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Simulating the Carbon Permit Market with Imperfect Observations of Emissions: Approaching Equilibrium through Sequential Bilateral Trade

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Approved by

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Abstract

It is still unclear how the Parties of the Kyoto Protocol will deal with emission trading and compliance given the fact that emissions of greenhouse gases are not perfectly observable and underreporting of emissions may occur. This paper gives an analytical and numerical analysis of the carbon permit market given imperfect observation of emission levels. Our setting is such that Parties must undershoot their emission targets to be able to verify compliance with the Protocol if unreported emissions are accounted for. Targets can be met by traditional emission abatement, by investing in monitoring (reducing unreported emission) or by trading in permits. The paper proves that sequential bilateral trade converges to an equilibrium where marginal abatement costs equal marginal monitoring costs across all Parties. The method is applied for the fossil fuel related carbon emissions of the major Parties of the Kyoto Protocol. Our numerical findings indicate that USA, Japan and the European Union could increase their compliance costs significantly when uncertainty in the emission levels is included. Although Central Eastern Europe, Russia, and Ukraine are assumed to have larger uncertainties in emission levels, their net costs may be reduced as they can sell emission reductions at a higher price. Compared to the no trade case, we find that emissions trading may lead to somewhat lower aggregate uncertainty in greenhouse gas accounts.

Key Words: emissions trading, carbon, monitoring, simulation, uncertainty

JEL classification: Q35, Q38

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Simulating the Carbon Permit Market with Imperfect Observations of Emissions: Approaching Equilibrium through Sequential Bilateral Trade.

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1. Introduction

The Kyoto Protocol was established in 1997 under the United Nations Framework Convention on Climate Change (UNFCCC 1992). The main objective of the Convention is to reduce the emissions of greenhouse gases (GHGs) to prevent dangerous anthropogenic interference with the climate system. For each country taking part, (referred to as Party), the Protocol specifies a specific emission level not to be exceeded in the period 2008-2012 (UNFCCC 1997). However, Article 17 allows for emission trading between the industrialized Parties to the Protocol. This means that each Party, or signatory of the Protocol, has the possibility to exceed their prescribed emission level given that another Party carries out an equivalent emission reduction such that the aggregate emission level remains constant.

Montgomery (1972) demonstrates that the least cost solution of reaching the aggregate target of pollution reduction agreements can be realized though trading in emission permits. The cost-effective solution can be computed and implemented if the abatement cost functions for all countries are known. However, if a permit buyer reveals its abatement cost function, the seller can use this information when bargaining on a permit price such that the buyer is worse off than she otherwise would be. Hence, Parties have incentives to keep this information private and the specific costs of emission reductions remain unknown. Acknowledging this information problem, Ermoliev et al. (2000) analyzed a scheme of sequential bilateral trade. The basic feature of this scheme is that two Parties (e.g. picked at random) meet, and, if possible, they exchange emission permits in a mutual beneficial way. A new pair is picked and the procedure is repeated. Ermoliev et al. (2000) prove that this dynamic process will lead the Parties to the least cost solution when the information of each Party's emission abatement cost function is private. In other words, the feasibility of bilateral trades to deal with incomplete (asymmetric) information was demonstrated.

In light of how markets often function, common objections to the above scheme are that Parties meet randomly and only trade bilaterally. However, it is not the purpose of our study to predict how the carbon market will evolve. We use the methodology of sequential bilateral trade to deal with uncertainties in emissions. The information requirements in this case are more realistic and relaxed compared to general equilibrium analysis giving conclusions that are less restricted.

Emissions of GHGs are in general not directly observable. On the basis of specific conversion factors, emissions can be estimated with information on GHG-emitting activities. These activities are monitored by the regulatory agency in each Party and emissions are reported to the Convention Secretariat according to specific guidelines developed by the Intergovernmental Panel on Climate Change (IPCC 1997). The accuracy of the estimated emissions is dependent on the quality of the monitoring system in each specific country and on the accuracy of the conversion factors used (c.f. Rypdal and Zhang 2000).

As emissions of greenhouse gases cannot be observed perfectly, we assume that Parties can underreport emissions either on purpose or because of uncertainty. The term unreported emissions used in this analysis therefore refers to the fact that activity data are "flexible", in the sense that GHG emitters within each Party can release carbon dioxide that is not included in the emissions reported by the regulatory agency to the Convention Secretariat. Conversion factors can also be manipulated, as many of them do not apply globally. Hence the uncertainty in emission levels can be exploited strategically giving rise to unreported emissions. We picture that when there is uncertainty involved in the activity data or conversion factors, the Protocol will require that the reported emissions plus the estimated unreported carbon emissions must be below the Kyoto target of that Party. Therefore, in bilateral trades the emission reduction must overshoot the level of uncertainty that provides incentives to reduce uncertainty before trading. In our scheme we follow the model proposed by Obersteiner et al. (2000) that is also based on early results by Jonas et al. (1999). It is assumed that the regulatory agency in each Party can reduce (the uncertainty surrounding) the amount of unreported emissions by investing in monitoring. Obersteiner et al. (2000) analyses the equilibrium conditions for the cost effective policies on the emission reduction and the monitoring. Our goal is to study the feasibility of the bilateral trade to achieve this equilibrium.

The purpose of this paper is, in fact, twofold. First, to examine analytically the conditions under which the carbon permit market converges to the equilibrium given the fact that emission levels are uncertain. Second, to apply this method using data on the major industrial Parties of the Kyoto Protocol.

The paper is organized as follows: Section Two describes the methodology of sequential bilateral trade with imperfect observations of emissions. In Section Three, data used in the analysis are presented. In Section Four we present and discuss the results. Concluding remarks are presented in Section Five. The proof of convergence of the adopted methodology is given in the Appendix.

2. Methodology

We first define the necessary set of variables. Let

- $i = 1,\dots, n$ be Parties (or sources) of the Kyoto Protocol;
- x_i:= the reported emissions at source i;
- u_i:= the unreported emissions at source i;

- $c_i(x_i)$:= the costs of holding reported emissions down to, x_i;
- $d_i(u_i)$:= the cost of holding unreported emissions down to u_i, (through investing in monitoring);
- y_i:= the amount of emission permits acquired by source i (y_i is negative if i is a net supplier of permits) and
- K_i:= the Kyoto target for source i.

In our scheme, we separate the decision problem each Party is faced with in two. Firstly, for a given amount of permits, each Party has to decide whether to spend resources on abating emissions or investing in monitoring. This individual decision problem involves choosing parameters that does not require the information from any other Party, and we assume that the Party therefore can perform a regular optimization on this problem. Secondly, the Party needs to decide whether or not to exchange permits with other Parties. This decision problem involves the cost functions of other Parties. In our scheme this information is private and we therefore adopt the methodology of sequential bilateral trade.

For the individual optimization problem discussed above, we define the least costs for Party i to comply with the Protocol for a given amount of permits, y_i as the minimization of emission reduction costs and monitoring costs.

$$f_i(y_i) := \min_{x_i, u_i} \left[c_i(x_i) + d_i(u_i) \right] \qquad (1)$$

s.t. $x_i + u_i \le K_i + y_i$, for all i (2)

Assume that the cost functions $c_i(x_i)$ and $d_i(u_i)$ are positive, decreasing, convex in x_i and u_i respectively and that the left- and right hand side derivatives are well defined. With this formulation, marginal costs $c_i'(x_i)$ and $d_i'(u_i)$ are negative in x_i and u_i respectively, hence being positive in reducing x_i and u_i. We note that constraint (2) will hold with equality in the realistic case where $\left| c_i'(0_i^+) \right|$ and $\left| d_i'(0_i^+) \right|$ are sufficiently large such that optimal u_i^* and x_i^* for given y_i are strictly positive. Furthermore, as $f_i(y_i)$ is the minimum of two convex functions subject to a linear constraint with respect to decision variables x_i, and u_i, then from general convexity analysis we know that the function $f_i(y_i)$ is convex. Hence, the reduced function $f_i(y_i)$ is positive, convex and decreasing. The convexity of $f_i(y_i)$ ensures that the left- and right hand derivatives are well defined, which is a sufficient requirement for our analysis. By substituting (2) in (1) through eliminating x_i we obtain:

$$f_i(y_i) = \min_{u_i} \left[c_i(K_i + y_i - u_i) + d_i(u_i) \right] \qquad (3)$$

Then, by making use of the envelope theorem on (3) we obtain

$$f_i'(y_i) = \frac{\partial}{\partial y_i} \min_{u_i} \left[c_i(K_i + y_i - u_i) + d_i(u_i) \right] = c_i'(x_i^*) \qquad (4)$$

If substituting (2) in (1) through eliminating u_i we would equivalently obtain

$$f_i'(y_i) = \frac{\partial}{\partial y_i} \min_{x_i} \left[c_i(x_i) + d_i(K_i + y_i - x_i) \right] = d_i'(u_i^*) \qquad (5)$$

Where u_i^*, x_i^* are optimal reported and unreported emissions for given y_i. As (4) is equivalent to (5), we obtain the standard condition for static optimization, namely that $c_i N(x_i^*) = d_i N(u_i^*)$. This states that in the cost-minimum for each source i, the marginal cost of holding emissions down to x_i^* will be equal to the marginal costs of holding unreported emissions down to u_i^*. If not, the total costs for Party i of reaching K_i for a given amount of permits, y_i, could be lowered.

