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**Interim Report**

**IR-03-018**

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## **A Game-Dynamic Model of Gas Transportation Routes and Its Application to the Turkish Gas Market**

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April 2003

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## Abstract

The purpose of this paper is to study an optimal structure of a system of international gas pipelines competing for a gas market. We develop a game-dynamic model of the operation of several interacting gas pipeline projects with project owners acting as players in the game. The model treats the projects' commercialization times as major players' controls. Current quantities of gas supply are modeled as approximations to Nash equilibrium points in instantaneous "gas supply games", in which each player maximizes his/her current netprofit due to the sales of gas. We use the model to analyze the Turkish gas market, on which gas routes originating from Russia, Turkmenistan and Iran are competing. The analysis is carried out in three steps. At step 1, we model the operation of the pipelines as planned and estimate the associated profits. At step 2, we optimize individual projects, with respect to their profits, assuming that the other pipelines operate as planned. At step 3, we find numerical Nash equilibrium commercialization policies for the entire group of the pipelines. The simulations show the degrees to which the planned regimes are not optimal compared to the Nash equilibrium ones. Another observation is that in equilibrium regimes the pipelines are not always being run at their full capacities, which implies that the proposed pipeline capacities might not be optimal. The simulation results turn out to be moderately sensitive to changes in the discount rate and highly sensitive to changes in the price elasticity of gas demand.

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# A Game-Dynamic Model of Gas Transportation Routes and Its Application to the Turkish Gas Market

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## 1 Introduction

The routing of oil and gas pipelines in Asia and especially in the Caspian region is at the center of the geopolitics of energy. Various countries in the Caspian region are playing the pipeline game to get access to one of the most promising markets in the region: Turkey. Turkey's gas demand is expected to quintuple by 2010 (see [EIA, 1999, 2000]). Russia's Gazprom proposes to build the "Blue Stream" pipeline under the Black Sea to expand its current gas deliveries to Turkey. Turkmenistan, backed by the USA, is heading for the Trans-Caspian gas pipeline to deliver gas to Turkey. This pipeline would flow underneath the Caspian Sea through Azerbaijan and Georgia on to Turkey (see [EIA, 1999, 2000]). Meanwhile, the Iranians have completed their own gas pipeline to the Turkish border and are awaiting the Turkish side of the pipeline to be completed (see [Ignatius, 2000]). It seems that some of countries are moving ahead fast so as to preempt the investments decisions of others, making it unattractive to build a new transmission pipeline since the market is not big enough. In addition gas could and actually is being shipped to Turkey in the form of LNG from Algeria and Egypt.

To increase the complication, Turkey is not the only relevant gas market for the three suppliers. Gazprom has expressed a desire for diversification of export routes. Gas market liberalization in Europe is expected to lower prices which lowers the rate of return of large scale investments in the Yamal peninsula or the Barents Sea. Gazprom is actively looking for new markets in the Asia Pacific (China) and the Middle-East (Turkey) (see [Makarov, 1999]), and several pipelines have been proposed to China. Turkmenistan could also deliver gas to China or to India and Pakistan (see [Klaassen et al., 2001a]). Iran is looking into the option to pipe gas to India as well.

The literature basically distinguishes three approaches to analyze gas infrastructure development and gas exports: partial equilibrium models, game-theoretical models, and energy-flow optimization models. Examples of the first modeling approach are [Golombek et al., 1995], [Van Oostvoorn, and Boots, 1999], and [Beltramo et al., 1986]. Game-theoretical models for the gas market have been developed in [Brekke et al., 1987, 1991], [Berg et al., 1998], [Haurie, Smeers, and Zaccour, 2000], and [Wolf, and Smeers, 1997].

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<sup>\*</sup>The author was partially supported by the Russian Fund for Fundamental Research, Grant 02-01-00769, and the Program for the Sponsorship of Leading Scientific Schools, Grant 791.2003.1.

Energy-flow optimization models are numerous. A few dealing explicitly with gas trade and infrastructure in Asia are [Sugiyama, 1999], and [Fujii, and Yamaji, 1999]. The strength of the partial equilibrium models is the fact that can easily deal with market liberalization aspects. Their weakness is the lack of dynamics in the path towards achieving the equilibrium. The game-theoretical models explicitly model the investment dynamics but results can be path dependent and the scope of the models (number of players and investments) is limited. The optimization models can deal with technological progress and investments but miss the effects of strategic investment decisions on prices and energy demand, and due to their single optimization objective are less able to deal with the effect of market development.

Against this background, the purpose of this paper is to examine the long-term prospects for gas infrastructure in Central Asia taking into account the effect of strategic behavior as well as the effect of different market arrangements (take-or-pay or liberalized markets). The suggested model explicitly incorporates lumpy investments in individual pipelines and individual countries as separate decision makers. The model differs from optimization models since it incorporates multiple actors which maximize profits taking into account the effect of their actions on prices and gas demand. The model is constructed on the basis of classical micro and macro patterns of mathematical economics (see [Arrow, 1985], [Intriligator, 1971]). It employs methods of financial arithmetics (see [Cartledge, 1993], and operates with constructions of financial mathematics and investments theory [Sharpe et al., 1999]). The model combines four microeconomic levels of dynamic optimization: assessment of the market potential innovation, selection of innovation scenarios, regulation of the future supply and optimization of the current investments, through the price formation mechanism into a macroeconomic system.

The approach expands the existing game-theoretic models (see [Pontryagin et al., 1962], [Krasovskii, Subbotin, 1988]) since it views investment decisions as a dynamic process rather than a yes/no decision making. A crucial decision is the choice of the commercialization time of the project, i.e., the time of finalizing the construction of the pipeline and starting gas deliveries. Elements of optimal timing models (see [Barzel, 1968], [Tarasyev, Watanabe, 2001]) constitute the basis for a reasonable optimization setting in terms of the commercialization time, or, equivalently, the delivery time. A right individual choice of the commercialization time could be the best response to the choices of the other competitors or a reasonable analogue of the best reply dynamics (see [Hofbauer, Sigmund, 1988], [Kryazhimskii, Osipov, 1995]). Accordingly, a collection of commercialization times is suitable to every competitor if and only if the commercialization time of every competitor responds best to the commercialization times of the other competitors. Such situations can be associated with Nash equilibria in the game between the competing gas pipeline projects. The analogous approach based on the best reply dynamics is employed in the paper to model Nash equilibrium of gas deliveries and market gas price. We prove rigorously the existence results for Nash equilibrium of gas deliveries and prices and outline approach for finding Nash equilibrium of commercialization times.

The approach was announced in [Klaassen, Roehrl, Tarasyev, 2001b] and partially tested in [Golovina, Klaassen, Roehrl, 2002]; a formal mathematical background (restricted so far to the case of 2 competing gas pipeline projects) was elaborated in [Klaassen, Kryazhimskii, Tarasyev, 2001c]

The paper has the following structure.

Section 2 describes the model.

Section 3 presents an approach to finding equilibrium commercialization strategies in game-type interactions between pipeline projects.

Section 4 summarizes the data used for the numerical analysis of the Turkey gas market. Section 5 discusses results of numerical simulations. Section 6 concludes the paper.

## 2 Model

### 2.1 Outline of Model

In the suggested model, pipeline projects competing for a gas market are numbered  $1, \dots, L$ . Throughout the paper,  $t$  stands for the time variable.

The life circle of every pipeline project  $i$  comprises the investment period and supply period. The investment period starts at time 0 and terminates (starting the supply period) at the commercialization time  $t_i$  when the accumulated investment  $x_i(t)$  in the project reaches a prescribed commercialization level  $\bar{x}_i$ . The duration of the supply period,  $T_i$ , is prescribed. During the supply period, at every time  $t$  gas of quantity  $z_i(t) \leq M_i$  is supplied through pipeline  $i$ ; here  $M_i$  is the maximum capacity of this pipeline. Supply  $z_i(t)$  is chosen so as to maximize the current profit,  $R_i(t)$ , due to the sales of gas. As other projects follow the same strategy of supply,  $z_i(t)$  constitutes the  $i$ th component of a Nash equilibrium point in the instantaneous “benefit maximization supply game” (see [Klaassen, Kryazhinskii, Tarasyev, 2001c]).

### 2.2 Project’s Parameters and Variables

For each gas pipeline project  $i$  the characteristic constants are  $t_i^0$  - starting time for the investment period,

- $\bar{x}_i^0$  - the starting capital invested at time  $t_i^0$ ,
- $\bar{x}_i > \bar{x}_i^0$  - the level of commercialization capital,
- $\sigma_i$  - rate of obsolescence of accumulated investments,
- $\lambda_i$  - investments discount rate,
- $\gamma_i$  - investments delay coefficient,
- $M_i$  - maximum capacity,
- $T_i$  - lifetime of the pipeline.

The dynamics of project  $i$  is expressed in terms of the following variables:

- $t_i \geq t_i^0$  - commercialization time,
- $r_i(t)$  - current investment at time  $t$ ,
- $x_i(t)$  - accumulated investments at time  $t$ ,
- $W_i(t_i)$  - investment cost,
- $z_i(t)$  - current supply,
- $y_i(t)$  - current accumulated supply,
- $C_i^1(t) = C_i^1(t, y_i(t))$  - current cost of extraction of gas,
- $C_i^2(t)$  - current cost of transportation of gas,
- $C_i^3(t)$  - transit fees,
- $C_i(t)$  - current overall cost of delivering gas to market,
- $R_i(t)$  - current profit due to sales of gas,
- $\Pi_i$  - overall profit.

### 2.3 Project's Dynamics: Investment Period

For each project  $i$ , the investment period starts at the fixed time  $t_i^0$  and terminates at the project's commercialization time  $t_i$  viewed as a changeable control variable. The process of growth of the accumulated investment,  $x_i(t)$ , is modeled by the differential equation

$$\dot{x}_i(t) = r_i(t) - \sigma_i x_i(t) \quad (2.1)$$

with the initial condition

$$x_i(t_i^0) = x_i^0 \quad (2.2)$$

and final condition

$$x_i(t_i) = \bar{x}_i. \quad (2.3)$$

The time-varying investment rate  $r_i(t)$  is chosen so as to minimize the total expenditure

$$E_i(t_i, r_i) = \int_{t_i^0}^{t_i} e^{-\lambda_i t} r_i^{\gamma_i}(t) dt.$$

In [Tarasyev, Watanabe, 2001] it was shown that the minimum expenditure, or *investment cost*, equals

$$W_i(t_i) = \rho^{\alpha_i - 1} \frac{e^{-\lambda_i t_i} (\bar{x}_i - \bar{x}_i^0 e^{-\sigma(t_i - t_i^0)})}{(1 - e^{-\rho_i(t_i - t_i^0)})^{\alpha_i - 1}}, \quad (2.4)$$

where

$$\rho_i = \frac{\alpha_i \sigma_i + \lambda_i}{\alpha_i - 1}, \quad \alpha_i = \frac{1}{\gamma_i},$$

and the minimizing investment rate is given by

$$r_i(t) = \rho \frac{e^{\sigma(t_i - t_i^0)} \bar{x}_i - \bar{x}_i^0}{e^{\rho_i(t_i - t_i^0)} - 1} \quad (t_i^0 \leq t \leq t_i) \quad (2.5)$$

### 2.4 Market's Parameters and Variables

Below, the abbreviation GDP stands for the "Gross Domestic Product of the gas market" and abbreviation LNG stands for "liquid natural gas". The gas market is characterized by the following constants and variables:  $e_y$  - GDP elasticity of gas demand,

$e_p$  - price elasticity of gas demand,

$M_{lng}$  - maximal capacity of LNG terminals,

$G(t)$  - GDP,

$P_{lng}(t)$  - world market price for LNG,

$d(t)$  - current demand of gas,

$z(t)$  - current total supply of gas,

$P_0(t) = P_0(t, z(t))$  - current price of gas, given no competition with LNG;

$P(t) = P(t, z(t))$  - actual current price of gas.

The evolution of the GDP,  $G(t)$ , and price for LNG,  $P_{lng}(t)$ , are formed exogenously.

