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APPLICATION OF THE KALMAN FILTER TO CYCLONE FORECASTING

3. HURRICANE FORECASTING
4. ADDITIONAL TYPHOON FORECASTING

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Preface

This is the second part of a two-part report for application of the Kalman filter to cyclone forecasting in which hurricane experiments and further typhoon experiments are presented. The first part was published as RM-76-9.

Abstract

This is the second part of a report on application of the Kalman filter to cyclone forecasting. Following the preliminary experiments of typhoon forecasting, this paper presents the results of hurricane experiments and further typhoon experiments.

The 12 and 24 hour forecasting NHC72 model and the 24 hour forecasting SNT model developed by the National Hurricane Center, NOAA, USA and the Japan Meteorological Agency, respectively, were examined. The improvements obtained by using the Kalman filter over the original models were found to be roughly 10% for hurricane forecasting and 20% for 24 hour typhoon forecasting, on the average, in terms of vector errors.

The conclusion drawn by the previous experiments was reconfirmed. That is, the application of the Kalman filter to utilize better simple linear regression models is effective when the original regression model gives consecutively biased forecasts for a considerably long time; it is not effective when the performance of the original model is poor, yet its residual errors are not highly correlated.

In addition to this conclusion, a statistical test of the validity of forecasting regression models showed that the structure of the model should be further improved before considering application of the Kalman filter.

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TABLE OF CONTENTS

I. INTRODUCTION

In the previous report of this study (Takeuchi, 1976), which will hereafter be referred to as Part I, the Arakawa's 12 hour SFC.700 mb model and the 24 hour SNT model were examined. Application of the Kalman filter to both of these models showed very successful results. The conclusion reached was that the Kalman filter is a powerful tool to make the simple linear regression model applicable to a nonstationary process, provided that the residual errors of the regression model are highly correlated.

Since the number of cases examined in the previous paper was limited, i.e., only to typhoons formed in the northern Pacific in August of 1974, it was necessary to examine further other cases.

The purpose of this paper is, then, to apply the same technique for hurricanes and other typhoons. The NHC 72 model developed by the National Hurricane Center, NOAA, in the USA, and the SNT model by the Japan Meteorological Agency were used in the experiment. The hurricanes formed during the period of 1945 through 1973 and the typhoons during the period of June through September of 1974 were used as data; the types obtained are listed in Table 3 of Part I. Not all of the available data were used; but those of cyclones which lasted for relatively long periods were arbitrarily chosen to reduce the computational burden. This included eight hurricanes (about 55 days all together) and seven typhoons (about 60 days). In the hurricane analyses, both 12 hour and 24 hour forecasts were examined, but in the typhoon analyses, only 24 hour forecasts were examined. This is because a large number (over 30) of 12 hour typhoon forecasts were examined in Part I, whereas the number of cases for 24 hour forecasts processed was only 10. Furthermore the formula used to obtain 500 mb gph values that were used as predictors of the 24 hour SNT model were later found inadequate and a revised calculation was necessary.

Section II presents the results of the hurricane analysis; the typhoon results follow in Section III. In Section II some modification to the original methodology described in Part I was added. This is an approach to compute the initial error covariance matrix $P_{0|0}$ of state variables in the Kalman filter directly from the original data set. A slight improvement was achieved by this approach. During the hurricane analyses, it was found that the original forecasting model, formulated as a regression equation, did not adequately represent the dynamics of cyclone advancement and that standard statistical tests did not justify the validity of the model. The discussion of this test is also included in Section II.

II. HURRICANE FORECASTING

The U.S. National Hurricane Center model NHC-72 was developed for four stratified data sets according to the initial motions of hurricanes. Two characteristics of initial storm motion, the direction θ and the speed v were considered, where θ was measured in degrees clockwise from the north and v was measured in knots. Let $x = v \sin \theta$ and $y = v \cos \theta$. Compute $Y_1 = \frac{2}{9} x + 6$ and $Y_2 = -\frac{9}{2} x - 3$. Storms were then classified into 1, 2, 3 and 4 quadrants, respectively (Neumann et al., 1972, p. 8) when

$$y \geq Y_1 \quad \text{and} \quad y \geq Y_2 \;,$$

$$y \leq Y_1 \quad \text{and} \quad y \geq Y_2 \;,$$

$$y < Y_1 \quad \text{and} \quad y < Y_2 \;, \text{ or}$$

$$y \geq Y_1 \quad \text{and} \quad y < Y_2 \;.$$

The hurricanes examined in this study are only those in the first quadrant. Roughly speaking, their initial motions are north-easterly in direction. The twelve hour and twenty-four hour forecasts were examined. The models used are described in Table 1.

Results Using the Same Procedure Described in Part I

Initial experiments were conducted using the unknown error covariance matrices Q, R, $P_{0|0}$, as described in Part I; namely, the variance was assumed to be proportional to the magnitude of the coefficients. This assumption can also be referred to as an equal coefficient of the variation assumption. This is because the variance σ_i^2 of the initial estimate $\hat{x}^i_{0|0}$ was assumed to have a coefficient of variation γ_1 for all i, or

$$\frac{\sigma i}{\hat{x}^i_{0|0}} = \gamma_1 \text{ for all } i \quad . \tag{1}$$

Using this assumption and the shaping filter only for the system disturbances, in other words, using the filtered model A (Part I, p. 19), some improvement of forecasts was found. For the 12 hour forecasts, 19% improvement in longitudinal motion (λ) and none (1%) in latitudinal motion (φ). For 24 hour forecasts, they are 9% and -1%, respectively. The vector error reductions were 9% for 12 hour forecasts and 4% for 24 hour forecasts. Prediction of some hurricanes was improved by the magnitude of 40% in one of the two components, but in some cases the filtered model forecasts were -15% worse than the original forecasts. Table 2 shows the summary of results. Also note that the significant improvement in the HUR134[1] is decisive for average improvement. Therefore, a glance only at the average improvement is somehow misleading. The optimal coefficients α, β, γ_1, and γ_2 were found in some cases, quite similar to and, in other cases, considerably different from those identified for typhoon studies in Part I.

The results obtained were unsatisfactory. What follows is the report of the efforts made to improve the forecasts and the analyses of the reasons why a greater improvement was not obtained by using the Kalman filter.

Further Analysis of Error Covariance Matrices

The assumption of equal coefficient of variation seems reasonable under an intuitive judgement. However, how reliable is it? How reasonable is the assumption of zero cross correlations? There is only one way to examine this assumption; this is to compute the error covariances of the state variables using the given observation data. The state variables are made up of two different sets of variables. One is the coefficient of the given regression model and the other is the system disturbances. Their covariance matrices are P_1 and P_2, forming a large matrix $P_{0|0}$ as

$$P_{0|0} = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \tag{2}$$

While P_2 and the covariance matrices Q of the system disturbances cannot be estimated from the given observation data, it is possible to estimate the P_1 and R of the measurement noise.

The original forecast model has a simple linear regression form:

$$y = x\beta + e \tag{3}$$

where y is the predictant, the displacement of a cyclone eye; x is the vector of the predictors; β is the corresponding coefficient vector and e is the noise. In the Kalman filter model, the coefficient β is treated as the state variable. The error covariance of the initial state variables is, therefore,

$$E(\hat{\beta} - \beta)(\hat{\beta} - \beta)^T \tag{4}$$

where $\hat{\beta}$ is the estimate of β. Let Y and X be the observation vector and matrix whose components are realization of y and x. Then

$$Y = X\beta + \epsilon \tag{5}$$

where ϵ is now an error vector. The least squared error estimate of β is

$$\hat{\beta} = (X^T X)^{-1} X^T Y \quad . \tag{6}$$

The error covariance associated with $\hat{\beta}$, or the expression (4) becomes

$$E(\hat{\beta} - \beta)(\hat{\beta} - \beta)^T = (X^T X)^{-1} X^T E(\epsilon\epsilon^T) X (X^T X)^{-1} \tag{7}$$

If each component of noise ϵ is considered independent and distributed identically with variance σ^2,

$$E(\epsilon\epsilon^T) = \sigma^2 I \tag{8}$$

where I is an identity matrix. Then (7) becomes

$$E(\hat{\beta} - \beta)(\hat{\beta} - \beta)^T = \sigma^2 (X^T X)^{-1} \quad . \tag{9}$$

σ^2 may be estimated by

$$\hat{\sigma}^2 = \frac{(Y - X\hat{\beta})^T (Y - X\hat{\beta})}{n-p} \tag{10}$$

where p is the dimension of vector β and n is the number of observations. Instead of assuming the equal coefficient of variation γ_1, (9) can be used for the initial estimate of error covariance P_1 in (2). Since the regression model (3) is used as the measurement equation in the Kalman filtering model, the measurement error R can also be replaced by $\hat{\sigma}^2$. Strictly speaking, this is not quite correct, because R is the measurement error variance after considering the dynamic changes of state variables. The estimate of σ^2 using (10) may, however, be postulated to be not very different from R. It should be noted that the artificial coefficients γ_1 and β are now set equal to unity, which greatly reduces the optimization procedure.

The next task is to assume the P_2 and Q. They are assumed to be similar to the procedure described in Part I; namely

$$\begin{aligned} P_2 &= \gamma_2^2 P_1 \\ Q &= \alpha^2 P_1 \end{aligned} \tag{11}$$

where γ_2 and α are the parameters to be optimized under the criterion described in Part I (p. 16). Another assumption for consideration is the off-diagonal elements of P_1. In P_1, the off-diagonal elements are used as they are estimated. However, in P_2 and Q, they are set equal to zero. In other words, only the diagonal elements of P_1 are introduced into P_2 and Q. This procedure was introduced because P_2 and Q were actually unknown and, above all, the off-diagonal elements of $P_{0|0}$, other than P_1 and P_2, are arbitrarily set to zero, or

$$P_{0|0} = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$$

which introduces additional errors. $P_{0|0}$ can, of course, be set out as

$$P_{0|0} = \begin{bmatrix} P_1 & \gamma_3^2 P_1 \\ \gamma_3^2 P_1 & \gamma_2^2 P_1 \end{bmatrix} .$$

But this assumption was not selected. The estimate of P_1 would, afterall not be so reliable as to be used intensively, and therefore creating a cumbersome optimization procedure for γ_3 is useless.

Table 3 lists the summary of results for 12 hour forecasts, 17% (19%) in λ and 8% (11%) in φ were improved. The vector error reduction was 11% (15%). For 24 hour forecasts, 11% (13%) in λ, -0.5% (0.7%) in φ, and 5% (8%) in the vector error. The figures in parentheses are the improvements in terms of root mean squares whereas the preceeding values are improvements in terms of the mean of the absolute errors.

The improvements of this trial over the previous trial are not great, but undoubtedly positive. A slight decline in the improvement of λ component of the 12 hour forecasts can be explained due to the incomplete optimization of the parameters α and γ_2. In fact, only the cases $\alpha = \gamma_2$ are examined; altogether only five cases. A case such as $\alpha < \gamma_2$ should certainly have been examined.

