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COMPUTER PROGRAMS FOR SPATIAL DEMOGRAPHIC ANALYSIS

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Preface

Interest in human settlement systems and policies has been a critical part of urban-related work at IIASA since its inception. Recently this interest has given rise to a concentrated research effort focusing on migration dynamics and settlement patterns. Four sub-tasks form the core of this research effort:

I. the study of spatial population dynamics;

II. the definition and elaboration of a new research area called demometrics and its application to migration analysis and spatial population forecasting;

III. the analysis and design of migration and settlement policy;

IV. a comparative study of national migration and settlement patterns and policies.

This paper, the sixth in the comparative study series, describes two computer programs for spatial demographic analysis. The first is a program which calculates a multiregional life table. The second program generates a multiregional population projection. Listings of the actual programs are included in the Appendices.

Related papers in the comparative study series, and other publications of the migration and settlement study, are listed on the back page of this report.

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July 1976

Abstract

This paper lays out the algorithms and presents the FORTRAN IV codes of computer programs for spatial demographic analysis. The programs compute the multiregional life table and perform the population projection for a multiregional demographic system. The focus of this paper is on the interpretation of the output and on the preparation of the input data.

Acknowledgements

The development of computer programs for spatial demographic analysis began at Northwestern University, Evanston, USA, in 1972. A number of former graduate students have collaborated in the project. In particular, we are indebted to Jacques Ledent, Richard Walz and Richard Raquillet who wrote earlier versions of the programs.

The programs as listed at the end of this paper have been written at IIASA. We made intensive use of IIASA's in-house computing facilities, a PDP-11/45, and benefited from some of the nice features of the UNIX time-sharing system. We are most grateful to Computer Services and in particular to Jim Curry and Mark Pearson. Although we have created many difficulties for them, they were always helpful and saw the problems through.

The manuscript has been typed by Linda Samide. She performed her task with great skill and good cheer.

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Computer Programs for Spatial Demographic Analysis

There is a growing awareness among researchers, planners and governments that population growth must be viewed in its spatial dimension. The decline of major central cities of the world, the continuing depopulation of rural areas in developing countries and the accelerating suburbanization movements in developed nations have led governments all over the world to examine the desirability of population distribution policies.

A basic requirement for an effective policy regarding population redistribution is a well-developed understanding of spatial population dynamics. The mathematics of spatial demographic growth has been studied by Rogers (1968, 1975). We have translated this body of knowledge into a set of FORTRAN computer programs, to provide the user with a ready tool for the analysis of his own data. It is hoped that the programs help researchers, students, planners, and policy makers to better understand the dynamic behavior of demographic systems.

The computer programs are the result of a considerable investment in time, nerves and money, both at Northwestern University (USA) and at IIASA. Before reaching their final versions, they have passed through a number of successive formulations. Jacques Ledent, Richard Walz and Richard Raquillet, former graduate students at Northwestern University, all wrote earlier versions. For consistency reasons, their programs have been completely rewritten. Their efforts, however, have guided and facilitated our work, and their contribution to this project therefore has been a significant one.

Previous publications of computer programs for population analysis and for operations research methods have guided us in our work. Most influential in this regard has been the book of Keyfitz and Flieger (1971). It served as our basic reference. Other references were Greenberg, Krueckeberg and Mautner (1973), and Land and Powell (1973).

This paper is organized into five sections. In an introductory section, our general approach to computer programming is clarified. The next three sections explain the output and the mathematical formulas of observed population statistics, the life table and the population projection. The final section describes the format in which the input data must be provided. To illustrate the programs and their use, the two-region system of Slovenia and Rest of Yugoslavia has been used. Some sample runs, in addition to the one which produced the tables in the text, are set out in the Appendices.

1. THE APPROACH

The concept underlying the programs is that of a modular system. It consists of a set of subroutines, each of which performs a specific task, such as matrix inversion, reading of data, computation of life table, printing of the output, and so on. The main program is kept very short. It coordinates the computations through CALL statements and transmits information from one subroutine to another through labeled COMMON statements. It may also create data files which serve as input files for later computations (stable population) and for plotting purposes. No major computations are performed in the main program.

Two general purpose subroutines are used frequently by a number of subroutines: MULTIP for matrix multiplication, and INVERT for matrix inversion. The other subroutines perform the following specific tasks:

- DATAS: reads in the data;
- PRINTD1: prints the data and some observed demographic characteristics, such as age composition, rates, mean ages, and features of the aggregated system;
- PROBR: computes the migration probabilities under the assumption of no multiple transitions;
- PROBSCH: computes the migration probabilities allowing multiple transitions;
- LIFE: computes the life table;

WHOLE:	aggregates the life table statistics over all the regions;
PRINTL:	prints the life table;
GROWTH:	computes the growth matrix;
PROJECT:	projects the population until stability is achieved. (It is equivalent to the power-method algorithm for eigenvalue computations.)

The purpose of separating each major task into subroutines is to keep the whole structure of the programs very clear and to enable the user to change parts of the programs to make it fit his needs better. Clarity and flexibility are major objectives we tried to keep in mind while writing the programs. Computational efficiency was of secondary importance. In a rapidly growing field such as multiregional demographic analysis, computer programs must be flexible and easy to adapt to new theoretical or methodological developments. The computer programs published here are not final products to be consumed, but working tools to produce useful numerical demographic results. The user is urged to adapt them to fit his own needs in order to get the most out of them.

Each subroutine that performs a major computational task is covered in detail. The outputs are organized into three parts: observed population characteristics, the multiregional life table and the multiregional population projection.[1] The focus of our exposition is on the clarification of the output. For a detailed mathematical treatment of the various topics, the user is referred to Rogers (1975).

2. OBSERVED POPULATION CHARACTERISTICS

Before proceeding to its main task of computing a life table and a population projection, the program first computes some observed population characteristics directly from the data. This task is performed by the subroutine PRINTD1. The data

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[1]The programs work equally well for a single region system. An illustration is given in Appendix 3.

are given in Table 1. The user should check the data carefully for any input error. The totals are computed and may therefore be used for a quick check. The preparation of the input data will be discussed later.

Table 2 gives the percentage distribution of the population, parents at time of childbearing, deaths, and migrants. The mean age is defined as

$$\bar{m}_i = \sum_x \left(x + \frac{NY}{2}\right) \cdot c_i(x)/100 \qquad (1)$$

where $c_i(x)$ is the percentage distribution,
NY is the age interval, and
$(x + \frac{NY}{2})$ is the average age of the interval.

The direct input to the life table consists of observed age-specific rates (Table 3). The death rates are computed by dividing the annual number of deaths by the mid-year population in each age group. Fertility and migration rates are derived in a similar fashion. If death, birth or migration data are not available on an annual basis, but for a five-year period, say, then the program reduces the data to an annual basis. The population must in this case be the population at the mid-period. The sum of the age-specific rates is called the gross rate. The gross fertility rate of Slovenia, for example, is 0.222562. To get the gross reproduction rate (GRR), one must multiply this by NY, the age interval (in this case, five). Similarly, the gross migraproduction rate (GMR) is the gross migration rate times five. The crude rate is the total number of births, deaths or outmigrants divided by the total mid-year population. For example, the crude birth rate of Slovenia is

$$0.017 = 14{,}159/832{,}800$$

Table 1

```
*************************************
*                                   *
*     Multiregional Life Table      *
*   Yugoslavia:  Two Regions 1961   *
*    Slovenia - Rest Yugoslavia     *
*************************************
```

list of parameters [(a)]
****

na	=	18	ny	=	5	nr	=	2
nzb	=	1	nzd	=	1	nzo	=	1
irunt	=	2	ioptg	=	2	ngro	=	1
init	=	1961	ka	=	1	kc	=	2
nu	=	3	10	=	2	npar1	=	0
npar2	=	2	npar3	=	2	npar4	=	7
npar5	=	2016	npar6	=	200	npar7	=	0
npar8	=	0						

(a) The meaning of the parameters is given in Section 5.

Table 1 (continued)

Observed Population Characteristics

Input-Data

****

region slovenia

age	population	births	deaths	migration from slovenia to slovenia	r.yugos.
0	67800.	0.	417.	0.	192.
5	74100.	0.	32.	0.	170.
10	70700.	5.	21.	0.	105.
15	60100.	953.	31.	0.	310.
20	62900.	4444.	47.	0.	451.
25	66500.	4204.	45.	0.	368.
30	67100.	2758.	67.	0.	252.
35	62900.	1438.	77.	0.	111.
40	39500.	308.	76.	0.	40.
45	47900.	34.	171.	0.	26.
50	51300.	15.	268.	0.	34.
55	46100.	0.	369.	0.	29.
60	39600.	0.	513.	0.	35.
65	29500.	0.	763.	0.	28.
70	21700.	0.	1036.	0.	19.
75	14400.	0.	1088.	0.	16.
80	7100.	0.	1041.	0.	5.
85	3600.	0.	733.	0.	4.
total	832800.	14159.	6795.	0.	2195.

region r.yugos.

age	population	births	deaths	migration from r.yugos. to slovenia	r.yugos.
0	847900.	0.	19051.	231.	0.
5	905200.	0.	606.	150.	0.
10	808100.	54.	386.	127.	0.
15	617400.	16335.	534.	419.	0.
20	725500.	63828.	885.	680.	0.
25	774000.	57477.	1227.	392.	0.
30	728400.	32261.	1277.	255.	0.
35	633300.	14903.	1313.	143.	0.
40	392400.	4729.	1127.	72.	0.
45	437100.	940.	1700.	41.	0.
50	453800.	324.	2896.	59.	0.
55	389300.	0.	3743.	80.	0.
60	325800.	0.	5492.	66.	0.
65	230600.	0.	6407.	36.	0.
70	180000.	0.	8652.	14.	0.
75	120900.	0.	8715.	12.	0.
80	61200.	0.	6843.	12.	0.
85	39300.	0.	5639.	3.	0.
total	8670200.	190851.	76493.	2792.	0.

Table 2

Percentage Distribution

region slovenia

age	population	births	deaths	migration from slovenia to slovenia	r.yugos.
0	8.1412	0.0000	6.1369	0.0000	8.7472
5	8.8977	0.0000	0.4709	0.0000	7.7449
10	8.4894	0.0353	0.3091	0.0000	4.7836
15	7.2166	6.7307	0.4562	0.0000	14.1230
20	7.5528	31.3864	0.6917	0.0000	20.5467
25	7.9851	29.6914	0.6623	0.0000	16.7654
30	8.0572	19.4788	0.9860	0.0000	11.4806
35	7.5528	10.1561	1.1332	0.0000	5.0569
40	4.7430	2.1753	1.1185	0.0000	1.8223
45	5.7517	0.2401	2.5166	0.0000	1.1845
50	6.1599	0.1059	3.9441	0.0000	1.5490
55	5.5355	0.0000	5.4305	0.0000	1.3212
60	4.7550	0.0000	7.5497	0.0000	1.5945
65	3.5423	0.0000	11.2288	0.0000	1.2756
70	2.6057	0.0000	15.2465	0.0000	0.8656
75	1.7291	0.0000	16.0118	0.0000	0.7289
80	0.8525	0.0000	15.3201	0.0000	0.2278
85	0.4323	0.0000	10.7873	0.0000	0.1822
total	100.0000	100.0000	100.0000	100.0000	100.0000
m.age	33.3796	27.6427	66.0931	0.0000	25.0376

region r.yugos.

age	population	births	deaths	migration from r.yugos. to slovenia	r.yugos.
0	9.7795	0.0000	24.9055	8.2736	0.0000
5	10.4404	0.0000	0.7922	5.3725	0.0000
10	9.3204	0.0283	0.5046	4.5487	0.0000
15	7.1209	8.5590	0.6981	15.0072	0.0000
20	8.3677	33.4439	1.1570	24.3553	0.0000
25	8.9271	30.1162	1.6041	14.0401	0.0000
30	8.4012	16.9038	1.6694	9.1332	0.0000
35	7.3043	7.8087	1.7165	5.1218	0.0000
40	4.5258	2.4778	1.4733	2.5788	0.0000
45	5.0414	0.4925	2.2224	1.4685	0.0000
50	5.2340	0.1698	3.7860	2.1132	0.0000
55	4.4901	0.0000	4.8933	2.8653	0.0000
60	3.7577	0.0000	7.1797	2.3639	0.0000
65	2.6597	0.0000	8.3759	1.2894	0.0000
70	2.0761	0.0000	11.3108	0.5014	0.0000
75	1.3944	0.0000	11.3932	0.4298	0.0000
80	0.7059	0.0000	8.9459	0.4298	0.0000
85	0.4533	0.0000	7.3719	0.1074	0.0000
total	100.0000	100.0000	100.0000	100.0000	100.0000
m.age	30.6024	27.1063	50.2401	26.0781	0.0000

Table 3

Observed Rates
****

death rates
****

age	slovenia	r.yugos.
0	0.006150	0.022468
5	0.000432	0.000669
10	0.000297	0.000478
15	0.000516	0.000865
20	0.000747	0.001220
25	0.000677	0.001585
30	0.000999	0.001753
35	0.001224	0.002073
40	0.001924	0.002872
45	0.003570	0.003889
50	0.005224	0.006382
55	0.008004	0.009615
60	0.012955	0.016857
65	0.025864	0.027784
70	0.047742	0.048067
75	0.075556	0.072084
80	0.146620	0.111814
85	0.203611	0.143486
gross	0.542112	0.473962
crude	0.008159	0.008823
m.age	79.1635	74.4001

fertility rates
****

age	slovenia	r.yugos.
0	0.000000	0.000000
5	0.000000	0.000000
10	0.000071	0.000067
15	0.015857	0.026458
20	0.070652	0.087978
25	0.063218	0.074260
30	0.041103	0.044290
35	0.022862	0.023532
40	0.007797	0.012051
45	0.000710	0.002151
50	0.000292	0.000714
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000
85	0.000000	0.000000
gross	0.222562	0.271501
crude	0.017002	0.022012
m.age	27.7683	27.4740

Table 3 (continued)

outmigration rates
****

migration from slovenia to

age	total	slovenia	r.yugos.
0	0.002832	0.000000	0.002832
5	0.002294	0.000000	0.002294
10	0.001485	0.000000	0.001485
15	0.005158	0.000000	0.005158
20	0.007170	0.000000	0.007170
25	0.005534	0.000000	0.005534
30	0.003756	0.000000	0.003756
35	0.001765	0.000000	0.001765
40	0.001013	0.000000	0.001013
45	0.000543	0.000000	0.000543
50	0.000663	0.000000	0.000663
55	0.000629	0.000000	0.000629
60	0.000884	0.000000	0.000884
65	0.000949	0.000000	0.000949
70	0.000876	0.000000	0.000876
75	0.001111	0.000000	0.001111
80	0.000704	0.000000	0.000704
85	0.000000	0.000000	0.000000
gross	0.037365	0.000000	0.037365
crude	0.002636	0.000000	0.002636
m.age	29.4835	0.0000	29.4835

migration from r.yugos. to

age	total	slovenia	r.yugos.
0	0.000272	0.000272	0.000000
5	0.000166	0.000166	0.000000
10	0.000157	0.000157	0.000000
15	0.000679	0.000679	0.000000
20	0.000937	0.000937	0.000000
25	0.000506	0.000506	0.000000
30	0.000350	0.000350	0.000000
35	0.000226	0.000226	0.000000
40	0.000183	0.000183	0.000000
45	0.000094	0.000094	0.000000
50	0.000130	0.000130	0.000000
55	0.000205	0.000205	0.000000
60	0.000203	0.000203	0.000000
65	0.000156	0.000156	0.000000
70	0.000078	0.000078	0.000000
75	0.000099	0.000099	0.000000
80	0.000196	0.000196	0.000000
85	0.000000	0.000000	0.000000
gross	0.004638	0.004638	0.000000
crude	0.000322	0.000322	0.000000
m.age	33.2519	33.2519	0.0000

The mean age given in this table is the mean age of the schedule. The mean age of the fertility schedule of Slovenia, for example, is

$$\bar{m}_1 = 27.77 = \frac{\sum_x \left(x + \frac{NY}{2}\right) F_1(x)}{\sum_x F_1(x)} \tag{2}$$

where $F_1(x)$ are the age-specific fertility rates of Slovenia and NY is five.

The mean age of the Slovenia to Rest-of-Yugoslavia migration schedule is 29.48 years. The mean age of the migrants is considerably less (25.04 years). This is due to the relatively young age composition of Slovenia's population. The age composition does not affect the migration schedule or its mean age.

A last table of observed characteristics gives information on the whole system (Table 4). It is an aggregation of the regional data. The migration column contains the number of migrants between the regions in the system. It is obvious that the number of migrants depends on the regional delineation.

3. THE MULTIREGIONAL LIFE TABLE

The multiregional life table is a device for exhibiting the mortality and mobility history of an artificial population, called a cohort, as it gradually decreases in size until all its members have died. The method of constructing such a life table is treated in detail in Rogers (1975, Chapter 3).

The cohort we deal with is a birth cohort, or radix. It is a group of people born at the same moment in time and in the same region. Their life history is of special interest because it provides the necessary input information to the numerical computations of the multiregional demographic growth models. The regional radices are provided by the user and stored in the labeled common area:

Table 4

Total Population System
****

age	population		births		deaths		migration		observed rates		
	absolute	percent	absolute	percent	absolute	percent	absolute	percent	birth	death	migration
0	915700.	9.6359	0.	0.0000	19468.	23.3743	423.	8.4821	0.000000	0.021260	0.000462
5	979300.	10.3052	0.	0.0000	638.	0.7660	320.	6.4167	0.000000	0.000651	0.000327
10	878800.	9.2476	59.	0.0288	407.	0.4887	232.	4.6521	0.000067	0.000463	0.000264
15	677500.	7.1293	17288.	8.4328	565.	0.6784	729.	14.6180	0.025517	0.000834	0.001076
20	788400.	8.2963	68272.	33.3018	932.	1.1190	1131.	22.6790	0.086596	0.001182	0.001435
25	840500.	8.8446	61681.	30.0868	1272.	1.5272	760.	15.2396	0.073386	0.001513	0.000904
30	795500.	8.3710	35019.	17.0816	1344.	1.6137	507.	10.1664	0.044021	0.001690	0.000637
35	696200.	7.3261	16341.	7.9708	1390.	1.6689	254.	5.0932	0.023472	0.001997	0.000365
40	431900.	4.5449	5037.	2.4570	1203.	1.4444	112.	2.2458	0.011662	0.002785	0.000259
45	485000.	5.1037	974.	0.4751	1871.	2.2464	67.	1.3435	0.002008	0.003858	0.000138
50	505100.	5.3152	339.	0.1654	3164.	3.7989	93.	1.8648	0.000671	0.006264	0.000184
55	435400.	4.5817	0.	0.0000	4112.	4.9371	109.	2.1857	0.000000	0.009444	0.000250
60	365400.	3.8451	0.	0.0000	6005.	7.2099	101.	2.0253	0.000000	0.016434	0.000276
65	260100.	2.7370	0.	0.0000	7170.	8.6087	64.	1.2833	0.000000	0.027566	0.000246
70	201700.	2.1225	0.	0.0000	9688.	11.6319	33.	0.6617	0.000000	0.048032	0.000164
75	135300.	1.4238	0.	0.0000	9803.	11.7700	28.	0.5615	0.000000	0.072454	0.000207
80	68300.	0.7187	0.	0.0000	7884.	9.4659	17.	0.3409	0.000000	0.115432	0.000249
85	42900.	0.4514	0.	0.0000	6372.	7.6506	7.	0.1404	0.000000	0.148531	0.000163
total	9503000.	100.0000	205010.	100.0000	83288.	100.0000	4987.	100.0000	0.267401	0.480391	0.007607
crude									0.021573	0.008764	0.000525
m.age		30.8458		27.1434		51.5334		25.6201	27.4908	74.8451	33.5908

```
COMMON/CRAD/RADIX(4),RADIXT
```

where RADIXT is the sum of all the radices. In most cases, radices of 100,000 will be used.

The computation of the multiregional life table begins with the estimation of age-specific outmigration and death probabilities (Rogers, 1975, p. 60). The probabilities are estimated by the subroutines PROBR (option 1) and PROBSCH (option 3). They will be discussed later. The probabilities are stored in a labeled common area:

```
COMMON/CPQ/Q(18,4),P(18,4,4),PMIGT(18,4)
```

where Q(X,I) is the probability that an individual in region I will die before reaching age X + h. (In our example, h = 5).

P(X,J,I) is the probability that an individual in region I at age X will survive and be in region J at age X + h.

$$PMIGT(X,I) = \sum_{j} P(X,J,I) \quad .$$

Table 5 contains the probabilities of dying and outmigrating of females of a two-region system of Yugoslavia for 1961. They have been computed using PROBSCH. Therefore, they deviate slightly from the probabilities presented by Rogers (1975, p. 66), which were computed by PROBR. As a consequence, all life-table statistics deviate from Rogers'. The PROBR probabilities and the associated life table is given in Appendix 2.

3.1 Life Histories

The life histories of the radices are computed by applying the age-specific probabilities of dying and outmigrating recursively to the radices. Assuming equal radices of 100,000 in Slovenia and the Rest of Yugoslavia, Table 6 gives the complete life history of these radices. Let the number of people in region i at exact age x, who are born in region j, be denoted by

Table 5

Probabilities of Dying and Migrating
****

region slovenia
****

age	death	migration from slovenia to slovenia	r.yugos.
0	0.030813	0.956084	0.013103
5	0.002164	0.986467	0.011370
10	0.001487	0.991131	0.007381
15	0.002598	0.972070	0.025332
20	0.003770	0.961262	0.034968
25	0.003439	0.969456	0.027105
30	0.005015	0.976525	0.018460
35	0.006121	0.985171	0.008708
40	0.009586	0.985426	0.004988
45	0.017694	0.979646	0.002660
50	0.025793	0.970993	0.003213
55	0.039248	0.957747	0.003005
60	0.062780	0.933124	0.004097
65	0.121486	0.874356	0.004157
70	0.213259	0.783257	0.003484
75	0.317728	0.678322	0.003949
80	0.536332	0.461658	0.002010
85	1.000000	0.000000	0.000000

region r.yugos.
****

age	death	migration from r.yugos. to slovenia	r.yugos.
0	0.106319	0.001261	0.892421
5	0.003341	0.000821	0.995838
10	0.002385	0.000781	0.996834
15	0.004312	0.003333	0.992355
20	0.006075	0.004571	0.989354
25	0.007889	0.002481	0.989630
30	0.008724	0.001721	0.989555
35	0.010310	0.001114	0.988575
40	0.014256	0.000904	0.984840
45	0.019259	0.000460	0.980282
50	0.031406	0.000630	0.967964
55	0.046941	0.000982	0.952077
60	0.080868	0.000939	0.918193
65	0.129895	0.000684	0.869422
70	0.214551	0.000309	0.785139
75	0.305390	0.000353	0.694257
80	0.436969	0.000560	0.562471
85	1.000000	0.000000	0.000000

Table 6

Life History of Initial Cohort
****

initial region of cohort slovenia
****

age	number of deaths in each region of residence slovenia	r.yugos.
0	3081.30	0.00
5	206.86	4.38
10	140.29	5.71
15	242.85	13.28
20	342.65	32.96
25	300.53	67.42
30	424.94	94.44
35	506.58	126.57
40	781.75	183.28
45	1422.15	251.69
50	2031.10	409.05
55	3001.31	603.69
60	4598.67	1008.75
65	8305.32	1526.74
70	12749.18	2253.45
75	14878.71	2581.98
80	17038.04	2645.71
85	14669.22	0.00
total	84721.46	11809.09

age	number of migrants from slovenia to slovenia	r.yugos.
0	95608.35	1310.34
5	94314.46	1087.03
10	93479.10	696.15
15	90870.02	2368.10
20	87359.77	3177.87
25	84715.48	2368.55
30	82747.46	1564.28
35	81538.78	720.73
40	80363.90	406.81
45	78739.57	213.79
50	76461.43	253.05
55	73238.53	229.80
60	68352.40	300.09
65	59774.59	284.21
70	46825.15	208.29
75	31764.76	184.94
80	14665.83	63.87
85	0.00	0.00
total	1240819.63	15437.88

Table 6 (continued)

age	number of migrants from r.yugos. to slovenia	r.yugos.
0	0.00	0.00
5	1.08	1304.89
10	1.87	2384.35
15	10.27	3056.95
20	24.80	5367.29
25	21.20	8456.54
30	18.63	10712.02
35	13.68	12136.05
40	11.62	12661.87
45	6.01	12810.99
50	8.21	12607.52
55	12.62	12244.25
60	11.71	11453.59
65	8.04	10218.90
70	3.25	8246.40
75	2.98	5869.73
80	3.39	3405.57
85	0.00	0.00
total	159.35	132936.91

initial region of cohort r.yugos.
****

age	number of deaths in each region of residence slovenia	r.yugos.
0	0.00	10631.87
5	0.27	298.18
10	0.29	211.97
15	0.69	382.04
20	2.09	534.14
25	3.21	686.42
30	5.62	751.40
35	7.61	878.96
40	12.65	1201.57
45	24.35	1598.75
50	35.76	2555.82
55	54.85	3697.97
60	88.89	6065.70
65	169.06	8946.78
70	269.53	12849.30
75	320.42	14361.21
80	375.78	14267.91
85	0.00	18367.18
total	1371.07	98287.20

Table 6 (continued)

age	number of migrants from slovenia to slovenia	r.yugos.
0	0.00	0.00
5	124.35	1.43
10	195.89	1.46
15	257.90	6.72
20	531.74	19.34
25	905.12	25.31
30	1094.63	20.69
35	1224.41	10.82
40	1300.17	6.58
45	1348.34	3.66
50	1346.28	4.46
55	1338.53	4.20
60	1321.17	5.80
65	1216.76	5.79
70	989.92	4.40
75	684.06	3.98
80	323.46	1.41
85	0.00	0.00
total	14202.73	126.06

age	number of migrants from r.yugos. to slovenia	r.yugos.
0	126.06	89242.06
5	73.29	88870.60
10	69.42	88590.65
15	295.28	87914.79
20	401.89	86985.48
25	215.83	86102.57
30	148.21	85228.27
35	94.99	84275.02
40	76.18	83008.09
45	38.16	81377.75
50	51.30	78774.29
55	77.34	75003.44
60	70.43	68871.51
65	47.10	59883.43
70	18.54	47021.38
75	16.59	32647.98
80	18.28	18365.78
85	0.00	0.00
total	1838.86	1242163.00

${}_{j0}\ell_i(x)$. Then

$$ {}_{j0}\ell_j(0) = \text{RADIX}(0,j) \quad . \tag{3}$$

The expected number of people alive in region i at exact age x, born in region j, who will die before reaching age x + h, is ${}_{j0}\ell_{i\delta}(x)$. The expected number of migrants from i to k among the people living in i at age x and born in j is ${}_{j0}\ell_{ik}(x)$. From the 100,000 babies born in Slovenia, 3081 will die before they reach age 5, i.e.,

$$ \begin{aligned} &100{,}000 * 0.030813 = 3081 \\ &{}_{10}\ell_1(0) * q_1(0) = {}_{10}\ell_{1\delta}(0) \quad , \end{aligned} \tag{4}$$

and 1310 will move to the Rest of Yugoslavia,

$$ \begin{aligned} &100{,}000 * 0.013103 = 1310 \\ &{}_{10}\ell_1(0) * p_{12}(0) = {}_{10}\ell_{12}(0) \quad . \end{aligned} \tag{5}$$

The residual, i.e.,

$$100{,}000 - 3081 - 1310 = 95{,}608$$

or

$$100{,}000 * 0.956084$$

$$ {}_{10}\ell_1(0) * p_{11}(0) = {}_{10}\ell_{11}(0) \tag{6}$$

remain in Slovenia, and are there at exact age 5. Therefore, from the females born in Slovenia, only 95.6% will still be there 5 years later.