## 2.5 Price Formation Mechanism

Note that for each pipeline  $i$  the supply period starts at time  $t_i$  and terminates at time  $t_i + T_i$ ; therefore, we set

$$z_i(t) = 0 \text{ if } t \leq t_i \text{ or } t > t_i + T_i. \quad (2.6)$$

Current total gas supply,  $z(t)$ , is the sum of current individual supplies:

$$z(t) = z_1(t) + \dots + z_L(t). \quad (2.7)$$

Current market gas demand,  $d(t)$ , is determined by the current GDP,  $G(t)$ , and the current price of gas,  $P(t)$ :

$$d(t) = AG^{e_y}(t)P^{-e_p}(t);$$

here  $A$  is a demand scale coefficient. The market equilibrium condition,  $z(t) = d(t)$ , gives rise to the price formation mechanism

$$P_0(t) = P_0(t, z(t)) = A^{1/e_p} \frac{G(t)^{e_y/e_p}}{z(t)^{1/e_p}}. \quad (2.8)$$

Taking into account that the price of gas,  $P(t)$ , can not exceed the price of LNG,  $P_{lng}(t)$ ,

$$P(t) \leq P_{lng}(t),$$

we come to the following final formula for the price of gas:

$$P(t) = P(t, z(t)) = \min\{P_0(t, z(t)), P_{lng}(t)\} = \min\left\{A^{1/e_p} \frac{G(t)^{e_y/e_p}}{z(t)^{1/e_p}}, P_{lng}(t)\right\}. \quad (2.9)$$

## 2.6 Instantaneous Supply Game

For pipeline  $i$ , the current overall cost of delivering a unit of gas to market is found as

$$C_i(t) = C_i^1(t, y_i(t)) + C_i^2(t) + C_i^3(t)$$

where the accumulated supply  $y_i(t)$  is subject to

$$\dot{y}_i(t) = z_i(t),$$

and the current profit due to sales of gas is given by

$$R_i(t) = R_i(t, z_1(t), \dots, z_L(t)) = z_i(t)(P(t, z(t)) - C_i(y_i(t))). \quad (2.10)$$

We suggest that at every time  $t$  the collection of current individual gas supplies,  $z_1(t), \dots, z_L(t)$ , forms a Nash equilibrium point in the *instantaneous supply game* between players  $1, \dots, L$ , in which each player  $i$  (responsible for pipeline  $i$ ) maximizes the current profit  $R_i(t) = R_i(t, z_1(t), \dots, z_L(t))$  choosing  $z_i(t)$  between 0 and  $M_i$  provided  $t_i + T_i \geq t \geq t_i$  (recall that  $z_i(t) = 0$  if  $t < t_i$  or  $t > t_i + T_i$ ).

**Proposition 2.1** *For every collection of commercialization times,  $t_1, \dots, t_L$ , and every  $t$  located between the starting time of supply,*

$$t_0 = \min\{t_1, \dots, t_L\},$$

*and the final time of supply,*

$$t^0 = \max\{t_1 + T_1, \dots, t_L + T_L\},$$

*the instantaneous supply game has a Nash equilibrium point.*

**Proof.** Fix an arbitrary  $t \in [t_0, t^0]$ . Recall that an amount  $z_i(t)$  of gas supply is admissible for player  $i$  (at time  $t$ ) if  $z_i(t) \in [0, M_i]$  provided  $t \in [t_i, t_i + T_i]$  and  $z_i(t) = 0$  otherwise. Substituting (2.9) in (2.10), we get

$$R_i(t, z_1(t), \dots, z_L(t)) = \min \left\{ A^{1/e_p} \frac{G(t)^{e_y/e_p} z_i(t)}{z(t)^{1/e_p}} - C_i(y_i(t)) z_i(t), \right. \\ \left. z_i(t) (P_{lng}(t) - C_i(y_i(t))) \right\}.$$

Clearly,  $R_i(t, z_1(t), \dots, z_L(t))$  is concave in  $z_i(t)$ . For every collection of admissible individual supplies,  $(z_1(t), \dots, z_L(t))$ , let  $B_i(z_1(t), \dots, z_L(t))$  be the set of all best reply supplies of player  $i$ , i.e., those maximizing the player's profit  $R_i(t, z_1(t), \dots, z_L(t))$  over all supplies  $z'_i(t)$  admissible for this player. The best reply map associates every collection of admissible individual supplies,  $(z_1(t), \dots, z_L(t))$ , with the product

$$B(z_1(t), \dots, z_L(t)) = B_1(z_1(t), \dots, z_L(t)) \times \dots \times B_L(z_1(t), \dots, z_L(t))$$

of players' best replies. The multi-valued function  $B(z_1(t), \dots, z_L(t))$  map the set  $\Pi$  of all collections  $(z_1(t), \dots, z_L(t))$  of admissible individual supplies into itself. The set  $\Pi$  is a convex compactum in the  $L$  - dimensional Euclidean space  $R^L$ . One can easily show that  $B(z_1(t), \dots, z_L(t))$  is upper semi-continuous (see [Kantorovich, Akilov, 1982], [Rudin, 1991]). Finally, due to the concavity of the profit functions  $R_i(t, z_1(t), \dots, z_L(t))$  in  $z_i(t)$ , the best reply sets  $B(z_1(t), \dots, z_L(t))$  are convex. Thus,  $B(z_1(t), \dots, z_L(t))$  is an upper-semicontinuous convex-valued map of a convex compactum  $\Pi$  into itself. By the Kakutani theorem (see [Kantorovich, Akilov, 1982], [Rudin, 1991], [Kakutani, 1941]). there exists a  $(z_1^*(t), \dots, z_L^*(t))$  such that  $(z_1^*(t), \dots, z_L^*(t)) \in B(z_1^*(t), \dots, z_L^*(t))$ . Clearly,  $(z_1^*(t), \dots, z_L^*(t))$  is a Nash equilibrium point in the instantaneous supply game.

The proposition is proved.

**Remark 2.1** Let  $I(t)$  be the set of the indices  $i$  of all pipelines operating at time  $t$  (i.e. such that  $t \in [t_i, t_i + T_i]$ ), and  $L(t)$  be the number of these pipelines. Set

$$z^*(t) = \left( \frac{AG(t)^{e_y/e_p}}{\sum_{i \in I(t)} C_i} (e_p L - 1) \right)^{e_p}, \quad (2.11)$$

$$z_i^*(t) = e_p z^*(t) \left( 1 - \frac{C_i(t) z^{*(1/e_p)}(t)}{AG(t)^{e_y/e_p}} \right) \quad (2.12)$$

for every  $i \in I(t)$  and  $z_i^*(t) = 0$  for every  $i \notin I(t)$ . One can show that if

$$e_p > \frac{1}{L(t)}, \quad (2.13)$$

$$P(t, z^*(t)) < P_{lng}(t) \quad (2.14)$$

and

$$z_i^*(t) \leq M_i \quad (2.15)$$

for all  $i \in I(t)$ , then  $(z_1^*(t), \dots, z_L^*(t))$  is a Nash equilibrium point in the instantaneous supply game.

**Proof.** Indeed, note, first, that by (2.15) collection  $(z_1^*(t), \dots, z_L^*(t))$  is admissible. By (2.12)

$$\sum_{i=1}^L z_i^*(t) = e_p z^*(t) L(t) - \frac{z^{*(1/e_p+1)}(t) \sum_{i \in I(t)} C_i(t)}{AG(t)^{e_y/e_p}};$$

hence, by (2.11) and by (2.13)

$$\sum_{i=1}^L z_i^*(t) = z^*(t).$$

Assumption (2.14) and expression (2.9) for price  $P(t)$  imply

$$P(t, z^*(t)) = A^{1/e_p} \frac{G(t)^{e_y/e_p}}{z^{*(1/e_p)}(t)}.$$

Substituting this in (2.10), we get

$$R_i(t, z_1(t), \dots, z_L(t)) = A^{1/e_p} \frac{G(t)^{e_y/e_p} z_i(t)}{z(t)^{1/e_p}} - C_i(y_i(t) z_i(t)). \quad (2.16)$$

for all admissible  $(z_1(t), \dots, z_L(t))$  varying in a neighborhood of  $(z_1^*(t), \dots, z_L^*(t))$ . Since  $R_i(t, z_1(t), \dots, z_L(t))$  is concave in  $z_i(t)$ , the equality

$$\frac{\partial R_i(t, z_1^*(t), \dots, z_L^*(t))}{\partial z_i(t)} = 0 \quad (2.17)$$

holding for all  $i \in I(t)$  is sufficient for  $(z_1^*(t), \dots, z_L^*(t))$  to be a Nash equilibrium point. Take an  $i \in I(t)$ . The differentiation of (2.16) brings (2.17) to the form

$$A \frac{G(t)^{e_y/e_p}}{z^{*(1/e_p)}} - C_i - \frac{z_i^*(t) A G(t)^{e_y/e_p}}{z^{*(1/e_p+1)}} = 0 \quad (2.18)$$

Substituting (2.12) we find that (2.18) is true. Thus,  $(z_1^*(t), \dots, z_L^*(t))$  is a Nash equilibrium point in the instantaneous supply game.

## 2.7 Best Reply Policy for Gas Supply Regulation

Generally, there is no clear analytic formula for a Nash equilibrium  $(z_1^*(t), \dots, z_L^*(t))$  in the instantaneous supply game. Therefore, we approximate  $(z_1^*(t), \dots, z_L^*(t))$  by a collection of players' *best reply* supply regulation policies. The best reply supply regulation policy for player  $i$  is modeled as

$$\begin{aligned} \dot{z}_i(t) &= S_i [-\text{sign}(z_i(t)) + \text{sign}(M_i - z_i(t)) + \\ &\text{sign}\left(\frac{\partial R_i(t, z_1(t), \dots, z_L(t))}{\partial z_i}\right)] \\ z_i(t_i) &= 0, \quad (t_i \leq t \leq t_i + T_i). \end{aligned} \quad (2.19)$$

This supply regulation policy prescribes supply  $z_i(t)$  to stay between 0 and  $M_i$ , and grow (respectively, decline) whenever  $\partial R_i(t, z_1(t), \dots, z_L(t))/\partial z_i$  is positive (respectively, negative). The positive parameter  $S_i$  characterizes the ability of player  $i$  to react to changes on market. A solution to each of the differential equations (2.19) with the discontinuous right hand side is understood in the sense of Filippov (see [Filippov, 1988]).

**Remark 2.2** If all  $S_i$  are sufficiently large then, typically, a collection of  $z_i(t)$ 's driven by (2.19) lies close to a Nash equilibrium  $(z_1^*(t), \dots, z_L^*(t))$ . Sufficient conditions for this are given in Remark 2.1. Here, we omit a mathematical justification for this fact and just view (2.19) as a rational individual gas supply policy intended to approximate the  $i$ th component,  $z_i^*(t)$ , of a Nash equilibrium point,  $(z_1^*(t), \dots, z_L^*(t))$ , in the current instantaneous gas supply game.

### 3 An Approach to Equilibrium Commercialization

#### 3.1 Overall Profits, Individual Best Replies and Equilibrium Commercialization Times

For each player  $i$  projects' commercialization times,  $t_1, \dots, t_L$ , determine the player's overall profit,  $\Pi_i$ , by

$$\Pi_i = \Pi_i(t_1, \dots, t_L) = -W_i(t_i) + \int_{t_i}^{t_i+T_i} e^{-\lambda t} R_i(t, z_1(t), \dots, z_L(t)) dt; \quad (3.20)$$

here  $W_i(t_i)$  and  $R_i(t_i)$  are given by (2.4) and (2.10), and  $z_1(t), \dots, z_L(t)$  are modeled by (2.19), (2.6). A commercialization time  $t_i^*$  of pipeline  $i$  is a *best reply* of player  $i$  to commercialization times  $t_1, \dots, t_{i-1}, t_{i+1}, \dots, t_L$  of other pipelines if  $t_i^*$  maximizes  $\Pi_i$  over all  $t_i > t_i^0$ :

$$\Pi_i(t_1, \dots, t_{i-1}, t_i^*, t_{i+1}, \dots, t_L) = \max_{t_i > 0} \Pi_i(t_1, \dots, t_{i-1}, t_i, t_{i+1}, \dots, t_L).$$

A collection  $t_1^*, \dots, t_L^*$  of pipelines' commercialization times constitutes a *Nash equilibrium* if for every player  $i$ ,  $t_i^*$  is a best reply of this player to the commercialization times  $t_1^*, \dots, t_{i-1}^*, t_{i+1}^*, \dots, t_L^*$  of pipelines  $1, \dots, i-1, i+1, \dots, L$ .

#### 3.2 Best Reply Policy for Investment Regulation

An individual best reply investment policy described in this subsection presents a model for individual reviewing current investment plans basing on a currently available information. In the next subsection we will show that the collection of individual best reply investment policies provides a tool to finding current Nash equilibrium commercialization times.