If solving (1) subject to (2) by setting up the Lagrangian, and applying the envelope theorem to this scheme we would obtain the condition that

$$f_i'(y_i) = -\lambda_i \qquad (6)$$

where λ_i is the Lagrangian multiplier and is interpreted as the shadow price, or the willingness to pay by Party i, for relaxing constraint (2) with one unit, i.e. the right to emit one more unit of reported or unreported emissions. We note that λ_i is strictly positive if $c_i(x_i)$ and $d_i(u_i)$ are stricktly decreasing. Otherwise, λ_i could be equal to zero. According to (6), the marginal change in the minimum cost of complying with the Protocol by a unit increase in y_i, (which is negative) is also equal to $-\lambda_i$. Hence according to (4) to (6), for given y_i, the value of one additional permit, is equal to the marginal cost of holding reported or unreported emissions down to the optimal level x_i^* or u_i^*. Outside equilibrium, λ_i will differ between two or more Parties; i.e. they have different willingness to pay for a permit, thus making trading in permits in a mutual beneficial way possible.

This brings us to the second optimization problem, which involves finding the permit vector, or distribution of permits that realizes the global least cost solution. We define:

$$F(y) := \sum_{i=1}^{n} f_i(y_i) \qquad (7)$$

as the total or social costs for reaching the agreement for a given vector of permits, y. $f_i(y_i)$ are defined in (1). As indicated in (1) and (7) it should be noted that we in our setting operate with reduced functions. If we had a social planner that knew $f_i(y_i)$ for all i she could minimize (7) subject to:

$$\sum_{i=1}^{n} y_i = 0 \tag{8}$$

by setting up the Lagrangian, which would yield the first order condition:

$$f_i'(y_i) = \mu \text{ for all i.} \tag{9}$$

Condition (9) states that the marginal value of a permit shall in equilibrium be equal to a specific level μ among all parties. However, in our setting, each Party only knows its own willingness to pay for a permit, λ_i, and as there is no central planner that know the cost functions of all Parties, the minimum value of (7) cannot be resolved by setting up the Lagrangian. We therefore turn to the scheme of sequential bilateral trade, which, as we show, will approach the global least cost solution where $f_i'(y_i)$ for all i converge to the same and unknown value of μ. Below is a discussion of how the scheme of sequential bilateral trade converges. The formal proofs follow the approach of Ermoliev et al. (2000), and are given in the Appendix.

Let $y^k = (y_1{}^k, ..., y_n{}^k)$ be the vector of emission permits after k trades, (k = 0, 1,…) which satisfies (8) but may not be overall cost effective. We then consider two sources i_k, j_k picked at random at step k. An exchange of permits between these two sources $i = i_k$ and $j = j_k$, $i \neq j$ leads to a new distribution of permits $y_i = y_i{}^{k+1}$, $y_j = y_j{}^{k+1}$ satisfying the constraint:

$$y_i{}^{k+1} + y_j{}^{k+1} = y_i{}^k + y_j{}^k \tag{10}$$

such that the sum of the number of permits of these two sources remains constant. If the permit vector $y^k = (y_1{}^k, ..., y_n{}^k)$ is not cost efficient, then there exists sources i,j having different marginal costs on reducing reported emissions (or monitoring). Without loss of generality, assume that j has higher marginal costs of reducing emissions than i. As $f_i'(y_i)$ are negative we therefore have

$$f_i'(y_i{}^k) - f_j'(y_j{}^k) > 0\,. \tag{11}$$

The exchange in permits is such that j increases - and i decreases the number of permits:

$$y_j^{k+1} = y_j^{k} + \Delta_k, \qquad y_i^{k+1} = y_i^{k} - \Delta_k, \qquad \Delta_k > 0$$

where Δ_k is the amount of permits that is exchanged from one step to the next. For small enough Δ_k, the new distribution of permits reduces the total costs (7) due to (11) by:

$$F(y^{k+1}) - F(y^{k}) = f_i(y_i^{k+1}) + f_j(y_j^{k+1}) - f_i(y_i^{k}) - f_j(y_j^{k}) =$$

$$\Delta_k \left[f_i'(y_i^{k}) - f_j'(y_j^{k}) \right] + o(\Delta_k) < 0,$$

which is in accordance with the mean-value theorem. Since the term $o(\Delta_k)/\Delta_k \rightarrow 0$ for $\Delta_k \rightarrow 0$, for small Δ_k we therefore have that

$$f_i(y_i^{k+1}) - f_i(y_i^{k}) < f_j(y_j^{k}) - f_j(y_j^{k+1}) \qquad (12)$$

This implies that the new distribution of permits reduces costs of j more than it increases the costs of i. Hence j is able to compensate i for the increased costs in a mutually beneficial way. Rearranging (12) yields

$$f_i(y_i^{k+1}) + f_j(y_j^{k+1}) < f_i(y_i^{k}) + f_j(y_j^{k}) \qquad (13)$$

On the left hand side of (13) we have the aggregate costs for sources i and j after co-operating from step k to step k+1. The right hand side of (13) is the costs before co-operation. Clearly (13) demonstrates that co-operation reduces the aggregate costs for sources i and j of reaching their aggregate target. In our scheme sources i and j do not search for the full potential cost saving at each step, but as long as marginal costs differ, as assumed in (11), these sources will take a small step in a direction where their aggregate cost are lower.

When programming this system a procedure for finding an adequate amount of traded permits, Δ_k, at each step is necessary. The following will suffice. Choose a specific value of $\Delta_{k,s}$, s = 1, 2, …where s is the number of trials at trade k before an acceptable stepsize is found. Then (12) is evaluated and if it holds, $\Delta_{k,s}$ is adopted. If $\Delta_{k,s}$ is too large too lead to a profitable trade, (12) is evaluated with a new $\Delta_{k,s+1} < \Delta_{k,s}$ according to

$$\Delta_{k,s+1} = \frac{1}{s+1}\Delta_{k,s}, \quad s = 1,2,\ldots..$$

until a small enough $\Delta_{k,s}$ is found and employed.

Before the first trade and after each consecutive step of trade, sources i, j solve for optimal x_i^* and u_i^* for given y_i individually.

In Figure 1 we give a graphical presentation of how we model unreported emissions. The vertical axis depicts the marginal abatement cost function for reducing x and u, (i.e. - cN(x), and - dN(u) respectively). Without unreported emissions and trading the source would have to reduce reported emissions to the level K. When unreported emissions are included in the emission inventory the source has to keep the sum of u and x below the Kyoto target. Then after optimizing between abating reported emissions and investing in monitoring, reported emissions increase to x^*, while the unreported decrease to u^*, satisfying the constraint $x^* + u^* = K$, $(y = 0)$ while minimizing the costs of reaching the target. In Figure 1 trading in permits would graphically be the same as changing the Kyoto target for this particular source.

