_1,...,A_S)$ for land quality types, $x = (x_1,...,x_N)$ for natural conditions.

We also define the vectors of purchasing prices for inputs p^v and selling prices for outputs p^y which are of dimension K and C, respectively. For sub-regions that are net purchasers, the purchasing price of goods governs the unit value of the crop on the local (sub-regional) market, and for net sellers, the farm gate price will perform this role. Program (1) does not determine land types, as these follow from intertemporal decisions. Usually the transformation function is taken to be quasiconvex continuous, nondecreasing in (Y, −V, −A) and linear homogeneous in (V,A). However, we will use a slightly more general formulation to accommodate a particular form that is often used by agronomists. First, we impose a separation between inputs and outputs:

$$T(Y,-V,-A,x) = Q(Y) - F(V,A;x) \tag{2}$$

where Q is linear homogeneous, convex nondecreasing aggregate output index and F(V,A;x) an input response or production function that is linear homogeneous and nondecreasing in (V,A). We postpone a discussion of the concavity properties of this function until the next section. Since Q(Y) is linear homogeneous, it defines the convex revenue index function $R(p^y)$:

$$R(p^y) = \max_{u \geq 0} \{p^y u \mid Q(u) = 1\} \tag{3}$$

Hence (1) reduces to:

$$\Pi(R,p^y;A,x) = \max_{V \geq 0} R\, F(V,A;x) - p^v V \tag{4}$$

The value function $\Pi(R,p^y;A,x)$ of this problem is the so-called restricted profit function and in applications the optimal input is often obtained as the derivative of this function w.r.t. p^y. However, to keep incorporation within a larger welfare program simple, we will maintain purely primal formulations of the input response function F(V,A;x), for which we still need to specify a functional form.

2.2 Analysis of a generalized Mitscherlich-Baule yield function

We analyze the properties of the *Mitscherlich-Baule yield function* (MB for short), a functional form for yield functions that has gained much popularity in the agronomy literature. For example, Llewelyn and Featherstone (1996) report that MB-yield function performs better for an extensive set of synthetic data on irrigated corn derived from a crop model. Franke et al. (1990) came to similar conclusions on the basis of data from experimental plots.

Even though agronomists usually apply this function in a purely experimental context, they derive recommendations on application of fertilizer and other inputs on the basis of this function. Hence, they at least implicitly assume the function to be usable in a context of constrained optimization. We will show that when there is more than one input the pure MB form exhibits increasing returns to scale, which are not based in empirical findings but only on the choice of functional form. Consequently, the agronomist who bases his recommendation on this function will suggest input applications that exploit returns to scale, and are therefore high by necessity. We will also propose introduction of additional parameters in this functional form which make it possible to measure the returns to scale. The pure *Mitscherlich-Baule* is a yield function for crop j:

$$y_j = \prod_k f(\beta_{kj} + \gamma_{kj} v_{kj}) \bar{y}_j(x) \tag{5}$$

where v_{kj}, y_j, $\bar{y}_j$ the application of input k, the realized yield and the maximum attainable yield, respectively, all measured per hectare. The maximal yield is given and was, say, computed on the basis of a crop model as a function of natural conditions x. The function f(A) is specified as $f(g) = 1 - e^{-g}$; it thus maps nonnegative values of g on the unit interval with f(0) = 0 and f(g) asymptotically approaching unity as g goes to infinity. Since the product multiplies values that lie on the unit interval, y_c will not exceed the value and approach it asymptotically as all inputs go to infinity. We already notice that the product form is purely arithmetic: there are no coefficients to describe the substitutability among inputs. The production function defines output as a function of applied inputs and land. The production function $F_j(V,A;x)$ that follows from (5) is:

$$Q_j = \prod_k f(\beta_{kj} + \gamma_{kj} V_{kj}/A_j)\ N(A_j; \bar{y}_j(x))$$

where V_{kj}, Q_j, N_j are the application of input k, the realized output and the maximal attainable output, respectively on the surface A_j, and $N(A_j; \bar{y}_j) = \bar{y}_j A_j$.