Of the 100,000 females born in Slovenia, at exact age 5, 95,608 will still be there, and 1,310 will be in the Rest of

Yugoslavia. From these 95,608, the number of females dying before reaching age 10 is

$$95{,}608 * 0.002164 = 207$$

$${}_{10}\ell_1(5) * q_1(5) = {}_{10}\ell_{1\delta}(5) \qquad (7)$$

and the number migrating to the Rest of Yugoslavia is

$$95{,}608 * 0.011370 = 1087$$

$${}_{10}\ell_1(5) * p_{12}(5) = {}_{10}\ell_{12}(5) \quad .$$

The residual is the number of females, who were in Slovenia at age 5 and are still there at age 10:

$$95{,}608 - 207 - 1087 = 94{,}314$$

or

$$95{,}608 * 0.986467$$

$${}_{10}\ell_1(5) * p_{11}(5) = {}_{10}\ell_{11}(5) \quad . \qquad (8)$$

What happens to the 1310 migrants born in Slovenia, but who are in the Rest of Yugoslavia at exact age 5? They die, move back to Slovenia or stay in the Rest of Yugoslavia. If one assumes that the mortality and migration behavior depends on the region of residence, then

$$1310 * 0.003341 = 4$$

$${}_{10}\ell_2(5) * q_2(5) = {}_{10}\ell_{2\delta}(5) \qquad (9)$$

females die before reaching age 10, and

$$1310 * 0.000821 = 1$$

$$_{10}\ell_{2}(5) * p_{21}(5) = {}_{10}\ell_{21}(5) \qquad (10)$$

move back to Slovenia, while

$$1310 * 0.995838 = 1305$$

remain in the Rest of Yugoslavia.

Pursuing this procedure until the last age group, we have a detailed description of the life history of the people born in Slovenia. The last age group 85 is open-ended, therefore all people who reach age 85 are expected to die in that age group, i.e. $q_i(85) = 1.0$, and hence

$$_{10}\ell_{1\delta}(85) = {}_{10}\ell_{1}(85) \quad . \qquad (11)$$

An analogue procedure is followed to derive the life history of the females born in the Rest of Yugoslavia.

The following life table statistics are various aggregations of Table 6.

3.2 Expected Number of Survivors at Exact Age x

Table 7 is an aggregation of Table 6. It gives the number of people by place of birth and place of residence. We saw already that of the 100,000 girls born in Slovenia, there are 1310 who at exact age 5 reside in the Rest of Yugoslavia. This number may also be found in Table 7. Of the people, born in Slovenia and residing in the Rest of Yugoslavia at age 10, for example, some were there already at age 5 and stayed there, while others moved in from Slovenia, i.e.

Table 7

Expected Number of Survivors
at Exact Age x in Each Region
****

age	aggregated
0	200000.
5	186287.
10	185777.
15	185419.
20	184780.
25	183868.
30	182811.
35	181534.
40	180014.
45	177835.
50	174538.
55	169507.
60	162149.
65	150387.
70	131439.
75	103317.
80	71175.
85	36848.

age	initial region of cohort slovenia		
	total	slovenia	r.yugos.
0	100000.	100000.	0.
5	96919.	95608.	1310.
10	96707.	94316.	2392.
15	96561.	93481.	3080.
20	96305.	90880.	5425.
25	95930.	87385.	8545.
30	95562.	84737.	10825.
35	95042.	82766.	12276.
40	94409.	81552.	12857.
45	93444.	80376.	13069.
50	91770.	78746.	13025.
55	89330.	76470.	12861.
60	85725.	73251.	12474.
65	80118.	68364.	11754.
70	70286.	59783.	10503.
75	55283.	46828.	8455.
80	37822.	31768.	6055.
85	18139.	14669.	3469.

age	initial region of cohort r.yugos.		
	total	slovenia	r.yugos.
0	100000.	0.	100000.
5	89368.	126.	89242.
10	89070.	198.	88872.
15	88857.	265.	88592.
20	88475.	553.	87922.
25	87938.	934.	87005.
30	87249.	1121.	86128.
35	86492.	1243.	85249.
40	85605.	1319.	84286.
45	84391.	1376.	83015.
50	82768.	1386.	81381.
55	80176.	1398.	78779.
60	76424.	1416.	75008.
65	70269.	1392.	68877.
70	61153.	1264.	59889.
75	48034.	1008.	47026.
80	33353.	701.	32652.
85	18709.	342.	18367.

$$2392 = 1305 + 1087$$

$$_{10}\ell_1(10) = {}_{10}\ell_{11}(5) + {}_{10}\ell_{21}(5) \tag{12}$$

where $_{j0}\ell_i(x)$ is the number of people in region i at exact age x, who are born in region j. Equation (12) is equivalent to:

$$_{10}\ell_1(10) = {}_{10}\ell_1(5)p_{11}(5) + {}_{10}\ell_2(5)p_{21}(5) \quad . \tag{13}$$

The total of 2392 is given in Table 7, while the components may be found in Table 6.

The computation of the expected number of survivors at exact age x in a multiregional system is more conveniently performed using matrix notation. For our two-region example, let

$$\underset{\sim}{\ell}(x) = \begin{bmatrix} _{10}\ell_1(x) & _{20}\ell_1(x) \\ _{10}\ell_2(x) & _{20}\ell_2(x) \end{bmatrix} \tag{14}$$

$$\underset{\sim}{P}(x) = \begin{bmatrix} p_{11}(x) & p_{21}(x) \\ p_{12}(x) & p_{22}(x) \end{bmatrix} \quad . \tag{15}$$

The matrix analogue of equation (13) is then

$$\underset{\sim}{\ell}(x + 5) = \underset{\sim}{P}(x)\underset{\sim}{\ell}(x) \quad . \tag{16}$$

For x = 5, we have

$$\begin{bmatrix} 94{,}316 & 198 \\ 2{,}392 & 88{,}872 \end{bmatrix} = \begin{bmatrix} 0.986467 & 0.000821 \\ 0.011370 & 0.995838 \end{bmatrix} \begin{bmatrix} 95{,}608 & 126 \\ 1{,}310 & 89{,}242 \end{bmatrix} .$$

The probability that an individual in region i at age x will survive and be in region j, n years later, is easily derived from the entries of Table 7. The probability of surviving from age x to x + n in the multiregional system is

$$\underset{\sim}{P}(x + n - 5)\ \underset{\sim}{P}(x + n - 10) \ldots \underset{\sim}{P}(x) \quad .$$

It follows from (16) that

$$\underset{\sim}{\ell}(x + n) = \underset{\sim}{P}(x + n - 5)\ \underset{\sim}{P}(x + n - 10) \ldots \underset{\sim}{P}(x)\ \underset{\sim}{\ell}(x) \quad . \qquad (17)$$

Hence,

$$\underset{\sim}{P}(x + n - 5)\ \underset{\sim}{P}(x + n - 10) \ldots \underset{\sim}{P}(x) = \underset{\sim}{\ell}(x + n)\ \underset{\sim}{\ell}^{-1}(x) \ . \qquad (18)$$

The probability that an individual in region i at age x will be in j, n years later, is therefore given by

$$\underset{\sim}{\ell}(x + n)\ \underset{\sim}{\ell}^{-1}(x) \qquad (19)$$

where the entries of $\underset{\sim}{\ell}(x + n)$ and $\underset{\sim}{\ell}(x)$ are found in Table 7. For example, if one knows the distribution of people at the time they enter the labor force or marriage, age 20 say, and denote this by $\{\underset{\sim}{w}(20)\}$, then their distribution at retirement age, 60 say, is given by

$$\{\underset{\sim}{w}(60)\} = \underset{\sim}{\ell}(60)\ \underset{\sim}{\ell}^{-1}(20)\ \{\underset{\sim}{w}(20)\} \qquad (20)$$

$$= \begin{bmatrix} 73{,}251 & 1{,}416 \\ 12{,}474 & 75{,}008 \end{bmatrix} \begin{bmatrix} 90{,}880 & 553 \\ 5{,}425 & 87{,}922 \end{bmatrix}^{-1} \{\underset{\sim}{w}(20)\}$$

$$= \begin{bmatrix} 0.805532 & 0.005043 \\ 0.137338 & 0.852580 \end{bmatrix} \{\underset{\sim}{w}(20)\} \quad .$$

The probability that an individual in Slovenia at age 20 will be in the Rest of Yugoslavia at retirement age is quite high, namely 13.7%.

3.3 Number of Years Lived in Each Region by the Initial Unit Cohort

The number of years individuals at age x may expect to live in the next five years on the average is

$$\underset{\sim}{L}(x) = \int_0^5 \underset{\sim}{\ell}(x + t)\, dt \tag{21}$$

where

$$\underset{\sim}{L}(x) = \begin{bmatrix} {}_{10}L_1(x) & {}_{20}L_1(x) \\ {}_{10}L_2(x) & {}_{20}L_2(x) \end{bmatrix} \tag{22}$$

with ${}_{j0}L_i(x)$ being the expected numbers of person-years lived in region i between x and x + 5, by an individual born in region j and now x years of age.

The numerical approximation of (21) has given rise to a number of variants of life table construction (Keyfitz, 1968, pp. 228). A simple approximation of $\underset{\sim}{L}(x)$ is a linear combination of the people at exact age x and the people at exact age x + 5:

$$\underset{\sim}{L}(x) = 5[a \cdot \underset{\sim}{\ell}(x) + (1 - a) \cdot \underset{\sim}{\ell}(x + 5)] \quad . \tag{23}$$

In the computer program, a is set equal to 0.5. Therefore,

$$\underset{\sim}{L}(x) = \frac{5}{2}[\underset{\sim}{\ell}(x) + \underset{\sim}{\ell}(x + 5)] \quad . \tag{24}$$

The person-years lived in each region in each age group per unit radices is

$$\underset{\sim}{L}(x) = \frac{5}{2}[\underset{\sim}{\ell}(x) + \underset{\sim}{\ell}(x + 5)] \; \underset{\sim}{\ell}^{-1}(0) \quad . \tag{25}$$

For example, $\underset{\sim}{L}(10)$ given in Table 8 is computed from the table as follows:

$$\underset{\sim}{L}(10) = \frac{5}{2}[\underset{\sim}{\ell}(10) + \underset{\sim}{\ell}(15)] \; \underset{\sim}{\ell}^{-1}(0)$$

$$\begin{bmatrix} 4.694913 & 0.011574 \\ 0.136810 & 4.436604 \end{bmatrix} = \frac{5}{2}\left[\begin{bmatrix} 94,316 & 198 \\ 2,392 & 88,872 \end{bmatrix} + \begin{bmatrix} 93,481 & 265 \\ 3,080 & 88,592 \end{bmatrix}\right]\begin{bmatrix} 100,000 & 0 \\ 0 & 100,000 \end{bmatrix}^{-1}$$

The terminal age interval in a life table is a half-open interval: z years and over. The probability of dying in this interval therefore is unity. It is assumed that people do not move in that age group. Since the length of the interval is infinite, $\underset{\sim}{L}(z + 5)$ is not available and (24) cannot be used to compute $\underset{\sim}{L}(z)$. Following the approach of Rogers (1975, p. 64) we set

$$\underset{\sim}{L}(z) = [\underset{\sim}{M}_{\delta}(z)]^{-1} \; \underset{\sim}{\ell}(z) \tag{26}$$

where $\underset{\sim}{M}_{\delta}(z)$ is a diagonal matrix with the regional death rates of the last age group in the diagonal.

Table 8

Number of Years Lived in Each Region
by the Initial Unit Cohort
****

age	aggregated
***	**********
0	4.828586
5	4.650800
10	4.639950
15	4.627487
20	4.608102
25	4.583485
30	4.554310
35	4.519358
40	4.473121
45	4.404669
50	4.300560
55	4.145690
60	3.906693
65	3.522819
70	2.934452
75	2.181154
80	1.350283
85	1.129550

age	initial region of cohort slovenia		
***	***********************************		
	total	slovenia	r.yugos.
0	4.922968	4.890209	0.032759
5	4.840654	4.748097	0.092557
10	4.831723	4.694913	0.136810
15	4.821670	4.609031	0.212639
20	4.805876	4.456621	0.349255
25	4.787288	4.303031	0.484256
30	4.765104	4.187569	0.577535
35	4.736290	4.107964	0.628327
40	4.696336	4.048200	0.648136
45	4.630364	3.978028	0.652337
50	4.527514	3.880381	0.647134
55	4.376385	3.743020	0.633365
60	4.146075	3.540382	0.605693
65	3.760088	3.203668	0.556420
70	3.139220	2.665276	0.473945
75	2.327638	1.964904	0.362734
80	1.399027	1.160924	0.238103
85	0.962249	0.720453	0.241796

age	initial region of cohort r.yugos.		
***	***********************************		
	total	slovenia	r.yugos.
0	4.734203	0.003152	4.731051
5	4.460945	0.008093	4.452853
10	4.448177	0.011574	4.436604
15	4.433303	0.020462	4.412841
20	4.410328	0.037170	4.373158
25	4.379682	0.051365	4.328318
30	4.343516	0.059095	4.284421
35	4.302426	0.064056	4.238370
40	4.249906	0.067394	4.182512
45	4.178973	0.069071	4.109902
50	4.073606	0.069602	4.004004
55	3.914996	0.070336	3.844660
60	3.667310	0.070187	3.597124
65	3.285549	0.066386	3.219163
70	2.729682	0.056808	2.672875
75	2.034671	0.042728	1.991944
80	1.301538	0.026060	1.275479
85	1.296852	0.016784	1.280068

The $\underset{\sim}{L}(x)$ matrix is further used for computations of stable population characteristics. Its values are stored in a labeled common area:

```
COMMON/CL/L(18,4,4),CLLT(18,4),CLLTOT(18)
```

where L(X,I,J) denotes the number of years lived in region J between ages x and x + 5 by an individual born in region I.

$$\text{CLLT}(X,I) = \sum_{J} L(X,I,J)$$

$$\text{CLLTOT}(X) = \sum_{I} \frac{\text{RADIX}(I)}{\text{RADIXT}} \cdot \text{CLLT}(X,I)$$

with RADIX(I) being the radix of region I, and

$$\text{RADIXT} = \sum_{I} \text{RADIX}(I) \quad .$$

3.4 Total Number of Years to be Lived

The number of years people at age x, may expect to live, is

$$\underset{\sim}{T}(x) = \sum_{y=x}^{z} \underset{\sim}{L}(y) \tag{27}$$

where z is the oldest age group. For example, the value of $\underset{\sim}{T}(10)$ in Table 9 is

$$\underset{\sim}{T}(10) = \begin{bmatrix} 55.264362 & 0.799076 \\ 7.448485 & 56.251442 \end{bmatrix} .$$

$$\begin{matrix} 62.712847 & 57.050518 \end{matrix}$$

Table 9

Total Number of Years to be Lived -T-
****

age	total	slovenia	r.yugos.
***	initial region of cohort slovenia		
0	72.476471	64.902672	7.573800
5	67.553497	60.012459	7.541042
10	62.712849	55.264362	7.448485
15	57.881126	50.569450	7.311675
20	53.059456	45.960419	7.099036
25	48.253582	41.503799	6.749781
30	43.466293	37.200768	6.265525
35	38.701191	33.013199	5.687990
40	33.964897	28.905235	5.059663
45	29.268562	24.857035	4.411527
50	24.638197	20.879007	3.759190
55	20.110683	16.998627	3.112056
60	15.734297	13.255606	2.478691
65	11.588223	9.715224	1.872998
70	7.828135	6.511556	1.316578
75	4.688913	3.846280	0.842633
80	2.361276	1.881377	0.479899
85	0.962249	0.720453	0.241796

age	total	slovenia	r.yugos.
***	initial region of cohort r.yugos.		
0	66.245667	0.810320	65.435349
5	61.511463	0.807169	60.704296
10	57.050518	0.799076	56.251442
15	52.602341	0.787502	51.814838
20	48.169033	0.767040	47.401993
25	43.758705	0.729870	43.028835
30	39.379025	0.678506	38.700520
35	35.035511	0.619411	34.416100
40	30.733084	0.555355	30.177729
45	26.483177	0.487961	25.995216
50	22.304205	0.418890	21.885315
55	18.230600	0.349288	17.881311
60	14.315604	0.278952	14.036652
65	10.648294	0.208766	10.439528
70	7.362744	0.142379	7.220365
75	4.633061	0.085571	4.547490
80	2.598390	0.042844	2.555547
85	1.296852	0.016784	1.280068

The number of years that a girl, just born in Slovenia, may expect to live beyond age 10 is 62.71. From this total, 55.26 years are expected to be lived in Slovenia and 7.45 years in the Rest of Yugoslavia. Similarly, a new born baby girl of Slovenia has ${}_{10}T(60)$ or 15.73 years of retirement to look forward to, 2.48 years of which will be spent in the Rest of Yugoslavia.

3.5 Expectations of Life

The most important life table statistic is the life expectancy. The expectation of life at age x is the number of years an individual may expect to live beyond age x, given that he reaches age x:

$$\underset{\sim}{e}(x) = \underset{\sim}{T}(x)\left[\underset{\sim}{\ell}(x)\ \underset{\sim}{\ell}^{-1}(0)\right]^{-1}, \tag{28}$$

where

$$\underset{\sim}{e}(x) = \begin{bmatrix} {}_{10}e_1(x) & {}_{20}e_1(x) \\ {}_{10}e_2(x) & {}_{20}e_2(x) \end{bmatrix} \tag{29}$$

with ${}_{i0}e_j(x)$ the expectation of life at age x of a person born in region i and expected to be lived in region j.

The life expectancy at each age, except the first, is higher than $\underset{\sim}{T}(x)$, since it is a conditional measure.

The life expectancies of 10-year old girls are (Table 10):

$$\underset{\sim}{e}(10) = \underset{\sim}{T}(10)\left[\underset{\sim}{\ell}(10)\ \underset{\sim}{\ell}^{-1}(0)\right]^{-1}$$

Table 10

Expectations of Life
****

age	aggregated
***	**********
0	69.361069
5	69.265259
10	64.449776
15	59.570431
20	54.769451
25	50.030788
30	45.309593
35	40.613621
40	35.938591
45	31.351742
50	26.897778
55	22.625443
60	18.543140
65	14.808808
70	11.588725
75	9.063486
80	7.016863
85	6.118346

age	initial region of cohort slovenia		
***	***********************************		
	total	slovenia	r.yugos.
0	72.476471	64.902672	7.573801
5	69.701202	61.920414	7.780792
10	64.847992	57.145912	7.702079
15	59.942261	52.370220	7.572042
20	55.095036	47.723652	7.371385
25	50.300976	43.264801	7.036173
30	45.485023	38.928505	6.556518
35	40.719925	34.735237	5.984688
40	35.976246	30.616957	5.359288
45	31.321970	26.600943	4.721028
50	26.847662	22.751362	4.096301
55	22.512747	19.028980	3.483767
60	18.354342	15.462904	2.891438
65	14.463982	12.126177	2.337805
70	11.137589	9.264408	1.873180
75	8.481641	6.957426	1.524215
80	6.243061	4.974238	1.268822
85	5.304964	3.971919	1.333045

age	initial region of cohort r.yugos.		
***	***********************************		
	total	slovenia	r.yugos.
0	66.245667	0.810320	65.435349
5	68.829315	0.903195	67.926117
10	64.051559	0.897136	63.154427
15	59.198593	0.886254	58.312340
20	54.443863	0.866960	53.576904
25	49.760601	0.829978	48.930622
30	45.134163	0.777668	44.356495
35	40.507317	0.716150	39.791168
40	35.900940	0.648739	35.252201
45	31.381514	0.578215	30.803299
50	26.947889	0.506102	26.441788
55	22.738134	0.435650	22.302485
60	18.731941	0.365008	18.366932
65	15.153636	0.297095	14.856541
70	12.039862	0.232824	11.807037
75	9.645331	0.178146	9.467185
80	7.790665	0.128456	7.662209
85	6.931729	0.089710	6.842019

$$\begin{bmatrix} 57.145912 & 0.897136 \\ 7.702079 & 63.154427 \end{bmatrix} = \begin{bmatrix} 55.264362 & 0.799076 \\ 7.448485 & 56.251442 \end{bmatrix} \begin{bmatrix} 0.94316 & 0.00198 \\ 0.02392 & 0.88872 \end{bmatrix}^{-1}$$

$$\begin{matrix} \overline{64.847992} & \overline{64.051559} \end{matrix}$$

where the matrix inverse is $\left[\ell(10)\ \ell^{-1}(0)\right]^{-1}$.

A girl, born in Slovenia, may expect to live another 64.85 years, <u>when</u> reaching 10 years of age. Of this, 7.70 years will be spent in the Rest of Yugoslavia, i.e. 12%. At age 65, however, 2.34 years of the future life time of 14.46 years will be spent in the Rest of Yugoslavia, i.e. 16%.

It is the special feature of the multiregional life table that the demographic measure of the expectation of life is decomposed according to <u>where</u> that life is spent. It introduces the spatial dimension into classical demographic analysis.

3.6 Survivorship and Outmigration Proportions

A most useful application of the multiregional life table is found in multiregional population projection. The assumption is that the survivorship and migration behavior exhibited by the stationary life table population adequately represents the survivorship and migration experience of the empirical population for which the life table was developed.

The necessary information for the projection of age groups beyond the first one, is given by age-specific matrices of survivorship proportions. The number of people in age group x + 5 in the stationary population is

$$\underset{\sim}{L}(x + 5) = \underset{\sim}{S}(x)\ \underset{\sim}{L}(x) \qquad (30)$$

where

$$\underset{\sim}{S}(x) = \begin{bmatrix} s_{11}(x) & s_{21}(x) \\ s_{12}(x) & s_{22}(x) \end{bmatrix} \qquad (31)$$

with $s_{ij}(x)$ being the proportion of individuals aged x to x + 4 who survived to be x + 5 to x + 9 years old 5 years later, by new places of residence.

For example, the number of people in the Rest of Yugoslavia at ages 15 to 19, who were born in Slovenia, per unit radix is (Table 11):

$$_{10}L_2(15) = s_{12}(10)\,_{10}L_1(10) + s_{22}(10)\,_{10}L_2(10)$$

$$0.212639 = 0.016308 * 4.694913 + 0.994601 * 0.136810 \quad .$$

The computation of $\underset{\sim}{L}(x)$ in the life table is not performed using (30) but by (24). In (30), the unknown is $\underset{\sim}{S}(x)$, therefore

$$\underset{\sim}{S}(x) = \underset{\sim}{L}(x + 5)\,\underset{\sim}{L}^{-1}(x) \quad . \tag{32}$$

For x = 10 in the Yugoslavia example, $\underset{\sim}{S}(x)$ is

$$\begin{bmatrix} 0.981648 & 0.002051 \\ 0.016308 & 0.994601 \end{bmatrix} = \begin{bmatrix} 4.609031 & 0.020462 \\ 0.212639 & 4.412841 \end{bmatrix} \begin{bmatrix} 4.694913 & 0.011574 \\ 0.136810 & 4.436604 \end{bmatrix}^{-1}$$

and

$$0.016308 = \frac{0.212639 * 4.436604 + 4.412841 * (-0.136810)}{4.694913 * 4.436604 - 0.011574 * 0.136810} \quad .$$

The number 0.016308 is the proportion of the girls residing in Slovenia and 10 to 14 years old, that will be alive and in the Rest of Yugoslavia 5 years from now.

Table 11

Survivorship Proportions
****

region slovenia
****

	total	slovenia	r.yugos.
0	0.983554	0.970932	0.012622
5	0.998175	0.988783	0.009392
10	0.997956	0.981648	0.016308
15	0.996813	0.966751	0.030062
20	0.996376	0.965259	0.031117
25	0.995779	0.972930	0.022849
30	0.994431	0.980794	0.013638
35	0.992151	0.985297	0.006854
40	0.986384	0.982557	0.003827
45	0.978290	0.975364	0.002925
50	0.967565	0.964468	0.003098
55	0.949213	0.945701	0.003512
60	0.908828	0.904756	0.004072
65	0.835603	0.831859	0.003743
70	0.740756	0.737167	0.003589
75	0.593737	0.590754	0.002983
80	0.622941	0.620487	0.002454

region r.yugos.
****

	total	slovenia	r.yugos.
0	0.942253	0.001064	0.941189
5	0.997136	0.000802	0.996334
10	0.996653	0.002051	0.994601
15	0.994808	0.003940	0.990868
20	0.993023	0.003541	0.989482
25	0.991694	0.002107	0.989587
30	0.990486	0.001423	0.989063
35	0.987727	0.001010	0.986717
40	0.983260	0.000682	0.982578
45	0.974728	0.000543	0.974184
50	0.960951	0.000801	0.960150
55	0.936506	0.000955	0.935551
60	0.895649	0.000802	0.894847
65	0.830716	0.000492	0.830224
70	0.745486	0.000318	0.745168
75	0.640665	0.000411	0.640255
80	1.004030	0.000482	1.003548

3.7 Estimation of Age-Specific Outmigration and Death Probabilities

The probabilities are derived from the observed annual age-specific rates of outmigration and death. The rates are computed by dividing the number of outmigrants and deaths in a certain age group by the mid-year population. The computation is performed in the subroutine PRINTD1 and the output is listed together with the other observed population characteristics. The death and outmigration rates are given in Table 3.

Starting from the observed rates, the probabilities of dying and outmigrating may be computed along two lines. The basic difference is the assumption about multiple transitions. In the original set-up of the probability estimation procedure, no multiple transitions were allowed (Rogers, 1975, p. 82). In other words, it was assumed that an individual only makes one move during a unit time period, five years say. This is consistent with the ideas underlying Markov chain analysis. In later developments, this assumption has been dropped, and multiple moves can be accommodated (Schoen, 1975; Rogers and Ledent, 1976). The estimation of the probabilities under the assumption of no multiple transitions is performed by the subroutine PROBR (option 1). The subroutine PROBSCH (option 3) does the estimation without this assumption. The numerical illustrations of the life table in the text are based on probabilities computed by PROBSCH.

a. Estimation under multiple transition.

The observed outmigration and death rates are arranged in the following matrix $\underset{\sim}{\overline{M}}(x)$.

$$\underset{\sim}{\overline{M}}(x) = \begin{bmatrix} \left[M_{1\delta}(x) + \sum\limits_{j\neq 1} M_{1j}(x)\right] & -M_{21}(x) & \cdots\cdots & -M_{n1}(x) \\ -M_{12}(x) & \left[M_{2\delta}(x) + \sum\limits_{j\neq 2} M_{2j}(x)\right] & & -M_{n2}(x) \\ \vdots & \vdots & & \vdots \\ -M_{1n}(x) & -M_{2n}(x) & & \left[M_{n\delta}(x) + \sum\limits_{j\neq n} M_{nj}(x)\right] \end{bmatrix} \qquad (33)$$

where n is the number of regions;

$M_{i\delta}(x)$ is the age-specific mortality rate in region i;

$M_{ij}(x)$ is the age-specific migration rate from region i to region j.

It can be shown that the probability matrix $\underset{\sim}{P}(x)$ is (Rogers and Ledent, 1976):

$$\underset{\sim}{P}(x) = [\underset{\sim}{I} + \frac{5}{2} \underset{\sim}{\bar{M}}]^{-1} [\underset{\sim}{I} - \frac{5}{2} \underset{\sim}{\bar{M}}] \qquad (34)$$

where, for a two-region case

$$\underset{\sim}{P}(x) = \begin{bmatrix} p_{11}(x) & p_{21}(x) \\ p_{21}(x) & p_{12}(x) \end{bmatrix} \qquad (35)$$

with $p_{ij}(x)$ being the probability that an individual in region i at exact age x will survive and be in region j five years later. The off-diagonal elements are migration probabilities analogous to transition probabilities in Markov theory. The diagonal element $p_{ii}(x)$ denotes the probability of surviving and remaining in region i. The elements of each column in $\underset{\sim}{P}(x)$ do not sum up to unity since mortality is accounted for. Rather, $\underset{\sim}{P}(x)$ is analogous to the transition matrix of an <u>absorbing</u> Markov chain. However, the interpretation of $\underset{\sim}{P}(x)$ is somewhat different. The element $p_{ij}(x)$ for example, does not denote the probability of making a move from i to j by a person living in i at the beginning of the transition period. What it represents is the probability that an individual in region i in the beginning of the time period is in region j in the beginning of the next period. During the period, several moves may have been made.

For example, the matrix of probabilities at age 10 is (Table 5):

$$\underset{\sim}{P}(10) = \begin{bmatrix} 0.991131 & 0.000781 \\ 0.007381 & 0.996834 \end{bmatrix} .$$

The probability that a female in Slovenia at age 10 will be in the Rest of Yugoslavia at age 15 is 0.007381.