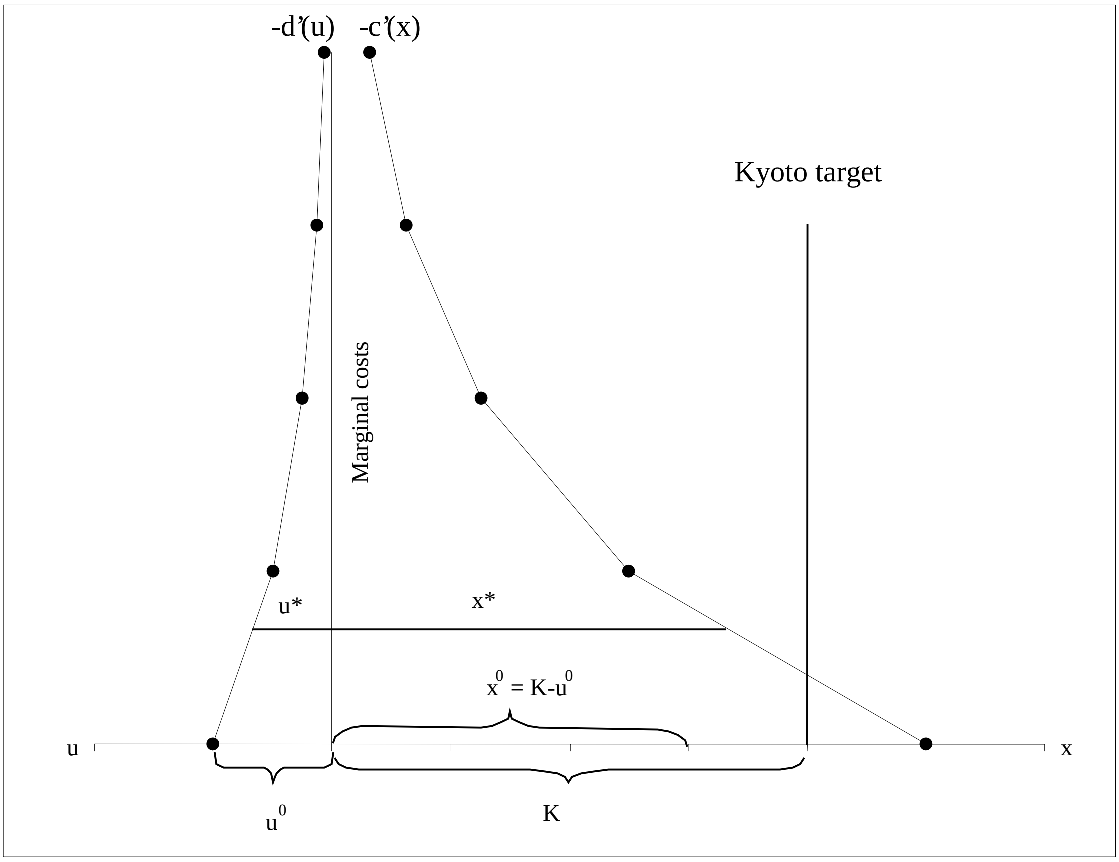

Figure 1. Graphical presentation of the setting.

Returning to the trading scheme, when sources i, j have implemented a successful exchange of permits, a new random pick of two Parties from i = 1,2,…, n is carried out and the trading procedure is repeated. The formal proof that this scheme converges to the minimum value of (7) is given in the Appendix.

In the above procedure sources i and j stop trading after one trade, even though their marginal costs after this trade may differ. A more optimistic procedure for simulating bilateral trade could be to assume that sources i, j continue to trade until the difference in marginal abatement costs is smaller than some prescribed ε. This would only require minor modifications to the model.

An implicit assumption of the above procedure is that sources i,j are able to agree on an exchange of permits if they have different marginal costs. Hence we assume no strategic behavior of the Parties, even though in reality some Parties could have the market power to exhibit such behavior. We also assume that there are no transaction costs and that we have perfect enforcement of the Protocol given the unreported emissions.

3. Data

To apply the model described in Section 2, we employ data on the costs of emissions reductions estimated from the POLES model (see Gusbin et al. 1999) for the countries (or group of countries) of USA, Japan, EU-15 and Central Eastern Europe (CEE)[1]. We also include Russia and Ukraine into the analysis. The cost functions for the latter two countries were derived from the results of POLES model for the Former Soviet Union, using additional information on emissions from Victor et al. (1998). All emission reduction cost functions employed in the numerical analysis only consider energy related carbon emissions reductions. Other carbon sources or GHG emissions are disregarded. The countries included in the analysis constitute the major participants of the Kyoto Protocol. Piecewise linear marginal cost functions were fitted to the dataset as shown in Figure 2.[2]

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[1] CEE includes Czech Republic, Hungary, Poland, Slovakia, Bulgaria and Rumania.

[2] We use the following notation: One metric ton (t), carbon (C), United States Dollars (USD), Million (M), and Billion (B).

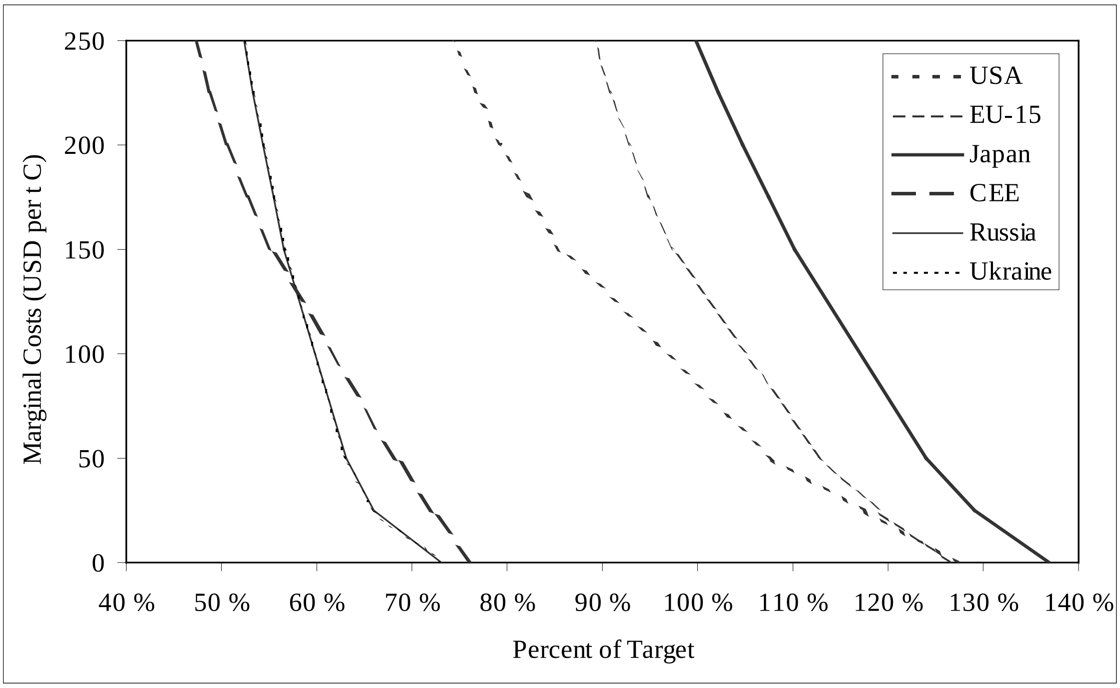

Figure 2. Marginal costs of reducing emissions as a function of the emission level relative to the Kyoto target (USD per tC).[3]

The availability of reliable data for unreported emissions and the costs of reducing the latitude for such activities is limited. We have therefore implemented some estimates for these figures. The IPPC reports global uncertainty ranges for fossil fuel emissions of around 10% (IPPC 1997). Estimates of the uncertainty for the USA vary between 4 and 10% whereas for Russia estimates range from 17 to 30% (Nilsson et al. 2000). For the other Parties we employ figures from Obersteiner (2000). We also made some simplifying assumptions on the costs of reducing them which fits well with the information on the monitoring costs of the sulfur emission-trading program in the USA (Klaassen and Nentjes 1997, p.135). The numerical results presented in Section 4 are based on these estimates and should therefore be considered as illustrative. The maneuvering room for underreporting emissions depends on the uncertainty on the emission estimates. We have employed unreported emission levels in year 2010 as a percentage of the Kyoto target according to the figures in Table 1.

Table 1. Unreported emissions as percentage of reported Kyoto target emission levels. Percent (%) and levels (MtC).

