International Institute for Applied Systems Analysis • A-2361 Laxenburg • Austria
Tel: +43 2236 807 • Fax: +43 2236 71313 • E-mail: info@iiasa.ac.at • Web: www.iiasa.ac.at

INTERIM REPORT IR-98-079/September

Generators of Seismic Events and Losses: Scenario-based Insurance Optimization

Boris V. Digas (digas@imm.uran.ru)

Approved by
Joanne Linnerooth-Bayer (bayer@iiasa.ac.at)
Co-Leader, *Risk, Modeling, and Policy Project*

Contents

Abstract

A principal difficulty in insuring catastrophic risks is the insufficiency of disaggregated historical data on the catastrophe losses. The lack of data and complex spatial and dynamic interdependencies between catastrophic events imply the necessity to base the insurance of property against natural hazards on catastrophe modeling. The purpose of this research is to develop a working tool for increasing capacity of insurance networks, which insure property against earthquakes. This includes a generator of earthquake scenarios and losses, which is based on seismic maps and geophysical formulas. The outputs of the generator serve as inputs for a procedure, which finds an optimal structure of a network of insurance companies. A "guaranteed" approach to finding coverage of the insurance companies is outlined.

Acknowledgments

The author would like to thank Professor Yurii Ermoliev, IIASA, and Professor Arkadii Kryazhimskii, IIASA, for their scientific advice and help during the YSSP studies. Special thanks go to Professor Aniello Amendola (IIASA's Risk, Modeling and Policy project) for his aid in obtaining data and fruitful comments on this paper. The author is also grateful to all IIASA staff and young colleagues for their friendly attitude and assistance.

About the Author

Boris V. Digas is an assistant scientific researcher at the Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences. Currently he works on his PhD thesis in mathematics. His research interests include inverse problems of dynamics, optimal control theory, seismic ray tomography, and block models of lithosphere dynamics. He was a participant of IIASA’s Young Scientists Summer Program 1998. For more information, see the following URL:

http://www.imm.intec.ru/GRAF/ENG/PERSONS/BIOGRAPH/DIGAS.HTM

Generators of Seismic Events and Losses: Scenario-based Insurance Optimization

Boris V. Digas (digas@imm.uran.ru).

1. Introduction

The complex socio-economic developments of the world have led to a dramatic increase of losses due to natural and anthropogenic catastrophes, e.g., earthquakes, floods, and nuclear accidents. It has been estimated that within 50 years, more than a third of the world's population will live in seismically and volcanically active zones (Bilham, 1988; Rundle, et al., 1996). Insurance is one of the most important tools for spreading catastrophe losses over society. The insurability of catastrophic risks is becoming an urgent socio-economic issue.

Approaches to insuring catastrophic risks differ from traditional insurance techniques. The latter operate with frequent, well-defined cases, which allow the insurers to use rich data collected over long periods. In the insurance of catastrophic risks, the decisions have to be made under a considerable lack of data. The existing historical data on earthquakes are insufficient for predicting seismic events at any particular location, although rich data on their occurrence and magnitudes may exist on an aggregated (say, regional) level. Potential damages in a particular location may be unlike anything that has been experienced in the past; moreover, the catastrophes produce highly correlated damages, and the correspondent claims strongly depend on the region of occurrence and other factors (Ermolieva, 1997; Ermolieva, et al., 1997).

In recent years, it has become clear that strategies for insuring property against natural hazards might usefully be based on catastrophe modeling. Catastrophe modeling can to some extent compensate for the lack of historical data.

This paper describes a data-based modeling approach aimed at supporting decisions on insuring property in seismically active regions. Seismic models have primarily been used for understanding the mechanisms of changes in seismically active zones, which may precede strong earthquakes. A large variety of models has been suggested. We divide them, roughly, into two categories: physical models and data-based models. Physical models represent natural processes in the Earth's crust. Most of the physical models refer to mechanical laws. Of importance is the mechanical model describing the dynamics of lattices of slider blocks (Burridge, and Knopoff, 1967). Other mechanical models have been developed, for instance, by Gabrielov, et al., 1986; Gabrielov, and Soloviev, 1997. Both deterministic and Monte Carlo models have been studied, and blocks with and without mass (inertia) have been used (Rundle, et al., 1986).

Data-based models deal with available historical catalogs of seismic events and employ statistical approaches for identifying earthquake precursors. Many efforts have been made to create intermediate-term earthquake prediction algorithms. Considerable success was achieved in the mid-eighties due to the M8 and CN algorithms (Healy, et al., 1992; Keilis-Borok, and Kossobokov, 1986; Keilis-Borok, and Kossobokov, 1990; Sadovskii, 1986), which showed their efficiency on a number of real catalogs. For processing statistical data, various optimization and randomization techniques have been developed (Molchan, 1991; Molchan, 1992; Kryazhimskii, et al., 1996). Approaches to the interpretation and processing of observed data have been suggested, and problems of seismic wave propagation theory have been considered, for instance, in Aki and Richards, (1980).

Unfortunately, the relative success in testing expectational prediction algorithms has not lead to real progress in the practice of predicting particular events. Seismologists agree that the results of efforts to develop reliable earthquake prediction methods over the last 30 years have been disappointing (Geller, 1991, 1996, 1997; Geller, et al., 1997; Kagan, 1997; Kagan and Jackson, 1994).

Although the prediction of particular earthquakes is a very difficult task, the existing data may give enough information for the estimation of possible scenarios of earthquakes. These estimates may, in turn, serve as a basis for making decisions on the insurability of property in seismically active regions.

In this paper, a data-based generator of scenarios of earthquakes and associated losses is described and a scenario-based approach to optimal pricing catastrophic insurance coverages is proposed. The generator forms inputs to insurance optimization algorithms. It is expected that a decisionmaker may carry out large-scale experiments by trying different insurance optimization algorithms and comparing their outputs. Therefore the structure of the scenario generator has been optimized; the latter gives out an aggregated information needed only for making decisions on optimal insurance pricing.

The generator of loss scenarios due to earthquakes has two levels. On the lower level, geophysical data is processed and earthquake scenarios are designed. On the upper level, the lower-level outputs are matched with data on property located in the affected zones, and the scenarios of losses are shaped. The latter scenarios serve as inputs to mathematically justified algorithms for optimizing the structure of the insurance networks (Figure 1).

The work reported here has also required the creation of software. The author worked out two Windows applications for PC, which can serve as working instruments for testing the proposed methodology and carrying out case studies. All illustrations given in this paper (except of Figures 1 and 4) are made by the use of this software.