At any rate, the improvement was not satisfactory and the next task is to analyze the reason.

Performance of Filtered Forecasts

Figure 1 shows the λ displacements of HUR72 of which the 24 hour filtered forecasts showed the worst decline of the entire 24 hour forecasting cases. From Figure 1a one can see that the overall NHC72 forecasts are not unreasonable. The first part is unsatisfactory, but in the latter part, from the time point 4 and up, the original forecasts are excellent. The filtered forecasts, on the other hand, are not at all good, especially at the time points 4 and 5.

Figure 1b illustrates the reason why this extraordinary misprediction occurred. For the time point 3, the NHC72 had forecast 120 n.m. to the east (B). In fact the observed displacement was 100 n.m. to the west (A). The filtered forecast was 70 n.m. to the east which realized a 190 n.m. easterly error (C). At this point, the forecast for the time point 4 should be made. The NHC72 indicates this should be 120 n.m. to the west (B´). Now, it is time for the filtered model to make the forecast based on the scheme of the NHC72 model. The NHC72 forecast indicates about a 220 n.m. westerly change as compared to the previous forecast. This is obviously due to the drastic change in meteorological synoptic conditions. Without having knowledge of the meteorological dynamics, it is necessary to rely on the NHC72 models' forecasting scheme. Namely, the filtered model also considers a 220 n.m. westerly change from its previous forecast, which leads to point C_1. This is because the filtered model does not have any mechanism to check whether the meteorological change observed really leads to such a large change in the hurricane motion. However, the filtered model can adjust itself by improving predictions based on the previous experience of the error. The previous error was 190 n.m. to the east; this is extremely important. The filtered model had been adjusting itself based on the preceeding errors. It had 190 n.m. biased to the east at time point 3. Naturally, the filtered model postulates that the current forecast may also have an easterly bias. In other words, the filtered model should adjust itself to forecast a displacement slightly more west than predicted. The Kalman filter indicates how much adjustment should be made--it is not 190 n.m. but 65 n.m. This figure is calculated by the Kalman gain vector based upon the error covariances P, Q and R. As a result, the new forecast of the filtered model is 65 n.m. west of point C_1, or point C'. Unfortunately however, the displacement was not at all easterly but in fact, westerly. Therefore, a 65 n.m. adjustment created more errors than would have been created with no adjustment.

One may suggest a lesser adjustment. This is not appropriate, however, if the cases at the time points 1, 2 and 3 are examined. In these cases, the adjustments are insufficient. In fact, the adjustment at time point 5 is also insufficient. The rate of adjustment should be based on an average. This rate is controlled by the assumptions of $P_{0|0}$, Q and R.[2]

From the examination of Figure 1 it may be obvious that the Kalman filter can adjust the forecasts correctly only when the original model gives consecutively biased forecasts in the same direction. In other words, if the errors of the original forecasts are highly correlated, the Kalman filter works well. This fact is most clear in the HUR134 in which the NHC72 gives easterly biased forecasts almost all of the time. Furthermore, as seen in Table 3, the 12 hour forecasts are improved more than the 24 hour forecasts. This is simply because the former has a higher sequential correlation of the forecast errors, since the time increment is shorter.

What the Kalman filter essentially does is to remove the bias of the errors by adjusting the state variables of a system to their short-term current mean values. If the errors are originally white noise, the Kalman filter cannot reduce them. This is clear from the fact that if the system performance is optimal, the innovation sequence is white noise as mentioned in Part I (p. 15). The Kalman filter is therefore often used to detect gradual or sudden changes in the system structure when the change extends over a considerable time period, e.g. space ships and aircraft controls. It is a powerful tool in such cases.

Figures 2 and 3 show all of the hurricane forecasts where only the prediction errors are plotted. It will be seen that any single forecast error can be explained by the rule described above. The hurricane tracts and the forecasts for HUR72 and HUR134 are plotted on maps, Figures 4 and 5. The HUR72 is an example of the worst case and HUR134 of the best case.

Examination of the NHC72 Model

It has been pointed out that application of the Kalman filter to better utilize a regression model is useful only when the errors of the original regression model show consecutive biases over considerably long time periods. It has also been pointed out that the NHC72 model produces errors which do not have properties mentioned above, although they are not necessarily white noises. Based upon these facts, it may be concluded that the Kalman filter is not very useful for hurricane forecasting, at least by the method applied in this analysis. There is, however, another task left. That is to consider why the NHC72 model has less correlated errors. An answer to this question was previously mentioned; namely, the underlying system structure of hurricane motions is not made up of gradually changing components or components which may change suddenly from time to time but last a long time once changed. Although this is quite possibly true, there may be other reasons. One of them may be the poor construction of the NHC72 model. This possibility is quite conceivable.[3] As seen in Figure 1, it often happens that while the hurricane moves a great distance in a particular direction, say to the north, the forecast indicates an opposite movement, or a southerly displacement. It also frequently happens that while a hurricane speed is accelerating to the north, the corresponding forecast indicates a southerly acceleration. In fact, the forecasts fluctuate by and large more than the real hurricane motions. If the situation is the opposite; namely, if forecasts do not fluctuate as much as the actual hurricane motions, the forecast errors would show a consecutively biased structure. The following is the result of re-examination of the NHC72 models.

Using 104 data which was used in forecasting analyses, the regression models were recalculated and the results were found to be extremely interesting. Table 4 lists the comparison between the original coefficients and the recalculated coefficients. The predictor number corresponds to the number of Table 1. The values in parentheses are the standard deviations of the estimated coefficients. Considerably large differences between the originally given coefficients and those recalculated can be observed. In some cases, even the signs are different. These differences are not strange at all if one considers the corresponding standard deviations. In many cases the standard deviations are large enough to question the validity of the coefficients.

Given the standard deviations, the confidence limits of coefficients can be calculated. In the general linear regression model (5) the least square estimates $\hat{\beta} = (X^TX)^{-1}X^TY$ are normally distributed with the mean β and variance $\sigma^2(X^TX)^{-1}$ if each component of ϵ in (5) is independently identically distributed with $N(0,\sigma^2)$. Joint $100(1-\alpha)\%$ confidence region for all the components β are then obtained from the equation

$$\frac{(\hat{\beta} - \beta)^T X^T X(\hat{\beta} - \beta)/p}{\hat{\sigma}^2} \leq F(p,n-p,1-\alpha) \qquad (12)$$

where $F(p,n-p,1-\alpha)$ is the $(1-\alpha)$ point of the $F(p,n-p)$ distribution, p is the number of components of β and $\hat{\sigma}^2$ is as defined by (10). This inequality provides the equation with an elliptically-shaped contour in a space of p dimensions. Individual confidence intervals for the various components separately, however, can be obtained from the formula

$$\hat{\beta}_i \pm t(n-p, 1-\tfrac{1}{2}\alpha) \cdot \hat{\sigma}_i \qquad (13)$$

where $\hat{\beta}_i$ is the ith component of $\hat{\beta}$ and $\hat{\sigma}_i$ is the square root of the ith diagonal element of $\hat{\sigma}^2$ matrix $(=\sigma^2(X^TX)^{-1})$ and $t(n-p, 1-\frac{1}{2}\alpha)$ is the $(1-\frac{1}{2}\alpha)$ point of the t distribution with (n-p) degrees of freedom. Roughly speaking, 95% confidence limits for β_i are given by $\hat{\beta}_i \pm 2\hat{\sigma}_i$. These confidence limits can also be thought of as a hypothesis test. If the zero point lies in between the confidence limits, then the hypothesis that $\beta_i = 0$ cannot be rejected at the 100α% failure level. In Table 4, the coefficients which have no * at the end correspond to such cases. These coefficients are not significantly different from zero. The coefficients with * are significant in the sense that the hypothesis m = 0 is rejected with a 5% probability of failure. In other words, the predictors with *'s are the only variables worth including in the regression model. It is surprising to see that so many predictors are insignificant. These predictors contribute to reduction of the error variances for the data sets used for the model construction, but do not contribute to better forecasts for the independent data.

This finding leads to another question. How good are the regression models without those insignificant predictors? The third and forth columns in Table 4 give the estimates of coefficients when only the variables with * are included in the regression model. With fewer predictors, the error variances naturally increase. In the 12 hour φ model for example, five variables out of a given 13 variables are considered and the standard error is increased from 52.6 n.m. to 54.8 n.m. It is now necessary to check whether or not the model with all 13 variables, namely the original model, is significantly better than the new model with only five variables. For this purpose an F test was conducted, considering the new model as the null hypothesis and the original model as the alternative hypothesis;

namely;

H_0: Coefficients of variables 1,6,7,8,9,10,12 and 13 are all zero.

H_1: Not all of these eight coefficients are zero.

The value

$$F = \frac{\left(\hat{\sigma}_5^2 - \hat{\sigma}_{13}^2\right)/(13 - 5)}{\hat{\sigma}_{13}^2/(104 - 13)} = 0.97 \qquad (14)$$

follows the F-distribution with degrees of freedom 8 and 99. $\hat{\sigma}_5^2$ and $\hat{\sigma}_{13}^2$ are the error variances of the model under H_0 and H_1, respectively. The total number of observations is 104. The F value for 95% confidence is 1.98 as indicated in parentheses in Table 4a. This means that the F value is in the 95% confidence limits and that the null hypothesis cannot be rejected. In other words, this test suggests that it is of no value to include an additional eight variables in the regression model. All the coefficients of the selected five variables are statistically significantly different from zero and accordingly, recalculation of the regression coefficients is terminated.

The same procedure was applied for all four models. In general only five variables were recognized as significant. The 24 hour φ model was an exception, where only the two variables (8th and 10th) remained at the final stage. But in this case, the F-test rejected, at the 95% confidence level, the null hypothesis that the variables without * are all zero. This is an example of how a univariate t-test misleads an inference. The reason for this failure would be that the confidence region of coefficients were irregularly-shaped in this case, which would not be exceptional in multivariate distributions. A more sophisticated reason is that the variables under examination are selected by various screening tests. In general, the standard statistical tests are not necessarily applicable to the variables that have already been screened through pretesting. Nevertheless, the many predictors are hardly justifiable in all the models. The second and third predictors are recognized in all models except in the 24 hour φ model, which coincides with the fact that they were reported to be the most reliable in the sense that they reduced the variance of the prediction errors more than the lower ordered predictors (Neumann et al, p. 12). A conclusion that could be drawn from this test would be that the predictors selected by the NHC72 model were not necessarily appropriate, at least for the data set used in this analysis. If the same test is applied to the original set of data that was used for the development of the NHC72 model, and if all of the predictors were found significant, such a large sampling variation would be surprising.

The last task is to examine how well the Kalman filter works for the new model using only the significant predictors. Unfortunately, the results were not better than the previous ones and the errors left were not correlated to a great extent.