The probabilities of dying are found by subtraction. The probability that an individual in region i at age x dies before reaching x + 5 is

$$q_i(x) = 1 - \sum_{j=1}^{n} p_{ij} \quad .$$

The probability of dying in the next five years for a 10 year old in Slovenia is

$$1 - 0.991131 - 0.007381 = 0.001487 \quad .$$

As the computer programs also work for a single region population system, equation (34) is equally valid in the single-region case. Since migration is ignored, (34) becomes

$$p(x) = \left[1 + \frac{5}{2}\,(M_\delta(x))\right]^{-1} \left[1 - \frac{5}{2}\,(M_\delta(x))\right]$$

$$= \frac{1 - \frac{5}{2} M_\delta(x)}{1 + \frac{5}{2} M_\delta(x)} \quad . \tag{36}$$

Formula (36) is equivalent to equation (1.1.9) of Keyfitz (1968, p. 14) and Keyfitz and Flieger (1971, p. 135). The probability of dying is

$$q(x) = 1 - p(x) = \frac{5\,M_\delta(x)}{1 + \frac{5}{2} M_\delta(x)} \tag{37}$$

b. Estimation under no multiple transition.

Rogers (1975, p. 82) shows that under the assumption of no multiple transition, the outmigration probability $p_{ij}(x)$ is given by

$$p_{ij}(x) = \frac{5\ M_{ij}(x)}{1 + \frac{5}{2}\ M_{i\delta}(x) + \frac{5}{2} \sum_{j \neq i} M_{ij}(x)} \quad . \qquad (38)$$

The probability of dying in region i is

$$q_i(x) = \frac{5\ M_{i\delta}(x)}{1 + \frac{5}{2}\ M_{i\delta}(x) + \frac{5}{2} \sum_{j \neq i} M_{ij}(x)} \quad . \qquad (39)$$

The probability of surviving and remaining in the region is found as a residual

$$p_{ii}(x) = 1 - q_i(x) - \sum_{j \neq i} p_{ij}(x) \quad . \qquad (40)$$

The probabilities computed by this method are given in Appendix 2. The matrix of probabilities at age 10 is

$$\underset{\sim}{P}(x) = \begin{bmatrix} 0.991129 & 0.000785 \\ 0.007393 & 0.996831 \end{bmatrix} \quad .$$

For a single region case, $p_{ij}(x) = 0$ and formula (34) reduces to (36). It is clear that the distinction between multiple transition and no multiple transition is irrelevant in a single-region situation, since one can die only once.

3.8 Aggregated Life Table Statistics

The life table statistics considered thus far refer to a multiregional system. The life table functions are basically matrix equations and give regional statistics. In order to aggregate the regional measures to yield the life table statistics for the whole system (country), regional weights must be introduced. The weights are the regional radices, specified by the user.

The aggregation of regional statistics is done in the subroutine WHOLE.[2] The reason for separating the aggregation procedure in a distinct subroutine is to enable the user to experiment with different radices without recomputing the whole life table. WHOLE is called in the main program and may easily be adapted for such experiments.

3.9 Life Table Output

The printing of the life table output is completely separated from its computation. All printing is done by the subroutine PRINTL, which is called by the main program. The life table measures are transferred to PRINTL, via the main program, by labeled COMMON statements. The user may suppress the output, parts of the output, or change its lay-out without disturbing the life table computations.

If the number of regions is less than three, summary life tables are produced. Table 12 summarizes the major life table statistics treated in this paper. A single region life table is given in Appendix 3.

4. MULTIREGIONAL POPULATION PROJECTION

The population growth process has been represented by demographers as a matrix multiplication or, equivalently, as a system of linear, first-order, homogeneous difference equations with constant coefficients. This approach was used by Leslie in 1945 to project populations composed of a number of age group. Rogers (1966) and later Feeney (1970) have generalized Leslie's idea to encompass multiregional population systems.

The general matrix expression of the multiregional growth process is (Rogers, 1975, pp. 122-123):

$$\{\underset{\sim}{K}^{(t+1)}\} = \underset{\sim}{G}\{\underset{\sim}{K}^{(t)}\} \tag{41}$$

---

[2] Warning: Unless regional radices are set in proportion to an estimate of appropriate life table births, the aggregated life table values will be incorrect. For example, setting all radices equal to 100,000 implies that regional births in the life table population are all equal in number. If in the empirical population they are not, then obviously the life table statistics in the aggregated column are wrong.

TABLE 12 - MULTIREGIONAL (TWO-REGION) LIFE TABLE OPTION 3

MORTALITY LEVELS $_{10}e_1(0) = 72.48$ $_{20}e_2(0) = 66.25$ MIGRATION LEVELS $_1\theta_2 = 0.1045$ $_2\theta_1 = 0.0122$

AGE	Q(X,1)	P(X,1,1)	P(X,2,1)	L(X,1,1)	L(X,2,1)	LL(X,1,1)	LL(X,2,1)	M(X,2,1)	MD(X,1)	S(X,1,1)	S(X,2,1)	E(X,1,1)	E(X,2,1)
0	0.030813	0.956084	0.013103	100000.	0.	4.89021	0.03276	0.002832	0.006150	0.970932	0.012622	64.90	7.57
5	0.002164	0.986467	0.011370	95608.	1310.	4.74810	0.09256	0.002294	0.000432	0.988783	0.009392	61.92	7.78
10	0.001487	0.991131	0.007381	94316.	2392.	4.69491	0.13681	0.001485	0.000297	0.981648	0.016308	57.15	7.70
15	0.002598	0.972070	0.025332	93481.	3080.	4.60903	0.21264	0.005158	0.000516	0.966751	0.030062	52.37	7.57
20	0.003770	0.961262	0.034968	90880.	5425.	4.45662	0.34926	0.007170	0.000747	0.965259	0.031117	47.72	7.37
25	0.003439	0.969456	0.027105	87385.	8545.	4.30303	0.48426	0.005534	0.000677	0.972930	0.022849	43.26	7.04
30	0.005015	0.976525	0.018460	84737.	10825.	4.18757	0.57753	0.003756	0.000999	0.980794	0.013638	38.93	6.56
35	0.006121	0.985171	0.008708	82766.	12276.	4.10796	0.62833	0.001765	0.001224	0.985297	0.006854	34.74	5.98
40	0.009586	0.985426	0.004988	81552.	12857.	4.04820	0.64814	0.001013	0.001924	0.982557	0.003827	30.62	5.36
45	0.017694	0.979646	0.002660	80376.	13069.	3.97803	0.65234	0.000543	0.003570	0.975364	0.002925	26.60	4.72
50	0.025793	0.970993	0.003213	78746.	13025.	3.88038	0.64713	0.000663	0.005224	0.964468	0.003098	22.75	4.10
55	0.039248	0.957747	0.003005	76470.	12861.	3.74302	0.63337	0.000629	0.008004	0.945701	0.003512	19.03	3.48
60	0.062780	0.933124	0.004097	73251.	12474.	3.54038	0.60569	0.000884	0.012955	0.904756	0.004072	15.46	2.89
65	0.121486	0.874356	0.004157	68364.	11754.	3.20367	0.55642	0.000949	0.025864	0.831859	0.003743	12.13	2.34
70	0.213259	0.783257	0.003484	59783.	10503.	2.66528	0.47394	0.000876	0.047742	0.737167	0.003589	9.26	1.87
75	0.317728	0.678322	0.003949	46828.	8455.	1.96490	0.36273	0.001111	0.075556	0.590754	0.002983	6.96	1.52
80	0.536332	0.461658	0.002010	31768.	6055.	1.16092	0.23810	0.000704	0.146620	0.620487	0.002454	4.97	1.27
85	1.000000	0.000000	0.000000	14669.	3469.	0.72045	0.24180	0.000000	0.203611	0.000000	0.000000	3.97	1.33

AGE	Q(X,2)	P(X,2,2)	P(X,1,2)	L(X,2,2)	L(X,1,2)	LL(X,2,2)	LL(X,1,2)	M(X,1,2)	MD(X,2)	S(X,2,2)	S(X,1,2)	E(X,2,2)	E(X,1,2)
0	0.106319	0.892421	0.001261	100000.	0.	4.73105	0.00315	0.000272	0.022468	0.941189	0.001064	65.44	0.81
5	0.003341	0.995838	0.000821	89242.	126.	4.45285	0.00809	0.000166	0.000669	0.996334	0.000802	67.93	0.90
10	0.002385	0.996834	0.000781	88872.	198.	4.43660	0.01157	0.000157	0.000478	0.994601	0.002051	63.15	0.90
15	0.004312	0.992355	0.003333	88592.	265.	4.41284	0.02046	0.000679	0.000865	0.990868	0.003940	58.31	0.89
20	0.006075	0.989354	0.004571	87922.	553.	4.37316	0.03717	0.000937	0.001220	0.989482	0.003541	53.58	0.87
25	0.007889	0.989630	0.002481	87005.	934.	4.32832	0.05136	0.000506	0.001585	0.989587	0.002107	48.93	0.83
30	0.008724	0.989555	0.001721	86128.	1121.	4.28442	0.05909	0.000350	0.001753	0.989063	0.001423	44.36	0.78
35	0.010310	0.988575	0.001114	85249.	1243.	4.23837	0.06406	0.000226	0.002073	0.986717	0.001010	39.79	0.72
40	0.014256	0.984840	0.000904	84286.	1319.	4.18251	0.06739	0.000183	0.002872	0.982578	0.000682	35.25	0.65
45	0.019259	0.980282	0.000460	83015.	1376.	4.10990	0.06907	0.000094	0.003889	0.974184	0.000543	30.80	0.58
50	0.031406	0.967964	0.000630	81381.	1386.	4.00400	0.06960	0.000130	0.006382	0.960150	0.000801	26.44	0.51
55	0.046941	0.952077	0.000982	78779.	1398.	3.84466	0.07034	0.000205	0.009615	0.935551	0.000955	22.30	0.44
60	0.080868	0.918193	0.000939	75008.	1416.	3.59712	0.07019	0.000203	0.016857	0.894847	0.000802	18.37	0.37
65	0.129895	0.869422	0.000684	68877.	1392.	3.21916	0.06639	0.000156	0.027784	0.830224	0.000492	14.86	0.30
70	0.214551	0.785139	0.000309	59889.	1264.	2.67287	0.05681	0.000078	0.048067	0.745168	0.000318	11.81	0.23
75	0.305390	0.694257	0.000353	47026.	1008.	1.99194	0.04273	0.000099	0.072084	0.640255	0.000411	9.47	0.18
80	0.436969	0.562471	0.000560	32652.	701.	1.27548	0.02606	0.000196	0.111814	1.003548	0.000482	7.66	0.13
85	1.000000	0.000000	0.000000	18367.	342.	1.28007	0.01678	0.000000	0.143486	0.000000	0.000000	6.84	0.09

where $\{\underset{\sim}{K}^{(t)}\}$ is the age and regional distribution of the population at time t,

$\underset{\sim}{G}$ is the multiregional matrix growth operator or generalized Leslie matrix.

The vector $\{\underset{\sim}{K}^{(t)}\}$ is partitioned as follows:

$$\{\underset{\sim}{K}^{(t)}\} = \begin{bmatrix} \{\underset{\sim}{K}^{(t)}(0)\} \\ \{\underset{\sim}{K}^{(t)}(5)\} \\ \vdots \\ \{\underset{\sim}{K}^{(t)}(z)\} \end{bmatrix} \quad \text{and} \quad \{\underset{\sim}{K}^{(t)}(x)\} = \begin{bmatrix} K_1^{(t)}(x) \\ K_2^{(t)}(x) \\ \vdots \\ K_n^{(t)}(x) \end{bmatrix} \tag{42}$$

where $K_i^{(t)}(x)$ denotes the population in region i at time t, who are x to x + 4 years of age, and

$\{\underset{\sim}{K}^{(t)}(x)\}$ is the regional distribution of the population in age group x to x + 4.

4.1 The Growth Matrix

The arrangement of the growth matrix $\underset{\sim}{G}$ is:

$$\underset{\sim}{G} = \begin{bmatrix} \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{B}(\alpha - 5) & \cdots & \underset{\sim}{B}(\beta - 5) & \underset{\sim}{0} & \cdots & \underset{\sim}{0} \\ \underset{\sim}{S}(0) & \underset{\sim}{0} & & & & & & \\ \underset{\sim}{0} & \underset{\sim}{S}(5) & \ddots & & & & & \\ \vdots & \vdots & & & \ddots & & & \\ \underset{\sim}{0} & \underset{\sim}{0} & & & & \cdot & \underset{\sim}{S}(z - 5) & \underset{\sim}{0} \end{bmatrix} \tag{43}$$

where α and β are the first and last age of childbearing respectively. The matrix of survivorship proportions $\underset{\sim}{S}(x)$ is computed in the multiregional life table. Recall that

$$\underset{\sim}{S}(x) = \begin{bmatrix} s_{11}(x) & s_{21}(x) \\ s_{12}(x) & s_{22}(x) \end{bmatrix} \tag{44}$$

where $s_{ij}(x)$ is the proportion of x to (x + 4)-year-old residents of region i at time t who are alive and x + 5 to x + 9 years old in region j five years later at time t + 1.

The first row of G is composed of matrices B(x):

$$\underset{\sim}{B}(x) = \begin{bmatrix} b_{11}(x) & b_{21}(x) \\ b_{12}(x) & b_{22}(x) \end{bmatrix} \tag{45}$$

where $b_{ij}(x)$ is the average number of babies born during the unit time interval <u>and alive</u> in region j at the end of that interval, per x to (x + 4)-year-old resident of region i at the beginning of that interval. The off-diagonal elements of $\underset{\sim}{B}(x)$ are measures of the mobility of children 0 to 4 years old, who were born to x to (x + 4)-year-old parents.

It can be shown that $\underset{\sim}{B}(x)$ obeys the relationship (Rogers, 1975, pp. 120-121):

$$\underset{\sim}{B}(x) = \frac{1}{2} \underset{\sim}{L}(0) \; \underset{\sim}{\ell}^{-1}(0) \; [\underset{\sim}{F}(x) + \underset{\sim}{F}(x + 5) \; \underset{\sim}{S}(x)]$$

whence

$$\underset{\sim}{B}(x) = \frac{5}{4} [\underset{\sim}{P}(0) + \underset{\sim}{I}] [\underset{\sim}{F}(x) + \underset{\sim}{F}(x + 5) \underset{\sim}{S}(x)] \tag{46}$$

since

$$\underset{\sim}{L}(0) = \frac{5}{2} [\underset{\sim}{\ell}(5) + \underset{\sim}{\ell}(0)] = \frac{5}{2} [\underset{\sim}{P}(0) + \underset{\sim}{I}] \underset{\sim}{\ell}(0)$$

where $\underset{\sim}{L}(0)$, $\underset{\sim}{\ell}(0)$, $\underset{\sim}{P}(0)$, and $\underset{\sim}{S}(x)$ are life table statistics. $\underset{\sim}{F}(x)$ is a diagonal matrix containing the annual regional birth-rates of people aged x to x + 4. The number of births in year t from people aged x to x + 4 at t is $\underset{\sim}{F}(x)\{\underset{\sim}{K}^{(t)}(x)\}$. The number of births during a five year period starting at t, from people aged x to x + 4 at t, is

$$\frac{5}{2} \left[\underset{\sim}{F}(x)\{\underset{\sim}{K}^{(t)}(x)\} + \underset{\sim}{F}(x + 5)\{\underset{\sim}{K}^{(t+1)}(x + 5)\}\right]$$

$$= \frac{5}{2} [\underset{\sim}{F}(x) + \underset{\sim}{F}(x + 5) \underset{\sim}{S}(x)] \{\underset{\sim}{K}^{(t)}(x)\} \quad .$$

Of these births, a proportion $\underset{\sim}{L}(0)[5\underset{\sim}{\ell}(0)]^{-1}$ will be surviving in the various regions at the end of the time interval. Because of the special structure of the generalized Leslie matrix, (41) may be written as two equation systems:

$$\{\underset{\sim}{K}^{(t+1)}(0)\} = \sum_{\alpha-5}^{\beta-5} \underset{\sim}{B}(x)\{\underset{\sim}{K}^{(t)}(x)\} \tag{47}$$

$$\{\underset{\sim}{K}^{(t+1)}(x + 5)\} = \underset{\sim}{S}(x)\{\underset{\sim}{K}^{(t)}(x)\} \quad , \tag{48}$$

$$\text{for } 5 \le x \le z - 5 \quad .$$

The relationships (47) and (48) are the basic components of the population projections as performed by the computer program. The growth matrix is created by the subroutine GROWTH.

The input information and the output are contained in two labeled COMMON statements:

```
COMMON/CSU/SU(18,4,4), SSU(18,4)
COMMON/CGROW/BR(18,4,4), PP1(4,4), POPR(18,4),
             POPPR(4), POPPRT
```

where SU(X,I,J) stands for $\underset{\sim}{S}(x)$. It denotes the proportion of x to (x + 4)-year-old people in region I, who will survive to be in J in the next time interval.

$$SSU(X,I) = \sum_J SU(X,I,J) \quad .$$

BR(X,J,I) stands for $\underset{\sim}{B}(x)$: first row of growth matrix. It denotes the number of children born in region I from a mother x to (x + 4) years of age, who will survive to be in region J in the first age group in the beginning of the next time interval.

PPI(J,I) stands for $\underset{\sim}{P}(0)$: the probability that a baby born in region I will survive and be in J at exact age 5 five years later.

POPR(X,I): population of age group x to (x + 4) in region I. At the beginning of the projection process, POPR(X,I) = POP(X,I), the base year population. During the projection process, POPR(X,I) contains the projected population. At the end of the process, it contains the stable equivalent population.

POPPR(I): total population in region I

$$POPPR(I) = \sum_X POPR(X,I) \quad .$$

POPPRT: total population

$$POPPRT = \sum_I POPPR(I) \quad .$$

The elements of the matrices $\underset{\sim}{B}(x)$ and $\underset{\sim}{S}(x)$ are given in Table 13. For example

$$\underset{\sim}{B}(20) = \frac{5}{4}\,[\underset{\sim}{P}(0) + \underset{\sim}{I}]\,[\underset{\sim}{F}(20) + \underset{\sim}{F}(25)\,\underset{\sim}{S}(20)]$$

$$\begin{bmatrix} 0.321960 & 0.000802 \\ 0.007623 & 0.381934 \end{bmatrix} = \frac{5}{4}\begin{bmatrix} 1.956084 & 0.001261 \\ 0.013103 & 1.892421 \end{bmatrix}\left[\begin{bmatrix} 0.070652 & 0 \\ 0 & 0.087978 \end{bmatrix} + \begin{bmatrix} 0.063218 & 0 \\ 0 & 0.074260 \end{bmatrix}\begin{bmatrix} 0.965259 & 0.003541 \\ 0.031117 & 0.989482 \end{bmatrix}\right].$$

4.2 Projection Process

The age- and region-specific population is projected forward in time by the equation systems (47) and (48) using constant coefficients. This procedure is equivalent to (41). The initial population is the observed base-year population. The projections are for unit time intervals of NY years (five, say) that are equal to the age-interval (Table 14).

For pragmatic reasons, a distinction is made between short-term and long-term projections. Short-term projection outputs are given for every NY years, whereas long-term projection outputs are listed for every NPAR6 years (100 or 200, say). The limit between short- and long-term projections is specified by the user. The purpose of the long-term projection is to identify the stable characteristics of the population system.

In addition to the regional and total population in each age group and the age composition, the output contains the mean age (M.AGE) of the population, the regional shares (SHA) of the total population, and the growth ratio (LAM) of the previous period, i.e. from (t - 1) to t.

Table 13

The Discrete Model of Multiregional Demographic Growth
****

multiregional projection matrix
****

region slovenia
****

age	first row: slovenia[a]	r.yugos.[b]
0	0.000000	0.000000
5	0.000172	0.000003
10	0.038235	0.001277
15	0.205784	0.007635
20	0.321960	0.007623
25	0.252357	0.004084
30	0.155328	0.001800
35	0.074684	0.000696
40	0.020770	0.000159
45	0.002432	0.000021
50	0.000714	0.000005
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000

age	survivorship proportions: slovenia[c]	r.yugos.[d]
0	0.970932	0.012622
5	0.988783	0.009392
10	0.981648	0.016308
15	0.966751	0.030062
20	0.965259	0.031117
25	0.972930	0.022849
30	0.980794	0.013638
35	0.985297	0.006854
40	0.982557	0.003827
45	0.975364	0.002925
50	0.964468	0.003098
55	0.945701	0.003512
60	0.904756	0.004072
65	0.831859	0.003743
70	0.737167	0.003589
75	0.590754	0.002983
80	0.620487	0.002454

Table 13 (continued)

region r.yugos.
****

age	first row slovenia$_e$	first row r.yugos.$_f$
0	0.000000	0.000000
5	0.000000	0.000158
10	0.000121	0.062408
15	0.000860	0.268805
20	0.000802	0.381934
25	0.000398	0.279344
30	0.000186	0.159826
35	0.000075	0.083794
40	0.000024	0.033507
45	0.000005	0.006734
50	0.000001	0.001689
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000

age	survivorship proportions slovenia$_g$	survivorship proportions r.yugos.$_h$
0	0.001064	0.941189
5	0.000802	0.996334
10	0.002051	0.994601
15	0.003940	0.990868
20	0.003541	0.989482
25	0.002107	0.989587
30	0.001423	0.989063
35	0.001010	0.986717
40	0.000682	0.982578
45	0.000543	0.974184
50	0.000801	0.960150
55	0.000955	0.935551
60	0.000802	0.894847
65	0.000492	0.830224
70	0.000318	0.745168
75	0.000411	0.640255
80	0.000482	1.003548

$a = b_{11}(x)$ $c = s_{11}(x)$ $e = b_{21}(x)$ $g = s_{21}(x)$

$b = b_{12}(x)$ $d = s_{12}(x)$ $f = b_{22}(x)$ $h = s_{22}(x)$

Table 14

Multiregional Population Projection
****

year 1961

population

age	total	slovenia	r.yugos.
0	915700.	67800.	847900.
5	979300.	74100.	905200.
10	878800.	70700.	808100.
15	677500.	60100.	617400.
20	788400.	62900.	725500.
25	840500.	66500.	774000.
30	795500.	67100.	728400.
35	696200.	62900.	633300.
40	431900.	39500.	392400.
45	485000.	47900.	437100.
50	505100.	51300.	453800.
55	435400.	46100.	389300.
60	365400.	39600.	325800.
65	260100.	29500.	230600.
70	201700.	21700.	180000.
75	135300.	14400.	120900.
80	68300.	7100.	61200.
85	42900.	3600.	39300.
tot	9503000.	832800.	8670200.

percentage distribution

age	total	slovenia	r.yugos.
0	9.6359	8.1412	9.7795
5	10.3052	8.8977	10.4404
10	9.2476	8.4894	9.3204
15	7.1293	7.2166	7.1209
20	8.2963	7.5528	8.3677
25	8.8446	7.9851	8.9271
30	8.3710	8.0572	8.4012
35	7.3261	7.5528	7.3043
40	4.5449	4.7430	4.5258
45	5.1037	5.7517	5.0414
50	5.3152	6.1599	5.2340
55	4.5817	5.5355	4.4901
60	3.8451	4.7550	3.7577
65	2.7370	3.5423	2.6597
70	2.1225	2.6057	2.0761
75	1.4238	1.7291	1.3944
80	0.7187	0.8525	0.7059
85	0.4514	0.4323	0.4533
total	100.0000	100.0000	100.0000
m.age	30.8458	33.3796	30.6024
sha	100.0000	8.7635	91.2365

Table 14 (continued)

year 1966

population

age	total	slovenia	r.yugos.
0	967579.	69924.	897655.
5	865621.	66731.	798890.
10	976572.	73995.	902577.
15	875950.	71060.	804890.
20	674103.	60534.	613569.
25	783110.	63284.	719826.
30	833790.	66331.	767460.
35	788196.	66848.	721349.
40	687934.	62615.	625319.
45	424793.	39079.	385715.
50	472913.	46957.	425956.
55	485716.	49841.	435875.
60	408341.	43969.	364372.
65	327792.	36090.	291702.
70	216213.	24653.	191560.
75	150262.	16054.	134208.
80	86006.	8557.	77450.
85	65870.	4435.	61435.
tot	10090763.	870955.	9219808.

percentage distribution

age	total	slovenia	r.yugos.
0	9.5888	8.0284	9.7362
5	8.5784	7.6619	8.6649
10	9.6779	8.4958	9.7895
15	8.6807	8.1589	8.7300
20	6.6804	6.9503	6.6549
25	7.7607	7.2660	7.8074
30	8.2629	7.6159	8.3240
35	7.8111	7.6752	7.8239
40	6.8175	7.1892	6.7823
45	4.2097	4.4869	4.1835
50	4.6866	5.3915	4.6200
55	4.8135	5.7225	4.7276
60	4.0467	5.0483	3.9521
65	3.2484	4.1437	3.1639
70	2.1427	2.8306	2.0777
75	1.4891	1.8432	1.4556
80	0.8523	0.9824	0.8400
85	0.6528	0.5092	0.6663
total	100.0000	100.0000	100.0000
m.age	31.6928	34.1771	31.4581
sha	100.0000	8.6312	91.3688
lam	1.061850	1.045815	1.063390

Table 14 (continued)

year 1971

population

age	total	slovenia	r.yugos.
0	988585.	71442.	917143.
5	914592.	68846.	845746.
10	863211.	66624.	796588.
15	973399.	74488.	898911.
20	871544.	71869.	799676.
25	669603.	60604.	608999.
30	776864.	63087.	713777.
35	826119.	66149.	759971.
40	778819.	66593.	712225.
45	676613.	61949.	614664.
50	414197.	38325.	375871.
55	454757.	45630.	409127.
60	455509.	47551.	407958.
65	366309.	40073.	326236.
70	272478.	30165.	242313.
75	161067.	18235.	142833.
80	95514.	9539.	85975.
85	83092.	5347.	77746.
tot	10642274.	906515.	9735759.

percentage distribution

age	total	slovenia	r.yugos.
0	9.2892	7.8809	9.4204
5	8.5940	7.5946	8.6870
10	8.1112	7.3494	8.1821
15	9.1465	8.2170	9.2331
20	8.1895	7.9280	8.2138
25	6.2919	6.6854	6.2553
30	7.2998	6.9593	7.3315
35	7.7626	7.2970	7.8060
40	7.3182	7.3461	7.3156
45	6.3578	6.8338	6.3135
50	3.8920	4.2278	3.8607
55	4.2731	5.0336	4.2023
60	4.2802	5.2454	4.1903
65	3.4420	4.4206	3.3509
70	2.5603	3.3276	2.4889
75	1.5135	2.0115	1.4671
80	0.8975	1.0523	0.8831
85	0.7808	0.5898	0.7986
total	100.0000	100.0000	100.0000
m.age	32.4294	34.8436	32.2046
sha	100.0000	8.5181	91.4819
lam	1.054655	1.040829	1.055961

Table 14 (continued)

year 1976

population

age	total	slovenia	r.yugos.
0	1049627.	74584.	975043.
5	934448.	70341.	864107.
10	912044.	68752.	843292.
15	860408.	67035.	793374.
20	968495.	75553.	892941.
25	865705.	72203.	793501.
30	664288.	60247.	604042.
35	769722.	62891.	706831.
40	816273.	65944.	750329.
45	765989.	65918.	700072.
50	659734.	60757.	598977.
55	398276.	37265.	361012.
60	426462.	43543.	382919.
65	408603.	43349.	365254.
70	304495.	33496.	270999.
75	202986.	22314.	180672.
80	102335.	10831.	91504.
85	92264.	5960.	86304.
tot	11202155.	940982.	10261173.

percentage distribution

age	total	slovenia	r.yugos.
0	9.3699	7.9262	9.5023
5	8.3417	7.4753	8.4211
10	8.1417	7.3065	8.2183
15	7.6807	7.1239	7.7318
20	8.6456	8.0292	8.7021
25	7.7280	7.6732	7.7330
30	5.9300	6.4025	5.8867
35	6.8712	6.6836	6.8884
40	7.2868	7.0080	7.3123
45	6.8379	7.0052	6.8225
50	5.8893	6.4567	5.8373
55	3.5554	3.9602	3.5182
60	3.8070	4.6274	3.7317
65	3.6475	4.6068	3.5596
70	2.7182	3.5597	2.6410
75	1.8120	2.3713	1.7607
80	0.9135	1.1510	0.8917
85	0.8236	0.6334	0.8411
total	100.0000	100.0000	100.0000
m.age	32.9680	35.3194	32.7524
sha	100.0000	8.4000	91.6000
lam	1.052609	1.038021	1.053967

4.3 Stable Equivalent

It is a well-known fact in demography that, in the long run, the age and spatial distribution of the population is independent of the current distribution, and is uniquely determined by the schedules of fertility, mortality and migration as represented by the growth matrix. Therefore, if one projects the population with a constant growth matrix for a long enough period of time, then the ultimate (stable) growth ratio and the ultimate (stable) distribution are independent of the current growth rate and population distribution. In fact, the stable growth ratio is the dominant eigenvalue of $\underset{\sim}{G}$ and the stable distribution is the right eigenvector associated with the dominant eigenvalue. The long-term projection process is totally equivalent to the power method for eigenvalue determination.