We now analyze a more general production function. First, we include various landtypes A_S, s = 1,...,S, as inputs. This facilitates the parameter estimation when the application of inputs V_k to various land-types is unknown. Secondly, we introduce positive parameters θ_k as exponents of the functions f(·) to measure returns to scale. Then, after dropping the subscript j, the production function becomes:

$$Q = \prod_k f(\beta_k + \gamma_k V_k / H(A))^{\theta_k}\ N(A; \bar{y}(x)) \tag{6}$$

where $A = (A_1, ..., A_S)$ is the vector of areas A_S, $\bar{y} = (\bar{y}_1, \ldots, \bar{y}_S)$ is the vector of maximal agronomic yields on landtype s (with $\bar{y}_S$ the maximum yield of a reference crop), $\beta_{k,}$ γ_k and θ_k are given positive parameters (to be estimated from empirical data);[1] H(A) is a concave, linear homogeneous area index (say a Constant Elasticity of Substitution (CES) function), and $N(A;\bar{y})$ is an index of maximal production capacity, that is concave in A while we write N(A) for short; (H(A) and $N(A;\bar{y})$ possibly also contain parameters to be estimated. For given land inputs A and natural conditions x, we consider the problem of optimal input application:

$$\max_{Q \geq 0, V \geq 0} \; Q - pV$$

subject to (7)

$$Q = F(V, A; x)$$

where the positive K-dimensional vector p is the given ratio of input prices to the price $R(p^y)$ of aggregate output. The following proposition further characterizes this problem.

Proposition: Let the production function F(V,A;x) be of the generalized MB-form (6) with returns to scale parameters, then

(i) *F(V,A;x) is strict log-concave in V,*
(ii) *Input demand in program (7) is nondecreasing in Q*
(iii) *if K = 1 or $\Sigma_k\theta_k \neq 1$ program (7) has a single stationary point, but for K > 1 or $\Sigma_k\theta_k > 1$ stationary points can be multiple, though not exceeding 2K,*
(iv) *the optimum is reached at the stationary point with the largest output value F.*

proof:

(i) For given A, let $q(V) = \Sigma_k\theta \ln(f(\beta_k+\gamma_k V_k/H(A))$, whose derivative w.r.t V is: $\partial q/\partial V_h =$ $\gamma^k\theta_k \exp(-(\beta_h+\gamma_h V_h/H(A)))/f(V_k)$ while the second derivative is: $\partial^2 q/\partial V_k\partial V_h = -\theta_k\gamma^k/H(A)$ $\exp(-(\beta_h+\gamma_h V_h/H(A))) < 0$. Hence the logarithm of F is strictly concave in V (and F is strict log concave in V). Note that since q(V) is concave in V, the cost function $C(p, q_0) = \min_{V\geq 0} pV \mid q(V) \geq q_0)$ is convex in q_0; solving program (7) amounts to $\max_{q\geq 0} [\exp(q) - C(p,q)]$.

(ii) We rewrite (6) as $F(V,A;x) = \exp(\Sigma_k\theta_k \ln f(g_k (V_k)))\, N(A; \bar{y}(x))$;
hence $\partial F/\partial V_k = F(\partial \ln f(g_k)/\partial g_k)\theta_k\gamma_k/H(A)$ and the first-order condition of profit maximization reads, for $V_k > 0$:

$$\zeta_k = F \exp(-g_k)/(1-\exp(-g_k)),$$
$$= F\, (1/f(g_k) - 1)$$

[1] These parameters can be made dependent on x. This also applies to the area index H(A).

hence

$$– \ln [\zeta_k/F + 1] = \ln f(g_k),$$

for $\zeta_k = p_k H(A)/(\gamma_k \theta_k)$, which shows that input demand in nondecreasing in F.