Concluding Remarks

The following is a list of conclusions and subsequent remarks which are not necessarily mentioned in the text:

(1) By replacing the equal coefficient of variation assumption with the estimate of error covariance matrix of the initial estimate of state variables, some improvement can be achieved. If data are available to estimate the error covariance matrix, it should by all means be used. By doing so, the number of parameters to be specified are also reduced from four to two.

(2) Application of the Kalman filter to a given regression model allows the regression coefficients to vary slowly and a significant improvement in performance of the model can be expected if the original errors of the regression model are from time to time highly biased consecutively. This is the case for typhoon forecast models. If, however, the original errors are not biased and close to the white noises, little improvement may be achieved. This is the case for all the hurricanes with the exception of HUR134.

(3) The NHC72 model was found to include many insignificant predictors, at least with respect to the data used in this analysis. The 12 hour φ model, for instance, was found to have only five significant predictors whereas 13 predictors were originally considered. The use of only the significant predictors, however, could not improve the performance of the filtered model. The validity of the original regression model and the utility of the Kalman filter are two different matters.

(4) Nonetheless, the Kalman filter showed its improvement potential up to 10% on an average, both at 12 and 24 hours. The use of this technique to predict hurricane movement should be seriously considered. One possible utilization would be the combination of filtered forecasts with the forecasts obtained through other models. The combination may lead to better performance since the Kalman filter forecasts include new information independent of other forecasts.

III. ADDITIONAL TYPHOON FORECASTING

As mentioned in the Introduction, only the 24 hour SNT forecasting model was examined. This model was fully described in Table 2 of Part I. Two kinds of predictors are used in this model: one is the persistence data and the other is the prognostic 500 mb gph data obtained from the numerical solutions of a three layer balance model of the atmosphere in the entire northern hemisphere. These numerical solutions are given only for 250, 550 and 850 mb gph values. In the previous analyses, 500 mb gph was computed using a simple linear interpolation formula (Part I, p. 27):

$$500 \text{ mb gph} = \frac{5}{6} \cdot 550 \text{ mb gph} + \frac{1}{6} \cdot 250 \text{ mb gph} \quad . \qquad (15)$$

Later, however, this formula was found to give biased height values. Formula (15) may be suitable for some purposes but not for use of the SNT model. As a result, the SNT forecasts using this formula appeared biased, as seen in Figures 7, 8 and 9 of Part I. Since biases were favorable to the Kalman filter, an incredible 70% improvement was achieved in the φ forecasting, which was not a fair comparison between the original and filtered forecasts.

Formula (15) was replaced by the following static equation of the atmosphere:

$$c_p \theta \frac{\partial}{\partial p} \left(\frac{p}{p_0}\right)^{\kappa} = -g\frac{\partial z}{\partial p} \tag{16}$$

where p is pressure (mb) at the geopotential height z(m), p_0 is 1000 mb, c_p is specific heat of dry air under constant pressure (1004 $m^2/sec^2/deg$), θ is potential temperature (deg), g is gravitational constant (9.8 m/sec^2) and $\kappa = R/c_p$ with R being gas constant (287 $m^2/sec^2/deg$).

Assuming the potential temperatures at z_{500} (500 mb gph) and z_{550} to be the same as θ_{550}, equation (16) can be rewritten, by differencing, as

$$z_{500} = z_{500} + \frac{c_p}{g} \theta_{500} \left[\left(\frac{p_{500}}{p_0}\right)^{\kappa} - \left(\frac{p_{500}}{p_0}\right)^{\kappa} \right]$$

$$= z_{500} + 2.315\ \theta_{550} \quad . \tag{17}$$

This formula gives better estimates of z_{500} than equation (15).

Results Using the Same Procedure Described in Part I

Using formula (17) for computing 500 mb gph, the 24 hour SNT forecastings were conducted for TYPH05, 08, 14, 16, 18, 21 and 22. The same model was also run through the Kalman filter. The improvements of the filtered forecasts over the original forecasts are summarized in Table 5 and appear substantially different from Table 8 of Part I, which is the counterpart of Table 5, by using formula (15) instead of (17) for 500 mb gph values. The φ forecasting improvement dropped from 70% to 27%.

The original φ error was 3.74 degrees in Table 8 of Part I, but is now 1.82 degrees. As the large biases disappeared from the original forecasts, the extremely favorable condition for applying the Kalman filter also disappeared. However, Table 5 shows a significant improvement. In terms of the mean of absolute errors, the λ errors improved from 1.46° to 1.20° (18%) and errors from 1.82° to 1.32° (27%). The vector errors improved from 264^{km} to 206^{km} (22%). In terms of root mean squared errors, the improvements are slightly less but still substantial.

The parameters of error covariances, α, β, γ_1 and γ_2 were set as indicated in Table 5 from the first trial and never adjusted because the performance was satisfactory. For this reason, all the cases listed are considered independent cases.

Examples of typhoon tracks with both the original and filtered forecasts are plotted in Figures 6 through 10. The reason for the greater improvements in predicting typhoons than in hurricanes are obvious from these figures. The consecutive biases are still present, although their magnitudes are not as great as in Figures 7 through 9 in Part I. These biases are sometimes easterly and sometimes westerly. Northern biases are more dominant than southern biases, which may be the result of the limited number of samples.

Before concluding the typhoon analyses, it should be further clarified why the typhoon experiments with the Kalman filter showed more improvement than the hurricane experiments. As was mentioned, this is because the original forecasting errors of the typhoon models are more highly correlated than those of the hurricane forecasting models. The reason for the high correlations present in the prediction errors in typhoon forecasting models is more difficult to explain. One probable answer to this question may be that the typhoon forecasting models are not developed for stratified data sets, whereas the hurricane models are separated into four classified equations, depending upon the initial hurricane motion. The typhoon forecasting models are therefore less specific to any particular atmospheric circumstances. As a result, a single model is applicable to all typhoons regardless of their origins or their initial motions. However, it may produce the estimates biased in a certain direc-

However, it may produce the estimates biased in a certain direction peculiar to each typhoon, depending upon the particular atmospheric conditions governing its motion.

It is important to note, nevertheless, that this fact alone does not necessarily justify the methodology of stratification. This is because the stratification of data is equivalent to including more parameters; these contribute to an explanation of the historical data, but do not necessarily provide more accurate forecasts. Too little evidence is available, however, to tell whether or not the NHC72 model is an example of such cases. All one can say is that if a stratification of data were used in a forecasting model to get rid of biased forecasts appearing in different directions from one cyclone to another, it would be better to avoid this and to try the Kalman filter as a means of filtering the biases out.

Concluding Remarks

(1) Formula (17) was used to obtain better estimates of 500 mb gph, replacing formula (15). The 24 hour SNT forecasts using the data obtained through this formula were found to be less biased. The Kalman filter, however, still improved the forecasts to a significant extent: 18% in λ and 27% in φ. The improvement in vector errors was 22%, which roughly corresponded to an error reduction from 260^{km} to 210^{km}, on the average.

(2) The accuracy of SNT forecasts is not substantially different from that of the NHC72 model, at least for the data examined in this analysis; but the SNT model was found to produce more highly-correlated forecasting errors than the NHC72 model, which formed an advantageous basis for the application of the Kalman filter.

(3) If consecutive biases in the forecasting errors develop in different directions from one cyclone to another, the Kalman filter would be a better means of decreasing them, rather than including additional parameters to a forecasting model.

Table 1 Description of NHC-72 Model

12 hours forecast for Quadrant 1

Meridional Motion (φ) / Zonal Motion (λ)

	Predictor	Coefficient		Predictor	Coefficient
1	constant	84.1900	1	constant	-3217.2300
2	H5(51)	-0.6326	2	H5(37)	0.7910
3	H5(54)	0.2085	3	DH10(97)	-0.9214
4	H5(76)	0.2869	4	DH5(56)	0.2531
5	DH10(102)	0.2973	5	H7(91)	-0.4460
6	H10(9)	0.2186	6	H10(38)	0.3125
7	DH7(10)	-0.1621	7	H10(8)	-0.1768
8	DH7(7)	-0.1692	8	H10(83)	-0.2890
9	DH5(65)	-0.3486			
10	H7(69)	0.3920			
11	H7(120)	-0.5247			
12	H10(105)	0.2602			
13	H7(92)	0.3609			

24 hours forecast for Quadrant 1

Meridional Motion (φ) / Zonal Motion (λ)

	Predictor	Coefficient		Predictor	Coefficient
1	constant	-1318.7000	1	constant	-4437.6900
2	DH5(51)	-0.6721	2	H5(37)	1.8303
3	DH10(71)	0.8779	3	H7(84)	-0.7772
4	DH7(92)	1.2525	4	DH5(56)	0.5491
5	DH5(65)	-0.7716	5	DH10(97)	-1.8328
6	H10(9)	0.3690	6	H10(38)	0.6131
7	H5(85)	0.3597	7	H10(106)	-1.0959
8	H7(51)	-1.2913	8	H5(66)	-0.6457
9	H7(69)	0.9108	9	H10(8)	-0.2473
10	H5(62)	0.4436			
11	DH7(10)	-0.1280			
12	DH7(7)	-0.2227			
13	H7(120)	-0.6741			

Note: H5, H7, H10 : 500, 700, 1000 mb gph (m)

DH5, DH7, DH10 : 24 hrs change of 500, 700 1000 mb gph (m)

Table 2 Improvements of the Filtered Forecasts over the NHC 72 Forecasts: equal c.v.

12 hrs forecast

$\alpha = 0.001$, $\beta = 0.5$, $\gamma_1 = 0.01$, $\gamma_2 = 0.001$

$\alpha = 0.0005$, $\beta = 1.0$, $\gamma_1 = 0.001$, $\gamma_2 = 0.0001$

HUR No.	λ error	φ error	Vector error
2	26%	-1%	15%
8	-5	19	4
17	9	-5	-2
54	-11	14	17
72	15	9	9
96	-3	-3	-3
112	3	-15	-7
134	42	-8	21
Average	19	1	9

24 hrs forecast

$\alpha = 0.001$, $\beta = 1.0$, $\gamma_1 = 0.01$, $\gamma_2 = 0.001$

$\alpha = 0.001$, $\beta = 2.0$, $\gamma_1 = 0.01$, $\gamma_2 = 0.001$

HUR No.	λ error	φ error	Vector error
2	5%	-3%	0%
8	-2	0	-1
17	9	-1	3
54	11	2	5
72	-4	-2	-3
96	-2	-1	-2
112	-1	0	-1
134	16	-1	12
Average	9	-1	4

Note: mean of the absolute errors are compared.