It is a characteristic of the stable population that it grows at a constant rate and that its age by region distribution remains constant. This feature underlies the stopping criterion for the projection process (or iteration in the power method). The user may choose between two options:

a. $\lambda_1(t) - \lambda_1(t - 1) \leq \text{TOLX}$ (NPAR3 = 3) (49)

b. $\lambda_{NR}(t) - \lambda_1(t) \leq \text{TOLX}$ (NPAR3 = 2) (50)

where NR is the number of regions,

$\lambda_1(t)$ is the growth ratio of region i in the period from (t - 1) to t,

TOLX is the tolerance level for the eigenvalue computation.

The user makes his choice by specifying the parameter NPAR3 (default value NPAR3 = 3). In the single-region case, option (a) is chosen automatically. The tolerance level TOLX is

$$\text{TOLX} = \frac{1}{10^{\text{NPAR4}}}$$

where NPAR4 is given by the user (default value NPAR4 = 6).

Once stability is achieved, the age and region composition of the population remains constant. All regions grow at the same constant ratio. The stable distribution $\{\underset{\sim}{x}\}$ is the characteristic vector associated with λ, the dominant eigenvalue of $\underset{\sim}{G}$. In other words, $\{\underset{\sim}{x}\}$ is the solution of the following system:

$$\underset{\sim}{G}\{\underset{\sim}{x}\} = \lambda\{\underset{\sim}{x}\} \quad . \tag{51}$$

The eigenvector $\{\underset{\sim}{x}\}$ is unique up to a scalar; therefore we may choose $\{\underset{\sim}{x}\}$ such that its elements sum up to unity, i.e.

$$\{\underset{\sim}{\mathbf{1}}\}'\{\underset{\sim}{x}\} = 1$$

with $\{\underset{\sim}{\mathbf{1}}\}$ a vector of ones.

The ultimate population is

$$\underset{\sim}{G}^n\{\underset{\sim}{K}\} = \underset{\sim}{G}^n Y\{\underset{\sim}{x}\} = \lambda^n Y\{\underset{\sim}{x}\} \tag{52}$$

for n large. The scalar Y is called the <u>stable equivalent</u> of the observed population (Keyfitz, 1969; Rogers, 1975, p. 38). It is the population which, if distributed as the stable population, would increase at the same rate and toward the same population as would, in the long run, the observed population under projection (41).

From (52) it follows that

$$\{\underset{\sim}{\mathbf{1}}\}'\underset{\sim}{G}^n\{\underset{\sim}{K}\} = \lambda^n Y\{\underset{\sim}{\mathbf{1}}\}'\{\underset{\sim}{x}\}$$
$$= \lambda^n Y$$

and

$$Y = \frac{1}{\lambda^n}\{\underset{\sim}{\mathbf{1}}\}'\underset{\sim}{G}^n\{\underset{\sim}{K}\} \quad . \tag{53}$$

The stable equivalent and the stable age and regional distributions are given in Table 15.

Table 15

Stable Equivalent to Original Population
****

age	total	slovenia	r.yugos.
0	941727.	45088.	896639.
5	862498.	43388.	819111.
10	834239.	42250.	791989.
15	806525.	41804.	764721.
20	778320.	42123.	736197.
25	749811.	41967.	707844.
30	721414.	41051.	680363.
35	693245.	39992.	653253.
40	664341.	38861.	625480.
45	633717.	37450.	596267.
50	599277.	35744.	563533.
55	558807.	33876.	524931.
60	508025.	31561.	476464.
65	441748.	28068.	413680.
70	356078.	22844.	333233.
75	257373.	16437.	240936.
80	159189.	9515.	149674.
85	151513.	5796.	145716.
tot	10717847.	597814.	10120033.

percentage distribution
- - - - - - - - - - - - - -

age	total	slovenia	r.yugos.
0	8.7865	7.5422	8.8600
5	8.0473	7.2578	8.0940
10	7.7836	7.0674	7.8260
15	7.5251	6.9928	7.5565
20	7.2619	7.0461	7.2746
25	6.9959	7.0200	6.9945
30	6.7310	6.8668	6.7229
35	6.4681	6.6897	6.4550
40	6.1985	6.5004	6.1806
45	5.9127	6.2644	5.8920
50	5.5914	5.9791	5.5685
55	5.2138	5.6667	5.1870
60	4.7400	5.2794	4.7081
65	4.1216	4.6951	4.0877
70	3.3223	3.8213	3.2928
75	2.4013	2.7495	2.3808
80	1.4853	1.5916	1.4790
85	1.4136	0.9696	1.4399
total	100.0000	100.0000	100.0000
m.age	35.1368	36.7636	35.0408
sha	100.0000	5.5777	94.4223
lam	1.030967	1.030967	1.030967

5. PREPARATION OF THE DATA DECK

There is a single data deck for the computation of all programs. All data are read in at the beginning of the set of programs by the subroutine DATAS. The advantage of concentrating all the READ statements in a single subroutine, is that it enables the user to change the FORMAT statements to fit particular data sets.

The data are read in fixed format from unit 5 (the conventional unit for cards in most computers).

The data deck used to produce the tables in this paper is given in Table 16. The card sequence is as follows:

1) Identification card

   The first card of the deck is an identification card. It may contain any information for the user. The identification card is not read in. Its only purpose is to simplify the handling of the data deck. Therefore, it is advisable to use a card of a specific color as the first card of the deck.

2) Parameter card
3) Title cards
4) Names of the regions
5) Regional radices
6) For each region:
   a) population
   b) births
   c) deaths
   d) migrants
7) The last card of the deck is a blank card. It may be a colored card to identify the end of the deck to the user.

1) Parameter card

The parameter card contains instructions to the program concerning some general characteristics of the data and concerning the desired computations.

Table 16

```
yugoslavia 1961 females  two regions  : data set                                    Identification card
18 2 5 3 1 1 1 2 2 1 1 2 0 0 2 0 019612016 200                                      Parameter card
    multiregional life table                                                        ┐
     yugoslavia : two regions 1961                                                  │ Title cards
     slovenia  - rest yugoslavia                                                    ┘
slovenia r.yugos.                                                                   Names
 100000. 100000.                                                                    Radices
    67800.    74100.    70700.    60100.    62900.    66500.    67100.    62900.   ┐
    39500.    47900.    51300.    46100.    39600.    29500.    21700.    14400.   │ Population of Slovenia
     7100.     3600.                                                               ┘
       .         .        5.      953.     4444.     4204.     2758.     1438.      308.       34.   ┐ Births in Slovenia
      15.        .         .         .         .         .         .         .         .         ┘
     417.       32.       21.       31.       47.       45.       67.       77.       76.      171.   ┐ Deaths in Slovenia
     268.      369.      513.      763.     1036.     1088.     1041.      733.                     ┘
     192.      170.      105.      310.      451.      368.      252.      111.       40.       26.   ┐ Migrants from Slovenia
      34.       29.       35.       28.       19.       16.        5.        4.                     ┘ to Rest of Yugoslavia
   847900.   905200.   808100.   617400.   725500.   774000.   728400.   633300.   ┐ Population of Rest
   392400.   437100.   453800.   389300.   325800.   230600.   180000.   120900.   │   of Yugoslavia
    61200.    39300.                                                               ┘
       .         .       54.    16335.    63828.    57477.    32261.    14903.     4729.      940.   ┐ Births in Rest of
     324.        .         .         .         .         .         .         .         .         ┘   Yugoslavia
   19051.      606.      386.      534.      885.     1227.     1277.     1313.     1127.     1700.   ┐ Deaths in Rest of
    2896.     3743.     5492.     6407.     8652.     8715.     6843.     5639.                     ┘   Yugoslavia
     231.      150.      127.      419.      680.      392.      255.      143.       72.       41.   ┐ Migration from Rest
      59.       80.       66.       36.       14.       12.       12.        3.                     ┘   of Yug. to Slovenia

                                                                                  -- Blank card
```

The following parameters are used in the life table computation and in the projection process:

a. Life table.

NA : number of age groups

NY : width of age group

NR : number of regions. The upper limit is 10 (for output reasons). When NR = 1, the single region life table and projection are computed.

NR : number of title cards.

Parameters specifying the time interval for which the data are given.

NZB : births

NZD : deaths

NZO : migration

e.g. NZO = 1: one-year migration data

NZO = 5: five-year migration data

IRUNT : option for estimating death and outmigration probabilities

IRUNT = 1: option 1 method only (PROBR)

IRUNT = 2: option 3 method only (PROBSCH)

IRUNT ≠ 1 or 2: both methods are computed

LØ : option for probability of dying.

LØ = 1: probability of dying depends on place of birth

LØ = 2: probability of dying depends on place of residence

IOPTG : computation desired.

IOPTG = 0: only life table is computed

IOPTG = 1: only stable (and/or stationary) population is computed. No life table output is given.

IOPTG ≠ 0 or 1: both life table and stable population are computed

b. Population projection.

NA, NR, NY

INIT : base-year (or middle year if three or five year data are used)

NPAR3 : choice of stopping criterion for stable population calculation

NPAR3 = 2: criterion is difference in growth ratio between the first and the last region

NPAR3 = 3: criterion is difference in growth ratio of the first region between the actual and the previous time period

NPAR4: tolerance level for stopping criterion. Tolerance level = $10^{(-NPAR4)}$.

NPAR5: time horizon of short term projections: year until which the detailed projection output is given at each time period.

NPAR6: time interval in years, for long-term projection output, e.g. NPAR6 = 100 - detailed projection output is given every 100 years.

c. Other parameters.

Free: KA, KC, NPAR1, NPAR7, NPAR8.
For later programs (zero growth; stable population): NGRO, NPAR2.

The parameter card is composed of 22 "fields" each requiring a specific piece of information as follows:

COLS	FORMAT	VAR. NAME
1-2	I2	NA
3-4	I2	NR
5-6	I2	NY
7-8	I2	NU
9-10	I2	NZB
11-12	I2	NZD
13-14	I2	NZO
15-16	I2	IRUNT
17-18	I2	IOPTG
19-20	I2	NGRO
21-22	I2	KA
23-24	I2	KC
25-26	I2	LØ
27-28	I2	NPAR1
29-30	I2	NPAR2
31-32	I2	NPAR3
33-34	I2	NPAR4
35-38	I4	INIT
39-43	I4	NPAR5
44-47	I4	NPAR6
48-51	I4	NPAR7
52-55	I4	NPAR8

2) Title cards

There are NU title cards. There is no limit on NU, as long as it is greater than zero. Each title card is printed out as it is read in. All 80 columns of the card may be used. The title is not stored.

3) Names of the regions

In listing the output, each region is identified by its name. Each name consists of a maximum of eight characters. Any character can be used. The names appear in sequence on the same card.

COLS	FORMAT	VAR. NAME
1-80	10A8	REG(I),I = 1,NR

4) Regional radices

The radices appear in sequence on one card.

COLS	FORMAT	VAR. NAME
1-80	10F8.0	RADIX(I),I = 1,NR

5) Population data, births, deaths and migrants

The data related to each region are given sequentially, i.e.

- observations for region 1
- observations for region 2

.
.
.

- observations for region NR.

The age structure is contained on one card followed by continuation cards. If no observations on births are available or required (as for life table computations), the number of births by age of mother is replaced by zero (i.e. 0.).

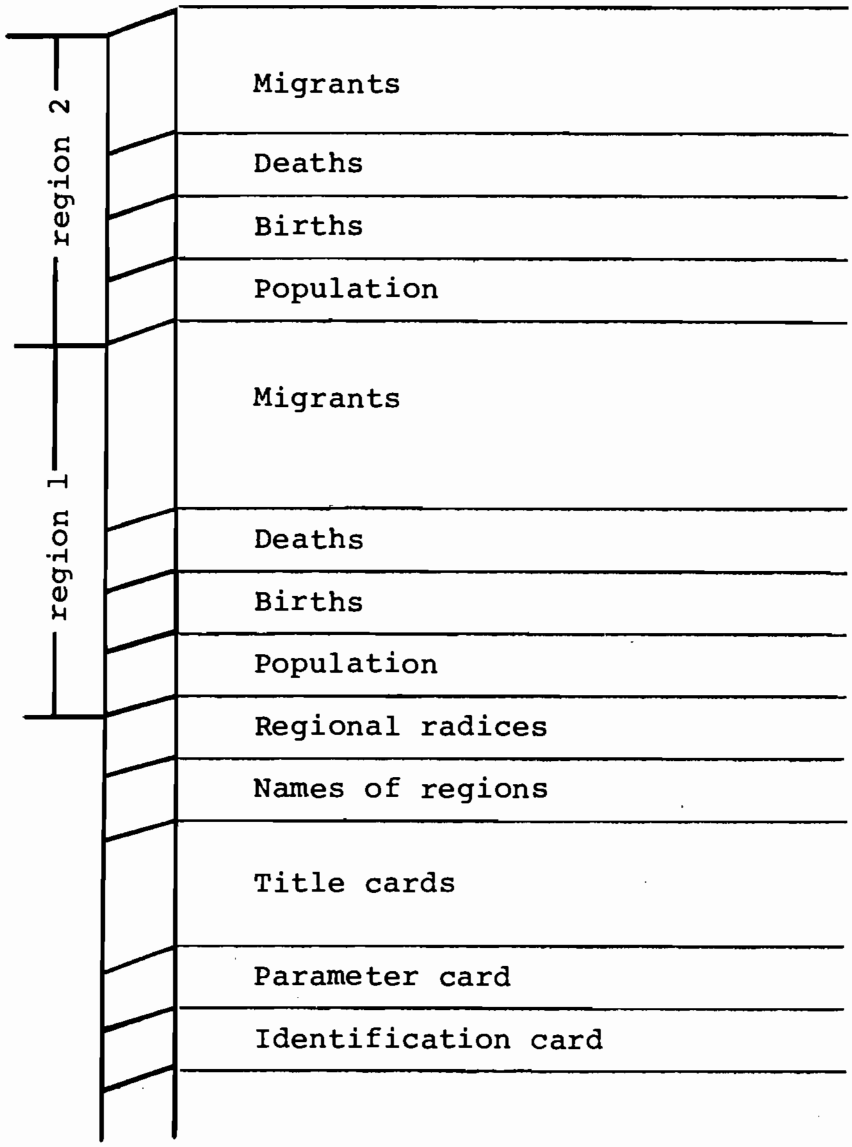
Since the model does not take into account intra-regional migration, such data are not needed. Intra-regional migration data are not read in and must be left out of the data deck.

The sequence of cards and the formats are as follows:

Observations for region I:

	CARD #	COLS	FORMAT	VAR. NAME
	1a	1-80	8F10.0	POP(X,I),X = 1,8
	1b	1-80	8F10.0	POP(X,I),X = 9,16
	1c	1-20	2F10.0	POP(X,I),X = 17,18
	2a	1-80	10F8.0	BIRTH(X,I),X = 1,10
	2b	1-64	8F8.0	BIRTH(X,I),X = 11,18
	3a	1-80	10F8.0	DEATH(X,I),X = 1,10
	3b	1-64	8F8.0	DEATH(X,I),X = 11,18
J = 1,NR J≠I	4a	1-80	10F8.0	OMIG(X,J,I),X = 1,10
	4b	1-64	8F8.0	OMIG(X,J,I),X = 11,18
	⋮			
	3+2(NR-1)a	1-80	10F8.0	OMIG(X,J,I),X = 1,10
	3+2(NR-1)b	1-64	8F8.0	OMIG(X,J,I),X = 11,18

The last card of the deck is a blank card.



<u>Figure 1</u>

Data Deck for Multiregional
Population Model

APPENDIX 1

Glossary of mathematical symbols and FORTRAN names of demographic variables.

Symbol	Page in Rogers (1975)	FORTRAN Name	Description
			Subscripts
		X	age group (1,2,3,...)
		IO(or I)	region of birth
		I	region of residence
		J	region of destination (in case of migration)
			Observations
$K_i(x)$	82	POP(X,I)	population by age and region
		BIRTH(X,I)	births " " " "
$D_i(x)$	82	DEATH(X,I)	deaths " " " "
$K_{ij}(x)$	82	OMIG(X,J,I)	migrants from I to J by age and region
$\ell_i(0)$	73	RADIX(I)	radix of region I
		REG(I)	name of region I
$M_{i\delta}(x)$	82	RATD(X,I)	age-specific death rates for region I
		RATF(X,I)	age-specific fertility rates for region I
$M_{ij}(x)$	83	RATM(X,J,I)	age-specific migration rates from I to J
		RATOT(X,I)	age-specific total outmigration rates of I

		RATDT(X)	age-specific death rates for whole system
		RATFT(X)	age-specific fertility rates for whole system
		RATMT(X)	age-specific migration rates for whole system
		POPC(X)	age distribution of population for total system

Life Table

$q_i(x)$	60	Q(X,I)	probability of dying in I
$p_{ij}(x)$	60	P(X,J,I)	probability of being in J at age X + h, while in I at X
		PMIGT(X,I)	probability of being in another region at age X + h, while in I at X
$_{i0}d_j(x)$	61	CDR(X,IO,I)	deaths in region I of people aged X to X + h and born in region IO
$_{i0}\ell_{jk}(x)$	60	CM(Z,I,J)	migrants from region I to J, aged X to X + h and born in region IO. Z = NA * (IO-1) + X
$_{i0}\ell_j(x)$	60	CL(X,I,J)	probability that an individual born in I will be in J at age X
		CLT(X,I)	probability that an individual born in I survives to age X
		CLTO(X)	average probability that a baby, just born, reaches age X in the system
$_{i0}L_j(x)$	61	L(X,I,J)	number of years lived in J between ages X and X + h by an individual born in I

		CLLT(X,I)	number of years lived between ages X and X + h by an individual born in I
		CLLTOT(X)	average number of years lived between ages X and X + h per unit radix
${}_{i0}T_j(x)$	63	T(X,I,J)	number of years expected to be lived in region J beyond age X, by an individual born in I
		TT(X,I)	number of years expected to be lived beyond age X by an individual born in I
${}_{i0}e_j(x)$	63	E(X,I,J)	expected number of years remained to be lived in region J by an individual born in I, and now of age X
		EE(X,I)	expected number of years remained to be lived by an individual born in I, and now of age X
		ETOTC(X)	average expected number of years remained to be lived by an individual of age X
$s_{ij}(x)$	79	SU(X,I,J)	proportion of people aged X to X + h in region I, surviving to be in region J and X + h to X + 2h years old h years later
		SSU(X,I)	proportion of people aged X to X + h in region I, surviving to be X + h to X + 2h years old h years later

Population Projection

$s_{ij}(x)$	118	SU(X,I,J)	see life table
$b_{ij}(x)$	118	BR(X,J,I)	average number of babies born during the unit time interval and alive in region J at end of that interval, per X to (X + h) year old residents of region I at beginning of that interval

$p_{ij}(0)$	60	PP1(J,I)	probability that an individual born in I survives to be in J h years later (at beginning of next interval)
$K_i^{(t)}(x)$	118	POPR(X,I)	projected population in age group X in region I
		POPPR(I)	total projected population in region I
		POPPRT	total projected population in whole system

APPENDIX 2

Multiregional Life Table and Population Projection

Option 1: No Multiple Transition

```
*********************************
*   MULTIREGIONAL LIFE TABLE    *
* YUGOSLAVIA : TWO REGIONS 1961 *
*  SLOVENIA - REST YUGOSLAVIA   *
*********************************
```

LIST OF PARAMETERS
****

```
NA     =   18       NY     =    5       NR     =    2

NZB    =    1       NZD    =    1       NZO    =    1

IRUNT  =    1       IOPTG  =    2       NGRO   =    1

INIT   = 1961       KA     =    1       KC     =    2

NU     =    3       L0     =    2       NPAR1  =    0

NPAR2  =    2       NPAR3  =    0       NPAR4  =    7

NPAR5  = 2016       NPAR6  =  200       NPAR7  =    0

NPAR8  =    0
```

OBSERVED POPULATION CHARACTERISTICS

INPUT-DATA

REGION SLOVENIA

AGE	POPULATION	BIRTHS	DEATHS	MIGRATION FROM SLOVENIA TO SLOVENIA	R.YUGOS.
0	67800.	0.	417.	0.	192.
5	74100.	0.	32.	0.	170.
10	70700.	5.	21.	0.	105.
15	60100.	953.	31.	0.	310.
20	62900.	4444.	47.	0.	451.
25	66500.	4204.	45.	0.	368.
30	67100.	2758.	67.	0.	252.
35	62900.	1438.	77.	0.	111.
40	39500.	308.	76.	0.	40.
45	47900.	34.	171.	0.	26.
50	51300.	15.	268.	0.	34.
55	46100.	0.	369.	0.	29.
60	39600.	0.	513.	0.	35.
65	29500.	0.	763.	0.	28.
70	21700.	0.	1036.	0.	19.
75	14400.	0.	1088.	0.	16.
80	7100.	0.	1041.	0.	5.
85	3600.	0.	733.	0.	4.
TOTAL	832800.	14159.	6795.	0.	2195.

REGION R.YUGOS.

AGE	POPULATION	BIRTHS	DEATHS	MIGRATION FROM R.YUGOS. TO SLOVENIA	R.YUGOS.
0	847900.	0.	19051.	231.	0.
5	905200.	0.	606.	150.	0.
10	808100.	54.	386.	127.	0.
15	617400.	16335.	534.	419.	0.
20	725500.	63828.	885.	680.	0.
25	774000.	57477.	1227.	392.	0.
30	728400.	32261.	1277.	255.	0.
35	633300.	14903.	1313.	143.	0.
40	392400.	4729.	1127.	72.	0.
45	437100.	940.	1700.	41.	0.
50	453800.	324.	2896.	59.	0.
55	389300.	0.	3743.	80.	0.
60	325800.	0.	5492.	66.	0.
65	230600.	0.	6407.	36.	0.
70	180000.	0.	8652.	14.	0.
75	120900.	0.	8715.	12.	0.
80	61200.	0.	6843.	12.	0.
85	39300.	0.	5639.	3.	0.
TOTAL	8670200.	190851.	76493.	2792.	0.

PERCENTAGE DISTRIBUTION
****

REGION SLOVENIA

AGE	POPULATION	BIRTHS	DEATHS	MIGRATION FROM SLOVENIA TO SLOVENIA	R.YUGOS.
0	8.1412	0.0000	6.1369	0.0000	8.7472
5	8.8977	0.0000	0.4709	0.0000	7.7449
10	8.4894	0.0353	0.3091	0.0000	4.7836
15	7.2166	6.7307	0.4562	0.0000	14.1230
20	7.5528	31.3864	0.6917	0.0000	20.5467
25	7.9851	29.6914	0.6623	0.0000	16.7654
30	8.0572	19.4788	0.9860	0.0000	11.4806
35	7.5528	10.1561	1.1332	0.0000	5.0569
40	4.7430	2.1753	1.1185	0.0000	1.8223
45	5.7517	0.2401	2.5166	0.0000	1.1845
50	6.1599	0.1059	3.9441	0.0000	1.5490
55	5.5355	0.0000	5.4305	0.0000	1.3212
60	4.7550	0.0000	7.5497	0.0000	1.5945
65	3.5423	0.0000	11.2288	0.0000	1.2756
70	2.6057	0.0000	15.2465	0.0000	0.8656
75	1.7291	0.0000	16.0118	0.0000	0.7289
80	0.8525	0.0000	15.3201	0.0000	0.2278
85	0.4323	0.0000	10.7873	0.0000	0.1822
TOTAL	100.0000	100.0000	100.0000	100.0000	100.0000
M.AGE	33.3796	27.6427	66.0931	0.0000	25.0376

REGION R.YUGOS.

AGE	POPULATION	BIRTHS	DEATHS	MIGRATION FROM R.YUGOS. TO SLOVENIA	R.YUGOS.
0	9.7795	0.0000	24.9055	8.2736	0.0000
5	10.4404	0.0000	0.7922	5.3725	0.0000
10	9.3204	0.0283	0.5046	4.5487	0.0000
15	7.1209	8.5590	0.6981	15.0072	0.0000
20	8.3677	33.4439	1.1570	24.3553	0.0000
25	8.9271	30.1162	1.6041	14.0401	0.0000
30	8.4012	16.9038	1.6694	9.1332	0.0000
35	7.3043	7.8087	1.7165	5.1218	0.0000
40	4.5258	2.4778	1.4733	2.5788	0.0000
45	5.0414	0.4925	2.2224	1.4685	0.0000
50	5.2340	0.1698	3.7860	2.1132	0.0000
55	4.4901	0.0000	4.8933	2.8653	0.0000
60	3.7577	0.0000	7.1797	2.3639	0.0000
65	2.6597	0.0000	8.3759	1.2894	0.0000
70	2.0761	0.0000	11.3108	0.5014	0.0000
75	1.3944	0.0000	11.3932	0.4298	0.0000
80	0.7059	0.0000	8.9459	0.4298	0.0000
85	0.4533	0.0000	7.3719	0.1074	0.0000
TOTAL	100.0000	100.0000	100.0000	100.0000	100.0000
M.AGE	30.6024	27.1063	50.2401	26.0781	0.0000

OBSERVED RATES
****

DEATH RATES
****

AGE	SLOVENIA	R.YUGOS.
0	0.006150	0.022468
5	0.000432	0.000669
10	0.000297	0.000478
15	0.000516	0.000865
20	0.000747	0.001220
25	0.000677	0.001585
30	0.000999	0.001753
35	0.001224	0.002073
40	0.001924	0.002872
45	0.003570	0.003889
50	0.005224	0.006382
55	0.008004	0.009615
60	0.012955	0.016857
65	0.025864	0.027784
70	0.047742	0.048067
75	0.075556	0.072084
80	0.146620	0.111814
85	0.203611	0.143486
GROSS	0.542112	0.473962
CRUDE	0.008159	0.008823
M.AGE	79.1635	74.4001

FERTILITY RATES
****

AGE	SLOVENIA	R.YUGOS.
0	0.000000	0.000000
5	0.000000	0.000000
10	0.000071	0.000067
15	0.015857	0.026458
20	0.070652	0.087978
25	0.063218	0.074260
30	0.041103	0.044290
35	0.022862	0.023532
40	0.007797	0.012051
45	0.000710	0.002151
50	0.000292	0.000714
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000
85	0.000000	0.000000
GROSS	0.222562	0.271501
CRUDE	0.017002	0.022012
M.AGE	27.7683	27.4740

OUTMIGRATION RATES
****

AGE	MIGRATION FROM SLOVENIA TO TOTAL	SLOVENIA	R.YUGOS.
0	0.002832	0.000000	0.002832
5	0.002294	0.000000	0.002294
10	0.001485	0.000000	0.001485
15	0.005158	0.000000	0.005158
20	0.007170	0.000000	0.007170
25	0.005534	0.000000	0.005534
30	0.003756	0.000000	0.003756
35	0.001765	0.000000	0.001765
40	0.001013	0.000000	0.001013
45	0.000543	0.000000	0.000543
50	0.000663	0.000000	0.000663
55	0.000629	0.000000	0.000629
60	0.000884	0.000000	0.000884
65	0.000949	0.000000	0.000949
70	0.000876	0.000000	0.000876
75	0.001111	0.000000	0.001111
80	0.000704	0.000000	0.000704
85	0.000000	0.000000	0.000000
GROSS	0.037365	0.000000	0.037365
CRUDE	0.002636	0.000000	0.002636
M.AGE	29.4835	0.0000	29.4835