In Section 2, a set of geophysical data needed for modeling earthquakes is introduced. Section 3 describes an earthquake scenario generation technique and illustrates it by an example of a seismically active region of Italy. Section 4 describes a dynamical generator of earthquake scenarios, which uses the method of Monte Carlo. In Section 5, the data on property, which is needed for the losses-scenario generator, is defined. Section 6 describes static and dynamic generators of losses. Section 7

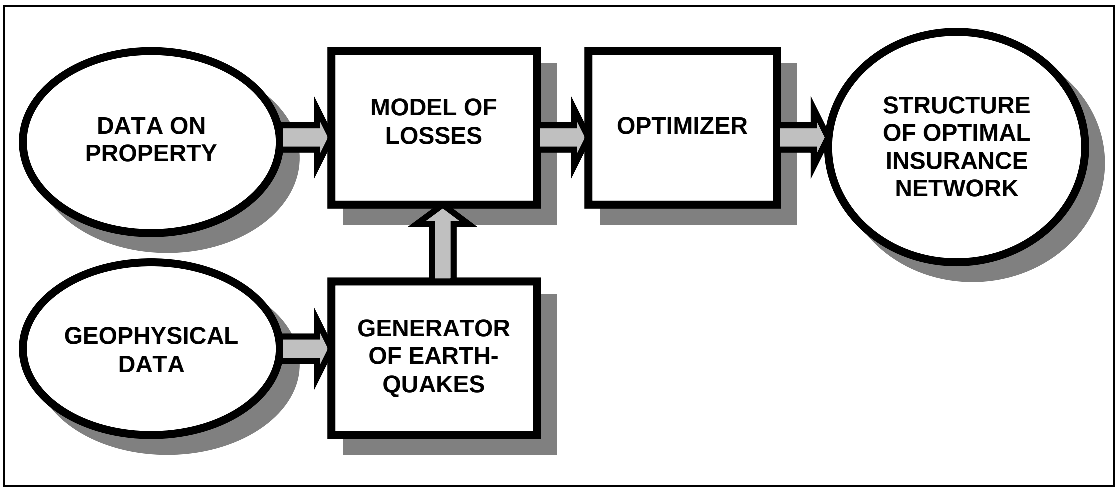

Figure 1. A Flowchart of the suggested approach.

overviews an insurance optimization technique linked to the scenario generator. The technique rests on the method of constraint aggregation (Ermoliev, et al., 1997) and solves a problem of guaranteed insurance optimization. An optimization algorithm uses outputs of the scenario generator as input data. Section 8 presents some concluding remarks on the perspectives of the suggested approach. Some questions on data needed for the successful implementation of the proposed technique are raised.

2. Geophysical Data

The suggested generator of earthquake and loss scenarios for a given region uses the following data on the region: the maximum observed macroseismic intensity, the seismic activity (the frequency of occurrence of strong events), the magnitude of the earthquakes, the scheme of the lineaments.

2.1. Macroseismic intensity

The intensity of an earthquake on the surface is measured in points of the seismic scale (Keilis-Borok, et al., 1980). The seismic scale is used in two aspects: it permits the estimation of the intensities of the earthquakes and serves as a basis for calculating the seismic vulnerability of buildings. The scale consists of two parts, descriptive (macroseismic) and instrumental. The instrumental part associates the intensities with the quantitative parameters of ground oscillations (the maximal acceleration of ground, the maximal oscillation velocity of ground). The descriptive part describes damages of buildings due to different shaking intensities.

2.2. Magnitude of Earthquakes

The occurrence of an earthquake is connected with the release of elastic deformation energy. A part of this energy produces seismic waves that affect buildings. Thus, the seismic energy is the main factor determining the intensity of surface shaking and outer seismic effects. Much effort has been directed to finding connections between the values of seismic energy of a source, characteristics of surface shaking, and seismic effects.

The magnitude of an earthquake is proportional to the energy released in the earthquake. The seismic energy of an earthquake source is usually defined by the level of the amplitude of seismic oscillations. The first research efforts in this field were made by B.B. Golitsyn in 1900's. The first scale classifying earthquakes by magnitude was suggested by C. Richter in 1935. Such scales are known as magnitude scales (Keilis-Borok, et al., 1980)

Nowadays, there are three main types of magnitude scales. A most widely used one classifies magnitudes with respect to the surface waves. It is usually accepted as a principal scale for seismic regioning.

2.3. Seismic activity

One of the important parameters of the long-term average seismicity in a region is seismic activity, the average frequency of the occurrence of earthquakes of a given magnitude.

If ***B*** is the average frequency then ***T=1/B*** is the average return period for strong earthquakes. These data makes it possible to calculate the probability of the occurrence of an earthquake at a given location for arbitrary expectation time (under appropriate assumptions on the character of further occurrences).

It is hardly possible to find the magnitude of a strongest possible earthquake at a given location using seismic observations at this location. The extreme events occur too rarely. For initial approximations to strongest possible events one can take strongest observed events. To get better estimates of strongest possible earthquakes one should take into account indirect factors and specific conditions of the earthquake occurrence at a given location.

2.4. Lineaments

Lineaments characterize the geological structure of a region. Usually, the lineaments are identified with faults. A geologic fault is a fracture zone, along which a relative movement of two sides of the neighboring surface has been registered. A fault trace is represented by the ruptures on the Earth's surface formed by the intersection of the surface with the fault (Collins, 1995).

Those faults where there have been sufficiently recent displacements are called active and have a significant potential for further displacements. The absence of faults is no proof that the region is not endangered by earthquakes, because there may be 'hidden' faults.

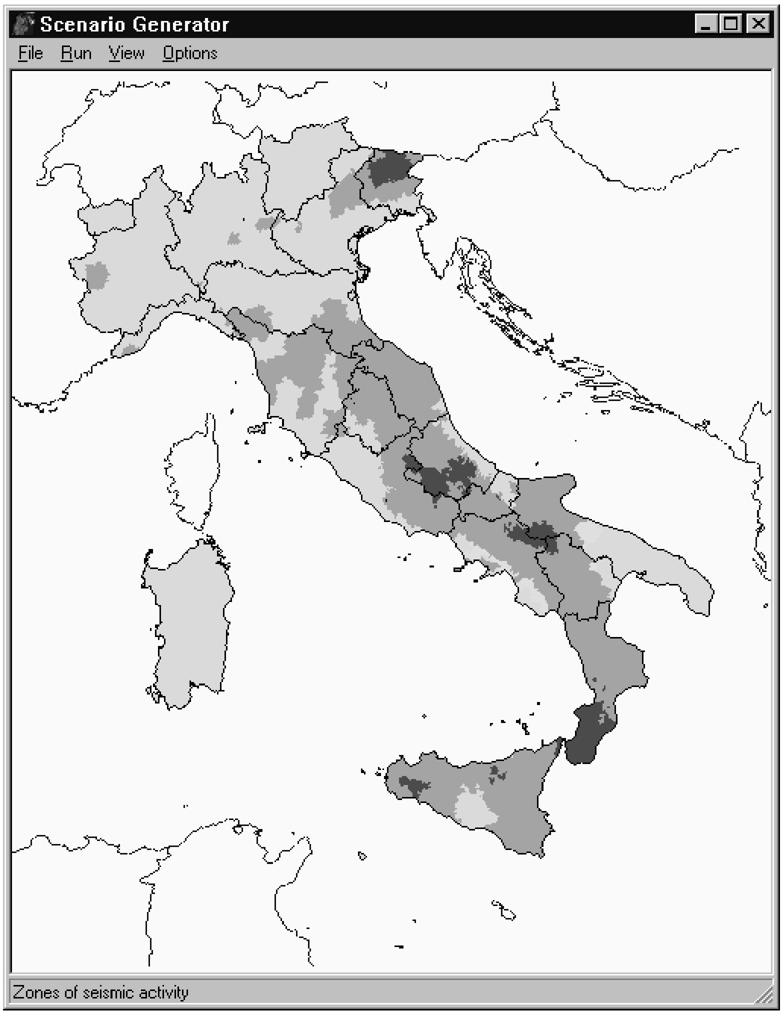

Figure 2. A map of seismic activity zones.