Table 3 Improvements of the Filtered Forecasts over the NHC 72 Forecasts: estimated $P_{0|0}$

12 hrs forecast

$$\begin{cases} \alpha = 0.01 \\ \beta = 1.0 \\ \gamma_1 = 1.0 \\ \gamma_2 = 0.01 \end{cases} \qquad \begin{cases} \alpha = 0.0003 \\ \beta = 1.0 \\ \gamma_1 = 1.0 \\ \gamma_2 = 0.0003 \end{cases}$$

HUR No.	λ error	φ error	Vector error
2	20% (18%)	0% (-2%)	11% (13%)
8	3 (-10)	12 (13)	4 (1)
17	-2 (4)	1 (1)	-6 (2)
54	2 (6)	13 (8)	13 (7)
72	-1 (10)	18 (17)	7 (14)
96	-15 (-25)	-4 (-3)	-8 (-11)
112	1 (-10)	2 (7)	-3 (2)
134	51 (43)	7 (12)	33 (30)
Average	17 (19)	8 (11)	11 (15)

24 hrs forecast

$$\begin{cases} \alpha = 0.003 \\ \beta = 1.0 \\ \gamma_1 = 1.0 \\ \gamma_2 = 0.003 \end{cases} \qquad \begin{cases} \alpha = 0.0 \\ \beta = 1.0 \\ \gamma_1 = 1.0 \\ \gamma_2 = 0.0 \end{cases}$$

HUR No.	λ error	φ error	Vector error
2	12% (7%)	-9% (-3%)	-1% (1.3%)
8	4 (2)	-1 (-2)	0 (0)
17	20 (11)	11 (4)	9 (6)
54	10 (9)	7 (1)	8 (5)
72	-17 (-5)	1 (5)	9 (0)
96	-1 (-7)	0 (-4)	-6 (-5)
112	-7 (-13)	-7 (-6)	-7 (-8)
134	34 (24)	3 (4)	27 (18)
Average	11 (13)	0 (1)	5 (8)

Note:

1. Values outside of the parenthesis are based on the mean of the absolute errors.
2. Values in the parenthesis are based on the root mean squared errors.
3. $\alpha = \gamma_2 = 0$ (24 hrs φ model) corresponds to the model without shaping filter.

Table 4a <u>Test of Regression Coefficients: 12 hours Forecast Model</u>

Predictor (φ)	Original	Recalculated (Stan. Dev.)	Recalculated
1	84.1900	478.5 (1469.)	
2	-0.6326	-0.8483 (0.1611)*	-0.8864 (0.1132)*
3	0.2085	0.5259 (0.1828)*	0.7358 (0.1475)*
4	0.2869	0.7935 (0.1970)*	0.7408 (0.1715)*
5	0.2973	0.7360 (0.3278)*	0.7818 (0.3079)*
6	0.2186	0.1190 (0.0766)	
7	-0.1631	-0.0714 (0.0801)	
8	-0.1692	-0.0632 (0.0881)	
9	-0.3486	-0.1883 (0.2065)	
10	0.3920	0.4382 (0.2666)	
11	-0.5247	-1.0800 (0.4046)*	-1.0764 (0.2371)*
12	0.2602	0.0607 (0.2847)	
13	0.3609	-0.3760 (0.4093)	
		$\hat{\sigma}$ = 52.6	$\hat{\sigma}$ = 54.8 F = 0.97(1.98)

Predictor (λ)	Original	Recalculated (Stan. Dev.)	Recalculated
1	-3217.2300	-5769. (1590.)*	-3405.9 (461.9)*
2	0.7910	0.6736 (0.0893)*	0.5752 (0.0809)*
3	-0.9214	-1.8901 (0.4518)*	-1.8109 (0.4483)*
4	0.2531	0.2233 (0.0959)*	0.2080 (0.0979)*
5	-0.4460	0.5608 (0.4718)	
6	0.3125	0.4226 (0.1732)*	0.3012 (0.1593)(*)
7	-0.1768	-0.1995 (0.1116)	
8	-0.2890	0.2557 (0.3590)	
		$\hat{\sigma}$ = 62.2	$\hat{\sigma}$ = 64.2 F = 2.09 (2.71)

Table 4b Test of Regression Coefficients: 24 hours Forecasting Model

Predictor (φ)	Original	Recalculated (s.d.)	Recalculated (s.d)	Recalculated (s.d)
1	-1318.7	2379.9 (2330.6)		
2	-0.6721	-0.7028 (0.4911)		
3	0.8779	0.2877 (0.3727)		
4	1.2525	0.8225 (0.7597)		
5	-0.7716	-0.5169 (0.4825)		
6	0.3690	0.2112 (0.1562)		
7	0.3597	-0.1098 (0.4356)		
8	-1.2913	-1.7600 (0.4814)*	-2.2949 (0.3188)*	-2.3074 (0.3174)*
9	0.9108	0.8911 (0.6354)		
10	0.4436	1.1253 (0.2376)*	1.3932 (0.2427)*	1.2629 (0.1719)*
11	-0.2480	-0.2515 (0.1617)		
12	-0.2227	-0.0871 (0.1883)		
13	-0.6741	-1.7418 (0.5825)*	-0.2521 (0.3325)	
		$\hat{\sigma}$ = 106.8	$\hat{\sigma}$ = 119.7 F = 2.32 (1.94)	$\hat{\sigma}$ = 120.0 F = 2.17 (1.90)

Predictor (λ)	Original	Recalculated (s.d.)	Recalculated (s.d)
1	-4437.7	-7876.2 (2431.5)*	-7332.0 (925.7)*
2	1.8303	1.2298 (0.2034)*	1.2361 (0.1621)*
3	-0.7772	-0.2193 (0.5581)	
4	0.5491	0.6197 (0.2105)*	0.5980 (0.1962)*
5	-1.8328	-2.5948 (0.9136)*	-2.6890 (0.8985)*
6	0.6131	0.9005 (0.3544)*	0.6789 (0.3192)*
7	-1.0959	0.0597 (0.7184)	
8	-0.6457	0.2183 (0.4093)	
9	-0.2473	-0.3168 (0.2328)	
		$\hat{\sigma}$ = 127.4	$\hat{\sigma}$ = 128.7 F = 0.49(2.71)

Table 5

Improvements of the Filtered Forecasts Over the SNT Forecasts:

24 hours

$\alpha = 0.001$

$\beta = 1.0$

$\gamma_1 = 0.01$

$\gamma_2 = 0.001$

TYPH	Number of forecasts	λ error	error	Vector error
05	6	37%(36%)	45%(41%)	38%(38%)
08	10	-2 (-1)	12 (20)	6 (15)
14	13	12 (3)	26 (10)	18 (8)
16	6	22 (22)	6 (8)	6 (0)
18	4	0 (12)	26 (26)	7 (16)
21	8	41 (35)	38 (38)	42 (36)
22	8	-5 (-1)	33 (10)	20 (7)
Average	$55_{(total)}$	18 (15)	27 (19)	22 (18)
Mean of the absolute error		1.46°	1.82°	264^{km}
is reduced to		1.20°	1.32°	206^{km}

Note:

1. Values outside of the parenthesis are based on the mean of the absolute errors.
2. Values in the parenthesis are based on the root mean squared errors.