We assume that during the investment period, each player  $i$  updates his/her current investments basing on the current information on the projects' commercialization times. Suppose player  $i$  makes his/her investment updates at times

$$\tau_i^0 = t_i^0, \tau_i^1 = t_i^0 + \delta, \tau_i^2 = \tau_i^1 + \delta, \dots,$$

with a small time step  $\delta$ . At each time  $\tau_i^k$  player  $i$  observes the current projected commercialization times of all projects,  $t_1 = t_1(\tau_i^k) = t_1^k, \dots, t_L = t_L(\tau_i^k) = t_L^k$ , and changes his/her own projected commercialization time  $t_i = t_i(\tau_i^k) = t_i^k$  to  $t_i = t_i(\tau_i^{k+1}) = t_i^{k+1}$ . In order to find a  $t_i^{k+1}$ , player  $i$  analyzes his/her profit subsequent to  $\tau_i^k$ ,

$$\Pi_i^k(t_i) = -W_i^k(t_i) + \int_{t_i}^{t_i+T_i} e^{-\lambda t} R_i(t, z_1^k(t), \dots, z_{i-1}^k(t), z_i(t), z_{i+1}^k(t), \dots, z_L^k(t)) dt, \quad (3.21)$$

as a function of a (changeable) projected commercialization time  $t_i \geq \tau_i^k$ . In (3.21)  $W_i^k(t_i)$ , is the investment subsequent to  $\tau_i^k$ ; a formula for  $W_i^k(t_i)$  is obtained from (2.4), if one replaces the initial time  $t_i^0$  and initial investment  $\bar{x}_i^0$  in (2.4) by the current time  $\tau_i^k$  and current investment

$$\bar{x}_i^k = x_i(\tau_i^k), \quad (3.22)$$

respectively:

$$W_i^k(t_i) = \rho^{\alpha_i-1} \frac{e^{-\lambda t_i} (\bar{x}_i - \bar{x}_i^k e^{-\sigma(t_i-\tau_i^k)})}{(1 - e^{-\rho_i(t_i-\tau_i^k)})^{\alpha_i-1}}. \quad (3.23)$$

The individual supply policies  $z_i(t)$  and  $z_j(t) = z_j^k(t)$ , ( $j \neq i$ ) in (3.21) are modeled by (2.19), (2.6) with  $t_j = t_j^k$ , ( $j \neq i$ ). As a result of the analysis of  $\Pi_i^k(t_i)$ , player  $i$  chooses a

new projected commercialization time,  $t_i^{k+1}$ , as a maximizer to  $\Pi_i^k(t_i)$ , or a best reply to the currently projected commercialization times of other players:

$$\Pi_i^k(t_i^{k+1}) = \max_{t_i \geq \tau_i^k} \Pi_i^k(t_i).$$

Finally, basing on the new projected commercialization time  $t_i^{k+1}$ , player  $i$  determines his/her investment plan  $r_i^k(t)$  for the period between  $\tau_i^k$  and  $\tau_i^{k+1}$ ; namely,  $r_i^k(t)$  is set to be the cost minimizing investment plan for the current time  $\tau_i^k$  and current accumulated investment  $\bar{x}_i^k$  (see (3.22)); a formula for  $r_i^k(t)$  is obtained from (2.5) if one replaces the initial time  $t_i^0$  and initial accumulated investment  $\bar{x}_i^0$  in (2.5) by  $\tau_i^k$  and  $\bar{x}_i^k$ , respectively:

$$r_i^k(t) = \rho \frac{e^{\sigma(t_i - \tau_i^k)} \bar{x}_i - \bar{x}_i^k}{e^{\rho_i(t_i - \tau_i^k)} - 1} \quad (\tau_i^k \leq t \leq \min\{\tau_i^{k+1}, t_i^{k+1}\}).$$

This finalizes the description of the *best reply investment policy* for player  $i$ .

### 3.3 Finding Equilibrium Commercialization Times

Assume that each player applies the best reply investment updating policy described in the previous subsection.

In what follows, we use the “universal” sequence of investment update instances,  $\tau^0, \tau^1, \dots$ :

$$\tau^0 = \min_{i=1, \dots, L} \tau_i^0 = \min_{i=1, \dots, L} t_i^0, \quad \tau^{k+1} = \tau^k + \delta.$$

We also assume that for any  $i$  we have

$$t_i^0 - \tau^0 = m_i \delta$$

with some integer  $m_i$ , or, equivalently,

$$t_i^0 = \tau^{m_i}.$$

Note that if  $\delta$  is sufficiently small, this assumption practically does not restrict us in generality. Then, clearly,

$$\tau_i^k = \tau^{m_i + k}.$$

Accordingly, we enumerate the projected players’ commercialization times  $t_i^k$ , introducing

$$\bar{t}_i^{m_i + k} = t_i^k, \quad \bar{t}_i^j = t_i^0 \quad (j = 0, \dots, m_i - 1).$$

At every current investment updating time  $\tau^k$ , the subsequent profit for player  $i$  is given by

$$\Pi_i^k(t_1, \dots, t_L) = -W_i^k(t_i) + \int_{t_i}^{t_i + T_i} e^{-\lambda t} R_i(t, z_1(t), \dots, z_L(t)) dt$$

(see (3.20)) where  $W_i^k(t_i)$  is defined by (3.23),  $R_i(t_i)$  is given by (2.10), and  $z_1(t), \dots, z_L(t)$  are modeled by (2.19), (2.6). A commercialization time  $t_i^{*k}$  of pipeline  $i$  is a *best reply* of player  $i$  to commercialization times  $t_1, \dots, t_{i-1}, t_{i+1}, \dots, t_L$  at time  $\tau^k$  if  $t_i^{*k}$  maximizes  $\Pi_i^k$  over all  $t_i > \tau^k$ :

$$\Pi_i^k(t_1, \dots, t_{i-1}, t_i^{*k}, t_{i+1}, \dots, t_L) = \max_{t_i > \tau^k} \Pi_i^k(t_1, \dots, t_{i-1}, t_i, t_{i+1}, \dots, t_L).$$

A collection  $t_1^{*k}, \dots, t_L^{*k}$  of pipelines’ commercialization times constitutes a *Nash equilibrium at time  $\tau^k$*  if for every player  $i$ ,  $t_i^{*k}$  is a best reply of this player at time  $\tau^k$  to the commercialization times  $t_1^{*k}, \dots, t_{i-1}^{*k}, t_{i+1}^{*k}, \dots, t_L^{*k}$  of pipelines  $1, \dots, i-1, i+1, \dots, L$ .

The following proposition can easily be proved.

Table 1: Overview of gas routes in the model.

	Regions	Comments
Supply	Russia	Russian, West Siberian and Caspian gas fields. The huge East Siberian gas amounts will be treated in a future extension of the model.
	“Central Asian Producers” (CAP)	Turkmenistan, Uzbekistan, Kazakhstan
	LNG from World Market	Algeria, Egypt, Libya, Nigeria
Demand	Turkey	Transition of gas to Western and Eastern Europe
Transit countries	Azerbaijan, Armenia, Georgia, Iran, Russia	Possible gas routes through the Ukraine

**Proposition 3.2** *Let each player apply the best reply investment policy and  $\bar{t}_i^0, \bar{t}_i^1, \dots$  be the associated sequence of updated commercialization times for player  $i$  ( $i = 1, 2, \dots, L$ ). If all these sequences stabilize at some step  $m$ , i.e.,  $\bar{t}_i^j = \bar{t}_i^m$  for all  $j = m+1, m+2, \dots$  and for all  $i = 1, 2, \dots, L$ , then  $\bar{t}_1^m, \dots, \bar{t}_L^m$  is a Nash equilibrium collection of commercialization times at every time  $\tau^j$  with  $j \geq m$ .*

Proposition 3.2 shows that the collection of individual best reply investment policies provides a tool to finding current Nash equilibrium commercialization times.

**Remark 3.3** If initial times  $t_1^0, \dots, t_L^0$  are close to each other and, for a stabilization iteration  $m$ ,  $\tau^m$  lies close to all of them, then Nash equilibrium commercialization times  $\bar{t}_1^m, \dots, \bar{t}_L^m$  at time  $\tau^m$  provide a reasonable approximation to seek Nash equilibrium times  $t_1^*, \dots, t_L^*$ .

## 4 Data for Numerical Analysis

### 4.1 Overview of Turkey Gas Market

The purpose of the first application of the model is to study the investment scenarios into gas pipeline routes to Turkey. The time horizon of the model is 1998-2050.

The future versions of the model may consider extensions to alternative demand gas markets such as China, India, Japan, and Western Europe. Another extension of the model may infer that a part of gas supply to Turkey can be exported to Western and Eastern Europe. Table 1 provides an overview of possible gas pipeline routes for the case study of the energy market in Turkey.

### 4.2 Gas Supply

Masters and Turner [1998] provide an illustration of the geographical location of gas fields in the Caspian Sea region. Russia and Iran (Iran holds 15% of the world’s gas reserves, see [EIA, 1999, 2000]) hold truly gigantic amounts. Amounts in Turkmenistan and Kazakhstan are much smaller and are of the similar order as the North Sea Gas Reserves.

Many energy-systems models (e.g., MESSAGE, see [Strubegger, and Messner, 1995]) use cumulative gas supply cost curves. The expected cost of gas extraction depend on the cumulative amount of extracted gas. We follow (just as in MESSAGE) the classification

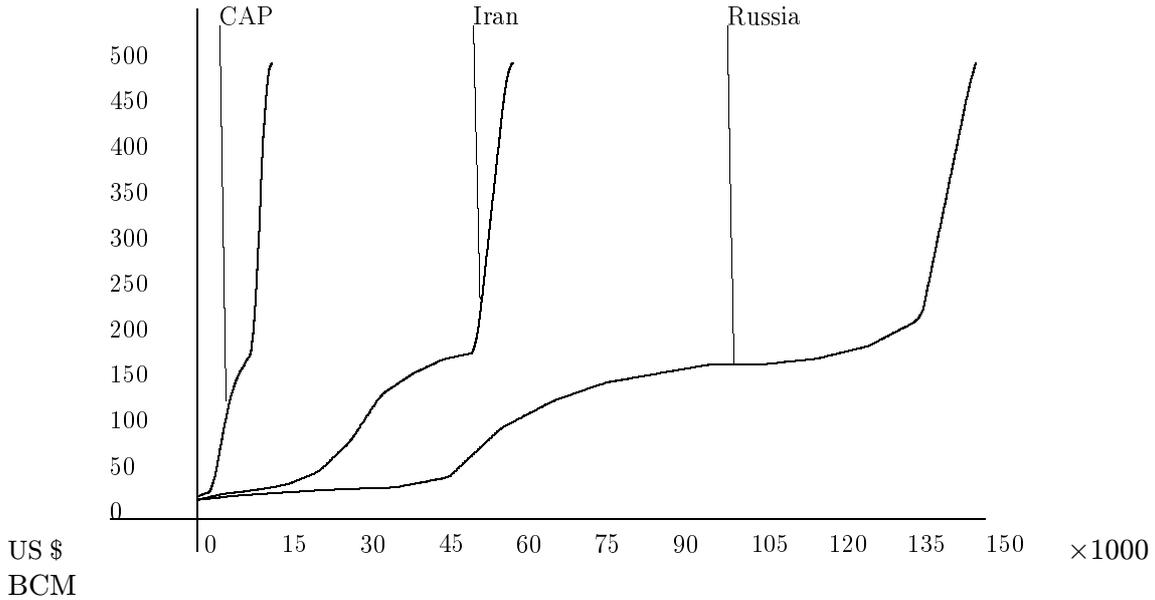


Figure 1: Cumulative gas supply cost curves.

method developed in [Rogner, 1997]. He distinguishes eight different cost categories for gas extraction according to their economic and technical feasibility. Technological progress in gas extraction continuously transforms higher categories into lower categories.

In our analysis we use the regional data from the World Energy Assessment Report (see [WEA, 2000]), This data distribution applied to natural gas categories I-VI generates cumulative gas supply functions indicated on Figure 1.

The model infers assumptions concerning the exogenous gas demand in Eastern Europe, Western Europe, Russia, Iran, Turkmenistan and Kazakhstan. These assumptions are based on recent global long-term energy scenarios (see [Riahi, and Roehl, 2000]) prepared for IPCC (Intergovernmental Panel on Climate Change). Such exogenous functions for the model simulation are shown on Figure 2.

As to LNG supply, it is assumed that the maximal capacity of LNG terminals is large enough to satiate the demand of the energy market in Turkey. The exogenous world market LNG price is derived from the IIASA-B2 scenario (see [Roehrl, 2000]) and its dynamics is presented at Figure 3. LNG supply in 2000 is assumed to be 2.8 bcm/year (billions of cubic meters).

Domestic gas production in Turkey is assumed to be constant at the current level of 0.2 bcm per year (see [EIA, 1999, 2000]).

### 4.3 Gas Demand

Figure 4 shows natural gas consumption and production in the 1990s for the examined regions. The difference between production and consumption constitutes the net exports. One can observe the strong dominance of Russia both in gas consumption and export. The huge gas resources based in Russia in comparison with the resources of CAP countries induce the dominance of the Russian gas export at the Euroasian gas markets in the long-term perspective.

