NEW SOCIETAL EQUATIONS (IV)

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A. The expression "societal equations" shall be considered only as a terminus technicus and not be given any philosophical meaning. What is meant is a set of equations for the description of the embedding of technological strategies, mostly into an economy in a broad sense. Yet the format of the equations is conceived in such a way that generalizations for their embedding into the environment and the sociosphere can be considered as well. This paper is a step in a series geared at the fuller understanding of embedding. The steps taken so far were the following:

1. "Objective Functions" by W. Häfele, WP-75-25.
This paper is a first conceptual attempt to link the topological evaluation of a phase portrait with a crude macro-economic model that allows for an interface with certain assumed behaviors of a society. Particular attention is given to the notion of resilience as conceived by C.S. Holling and previous members of the IIASA Ecology Group.

2. "New Societal Equations" by R. Avenhaus, D. Bell, H.R. Grümm, W. Häfele, L. Schrattenholzer and C. Winkler, WP-75-67.
This paper is a generalization of 1.,mostly designed to study the phenomena of separatrix interfaces in three dimensions, as in this way the richness of the phenomenon can be seen more clearly than in only two dimensions (as is the case in 1.).

3. "An Attempt of Long-Range Macroeconomic Modeling in View of Structural and Technological Change" by R. Bürk and W. Häfele, RM-76- in preparation.

This paper concentrates on the conception of a model that starts to have some economic meaning. Again it considers energy, capital and labor as the principal variables. But it introduces a distinction between two types of energy: cheap energy with limited fuel supply (oil, gas) and capital-intensive energy with virtually unlimited supply of fuel (breeder, solar power). Attention is being given to the distinction of various strata, in particular the stratum of fast variables and that of slow variables.

-. "Collected Aide-Memoires of the Workshop on Resilience", notes of the resilience workshop of the IIASA Energy and Ecology groups held at Baden, January, 1976.
These are only notes, but they are extremely helpful in drawing the distinction between a descriptive and a prescriptive (policy) part of an overall model. They also give some hints as to how to do the modelling so as to allow for the application of differential-topology methods.

As mentioned above, the present paper is the fourth step in this sequence. Other steps will follow. From now on we will always use the title "New Societal Equations" and enumerate them.

B. The starting point is always the equation that balances production of a gross national product (GNP) and its spending by consumption and investment. Consistently with paper 3. (Bürk, Häfele), we start with the following simple observation:

$$\left(A\,E^{\hat{\alpha}} \cdot K^{\hat{\beta}} \cdot L^{\hat{\gamma}} - e_o^E\,E - e_o^K\,K - e_o^L\,L\right) = cP + I \quad ; \qquad (1)$$

$$I = e_1^E\,\dot{E} + e_1^K\,\dot{K} + e_1^L\,\dot{L} \quad ; \qquad (2)$$

E: energy, K: capital, L: labor, P: total population,
I: total investments
A, $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$, e_m^n, C are all constants, or to be more precise: slow variables.

For the moment, let us assume that P is given as a function of time. We will later drop this assumption.

In all steps taken so far we have implicitly assumed that investments are only constrained (or characterized) by the amount of money, I, that can be put into it. In this paper we now reflect on the fact that investments also require the engagement of production factors: the construction of a power plant ($\dot{E}$) requires an investment of energy, capital and labor and so do the investments in the capital stock K and the labor force L. We therefore have to introduce a partition. If E_{tot} is the total production factor energy, $\alpha_E E_{tot}$ shall be the share that goes into the production of added value (left part of (1) and $(1-\alpha_E)E_{tot}$ shall be the share that goes into the investments for all production factors E, K and L. Similarly, partitions α_K, $(1-\alpha_K)$ and α_L, $(1-\alpha_L)$ shall hold for K and L respectively. For the moment we assume

$$\alpha_E = \alpha_K = \alpha_L .$$

Later we will drop that assumption.
We then get the following relations instead of (1) and (2):

$$\left(A \cdot \alpha^{\hat{\alpha}+\hat{\beta}+\hat{\gamma}} \cdot E_{tot}^{\hat{\alpha}} \cdot K_{tot}^{\hat{\beta}} \cdot L_{tot}^{\hat{\gamma}} - e_o^E\, \alpha E_{tot} - e_o^K\, \alpha K_{tot} - e_o^L\, \alpha L_{tot}\right) = cP + I ; \tag{1a}$$

$$I = e_1^E\, \dot{E}_{tot} + e_1^K\, \dot{K}_{tot} + e_1^L\, \dot{L}_{tot} . \tag{2a}$$