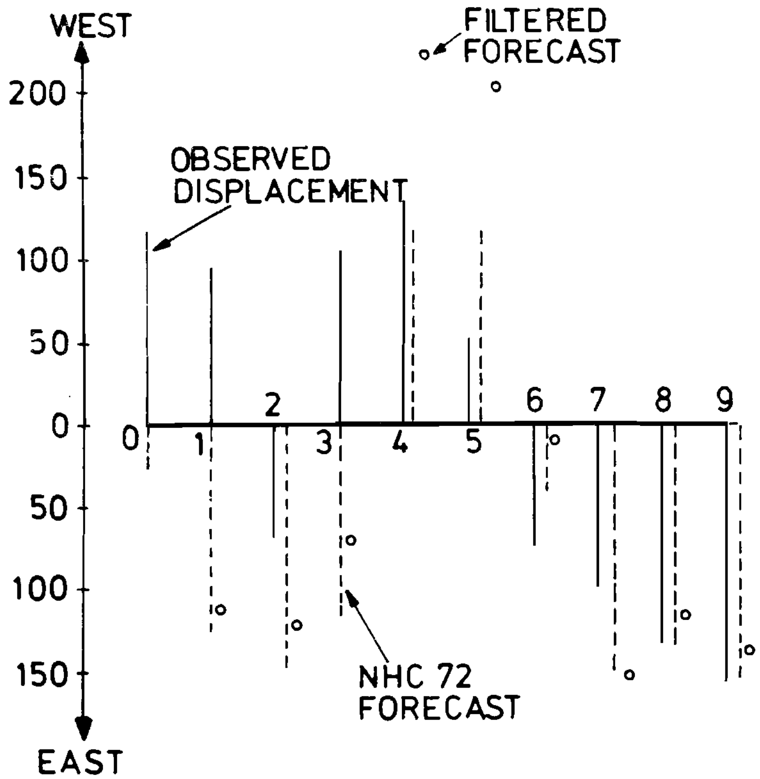


FIG. 1a. HUR 72 : 24 HRS FORECAST

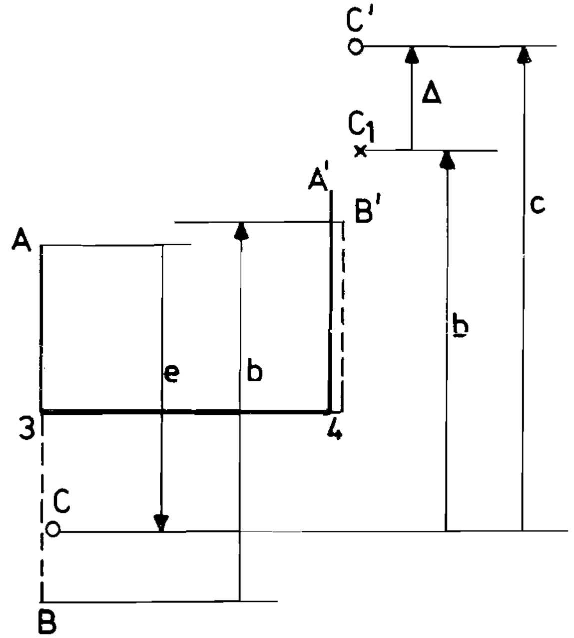


FIG.1b. ILLUSTRATION MECHANISM OF FILTERED FORECAST

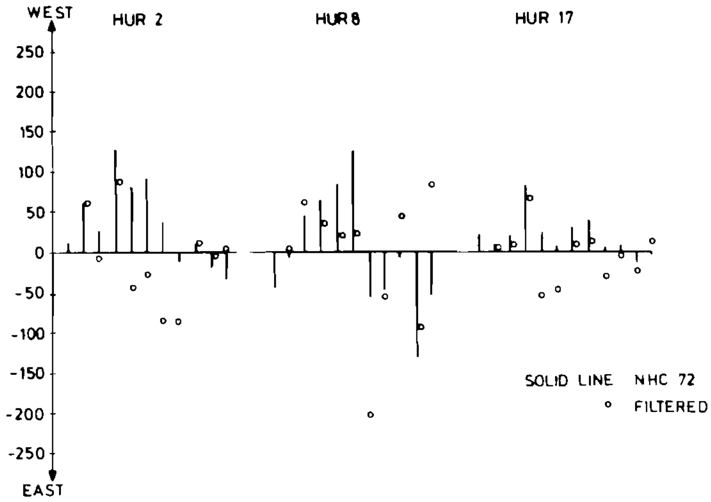


FIG.2a. 12 HRS FORECASTING ERRORS FOR λ

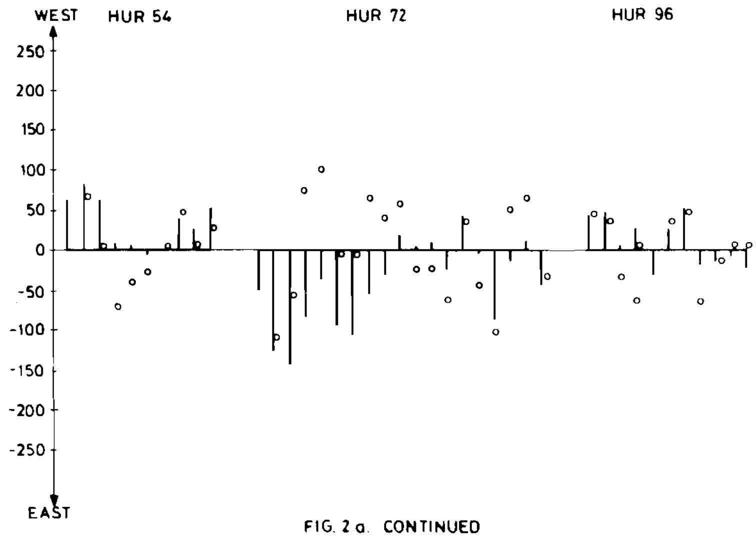


FIG. 2a. CONTINUED

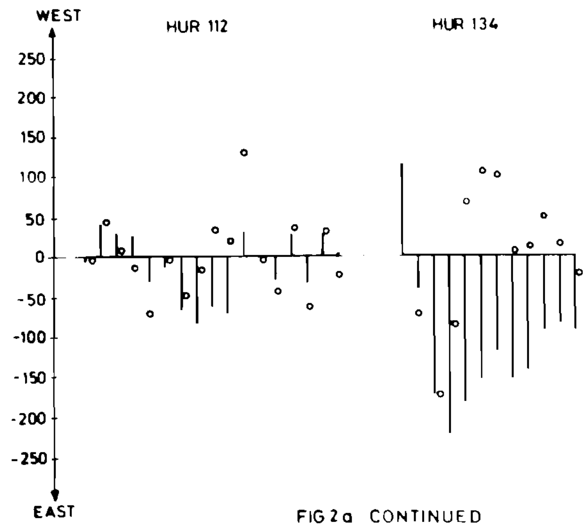


FIG 2a CONTINUED

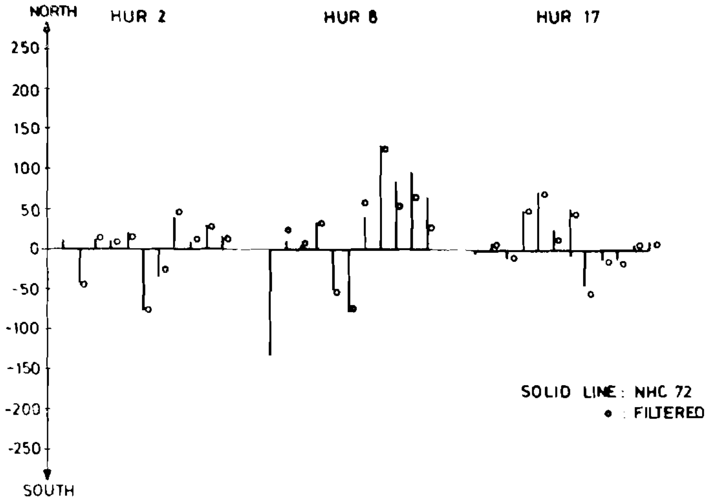


FIG. 2 b. 24 HRS FORECASTING ERRORS FOR φ

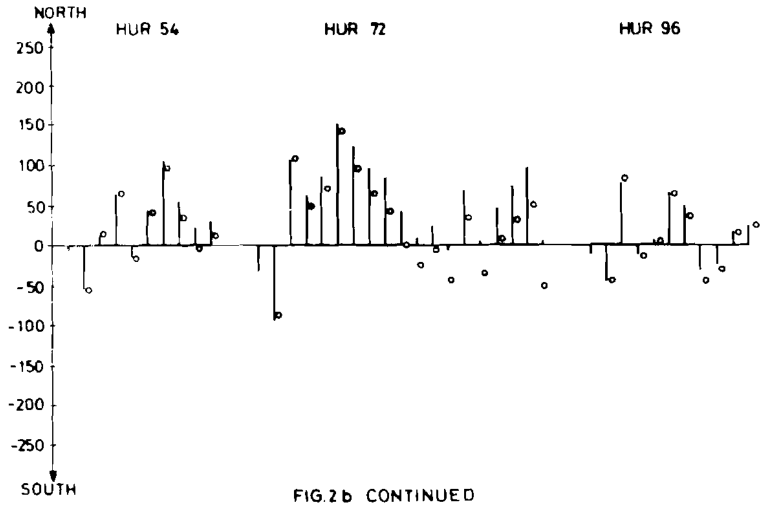


FIG. 2 b CONTINUED

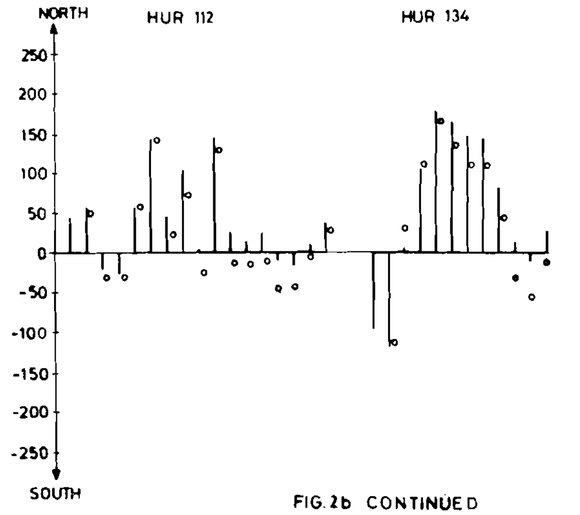


FIG. 2b CONTINUED

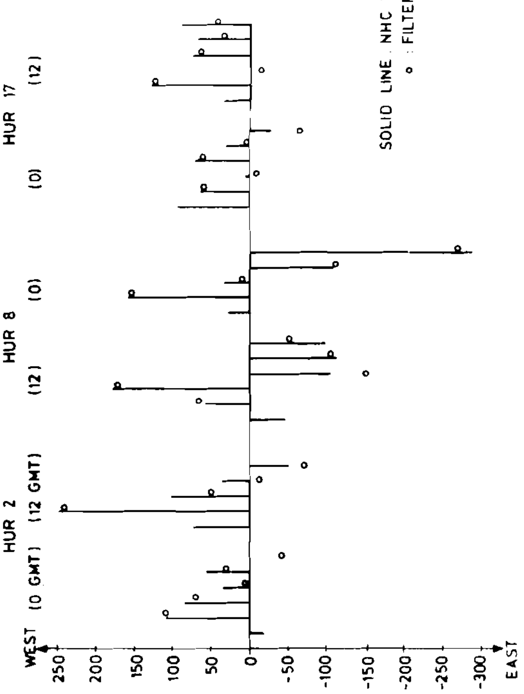


FIG 3a. 24 HRS FORECASTING ERRORS FOR λ