Demand for natural gas in Turkey is projected to quadruple within 20 years, and several independent estimates foresee Turkey's annual consumption reaching 1.4 tcf (trillions of

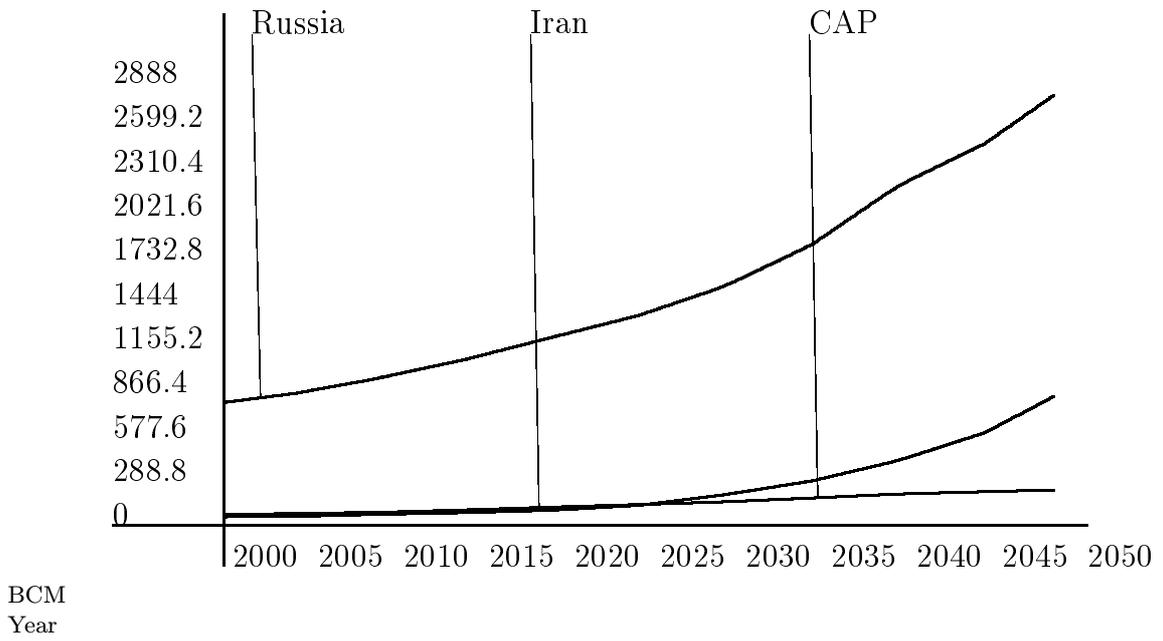


Figure 2: Exogenous gas demand functions.

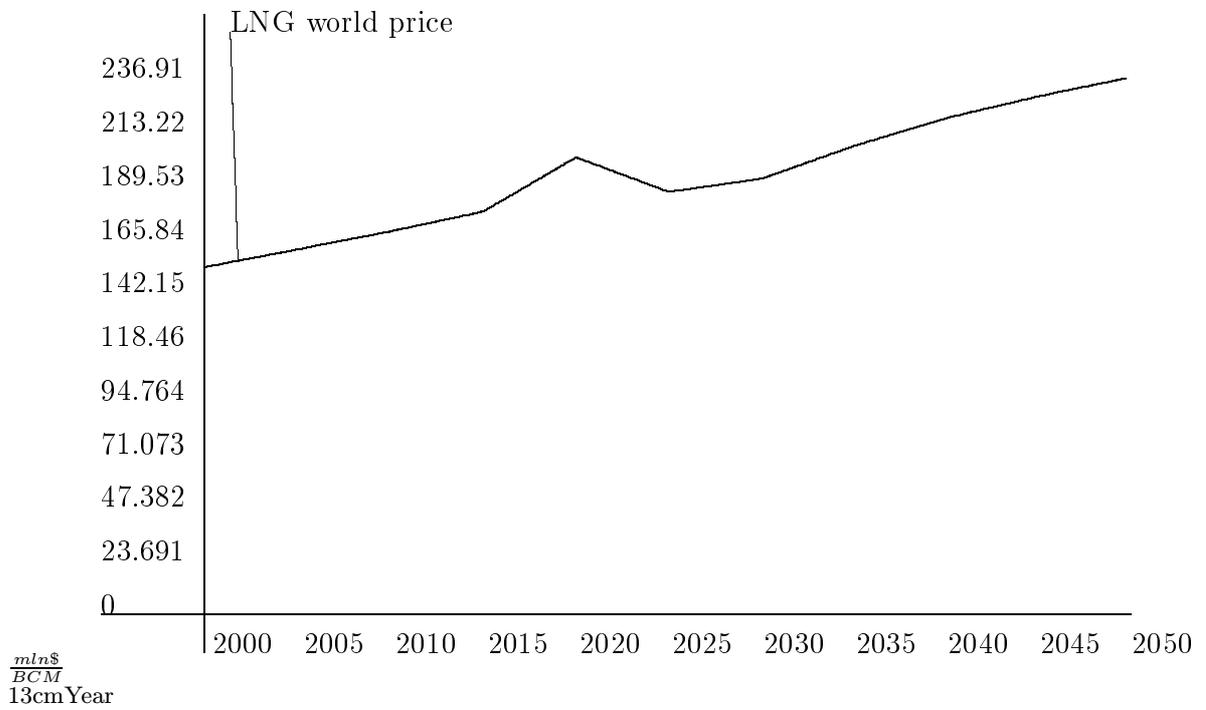


Figure 3: Dynamics of LNG world market price.

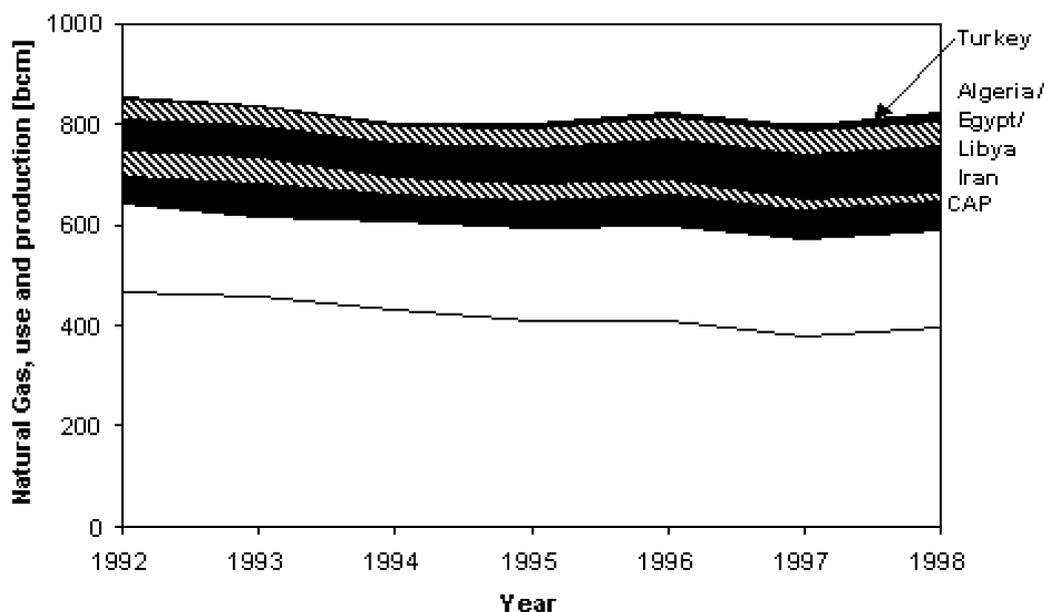


Figure 4: Natural gas consumption and production in 1990s.

cubic feet) by 2020. Gas consumption in Turkey has risen from 1.1 bcm in 1988 to 9.9 bcm in 1998. In 1998 6.8 bcm were imported from Russia through existing pipelines, 3.0 bcm were imported from Algeria in the form of LNG import. Domestic production was around 0.2 bcm in 1999 (see [EIA, 1999, 2000]) or 2.8% of domestic consumption. Gas consumption is projected to increase to 25.2 bcm in 2010 and to 165.2 bcm in 2020 (see [EIA, 1999, 2000]).

We assume in the model that the gas demand depends on GDP (national income) and the price of gas. Typical price elasticity of the gas demand is around -0.7 for the industry in Western Europe. In the paper [Golombek et al., 1995] the aggregated price elasticity covers industrial sectors and markets in 6 countries and is estimated around -0.93. In the paper [Brekke et al., 1987] the authors used price elasticity of -0.7 and income elasticity of 0.8 for countries in Western Europe. Preliminary data for Turkey shows that price elasticity and income elasticity for Turkey can be estimated at quite different levels than in Western Europe:<sup>1</sup> price elasticity is around -0.12 and income elasticity is around 2.93. In other words, the gas demand is driven mainly by GDP growth and is practically insensitive to price changes. That is not surprising for an emerging economy with a weak energy infrastructure and rapidly growing energy demand. The fastest growth of gas consumption in both developed and developing countries is expected in the power industry, which scales perfectly with GDP growth (see, e.g., [Roehrl, 2000]). Note that the growth of electricity consumption in Turkey over the next 15 years is expected to reach at least 8% per year. Currently, 4.2 GW of gas-fired capacity is seeking investment (mainly according to the Turkish “Build-Operate-Transfer”-Model). Diffusion of gas demand to other sectors of Turkish economy is also expected. Taking into account the most probable liberalization of Turkish markets, convergence to price elasticity and income elasticity of today’s Western European economy might be expected.

Figure 5 shows GDP projection for the period from 1998 till 2050 for the considered

<sup>1</sup>Note that the average gas price for Turkey in 1998 was about 170 US\$/1000 cm.

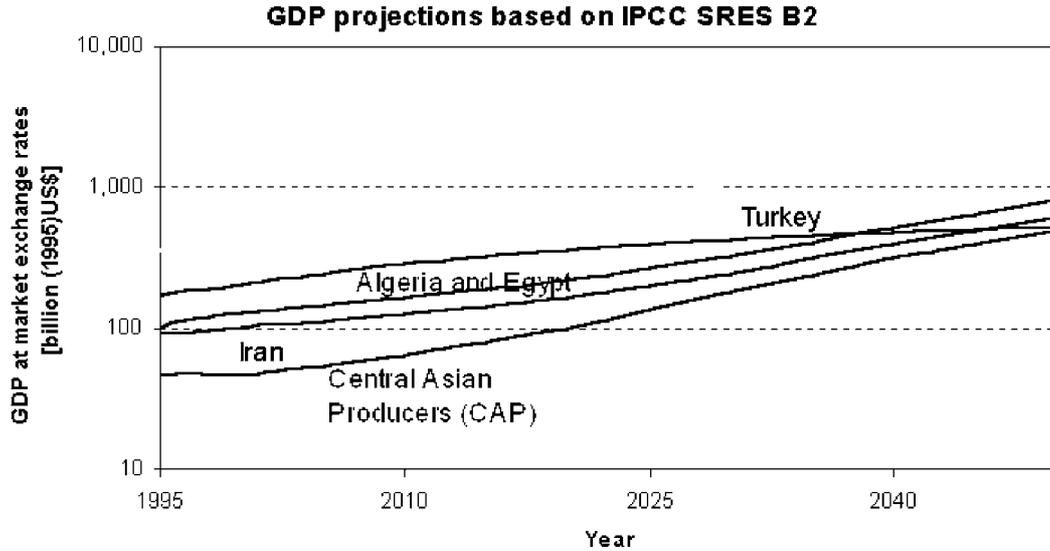


Figure 5: GDP projection for the period 1998-2050.

regions based on scenario *B2* from the new energy scenarios catalogue developed for IPCC (see [Riahi, and Roehrl, 2000]). Scenario *B2* is a “dynamics-as-usual” scenario, which follows a central path of scenarios set. The same trend will be most likely true for gas demand.

## 4.4 Gas Pipelines

### 4.4.1 Overview

Exporting through the Russian gas pipeline system was the unique option available for the Caspian region until 1997. Although over 2 tcf of Caspian-region gas was exported via this system in 1990, exports fell down to 0.3 tcf in 1997 due to disputes between Turkmenistan gas companies and their competitor – the Russian gas company Gazprom, who owns gas pipelines. In 1999, Turkmenistan gas companies and Gazprom came to an agreement allowing Turkmenistan to resume gas exports to the Ukraine. Turkmenistan is also developing an alternative export route by constructing a new pipeline from Ekarem (Turkmenistan) to the Iranian border. Limited export through this pipeline started in 1997; the ultimate capacity of the pipeline would be 0.5 tcf.

It is planned to construct several new pipelines. Most of the proposals call for pipelines with a capacity of 1 tcf each. Three or more pipelines could be built to satiate the gas demand depending on the extent to which the existing Russian system will supply gas to European markets. One can mention “Blue Stream” of Russian Gazprom, “Trans-Caspian” pipeline from Turkmenistan and “Trans-Iranian” pipeline among the proposed new pipelines bringing gas to Turkey which are in the most advanced planning stage. These competing pipelines can transport about 3 tcf of natural gas annually to Turkey and European markets.

## 4.5 Existing and Planned Pipeline Projects

Supply to Turkey can come from Russia, Iran, Turkmenistan, Kazakhstan, Azerbaijan, as well as from Egypt and Algeria. This Section provides a brief overview of the major proposed pipeline projects (see Figure 6).

### **Gas Export Projects to Turkey.**

#### a) Russia-Ukraine-Bulgaria-Turkey (existing pipeline “Bulgarpipe”).

Through this existing pipeline, 280 bcf were delivered to Turkey in 1998. In December 1997 another 25 year deal was signed between Gazprom and Turkey to deliver 500 bcf annually by early 2005. To deliver these large amounts either large investments in the existing pipeline route and contracts with transit countries are needed or new pipeline connections will be built, e.g. “Blue Stream”.

#### b) “Blue Stream”.

This project proposes a direct connection between Russia and Turkey under the Black Sea (to avoid trouble with transit countries). This 758 miles long connection would be the deepest underwater gas pipeline in the world. It is a technically challenging project involving huge investments – 4 US \$ billion.