The share $(1-\alpha)E_{tot}$ is the energy E_{in} that goes into investment. We therefore have:

$$(1-\alpha)E_{tot} = E_{in} = \left(\frac{E_{in}}{\dot{E}}\right) \cdot \dot{E}_{tot} + \left(\frac{E_{in}}{\dot{K}}\right) \dot{K}_{tot} + \left(\frac{E_{in}}{\dot{L}}\right) \dot{L}_{tot} , \tag{3}$$

and accordingly:

$$(1-\alpha)K_{tot}=K_{in}=(\frac{K_{in}}{\dot{E}})\dot{E}_{tot}+(\frac{K_{in}}{\dot{K}})\dot{K}_{tot}+(\frac{K_{in}}{\dot{L}})\dot{L}_{tot} \; ; \qquad (4)$$

$$(1-\alpha)L_{tot}=L_{in}=(\frac{L_{in}}{\dot{E}})\dot{E}_{tot}+(\frac{L_{in}}{\dot{K}})\dot{K}_{tot}+(\frac{L_{in}}{\dot{L}})\dot{L}_{tot} \; . \qquad (5)$$

We have an input/output matrix that relates input $(1-\alpha)\,\vec{X}$, $\vec{X} = \{E_{tot}, K_{tot}, L_{tot}\}$ and output $\dot{\vec{X}}$. Its coefficients, e.g. $(\frac{E_{in}}{\dot{E}})$, $(\frac{K_{in}}{\dot{L}})$ ... are constants (or slow variables) that can be provided, probably best, by more disaggregated submodels. In fact, the matrix elements $(\frac{E_{in}}{\dot{E}})$, $(\frac{E_{in}}{\dot{K}})$, $(\frac{E_{in}}{\dot{L}})$ are what "energy analysis" is all about. It should be equally noted that this formalism allows for the analysis of energy conservation strategies. Such strategies save energy but require investments in capital, labor and energy, too. Such conservation strategies are on the stratum of slow variables.

Let us also consider costs. The term $e_1^E \dot{E}_{tot}$ is the total cost for energy investments and is spent as follows:

$$e_1^E \dot{E}_{tot} = t_E^E E_{in}^E + t_K^E K_{in}^E + t_L^E L_{in}^E \; . \qquad (6)$$

t_L^E, for instance, is the cost per labor unit that goes into the construction of power plants, and this is also the interpretation of the other matrix elements t_m^n. They too are constants (or slow variables) that can be provided, probably best, by more disaggregated submodels.

We have of course:

$$E_{in}^E = (\frac{E_{in}}{\dot{E}})\, \dot{E}_{tot} \; ; \qquad (7)$$

$$K_{in}^E = (\frac{K_{in}}{\dot{E}})\, \dot{E}_{tot} \; ; \qquad (8)$$

$$L^E_{in} = \left(\frac{L_{in}}{\dot{E}}\right) \dot{E}_{tot} \; . \qquad (9)$$

We therefore find from (6) - (9) the following relation:

$$e^E_1 = t^E_E \left(\frac{E_{in}}{\dot{E}}\right) + t^E_K \left(\frac{K_{in}}{\dot{E}}\right) + t^E_L \left(\frac{L_{in}}{\dot{E}}\right) \; , \qquad (10)$$

and accordingly we have:

$$e^K_1 = t^K_E \left(\frac{E_{in}}{\dot{K}}\right) + t^K_K \left(\frac{K_{in}}{\dot{K}}\right) + t^K_L \left(\frac{K_{in}}{\dot{K}}\right) \; ; \qquad (11)$$

$$e^L_1 = t^L_E \left(\frac{E_{in}}{\dot{L}}\right) + t^L_K \left(\frac{K_{in}}{\dot{L}}\right) + t^L_L \left(\frac{L_{in}}{\dot{L}}\right) \; . \qquad (12)$$

After inserting (2a) in (1a) and then making use of equations (1a), (3), (4) and (5), we have four equations for the following four unknowns:

$$E_{tot}(t),\; K_{tot}(t),\; L_{tot}(t),\; \alpha(t) \; .$$

We thereby can explicitly study the evolution of technological strategies $\dot{E}_{tot}$ in conjunction with the strategies for $\dot{K}_{tot}$ and $\dot{L}_{tot}$. That is what is meant by the embedding of technological strategies into the here-provided context. $(1-\alpha(t))$ explicitly describes the investment share, that is that share of an economy activity that can go into a transition from a current state into a desired state.