3. Earthquake Scenario Generator

The described earthquake scenario generator uses regional maps of seismic activity zones and the maximum macroseismic intensity as input data. For some regions, for example Italy, there are maps that generalize the experience of observations during thousands of years (Boschi, et al., 1995; Molin, et al., 1996). Figure 2 shows a map of seismic activity zones in Italy. (The maps given in Figures 2 and 3 are available on the Internet at the URL http://www.dstn.pcm.it/ssn/). The zones with different average return periods of strong earthquakes are shown in different colors. On the basis of a map of seismic activity zones, the generator simulates the over-time occurrence of earthquake scenarios, data on the event occurrence in every point of the region is given out. If needed, data on the probabilities of the event occurrence is also provided.

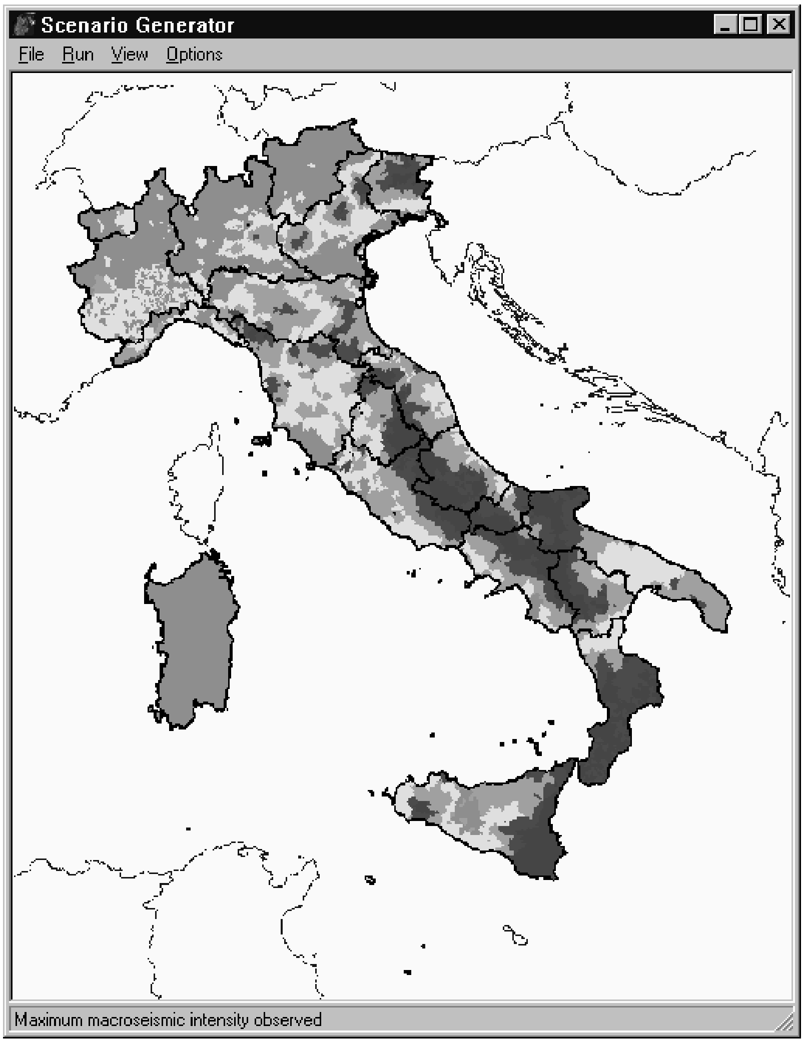

Figure 3. A map of maximum observed macroseismic intensity.

Another input exposes the maximum macroseismic intensities observed in every point of the region (Molin, et al., 1996). In Figure 3, the maximum intensities between 6 and 10 points are shown in different colors. The generator treats the maximum observed intensities as the estimates of the maximum expected shaking intensities at the epicenters of the earthquakes.

To calculate shaking intensities on the Earth's surface, we need models of isoseists (Keilis-Borok, et al., 1980; Keilis-Borok, et al., 1984). An isoseist $\boldsymbol{A_I}$ is a domain in which the shaking intensity is no less than I. The simplest model of an isoseist domain is an ellipse. The ratio between its axes is determined by the parameters of the region, magnitude, and earthquake mechanism. The affected area increases as the earthquake's magnitude grows. General models of isoseists are built on the basis of the estimates of the average radii of isoseists for well studied earthquakes. A relation between the magnitude M and shaken area Q (in sq. km.) is represented in the form of equations of the linear regression,

$$\log Q_I(M) = d_I + f_I M. \tag{3.1}$$

Parameters d and f are given in Table 1. Usually, the ratio between the axes of the isoseists is set 1:1,5. The main axes of the isoseist ellipses go along the tectonic structures or seismically active zones. An example of a system of isoseists is shown in Figure 4. In case of absence of data on tectonic structure of the region, circles can be taken as isoseiets.

Table 1. Parameters of regression $\log Q_I(M) = d_I + f_I M$.

I	d_I	f_I
6	0.06	0.55
7	–1.87	0.77
8	–1.31	0.6
9	–4.52	1

The systems of isoseist domains associated to every point in the region go to the output of the earthquake scenario generator.

Practically, the region under consideration should be represented as a collection into cells. An example of a rectangular region is shown in Figure 5. The region is split into 100 cells. Figures 6a and 6b show a set of scenarios generated for this region.

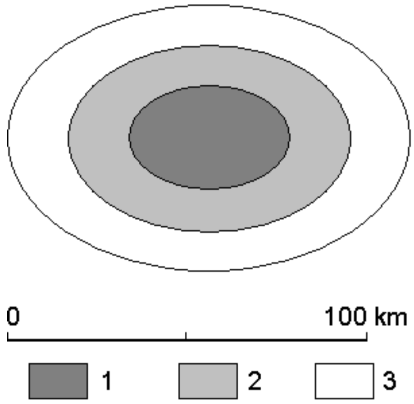

Figure 4. Isoseist domains for magnitude ***M***=7.0.
Intensity (in points of the seismic scale): 9–color 1, 8–color 2, 7-color 3.