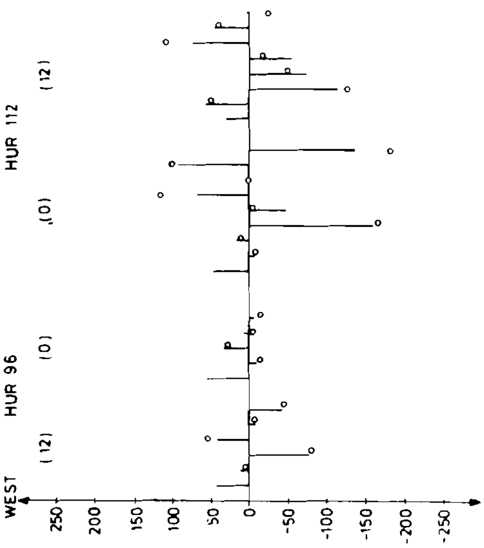


FIG 3a CONTINUED

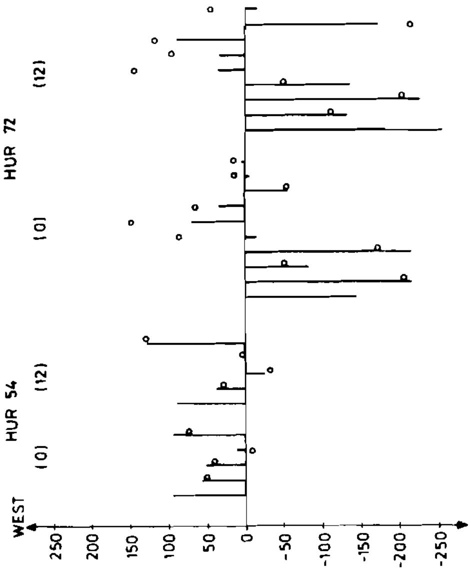


FIG 3a CONTINUED

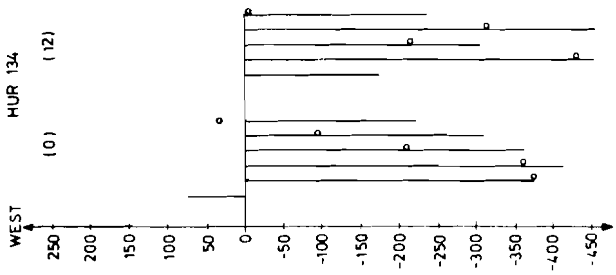


FIG.3a CONTINUED

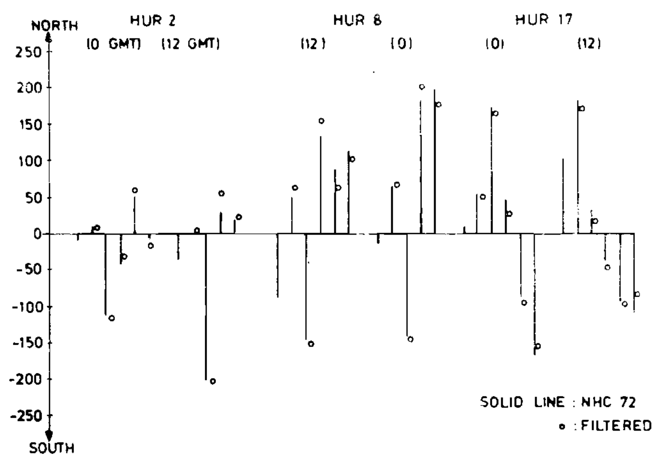


FIG 3 b. 24 HRS FORECASTING ERRORS FOR y

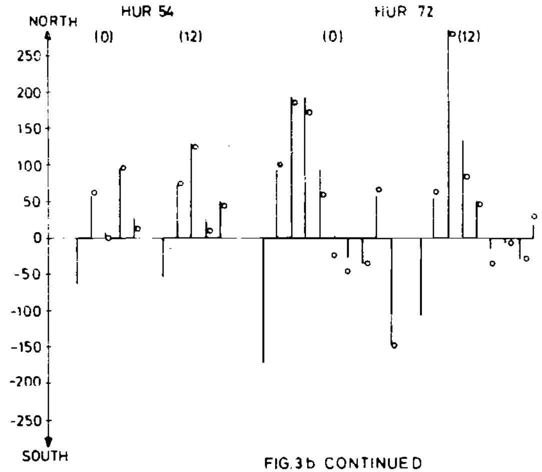


FIG.3b CONTINUED

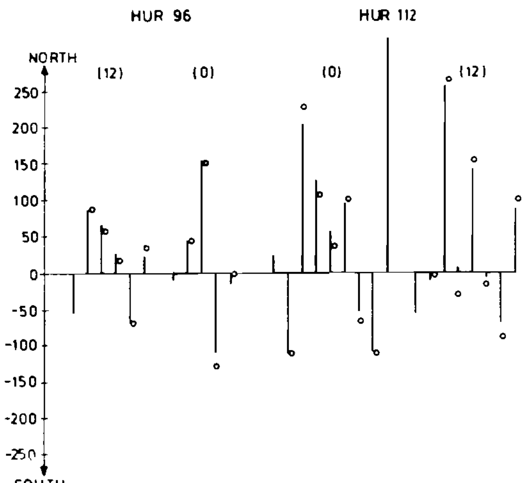


FIG.3b CONTINUED

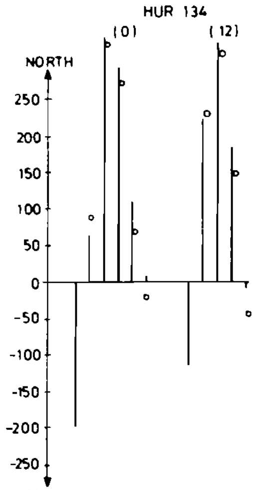


FIG.3b CONTINUED

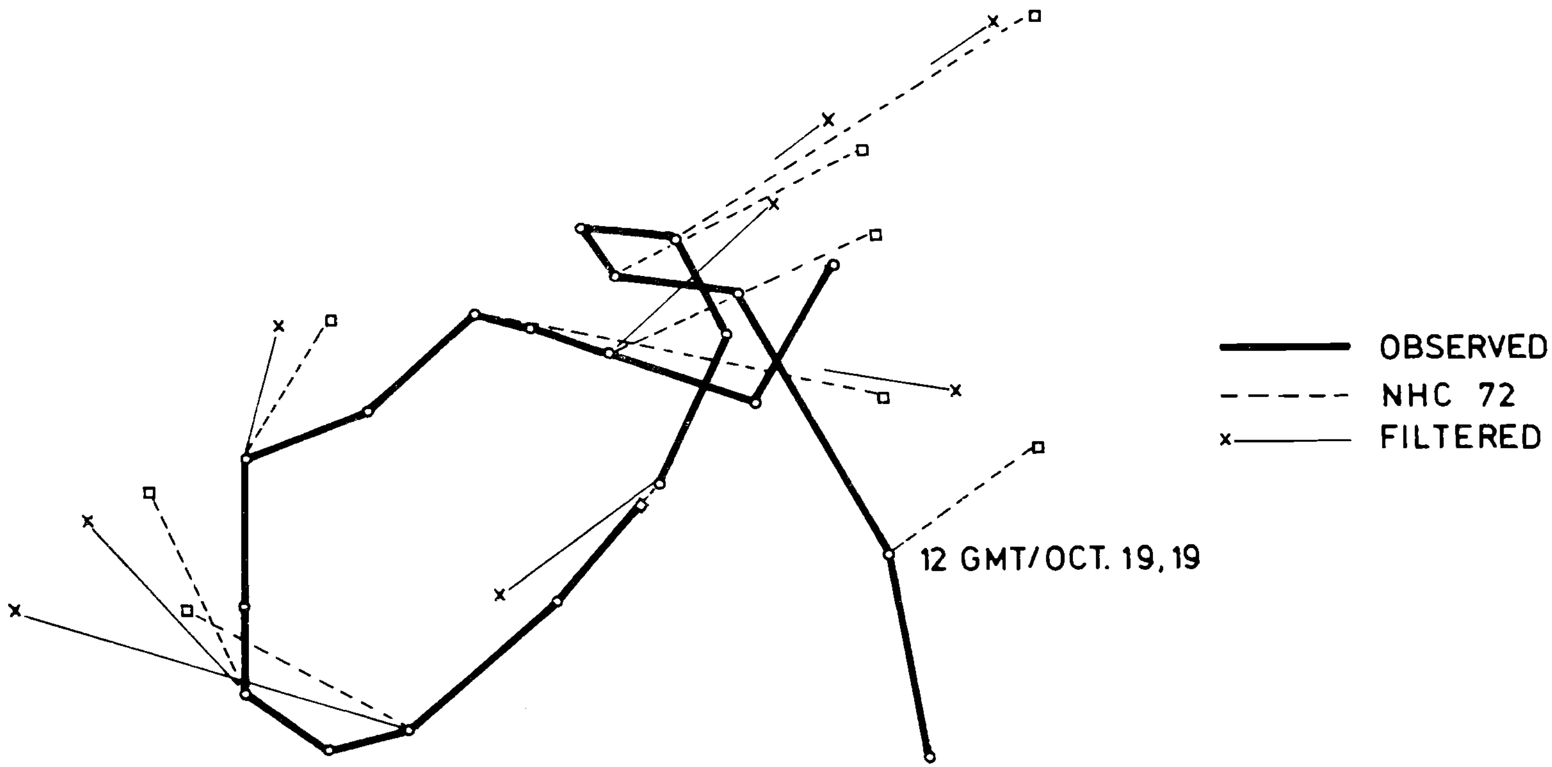


FIG. 4. HUR 72 : 24 HRS FORECAST (THE WORST CASE)

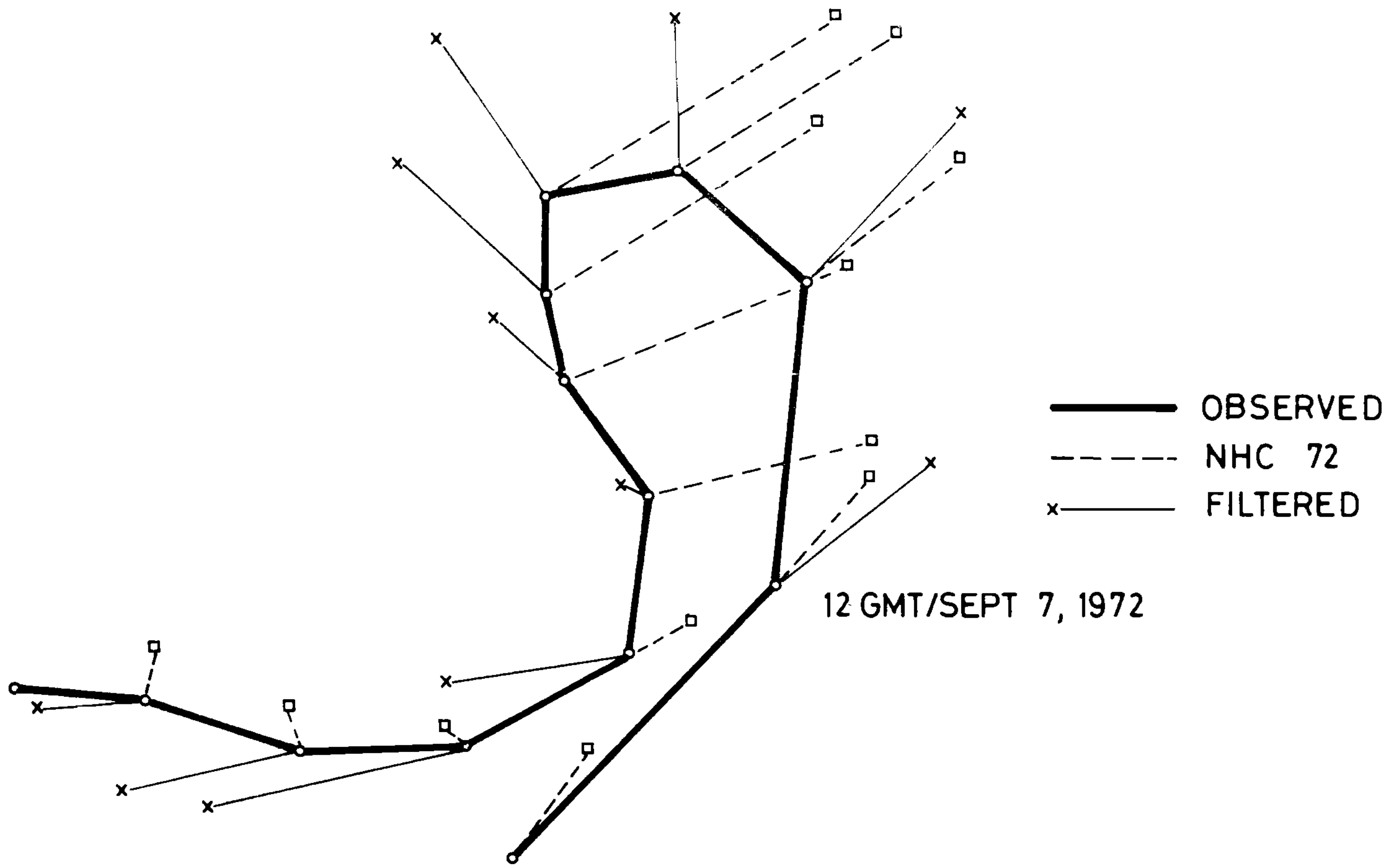


FIG.5. HUR 134 : 12 HRS FORECAST (THE BEST CASE)