	USA	EU	Japan	CEE	Russia	Ukraine	Total
Share of target, σ (%)	10%	20%	15%	25%	30%	30%	19%
Level (Mt C)	133	173	44	67	195	53	665

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[3] Russia and Ukraine are difficult to separate, as they are almost superimposed on each other.

We also assume that the amount of unreported emissions in 1990 are zero. This seems reasonable as the Convention adopted this year as the base year for the commitments in order to avoid strategic behavior in emissions reporting. As the emission targets given in the Kyoto Protocol, are specified as a percentage of emissions in 1990, Parties have the incentive to overestimate rather than underestimate the emissions in this year.

The piecewise linear marginal cost functions of investing in monitoring infrastructure are parameterized in a very simplistic manner. For illustration, see Figure 1. Consider the marginal cost function of reducing reported emissions; $c_iN(x_i) = a_i + b_i*x_i$, and the marginal cost functions of reducing unreported emissions; $d_iN(u_i) = p_i + q_i*u_i$. We assume that the marginal cost of reducing unreported emissions at the initial levels defined in Table 1 are zero. Then we assume that the marginal cost of reducing the unreported emissions at any percent of the initial level ($u_i^0 = \sigma_i*K_i$) is the same as the marginal cost of reducing the reported emissions with the same percentage of the initial level (BAU_i). When employing this scheme, the values of the parameters p_i and q_i are such that $p_i = a_i$ and $q_i = -a_i/(\sigma* K_i)$. This formulation of $d_iN(u_i)$ represents therefore just a rescaling of the slope of $c_iN(x_i)$, such that $d_i(u_i^*) = \sigma[c_i(x_i^*]$ for all i.

4. Results

Below we present the results of various simulations. First we simulate the market in a traditional way excluding the unreported emissions. This is therefore just a numerical application on the scheme described in Ermoliev et at. (2000). Both the initial and equilibrium states as well as the dynamic process itself are illustrated. Then we include the unreported emissions, and, as we show in the base case, the end conditions after trade are independent of the particular random picks in each simulation provided that a sufficient number of bilateral trades are made; hence we focus here only on the initial and equilibrium states. Finally, we present some sensitivity analysis where only parts of the unreported emissions are included.

4.1. Perfect Observations of Emissions – Base Case

The base case is where we assume that unreported emissions are not included in the agreement. Parties can then comply with the protocol only by reducing emissions and by trading permits. Some of the key figures before the first and after the last trade are presented in Table 2.

Table 2. Emissions, marginal costs and total costs before and after trade.

	USA	EU	Japan	CEE	Russia	Ukraine	**Total**
Emissions (Mt C)							
Kyoto target	1,325	867	295	267	650	178	3,582
BAU	1,690	1,097	404	203	475	130	3,999
After trade	1,487	1,003	373	187	418	114	3,582
Traded	162	136	78	-80	-232	-64	0
Marginal costs (USD per tC)							
Before trade	85	133	248	0	0	0	
After trade	38.5	38.5	38.5	38.5	38.5	38.5	
Abatement costs (MUSD per yr)							
Em. red. costs no trade	13,468	13,032	10,873	0	0	0	37,373
Em. red. costs after trade	3,907	1,722	556	308	915	248	7,658
Total savings after trade							29,698

From Table 2 we see that CEE, Russia and Ukraine can meet their targets without implementing any control measures, as their targets are higher than their business as usual (BAU) emissions, giving rise to so-called “hot air”. Before trading, Japan has marginal costs of emission reduction of around 250 USD per tC, the EU of 130 and the USA 85 USD per tC. Total costs before trading are 37,400 MUSD. After trading, marginal costs settle around 38.5 USD per tC in all countries. More than 50% of the committed reductions in the USA, EU and Japan are bought from CEE, Russia and Ukraine during trade.

In the model, trade stops when the difference between the highest and lowest marginal costs among all countries is smaller than some prescribed value, here set to 0.1 USD per tC. The choice of first traded volume at each step k, $\Delta_{k,s=0}$ (see Section 2) was set to 10 MtC. It is tempting to interpret the marginal cost on emission reductions after trading as the permit price. However, in the sequential bilateral trading scheme there is no unique permit price that applies to all Parties. When Parties reach similar marginal costs, the motivation for trade is brought towards an end. The financial transfers that accompany the emissions transfers during the trading process determine the permit price in that particular trade, and are in our scheme, not specified. This will depend on the specific pair of Parties that are actually picked during the trading process and how they agree on a price. What we do know however is that the sum of the permit expenses plus revenues across all Parties is zero. Hence the total savings realized from trade is computable, and estimated to be 29,700 MUSD per year, or, total costs of reaching the aggregate target are reduced by approximately 80% as a result of trading. The relatively large reduction in total costs illustrates why carbon trading is attractive in an economic context. Figure

3 shows the dynamic process of a particular simulation and how total costs are reduced as the number of trades increase.

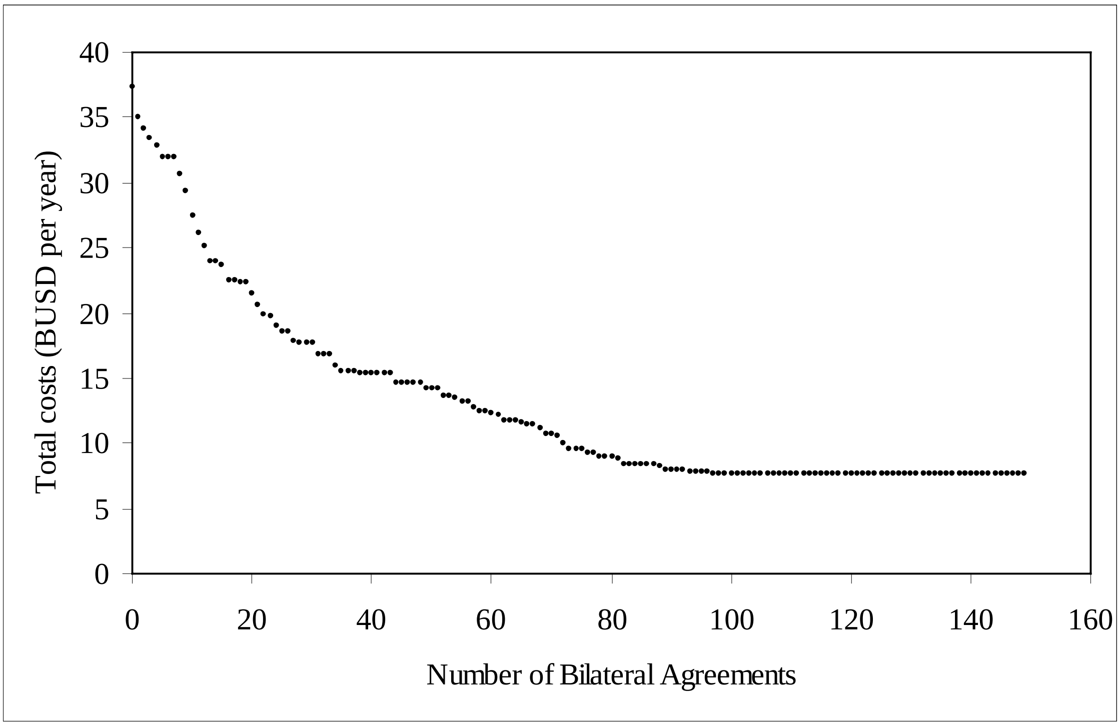

Figure 3. Total emission reduction costs as trade proceed (BUSD per year).

In Figure 3 we see that the cost reduction from each trade is larger in the early stages of trade than towards the end. The reason for this is illustrated in Figure 4. Parties have larger differences in marginal costs in the beginning, making the cost saving potentials greater than towards the end. This observation is typical for trading in many commodities. The total benefits from trading are larger the more different Parties are.

Figure 4 shows how the countries with the maximum and minimum level of marginal costs before each trade change as the number of bilateral agreements increase.