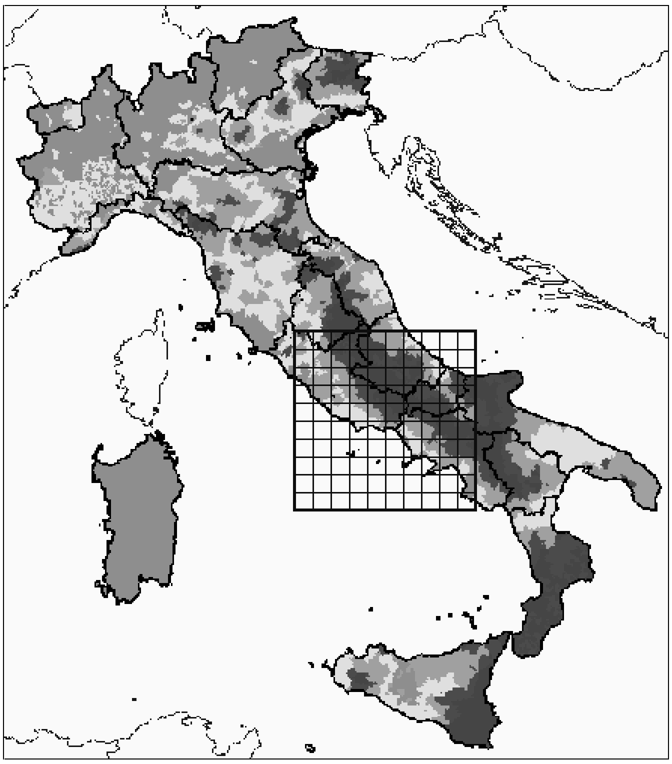

Figure 5. Region under consideration divided into cells.

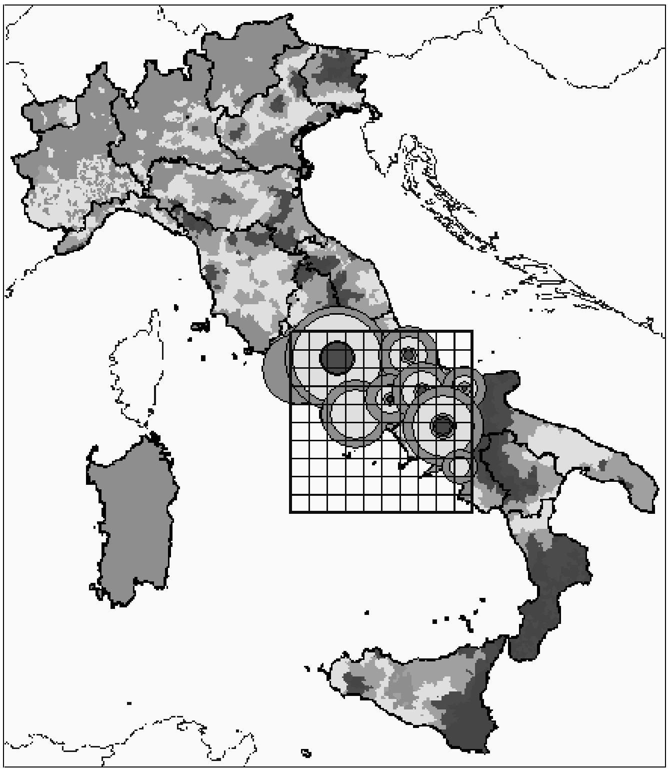

Figure 6a. Examples of generated scenarios of earthquakes.

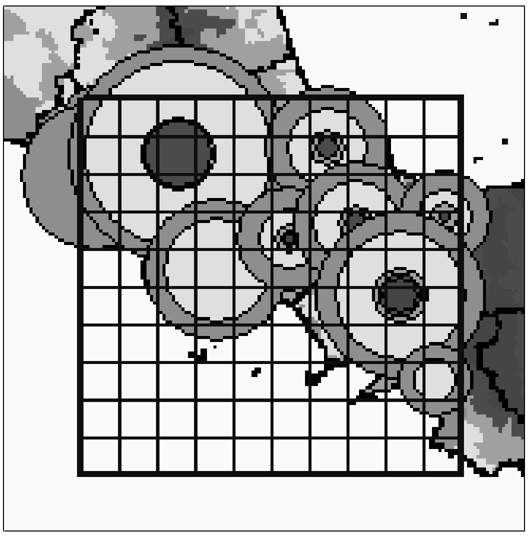

Figure 6b. Examples of generated scenarios of earthquakes: detailed view.

4. Dynamical Generator of Earthquakes

The dynamical generator of earthquakes complements the earthquake scenario generator and serves as the lower level of the dynamical generator of losses. It is being launched every time upon the requests of a dynamical insurance optimization procedure. A dynamical procedure of insurance optimization in a geographic region $\boldsymbol{G}$ is thought of as a process of step-by-step decisionmaking in time. We split region $\boldsymbol{G}$ into small cells $\boldsymbol{G_i}$, $\boldsymbol{i = 1,...,N}$ and assign all features of a cell's center, $\boldsymbol{r_i}$, to the whole cell. Let $\boldsymbol{B(r_i)}$ be the average frequency of earthquake occurence at point $\boldsymbol{r_i}$. The average return period for earthquakes is then $\boldsymbol{T(r_i) = 1 / B(r_i)}$. We set the probability of the occurrence of an earthquake during period $\Delta \boldsymbol{t}$ to be equal to $\boldsymbol{P(\Delta t, r_i) = \Delta t / T(r_i)}$. Let $\Delta \boldsymbol{t}$ be the duration of a time step in the process of decisionmaking. To model all earthquake scenarios that may occur in region $\boldsymbol{G}$ during one step, we find the value of the following logical (Boolean) variable* for every cell $\boldsymbol{G_i}$:

$$\boldsymbol{C_i = C(\Delta t, r_i) = \{P(\Delta t, r_i) < \mathbf{rand(1)}\}}, \qquad (4.1)$$

where **rand(1)** is a random real from the interval **(0,1)**. Each point $\boldsymbol{r_i}$, for which $\boldsymbol{C_i}$ is **true**, is taken for an epicenter of an earthquake. A detailed description of the process of dynamical generating scenarios of losses, which is based on the dynamical generator of earthquakes, is given in Section 6.

5. Data on property and model of losses

All property in a region is usually subdivided into several categories according to its vulnerability to earthquakes. Given a macroseismic intensity, a distribution of buildings of different categories generates a probability distribution of damages (Keilis-Borok, et al., 1984). Therefore, data on property located in the region and earthquake scenarios permit the modelling of losses due to the earthquakes. The next formula gives the probability of the damage level $\boldsymbol{d}$ for category $\boldsymbol{c}$ of buildings and shaking intensity $\boldsymbol{I}$ (Keilis-Borok, et al., 1984):

$$\boldsymbol{p_d(c, I) = p_{d+c-I+6}}, \quad \boldsymbol{d = 1,...,4.} \qquad (6.1)$$

* Logical (Boolean) variables take one of the two values: "**true**" and "**false**".