AGE	MIGRATION FROM R.YUGOS. TO TOTAL	SLOVENIA	R.YUGOS.
0	0.000272	0.000272	0.000000
5	0.000166	0.000166	0.000000
10	0.000157	0.000157	0.000000
15	0.000679	0.000679	0.000000
20	0.000937	0.000937	0.000000
25	0.000506	0.000506	0.000000
30	0.000350	0.000350	0.000000
35	0.000226	0.000226	0.000000
40	0.000183	0.000183	0.000000
45	0.000094	0.000094	0.000000
50	0.000130	0.000130	0.000000
55	0.000205	0.000205	0.000000
60	0.000203	0.000203	0.000000
65	0.000156	0.000156	0.000000
70	0.000078	0.000078	0.000000
75	0.000099	0.000099	0.000000
80	0.000196	0.000196	0.000000
85	0.000000	0.000000	0.000000
GROSS	0.004638	0.004638	0.000000
CRUDE	0.000322	0.000322	0.000000
M.AGE	33.2519	33.2519	0.0000

TOTAL POPULATION SYSTEM
****

AGE	POPULATION ABSOLUTE	POPULATION PERCENT	BIRTHS ABSOLUTE	BIRTHS PERCENT	DEATHS ABSOLUTE	DEATHS PERCENT	MIGRATION ABSOLUTE	MIGRATION PERCENT	OBSERVED RATES BIRTH	OBSERVED RATES DEATH	OBSERVED RATES MIGRATION
0	915700.	9.6359	0.	0.0000	19468.	23.3743	423.	8.4821	0.000000	0.021260	0.000462
5	979300.	10.3052	0.	0.0000	638.	0.7660	320.	6.4167	0.000000	0.000651	0.000327
10	878800.	9.2476	59.	0.0288	407.	0.4887	232.	4.6521	0.000067	0.000463	0.000264
15	677500.	7.1293	17288.	8.4328	565.	0.6784	729.	14.6180	0.025517	0.000834	0.001076
20	788400.	8.2963	68272.	33.3018	932.	1.1190	1131.	22.6790	0.086596	0.001182	0.001435
25	840500.	8.8446	61681.	30.0868	1272.	1.5272	760.	15.2396	0.073386	0.001513	0.000904
30	795500.	8.3710	35019.	17.0816	1344.	1.6137	507.	10.1664	0.044021	0.001690	0.000637
35	696200.	7.3261	16341.	7.9708	1390.	1.6689	254.	5.0932	0.023472	0.001997	0.000365
40	431900.	4.5449	5037.	2.4570	1203.	1.4444	112.	2.2458	0.011662	0.002785	0.000259
45	485000.	5.1037	974.	0.4751	1871.	2.2464	67.	1.3435	0.002008	0.003858	0.000138
50	505100.	5.3152	339.	0.1654	3164.	3.7989	93.	1.8648	0.000671	0.006264	0.000184
55	435400.	4.5817	0.	0.0000	4112.	4.9371	109.	2.1857	0.000000	0.009444	0.000250
60	365400.	3.8451	0.	0.0000	6005.	7.2099	101.	2.0253	0.000000	0.016434	0.000276
65	260100.	2.7370	0.	0.0000	7170.	8.6087	64.	1.2833	0.000000	0.027566	0.000246
70	201700.	2.1225	0.	0.0000	9688.	11.6319	33.	0.6617	0.000000	0.048032	0.000164
75	135300.	1.4238	0.	0.0000	9803.	11.7700	28.	0.5615	0.000000	0.072454	0.000207
80	68300.	0.7187	0.	0.0000	7884.	9.4659	17.	0.3409	0.000000	0.115432	0.000249
85	42900.	0.4514	0.	0.0000	6372.	7.6506	7.	0.1404	0.000000	0.148531	0.000163
TOTAL	9503000.	100.0000	205010.	100.0000	83288.	100.0000	4987.	100.0000	0.267401	0.480391	0.007607
CRUDE									0.021573	0.008764	0.000525
M.AGE		30.8458		27.1434		51.5334		25.6201	27.4908	74.8451	33.5908

MULTIREGIONAL LIFE TABLE OPTION 1

PROBABILITIES OF DYING AND MIGRATING

REGION SLOVENIA

AGE	DEATH	MIGRATION FROM SLOVENIA TO SLOVENIA	R.YUGOS.
0	0.030077	0.956075	0.013848
5	0.002145	0.986462	0.011393
10	0.001479	0.991129	0.007393
15	0.002543	0.972027	0.025430
20	0.003664	0.961182	0.035155
25	0.003332	0.969422	0.027246
30	0.004934	0.976509	0.018557
35	0.006075	0.985166	0.008758
40	0.009550	0.985423	0.005026
45	0.017668	0.979646	0.002686
50	0.025742	0.970992	0.003266
55	0.039176	0.957745	0.003079
60	0.062607	0.933122	0.004271
65	0.121198	0.874355	0.004448
70	0.212840	0.783256	0.003903
75	0.317016	0.678322	0.004662
80	0.535769	0.461657	0.002573
85	1.000000	0.000000	0.000000

REGION R.YUGOS.

AGE	DEATH	MIGRATION FROM R.YUGOS. TO SLOVENIA	R.YUGOS.
0	0.106299	0.001289	0.892412
5	0.003340	0.000827	0.995833
10	0.002385	0.000785	0.996831
15	0.004308	0.003380	0.992312
20	0.006067	0.004661	0.989272
25	0.007885	0.002519	0.989596
30	0.008720	0.001741	0.989539
35	0.010307	0.001123	0.988570
40	0.014251	0.000910	0.984838
45	0.019255	0.000464	0.980281
50	0.031397	0.000640	0.967963
55	0.046922	0.001003	0.952076
60	0.080837	0.000971	0.918191
65	0.129850	0.000730	0.869420
70	0.214514	0.000347	0.785139
75	0.305323	0.000420	0.694256
80	0.436764	0.000766	0.562470
85	1.000000	0.000000	0.000000

DEATH RATE AND MIGRATION RATES
****

REGION SLOVENIA
****

AGE	DEATH	MIGRATION FROM SLOVENIA TO SLOVENIA	R.YUGOS.
0	0.006150	0.195509	0.002832
5	0.000432	0.198637	0.002294
10	0.000297	0.199109	0.001485
15	0.000516	0.197163	0.005158
20	0.000747	0.196041	0.007170
25	0.000677	0.196895	0.005534
30	0.000999	0.197623	0.003756
35	0.001224	0.198506	0.001765
40	0.001924	0.198532	0.001013
45	0.003570	0.197944	0.000543
50	0.005224	0.197057	0.000663
55	0.008004	0.195683	0.000629
60	0.012955	0.193081	0.000884
65	0.025864	0.186593	0.000949
70	0.047742	0.175691	0.000876
75	0.075556	0.161667	0.001111
80	0.146620	0.126338	0.000704
85	0.203611	0.000000	0.000000

REGION R.YUGOS.
****

AGE	DEATH	MIGRATION FROM R.YUGOS. TO SLOVENIA	R.YUGOS.
0	0.022468	0.000272	0.188630
5	0.000669	0.000166	0.199582
10	0.000478	0.000157	0.199683
15	0.000865	0.000679	0.199228
20	0.001220	0.000937	0.198921
25	0.001585	0.000506	0.198954
30	0.001753	0.000350	0.198948
35	0.002073	0.000226	0.198850
40	0.002872	0.000183	0.198472
45	0.003889	0.000094	0.198008
50	0.006382	0.000130	0.196744
55	0.009615	0.000205	0.195090
60	0.016857	0.000203	0.191470
65	0.027784	0.000156	0.186030
70	0.048067	0.000078	0.175928
75	0.072084	0.000099	0.163908
80	0.111814	0.000196	0.143995
85	0.143486	0.000000	0.000000

LIFE HISTORY OF INITIAL COHORT
****

INITIAL REGION OF COHORT SLOVENIA
****

NUMBER OF DEATHS IN EACH REGION OF RESIDENCE

AGE	SLOVENIA	R.YUGOS.
0	3007.68	0.00
5	205.04	4.63
10	139.45	5.89
15	237.72	13.60
20	332.93	33.43
25	291.10	68.18
30	418.02	95.37
35	502.76	127.75
40	778.71	184.95
45	1419.84	253.98
50	2026.73	412.77
55	2995.29	609.16
60	4585.27	1018.21
65	8284.27	1542.38
70	12722.26	2280.53
75	14843.30	2619.75
80	17018.13	2697.09
85	14668.76	0.00
TOTAL	84477.26	11967.68

NUMBER OF MIGRANTS FROM SLOVENIA TO

AGE	SLOVENIA	R.YUGOS.
0	95607.49	1384.83
5	94313.16	1089.29
10	93477.62	697.25
15	90864.69	2377.15
20	87347.73	3194.70
25	84701.73	2380.59
30	82733.25	1572.25
35	81524.79	724.75
40	80350.15	409.85
45	78726.24	215.88
50	76448.52	257.12
55	73226.23	235.40
60	68341.15	312.84
65	59765.10	304.01
70	46818.18	233.32
75	31760.29	218.28
80	14664.04	81.74
85	0.00	0.00
TOTAL	1240670.25	15689.25

AGE	NUMBER OF MIGRANTS FROM R.YUGOS. TO SLOVENIA	R.YUGOS.
0	0.00	0.00
5	1.15	1379.06
10	1.94	2460.53
15	10.67	3133.50
20	25.69	5451.53
25	21.78	8556.27
30	19.04	10822.45
35	13.91	12253.03
40	11.82	12781.01
45	6.13	12930.75
50	8.41	12725.45
55	13.02	12360.39
60	12.24	11565.35
65	8.67	10327.14
70	3.69	8346.92
75	3.61	5956.89
80	4.73	3473.35
85	0.00	0.00
TOTAL	166.48	134523.63

INITIAL REGION OF COHORT R.YUGOS.
****

AGE	NUMBER OF DEATHS IN EACH REGION OF RESIDENCE SLOVENIA	R.YUGOS.
0	0.00	10629.89
5	0.28	298.10
10	0.30	211.92
15	0.68	381.65
20	2.05	533.35
25	3.16	685.95
30	5.62	750.91
35	7.67	878.51
40	12.79	1200.98
45	24.67	1598.13
50	36.20	2554.70
55	55.54	3695.76
60	89.95	6062.34
65	171.32	8942.18
70	273.75	12844.95
75	325.96	14355.86
80	384.26	14259.34
85	0.00	18365.20
TOTAL	1394.21	98249.70

AGE	NUMBER OF MIGRANTS FROM SLOVENIA TO SLOVENIA	R.YUGOS.
0	0.00	0.00
5	127.15	1.47
10	199.15	1.49
15	261.35	6.84
20	539.04	19.72
25	919.83	25.85
30	1112.22	21.14
35	1243.44	11.05
40	1319.60	6.73
45	1367.91	3.75
50	1365.65	4.59
55	1357.79	4.36
60	1340.70	6.14
65	1235.94	6.29
70	1007.42	5.02
75	697.45	4.79
80	331.11	1.85
85	0.00	0.00
TOTAL	14425.75	131.07

AGE	NUMBER OF MIGRANTS FROM R.YUGOS. TO SLOVENIA	R.YUGOS.
0	128.89	89241.22
5	73.79	88869.34
10	69.72	88589.16
15	299.46	87909.55
20	409.80	86973.23
25	219.15	86087.86
30	149.95	85212.86
35	95.68	84259.80
40	76.73	82993.15
45	38.54	81363.21
50	52.05	78760.22
55	78.99	74990.06
60	72.85	68859.23
65	50.24	59872.95
70	20.78	47013.50
75	19.77	32642.90
80	25.01	18363.35
85	0.00	0.00
TOTAL	1881.40	1242001.63

EXPECTED NUMBER OF SURVIVORS AT EXACT AGE x IN EACH REGION
****

AGE	AGGREGATED	AGE	INITIAL REGION OF COHORT SLOVENIA		
***	**********	***	TOTAL	SLOVENIA	R.YUGOS.
0	200000.	0	100000.	100000.	0.
5	186362.	5	96992.	95607.	1385.
10	185854.	10	96783.	94314.	2468.
15	185497.	15	96637.	93480.	3158.
20	184863.	20	96386.	90875.	5511.
25	183961.	25	96020.	87373.	8646.
30	182913.	30	95660.	84724.	10937.
35	181643.	35	95147.	82752.	12395.
40	180126.	40	94516.	81539.	12978.
45	177949.	45	93553.	80362.	13191.
50	174652.	50	91879.	78732.	13147.
55	169622.	55	89440.	76457.	12983.
60	162266.	60	85835.	73239.	12596.
65	150511.	65	80232.	68353.	11878.
70	131570.	70	70405.	59774.	10631.
75	103449.	75	55402.	46822.	8580.
80	71304.	80	37939.	31764.	6175.
85	36945.	85	18224.	14669.	3555.

AGE	INITIAL REGION OF COHORT R.YUGOS.		
***	TOTAL	SLOVENIA	R.YUGOS.
0	100000.	0.	100000.
5	89370.	129.	89241.
10	89072.	201.	88871.
15	88860.	269.	88591.
20	88477.	561.	87916.
25	87942.	949.	86993.
30	87253.	1139.	86114.
35	86496.	1262.	85234.
40	85610.	1339.	84271.
45	84396.	1396.	83000.
50	82773.	1406.	81367.
55	80183.	1418.	78765.
60	76431.	1437.	74994.
65	70279.	1414.	68865.
70	61165.	1286.	59879.
75	48047.	1028.	47019.
80	33365.	717.	32648.
85	18721.	356.	18365.

NUMBER OF YEARS LIVED IN EACH REGION BY THE INITIAL UNIT COHORT
****

AGE	AGGREGATED	AGE	INITIAL REGION OF COHORT SLOVENIA		
			TOTAL	SLOVENIA	R.YUGOS.
0	4.829531	0	4.924808	4.890187	0.034621
5	4.652711	5	4.844375	4.748045	0.096330
10	4.641891	10	4.835500	4.694847	0.140653
15	4.629501	15	4.825583	4.608873	0.216711
20	4.610308	20	4.810142	4.456220	0.353922
25	4.585932	25	4.792001	4.302423	0.489577
30	4.556953	30	4.770185	4.186895	0.583289
35	4.522120	35	4.741587	4.107275	0.634312
40	4.475944	40	4.701733	4.047517	0.654216
45	4.407518	45	4.635796	3.977358	0.658437
50	4.303431	50	4.532963	3.879733	0.653230
55	4.148604	55	4.381864	3.742405	0.639459
60	3.909710	60	4.151666	3.539816	0.611850
65	3.526011	65	3.765912	3.203179	0.562733
70	2.937740	70	3.145176	2.664891	0.480285
75	2.184411	75	2.333530	1.964644	0.368886
80	1.353114	80	1.404073	1.160817	0.243257
85	1.132808	85	0.968196	0.720430	0.247766

AGE	INITIAL REGION OF COHORT R.YUGOS.		
	TOTAL	SLOVENIA	R.YUGOS.
0	4.734253	0.003222	4.731030
5	4.461046	0.008246	4.452801
10	4.448281	0.011745	4.436536
15	4.433418	0.020742	4.412676
20	4.410475	0.037741	4.372734
25	4.379862	0.052195	4.327667
30	4.343721	0.060028	4.283692
35	4.302654	0.065032	4.237622
40	4.250155	0.068386	4.181769
45	4.179241	0.070069	4.109171
50	4.073898	0.070604	4.003294
55	3.915343	0.071362	3.843981
60	3.667754	0.071258	3.596495
65	3.286109	0.067493	3.218615
70	2.730304	0.057860	2.672444
75	2.035291	0.043635	1.991655
80	1.302156	0.026833	1.275322
85	1.297419	0.017490	1.279930

SURVIVORSHIP PROPORTIONS
****

REGION SLOVENIA
****

	TOTAL	SLOVENIA	R.YUGOS.
0	0,983961	0.970926	0,013035
5	0,998189	0.988779	0,009410
10	0,997988	0.981625	0,016363
15	0,996893	0.966690	0,030203
20	0,996483	0.965201	0,031282
25	0,995874	0.972905	0,022969
30	0,994495	0.980783	0,013711
35	0,992191	0.985294	0,006898
40	0,986414	0.982555	0,003859
45	0,978328	0.975364	0,002965
50	0,967627	0.964466	0,003160
55	0,949334	0.945699	0,003635
60	0,909055	0.904755	0,004300
65	0,835945	0.831858	0,004087
70	0,741291	0.737166	0,004125
75	0,594356	0.590754	0,003602
80	0,623627	0.620486	0,003141

REGION R.YUGOS.
****

	TOTAL	SLOVENIA	R.YUGOS.
0	0,942263	0.001082	0,941182
5	0,997137	0.000807	0,996330
10	0,996655	0.002077	0,994579
15	0,994815	0.004009	0,990806
20	0,993030	0.003606	0,989424
25	0,991699	0.002137	0,989562
30	0,990490	0.001437	0,989053
35	0,987731	0.001017	0,986714
40	0,983264	0.000688	0,982576
45	0,974733	0.000550	0,974183
50	0,960965	0.000816	0,960149
55	0,936531	0.000981	0,935550
60	0,895686	0.000840	0,894846
65	0,830756	0.000533	0,830223
70	0,745535	0.000368	0,745167
75	0,640784	0.000530	0,640254
80	1,004205	0.000659	1,003546

TOTAL NUMBER OF YEARS TO BE LIVED -T-
****

AGE INITIAL REGION OF COHORT SLOVENIA

AGE	TOTAL	SLOVENIA	R.YUGOS.
0	72.565094	64.895561	7.669532
5	67.640289	60.005375	7.634912
10	62.795914	55.257332	7.538582
15	57.960411	50.562481	7.397929
20	53.134823	45.953606	7.181219
25	48.324684	41.497387	6.827297
30	43.532681	37.194962	6.337719
35	38.762497	33.008068	5.754430
40	34.020908	28.900789	5.120118
45	29.319176	24.853273	4.465902
50	24.683380	20.875916	3.807465
55	20.150415	16.996181	3.154235
60	15.768553	13.253778	2.514775
65	11.616887	9.713962	1.902926
70	7.850976	6.510783	1.340193
75	4.705799	3.845891	0.859908
80	2.372269	1.881247	0.491022
85	0.968196	0.720430	0.247766

AGE INITIAL REGION OF COHORT R.YUGOS.

AGE	TOTAL	SLOVENIA	R.YUGOS.
0	66.251396	0.823944	65.427452
5	61.517136	0.820722	60.696415
10	57.056091	0.812476	56.243614
15	52.607807	0.800731	51.807076
20	48.174381	0.779989	47.394394
25	43.763908	0.742248	43.021660
30	39.384045	0.690053	38.693993
35	35.040321	0.630024	34.410297
40	30.737667	0.564992	30.172676
45	26.487514	0.496606	25.990910
50	22.308275	0.426536	21.881739
55	18.234377	0.355932	17.878445
60	14.319031	0.284570	14.034461
65	10.651278	0.213312	10.437966
70	7.365170	0.145818	7.219352
75	4.634866	0.087959	4.546907
80	2.599575	0.044323	2.555252
85	1.297419	0.017490	1.279930

EXPECTATIONS OF LIFE

AGE	AGGREGATED	AGE	INITIAL REGION OF COHORT SLOVENIA		
			TOTAL	SLOVENIA	R.YUGOS.
0	69.408241	0	72.565094	64.895561	7.669532
5	69.285957	5	69.737778	61.866112	7.871666
10	64.469887	10	64.883430	57.094246	7.789187
15	59.590294	15	59.977249	52.321896	7.655354
20	54.787739	20	55.127113	47.676636	7.450478
25	50.046268	25	50.327911	43.217598	7.110312
30	45.322731	30	45.507538	38.882309	6.625229
35	40.625217	35	40.739594	34.691658	6.047937
40	35.949493	40	35.994682	30.577511	5.417170
45	31.362213	45	31.339705	26.566034	4.773669
50	26.908054	50	26.865095	22.721096	4.143999
55	22.635374	55	22.529661	19.002993	3.526668
60	18.552647	60	18.370762	15.440985	2.929777
65	14.817460	65	14.479198	12.107406	2.371792
70	11.596285	70	11.151175	9.247625	1.903550
75	9.070240	75	8.493897	6.941777	1.552121
80	7.022092	80	6.252840	4.958601	1.294239
85	6.121485	85	5.312795	3.953228	1.359568

AGE	INITIAL REGION OF COHORT R.YUGOS.		
	TOTAL	SLOVENIA	R.YUGOS.
0	66.251396	0.823944	65.427452
5	68.834137	0.918341	67.915794
10	64.056335	0.912160	63.144176
15	59.203339	0.901120	58.302219
20	54.448364	0.881571	53.566792
25	49.764626	0.844022	48.920605
30	45.137917	0.790867	44.347050
35	40.510841	0.728384	39.782459
40	35.904305	0.659960	35.244343
45	31.384720	0.588422	30.796299
50	26.951014	0.515306	26.435707
55	22.741089	0.443903	22.297186
60	18.734533	0.372322	18.362211
65	15.155722	0.303522	14.852200
70	12.041394	0.238400	11.802995
75	9.646582	0.183069	9.463512
80	7.791345	0.132844	7.658501
85	6.930174	0.093422	6.836752

TABLE - MULTIREGIONAL (TWO-REGION) LIFE TABLE OPTION 1

MORTALITY LEVELS = 72.57 = 66.25 MIGRATION LEVELS = 0.1057 = 0.0124

AGE	Q(X,1)	P(X,1,1)	P(X,2,1)	L(X,1,1)	L(X,2,1)	LL(X,1,1)	LL(X,2,1)	M(X,2,1)	MD(X,1)	S(X,1,1)	S(X,2,1)	E(X,1,1)	E(X,2,1)
0	0.030077	0.956075	0.013848	100000.	0.	4.89019	0.03462	0.002832	0.006150	0.970926	0.013035	64.90	7.67
5	0.002145	0.986462	0.011393	95607.	1385.	4.74805	0.09633	0.002294	0.000432	0.988779	0.009410	61.87	7.87
10	0.001479	0.991129	0.007393	94314.	2468.	4.69485	0.14065	0.001485	0.000297	0.981625	0.016363	57.09	7.79
15	0.002543	0.972027	0.025430	93480.	3158.	4.60887	0.21671	0.005158	0.000516	0.966690	0.030203	52.32	7.66
20	0.003664	0.961182	0.035155	90875.	5511.	4.45622	0.35392	0.007170	0.000747	0.965201	0.031282	47.68	7.45
25	0.003332	0.969422	0.027246	87373.	8646.	4.30242	0.48958	0.005534	0.000677	0.972905	0.022969	43.22	7.11
30	0.004934	0.976509	0.018557	84724.	10937.	4.18690	0.58329	0.003756	0.000999	0.980783	0.013711	38.88	6.63
35	0.006075	0.985166	0.008758	82752.	12395.	4.10728	0.63431	0.001765	0.001224	0.985294	0.006898	34.69	6.05
40	0.009550	0.985423	0.005026	81539.	12978.	4.04752	0.65422	0.001013	0.001924	0.982555	0.003859	30.58	5.42
45	0.017668	0.979646	0.002686	80362.	13191.	3.97736	0.65844	0.000543	0.003570	0.975364	0.002965	26.57	4.77
50	0.025742	0.970992	0.003266	78732.	13147.	3.87973	0.65323	0.000663	0.005224	0.964466	0.003160	22.72	4.14
55	0.039176	0.957745	0.003079	76457.	12983.	3.74240	0.63946	0.000629	0.008004	0.945699	0.003635	19.00	3.53
60	0.062607	0.933122	0.004271	73239.	12596.	3.53982	0.61185	0.000884	0.012955	0.904755	0.004300	15.44	2.93
65	0.121198	0.874355	0.004448	68353.	11878.	3.20318	0.56273	0.000949	0.025864	0.831858	0.004087	12.11	2.37
70	0.212840	0.783256	0.003903	59774.	10631.	2.66489	0.48028	0.000876	0.047742	0.737166	0.004125	9.25	1.90
75	0.317016	0.678322	0.004662	46822.	8580.	1.96464	0.36889	0.001111	0.075556	0.590754	0.003602	6.94	1.55
80	0.535769	0.461657	0.002573	31764.	6175.	1.16082	0.24326	0.000704	0.146620	0.620486	0.003141	4.96	1.29
85	1.000000	0.000000	0.000000	14669.	3555.	0.72043	0.24777	0.000000	0.203611	0.000000	0.000000	3.95	1.36

AGE	Q(X,2)	P(X,2,2)	P(X,1,2)	L(X,2,2)	L(X,1,2)	LL(X,2,2)	LL(X,1,2)	M(X,1,2)	MD(X,2)	S(X,2,2)	S(X,1,2)	E(X,2,2)	E(X,1,2)
0	0.106299	0.892412	0.001289	100000.	0.	4.73103	0.00322	0.000272	0.022468	0.941182	0.001082	65.43	0.82
5	0.003340	0.995833	0.000827	89241.	129.	4.45280	0.00825	0.000166	0.000669	0.996330	0.000807	67.92	0.92
10	0.002385	0.996831	0.000785	88871.	201.	4.43654	0.01175	0.000157	0.000478	0.994579	0.002077	63.14	0.91
15	0.004308	0.992312	0.003380	88591.	269.	4.41268	0.02074	0.000679	0.000865	0.990806	0.004009	58.30	0.90
20	0.006067	0.989272	0.004661	87916.	561.	4.37273	0.03774	0.000937	0.001220	0.989424	0.003606	53.57	0.88
25	0.007885	0.989596	0.002519	86993.	949.	4.32767	0.05220	0.000506	0.001585	0.989562	0.002137	48.92	0.84
30	0.008720	0.989539	0.001741	86114.	1139.	4.28369	0.06003	0.000350	0.001753	0.989053	0.001437	44.35	0.79
35	0.010307	0.988570	0.001123	85234.	1262.	4.23762	0.06503	0.000226	0.002073	0.986714	0.001017	39.78	0.73
40	0.014251	0.984838	0.000910	84271.	1339.	4.18177	0.06839	0.000183	0.002872	0.982576	0.000686	35.24	0.66
45	0.019255	0.980281	0.000464	83000.	1396.	4.10917	0.07007	0.000094	0.003889	0.974183	0.000550	30.80	0.59
50	0.031397	0.967963	0.000640	81367.	1406.	4.00329	0.07060	0.000130	0.006382	0.960149	0.000816	26.44	0.52
55	0.046922	0.952076	0.001003	78765.	1418.	3.84398	0.07136	0.000205	0.009615	0.935550	0.000981	22.30	0.44
60	0.080837	0.918191	0.000971	74994.	1437.	3.59650	0.07126	0.000203	0.016857	0.894846	0.000840	18.36	0.37
65	0.129850	0.869420	0.000730	68865.	1414.	3.21862	0.06749	0.000156	0.027784	0.830223	0.000533	14.85	0.30
70	0.214514	0.785139	0.000347	59879.	1286.	2.67244	0.05786	0.000078	0.048067	0.745167	0.000368	11.80	0.24
75	0.305323	0.694256	0.000420	47019.	1028.	1.99166	0.04364	0.000099	0.072084	0.640254	0.000530	9.46	0.18
80	0.436764	0.562470	0.000766	32648.	717.	1.27532	0.02683	0.000196	0.111814	1.003546	0.000659	7.66	0.13
85	1.000000	0.000000	0.000000	18365.	356.	1.27993	0.01749	0.000000	0.143486	0.000000	0.000000	6.84	0.09

THE DISCRETE MODEL OF MULTIREGIONAL DEMOGRAPHIC GROWTH
****
****

MULTIREGIONAL PROJECTION MATRIX
****

REGION SLOVENIA
****

AGE	FIRST ROW SLOVENIA	FIRST ROW R.YUGOS.
0	0.000000	0.000000
5	0.000172	0.000003
10	0.038234	0.001295
15	0.205773	0.007742
20	0.321949	0.007774
25	0.252353	0.004193
30	0.155327	0.001863
35	0.074684	0.000725
40	0.020770	0.000167
45	0.002432	0.000022
50	0.000714	0.000005
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000

AGE	SURVIVORSHIP PROPORTIONS SLOVENIA	SURVIVORSHIP PROPORTIONS R.YUGOS.
0	0.970926	0.013035
5	0.988779	0.009410
10	0.981625	0.016363
15	0.966690	0.030203
20	0.965201	0.031282
25	0.972905	0.022969
30	0.980783	0.013711
35	0.985294	0.006898
40	0.982555	0.003859
45	0.975364	0.002965
50	0.964466	0.003160
55	0.945699	0.003635
60	0.904755	0.004300
65	0.831858	0.004087
70	0.737166	0.004125
75	0.590754	0.003602
80	0.620486	0.003141

REGION R.YUGOS.
****

AGE	FIRST ROW SLOVENIA	R.YUGOS.
0	0.000000	0.000000
5	0.000000	0.000158
10	0.000123	0.062407
15	0.000876	0.268792
20	0.000818	0.381923
25	0.000405	0.279340
30	0.000189	0.159825
35	0.000076	0.083794
40	0.000024	0.033506
45	0.000005	0.006734
50	0.000001	0.001689
55	0.000000	0.000000
60	0.000000	0.000000
65	0.000000	0.000000
70	0.000000	0.000000
75	0.000000	0.000000
80	0.000000	0.000000

AGE	SURVIVORSHIP PROPORTIONS SLOVENIA	R.YUGOS.
0	0.001082	0.941182
5	0.000807	0.996330
10	0.002077	0.994579
15	0.004009	0.990806
20	0.003606	0.989424
25	0.002137	0.989562
30	0.001437	0.989053
35	0.001017	0.986714
40	0.000688	0.982576
45	0.000550	0.974183
50	0.000816	0.960149
55	0.000981	0.935550
60	0.000840	0.894846
65	0.000533	0.830223
70	0.000368	0.745167
75	0.000530	0.640254
80	0.000659	1.003546

MULTIREGIONAL POPULATION PROJECTION
****

YEAR 1961

POPULATION
- - - - -

AGE	TOTAL	SLOVENIA	R.YUGOS.
0	915700.	67800.	847900.
5	979300.	74100.	905200.
10	878800.	70700.	808100.
15	677500.	60100.	617400.
20	788400.	62900.	725500.
25	840500.	66500.	774000.
30	795500.	67100.	728400.
35	696200.	62900.	633300.
40	431900.	39500.	392400.
45	485000.	47900.	437100.
50	505100.	51300.	453800.
55	435400.	46100.	389300.
60	365400.	39600.	325800.
65	260100.	29500.	230600.
70	201700.	21700.	180000.
75	135300.	14400.	120900.
80	68300.	7100.	61200.
85	42900.	3600.	39300.
TOT	9503000.	832800.	8670200.