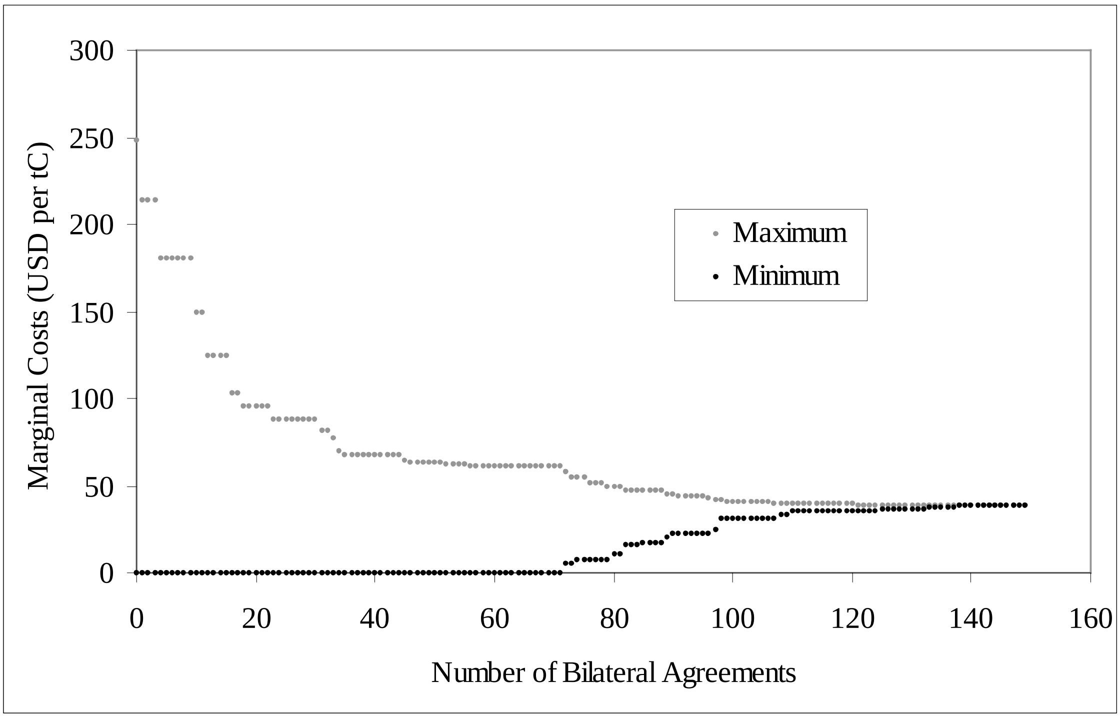

Figure 4. Highest and lowest marginal costs among the countries, as trade proceed (USD per tC).

There is “hot air” in the system as long as one country has zero marginal costs. In this particular simulation the hot air was not absorbed before the 72nd agreement. The shape of the graphs in Figure 3 and Figure 4 are dependent on choices of random picks. To give a better view of how the path towards equilibrium varies in different simulations, the model was run 50 times. To make the simulations less time consuming, trade stopped when the maximum difference in marginal costs was 1 USD per tC. The results are shown in Figure 5. In the simulation with the slowest convergence trade stopped after 153 bilateral agreements, the fastest at 80 trades, 90% being between 93 and 143 trades. Half of the cost reduction potentials were on average obtained after 19 bilateral agreements, again illustrating that trade is most efficient in the beginning of the process.

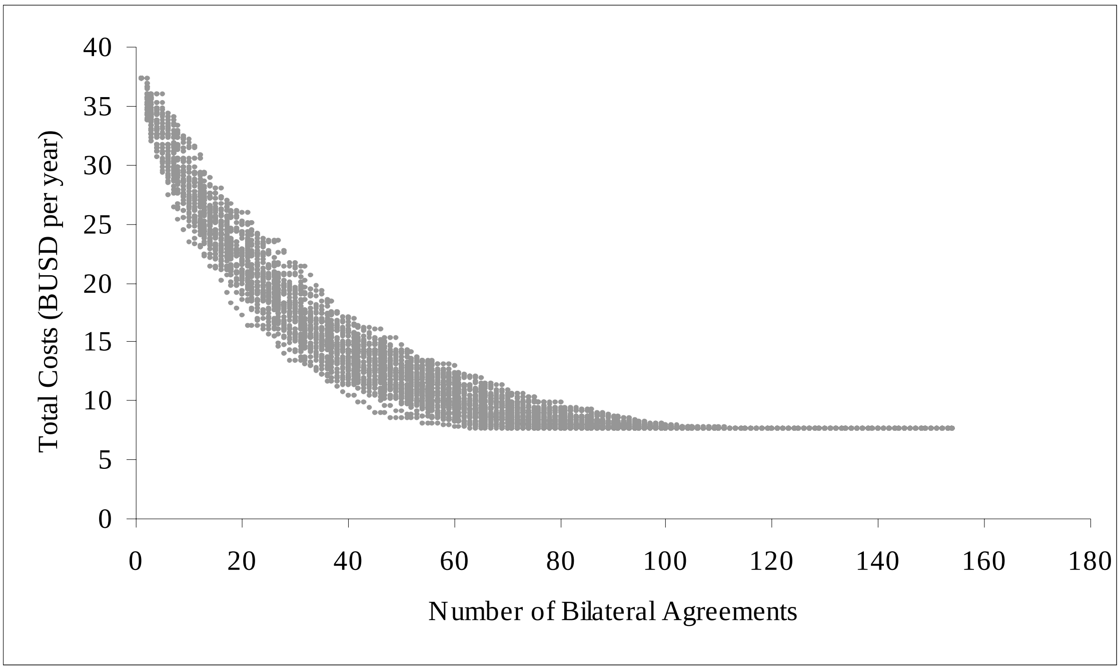

Figure 5. Change in total costs as a function of the number of bilateral agreements in 50 simulations (BUSD per year).

To get a more precise picture of how the variation changes, the standard deviation of the total costs on emission reduction and monitoring as trading proceeds, is plotted in Figure 6.

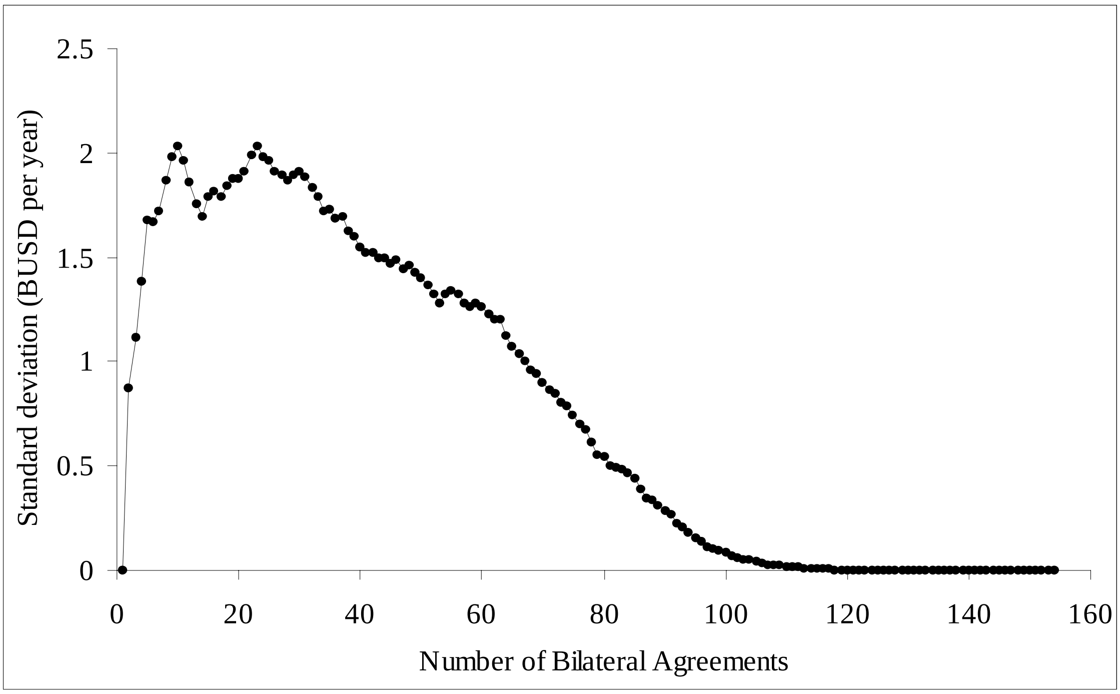

Figure 6. Standard deviation as a function of the number of bilateral agreements in 50 simulations.