In the right hand side, the distribution $\{p_l\}$ is a discrete approximation to the normal distribution. For practical calculations, it is assumed that

$$p_1 = 0.05, \quad p_2 = 0.40, \quad p_3 = 0.50, \quad p_4 = 0.05,$$

and for all other indices $p_l \equiv 0$.

Values $c = 1,2,3$ correspond to three categories, A, B, C, of buildings: A– stone-made buildings, B–brick or block buildings, C– wooden houses. Table 2 characterizes losses corresponding to different levels of damage.

Table 2. Levels of damage for the three categories of buildings.

Category of buildings	Level of damage, %				
	1	2	3	4	5
A	1–5	10–15	20–25	30–60	100
B	1–5	10–15	20–25	30–60	100
C	0.1–0.5	1–6	6–12	12–20	–

6. Generators of losses.

In this section, we give an algorithmic description of the dynamical and static generators of losses. The algorithms process the outputs of the earthquake scenario generators (Sections 3 and 4) and data on regional property (Section 5). The outputs go to algorithms of insurance optimization.

6.1. Dynamical generator of losses

The dynamical generator is being launched upon request of a deterministic or stochastic dynamical optimization procedure at the beginning of every step of the optimization process.

INPUT.

1) A Geographic region $\boldsymbol{G}$ divided into cells G_i, $i = 1,...,N$, with centers r_i.

2) Map 1: Zones of seismic activity in region $\boldsymbol{G}$.

3) Map 2: Maximum observed macroseismic intensities in region $\boldsymbol{G}$.

4) Map 3: Magnitudes of observed earthquakes in region $\boldsymbol{G}$. The usage of a catalog of earthquakes is also admissible.

5) Map 4: Scheme of lineaments in region $\boldsymbol{G}$.

6) Distributions of property buildings of categories A, B, and C, $\boldsymbol{D}_i^{\mathrm{A}}$, $\boldsymbol{D}_i^{\mathrm{B}}$, and $\boldsymbol{D}_i^{\mathrm{C}}$, $\boldsymbol{i} = \mathbf{1,...,}\boldsymbol{N}$.

7) Step duration, $\Delta \boldsymbol{t}$.

OUTPUT.

Values of losses in every cell, $\boldsymbol{W}_i$, $\boldsymbol{i} = \mathbf{1,...,}\boldsymbol{N}$.

ALGORITHM.

Set $\boldsymbol{W}_i = \mathbf{0}$ for all $\boldsymbol{i} = \mathbf{1,...,}\boldsymbol{N}$.

For every $\boldsymbol{i}$, ($\boldsymbol{i} = \mathbf{1,...,}\boldsymbol{N}$) do the following:

Using Map 1, find the value of the logical variable (4.1) $\boldsymbol{C}_i = \boldsymbol{C}(\Delta \boldsymbol{t}, \boldsymbol{r}_i)$.

If $\boldsymbol{C}_i = \mathbf{true}$, find the system of isoseists for the event with epicenter $\boldsymbol{r}_i$, and the associated values of losses, namely,

1) using Map 2, compute the shaking intensity $\boldsymbol{I}^*$ at the epicenter;
2) using Map 3 (or catalog), compute the magnitude $\boldsymbol{M}$ of the generated earthquake;
3) using Map 4, orient the greater axis of the isoseists centered at $\boldsymbol{r}_i$ parallel to a nearest lineament;
4) using formula (3.1) and Table 1, calculate the areas of the isoseists $\boldsymbol{A}_6, \dots, \boldsymbol{A}_{I*}$;
5) for every $\boldsymbol{I} = \mathbf{1}, \dots, \boldsymbol{I}^*$ compose the set $\boldsymbol{J}^{\boldsymbol{I}} = \{\boldsymbol{l}: \boldsymbol{G}_l \subset \boldsymbol{A}_I\}$ and for every cell $\boldsymbol{G}_l$, where $\boldsymbol{l} \in \boldsymbol{J}^{\boldsymbol{I}}$, find the most probable levels $\boldsymbol{d}_l^{\mathrm{A}}$, $\boldsymbol{d}_l^{\mathrm{B}}$, $\boldsymbol{d}_i^{\mathrm{C}}$ of damage (i.e. the shares of damaged property) for categories A, B, and C of buildings; here formula (6.1) and Table 2 are used.
6) for every $\boldsymbol{I} = \mathbf{1}, \dots, \boldsymbol{I}^*$ and every $\boldsymbol{l} \in \boldsymbol{J}^{\boldsymbol{I}}$ increase the value of $\boldsymbol{W}_l$ by the following value:

$$\boldsymbol{d}_l^{\mathrm{A}} \boldsymbol{D}_l^{\mathrm{A}} + \boldsymbol{d}_l^{\mathrm{B}} \boldsymbol{D}_l^{\mathrm{B}} + \boldsymbol{d}_l^{\mathrm{C}} \boldsymbol{D}_l^{\mathrm{C}}.$$

6.1. Static Generator of Losses

The static generator of losses forms collections of admissible scenarios of losses, which serve as input data for guaranteed insurance optimization procedures (see Section 7).

INPUT.

1) A geographic region $\boldsymbol{G}$ divided into cells $\boldsymbol{G}_i$, $\boldsymbol{i} = \mathbf{1,...,}\boldsymbol{N}$, with centers $\boldsymbol{r}_i$.

2) Map 1: Maximum observed macroseismic intensities in region ***G***.

3) Map 2: Magnitudes of observed earthquakes in region ***G***. The usage of a catalog of earthquakes is also admissible.

4) Map 3: Scheme of lineaments in region ***G***.

6) Level $\boldsymbol{I}_*$ of shaking intensity which is treated as critical for property in region ***G***.

OUTPUT.

A set $\boldsymbol{J} = \{\boldsymbol{J}_1, \ldots, \boldsymbol{J}_S\}$ of admissible scenarios of losses. Each scenario is a collection of cells which might be damaged due to an earthquake.

ALGORITHM.

Set $\boldsymbol{S} = \mathbf{0}$, $\boldsymbol{J} = \varnothing$.

For every ***i*** ($\boldsymbol{i} = \mathbf{1,...,}\boldsymbol{N}$) do the following:

Generate a scenario with epicenter at $\boldsymbol{r}_i$, namely,

1) using Map 1, find the shaking intensity $\boldsymbol{I}^*$ at the epicenter;
2) if $\boldsymbol{I}^* \geq \boldsymbol{I}_*$ increase ***S*** by 1, else go to step 1. for the next ***i***;
3) using Map 2 (or the catalog), compute the magnitude ***M*** of the generated earthquake;
4) using Map 3, orient the greater axis of the isoseists centered at $\boldsymbol{r}_i$ parallel to a nearest lineament;
5) using formula (3.1) and Table 1, calculate the area ***Q*** of the isoseist $\boldsymbol{A}_{I_*}$,
6) determine the set $\boldsymbol{J}_S = \{\boldsymbol{l}: \boldsymbol{G}_l \subset \boldsymbol{A}_{I_*}\}$; add $\boldsymbol{J}_S$ to set ***J***.