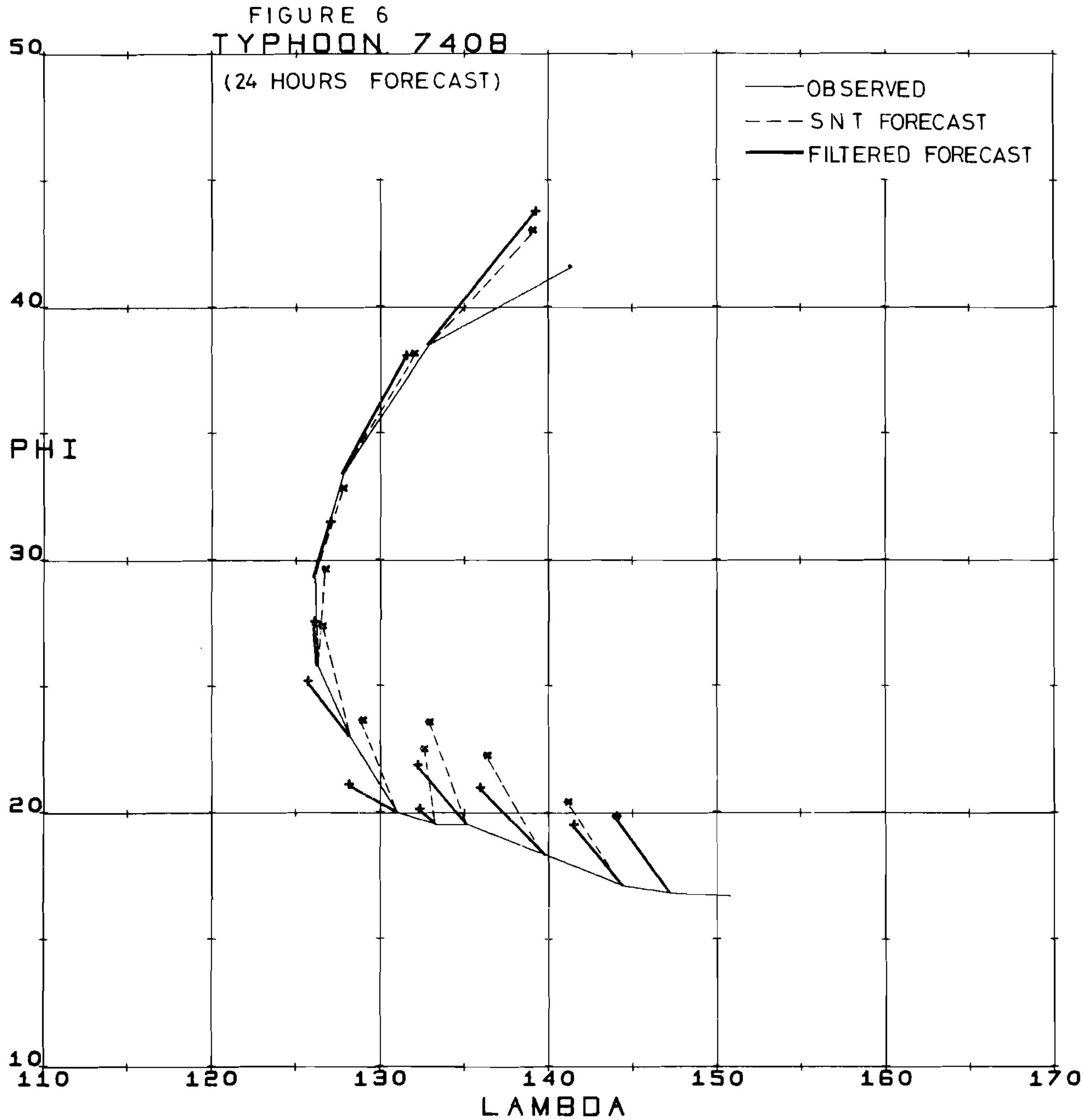


FIGURE 6
TYPHOON 7408
(24 HOURS FORECAST)
OBSERVED
SNT FORECAST
FILTERED FORECAST
PHI
50
40
30
20
10
110
120
130
140
150
160
170
LAMBDA

FIGURE 7

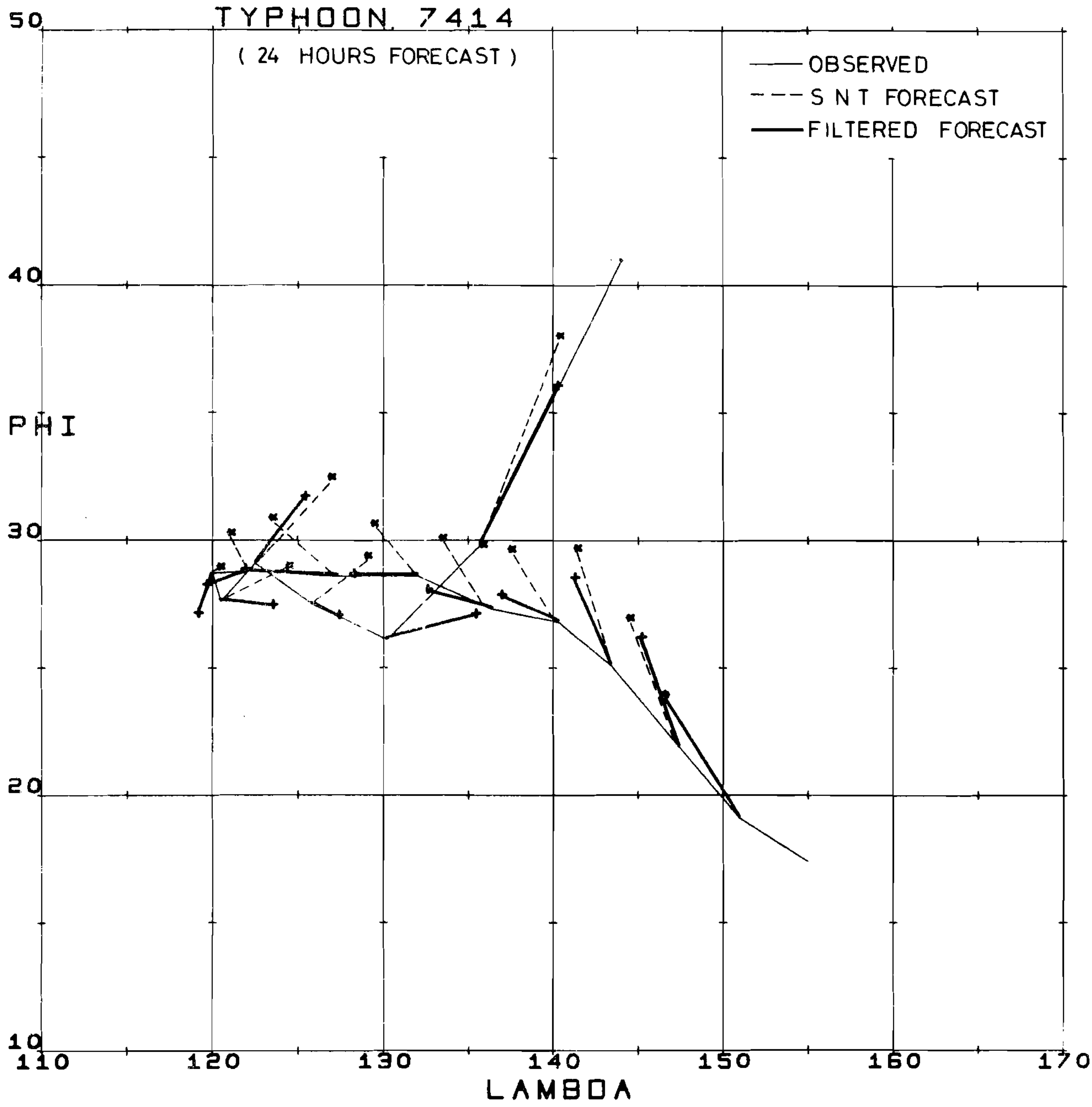

TYPHOON 7414
(24 HOURS FORECAST)
OBSERVED
S N T FORECAST
FILTERED FORECAST
50
40
30
20
10
PHI
110
120
130
140
150
160
170
LAMBDA

FIGURE 8

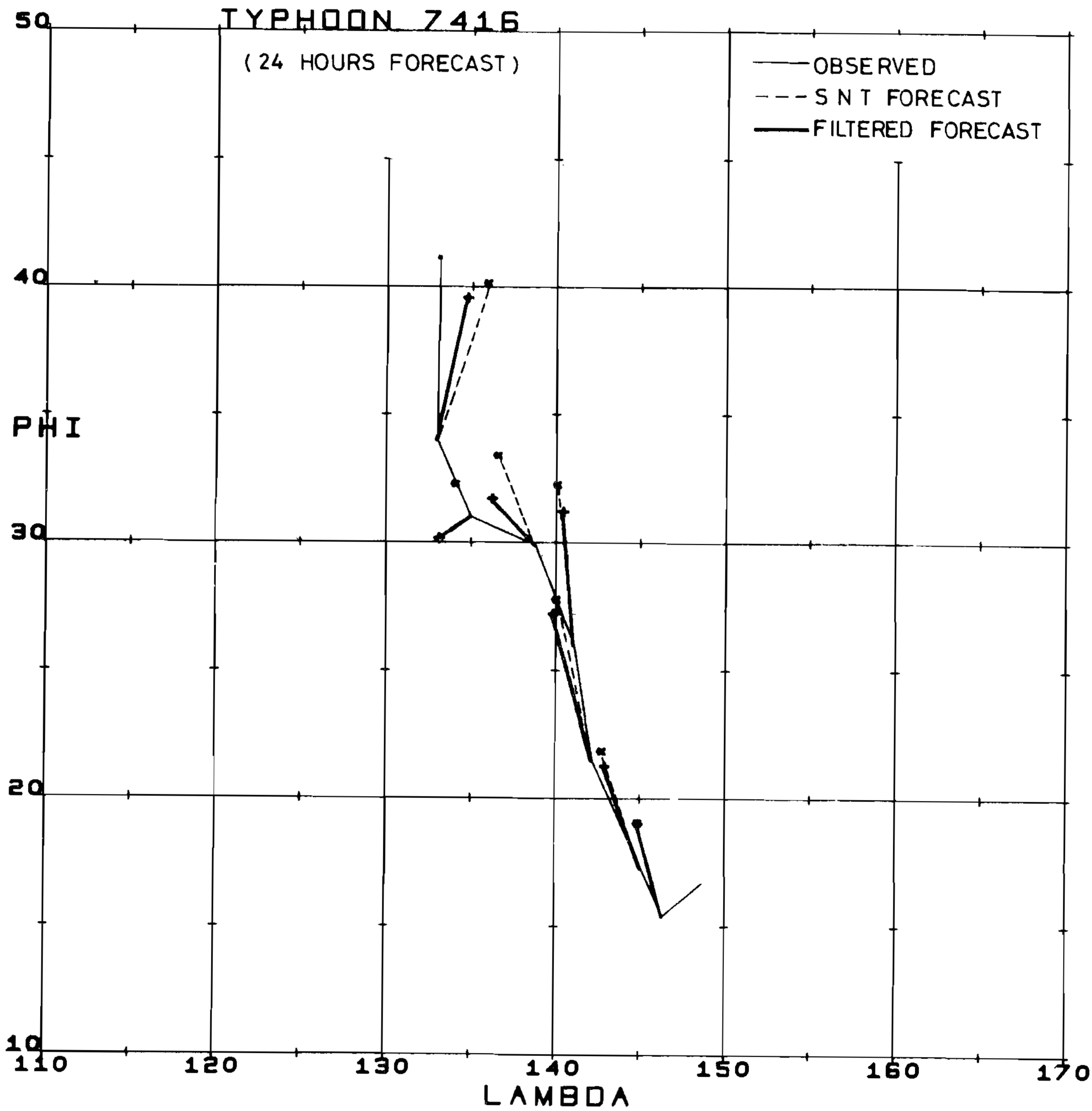

TYPHOON 7416
(24 HOURS FORECAST)
OBSERVED
S N T FORECAST
FILTERED FORECAST
PHI
50
40
30
20
10
110
120
130
140
150
160
170
LAMBDA