#### c) Turkmenistan into Iran (existing pipeline).

This short connection (only about 190 mill. US \$) already exists. Gas export to Iran began in late 1997 (141 bcf). A capacity extension to 283 bcf is being planned for 2006 together with an extension to Teheran.

#### d) SWAP deal between Turkmenistan, Iran and Turkey.

Example c) may become a part of a large-scale SWAP deal between Turkmenistan, Iran and Turkey. This plan would require three new pipelines to Turkey (46 inches each). Although economically reasonable, it faces fierce US opposition which is targeted to continue to isolate Iran in the region. Supporters of the SWAP deal however claim that it would not violate the American Iran-Libya Sanctions Act (ILSA) because Iran would only receive transit fees for moving gas to Turkey rather than exporting gas.

#### e) Agreement between Turkey and Iran.

In 1996, Iran and Turkey signed a 20 \$ - billion agreement that commits Iran to provide Turkey with natural gas over a period of 22 years. Export of Iranian gas to Turkey was slated to start in 1999 at initial rate of 300 mmcf/d (millions of cubic feet per day) and then it is planned to rise rate to a level of 1,000 mmcf/d in 2005. In November 1998, Turkey began construction of a 623-mile pipeline that could transport gas westward from Iran. In January 2000 Iran announced that it accepted Turkey’s request to delay the purchase of Iranian natural gas until September 2001. Turkey declared that it had been unable to complete its portion of the pipeline due to economic problems. It should be noted that a part of the pipeline that runs on the Iranian territory has been completed (see [Ignatius, 2000]). Construction on the Turkish side of the 160 mile of the pipeline routing from the Iranian border to Erzurum has been slowed.

#### f) US priority: “Trans-Caspian” Gas Pipeline (TCGP).

For obvious geo-political goals the US government pushes the TCGP project. This would be a 1050 miles gas pipeline from Turkmenistan underneath the Caspian Sea to Baku, through Azerbaijan and Georgia to Turkey for approximately 2.5 billion US \$. The first agreement was signed in May 1999 to deliver 565 bcf gas by 2002. A part of these

large amounts are assigned to be exported further to Europe. In parallel with the US plans this route is also a Turkish priority. However, recently a new large natural gas field was found in Azerbaijan which makes this potential transit country a direct competitor for Turkmenian gas export. It should be noted that Iran strongly objects to this gas route on “environmental” grounds and is exerting pressure on Turkmenistan.

g) North Caucasian-Transcaspian Gas Pipeline (NCTGP).

Pipeline NCTGP provides a link by the route Georgia-Armenia-Turkey passing the Russian pipeline in the Caspian region. Deliveries through this pipeline were cut off in 1997 due to lack of payments by Georgia for gas deliveries. Furthermore, the link from Georgia to Armenia was destroyed during the war in 1995. Investments of only 250 million US \$ would be needed to upgrade the pipelines and supply 425 bcf/year of gas to Turkey. Due to US pressure, Royal Dutch/Shell Company has recently switched from this project to the TCGP project. Consequently Gazprom now favors the much more expensive “Blue Stream” project.

h) LNG ports planned in Kazakhstan, Turkmenistan, Baku, and Georgia.

With the intention to export LNG through the Black Sea to Europe a number of LNG ports are being planned in the region. The planned investments constitute the amount of about 250 million US \$.

i) *Memorandum of Understanding (MOU) signed between Turkey and Egypt.*

In June 1998 a Memorandum of Understanding (MOU) was signed between Turkey and Egypt to build several objects:

- Offshore pipeline under the Mediterranean to deliver Egyptian gas to Gaza, Israel, Egypt, Lebanon, Syria, and to the South-East of Turkey.
- LNG ports to deliver Egyptian LNG to Izmir. This project would require to construct a liquefaction terminal with construction cost of 1.2 billion US \$.

Apart from these projects a number of other important projects could be taken into account in a future model extension, e.g.:

- China pipeline: 5000 miles from Turkmenistan to China with a possible extension to Japan, capacity - 1 tcf/year, cost - 8.5 billion US \$.
- Centgas: Pipeline from Turkmenistan via Afganistan to Pakistan and probably further to India; length - 900 miles, cost - 2-2.5 billion US \$.
- Pipelines directed to the South of Iran with LNG terminals to export LNG to Asian countries including Japan.
- Pipeline Iran-Pakistan-India for which MOU was signed in the spring 2000.

Construction cost of new pipelines is assumed to be a function of construction time. Figure 7 shows graphs of these functions.

An overview of pipelines projects (both oil and gas) in the Caspian region can be found on the Internet site <http://www.eia.doe.gov> (source: [EIA, 1999, 2000]).

Table 2 presents the cost estimations and capacity data for pipelines which has been actually used in the model simulations.

Finally, Figure 8 provides an overview of the main existing pipeline network to deliver gas from the countries of the Former Soviet Union (mainly Russia, but also Kazakhstan and Turkmenistan) to Western and Eastern Europe.

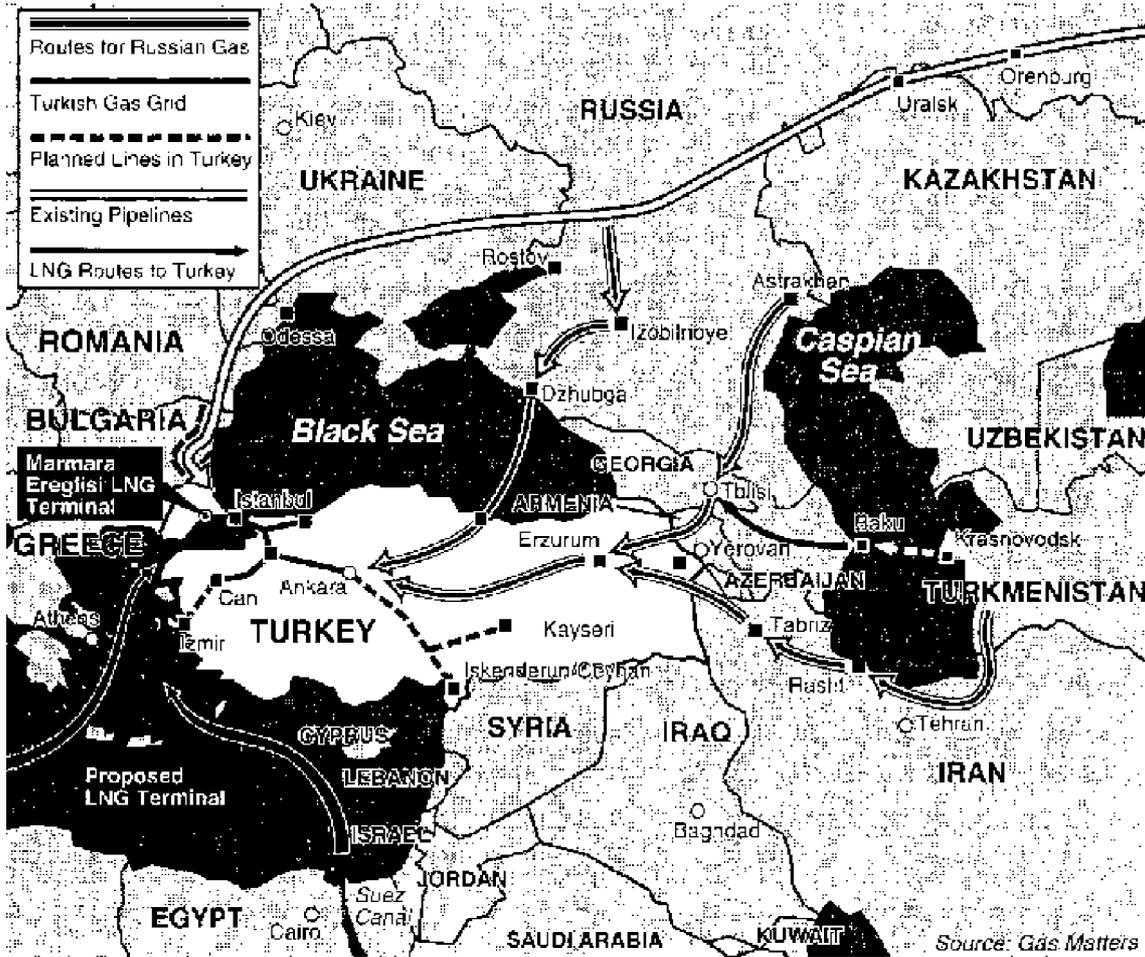


Figure 6: Gas Pipeline Routes to Turkey (source: Gas Matters).

Table 2: Cost estimations and capacities for major projects.

Location	Name	Year of start	Gas capacity [bcm]	Length [miles]	Inv. Low Est. [mln US\$]	Inv. high est. [mln US\$]	O&M <sup>2</sup> costs US\$ / bcm	Transit fees US\$ / bcm
f) TCGP	Trans-caspian	2002	31.15	1696	2000	3000	8.0	16.9
e) Iran - Turkey	Iranpipe	2010	28	2400	3900	4100	30.0	0
d) Turkmenistan-Iran - Turkey	SWAP	2009	28.3	2172	3800	4000	13.8	21.6
b) Russia-Turkey	Blue Stream	2003	14.16	1220	4000	6000	14.1	0
a) Russia-Balkan-Turkey	Bulgarpipeline	exists	10.2	3500	exists	2500	30	10

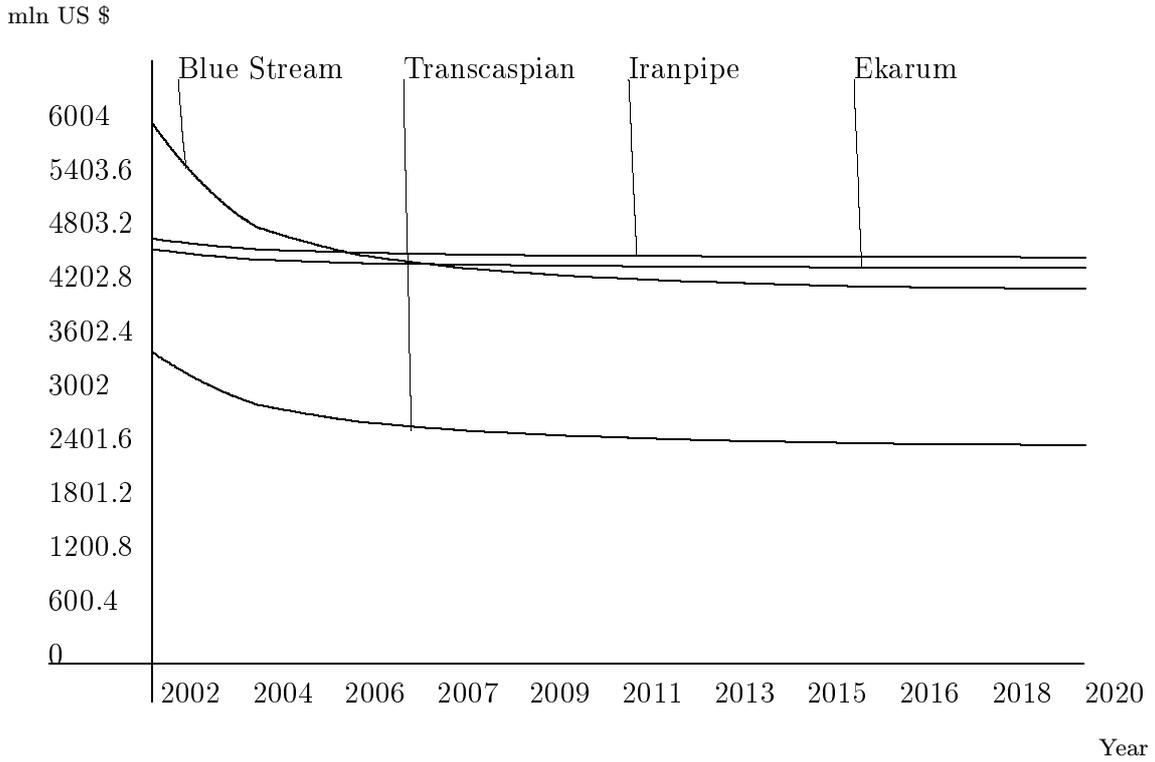


Figure 7: Costs of pipeline construction as a function of construction time.