PERCENTAGE DISTRIBUTION
- - - - - - - - - - - -

AGE	TOTAL	SLOVENIA	R.YUGOS.
0	9.6359	8.1412	9.7795
5	10.3052	8.8977	10.4404
10	9.2476	8.4894	9.3204
15	7.1293	7.2166	7.1209
20	8.2963	7.5528	8.3677
25	8.8446	7.9851	8.9271
30	8.3710	8.0572	8.4012
35	7.3261	7.5528	7.3043
40	4.5449	4.7430	4.5258
45	5.1037	5.7517	5.0414
50	5.3152	6.1599	5.2340
55	4.5817	5.5355	4.4901
60	3.8451	4.7550	3.7577
65	2.7370	3.5423	2.6597
70	2.1225	2.6057	2.0761
75	1.4238	1.7291	1.3944
80	0.7187	0.8525	0.7059
85	0.4514	0.4323	0.4533
TOTAL	100.0000	100.0000	100.0000
M.AGE	30.8458	33.3796	30.6024
SHA	100.0000	8.7635	91.2365

YEAR 1966

POPULATION

AGE	TOTAL	SLOVENIA	R.YUGOS.
0	967617.	69954.	897663.
5	865658.	66746.	798912.
10	976574.	73999.	902575.
15	875955.	71079.	804876.
20	674112.	60573.	613539.
25	783122.	63327.	719795.
30	833801.	66352.	767448.
35	788203.	66857.	721346.
40	687939.	62619.	625320.
45	424796.	39081.	385715.
50	472918.	46960.	425957.
55	485725.	49847.	435878.
60	408356.	43979.	364377.
65	327813.	36102.	291711.
70	216233.	24663.	191570.
75	150282.	16063.	134220.
80	86030.	8571.	77459.
85	65885.	4446.	61439.
TOT	10091021.	871219.	9219802.

PERCENTAGE DISTRIBUTION

AGE	TOTAL	SLOVENIA	R.YUGOS.
0	9.5889	8.0294	9.7363
5	8.5785	7.6612	8.6652
10	9.6777	8.4937	9.7895
15	8.6805	8.1586	8.7299
20	6.6803	6.9527	6.6546
25	7.7606	7.2688	7.8071
30	8.2628	7.6160	8.3239
35	7.8109	7.6740	7.8239
40	6.8173	7.1875	6.7824
45	4.2096	4.4858	4.1836
50	4.6865	5.3902	4.6200
55	4.8134	5.7216	4.7276
60	4.0467	5.0479	3.9521
65	3.2486	4.1438	3.1640
70	2.1428	2.8308	2.0778
75	1.4893	1.8437	1.4558
80	0.8525	0.9838	0.8401
85	0.6529	0.5103	0.6664
TOTAL	100.0000	100.0000	100.0000
M.AGE	31.6931	34.1776	31.4583
SHA	100.0000	8.6336	91.3664
LAM	1.061877	1.046132	1.063390

YEAR 1971

POPULATION

AGE	TOTAL	SLOVENIA	R.YUGOS.
0	988630.	71505.	917125.
5	914668.	68891.	845777.
10	863250.	66642.	796608.
15	973407.	74514.	898893.
20	871561.	71938.	799623.
25	669623.	60678.	608945.
30	776886.	63150.	713736.
35	826137.	66180.	759957.
40	778831.	66608.	712224.
45	676623.	61957.	614666.
50	414203.	38330.	375873.
55	454770.	45639.	409131.
60	455535.	47568.	407967.
65	366347.	40096.	326251.
70	272520.	30187.	242333.
75	161104.	18251.	142853.
80	95553.	9560.	85992.
85	83129.	5369.	77760.
TOT	10642777.	907064.	9735713.

PERCENTAGE DISTRIBUTION

AGE	TOTAL	SLOVENIA	R.YUGOS.
0	9.2892	7.8831	9.4202
5	8.5943	7.5950	8.6874
10	8.1111	7.3470	8.1823
15	9.1462	8.2149	9.2329
20	8.1892	7.9309	8.2133
25	6.2918	6.6895	6.2548
30	7.2997	6.9620	7.3311
35	7.7624	7.2961	7.8059
40	7.3179	7.3432	7.3156
45	6.3576	6.8305	6.3135
50	3.8919	4.2258	3.8608
55	4.2730	5.0315	4.2024
60	4.2802	5.2442	4.1904
65	3.4422	4.4204	3.3511
70	2.5606	3.3280	2.4891
75	1.5137	2.0121	1.4673
80	0.8978	1.0540	0.8833
85	0.7811	0.5919	0.7987
TOTAL	100.0000	100.0000	100.0000
M.AGE	32.4300	34.8435	32.2051
SHA	100.0000	8.5228	91.4772
LAM	1.054678	1.041143	1.055957

YEAR 1976

POPULATION

AGE	TOTAL	SLOVENIA	R.YUGOS.
0	1049682.	74678.	975004.
5	934532.	70418.	864114.
10	912121.	68801.	843321.
15	860452.	67072.	793380.
20	968515.	75636.	892879.
25	865735.	72318.	793416.
30	664317.	60335.	603982.
35	769750.	62962.	706789.
40	816296.	65980.	750317.
45	766006.	65936.	700071.
50	659749.	60769.	598981.
55	398290.	37275.	361015.
60	426491.	43562.	382928.
65	408652.	43380.	365272.
70	304553.	33528.	271025.
75	203045.	22342.	180703.
80	102386.	10858.	91528.
85	92316.	5989.	86327.
TOT	11202889.	941837.	10261052.

PERCENTAGE DISTRIBUTION

AGE	TOTAL	SLOVENIA	R.YUGOS.
0	9.3697	7.9290	9.5020
5	8.3419	7.4767	8.4213
10	8.1418	7.3049	8.2187
15	7.6806	7.1214	7.7320
20	8.6452	8.0306	8.7016
25	7.7278	7.6784	7.7323
30	5.9299	6.4061	5.8862
35	6.8710	6.6850	6.8881
40	7.2865	7.0054	7.3123
45	6.8376	7.0008	6.8226
50	5.8891	6.4521	5.8374
55	3.5552	3.9577	3.5183
60	3.8070	4.6253	3.7319
65	3.6477	4.6059	3.5598
70	2.7185	3.5599	2.6413
75	1.8124	2.3722	1.7611
80	0.9139	1.1528	0.8920
85	0.8240	0.6359	0.8413
TOTAL	100.0000	100.0000	100.0000
M.AGE	32.9687	35.3179	32.7531
SHA	100.0000	8.4071	91.5929
LAM	1.052628	1.038337	1.053960

STABLE EQUIVALENT TO ORIGINAL POPULATION
****

AGE	TOTAL	SLOVENIA	R.YUGOS.
0	942199.	45765.	896433.
5	862983.	44041.	818942.
10	834709.	42880.	791829.
15	806984.	42423.	764561.
20	778771.	42751.	736020.
25	750256.	42598.	707658.
30	721851.	41666.	680186.
35	693671.	40586.	653086.
40	664755.	39432.	625323.
45	634117.	37997.	596119.
50	599660.	36266.	563394.
55	559178.	34373.	524805.
60	508383.	32029.	476354.
65	442089.	28496.	413593.
70	356379.	23207.	333173.
75	257617.	16712.	240905.
80	159366.	9700.	149666.
85	151648.	5934.	145714.
TOT	10724616.	606854.	10117762.

PERCENTAGE DISTRIBUTION
- - - - - - - - - - - -

AGE	TOTAL	SLOVENIA	R.YUGOS.
0	8.7854	7.5414	8.8600
5	8.0468	7.2572	8.0941
10	7.7831	7.0659	7.8261
15	7.5246	6.9906	7.5566
20	7.2615	7.0446	7.2745
25	6.9956	7.0195	6.9942
30	6.7308	6.8659	6.7227
35	6.4680	6.6879	6.4548
40	6.1984	6.4977	6.1805
45	5.9127	6.2614	5.8918
50	5.5914	5.9761	5.5684
55	5.2140	5.6641	5.1870
60	4.7403	5.2779	4.7081
65	4.1222	4.6957	4.0878
70	3.3230	3.8241	3.2930
75	2.4021	2.7539	2.3810
80	1.4860	1.5984	1.4792
85	1.4140	0.9778	1.4402
TOTAL	100.0000	100.0000	100.0000
M.AGE	35.1391	36.7734	35.0411
SHA	100.0000	5.6585	94.3415
LAM	1.030968	1.030968	1.030968

APPENDIX 3

Single Region Life Table and Population Projection

```
YUGOSLAVIA  1961  FEMALES  SINGLE REGION  :  DATA SET
18 1 5 3 1 1 1 2 2 1 1 2 0 0 2 1 719612016 200
     SINGLE REGION LIFE TABLE
     YUGOSLAVIA  1961

YUGOSLAV
 100000.
   915700.   979300.   878800.   677500.   788400.   840500.   795500.   696200.
   431900.   485000.   505100.   435400.   365400.   260100.   201700.   135300.
    68300.    42900.
       .        .      59.  17288.  68272.  61681.  35019.  16341.   5037.    974.
    339.        .       .       .       .       .       .       .
  19468.     638.     407.     565.     932.    1272.    1344.    1390.    1203.   1871.
   3164.    4112.    6005.    7170.    9688.    9803.    7884.    6372.
```

3a - Input Data

```
****************************
* SINGLE REGION LIFE TABLE *
*                          *
*      YUGOSLAVIA 1961     *
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LIST OF PARAMETERS
****

NA	=	18	NY	=	5	NR	=	1
NZB	=	1	NZD	=	1	NZO	=	1
IRUNT	=	2	IOPTG	=	2	NGRO	=	1
INIT	=	1961	KA	=	1	KC	=	2
NU	=	3	L0	=	2	NPAR1	=	0
NPAR2	=	2	NPAR3	=	1	NPAR4	=	7
NPAR5	=	2016	NPAR6	=	200	NPAR7	=	0
NPAR8	=	0						

3b - Life Table

OBSERVED POPULATION CHARACTERISTICS
==================================

INPUT-DATA
****

AGE	POPULATION ABSOLUTE	POPULATION PERCENT	BIRTHS ABSOLUTE	BIRTHS PERCENT	DEATHS ABSOLUTE	DEATHS PERCENT	MIGRATION ABSOLUTE	MIGRATION PERCENT	OBSERVED RATES BIRTH	OBSERVED RATES DEATH	OBSERVED RATES MIGRATION
0	915700.	9.6359	0.	0.0000	19468.	23.3743	0.	0.0000	0.000000	0.021260	0.000000
5	979300.	10.3052	0.	0.0000	638.	0.7660	0.	0.0000	0.000000	0.000651	0.000000
10	878800.	9.2476	59.	0.0288	407.	0.4887	0.	0.0000	0.000067	0.000463	0.000000
15	677500.	7.1293	17288.	8.4328	565.	0.6784	0.	0.0000	0.025517	0.000834	0.000000
20	788400.	8.2963	68272.	33.3018	932.	1.1190	0.	0.0000	0.086596	0.001182	0.000000
25	840500.	8.8446	61681.	30.0868	1272.	1.5272	0.	0.0000	0.073386	0.001513	0.000000
30	795500.	8.3710	35019.	17.0816	1344.	1.6137	0.	0.0000	0.044021	0.001690	0.000000
35	696200.	7.3261	16341.	7.9708	1390.	1.6689	0.	0.0000	0.023472	0.001997	0.000000
40	431900.	4.5449	5037.	2.4570	1203.	1.4444	0.	0.0000	0.011662	0.002785	0.000000
45	485000.	5.1037	974.	0.4751	1871.	2.2464	0.	0.0000	0.002008	0.003858	0.000000
50	505100.	5.3152	339.	0.1654	3164.	3.7989	0.	0.0000	0.000671	0.006264	0.000000
55	435400.	4.5817	0.	0.0000	4112.	4.9371	0.	0.0000	0.000000	0.009444	0.000000
60	365400.	3.8451	0.	0.0000	6005.	7.2099	0.	0.0000	0.000000	0.016434	0.000000
65	260100.	2.7370	0.	0.0000	7170.	8.6087	0.	0.0000	0.000000	0.027566	0.000000
70	201700.	2.1225	0.	0.0000	9688.	11.6319	0.	0.0000	0.000000	0.048032	0.000000
75	135300.	1.4238	0.	0.0000	9803.	11.7700	0.	0.0000	0.000000	0.072454	0.000000
80	68300.	0.7187	0.	0.0000	7884.	9.4659	0.	0.0000	0.000000	0.115432	0.000000
85	42900.	0.4514	0.	0.0000	6372.	7.6506	0.	0.0000	0.000000	0.148531	0.000000
TOTAL	9503000.	100.0000	205010.	100.0000	83288.	100.0000	0.	100.0000	0.267401	0.480391	0.000000
CRUDE									0.021573	0.008764	0.000000
M.AGE		30.8458		27.1434		51.5334		0.0000	27.4908	74.8451	0.0000

TABLE - SINGLE REGION LIFE TABLE YUGOSLAV MORTALITY LEVEL = 66.68

AGE	Q(X)	L(X)	D(X)	LL(X)	M(X)	S(X)	T(X)	E(X)
0	0.100936	100000.	10094.	4.747659	0.021260	0.945310	66.682762	66.682762
5	0.003252	89906.	292.	4.488008	0.000651	0.997217	61.935097	68.888443
10	0.002313	89614.	207.	4.475517	0.000463	0.996764	57.447090	64.105064
15	0.004161	89407.	372.	4.461034	0.000834	0.994975	52.971573	59.247883
20	0.005893	89035.	525.	4.438616	0.001182	0.993287	48.510536	54.485001
25	0.007538	88510.	667.	4.408817	0.001513	0.992027	44.071922	49.793186
30	0.008412	87843.	739.	4.373664	0.001689	0.990830	39.663101	45.152397
35	0.009933	87104.	865.	4.333560	0.001997	0.988128	35.289440	40.514236
40	0.013830	86239.	1193.	4.282111	0.002785	0.983551	30.955877	35.895622
45	0.019104	85046.	1625.	4.211675	0.003858	0.975086	26.673769	31.363981
50	0.030838	83421.	2573.	4.106743	0.006264	0.961635	22.462091	26.926147
55	0.046132	80849.	3730.	3.949188	0.009444	0.937858	18.355349	22.703360
60	0.078928	77119.	6087.	3.703776	0.016434	0.897091	14.406162	18.680452
65	0.128945	71032.	9159.	3.322624	0.027566	0.831266	10.702386	15.066969
70	0.214412	61873.	13266.	2.761986	0.048032	0.744979	7.379761	11.927299
75	0.306713	48607.	14908.	2.057622	0.072454	0.635479	4.617775	9.500312
80	0.447904	33698.	15094.	1.307576	0.115432	0.489259	2.560153	7.597274
85	1.000000	18605.	18605.	1.252577	0.148531	0.000000	1.252577	6.732580

THE DISCRETE MODEL OF MULTIREGIONAL DEMOGRAPHIC GROWTH
****
****

MULTIREGIONAL PROJECTION MATRIX
****

REGION YUGOSLAV
****

AGE	FIRST ROW YUGOSLAV
0	0.000000
5	0.000159
10	0.060536
15	0.265104
20	0.378601
25	0.277871
30	0.159706
35	0.083073
40	0.032372
45	0.006320
50	0.001593
55	0.000000
60	0.000000
65	0.000000
70	0.000000
75	0.000000
80	0.000000

AGE	SURVIVORSHIP PROPORTIONS YUGOSLAV
0	0.945310
5	0.997217
10	0.996764
15	0.994975
20	0.993287
25	0.992027
30	0.990830
35	0.988128
40	0.983551
45	0.975086
50	0.961635
55	0.937858
60	0.897091
65	0.831266
70	0.744979
75	0.635479
80	0.489259

3c - Population Projection

MULTIREGIONAL POPULATION PROJECTION
****

YEAR 1961

POPULATION
- - - - -

AGE	TOTAL	YUGOSLAV
0	915700.	915700.
5	979300.	979300.
10	878800.	878800.
15	677500.	677500.
20	788400.	788400.
25	840500.	840500.
30	795500.	795500.
35	696200.	696200.
40	431900.	431900.
45	485000.	485000.
50	505100.	505100.
55	435400.	435400.
60	365400.	365400.
65	260100.	260100.
70	201700.	201700.
75	135300.	135300.
80	68300.	68300.
85	42900.	42900.
TOT	9503000.	9503000.

PERCENTAGE DISTRIBUTION
- - - - - - - - - - - -

AGE	TOTAL	YUGOSLAV
0	9.6359	9.6359
5	10.3052	10.3052
10	9.2476	9.2476
15	7.1293	7.1293
20	8.2963	8.2963
25	8.8446	8.8446
30	8.3710	8.3710
35	7.3261	7.3261
40	4.5449	4.5449
45	5.1037	5.1037
50	5.3152	5.3152
55	4.5817	4.5817
60	3.8451	3.8451
65	2.7370	2.7370
70	2.1225	2.1225
75	1.4238	1.4238
80	0.7187	0.7187
85	0.4514	0.4514
TOTAL	100.0000	100.0000
M.AGE	30.8458	30.8458
SHA	100.0000	100.0000

YEAR 1966

POPULATION

AGE	TOTAL	YUGOSLAV
0	967735.	967735.
5	865620.	865620.
10	976575.	976575.
15	875956.	875956.
20	674096.	674096.
25	783107.	783107.
30	833799.	833799.
35	788205.	788205.
40	687935.	687935.
45	424796.	424796.
50	472917.	472917.
55	485722.	485722.
60	408343.	408343.
65	327797.	327797.
70	216212.	216212.
75	150262.	150262.
80	85980.	85980.
85	33416.	33416.
TOT	10058471.	10058471.

PERCENTAGE DISTRIBUTION

AGE	TOTAL	YUGOSLAV
0	9.6211	9.6211
5	8.6059	8.6059
10	9.7090	9.7090
15	8.7086	8.7086
20	6.7018	6.7018
25	7.7856	7.7856
30	8.2895	8.2895
35	7.8362	7.8362
40	6.8394	6.8394
45	4.2233	4.2233
50	4.7017	4.7017
55	4.8290	4.8290
60	4.0597	4.0597
65	3.2589	3.2589
70	2.1496	2.1496
75	1.4939	1.4939
80	0.8548	0.8548
85	0.3322	0.3322
TOTAL	100.0000	100.0000
M.AGE	31.5122	31.5122
SHA	100.0000	100.0000
LAM	1.058452	1.058452

YEAR 1971

POPULATION

AGE	TOTAL	YUGOSLAV
0	988640.	988640.
5	914809.	914809.
10	863211.	863211.
15	973414.	973414.
20	871554.	871554.
25	669570.	669570.
30	776864.	776864.
35	826153.	826153.
40	778848.	778848.
45	676619.	676619.
50	414212.	414212.
55	454773.	454773.
60	455538.	455538.
65	366321.	366321.
70	272487.	272487.
75	161074.	161074.
80	95489.	95489.
85	42067.	42067.
TOT	10601644.	10601644.

PERCENTAGE DISTRIBUTION

AGE	TOTAL	YUGOSLAV
0	9.3253	9.3253
5	8.6289	8.6289
10	8.1422	8.1422
15	9.1817	9.1817
20	8.2209	8.2209
25	6.3157	6.3157
30	7.3278	7.3278
35	7.7927	7.7927
40	7.3465	7.3465
45	6.3822	6.3822
50	3.9071	3.9071
55	4.2896	4.2896
60	4.2969	4.2969
65	3.4553	3.4553
70	2.5702	2.5702
75	1.5193	1.5193
80	0.9007	0.9007
85	0.3968	0.3968
TOTAL	100.0000	100.0000
M.AGE	32.2158	32.2158
SHA	100.0000	100.0000
LAM	1.054002	1.054002

YEAR 1976

POPULATION

AGE	TOTAL	YUGOSLAV
0	1049332.	1049332.
5	934571.	934571.
10	912263.	912263.
15	860418.	860418.
20	968523.	968523.
25	865704.	865704.
30	664232.	664232.
35	769740.	769740.
40	816345.	816345.
45	766036.	766036.
50	659762.	659762.
55	398321.	398321.
60	426513.	426513.
65	408659.	408659.
70	304510.	304510.
75	202997.	202997.
80	102359.	102359.
85	46719.	46719.
TOT	11157004.	11157004.

AGE	TOTAL	YUGOSLAV
0	9.4051	9.4051
5	8.3765	8.3765
10	8.1766	8.1766
15	7.7119	7.7119
20	8.6809	8.6809
25	7.7593	7.7593
30	5.9535	5.9535
35	6.8992	6.8992
40	7.3169	7.3169
45	6.8660	6.8660
50	5.9134	5.9134
55	3.5701	3.5701
60	3.8228	3.8228
65	3.6628	3.6628
70	2.7293	2.7293
75	1.8195	1.8195
80	0.9174	0.9174
85	0.4187	0.4187
TOTAL	100.0000	100.0000
M.AGE	32.7463	32.7463
SHA	100.0000	100.0000
LAM	1.052384	1.052384

STABLE EQUIVALENT TO ORIGINAL POPULATION
****

AGE	TOTAL	YUGOSLAV
0	953529.	953529.
5	874781.	874781.
10	846603.	846603.
15	818961.	818961.
20	790799.	790799.
25	762311.	762311.
30	733918.	733918.
35	705728.	705728.
40	676769.	676769.
45	645993.	645993.
50	611312.	611312.
55	570514.	570514.
60	519273.	519273.
65	452085.	452085.
70	364709.	364709.
75	263682.	263682.
80	162623.	162623.
85	77219.	77219.
TOT	10830807.	10830807.

PERCENTAGE DISTRIBUTION
- - - - - - - - - - - -

AGE	TOTAL	YUGOSLAV
0	8.8039	8.8039
5	8.0768	8.0768
10	7.8166	7.8166
15	7.5614	7.5614
20	7.3014	7.3014
25	7.0384	7.0384
30	6.7762	6.7762
35	6.5159	6.5159
40	6.2486	6.2486
45	5.9644	5.9644
50	5.6442	5.6442
55	5.2675	5.2675
60	4.7944	4.7944
65	4.1741	4.1741
70	3.3673	3.3673
75	2.4346	2.4346
80	1.5015	1.5015
85	0.7130	0.7130
TOTAL	100.0000	100.0000
M.AGE	34.8357	34.8357
SHA	100.0000	100.0000
LAM	1.030407	1.030407

APPENDIX 4

Computer Programs

LIFE4R: main program for life table computation (option 1)

```
c computer programs for spatial demographic analysis
c
      common /csu/ su(18,4,4),ssu(18,4)
c     common /cgrow/ br(18,4,4),pp1(4,4),popr(18,4),poppr(4),popprt
      common /clife2/ dr(4),cl(18,4,4),e(18,4,4),t(18,4,4)
      common /cl/ l(18,4,4),cllt(18,4),clltot(18)
      common /clife3/ clt(18,4),ee(18,4),tt(18,4)
      common /cpq/ q(18,4),p(18,4,4),pmigt(18,4)
      common /cmul/ a1(4,4),b(4,4),c(4,4)
      common /crad/ radix(4),radixt
      common /crate/ ratd(18,4),ratm(18,4,4),ratf(18,4)
      common /cratet/ ratot(18,4)
      common /creg/ reg(4)
      common /cwhole/ clto(18),etotc(18)
      common /clif2/ cm(72,4,4),cdr(18,4,4)
      common /c1/ pop(18,4),birth(18,4),death(18,4),omig(18,4,4)
      common /c4/ nage(18)
c     common /cmat/ patfun(18,4,4)
      common /crats/ brate(4),drate(4),orate(4),oirate(4)
c     common /cque/ que(4),quet,sha(4)
      common /ctot/ birthc(18),deathc(18),cmigc(18)
      common /ctotrat/ popc(18),ratdt(18),ratft(18),ratmt(18)
      double precision reg
      integer x,xx
      real l
      call setfil (7,"outlif ")
  100  continue
      call datas (na,ny,nr,init,nzb,nzd,nzo,ngro,ka,kc,irunt,l0,
&     ioptg,npar1,npar2,npar3,npar4,npar5,npar6,npar7,npar8)
      naa=na-1
      zfny=float(ny)
      do 7 x=1,na
    7 nage(x)=ny*(x-1)
      call printd1 (na,ny,nr,nzb,nzd,nzo)
      do 66 j=1,nr
   66 dr(j)=ratd(na,j)
      do 500 irun=1,2
      if (irunt.eq.2) irun=2
       if (irun.eq.1) call probr(na,ny,nr)
c     if (irun.eq.2) call probsch (na,ny,nr)
      call life (na,ny,nr,l0,irun,ioptg)
      call whole (na,ny,nr)
      call printl (na,ny,nr,irun,ka)
  104 continue
      write (7,20) na,nr,ny,nzb,nzd,nzo,irunt,ioptg,ngro,ka,kc,l0,
&     npar1,npar2,npar3,npar4,init,npar5,npar6,npar7,npar8
   20 format (16i2,5i4)
      write (7,22) (reg(j),j=1,nr)
   22 format (9a8)
      write (7,23) radixt,(radix(j),j=1,nr)
   23 format (10f8.0)
      do 50 i=1,nr
      write (7,24) (p(1,j,i),j=1,nr)
      do 50 x=1,na
      write (7,24) (su(x,i,j),j=1,nr)
   24 format (9f8.6)
```

```
      write (7,24) (l(x,i,j),j=1,nr)
 50   continue
      do 51 i=1,nr
      write (7,25) (pop(x,i),x=1,na)
 25   format (8f10.0/8f10.0/2f10.0)
      write 7,26) (ratf(x,i),x=1,na)
 26   format (10f8.6/8f8.6)
      write (7,26) (ratd(x,i),x=1,na)
      write (7,26) (ratot(x,i),x=1,na)
      do 54 j=1,nr
 54   write (7,26) (ratm(x,j,i),x=1,na)
      write (7,24) (cllt(x,i),x=1,na)
 51   continue
      write (7,26) (ratdt(x),x=1,na)
      write (7,26) (ratft(x),x=1,na)
       write (7,26) (ratmt(x),x=1,na)
       write (7,24) (clltot(x),x=1,na)
      if (irunt.eq.1) irun=2
500   continue
600   continue
      read (5,101) ncontin
101   format (i1)
      if (ncontin.eq.1) go to 100
      stop
      end
```

LIFE4SCH: main program for life table computation (option 3)