Figures 7 and 8 show an example of generating a scenario of losses.

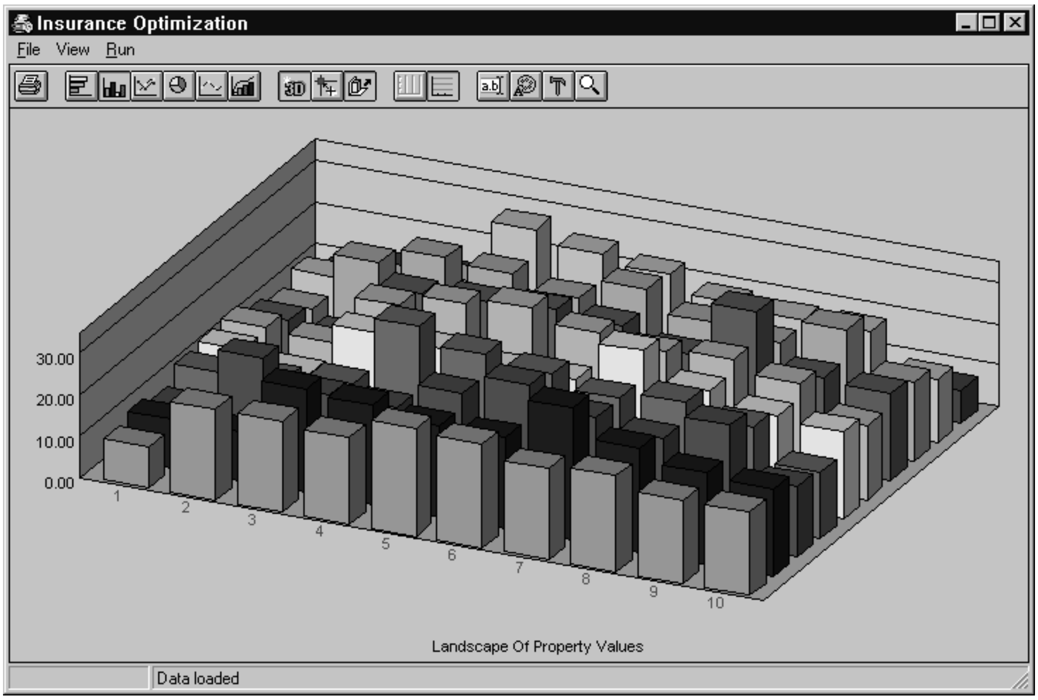

Figure 7. A regional property landscape.

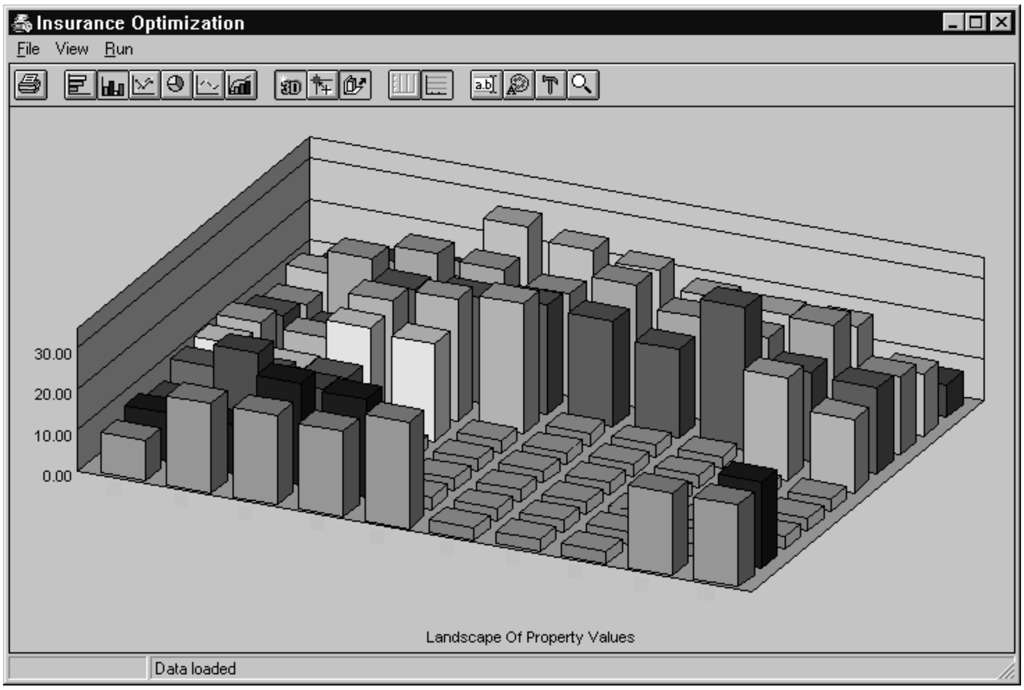

Figure 8. A scenario of losses on a regional property landscape.

7. Insurance Optimization Technique

At the moment, two approaches to the insurance optimization are being developed at the IIASA's Risk, Modeling and Policy Project and Dynamic Systems Project. One of them is based on probability theory and deals with stochastic optimization technique. It aids decision making on the spatial diversification of contracts, insurance premiums, reinsurance requirements, effects of mitigation measurements and the use of other financial mechanisms. The algorithm based on this approach uses sequential catastrophe modeling and adjusts policy coverage. The stochastic insurance optimization technique is described in detail in Ermolieva, 1997, Ermolieva, et al., 1997.

In what follows, we give an outline of a guaranteed insurance optimization method, which is described in detail in Digas, et al., 1998. The method is based on the constraint aggregation technique suggested in Ermoliev, et al., 1997. A bundle of admissible scenarios of losses formed by the static generator of losses (Section 6) is a principal input to the optimization algorithm. The latter finds a minimum premium and an optimal distribution of contracts between the insurers. This provides an insurance model, which survives "in the worst case", given a set of admissible scenarios.

A guaranteed insurance optimization problem is stated as follows. A geographic region G is divided into a number of cells, G_i, $i = 1,...,N$. Each cell G_i carries property whose total value is D_i. A group of insurance companies (we refer to them as companies $1,...,M$) insures the property in region G against earthquakes so that the whole property value in G is distributed between the companies. We denote by x_{ij} the share of property in cell G_i insured by company j. Obviously,

$$x_{ij} \geq 0, \qquad \sum_{j=1}^{M} x_{ij} = 1. \tag{7.1}$$

We deal with the distribution matrix

$$X = \begin{pmatrix} x_{11} & \dots & x_{1M} \\ & \dots & \\ x_{N1} & \dots & x_{NM} \end{pmatrix}. \tag{7.2}$$

Let K_j be the starting capital of company j and c_{ij} be the transaction cost, which company j pays for the right to insure a unit of property in cell G_i. We assume that the premium for a unit of the insured property, p, is the same for all companies, and in each cell G_i only the full damage (which costs D_i) is insured.