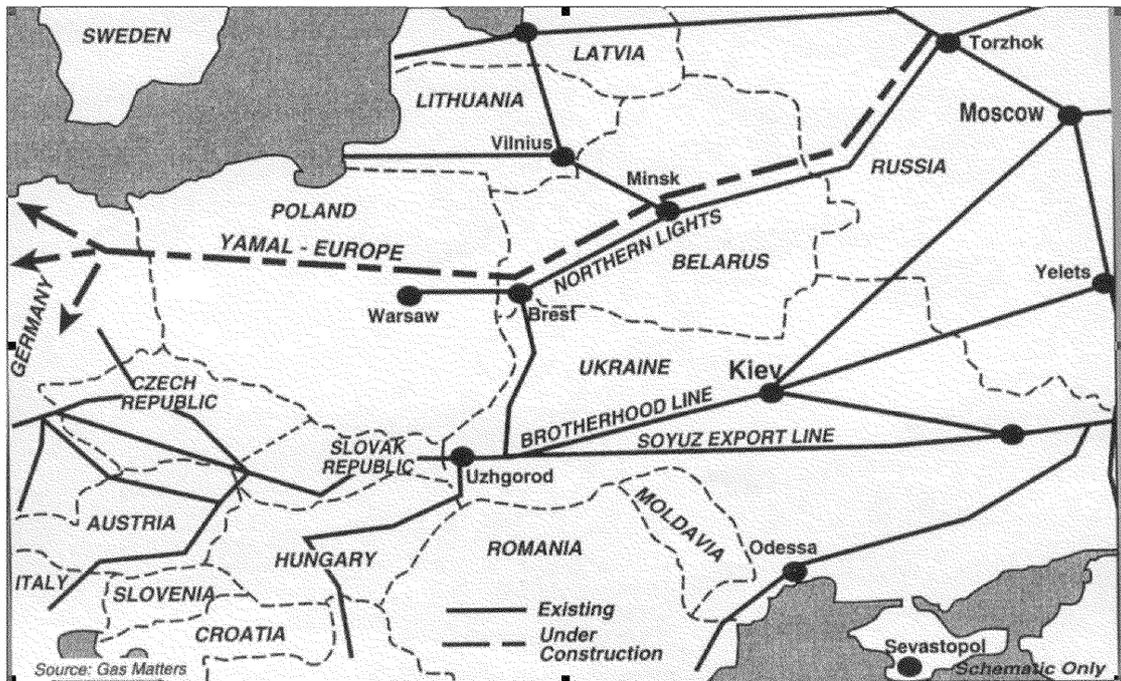


Figure 8: Main export pipelines from the countries of the Former Soviet Union (FSU) to Western and Eastern Europe (source: Gas Matters).

It is necessary to stress that geo-political aspects seem to be of the same importance as economic and financial issues for pipeline projects in the Caspian region. Other risks include uncertainties which can arise due to non-payments for gas deliveries, as well as due to unresolved legal issues concerning maritime boundaries in the Caspian Sea. Is the Caspian Sea an inland lake or is it governed by the International Law of the Sea? The legal status of an inland lake would support joint development projects. However, if the Caspian Sea is governed by the International Law of the Sea, the law of maritime boundaries would be applied to pipeline projects.

## 5 Results of Numerical Simulations

### 5.1 Basic Scenarios

On the basis of the described model a software package has been developed for numerical analysis of investments into gas pipeline projects. The package allows to analyze different scenarios of development of gas markets, to estimate optimal profits of investments and to implement the sensitivity analysis with respect to model parameters.

This section presents the data and results for a number of scenarios when the model is applied to gas pipeline projects in the Caspian region. Specifically, we focus on the international gas transmission routes from Russia, Iran and Turkmenistan to the rapidly expanding gas market in Turkey.

Following discussions of section 4, two alternative assumptions about the value of price elasticity at the gas market in Turkey is analyzed:

- case A: Price elasticity  $e_p = -0.7$ .
- case B: Price elasticity  $e_p = -0.3$ .

For both cases we consider the following scenarios.

- Scenario 1: Nowadays plans of timing and construction of gas pipelines.
- Scenario 2: Time optimization for starting supply through “Blue Stream”.
- Scenario 3: Time optimization for starting supply through “Transcaspian”.
- Scenario 4: Game optimization of profits for each player at all stages of the model: investments, commercialization, supply.

Model simulations show that pipelines run not at the maximal capacity for major period of their lifetimes. The question of optimizing maximal capacity of the planned pipelines is analyzed in the following scenarios.

- Scenario 6: Optimization of maximal capacity for “Transcaspian”.
- Scenario 5: Optimization of maximal capacity for “Blue Stream”.

### 5.2 Case A, Scenario 1: Analysis of Nowadays Plans of Pipeline Projects

Let us consider a scenario in which nowadays plans (see Table 2) of pipelines construction with the announced investments, commercialization times, supply capacities are analyzed in order to estimate net present values (NPVs) of the projects. The results of this analysis are shown on Figures 9,10 and 11. Figure 9 demonstrates that all gas pipelines except the

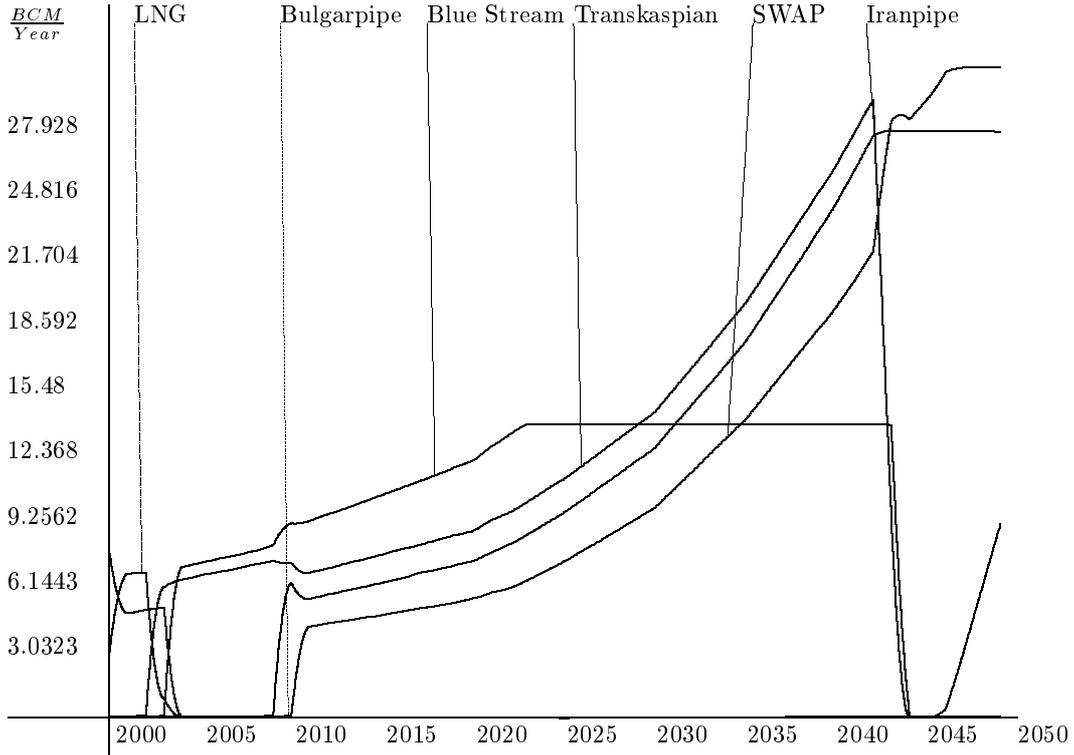


Figure 9: Case A, Scenario 1: Gas supply (bcm/year).

“Blue Stream” will operate at a lower capacity than the maximal capacity. Figures 10 displays graphs of NPVs for gas pipeline projects. The dynamics of gas price is depicted on Figure 10.

We use the following data in this scenario.

“Blue stream” delivery time	2003
“Transcaspian” delivery time	2002
“SWAP” delivery time	2010
“Iranpipe” delivery time	2009
Price elasticity $e_p$	-0.7
GDP elasticity $e_y$	1.25
Demand scale $A$	400

Figure 9 elucidates the following effects. First, LNG supply becomes unprofitable and disappears as soon as the “Transcaspian” pipeline starts its gas supply. Second, completion of the “Blue Stream” pipeline makes it unprofitable for Russia to supply gas through “Bulgarpipe”. Third, in 2020 gas deliveries of “Blue Stream” will reach the maximal capacity, but other new pipelines will not reach their maximal capacities, and their supply will be increasing gradually due to growth of the market demand not reaching the maximal scheduled capacity during their lifetimes.

Figure 10 depicts dynamics of net present values of gas pipeline projects. The analysis shows that investments into “Blue Stream” will be paid back in 2022. For “Transcaspian” the payback time is 2015, for “Ekarum” – 2035, for “Iranpipe” – 2027. “Ekarum” and “Iranpipe” projects will bring profits when lifetimes of “Blue stream” and “Transcaspian” will expire. But this would happen only if during such a long time period no new pipelines would be constructed to replace the old gas routes.

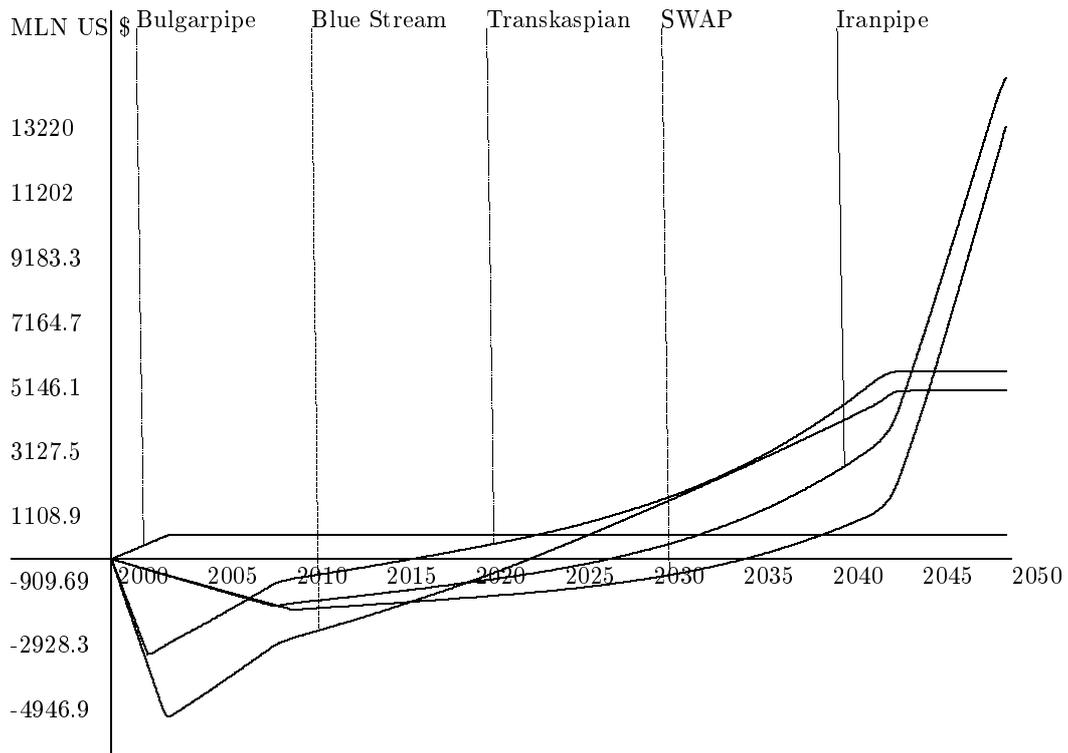


Figure 10: Case A, Scenario 1: Net present values of gas pipeline projects.

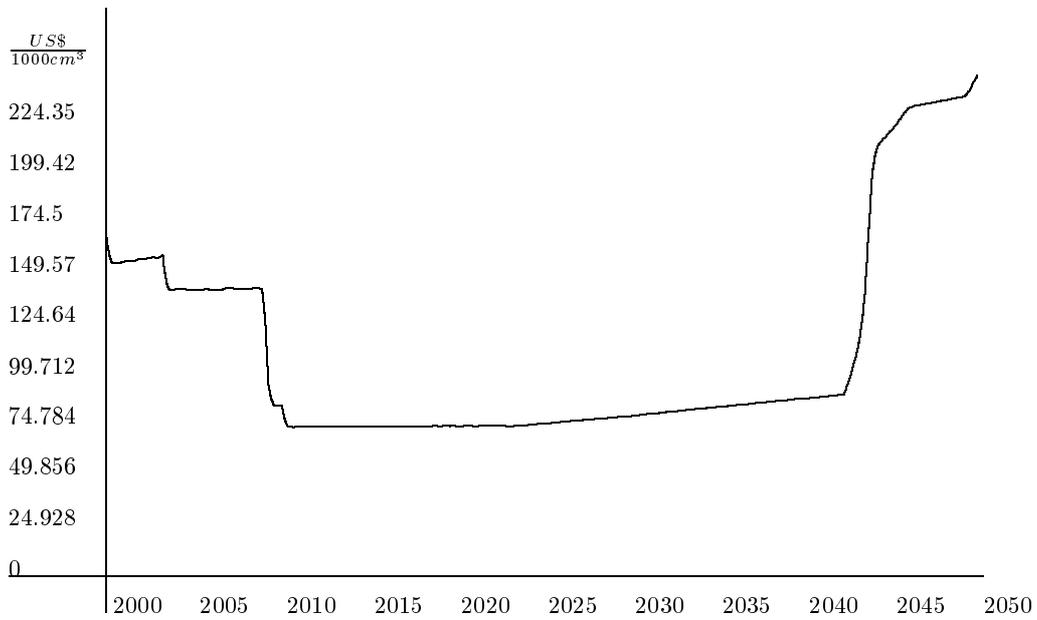


Figure 11: Case A, Scenario 1: Gas price.