From Figure 6 we see that the standard deviation towards the end of the simulations is zero. Hence, the equilibrium state is independent on the nature of each specific simulation. The maximum spread is in the beginning of the trading, as the total costs here are very dependent on the specific picks that are carried out. As trading proceeds and Parties close up on differences in marginal costs and the reduction of costs is less dependant on the particular pair picked, hence the spread decreases.

4.2 *Imperfect Observations of Emissions*

We now introduce imperfect observation of emissions into the calculations. The rules of the agreement are now such that Parties need to find the least cost combination of reducing reported emissions, x, reducing the unreported emissions, u, (by investing in monitoring) and by trading in emission permits, y, such that constraint (8) is satisfied. In Table 3 we present the initial situation before trading starts but after each Party has optimized between reducing reported and unreported emissions.

Table 3. Emissions, marginal costs and total costs before trading with imperfect observation of emissions.

	USA	EU	Japan	CEE	Russia	Ukraine	Total
Emissions, (MtC)							
Reported	1,222	744	263	201	461	126	3,017
Unreported	103	123	32	66	189	52	565
Total emissions	1,325	867	295	267	650	178	3,582
Marginal costs (USD per tC)							
Of emission reduction	119.3	223.0	360.0	5.2	7.8	7.4	
Of monitoring	119.3	223.0	360.0	5.2	7.8	7.4	
Costs (MUSD per year)							
Emission reduction costs	23,946	38,152	20,822	5	55	14	82,994
Monitoring costs	2,395	7,630	3,123	1	16	4	13,170
Total costs (excl. permits)	26,341	45,782	23,945	6	71	18	96,164

Table 3 shows that marginal cost within each country on reducing reported emissions and on investing in monitoring are equal, in accordance with the outline in Section 2. The marginal costs are in the range of 5 –360 USD per tC, considerably higher than when only reported emissions are included in the targets. This is because the need for reductions in reported emissions is now considerably larger. We also note that the “hot air” we had in the previous simulation now is eliminated since the unreported emissions are now added to the baseline emissions which implies that e.g. for Russia the sum of the unreported and reported emissions now exactly equals the Kyoto target. Table 3 also shows that without trading the monitoring costs would be 13.2 billion USD and abatement costs around 83 billion USD. This implies that the monitoring costs would be around 16% of the abatement costs. This estimate fits very well with the costs of (continuous emission) monitoring of the sulfur trading program in the USA that were estimated to range from 8 to 13% of the abatement costs (Klaassen and Nentjes 1997).

Table 4. Emissions, marginal costs and aggregate costs after all trades with imperfect observation of emissions.

	USA	EU	Japan	CEE	Russia	Ukraine	Total
Emissions, (MtC)							
Reported	1,193	871	333	154	376	103	3,031
Unreported	100	144	38	52	170	47	551
Total emissions	1,293	1,016	371	206	547	150	3,582
Amount traded	-32	149	76	-61	-103	-28	0
Marginal costs (USD per tC)							
Of emission reduction	129.3	129.3	129.4	129.4	129.4	129.3	
Of monitoring	129.3	129.3	129.4	129.4	129.4	129.3	
Costs (MUSD per year)							
Emission reduction costs	27,660	12,446	3,801	3,001	4,288	1,159	52,355
Monitoring costs	2,766	2,489	570	750	1,287	348	8,210
Total costs (excl. permits)	30,426	14,935	4,371	3,751	5,575	1,507	60,565
Total savings due to trade							35,600

After trading, marginal costs reach 129 USD per tC or about 3 times higher than in the base case. This is because all Parties now have to do considerably more abatement, as unreported emissions have to be accounted for. An important difference compared to the case of perfect emission observation is that the USA now becomes a net permit supplier; even though the quantity is small. This is because the rate of change in marginal costs in USA is lower than in the EU and Japan and because the assumed uncertainty (the share of the unreported emissions in the USA) is lower. Comparing the situation before and after trade we see that the amount of unreported emissions decreases by approximately 2.5 %. Trading in permits leads to increased emission abatement and monitoring investments in countries with relatively large unreported emissions. This effect on total unreported emissions is larger than the effect of higher unreported emission levels in permit importing countries, because they have relatively smaller uncertainties. The total amount of permits sold on the market is lowered from 376 to 225 MtC as compared to the base case simulation. This has to do with the fact that unreported emission levels in countries with low marginal costs are larger than in the high abatement cost countries, leading to a reduction of the optimal amount of emission traded in order to reach equilibrium where marginal costs are equal. Total abatement costs after trade are approximately 8 times higher compared to the situation where (the estimated) unreported emissions would have been excluded. The marginal cost functions on emission reduction are, as shown in Figure 2, quite linear in the range 38.5 to 129 USD per tC. Therefore when the marginal costs in the equilibrium state increase by approximately three times, the level of costs, i.e. the area under the marginal cost curve would be expected to increase about nine times, as this relationship is quadratic. However the additional option to meet targets through investing in monitoring and thus reducing unreported emissions reduces this factor from nine to eight. The cost savings due to trading in this case are 35.6 BUSD, or 37% of the no trade costs.

Unreported emissions will be present whether they are included in the commitments or not. However, when they were to be included in the Protocol, the total emission levels without abatement would be higher and the required emission reductions larger, changing the constraint of the minimization problem (equation (2) in Section 2). The results in Table 2, 3 and 4 should therefore be compared cautiously as they describe solutions of two very different minimization problems.

4.3. Sensitivity Analysis

Bearing in mind the reluctance of some Parties to ratify the Kyoto Protocol as it currently is formulated, it seems quite unrealistic that these Parties will agree to a scheme where all estimated unreported emissions are included in each Party’s target as described above. The costs of complying with the agreement will increase dramatically for some Parties. We have assumed that the latitude for strategic underreporting depends on the level of uncertainty in the emission accounts for the commitment period. However, as the applied conversion factors when estimating emission levels are likely to apply to both 2010 and 1990 emission levels, one could argue that the uncertainty in emissions trends might be a better indicator for unreported emissions. For example, if a Party uses a relatively high conversion factor on a particular emission activity on the 1990 data, implicitly increasing their Kyoto Target, this same conversion factor will be used on 2010 emission levels, reducing the benefits of increasing the 1990 emission levels. Rypdal and Zhang (2000) estimate that the level of uncertainty on Norwegian emission levels in 2010 is 17% (within two standard deviations) whereas the trend in uncertainty from 1990 to 2010 is only 4%.

On this basis we explore some more moderate versions of the above scheme. We simulate the model assuming that only a specific part of the unreported emissions are included (10%, 20%…). This is equivalent to saying that given that there are different estimates surrounding the uncertainty, instead of the using the highest estimate the Parties to the Protocol may agree on using the median or even lower estimates. The final marginal cost in each case is shown in Figure 7.

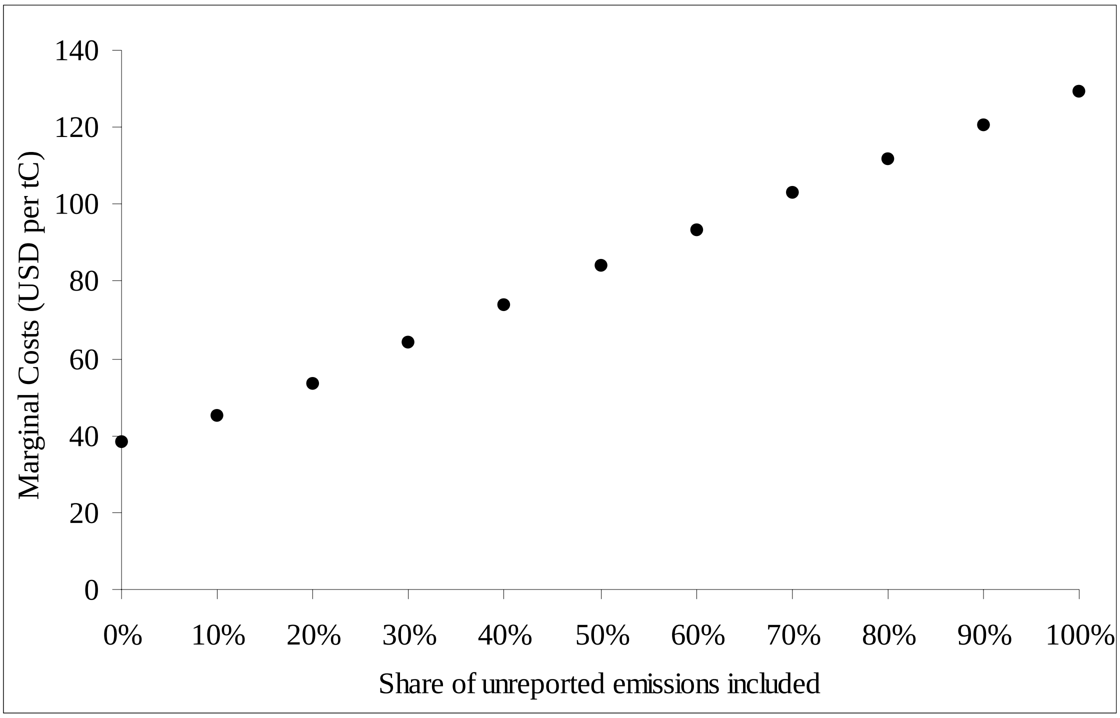

Figure 7. Marginal costs after trade versus amount of unreported emissions included in the agreement (USD per tC).