An earthquake may damage several cells $\boldsymbol{G_i}$. The collection of the numbers, ***i***, of all damaged cells, $\boldsymbol{G_i}$, represents a scenario of losses (see Section 6). In region ***G*** several scenarios are admissible. Let us denote by ***J*** the set of all admissible scenarios of losses. We define the risk of company ***j*** under scenario ***I*** to be the difference between the company's expenditure and income:

$$\boldsymbol{r_j^I(p,X) = \sum_{i\in I} D_i x_{ij} + \sum_{i=1}^{N} c_{ij} x_{ij} - (K_j + \sum_{i=1}^{N} p x_{ij})}\,. \qquad (7.3)$$

Here we indicate the dependence on the premium, ***p***, and the distribution matrix, ***X*** (7.2). The inequality

$$\boldsymbol{r_j^I(p,X) \le 0} \qquad (7.4)$$

reflects the fact that company ***j*** survives under scenario ***I***. A pair of control variables, $\boldsymbol{(p,X)}$, guarantees survival of all companies under all admissible scenarios if (7.4) holds for all $\boldsymbol{j = 1,\dots,M}$ and all $\boldsymbol{I \in J}$. We pose the insurance optimization problem as follows: Find the minimum of the premium ***p*** for which there exists a distribution matrix ***X*** such that $\boldsymbol{(p,X)}$ guarantees survival of all companies under all admissible scenarios. In standard notations of optimization theory the problem reads:

$$\text{minimize } \boldsymbol{p}, \qquad (7.5)$$

$$\boldsymbol{r_j^I(p,X) \le 0} \quad (\boldsymbol{j = 1,\dots,M},\ \ \boldsymbol{I \in J}), \qquad (7.6)$$

$$\boldsymbol{p \ge 0},\quad \boldsymbol{X \in \mathbf{X}}; \qquad (7.7)$$

here **X** is the set of all distribution matrices, i.e., matrices ***X*** (7.2) satisfying (7.1).

Let us note that the same type of problems arises when premiums depend on ***i*** and have the structure $\boldsymbol{p_i = p\gamma_i}$, where γ_i, $\boldsymbol{i = 1,\dots,N}$, are given numbers.

We assume that there exists a pair $\boldsymbol{(p,X)}$ satisfying the constraints (7.6), (7.7). Then the insurance optimization problem (7.5)–(7.7) has a solution. By $\boldsymbol{p_*}$ we denote the optimal premium, i.e., the minimum value in the problem (7.5)–(7.7). For every solution of (7.5)–(7.7), $\boldsymbol{(p_*,X_*)}$, we call $\boldsymbol{X_*}$ an optimal distribution matrix. For the set of all optimal distribution matrices we use the notation $\mathbf{X}_*$.

We propose the following sequential algorithm for solving the insurance optimization problem (7.5)–(7.7).

At step **0** we set $\boldsymbol{p}^1 = \mathbf{0}$ and fix an arbitrary distribution matrix $\boldsymbol{X}^1$. At step $\boldsymbol{k}$ ($\boldsymbol{k} = \mathbf{1}, \ldots$) we transform the pair $(\boldsymbol{p}^k, \boldsymbol{X}^k)$ into $(\boldsymbol{p}^{k+1}, \boldsymbol{X}^{k+1})$. We define $\boldsymbol{p}^{k+1}$ as the first component of

$$(\boldsymbol{p}^{k+1}, \boldsymbol{U}^{k+1}), \quad \text{a solution of the problem} \tag{7.8}$$

$$\text{minimize } \boldsymbol{p}, \tag{7.9}$$

$$\boldsymbol{p} \geq \boldsymbol{p}^k, \tag{7.10}$$

$$\sum_{I \in J} \sum_{j=1}^{M} \boldsymbol{r}_j^I(\boldsymbol{p}^k, \boldsymbol{X}^k)_+ \boldsymbol{r}_j^I(\boldsymbol{p}, \boldsymbol{U}) \leq \mathbf{0}, \tag{7.11}$$

$$\boldsymbol{U} \in \mathbf{X}; \tag{7.12}$$

here

$$\boldsymbol{r}_j^I(\boldsymbol{p}^k, \boldsymbol{X}^k)_+ = \mathbf{max\{0,} \boldsymbol{r}_j^I(\boldsymbol{p}^k, \boldsymbol{X}^k)\mathbf{\}}.$$

Next, we compute $\boldsymbol{X}^{k+1}$ from

$$\boldsymbol{X}^{k+1} = \boldsymbol{X}^k + \tau_{k+1}(\boldsymbol{U}^{k+1} - \boldsymbol{X}^k) \tag{7.13}$$

where

$$\tau_{k+1} = \underset{0 \leq \tau \leq 1}{\mathbf{argmin}} \left(\sum_{I \in J} \sum_{j=1}^{M} \boldsymbol{r}_j^I(\boldsymbol{p}^k, \boldsymbol{X}^k + \tau(\boldsymbol{U}^{k+1} - \boldsymbol{X}^k))_+^2 \right) \tag{7.14}$$

In Digas, et al. (1998), it is proven that algorithm (7.8)–(7.14) solves the insurance optimization problem (7.5)–(7.7), namely, $\boldsymbol{p}^k$ converges to the minimum premium $\boldsymbol{p}$ and $\boldsymbol{X}^k$ converges to an associated distribution matrix $\boldsymbol{X}$.

Figure 8 gives an example of the optimization process for a network of four companies. The companies' total risks are decreasing whereas the premium is approaching a certain optimal level.

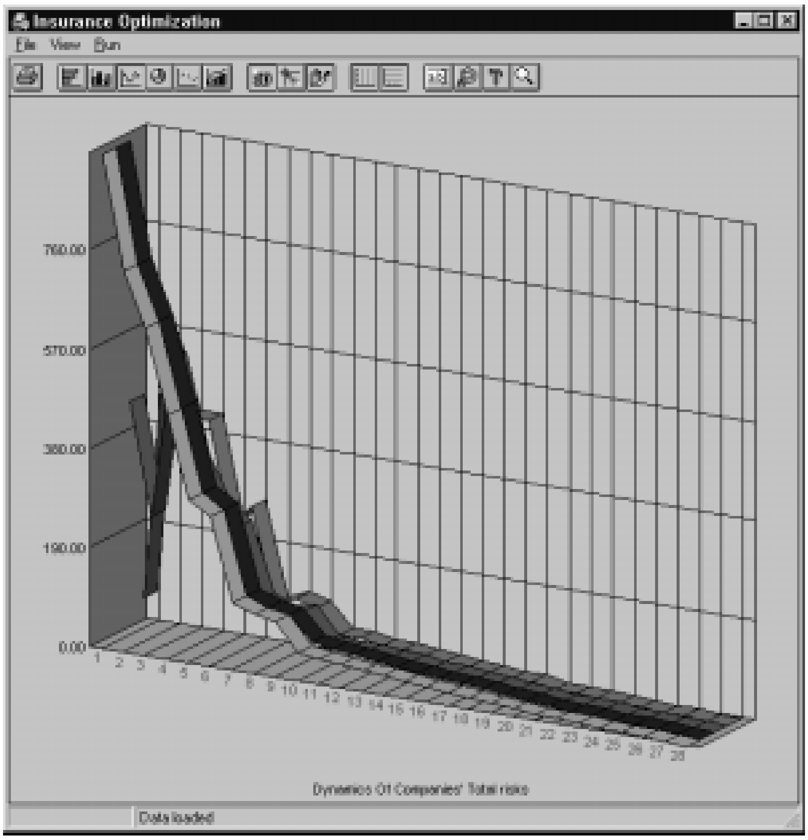

Figure 8. An example of the optimization process.