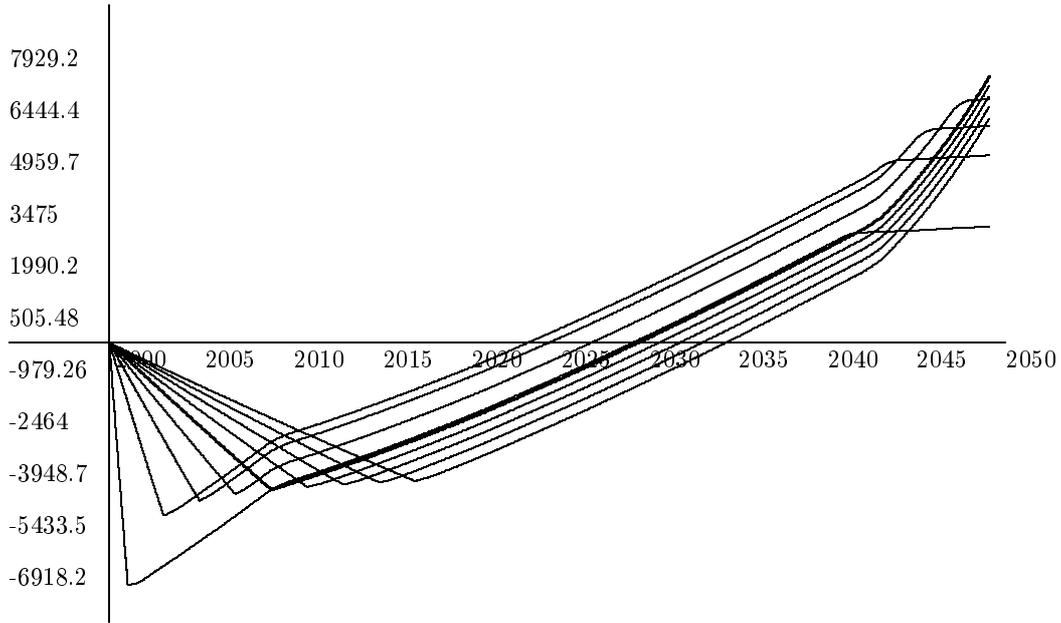


Figure 12: Case A, Scenario 2: Net present values of “Blue Stream” depending on delivery time.

Figure 11 demonstrates the dynamics of gas price at the gas market in Turkey if the nowadays pipeline plans would be implemented. The dynamics shows that at the time moments when new suppliers enter the gas market the price falls down significantly from the level approximately equal to the world LNG price (60\$/1000cm) and remains at this low level until the “Transcaspian” pipeline completes its supply (the lifetime of “Transcaspian” is 40 years).

### 5.3 Case A, Scenario 2: Time Optimization of Gas Deliveries for “Blue Stream”

In this scenario the net present value of one pipeline is maximized under the assumption that construction plans for other pipelines are fixed. The “Blue Stream” and “Transcaspian” pipelines are chosen as the main objects of optimization. The basic control variable of optimization is the time of delivery – the commercialization time of investments into a pipeline project.

We use the following data in this scenario.

“Blue stream” delivery time	optimization
“Transcaspian” delivery time	2002
“Ekarum” delivery time	2010
“Iranpipe” delivery time	2009
Price elasticity $e_p$	-0.7
GDP elasticity $e_y$	1.25
Demand scale $A$	400

Figure 12 shows net present values of “Blue Stream” depending on delivery time. Each curve corresponds to delivery time in the interval 2002-2018 partitioned with the step 2

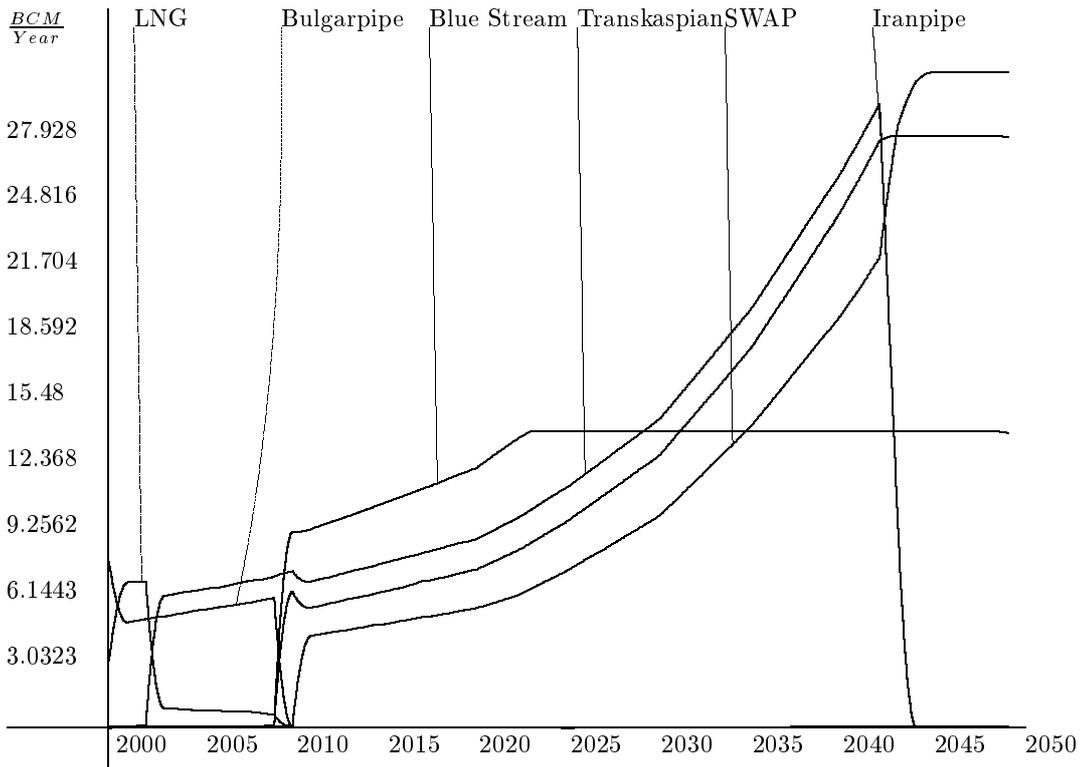


Figure 13: Case A, Scenario 2: Gas deliveries if construction of “Blue Stream” is completed in 2009.

years. The curve corresponding to the optimal net present value of “Blue Stream” is depicted by the bold line. This analysis shows that if competitors of “Blue Stream” fix their nowadays construction plans then it is optimal to start operation of “Blue stream” in 2009. The graphs of supply of gas suppliers are shown on Figure 13. In this scenario Russia is exploiting the “Bulgarpipe” until 2009; then gas deliveries are maintained only through “Blue Stream”.

Net present values of gas suppliers are shown on Figure 14. Rapid growth of profits on the interval 2040-2050 of “Blue Stream”, “Iranpipe”, and “SWAP” pipeline is caused by expiration of lifetime of “Transcaspian” pipeline.

#### 5.4 Case A, Scenario 3: Time Optimization of Gas Deliveries for “Transcaspian”

We use the following data in this scenario.

“Blue Stream” delivery time	2003
“Transcaspian” delivery time	optimization
“Ekarum” delivery time	2010
“Iranpipe” delivery time	2009
Price elasticity $e_p$	-0.7
GDP elasticity $e_y$	1.25
Demand scale $A$	400

Results of optimization procedure for “Transcaspian” are shown on Figure 15.

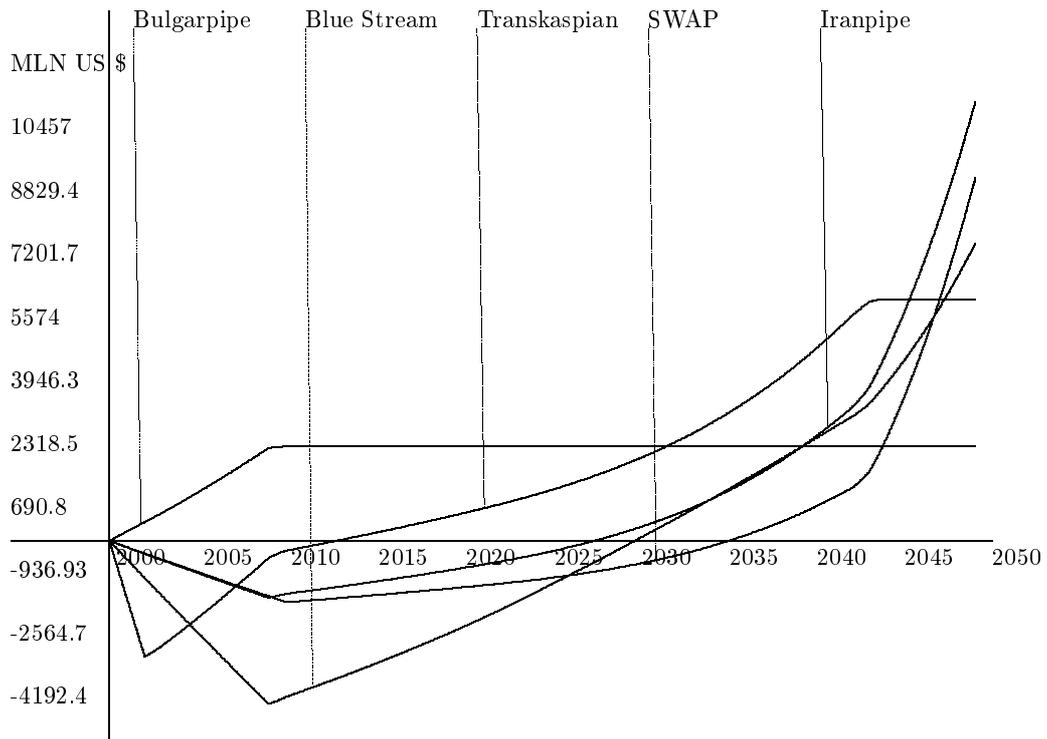


Figure 14: Case A, Scenario 2: Net present values of gas suppliers if construction of “Blue Stream” is completed in 2009.

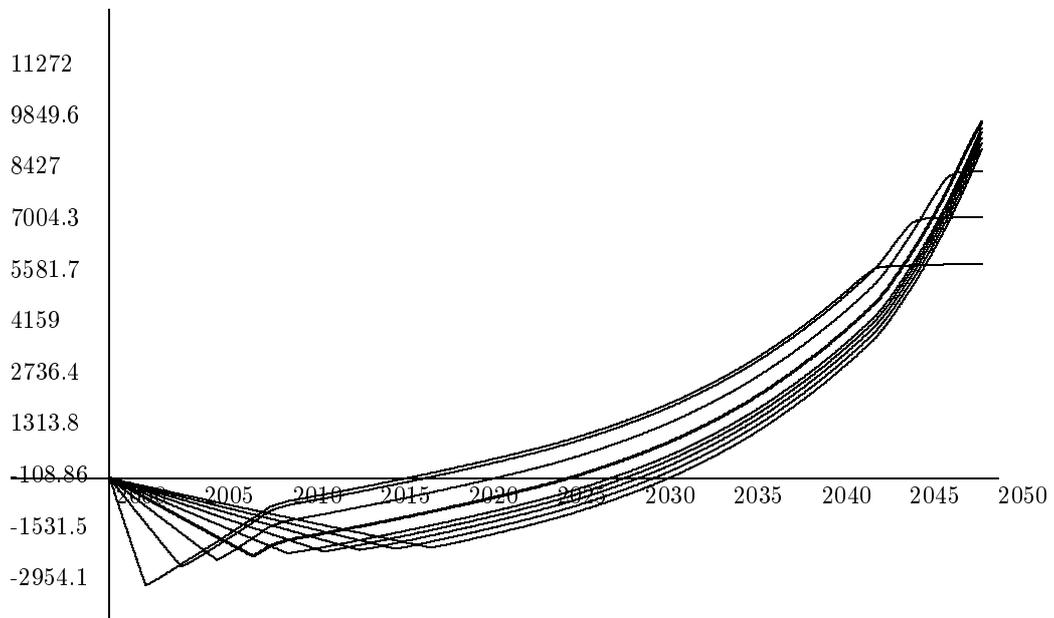


Figure 15: Case A, Scenario 3: Net present values of “Transcaspian” depending on delivery time.

Each curve on Figure 15 corresponds to delivery time in the interval 2002-2018 partitioned with the step 2 years. The optimal time of gas deliveries for “Transcaspian” is also 2009 as in Scenario 2 for “Blue Stream”. The curve of optimal net present value for “Transcaspian” is depicted by the bold line.

### 5.5 Case A, Scenario 4: Game-Optimization of Commercialization Times

We use the following data in this scenario.