The points in Figure 7 between 50 and 150 USD per tC lay on a straight line, because the piecewise linear cost functions in this range as presented in Figure 2 include only one line. With a finer grid on these functions in this interval, the curve in Figure 7 would increase more rapidly.

Furthermore, to give some indications on how the various Parties may gain or lose from including various amounts of the unreported emissions into the scheme, we estimate the total costs for each Party including the expenses/revenues from the permit trade. As pointed out above, we have no unique permit price in our scheme. In the following calculations we therefore apply the marginal cost after trade as a proxy for this parameter.

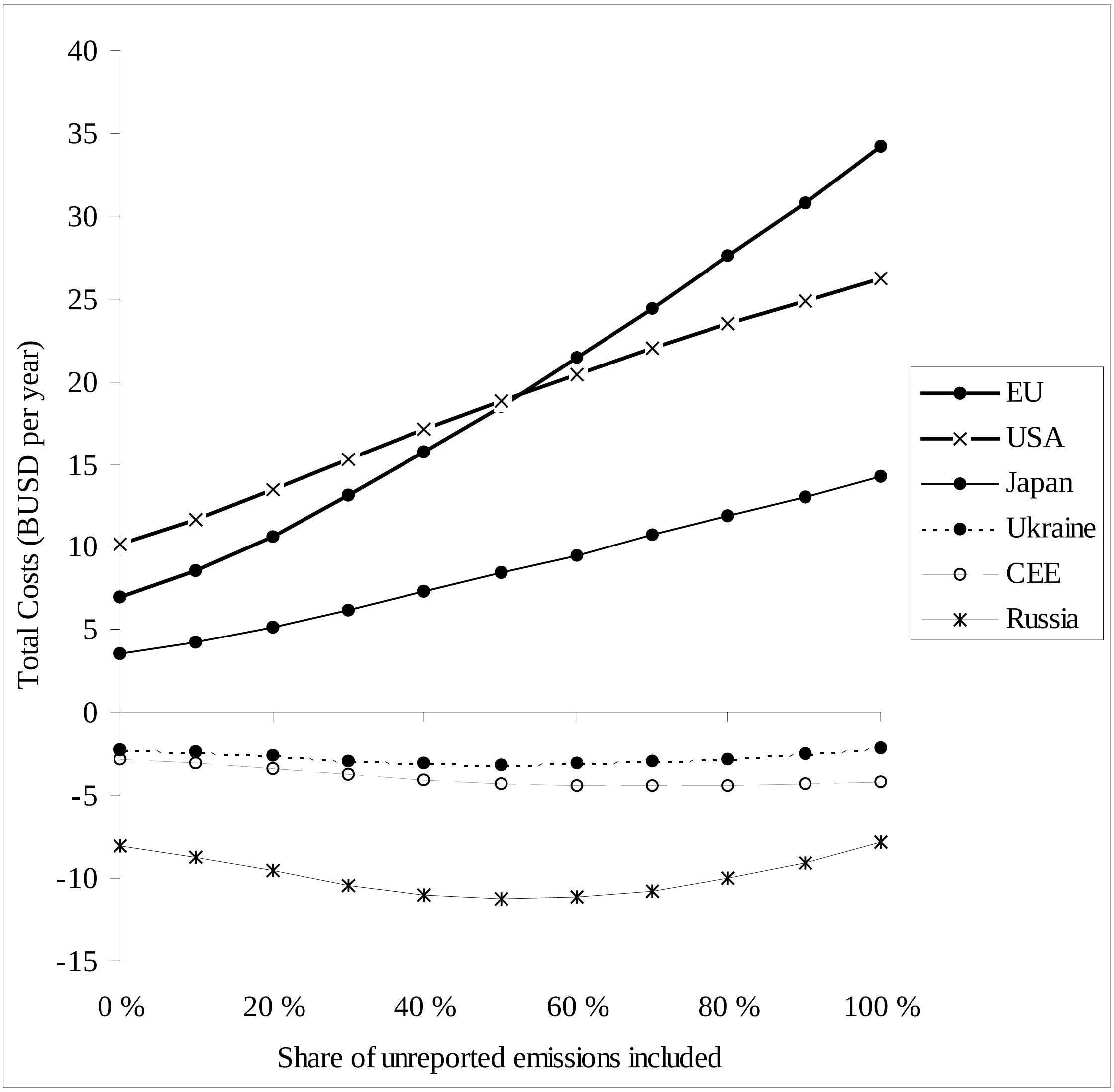

Figure 8. Change in the net total costs as a function of the share of unreported emissions included in the Protocol after trading, (BUSD per year).

Figure 8 shows that as the amount of unreported emissions increases, the costs raise quite rapidly for USA, EU and Japan. In these cases, the costs increase both because more reductions will be carried out at home and because the expenses on emission permits purchased increases as all Parties have higher marginal costs. In terms of total costs, EU is the most severely affected due to the assumption of larger unreported emissions in combination with a steep marginal cost function.

For Russia, Ukraine and CEE, the (net) costs of complying with the Protocol before including unreported emissions are negative, i.e. they make positive profits. This is because the value of the permits sold is higher than the costs of emission reductions and monitoring investments. The more surprising finding here is that these countries may actually benefit from including unreported emissions in the agreement even though these countries are assumed to have the highest level of unreported emissions. As unreported emissions are gradually included, this negative effect on profits is dominated by the increase in permit price, which contributes to higher profits on permits sold. In the Russian and Ukraine cases, profits are increased to its maximum (being 50% higher than in the base case without unreported emissions) when half of the unreported emissions were to be included in the Protocol. From this point, the effect of the increased permit price is less important than the need for reductions, making profits fall again approximately back to the initial level. For CEE however, the positive effect from the higher permit price dominates in all cases the effect of the need for larger reductions. This is because CEE is assumed to have a lower share of unreported emissions than Russia and Ukraine.

5. Concluding Remarks

The objective of this paper was to examine analytically the convergence of the carbon permit market given imperfect observations of emissions. On this scheme we applied data for the major Parties of the Kyoto Protocol. We have also provided a numerical illustration of the path to convergence based on the scheme of sequential bilateral trade with perfect observations of emissions as described by Ermoliev et al. (2000). To meet our objective, this scheme was then broadened to include unreported emissions. Proof of convergence was given. Numerical calculations on this scheme were also provided.

Our main finding when unreported emissions are included in the agreement and when countries can reduce these through investing in monitoring is that marginal emission reduction costs increase. Compliance costs may increase significantly in the USA, the EU and Japan since the options for buying cheap emission permits are restricted. Quite surprisingly we find that Russia, Ukraine and CEE might experience financial gains when unreported emissions are included in the Protocol commitment. This is so because the resulting rise in the permit price and associated revenues outbalances the need for additional monitoring and domestic emission reduction costs due the larger unreported emissions. We also find that trading in carbon permits may also lead to a reduction in the amount of unreported emissions.

It is important to note the limitations of this analysis. First of all, not all Parties of the Kyoto Protocol are included in our numerical analysis. The omitted (industrialized) countries are in aggregate likely to be net buyers of permits, which gives rise to higher equilibrium marginal cots than our results indicate. Moreover, the opposite effect would be expected if emission trading would be expanded to developing countries since these are likely to act as net sellers thus reducing the permit price (compare Gusbin et al. 2000). Including more carbon sources and sinks, as well as emissions of other greenhouse gases would also improve the numerical analysis.