8. Concluding Remarks

In this paper, a generator of earthquake scenarios and associated losses is described, and an approach to pricing catastrophic insurance coverage is presented. The suggested technique is aimed at supporting decisionmaking in insuring property against earthquakes. Two Windows applications realizing these approaches have been created.

The presented work contributes to a joint RMP-DYN research effort on data-based methodological support for decisionmaking in the insurance of catastrophic risks. In order to create more realistic scenario generators, one has to analyze a large amount of geophysical data on the region under consideration. All the parameters of the generators should be carefully adjusted with real data, which is a very time-consuming process.

We see the following perspectives of further development of the suggested model. It will be useful to take into account the impact of soil and subsoil structures, dynamical features of the processes taking place in the seismoactive region, and other patterns, such as migration of earthquake damages along tectonic structures, clustering, and seismic cycle.

For these purposes, regional seismic catalogs must be available, and a complex analysis of them must be carried out, which will permit us to take account of time interdependencies between the catastrophic events in the process of modeling. For case studies, detailed data on regional property will be needed.

9. References

1. Aki, K., Richards, P.G. (1980) Quantitative Seismology: Theory and Methods. Volumes I, II. W.H.Freeman and Company, San Francisco.
2. Bilham, R. (1988), Nature 336, 1988: 625.
3. Boschi, E., Ferrari, G., Gasperini, P., Guidoboni, E., Smiriglio, G., Valensise, G., (1995). Catalogo dei forti terremoti in Italia dal 461 a.C. al 1980. Publicato da I.N.G. e S.G.A., Bologna.
4. Burridge, R., Knopoff, L. (1967). Bull. Seism. Soc. Am. 57 (1967): 341.
5. Collins, C. (1995). Earthquake for insurers: a personal view. Royal & Sun Alliance. Foxton, Cambridge.
6. Digas, B.V., Ermoliev, Y.M., Kryazhimskii, A.V. (1998). Guaranteed Optimization in Insurance of Catastrophic Risks. IIASA Interim Report, IR-98-xxx.
7. Ermoliev, Y.M., Kryazhimskii, A.V., Ruszczyński, A. (1997). Constraint aggregation principle in convex optimization, Mathematical Programming, Series B, 76, 353–372.
8. Ermolieva, T.Y., Ermoliev, Y.M., Norkin, V.I. (1997). Spatial Stochastic Model for Optimization Capacity of Insurance Networks under Dependent Catastrophic Risks: Numerical Experiments. IIASA Interim Report, IR-97-028.
9. Ermolieva, T.Y. (1997). The design of Optimization Insurance Decisions in the Presence of Catastrophic Risks. IIASA Interim Report, IR-97-068.
10. Gabrielov, A.M., Keilis-Borok, V.I., Levshina, T.A., Shaposhnikov, V.A. (1986). Block model of lithosphere dynamics. In: Mathematical methods in seismology and geodynamics, Computational seismology, 19, 1986 Moscow, Nauka Publ., 168–178.
11. Gabrielov, A.M., Soloviev, A.A. (1997). Modeling of block structure dynamics. Fourth Workshop on Non-Linear Dynamics and Earthquake Prediction, ICTP, Trieste, 6–24 October 1997.
12. Geller, R.J. (1991). Shake-up for earthquake prediction. Nature, 352, 275–276.
13. Geller, R.J. (1996). VAN: A critical evaluation. In: Critical Review of VAN, Earthquake Prediction from Seismic Electrical Signals, 155–238, World Scientific, Singapore.

14. Geller, R.J. (1997). Earthquake prediction: a critical review. Geophys. J. Int., 131, 425–450.

15. Geller, R.J., Jackson, D.D., Kagan, Y.Y., Mulargia, F. (1997). Earthquake cannot be predicted, Science, 275, 1616–1617.

16. Healy, J.H., Kossobokov, V.G., Dewey, J.W. (1992). A test to evaluate the earthquake prediction algorithm M8. U.S. Geological Survey Open-File Report 92-401, 1992. 121 p.

17. Kagan, Y.Y. (1997). Are earthquakes predictable? Geophys. J. Int., 131, 505–525.

18. Kagan, Y.Y., Jackson, D.D. (1994). Earthquake prediction: a sorrowful tale. EOS. Trans. Am. Geophys. Un., Suppl., 75 (25), 57.

19. Keilis-Borok, V.I., Molchan, G.M., Gotsadze, O.D., Koridze, A.H., Kronrod T.L. (1984). Experience of estimation of seismic risk for inhabited buildings in countryside of Georgia. J. Computational Seismology, 17, 1984, 58–67.

20. Keilis-Borok, V.I., Kossobokov, V.G., (1986). Periods of high probability of occurrence of world's strongest earthquakes. Math. Methods in Seismology and Geodynamics. 19, 1986, 48–58.

21. Keilis-Borok, V.I., Kossobokov, V.G., (1990). Premonitory activation of seismic flow: algorithm M8. Phys. Earth and Planet. Inter. 61, 1990, 73–83.

22. Keilis-Borok, V.I., Kronrod, T.L., Molchan G.M. (1980). Calculation of seismic risk. In: Seismic regioning of territory of the USSR. Moscow, Nauka, 69–82.

23. Kryazhimskii, A.V., Maksimov, V.I., Soloviev, A.A., Chentsov, A.G. (1996) On a stochastic approach to quantitative description of dynamics of natural processes.

24. Molchan G.M. (1991). Optimal strategies of earthquakes. Computational Seismology. 24, 1991. 3–19.

25. Molchan G.M. (1992). Models of optimization of earthquake forecast. Computational Seismology. 25, 1992. 7–28.

26. Molin, D., Stucchi, M, Valensise, G., (1996). Massime intensita macrosismiche osservate nei comuni italiani. G.N.D.T.–I.N.G.–S.S.N (National Earthquake Defense Group–National Seismic Service–National Geophysical Institute) for Department of Civil Protection, Italy. (http://www.dstn.pcm.it/ssn/)

27. Rundle, J.B., Turcotte, D.L., Klein, W. (eds.), (1996). Reduction and predictability of natural disasters. Addison-Wesley, Reading, MA.

28. Sadovskii, M.A. (ed.), (1986). Long-term earthquake forecast: Methodical recommendations. Inst. Earth's Physics, Moscow.