FIGURE 9

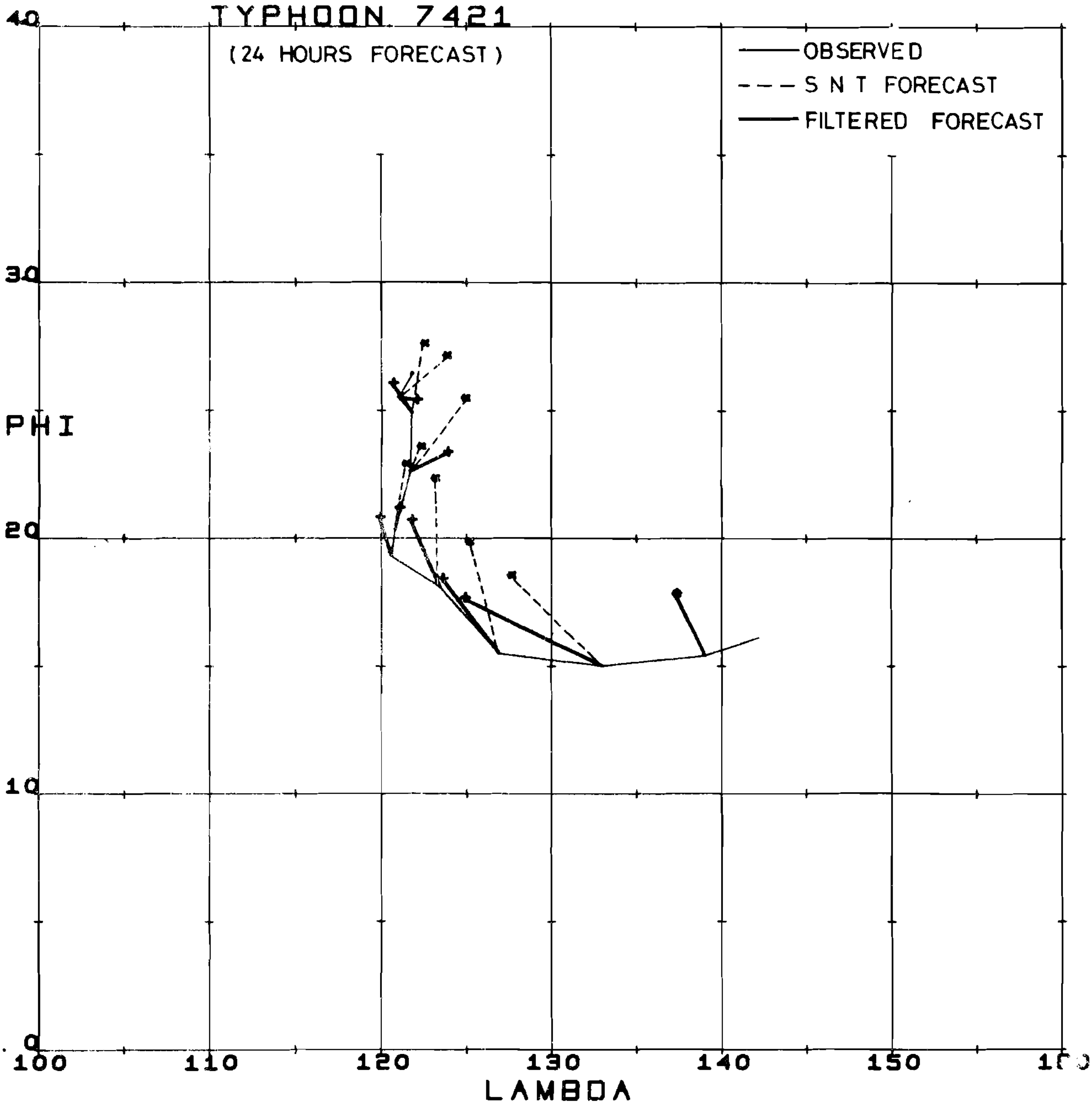
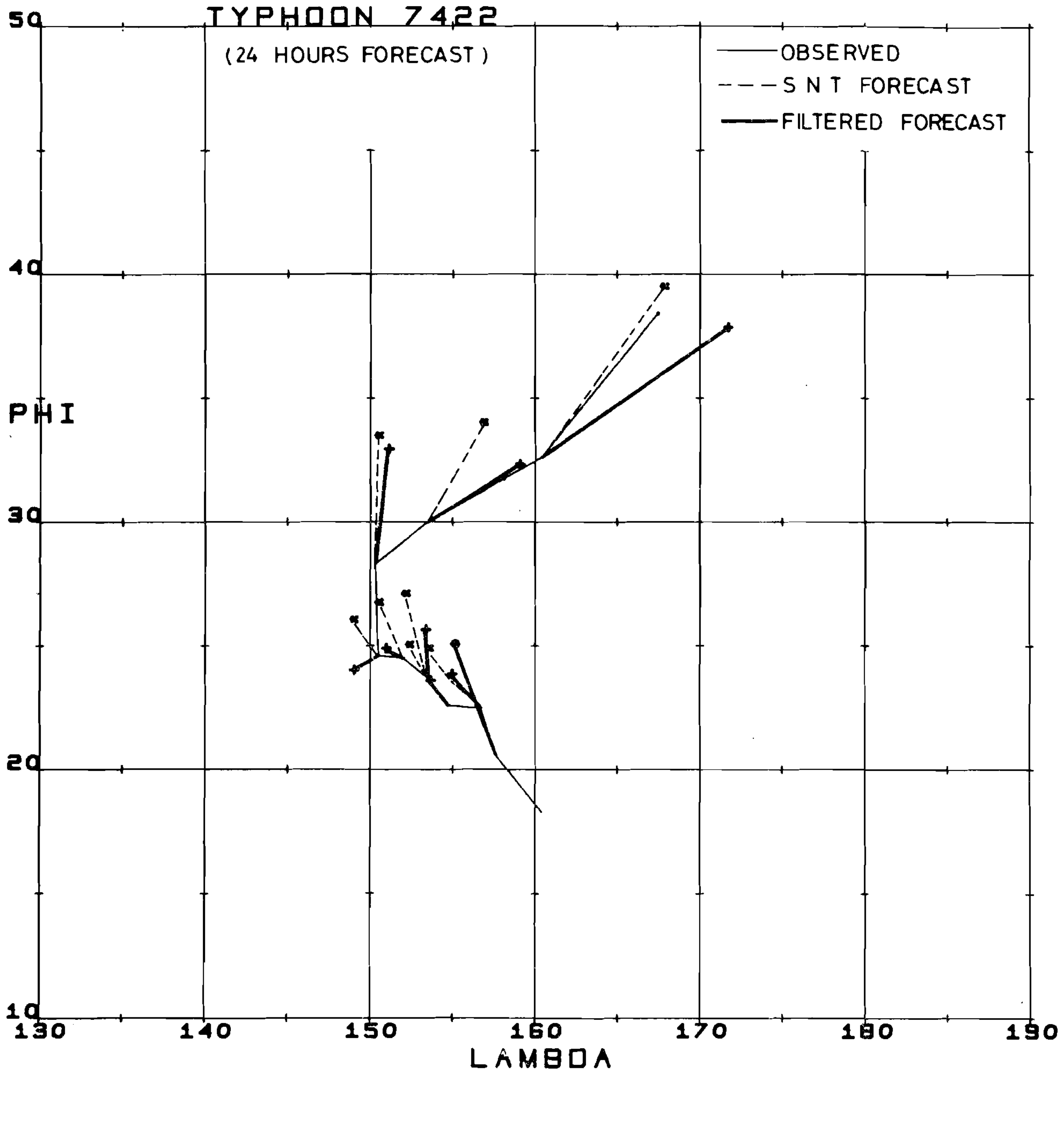

TYPHOON 7421
(24 HOURS FORECAST)
OBSERVED
S N T FORECAST
FILTERED FORECAST
40
30
20
10
0
PHI
100
110
120
130
140
150
LAMBDA

FIGURE 10


TYPHOON 7422
(24 HOURS FORECAST)
OBSERVED
S N T FORECAST
FILTERED FORECAST
PHI
LAMBDA
50
40
30
20
10
130
140
150
160
170
180
190

FOOTNOTES

1 The validity of the data during the time of this hurricane (Oct. 10~18, 1972) was carefully checked.

2 It can easily be seen from the Kalman filter formulation described in Part I (p. 9) that an adjustment is large if diagonal elements of $P_{0|0}$ and Q are large compared to R and it is small if R is larger. In the current case, the parameters are only α and γ_2 which control the lower part of the diagonal elements of $P_{0|0}$ and diagonal elements of Q. Therefore, the rate of adjustment is large as α and σ_2 increase. An intuitive explanation is given below. The large $P_{0|0}$ implies that the error involved in the original estimate $\hat{\beta}$ of β in (3) is large. Then the Kalman filter does not believe the value of $\hat{\beta}$; and once the error is observed, the Kalman filter changes the $\hat{\beta}$ a great deal to adjust the system to the observed value. The large Q implies the same, that is, if the rate of system disturbances is large, it tends to adjust $\hat{\beta}$ to a large extent. The large R, on the other hand, implies the opposite effect. This is the variance in the observation noise. If the variance of observation noise is large compared to Q and P, then the Kalman filter considers that the prediction error or the difference between the observed and the predicted is not due to the system error but due to the measurement error. Therefore, the Kalman filter tends to believe the state variables estimated so far and does not change them much, even when a large prediction error is present.

It must be clear now what small $P_{0|0}$, Q and R imply. The smaller $P_{0|0}$ and Q compared to R imply that the estimated state variables are reliable so that the Kalman filter will not adjust the state variables. The small R implies the small observation error and the Kalman filter fully understands the prediction errors due to the system error, according to large changes to the state variable estimates.

3 C. Neuman (personal communication, April 1976) pointed out that the hurricanes in the first quadrant were the most erratic of all cyclones in behavior. This in turn would imply that the NHC72 model for quadrant 1 was the least accurate model for forecasting.

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Neumann, C.J., Hope, J.R. and Miller, B.I.: "*A Statistical Method of Combining Synoptic and Empirical Tropical Cyclone Prediction Systems*", NOAA Technical Memorandum, NWS SR-63, 1972.

Takeuchi, K.: "*Application of the Kalman Filter to Cyclone Forecasting: 1 Methodology, 2 Typhoon Forecasting*", RM-76-9, IIASA, Laxenburg, Austria.