(iii) As the cost function is strict log-concave, the input combination associated with a given value of F is unique. Therefore, we can at every value F (and for fixed ζ) determine a unique set L(F) such that V_k is positive for each $k \in L(F)$. By (ii) the use of positive inputs can only increase with F; therefore, the set L(F) will not decrease with F. After multiplication by θ_k, summation over k yields, in the stationary point:

$$M^*(F) = \Sigma_k\theta_k \ln [\max(f(\beta_k), \zeta_k/F) + 1] + \ln F/N = 0 \quad (8)$$

After differentiation:

$$dM = \Sigma_{k\in L(F)} \theta_k\, d\ln [\zeta_k/F + 1] + d\ln F$$
$$dM = \{\Sigma_{k\in L(F)} \theta_k/[\zeta_k /F + 1] (-\zeta_k/F^2)\} + 1/F)\, dF;$$

hence

$$dM/dF = \{\Sigma_{k\in L(F)} –\theta_k/[F/\zeta_k + 1] + 1\}/F \quad (9)$$

and $d^2M/dF^2 > 0$

Note that if $\Sigma_{k\in L(F)} \theta_k < 1$ (e.g. if there is a single positive input), the derivative dM/dF will be positive for all $F > 0$. However, if $\Sigma_{k\in L(F)} \theta_k > 1$, it will be negative at small F, and then turn positive. Since d^2M/dF^2 is positive there will then exist two stationary points for every index set L(F). It follows from (8) that M*(F) will be continuous in F. The effect of a shift in F is only a downward jump in the derivative. This may, for $\Sigma_k\theta_k > 1$, lead to a large number of intersections, that will, however not exceed 2K. The value F* at which M*(F) is zero and F – pV is largest, will be the global optimum.

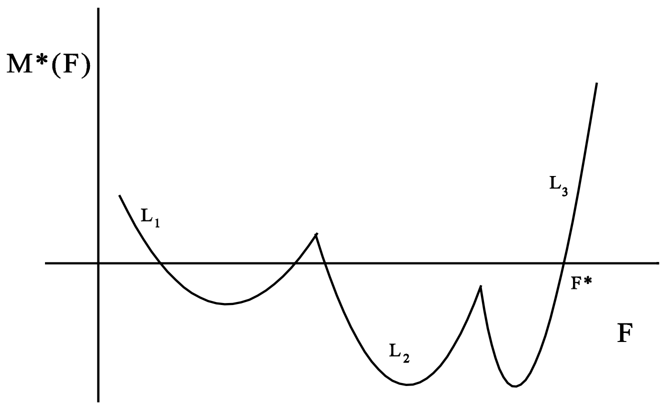

Figure 1. The function M*(F)

Figure 1 maps out the relationship between M*(F) and F; L_1, L_2 and L_3 denote the convex segments of the curve, associated to three consecutive values of the index set L(F). Computation can proceed via a line search that increases F, starting at a small positive value and until a right-hand intersection is found where M*(F) = 0. The calculation proceeds until L(F) contains all inputs and the inter-section with the largest value for profits is chosen. This analysis of the Mitscherlich-Baule function shows that it has a part with increasing returns to scale, due to the multiplication of f-functions. We conclude that since in the pure MB-function the (unit) exponents of this multiplication are not the result of estimation, recommendations about input applications that are based on this function have an a priori bias towards high input intensities.