```
c computer programs for spatial demographic analysis
c
      common /csu/ su(18,4,4),ssu(18,4)
c     common /cgrow/ br(18,4,4),pp1(4,4),popr(18,4),poppr(4),popprt
      common /clife2/ dr(4),cl(18,4,4),e(18,4,4),t(18,4,4)
      common /cl/ l(18,4,4),cllt(18,4),clltot(18)
      common /clife3/ clt(18,4),ee(18,4),tt(18,4)
      common /cpq/ q(18,4),p(18,4,4),pmigt(18,4)
      common /cmul/ a1(4,4),b(4,4),c(4,4)
      common /crad/ radix(4),radixt
      common /crate/ ratd(18,4),ratm(18,4,4),ratf(18,4)
      common /cratet/ ratot(18,4)
      common /creg/ reg(4)
      common /cwhole/ clto(18),etotc(18)
      common /clif2/ cm(72,4,4),cdr(18,4,4)
      common /c1/ pop(18,4),birth(18,4),death(18,4),omig(18,4,4)
      common /c4/ nage(18)
c     common /cmat/ patfun(18,4,4)
      common /crats/ brate(4),drate(4),orate(4),oirate(4)
c     common /cque/ que(4),quet,sha(4)
      common /ctot/ birthc(18),deathc(18),cmigc(18)
      common /ctotrat/ popc(18),ratdt(18),ratft(18),ratmt(18)
      double precision reg
      integer x,xx
      real l
      call setfil (7,"outlif ")
  100  continue
      call datas (na,ny,nr,init,nzb,nzd,nzo,ngro,ka,kc,irunt,l0,
&     ioptg,npar1,npar2,npar3,npar4,npar5,npar6,npar7,npar8)
      naa=na-1
      zfny=float(ny)
      do 7 x=1,na
    7 nage(x)=ny*(x-1)
      call printd1 (na,ny,nr,nzb,nzd,nzo)
      do 66 j=1,nr
   66 dr(j)=ratd(na,j)
      do 500 irun=1,2
      if (irunt.eq.2) irun=2
c      if (irun.eq.1) call probr(na,ny,nr)
      if (irun.eq.2) call probsch (na,ny,nr)
      call life (na,ny,nr,l0,irun,ioptg)
      call whole (na,ny,nr)
      call printl (na,ny,nr,irun,ka)
  104 continue
      write (7,20) na,nr,ny,nzb,nzd,nzo,irunt,ioptg,ngro,ka,kc,l0,
&     npar1,npar2,npar3,npar4,init,npar5,npar6,npar7,npar8
   20 format (16i2,5i4)
      write (7,22) (reg(j),j=1,nr)
   22 format (9a8)
      write (7,23) radixt,(radix(j),j=1,nr)
   23 format (10f8.0)
      do 50 i=1,nr
      write (7,24) (p(1,j,i),j=1,nr)
      do 50 x=1,na
      write (7,24) (su(x,i,j),j=1,nr)
   24 format (9f8.6)
```

```
      write (7,24) (l(x,i,j),j=1,nr)
 50   continue
      do 51 i=1,nr
      write (7,25) (pop(x,i),x=1,na)
 25   format (8f10.0/8f10.0/2f10.0)
      write (7,26) (ratf(x,i),x=1,na)
 26   format (10f8.6/8f8.6)
      write (7,26) (ratd(x,i),x=1,na)
      write (7,26) (ratot(x,i),x=1,na)
      do 54 j=1,nr
 54   write (7,26) (ratm(x,j,i),x=1,na)
      write (7,24) (cllt(x,i),x=1,na)
 51   continue
      write (7,26) (ratdt(x),x=1,na)
      write (7,26) (ratft(x),x=1,na)
       write (7,26) (ratmt(x),x=1,na)
       write (7,24) (clltot(x),x=1,na)
      if (irunt.eq.1) irun=2
500   continue
600   continue
      read (5,101) ncontin
101   format (i1)
      if (ncontin.eq.1) go to 100
      stop
      end
```

LIFE4PROJ: main program for population projection

```
c computer programs for demographic analysis : projection
c
      common /csu/ su(18,4,4),ssu(18,4)
      common /cgrow/ br(18,4,4),pp1(4,4),popr(18,4),poppr(4),popprt
      common /clife2/ dr(4),cl(18,4,4),e(18,4,4),t(18,4,4)
      common /cl/ l(18,4,4),cllt(18,4),clltot(18)
      common /clife3/ clt(18,4),ee(18,4),tt(18,4)
      common /cpq/ q(18,4),p(18,4,4),pmigt(18,4)
      common /cmul/ a1(4,4),b(4,4),c(4,4)
      common /crad/ radix(4),radixt
      common /crate/ ratd(18,4),ratm(18,4,4),ratf(18,4)
      common /cratet/ ratot(18,4)
      common /creg/ reg(4)
      common /cwhole/ clto(18),etotc(18)
      common /clif2/ cm(72,4,4),cdr(18,4,4)
      common /c1/ pop(18,4),birth(18,4),death(18,4),omig(18,4,4)
      common /c4/ nage(18)
      common /cmat/ patfun(18,4,4)
      common /crats/ brate(4),drate(4),orate(4),oirate(4)
      common /cque/ que(4),quet,sha(4)
      common /ctot/ birthc(18),deathc(18),cmigc(18)
      common /ctotrat/ popc(18),ratdt(18),ratft(18),ratmt(18)
      common /ceigen/ ce(4,4),root(4),vect(4),vectl(4)
      common /cpsi/ psi(4,4),vrpsi(4),vlpsi(4)
      double precision reg
      integer x,xx
      real l
      call setfil (7,"outlif ")
      ioptg=1
      call setfil (8,"outproj ")
 100  continue
      read (7,20) na,nr,ny,nzb,nzd,nzo,irunt,ioptg,ngro,ka,kc,l0,
&     npar1,npar2,npar3,npar4,init,npar5,npar6,npar7,npar8
  20  format (16i2,5i4)
      if (na.eq.0) stop
      nproj=npar5
      iprojp=npar6
      iprintp=npar1
      nstadc=npar2
      read (7,22) (reg(j),j=1,nr)
      print 260
 260  format (1h1,20x,27hpop. projection  parameters/21x,27(1h*))
      print 261
 261  format (/10x,2hna,3x,2hnr,1x,5hnpar3,1x,5hnpar1,1x,5hnpar2,2x,
&     4hinit,1x,5hnpar5,1x,5hnpar6,1x,5hnpar7/)
      print 262, na,nr,npar3,npar1,npar2,init,npar5,npar6,npar7
 262  format (7x,2i5,7i6)
  22  format (9a8)
      read (7,23) radixt,(radix(j),j=1,nr)
  23  format (10f8.0)
      do 50 i=1,nr
      read (7,24) (p(1,j,i),j=1,nr)
      do 50 x=1,na
      read (7,24) (su(x,i,j),j=1,nr)
      read (7,24) (l(x,i,j),j=1,nr)
  24  format (9f8.6)
```

```
 50   continue
      do 51 i=1,nr
      read (7,25) (pop(x,i),x=1,na)
 25   format (8f10.0/8f10.0/2f10.0)
      read (7,26) (ratf(x,i),x=1,na)
 26   format (10f8.6/8f8.6)
      read (7,26) (ratd(x,i),x=1,na)
      read (7,26) (ratot(x,i),x=1,na)
      do 54 j=1,nr
 54   read (7,26) (ratm(x,j,i),x=1,na)
      read (7,24) (cllt(x,i),x=1,na)
 51   continue
      read (7,26) (ratdt(x),x=1,na)
      read (7,26) (ratft(x),x=1,na)
      read (7,26) (ratmt(x),x=1,na)
      read (7,24) (clltot(x),x=1,na)
      naa=na-1
      zfny=float(ny)
      do 7 x=1,na
  7   nage(x)=ny*(x-1)
      do 6 i=1,nr
      do 14 x=1,na
 14   popr(x,i)=pop(x,i)
      popt=0.
      do 42 x=1,na
 42   popt=popt+pop(x,i)
  6   poppr(i)=popt
      popprt=0.
      do 154 j=1,nr
 154   popprt=popprt+poppr(j)
      do 84 i=1,nr
      do 84 j=1,nr
      pp1(j,i)=p(1,j,i)
 84   continue
      call growth (na,ny,zfny,nr)
      call project (na,ny,zfny,nr,init,nproj,iprojp,npar3,npar4,zlamda)
      do 95 i=1,nr
      do 95 j=1,nr
 95   write (8,26) (br(x,j,i),x=1,na)
      do 52 i=1,nr
 52   write (8,25) (popr(x,i),x=1,na)
      write (8,25) (poppr(j),j=1,nr),popprt
      write (8,55) zlamda
 55   format (e14.7)
500   continue
600   continue
      stop
      end
```

DATAS

```
      subroutine datas (na,ny,nr,init,nzb,nzd,nzo,ngro,ka,kc,irunt,l0,
&     ioptg,npar1,npar2,npar3,npar4,npar5,npar6,npar7,npar8)
       dimension title(10)
      common /c1/ pop(18,4),birth(18,4),death(18,4),omig(18,4,4)
      common /creg/ reg(4)
      common /crad/ radix(4),radixt
      double precision reg
       double precision title
      integer x
      read (5,222)
 222  format (1x)
      read (5,2) na,nr,ny,nu,nzb,nzd,nzo,irunt,ioptg,ngro,ka,kc,l0,
&     npar1,npar2,npar3,npar4,init,npar5,npar6,npar7,npar8
   2  format (17i2,5i4)
c   default values
       if (nzb.eq.0) nzb=1
       if (nzo.eq.0) nzo=1
       if (npar4.eq.0) npar4=7
       if (l0.eq.0) l0=2
      print 17
  17  format (1h1,1x)
      do 15 j=1,3
  15  print 16
  16  format  (//1x)
      print 22
  22  format (1x/4(20x,90(1h*)/),20x,5(1h*),80x,5(1h*))
      do 4 i=1,nu
      read (5,14) (title(j),j=1,10)
  14  format (10a8)
   4  print 5, (title(j),j=1,10)
   5  format (20x,5(1h*),10a8,5(1h*))
      print 24
  24  format (20x,5(1h*),80x,5(1h*)/4(20x,90(1h*)/))
      read (5,14) (reg(j),j=1,nr)
      read (5,13) (radix(j),j=1,nr)
  13  format (10f8.0)
      radixt=0.
      do 66 i=1,nr
  66  radixt=radixt+radix(i)
      do 18 j=1,3
  18  print 16
      print 19
  19  format (20x,18hlist of parameters/20x,18(1h*)/)
      print 20, na,ny,nr
  20  format (/10x,2hna,4x,1h=,i5,8x,2hny,4x,1h=,i5,8x,2hnr,4x,1h=,i5)
      print 21, nzb,nzd,nzo
  21  format (/10x,3hnzb,3x,1h=,i5,8x,3hnzd,3x,1h=,i5,8x,3hnzo,3x,1h=,
&     i5)
      print 25, irunt,ioptg,ngro
  25  format (/10x,5hirunt,1x,1h=,i5,8x,5hioptg,1x,1h=,i5,8x,4hngro,2x,
&     1h=,i5)
      print 26, init,ka,kc
  26  format (/10x,4hinit,2x,1h=,i5,8x,2hka,4x,1h=,i5,8x,2hkc,4x,1h=,i5)
      print 27, nu,l0,npar1
  27  format (/10x,2hnu,4x,1h=,i5,8x,2hl0,4x,1h=,i5,8x,5hnpar1,1x,1h=,
&     i5)
```

```
      print 28, npar2,npar3,npar4,npar5,npar6,npar7,npar8
   28 format (/10x,5hnpar2,1x,1h=,i5,8x,5hnpar3,1x,1h=,i5,8x,5hnpar4,
&     1x,1h=,i5//10x,5hnpar5,1x,1h=,i5,8x,5hnpar6,1x,1h=,i5,8x,5hnpar7,
&     1x,1h=,i5//10x,5hnpar8,1x,1h=,i5)
      do 10 i=1,nr
      read (5,3) (pop(x,i),x=1,na)
    3 format (8f10.0/8f10.0/2f10.0)
      read (5,31) (birth(x,i),x=1,na)
   31 format (10f8.0/8f8.0)
      read (5,31) (death(x,i),x=1,na)
      do 34 x=1,na
   34 omig(x,i,i)=0.
      if (nr.eq.1) go to 33
      do 32 j=1,nr
      if (i.eq.j) go to 32
      read (5,31) (omig(x,j,i),x=1,na)
   32 continue
   33 continue
   10 continue
      return
      end
```

PRINTD1

```
      subroutine printd1(na,ny,nr,nzb,nzd,nzo)
      dimension popc2(18),deathc2(18),birthc2(18),zmigc2(18)
      dimension pop2(18,4),death2(18,4),birth2(18,4),omig2(18,4,4)
      dimension omigt(4)
      dimension grd(4),grr(4),gro(4,4),grot(4)
      dimension crudd(4),crudf(4),crudo(4,4),crudot(4)
      dimension agep(4),aged(4),agef(4),ageo(4,4),ageot(4)
      common /creg/ reg(4)
      common /crate/ ratd(18,4),ratm(18,4,4),ratf(18,4)
      common /cratet/ ratot(18,4)
      common /c1/ pop(18,4),birth(18,4),death(18,4),omig(18,4,4)
      common /c4/ nage(18)
      common /ctot/ birthc(18),deathc(18),cmigc(18)
      common /ctotrat/ popc(18),ratdt(18),ratft(18),ratmt(18)
      double precision reg
      integer x,xx
        real l
       xzb=float(nzb)
      xzd=float(nzd)
      xzo=float(nzo)
      naa=na-1
      print 65
   65 format (1h1,5x,35hobserved population characteristics/6x,
     & 351h=)/)
      print 265
  265 format (/11x,10hinput-data/11x,10(1h*)/)
      if (nr.eq.1) go to 103
      zmm4=0.
      iskip=3
      do 6 i=1,nr
       if (iskip.ne.i) go to 164
       print 165
  165 format (1h1/1x)
      iskip=iskip+2
  164 continue
      print 15, reg(i)
   15 format (//5x,6hregion,3x,a8/5x,17(1h-))
      print 16,reg(i)
   16 format (3x,3hage,1x,10hpopulation,4x,6hbirths,4x,6hdeaths,5x,
     & 14hmigration from,1x,a8,1x,2hto)
      print 17,(reg(j),j=1,nr)
   17 format (37x,10(1x,a8))
      print 66
   66 format (1x)
      do 14 x=1,na
c     popr(x,i)=pop(x,i)
   14 print 8, nage(x),pop(x,i),birth(x,i),death(x,i),(omig(x,j,i),
     & j=1,nr)
    8 format (3x,i3,1x,3f10.0,10f9.0)
      popt=0.
      deatht=0.
      birtht=0.
      do 41 j=1,nr
   41 omigt(j)=0.
      do 42 x=1,na
      popt=popt+pop(x,i)
```

```
      deatht=deatht+death(x,i)
      birtht=birtht+birth(x,i)
      do 42 j=1,nr
      omigt(j)=omigt(j)+omig(x,j,i)
   42 continue
      do 43 x=1,na
      death2(x,i)=0.
      birth2(x,i)=0.
      pop2(x,i)=100.*pop(x,i)/popt
      if (deatht.ne.0.) death2(x,i)=100.*death(x,i)/deatht
      if (birtht.ne.0.) birth2(x,i)=100.*birth(x,i)/birtht
      do 43 j=1,nr
      omig2(x,j,i)=0.
      if (omigt(j).ne.0.) omig2(x,j,i)=100.*omig(x,j,i)/omigt(j)
   43 continue
      print 40, popt,birtht,deatht,(omigt(j),j=1,nr)
   40 format (/1x,5htotal,1x,3f10.0,10f9.0)
c     poppr(i)=popt
      crudd(i)=deatht/(popt*xzd)
      crudf(i)=birtht/(popt*xzb)
      z=0.
      do 69 j=1,nr
      z=z+omigt(j)
   69 crudo(j,i)=omigt(j)/(popt*xzo)
      zmm4=zmm4+z
      crudot(i)=z/(popt*xzo)
    6 continue
      print 44
   44 format (1h1,10x,23hpercentage distribution/11x,23(1h*)/)
      iskip=3
      do 45 i=1,nr
      if (iskip.ne.i) go to 166
      print 165
      iskip=iskip+2
  166 continue
      print 15, reg(i)
      print 16, reg(i)
      print 17, (reg(j),j=1,nr)
       print 66
      do 46 x=1,na
   46 print 47, nage(x),pop2(x,i),birth2(x,i),death2(x,i),(omig2(x,j,i),
     & j=1,nr)
   47 format (3x,i3,1x,3f10.4,10f9.4)
      z=100.
      nnn=nr+3
      print 147, (z,j=1,nnn)
  147 format (/2x,5htotal,3f10.4,10f9.4)
       agep(i)=0.
      aged(i)=0.
      agef(i)=0.
       do 68 j=1,nr
   68 ageo(j,i)=0.
      do 67 x=1,na
      k=(x-1)*ny
      z=float(k)+float(ny)*0.5
      z=z/100.
```

```
       agep(i)=agep(i)+z*pop2(x,i)
      aged(i)=aged(i)+z*death2(x,i)
      agef(i)=agef(i)+z*birth2(x,i)
      do 67 j=1,nr
   67 ageo(j,i)=ageo(j,i)+z*omig2(x,j,i)
      print 38, agep(i),agef(i),aged(i),(ageo(j,i),j=1,nr)
   38 format (1x,5hm.age,1x,3f10.4,10f9.4)
   45 continue
  103 continue
      do 5  i=1,nr
      do 5 x=1,na
      ratm(x,i,i)=0.
      ratd(x,i)=death(x,i)/(pop(x,i)*xzd)
      ratf(x,i)=birth(x,i)/(pop(x,i)*xzb)
      if (nr.eq.1) go to 5
      do 21 j=1,nr
   21 ratm(x,j,i)=omig(x,j,i)/(pop(x,i)*xzo)
    5 continue
      do 55 i=1,nr
      do 55 j=1,nr
      ratm(na,j,i)=0.
   55 continue
      do 35 i=1,nr
      grd(i)=0.
      grr(i)=0.
      do 36 j=1,nr
   36 gro(j,i)=0.
      do 35 x=1,na
      grd(i)=grd(i)+ratd(x,i)
      grr(i)=grr(i)+ratf(x,i)
      do 35 j=1,nr
      gro(j,i)=gro(j,i)+ratm(x,j,i)
   35 continue
      if (nr.gt.1) print 20
   20 format (1h1,5x,14hobserved rates/6x,14(1h*))
      do 33 i=1,nr
      do 77 x=1,na
      ratot(x,i)=0.
      do 77 j=1,nr
      if (i.eq.j) go to 77
      ratot(x,i)=ratot(x,i)+ratm(x,j,i)
   77 continue
      grot(i)=0.
      do 78 j=1,nr
      if (j.eq.i) go to 78
      grot(i)=grot(i+gro(j,i)
   78 continue
      aged(i)=0.
      agef(i)=0.
      ageot(i)=0.
      do  30 j=1,nr
   30 ageo(j,i)=0.
      do 48 x=1,na
      k=(x-1)*ny
      z=float(k)+float(ny)*0.5
      aged(i)=aged(i)+z*ratd(x,i)/grd(i)
      if (grr(i).gt.0.) agef(i)=agef(i)+z*ratf(x,i)/grr(i)
```

```
      if (grot(i).gt.0.) ageot(i)=ageot(i)+z*ratot(x,i)/grot(i)
      do 48 j=1,nr
 48   if (gro(j,i).gt.0.) ageo(j,i)=ageo(j,i)+z*ratm(x,j,i)/gro(j,i)
 33   continue
      if (nr.eq.1) go to 102
      print 31
 31   format (/20x,11hdeath rates/20x,11(1h*)/)
      print 32, (reg(j),j=1,nr)
 32   format (2x,3hage,5x,10(2x,a8))
      print 66
      do 18 x=1,na
 18   print 19, nage(x),(ratd(x,j),j=1,nr)
 19   format (2x,i3,5x,11f10.6)
      print 37, (grd(j),j=1,nr)
 37   format (/1x,5hgross,5x,f9.6,10f10.6)
      print 39, (crudd(j),j=1,nr)
 39   format (1x,5hcrude,5x,f9.6,10f10.6)
      print 49, (aged(j),j=1,nr)
 49   format (1x,5hm.age,5x,f9.4,10f10.4)
      print 171
171   format (//20x,15hfertility rates/20x,15(1h*)/)
      print 32, (reg(j),j=1,nr)
      print 66
      do 72 x=1,na
 72  print 19, nage(x),(ratf(x,j),j=1,nr)
      print 37, (grr(j),j=1,nr)
      print 39, (crudf(j),j=1,nr)
      print 49, (agef(j),j=1,nr)
      print 73
 73   format (1h1,19x,18houtmigration rates/20x,18(1h*))
       iskip=3
      do 79 i=1,nr
       if (iskip.ne.i) go to 167
       print 165
       iskip=iskip+2
167   continue
      print 74, reg(i)
 74   format (/20x,14hmigration from,1x,a8,1x,2hto)
      print 75, (reg(j),j=1,nr)
 75   format (2x,3hage,10x,5htotal,10(2x,a8))
      print 66
      do 76 x=1,na
 76   print 19, nage(x),ratot(x,i),(ratm(x,j,i),j=1,nr)
      print 37, grot(i),(gro(j,i),j=1,nr)
      print 39, crudot(i),(crudo(j,i),j=1,nr)
      print 49, ageot(i),(ageo(j,i),j=1,nr)
      print 66
 79   continue
708   continue
      print 51
 51   format (1h1,10x,23htotal population system/11x,23(1h*)/)
102    continue
      print 52
 52   format (3x,3hage,8x,10hpopulation,14x,6hbirths,15x,6hdeaths,12x,
&     9hmigration,17x,14hobserved rates)
      print 59
```

```
 59   format (6x,4(4x,8habsolute,2x,7hpercent),9x,5hbirth,6x,5hdeath,
&      2x,9hmigration/)
      pp4=0.
      bb4=0.
      dd4=0.
      do 54 x=1,na
      popc(x)=0.
      deathc(x)=0.
      birthc(x)=0.
      cmigc(x)=0.
      do 53 i=1,nr
      popc(x)=popc(x)+pop(x,i)
      deathc(x)=deathc(x)+death(x,i)
      birthc(x)=birthc(x)+birth(x,i)
      do 53 j=1,nr
      cmigc(x)=cmigc(x)+omig(x,j,i)
 53   continue
      pp4=pp4+popc(x)
      dd4=dd4+deathc(x)
      bb4=bb4+birthc(x)
      ratdt(x)=deathc(x)/(popc(x)*xzd)
      ratft(x)=birthc(x)/(popc(x)*xzb)
      ratmt(x)=cmigc(x)/(popc(x)*xzo)
 54   continue
      radd4=0.
      raff4=0.
      radc4=0.
      rafc4=0.
      ramc4=0.
      ramm4=0.
      do 58 x=1,na
      deathc2(x)=0.
      birthc2(x)=0.
      radd4=radd4+ratdt(x)
      raff4=raff4+ratft(x)
      ramm4=ramm4+ratmt(x)
      p25=popc(x)/pp4
      rafc4=rafc4+ratft(x)*p25
      radc4=radc4+ratdt(x)*p25
      ramc4=ramc4+ratmt(x)*p25
      popc2(x)=100.*popc(x)/pp4
      if (dd4.ne.0.) deathc2(x)=100.*deathc(x)/dd4
      if (bb4.ne.0.) birthc2(x)=100.*birthc(x)/bb4
      if (zmm4.ne.0.) zmigc2(x)=100.*cmigc(x)/zmm4
      print 63, nage(x),popc(x),popc2(x),birthc(x),birthc2(x),deathc(x),
&     deathc2(x),cmigc(x),zmigc2(x),ratft(x),ratdt(x),ratmt(x)
 63   format (3x,i3,4(2x,f10.0,f9.4),3x,3(1x,f10.6))
 58   continue
      zz=100.
      print 64, pp4,zz,bb4,zz,dd4,zz,zmm4,zz,raff4,radd4,ramm4
 64   format (/1x,5htotal,4(2x,f10.0,f9.4),3x,3(1x,f10.6))
      print 71, rafc4,radc4,ramc4
 71   format (1x,5hcrude,87x,3(1x,f10.6))
      agepc4=0.
      agefc4=0.
       agedc4=0.
```

```
      agemc4=0.
      agefr4=0.
      agedr4=0.
      agemr4=0.
      do 81 x=1,na
      k=(x-1)*ny
      z=float(k)+2.5
      z1=z/100.
      agepc4=agepc4+z1*popc2(x)
      agefc4=agefc4+z1*birthc2(x)
       agedc4=agedc4+z1*deathc2(x)
      agemc4=agemc4+z1*zmigc2(x)
      if (raff4.gt.0.) agefr4=agefr4+z*ratft(x)/raff4
       agedr4=agedr4+z*ratdt(x)/radd4
      if (ramm4.gt.0.) agemr4=agemr4+z*ratmt(x)/ramm4
81    continue
      print 82, agepc4,agefc4,agedc4,agemc4,agefr4,agedr4,agemr4
82    format (1x,5hm.age,4(11x,f10.4),3x,3f11.4)
      popprt=pp4
      if (nr.gt.1) go to 70
      crudd(1)=dd4/(pp4*xzd)
      crudf(1)=bb4/(pp4*xzb)
      crudo(1,1)=0.
70    continue
      return
      end
```

LIFE

```
      subroutine life (na,ny,nr,nop,irun,ioptg)
      dimension zkadj(4)
      dimension cc(4,4)
      common /csu/ su(18,4,4),ssu(18,4)
      common /cmul/ a1(4,4),b(4,4),c(4,4)
      common /crad/ radix(4),radixt
      common /cpq/ q(18,4),p(18,4,4),pmigt(18,4)
      common /clife2/ dr(4),cl(18,4,4),e(18,4,4),t(18,4,4)
      common /cl/ l(18,4,4),cllt(18,4),clltot(18)
      common /clife3/ clt(18,4),ee(18,4),tt(18,4)
      common /cwhole/ clto(18),etotc(18)
      common /c4/ nage(18)
      common /clif2/ cm(72,4,4),cdr(18,4,4)
      common /crate/ ratd(18,4),ratm(18,4,4),ratf(18,4)
      real l
      integer x,xx,xy,xz1
      integer xo
      xo=1
      do 1 i=1,nr
      zkadj(i)=float(ny)*0.5
1     continue
506    format (1x,8f14.2)
509   format (//1x)
      do 50 io=1,nr
      do 7 i=1,nr
      if (i.ne.io) cl(1,io,i)=0.
      if (i.eq.io) cl(1,io,i)=radix(io)
7     continue
      naa=na-1
      do 5 x=xo,naa
```

```
      xx=x+1
      xz1=na*(io-1)+x
      do 19 i=1,nr
      if (nop.eq.1) cdr(x,io,i)=cl(x,io,i)*q(x,io)
      if (nop.eq.2) cdr(x,io,i)=cl(x,io,i)*q(x,i)
      do 3 j=1,nr
    3 cm(xz1,i,j)=cl(x,io,i)*p(x,j,i)
      cl(xx,io,i)=0.
   19 continue
      do 5 i=1,nr
      do 4 j=1,nr
      xz1=(io-1)*na+x
    4 cl(xx,io,i)=cl(xx,io,i)+cm(xz1,j,i)
    5 continue
   17 continue
      do 6 i=1,nr
      if (i.eq.io) cdr(na,io,i)=cl(na,io,i)
      if (i.ne.io) cdr(na,io,i)=0.
      do 6 j=1,nr
      xz1=io*na
    6 cm(xz1,i,j)=0.
      do 8 x=1,na
      clt(x,io)=0.
      do 8 j=1,nr
      clt(x,io)=clt(x,io)+cl(x,io,j)
    8 continue
c     if (iadj.ge.2) go to 120
      if (xo.gt.naa) go to 18
      do 10 x=xo,naa
      if (x.eq.1) zz=zkadj(io)
      if (x.gt.1) zz=float(ny)/2.
      xx=x+1
      cllt(x,io)=0.
      do 9 i=1,nr
      l(x,io,i)=zz*(cl(x,io,i)+cl(xx,io,i))/radix(io)
    9 cllt(x,io)=cllt(x,io)+l(x,io,i)
   10 continue
   18 continue
      cllt(na,io)=0.
      do 11 i=1,nr
      if (nop.eq.1) l(na,io,i)=cl(na,io,i)/(dr(io)*radix(io))
      if (nop.eq.2) l(na,io,i)=cl(na,io,i)/(dr( i)*radix(io))
   11 cllt(na,io)=cllt(na,io)+l(na,io,i)
  120 continue
   50 continue
      if (nr.eq.1) go to 67
      do 61 x=1,naa
      xx=x+1
      do 74 io=1,nr
      do 74 j=1,nr
   74 cc(io,j)=l(x,io,j)
      call invert (cc,nr)
      do 75 io=1,nr
      do 75 j=1,nr
      su(x,io,j)=0.
      do 75 jj=1,nr
```