Furthermore, it is important to bear in mind that the data on business as usual emissions are uncertain, as are the marginal cost functions for reducing reported emissions. Especially the figures used for the unreported emissions are uncertain. This is also, but perhaps to a smaller degree, valid for our estimate of the monitoring costs since these correspond reasonable well to estimates of the monitoring costs of the existing sulfur trading program in the USA. Moreover, in our numerical analysis we have assumed that unreported emissions come additional to business as usual projections for reported emissions. This is not obvious, but given the lack of available data we consider this assumption to be as good as any other particular one. Finally, we would also expect that the cost functions for reducing reported emissions are dependent on how unreported emissions are modelled. Such changes in cost functions were not considered here. Although the actual levels of costs when unreported emissions are included should be considered as illustrative, our findings on the direction of change are probably more reliable.

Nevertheless, it is good to recall that the treatment of the unreported emissions and the associated compliance is still an open issue in the Kyoto Protocol. In spite of the above limitations, this study has given some insights on how the distribution and level of compliance costs may change (and perhaps significantly) if unreported and uncertain emissions were to be included in the Kyoto Protocol.

References

Ermoliev, Y., M. Michalevich and A. Nentjes (2000), ‘Markets for Tradable Emission and Ambient Permits: A dynamic Approach’. *Environmental and Resource Economics* 15: 39--56.

Gusbin, D., G. Klaassen and N. Kouvaritakis (1999), ‘The costs of ceiling on Kyoto flexibility’. *Energy Policy.* 27 (14), 833--844.

IPCC (1997), *Revised 1996 Guidelines for National Greenhouse Gas Inventories* (3 Volumes). Intergovernmental Panel on Climate Change, WGI Technical Support Unit, Bracknell, UK.

Jonas, M., S. Nilsson, M. Obersteiner, M. Gluck, and Y. Ermoliev (1999), ‘Verification Times Underlying the Kyoto Protocol: Global Benchmark Calculations’. Interim Report, IR-99-062, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Klaassen, G. and A. Nentjes (1997), ‘Creating markets for air pollution control in Europe and the USA’. *Environmental and Resource Economics,* 10: 125--146.

Montgomery, D.W. (1972), ‘Markets in Licenses and Efficient Pollution Control Programs’. *Journal of Economic Theory,* 5, 395--418.

Nilsson, S., A. Svidenko, V. Stolbovoi, M. Gluck, M. Jonas and M. Obersteiner (2000), ‘Full carbon account for Russia’. IR-00-021, IIASA, Laxenburg, Austria. Available at http://www.iiasa.ac.at.

Obersteiner, M. (2000), Personal Communication. IIASA, Laxenburg, Austria.

Obersteiner, M., Y.M. Ermoliev, M. Gluck, M. Jonas, S. Nilsson and A. Shvidenko (2000), ‘Avoiding a Lemons Market by Including Uncertainty in the Kyoto Protocol: Same Mechanism - Improved Rules’. Interim Report, IR-00-043, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Rypdal, K. and L-C. Zhang (2000), ‘Uncertainties in the Norwegian Greenhouse Gas Emission Inventory’. Reports 2000/13, Statistics Norway, Oslo, Norway.

UNFCCC (1997), ‘Kyoto Protocol to the United Nations Framework Convention on Climate Change’. Available at http://www.unfccc.de/.

UNFCCC (1992), ‘United Nations Framework Convention on Climate Change’. Available at http://www.unfccc.de/.

Victor, D., N. Nakicenovic and N. Victor (1998), ‘The Kyoto Protocol Carbon Bubble: Implications for Russia, Ukraine and Emission Trading’. IR-98-094, International Institute for Applied System Analysis, Laxenburg, Austria. Available at http://www.iiasa.ac.at.

Appendix: Convergence Analysis

In this Section we provide proof of the convergence of the sequential bilateral trading procedure. The setup follows Ermoliev et al. (2000). Given sequences of permits $y^k = (y_1{}^k,...,y_n{}^k)$ and the social cost values $F(y^k) := \sum_{i=1}^{n} f_i(y_i^k)$ described in Section 2. Consider the social objective to find the minimum value of the total costs $F(y)$ subject to $\sum_{i=1}^{n} y_i = 0$ in a decentralized manner. To accomplish this, we adopt the procedure of sequential bilateral trade as described in Section 2 with, random matching.

Theorem. Convergence of y to the social minimum of F.

Let Y^* be the set of cost effective permit allocations satisfying constraint (8) and $\{y^k\}$ be the result of iterated bilateral trade up to step k. Assume that the set of cost effective permit allocations is non-empty. Then either

(i) $y^k \in Y^*$ after a finite number of steps, or

(ii) the sequence $\{F(y^k)\}$ converges to its minimum value and all cluster points of $\{y^k\}$belong to Y^* or

(iii) if Y^* contains only a single point Y^*, then $\{y^k\}$ converges to this point.

Proof:

Throughout we assume that $K_i + y_i{}^k > \varepsilon$ and similarly, $K_i + y_i{}^* > \varepsilon$ for some $\varepsilon > 0$ and i = 1,…,n, k = 0,1,…The optimality condition for y^* are the following: $y^* = (y_1{}^*,...,y_n{}^*)$ is an optimal vector of permits if and only if

$$f_i'(y_i{}^*) = f_j'(y_j{}^*) \text{ for all i,j.} \qquad (14)$$

As it was shown in Section 2, the sequence $\{F(y^k)\}$, k = 0,1,…is monotonically decreasing. Since the sequence $\{F(y^k)\}$ is bounded, there exists a limit $F^* = \lim_k F(y^k)$. Let us prove that $F^* = F(y^*)$. Note that if $f_i'(y_i{}^k) = f_j'(y_j{}^k)$ for all i,j =1,…,n, then according to (14) the current distribution of permits is optimal and (i) is proved. Assume that $F^* \neq F(y^*)$. Then there exists a $\delta > 0$ and N such that

$$\max_{i,j}\left[f_i'(y_i{}^k) - f_j'(y_j{}^k)\right] > \delta \text{ for all } k > N$$

According to the simplifying assumption that $K_i + y_i^k > \varepsilon$, we can choose a small enough $\alpha > 0$ such that the vector y^{k+1} given by:

$$y_i^{k+1} = y_i^k, i \neq i_k, j_k \text{ and for } i = i_k, y_i^{k+1} = y_i^k - \frac{\alpha}{k+1}, i = j_k, y_j^{k+1} = y_j^k + \frac{\alpha}{k+1}$$

satisfies constraint (8). In general we have

$$f_j'(y_j^k) = \lim_{\Delta_k \to 0} \frac{f_j(y_j^k + \alpha/(k+1)) - f_j(y_j^k)}{\alpha/(k+1)},$$
$$f_i'(y_i^k - \Delta_k) = \lim_{\Delta_k \to 0} \frac{f_i(y_i^k) - f_i(y_i^k - \alpha/(k+1))}{\alpha/(k+1)}.$$

In accordance with the mean value theorem (similar to Section 2) it follows that

$$F(y^{k+1}) = F(y^k) - \frac{\alpha}{k+1}[f_i'(y_i^k) - f_j'(y_j^k)] + o(\frac{1}{k+1}) \qquad (15)$$

where the value of: $o(\frac{1}{k+1}) = \sum_{l=1}^{\infty} (-\Delta_k)^l \frac{1}{l!} f_i^{l+1}(y_i^k)$, is negligible.

Then from (15) it follows that for k > N

$$F(y^k) < F(y^N) - \delta\alpha \sum_{S=N}^{K-1} \frac{1}{s+1} \qquad (16)$$

Since $\sum_{S=0}^{\infty} \frac{1}{s+1} = \infty$ it follows from (16) that $F(y^k) \to -\infty$ for $k \to \infty$ what contradicts the convergence of $\{F(y^k)\}$. Thus $F(y^k) \to F(y^*)$. From this follows that all cluster points of the bounded sequence $\{y^k\}$ belong to Y^*. Hence if Y^* is a singleton, then $\{y^k\}$ converges to y^*.