C. We now drop the condition

$$\alpha_E = \alpha_K = \alpha_L \; ,$$

and therefore have two free variables. Instead of (1a) and (3), (4), (5) we have:

$$\left(A\cdot\alpha_E^{\hat{\alpha}}\cdot\alpha_K^{\hat{\beta}}\cdot\alpha_L^{\hat{\gamma}}\; E_{tot}^{\hat{\alpha}}\cdot K_{tot}^{\hat{\beta}}\cdot L_{tot}^{\hat{\gamma}} - e_o^E\alpha_E\; E_{tot} - e_o^K\alpha_K\; K_{tot} - e_o^L\alpha_L\; L_{tot}\right)$$

$$= cP + I\;. \qquad (1')$$

For I see equation (2a).

$$(1-\alpha_E)\; E_{tot} = E_{in} = \left(\frac{E_{in}}{\dot{E}}\right)\dot{E}_{tot} + \left(\frac{E_{in}}{\dot{K}}\right)\dot{K}_{tot} + \left(\frac{E_{in}}{\dot{L}}\right)\dot{L}_{tot} \qquad (3')$$

$$(1-\alpha_K)\; K_{tot} = K_{in} = \left(\frac{K_{in}}{\dot{K}}\right)\dot{E}_{tot} + \left(\frac{K_{in}}{\dot{K}}\right)\dot{K}_{tot} + \left(\frac{K_{in}}{\dot{L}}\right)\dot{L}_{tot} \qquad (4')$$

$$(1-\alpha_L)\; L_{tot} = L_{in} = \left(\frac{L_{in}}{\dot{E}}\right)\dot{E}_{tot} + \left(\frac{L_{in}}{\dot{K}}\right)\dot{K}_{tot} + \left(\frac{L_{in}}{\dot{L}}\right)\dot{L}_{tot} \qquad (5')$$

We have an allocation problem. If we employ for instance the observation Max $(\dot{\text{GNP}})$, we would probably describe a market economy (see the above quoted notes of the IIASA resilience workshop, January 1976). We can then evaluate the evolution of all six variables:

$$E_{tot}(t),\; K_{tot}(t),\; L_{tot}(t),\; \alpha_E(t),\; \alpha_K(t),\; \alpha_L(t).$$

As one of the next steps, we will pay attention to the kind of programming that can do this job. Constraints have to be observed. For instance:

$$\left\{\begin{array}{llll}
K & \geq & K_o & \text{minimum infrastructure} \\
E\;dt & \leq & R_o & \text{(finite resources)} \\
L_{tot} & \leq & \delta\cdot P & \text{(finite maximum share of labor for a given population P)} \\
\dot{E} & \leq & (\dot{E})_o & \\
\dot{K} & \leq & (\dot{K})_o & \text{finite speed of transition, G market penetration.} \\
\dot{L} & \leq & (\dot{L})_o &
\end{array}\right. \qquad (13)$$

D. We now address the problem of transition from one energy system (infrastructure) to another energy system. As mentioned in the introduction, the most salient point there is the fact that the present energy systems provide for cheap but finite energy E_f (oil, gas), while in the future we probably have to face capital-extensive energy E_i that is essentially infinite in fuel supply (breeder, solar power). We therefore put:

$$E = E_i + E_f \; . \tag{14}$$

We then have the following relation instead of (1'):

$$\left(A\cdot\alpha_E^{\hat{\alpha}}\cdot\alpha_K^{\hat{\beta}}\cdot\alpha_L^{\hat{\gamma}}\;(E_i+E_f)_{tot}^{\hat{\alpha}}\cdot K_{tot}^{\hat{\beta}}\cdot L_{tot}^{\hat{\gamma}} - e_o^E\alpha_E(E_i+E_f) - e_o^K\alpha_K K_o - e_o^L\alpha_L L)\right)$$

$$= c \cdot P + I \; .$$

For I see equation (2).
Instead of (3'), (4'), (5') we have:

$$(1-\alpha_E)(E_i+E_f)_{tot} = E_{in} = \left(\frac{E_{in}}{\dot{E}_f}\right)\dot{E}_{f\;tot} + \left(\frac{E_{in}}{\dot{E}_i}\right)\dot{E}_{i\;tot} + \left(\frac{E_{in}}{\dot{K}}\right)\dot{K}_{tot} + \left(\frac{E_{in}}{\dot{L}}\right)\dot{L}_{tot}; \tag{3''}$$