2.3 Embedding within the intertemporal optimization model

Note that since A is given, concavity in (V,A) was of no concern in (7). Yet the static model will eventually be embedded within a dynamic one where A becomes a variable. For $\Sigma_k\theta_k < 1$, linear homogeneity in (V,A) ensures that we can derive a unit value function $W(R,p^v)$ that maximizes the value per unit of N(A) (a concave function) and work with it in the land allocation problem. For $\Sigma_k\theta_k > 1$ there may be discontinuities as prices change. In this case embedding within a wider optimization becomes problematic. We also mention that the MB-form is only one out of several alternatives. It is for example possible to use other functions, such as:

$$Q(Y) = g(Z(V)/H(A))\ N(A) \tag{10}$$

where Z(V) is a Cobb Douglas, CES or other constant returns production function. The same holds for the index function H(A). This form has the advantage of allowing for a more flexible representation of substitution effects among inputs k and it avoids the convexity problems. Since the number of options is so large, a definite choice will have to be made on the basis of data availability and observed patterns, using a nested procedure as described in Llewelyn and Featherstone (op. cit.).

3. Cross section estimation

Let us briefly discuss the procedure for estimating the parameters of functions (4) and (6), or (10). The variable $\bar{y}$ (and x, if introduced) will often be autocorrelated spatially, and this has to be accounted for via spatial regression (see e.g.. Keyzer (1996)). The cross section is to be done across counties r, possibly with some time index to address the issue of productivity shifts under land-cover change. Hence, we distinguish the following index sets:

C crop rotations (land use types)
R sub-regions (counties)

Associated to C and R are the subscripts c and r. The focus will lie on estimation of parameters α of the output index Q(Y) and β, γ and θ of the input function F(V,A;x). It might be necessary to specify the parameters of the aggregations functions H and N on a priori grounds, to ensure consistent aggregation, as will be discussed in the next section.

It follows from the linear homogeneity of the functions Q and F that the optimal ratios Y_{cr}/N_r and V_{kr}/H_r will be constant across r. Therefore, if we assume that this conforms perfectly with the data, we may write:

$$Y_{cr} = \nu_c N_r \tag{11a}$$
$$V_{kr} = \phi_k H_r \tag{11b}$$

where N_r and H_r are evaluations of the functions $N(A_r)$ and $H(A_r)$, and ν_c and ϕ_k. These values can be obtained as follows. After substitution of the production function (6), the original problem (1) becomes:

$$\max_{V \geq 0,\ Y \geq 0} p^y Y - p^v V$$

subject to

$$Q(Y) = \prod_k f(\beta_k + \gamma_k V_k / H(A))^{\theta_k} N(A;\bar{y}(x))$$

This problem can be partitioned in two parts: the determination of product mix (as in (3)) and the determination of output scale and input demand (as in (7)):

$$R(p^y) = \max_{u \geq 0} \{p^y u \mid Q(u) = 1\} \tag{12a}$$

The second problem can be reformulated into:

$$\max_{Q \geq 0, V \geq 0} R(p^y)Q - p^v V$$

subject to (12b)

$$Q/N(A;\bar{y}) = \prod_k f(\beta_k + \gamma_k V_k / H(A))^{\theta_k}$$

This enables us to write:

$$\max_{q \geq 0, \phi \geq 0} q - p\phi$$

subject to (12c)

$$q = \prod_k f(\beta_k + \gamma_k \phi_k)^{\theta_k}$$

after the substitutions $q = Q/N(A;\bar{y})$, $\phi_k = V_k/H(A)$ and $p_k = p_k^v H(A)/(R(p^y)N(A;\bar{y}))$. For $\nu_c = u_c q$, the optimal values for outputs and inputs will be determined as in (11a,b).

Estimation of the relationships (12a) can proceed in the standard way, with a regression on:

$$[\partial Q(Y_r;\alpha)/ \partial Y_{cr})]/[\partial Q(Y_r;\alpha)/ \partial Y_{1r})] = p^y_{cr}/p^y_{1r} \quad (13a)$$

or, alternatively, if the function Q is a (convex) CES-function on:

$$[\partial R(p^y;\alpha)/ \partial p^y_{cr})]/[\partial R(p^y;\alpha)/ \partial p^y_{1r})] = Y_{cr}/Y_{1r} \quad (13b)$$

For (12b) there is a choice. The estimation can either be applied to the first-order conditions of the cost minimization problem, in which case the problem of the endogeneity of Q has to be addressed, or on (12b) directly. The parameters of the functions H(A) and $N(A;\bar{y})$ can be estimated simultaneously or must be given a priori.