```
      su(x,io,j)=su(x,io,j)+cc(io,jj)*l(xx,jj,j)
75    continue
      do 62 io=1,nr
      ssu(x,io)=0.
      do 62 j=1,nr
      ssu(x,io)=ssu(x,io)+su(x,io,j)
62    continue
61    continue
      go to 23
 67   naaa=na-2
      do 25 x=1,naaa
      xx=x+1
      su(x,1,1)=0.
      su(x,1,1)=l(xx,1,1)/l(x,1,1)
25    continue
      su(naa,1,1)=l(na,1,1)/(l(naa,1,1)+l(na,1,1))
      su(na,1,1)=0.
      do 24 x=1,naa
24    ssu(x,1)=su(x,1,1)
23    continue
      if (ioptg.eq.1) return
      do 51 io=1,nr
      do 14 x=xo,na
      tt(x,io)=0.
      do 13 i=1,nr
      t(x,io,i)=0.
      do 12 xy=x,na
12    t(x,io,i)=t(x,io,i)+l(xy,io,i)
13    tt(x,io)=tt(x,io)+t(x,io,i)
14    continue
      do 16 x=xo,na
      ee(x,io)=0.
      czz=clt(x,io)
      if (czz.eq.0.) go to 20
      go to 22
20    do 21 i=1,nr
21    e(x,io,i)=0.
22    continue
      do 15 i=1,nr
      e(x,io,i)=(t(x,io,i)/clt(x,io))*radix(io)
      ee(x,io)=ee(x,io)+e(x,io,i)
15    continue
16    continue
51    continue
      if (irun.eq.2) go to 31
      do 30 x=1,naa
      do 30 i=1,nr
      z1=zz*(1.+p(x,i,i))
      ratd(x,i)=q(x,i)/z1
      do 32 j=1,nr
32    ratm(x,j,i)=p(x,j,i)/z1
30    continue
      do 43 i=1,nr
     ratd(na,i)=dr(i)
      do 43 j=1,nr
43    ratm(na,j,i)=0.
```

```
      go to 34
31    do 40 x=1,na
      do 35 i=1,nr
      do 35 j=1,nr
      if (i.eq.j) a1(j,i)=1.-p(x,j,i)
      if (i.ne.j) a1(j,i)=-p(x,j,i)
      b(j,i)=cl(x,i,j)/radix(i)
35    continue
      call multip (nr,nr,nr)
      do 36 i=1,nr
      do 36 j=1,nr
36    cc(j,i)=l(x,i,j)
      call invert (cc,nr)
      do 37 i=1,nr
      do 37 j=1,nr
   a1(j,i)=c(j,i)
37    b(j,i)=cc(j,i)
      call multip (nr,nr,nr)
      do 39 i=1,nr
      z=0.
      do 38 j=1,nr
      if (i.eq.j) go to 38
      z=z-c(j,i)
38    continue
      do 39 j=1,nr
      ratm(x,j,i)=0.
      if (i.ne.j) ratm(x,j,i)=-c(j,i)
      if (i.eq.j) ratd(x,i)=c(i,i)-z
39    continue
40    continue
34    continue
      return
      end
```

MULTIP

```
      subroutine multip (n,k,l)
      common /cmul/ a1(4,4),b(4,4),c(4,4)
      do 3 i=1,n
      do 3 j=1,l
      c(i,j)=0.
      do 3 jj=1,k
      c(i,j)=c(i,j)+a1(i,jj)*b(jj,j)
 3    continue
       return
      end
```

INVERT

```
      subroutine invert (cc,nr)
      dimension pivot(4),cc(4,4)
      do 606 i=1,nr
      pivot(i)=cc(i,i)
      cc(i,i)=1.0
      do 607 j=1,nr
      cc(i,j)=cc(i,j)/pivot(i)
607   continue
      if (nr.eq.1) go to 10
      do 608 k=1,nr
      if (k.eq.i) go to 608
      h=cc(k,i)
      cc(k,i)=0.
      do 609 l=1,nr
      cc(k,l)=cc(k,l)-cc(i,l)*h
609   continue
608   continue
606   continue
10    continue
      return
       end
```

WHOLE

```
      subroutine whole (na,ny,nr)
      common /crad/ radix(4),radixt
      common /cl/ l(18,4,4),cllt(18,4),clltot(18)
      common /clife3/ clt(18,4),ee(18,4),tt(18,4)
      common /cwhole/ clto(18),etotc(18)
      integer x,xx
      do 46 x=1,na
      clltot(x)=0.
      clto(x)=0.
      etotc(x)=0.
       do 47 j=1,nr
      clltot(x)=clltot(x)+(radix(j)/radixt)*cllt(x,j)
      clto(x)=clto(x)+clt(x,j)
      etotc(x)=etotc(x)+ee(x,j)*radix(j)
47    continue
46    etotc(x)=etotc(x)/radixt
      return
      end
```

PROBR

```
      subroutine probr (na,ny,nr)
      common /crate/ ratd(18,4),ratm(18,4,4),ratf(18,4)
      common /cpq/ q(18,4),p(18,4,4),pmigt(18,4)
      integer x,xx
      naa=na-1
      z1=float(ny)/2.
      z3=float(ny)
      do 8 i=1,nr
      do 8 x=1,na
      z2=0.
      do 6 j=1,nr
      if (i.eq.j) go to 6
      z2=z2+ratm(x,j,i)
    6 continue
      z=1.+z1*ratd(x,i)+z1*z2
      q(x,i)=z3*ratd(x,i)/z
      do 7 j=1,nr
      if (i.eq.j) go to 7
      p(x,j,i)=z3*ratm(x,j,i)/z
    7 continue
    8 continue
      do 10 x=1,naa
      do 10 i=1,nr
      pmigt(x,i)=0.
      do 9 j=1,nr
      if (i.eq.j) go to 9
      pmigt(x,i)=pmigt(x,i)+p(x,j,i)
    9 continue
      p(x,i,i)=1.-q(x,i)-pmigt(x,i)
   10 continue
      do 11 i=1,nr
      q(na,i)=1.
      do 11 j=1,nr
      p(x,j,i)=0.
   11 continue
      return
      end
```

PROBSCH

```
      subroutine probsch (na,ny,nr)
      dimension rm(4,4),cc(4,4)
      common /crate/ ratd(18,4),ratm(18,4,4),ratf(18,4)
      common /cpq/ q(18,4),p(18,4,4),pmigt(18,4)
      common /cmul/ a1(4,4),b(4,4),c(4,4)
      integer x
c   compute rm(x)
      naa=na-1
      zzz=float(ny)/2.
      do 100 x=1,naa
      if (nr.eq.1) go to 50
      do 5 i=1,nr
      z=ratd(x,i)
      do 4 j=1,nr
      if (i.eq.j) go to 4
      z=z+ratm(x,j,i)
    4 continue
      rm(i,i)=(-1.)*z
      do 6 j=1,nr
      if (j.eq.i) go to 6
      rm(j,i)=ratm(x,j,i)
    6 continue
    5 continue
      do 7 i=1,nr
      do 7 j=1,nr
      if (i.eq.j) cc(j,i)=1.-zzz*rm(j,i)
    7 if (i.ne.j) cc(j,i)=-zzz*rm(j,i)
      call invert (cc,nr)
      do 8 i=1,nr
      do 8 j=1,nr
       a1(j,i)=cc(j,i)
      if (j.eq.i) b(j,i)=1.+zzz*rm(j,i)
    8 if (j.ne.i) b(j,i)=zzz*rm(j,i)
      call multip (nr,nr,nr)
      do 9 i=1,nr
      do 9 j=1,nr
    9 p(x,j,i)=c(j,i)
      go to 100
   50 do 51 x=1,naa
   51 p(x,1,1)=(1.-zzz*ratd(x,1))/(1.+zzz*ratd(x,1))
  100 continue
      do 10 i=1,nr
      do 10 j=1,nr
   10 p(na,j,i)=0.
      do 12 x=1,na
      do 12 i=1,nr
      pmigt(x,i)=0.
      do 11 j=1,nr
      if (i.eq.j) go to 11
      pmigt(x,i)=pmigt(x,i)+p(x,j,i)
   11 continue
      q(x,i)=1.-p(x,i,i)-pmigt(x,i)
   12 continue
       return
      end
```

PRINTL

```
      subroutine printl (na,ny,nr,irun,ka)
      dimension hulp(5)
      common /crate/ ratd(18,4),ratm(18,4,4),ratf(18,4)
      common /cpq/ q(18,4),p(18,4,4),pmigt(18,4)
      common /csu/ su(18,4,4),ssu(18,4)
      common /creg/ reg(4)
      common /crad/ radix(4),radixt
      common /clife2/ dr(4),cl(18,4,4),e(18,4,4),t(18,4,4)
      common /cl/ l(18,4,4),cllt(18,4),clltot(18)
      common /clife3/ clt(18,4),ee(18,4),tt(18,4)
      common /clif2/ cm(72,4,4),cdr(18,4,4)
      common /cwhole/ clto(18),etotc(18)
      common /c4/ nage(18)
      double precision reg
      real l
      integer x,xx
      iop=irun
      if (irun.eq.2) iop=3
      if (irun.eq.3) iop=2
      print 3,iop
    3 format (1h1/,5x,31hmultiregional life table option,i2/
     & 6x,33(1h=)/)
 9999 format (1h1//1x)
      print 4500
 4500 format (20x,36hprobabilities of dying and migrating/20x,36(1h*))
       iskip=3
      do 726 i=1,nr
      if (iskip.ne.i) go to 121
      print 120
  120 format (1h1/1x)
       iskip=iskip+2
  121 continue
      print 9001, reg(i)
 9001 format (//20x,6hregion,2x,a8,1x/20x,16(1h*)/)
      print 9011, reg(i)
 9011 format (19x,3hage,6x,5hdeath,5x,14hmigration from,1x,a8,1x,2hto)
      print 9020, (reg(j),j=1,nr)
 9020 format (33x,10(2x,a8))
      print 66
   66 format (1x)
      do 726 x=1,na
      print 9103, nage(x),q(x,i),(p(x,j,i),j=1,nr)
 9103 format (19x,i3,1x,11f10.6)
  726 continue
       if (irun.eq.2) go to 159
      print 9999
      print 4833
 4833 format (20x,30hdeath rate and migration rates/20x,30(1h*))
      iskip=3
      do 59 i=1,nr
      if (iskip.ne.i) go to 122
      print 120
      iskip=iskip+2
  122 continue
      print 9001, reg(i)
      print 9011, reg(i)
```

```
      print 9020, (reg(j),j=1,nr)
      print 66
      do 59 x=1,na
      print 9103, nage(x),ratd(x,i),(ratm(x,j,i),j=1,nr)
   59 continue
  159 continue
      if (ka.eq.0) go to 160
      print 9201
 9201  format (1h1,/20x,30hlife history of initial cohort/20x,30(1h*))
      do 9210 i=1,nr
       if (i.ne.1) print 9211
 9211 format (1h1,1x)
      print 9202, reg(i)
 9202 format (1h0,20x,24hinitial region of cohort,2x,a8/21x,
     & 34(1h*)/)
      print 9203
 9203 format (10x,44hnumber of deaths in each region of residence)
      print 9204, (reg(j),j=1,nr)
 9204 format (1x,3hage,2x,10(4x,a8))
      print 66
      do 9205 x=1,na
 9205 print 9206, nage(x),(cdr(x,i,j),j=1,nr)
 9206 format (1x,i3,2x,10f12.2)
      do 10 j=1,nr
      hulp(j)=0.
      do 10 x=1,na
   10 hulp(j)=hulp(j)+cdr(x,i,j)
      print 11, (hulp(j),j=1,nr)
   11 format (/1x,5htotal,10f12.2)
      do 9210 io=1,nr
      print 9207, reg(io)
 9207 format (/10x,23hnumber of migrants from,1x,a8,1x,2hto)
      print 9204, (reg(j),j=1,nr)
      print 66
      do 12 j=1,nr
   12 hulp(j)=0.
      do 9208 x=1,na
      xx=(i-1)*na+x
      print 9206, nage(x),(cm(xx,io,j),j=1,nr)
      do 13 j=1,nr
   13 hulp(j)=hulp(j)+cm(xx,io,j)
 9208 continue
      print 11, (hulp(j),j=1,nr)
 9210 continue
  160 continue
      print 9999
      print 4831
 4831 format (20x,51hexpected number of survivors at exact age x in each
     & 7h region,/20x,52(1h*))
      iskip=3
      do 50 i=1,nr
      if (iskip.ne.i) go to 123
      print 120
      iskip=iskip+2
  123 continue
      if (i.eq.1) print 9501, reg(i)
```

```
 9501 format (//1x,3hage,1x,10haggregated,4x,3hage,6x,
&      24hinitial region of cohort,2x,a8/1x,3(1h*),1x,10(1h*),4x,3(1h*),6x,
&      34(1h*)/)
      if (i.gt.1) print 9502,reg(i)
 9502 format (//19x,3hage,6x,24hinitial region of cohort,2x,a8/19x,
&      3(1h*),6x,34(1h*)/)
      print 9100, (reg(j),j=1,nr)
 9100  format (28x,5htotal,10(2x,a8))
       print 66
      do 50 x=1,na
      if (i.eq.1) print 9106, nage(x),clto(x),nage(x),clt(x,i),
&      (cl(x,i,j),j=1,nr)
 9106 format (1x,i3,1x,f10.0,4x,i3,11f10.0)
      if (i.gt.1) print 9101, nage(x),clt(x,i),(cl(x,i,j),j=1,nr)
 9101 format (19x,i3,1x,11f10.0)
  50  continue
      print 9999
      print 4832
 4832 format (10x,51hnumber of years lived in each region by the initial
&      ,6h unit ,7h cohort,/10x,64(1h*))
       iskip=3
      do 34 i=1,nr
      if (i.ne.iskip) go to 124
      print 120
      iskip=iskip+2
 124  continue
      if (i.eq.1) print 9501, reg(i)
      if (i.gt.1) print 9502, reg(i)
      print 9100, (reg(j),j=1,nr)
      print 66
      do 58 x=1,na
      if (i.eq.1) print 9114, nage(x),clltot(x),nage(x),cllt(x,i),
&      (l(x,i,j),j=1,nr)
 9114 format (1x,i3,1x,f10.6,4x,i3,1x,11f10.6)
      if (i.gt.1) print 9103, nage(x),cllt(x,i),(l(x,i,j),j=1,nr)
  58  continue
  34  continue
      print 9999
      print 4834
 4834 format (30x,24hsurvivorship proportions/30x,24(1h*))
       iskip=3
      do 62 i=1,nr
       if (iskip.ne.i) go to 125
       print 120
       iskip=iskip+2
 125  continue
      print 9001, reg(i)
      print 9100, (reg(j),j=1,nr)
      naa=na-1
       print 66
      do 63 x=1,naa
  63  print 9103, nage(x),ssu(x,i),(su(x,i,j),j=1,nr)
  62  continue
      print 9999
      print 4835
 4835 format (10x,37htotal number of years to be lived -t-/10x,37(1h*))
```

```
       iskip=3
      do 69 i=1,nr
        if (iskip.ne.i) go to 126
         print 120
         iskip=iskip+2
  126  continue
       print 9502, reg(i)
      print 9100, (reg(j),j=1,nr)
      print 66
      do 69 x=1,na
      print 9103, nage(x),tt(x,i),(t(x,i,j),j=1,nr)
  69  continue
      print 9999
      print 4830
 4830 format (30x,20hexpectations of life/30x,20(1h*))
        iskip=3
      do 52 i=1,nr
        if (iskip.ne.i) go to 127
        print 120
       iskip=iskip+2
  127 continue
      if (i.eq.1) print 9501, reg(i)
      if (i.gt.1) print 9502, reg(i)
      print 9100, (reg(j),j=1,nr)
      print 66
      do 51 x=1,na
      if (i.eq.1) print 9114, nage(x),etotc(x),nage(x),ee(x,i),
&     (e(x,i,j),j=1,nr)
      if (i.gt.1) print 9103, nage(x),ee(x,i),(e(x,i,j),j=1,nr)
  51  continue
  52  continue
      if (nr.ne.2) go to 5060
      print 9999
      print 5054, iop
 5054 format (7x,5htable,4x,39h- multiregional (two-region) life table,
&     1x,6hoption,i2/7x,5(1h-),4x,48(1h-)/)
      z1=e(1,1,2)/ee(1,1)
      z2=e(1,2,1)/ee(1,2)
      print 5055, (ee(1,i),i=1,nr),z1,z2
 5055 format (10x,16hmortality levels,8x,1h=,f6.2,8x,1h=,f6.2,7x,
&     16hmigration levels,8x,1h=,f7.4,7x,1h=,f7.4)
      i=1
      j=2
      print 5059, i,((i,i,j,i),k=1,3),j,i,i,((i,i,j,i),k=1,2)
 5059 format (//1x,3hage,4x,4hq(x,,i1,1h),2(2x,4hp(x,,i1,1h,,i1,1h)),
&     2(1x,4hl(x,,i1,1h,,i1,1h)),2(1x,5hll(x,,i1,1h,,i1,1h)),
&     2x,4hm(x,,i1,1h,,i1,1h),3x,5hmd(x,,i1,1h),2(2x,4hs(x,,i1,
&     1h,,i1,1h)),2(1x,4he(x,,i1,1h,,i1,1h))/)
      do 5057 x=1,na
 5057 print 5056, nage(x),q(x,1),p(x,1,1),p(x,2,1),cl(x,1,1),cl(x,1,2),l(x,1,1)
&     ,l(x,1,2),ratm(x,2,1),ratd(x,1),su(x,1,1),su(x,1,2),e(x,1,1),
&     e(x,1,2)
 5056 format (1x,i3,3f10.6,2f9.0,2f10.5,4f10.6,2f9.2)
      print 66
      print 5059, j,((j,j,i,j),k=1,3),i,j,j,((j,j,i,j),k=1,2)
      do 5058 x=1,na
```

```
 5058 print 5056, nage(x),q(x,2),p(x,2,2),p(x,1,2),cl(x,2,2),cl(x,2,1),l(x,2,2)
&       ,l(x,2,1),ratm(x,1,2),ratd(x,2),su(x,2,2),su(x,2,1),e(x,2,2),
&       e(x,2,1)
 5060 continue
      if (nr.ne.1) go to 5070
      print 5061, reg(1),ee(1,1)
 5061 format (1h1,//10x,5htable,4x,26h- single region life table,3x,
&       a8,5x,17hmortality level =,f6.2//)
      print 5062
 5062 format (1x,3hage,8x,4hq(x),8x,4hl(x),8x,4hd(x),7x,5hll(x),8x,
&       4hm(x),8x,4hs(x),8x,4ht(x),8x,4he(x)/)
      do 5063 x=1,na
 5063 print 5064, nage(x),q(x,1),clt(x,1),cdr(x,1,1),
&       cllt(x,1),ratd(x,1),ssu(x,1),tt(x,1),ee(x,1)
 5064 format (1x,i3,f12.6,2f12.0,5f12.6)
 5070 continue
      return
      end
```

GROWTH

```
      subroutine growth (na,ny,zfny,nr)
c  su is in conventional matrix notation, not in transpose
      common /csu/ su(18,4,4),ssu(18,4)
      common /cgrow/ br(18,4,4),pp1(4,4),popr(18,4),poppr(4),popprt
      common /cmul/ a1(4,4),b(4,4),c(4,4)
      common /crate/ ratd(18,4),ratm(18,4,4),ratf(18,4)
      common /creg/ reg(4)
      common /c4/ nage(18)
      double precision reg
      integer x,xx
      real l
      naa=na-1
      zz=zfny/4.
      do 4 x=1,naa
      xx=x+1
      do 3 i=1,nr
      do 3 j=1,nr
      if(i.eq.j) a1(j,i)=ratf(xx,i)
      if(i.ne.j) a1(j,i)=0.
    3 b(j,i)=su(x,i,j)
      call multip (nr,nr,nr)
      do 5 i=1,nr
      do 5 j=1,nr
      if (i.eq.j) b(j,i)=ratf(x,i)+c(j,i)
    5 if (i.ne.j) b(j,i)=c(j,i)
      do 7 i=1,nr
      do 7 j=1,nr
      if (i.eq.j) a1(j,i)=zz*(pp1(j,i)+1.)
      if (i.ne.j) a1(j,i)=zz*pp1(j,i)
    7 continue
      call multip (nr,nr,nr)
      do 8 i=1,nr
      do 8 j=1,nr
    8 br(x,j,i)=c(j,i)
    4 continue
c   print growth matrix
      print 10
   10 format (1h1,5x,48hthe discrete model of multiregional demographic
&     ,6hgrowth/6x,54(1h*)/6x,54(1h*)/)
      print 11
   11 format (/5x,31hmultiregional projection matrix/5x,31(1h*))
      do 20 i=1,nr
       if (i.ne.1) print 120
  120 format (1h1,1x)
      print 12, reg(i)
   12 format (//20x,6hregion,2x,a8/20x,16(1h*))
      print 13
   13 format (/5x,3hage,8x,9hfirst row)
      print 14, (reg(j),j=1,nr)
   14 format (11x,10(2x,a8))
      print 15
   15 format (1x)
      do 16 x=1,naa
   16 print 17, nage(x), (br(x,j,i),j=1,nr)
   17 format (5x,i3,3x,10f10.6)
      print 18
```

```
18  format (/5x,3hage,8x,24hsurvivorship proportions)
    print 14, (reg(j),j=1,nr)
    print 15
    do 19 x=1,naa
19  print 17, nage(x),(su(x,i,j),j=1,nr)
20  continue
    return
    end
```

PROJECT

```
      subroutine project (na,ny,zfny,nr,init,nproj,iprojp,
&     npar3,npar4,zlamda)
      dimension zmin1(4),z4(4),popt8(18),pop8(18,4),
&     zlamb(4),agem(4),zr(4)
      common /csu/ su(18,4,4),ssu(18,4)
      common /cgrow/ br(18,4,4),pp1(4,4),popr(18,4),poppr(4),popprt
      common /cmul/ a1(4,4),b(4,4),c(4,4)
      common /crate/ ratd(18,4),ratm(18,4,4),ratf(18,4)
      common /creg/ reg(4)
      common /c4/ nage(18)
      common /ctotrat/ popc(18),ratdt(18),ratft(18),ratmt(18)
      double precision reg
      integer x,x1,x2
      jgo=0
       if (npar3.ne.2) npar3=3
       if (npar4.eq.0) npar4=6
      iproj=0
      npar411=(-1)*npar4
      tolx=10.**(npar411)
       if (iprojp.eq.0) iprojp=200
      nn87=nproj
      nproj=(nproj-init)/ny
      ipro50=(nn87-init)/iprojp+1
       if (nproj.eq.0) nproj=10
      naa=na-1
      print 1
    1 format (1h1,5x,35hmultiregional population projection/6x,
&     35(1h*)/)
       zzlam=10.
      nyear1=init
      go to 509
  500 continue
      iproj=iproj+1
      nyear1=init+iproj*ny
      do 3 i=1,nr
      z4(i)=0.
    3 zmin1(i)=poppr(i)
      zmint=popprt
      do 2 x=1,na
      do 4 j=1,nr
      b(j,1)=popr(x,j)
      do 4 i=1,nr
    4 a1(j,i)=br(x,j,i)
      call multip (nr,nr,1)
      do 5 j=1,nr
    5 z4(j)=z4(j)+c(j,1)
    2 continue
      do 6 x=1,naa
      x1=na-x
      x2=x1+1
      do 7 j=1,nr
      b(j,1)=popr(x1,j)
      do 7 i=1,nr
    7 a1(j,i)=su(x1,i,j)
      call multip (nr,nr,1)
      do 8 j=1,nr
```

```
  8   popr(x2,j)=c(j,1)
  6   continue
      do 9 j=1,nr
  9   popr(1,j)=z4(j)
509   continue
c  compute total popul ation
      do 10 x=1,na
      popc(x)=0.
      do 11 j=1,nr
 11   popc(x)=popc(x)+popr(x,j)
 10   continue
      do 12 j=1,nr
 12   poppr(j)=0.
      do 13 x=1,na
      do 13 j=1,nr
 13   poppr(j)=poppr(j)+popr(x,j)
      popprt=0.
      do 17 j=1,nr
 17   popprt=popprt+poppr(j)
       if (iproj.eq.0) go to 506
      if (jgo.ge.1) go to 505
      do 62 j=1,nr
 62   zlamb(j)=poppr(j)/zmin1(j)
       zz=popprt/zmint
      ztolx=zzlam-zz
      if (npar3.eq.3) ztolx=zlamb(1)-zzlam
      if ((npar3.eq.2).and.(nr.ge.2)) ztolx=zlamb(nr)-zlamb(1)
      zzlam=zz
      if (npar3.eq.3) zzlam=zlamb(1)
      ttolx=-tolx
505   continue
      ipro49=iproj*ny
      ipro51=ipro49/ipro50
      if ((iproj.gt.nproj).and.(ipro51.ne.iprojp)) go to 501
      if (iproj.gt.nproj) ipro50=ipro50+1
      print 51
506   continue
 51   format (1h1,1x)
      print 52, nyear1
 52   format (5x,4hyear,1x,i5/5x,10(1h-)/)
       print 253
253   format (10x,10hpopulation/10x,5(2h- )/)
578   print 53, (reg(j),j=1,nr)
 53   format (1x,3hage,2x,6x,5htotal,10(3x,a8))
      print 54
 54   format (1x)
      do 55 x=1,na
 55   print 56, nage(x),popc(x),(popr(x,j),j=1,nr)
 56   format (1x,i3,2x,11f11.0)
       print 54
      print 57, popprt,(poppr(j),j=1,nr)
 57   format (1x,3htot,2x,11f11.0)
      agemt=0.
c  mean age
       do 21 j=1,nr
 21   agem(j)=0.
```

```
      do 20 x=1,na
       n9=(x-1)*ny
      a9=float(n9)+zfny/2.
      agemt=agemt+a9*popc(x)/popprt
      do 20 j=1,nr
   20 agem(j)=agem(j)+a9*popr(x,j)/poppr(j)
c  percentage distribution
      do 14 x=1,na
      popt8(x)=popc(x)*100./popprt
      do 14 j=1,nr
   14 pop8(x,j)=popr(x,j)*100./poppr(j)
      print 58
   58 format (//10x,23hpercentage distribution/10x,12(2h- )/)
      print 53, (reg(j),j=1,nr)
      print 54
      do 59 x=1,na
   59 print 60, nage(x),popt8(x),(pop8(x,j),j=1,nr)
   60 format (1x,i3,2x,11f11.4)
      z=100.
      print 761, z,(z,jj=1,nr)
  761 format (/1x,5htotal,11f11.4)
      print 22, agemt,(agem(j),j=1,nr)
   22 format (1x,5hm.age,11f11.4)
      z=0.
      do 16 j=1,nr
      z4(j)=(poppr(j)/popprt)*100.
   16 z=z+z4(j)
      print 63, z,(z4(j),j=1,nr)
c population distribution and growth
      if (iproj.eq.0) go to 500
   63 format (1x,3hsha,2x,11f11.4)
   64 format (1x,3hlam,2x,11f11.6)
      print 64, zz,(zlamb(j),j=1,nr)
  501 continue
      if ((ztolx.gt.tolx).or.(ztolx.lt.ttolx)) go to 500
      zlamda=zzlam
      jgo=jgo+1
      if (jgo.eq.1) jgo=2
      if (jgo.eq.2) go to 503
      if (jgo.eq.3) go to 504
  503 continue
      print 18, tolx
   18 format (1h0,1x,30htolerance level for eigenvalue,e14.4)
      print 65, iproj
   65 format (2x,39hnumber of iterations to reach stability,i7)
c  stable equivalent
      zs=zlamda**iproj
      do 66 j=1,nr
      poppr(j)=poppr(j)/zs
      do 66 x=1,na
   66 popr(x,j)=popr(x,j)/zs
      do 68 x=1,na
   68 popc(x)=popc(x)/zs
      popprt=popprt/zs
      print 69
   69 format (1h1,1x,40hstable equivalent to original population/2x,
```

```
&     40(1h*)/)
      go to 578
  504 continue
      return
      end
```

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