$$(1-\alpha_K)K_{tot} = K_{in} = \left(\frac{K_{in}}{\dot{E}_f}\right)\dot{E}_{f\;tot} + \left(\frac{K_{in}}{\dot{E}_i}\right)\dot{E}_{i\;tot} + \left(\frac{K_{in}}{\dot{K}}\right)K_{tot} + \left(\frac{K_{in}}{\dot{L}}\right)\dot{L}_{tot}; \tag{4''}$$

$$(1-\alpha_L)L_{tot} = L_{in} = \left(\frac{L_{in}}{\dot{E}_f}\right)\dot{E}_{f\;tot} + \left(\frac{L_{in}}{\dot{E}_i}\right)\dot{E}_{i\;tot} + \left(\frac{L_{in}}{\dot{K}}\right)\dot{K}_{tot} + \left(\frac{L_{in}}{\dot{L}}\right)\dot{L}_{tot}. \tag{5''}$$

Again we have four equations, but this time seven variables to be determined by an allocation programming technique:

$$E_{f\,tot}(t),\; E_{i\,tot}(t),\; K_{tot}(t),\; L_{tot}(t),\; \alpha_E(t), \alpha_K(t), \alpha_L(t) \; .$$

Constraints of the kind mentioned in (13) have to be taken into account.

E. Besides envisaging an allocation programming technique we equally envisage the differential topology techniques that consider phase portraits. Both should at least be consistent. A condition for that technique is to have only state variables (and no time lags, integrals or variables that explicitly depend on the time).

H.-R.Grümm has therefore proposed the following submodel. It indeed engages only state variables and forces the transition from E_f to E_i as the resources for E_f decline.

$$\dot{R} = - \eta E_f , \qquad R \geq 0 ; \tag{15}$$

$$E_f = \frac{KR}{1+KR} \cdot E_{tot} ; \tag{16}$$

$$E_i = \frac{1}{1+KR} \cdot E_{tot} ; \tag{17}$$

K is a constant, a slow variable.

Equivalent to (16) and (17) is:

$$\dot{E}_i = \frac{1}{1+KR} \dot{E}_{tot} + \frac{\eta K E_f}{(1+KR)^2} E_{tot} \tag{18}$$

$$\dot{E}_f = \frac{KR}{1+KR} \dot{E}_{tot} - \frac{\eta K E_f}{(1+KR)^2} E_{tot} \tag{19}$$

Various transition policies may or may not be sufficiently reflected by a proper choice of K and its evolution as a slow variable.

Earlier we assumed that $P = P(t)$ would be given as a function of time. We can internalize that and make P a state variable by assuming, for instance:

$$\dot{P} = P \cdot \rho - \sigma G , \tag{20}$$

ρ, σ being constants (slow variables) as it has been done earlier.

Obviously there is room for the improvement of (20). We should carefully examine the existing population models such as the Pestel-Mesarovic population model or others. But the point in the presently considered context is that by virtue of (2), P becomes a state variable and therefore allows for the application of the differential-topology techniques mentioned above.

Finally, it is often assumed that the constant A of equation (1) (and similar equation (1') and (1")) is a function of time too, thus reflecting technological progress. For instance, one often finds:

$$A = A_o e^{\lambda t} . \tag{21}$$

(21) could be expressed by the differential equation

$$\dot{A} = \lambda A , \tag{22}$$

and thereby becomes a state variable also. Again, there is room for the improvement of (22) much in the same sense as there was room for the improvement of (20). In the context of IIASA's differential-topology modelling exercise, we would therefore have the following variables

$E_{tot}(t)$, $E_{i_{tot}}(t)$, $E_{f_{tot}}(t)$, $K_{tot}(t)$, $L_{tot}(t)$, $R(t)$, $\alpha_E(t)$, $\alpha_K(t)$, $\alpha_L(t)$,

and P(t) and possibly A(t), altogether eleven variables and the nine equations (1), (3), (4), (5), (20), (22) as well as (15), (16) and (17).

It remains to be seen whether the existing fixed point algorithms can handle this problem.

F. The outputs $E_{i_{tot}}(t)$, $E_{f_{tot}}(t)$, $K_{in}(t)$ etc. should be used as inputs for more disaggregated technology models of the Häfele/Manne type. Such models would essentially disaggregate, for instance $E_{i_{tot}}(t)$, into more detailed technological components in some optimal fashion.