4. Aggregation to an economic region

The LUC-welfare model will be specified at the level of the economic region. We investigate the possibilities for consistent aggregation via a representative producer construction. After summation over r and optimal ν_c and ϕ_k:

$$Y_c = \nu_c\Sigma_r N_r \quad (14a)$$
$$V_k = \phi_k\Sigma_r H_r \quad (14b)$$

Now assume that the original distribution of land uses over sub-regions is kept fixed within a particular time period

$$A_{sr} = A_s\alpha_{sr} \quad (15)$$

where A_s is the regional total over sub-regions. Also assume that the functions H_r and N_r are both of the Cobb Douglas form, then, we can, by homogeneity of these functions, define aggregate functions N° and H° in terms of the aggregates A_s, of the form:

$$N°(A_1, ..., A_S) = \kappa N(A_1, ..., A_S) \quad (16a)$$
$$H°(A_1, ..., A_S) = \eta H(A_1, ..., A_S) \quad (16b)$$

these functions describe a so-called representative (rather than average) index. Next we assume that econometric estimation shows a multiplicative "fixed effect" that creates heterogeneity. This can be accommodated by multiplying the functions $N(A_r)$ and $H(A_r)$ by a correction term:

$$Y_{cr} = \nu_c\varepsilon_{cr}N(A_r) \quad (17a)$$
$$V_{kr} = \phi_k\zeta_{kr}H(A_r) \quad (17b)$$

This enables us to perform consistent aggregation, while at the same time accounting for variations within the sample. Alternatively, we can seek to generalize the specification beyond the Cobb Douglas form. One easy way is to restrict the model described so far to some fraction of land and output and specify exogenously some "committed" land utilization, input demand and production. Furthermore, we can decide to compute the Cobb Douglas coefficients of the aggregate function N° and H°, not through direct summation over Cobb Douglas forms but through first-order approximation in the logarithms of the analytical form $\Sigma_r N(A_r)$, $\Sigma_r H(A_r)$ with respect to aggregate utilization A_s using (14) at observed values, but this will only yield an approximate correspondence that may become imprecise when the values of A_s change.

For any of these options, once we have computed a solution of the national welfare program, the consistent formulation makes it possible to recover all the necessary production relationships at the sub-regional level and map these out within a geographical information system. We conclude with two remarks.

The first is that the Cobb Douglas formulation can only be operational if all land types A_s are positive. In practice this will not be the case. Furthermore, the Cobb Douglas formulation leads to rigid patterns of elasticities. Both difficulties can be alleviated by generating A_{rs} as a transformed quantity:

$$A_r = T\,L_r \tag{18}$$

where L has the same dimension as A and denotes the original (true) land types and T is a semipositive matrix, thus ensuring positivity of A_r. Calculations at sub-regional level can proceed in terms of the transformed land types with the transformation from L to A occurring in the regional model. Since land balances need only be maintained in terms of L, the transformed values A_{rs} can, for example, be generated as $A_{rs} = L_{rs} + \Sigma_s\, l L_{rs}$, where l is some positive fraction. Such a form ensures that nonnegative values of L_{rs} can be recovered after applying the sharing rule (15) to the regional values. In addition, the larger the value of the parameter l, the smaller the effect of transformation of land types on production. Obviously, the formulation (18) can be generalized in various ways.

Finally, the land types A_{rs} will change in a lagged manner as a result of land conversion activities. As the sharing relationship (15) can be viewed as a production function of a special kind, it is possible to treat it as the outcome of an intertemporal revenue maximization that only specifies exogenously the distribution of land transformation activities over sub-regions.

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