“Blue Stream” delivery time	optimization
“Transcaspian” delivery time	optimization
“Ekarum” delivery time	optimization
“Iranpipe” delivery time	optimization
Price elasticity $e_p$	-0.7
GDP elasticity $e_y$	1.25
Demand scale $A$	400

The recursive game-optimization procedure for finding Nash equilibrium in the space of commercialization times (see Section 2) can be tracked by steps. At the first step of optimization the commercialization times shift to the the following values: 9 years for “Blue Stream” instead of 3-4 years of nowadays plans, and 8.5 years for “Transcaspian” instead of the planned 2 years. Shift of the commercialization time for “Transcaspian” is quite essential. This circumstance drastically changes the situation at the second step for “Blue Stream”, and its commercialization is accelerated to 2004. This happens because “Blue Stream” does not compete with the existing pipeline “Bulgarpipeline” from Russia to Turkey. Controlling supply through both pipelines Gazprom is able to keep gas price at the LNG world price level, and thus make “Blue Stream” gas deliveries highly profitable. In this scenario “Blue Stream” does not operate at its maximal capacity during the first 5 years and LNG deliveries are profitable till 2009 when other gas suppliers enter the market.

Figure 16 shows gas deliveries of competitive pipelines.

Figure 17 depicts net present values of gas suppliers in the game scenario.

One can see from Figure 17 that the payback time of investments for “Blue Stream” is 2020. For other suppliers the time interval between the delivery time and the payback time is approximately the same – around 20 years.

The dynamics of gas price in the game scenario is shown on Figure 18.

One can see that the market gas price stabilizes at the level of approximately 73\$/1000cm in the period 2009–20045 when gas is delivered by several competitors. But in the period 2004–2009 when “Blue Stream” is a monopolist of gas deliveries gas prices oscillates around the level of the world LNG price.

### 5.6 Optimization of Maximal Capacity

Let us consider the inverse problem: to optimize the maximal capacity of pipelines if the commercialization times are fixed. We implement such optimization for two main competitors – “Blue Stream” and “Transcaspian.”

In the optimization procedure the net present value of a gas project is considered as a function of pipeline capacity. More precisely, it is assumed that the cost of pipeline construction  $W_{li}$  is a linear function of its capacity  $M_{li}$ .

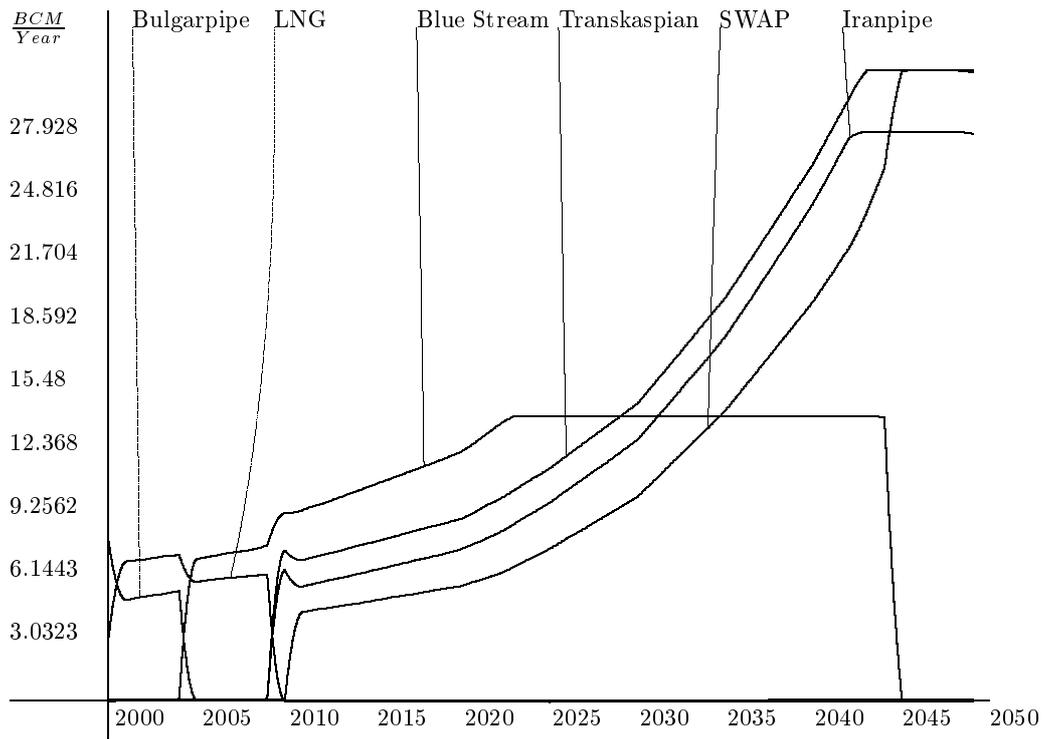


Figure 16: Case A, Scenario 4: Gas deliveries of competitive pipelines.

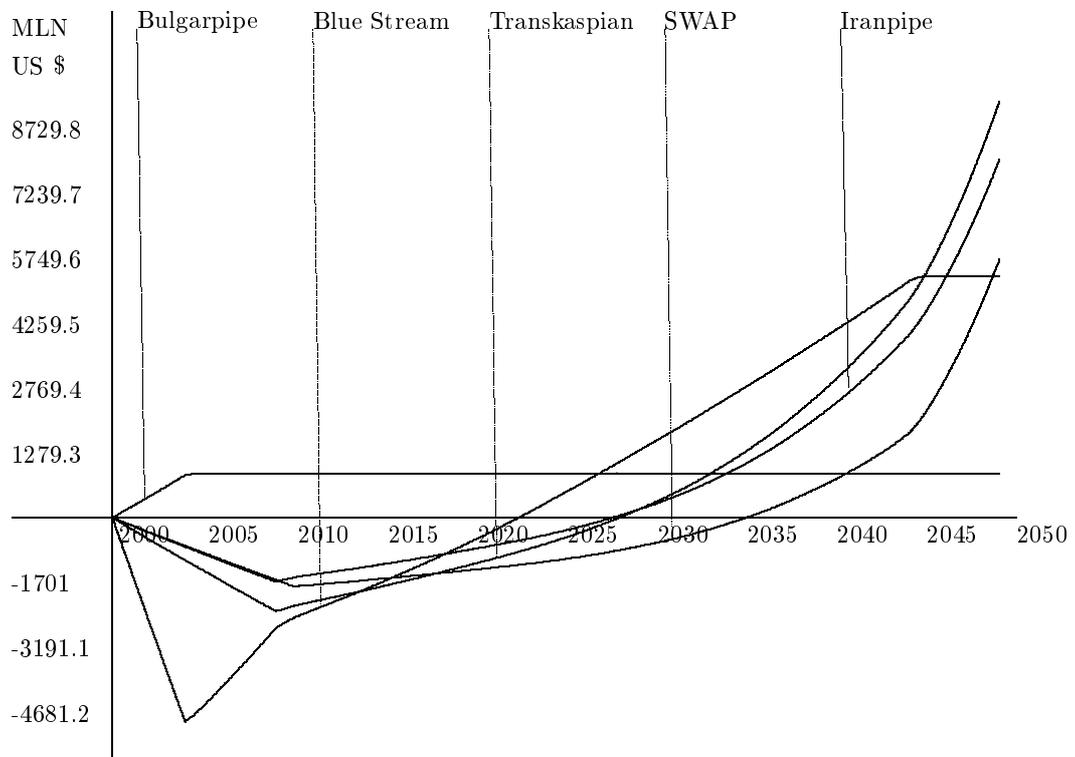


Figure 17: Case A, Scenario 4: Net present values of gas suppliers in the game scenario.

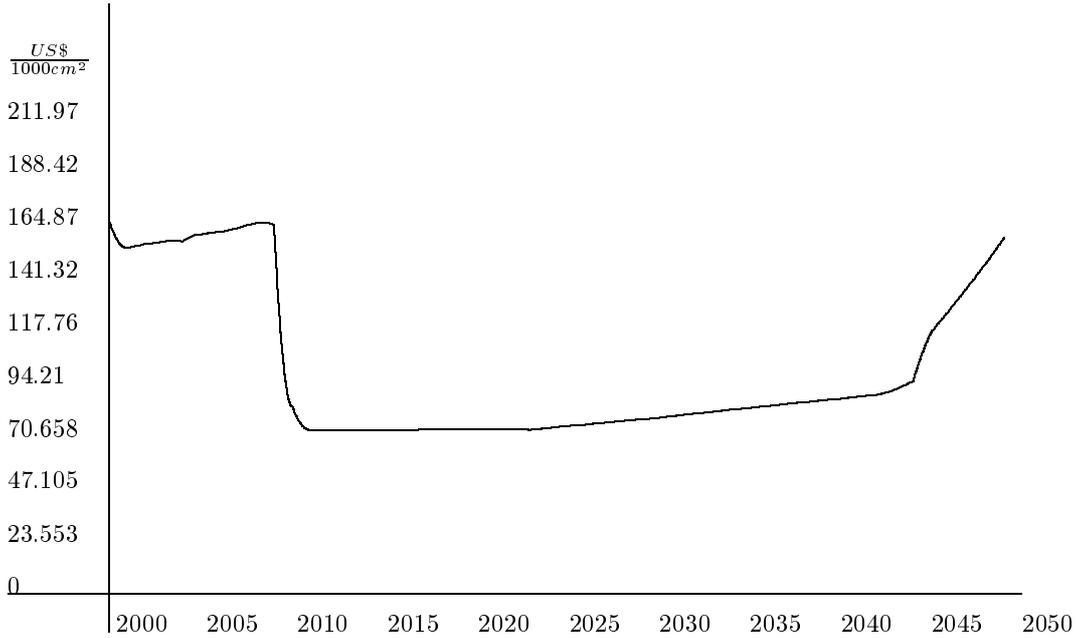


Figure 18: Case A, Scenario 4: Dynamics of gas price in the game scenario.

### 5.6.1 Scenario 5: Optimization of “Blue Stream” Maximal Capacity

The specific data for this scenario is given in the following table.

“Blue Stream” delivery time	2004
“Transcaspian” delivery time	2009
“Ekarum” delivery time	2010
“Iranpipe” delivery time	2009
Price elasticity $e_p$	-0.7
GDP elasticity $e_y$	1.25
Demand scale $A$	400
“Blue Stream” capacity	optimization
“Transcaspian” capacity	31 bcm/year

The analysis shows that optimal capacity of “Blue Stream” is 15 bcm/year instead of the proposed 14 bcm/year, but it will operate at this capacity only starting from 2025 (see Figure 19).

### 5.6.2 Scenario 6: Optimization of “Transcaspian” Maximal Capacity

We use the following data for this scenario.

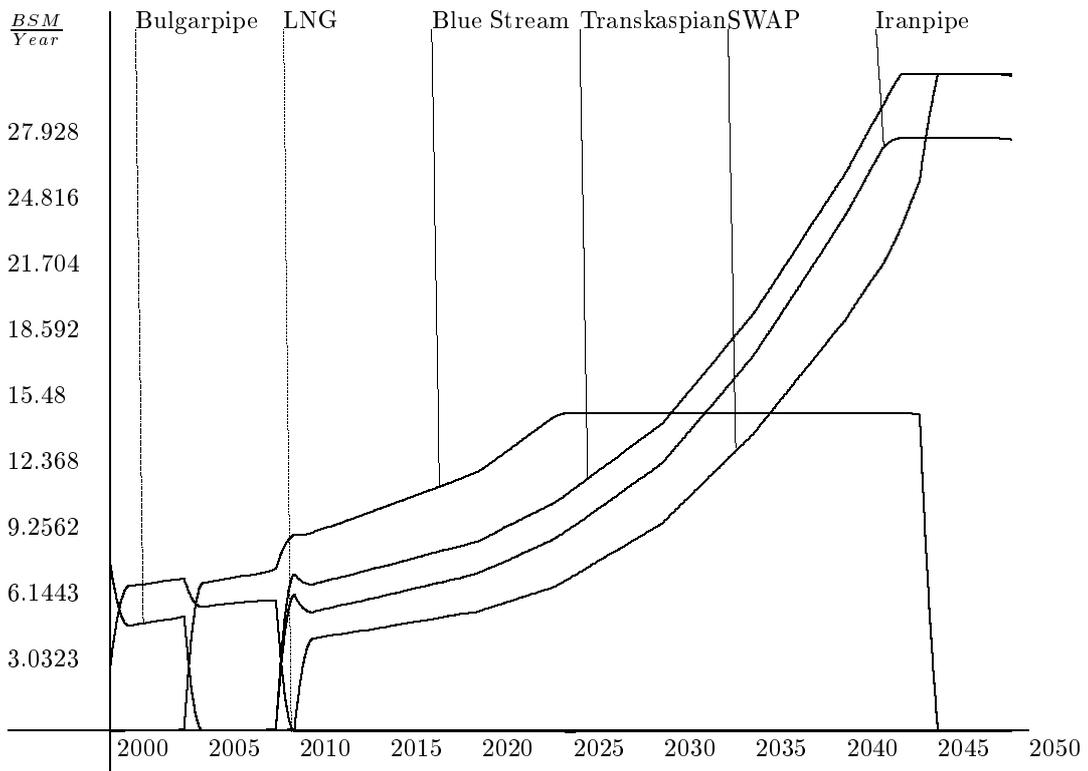


Figure 19: Case A, Scenario 5: Supply by competitive pipelines under optimization of “Blue Stream” maximal capacity.

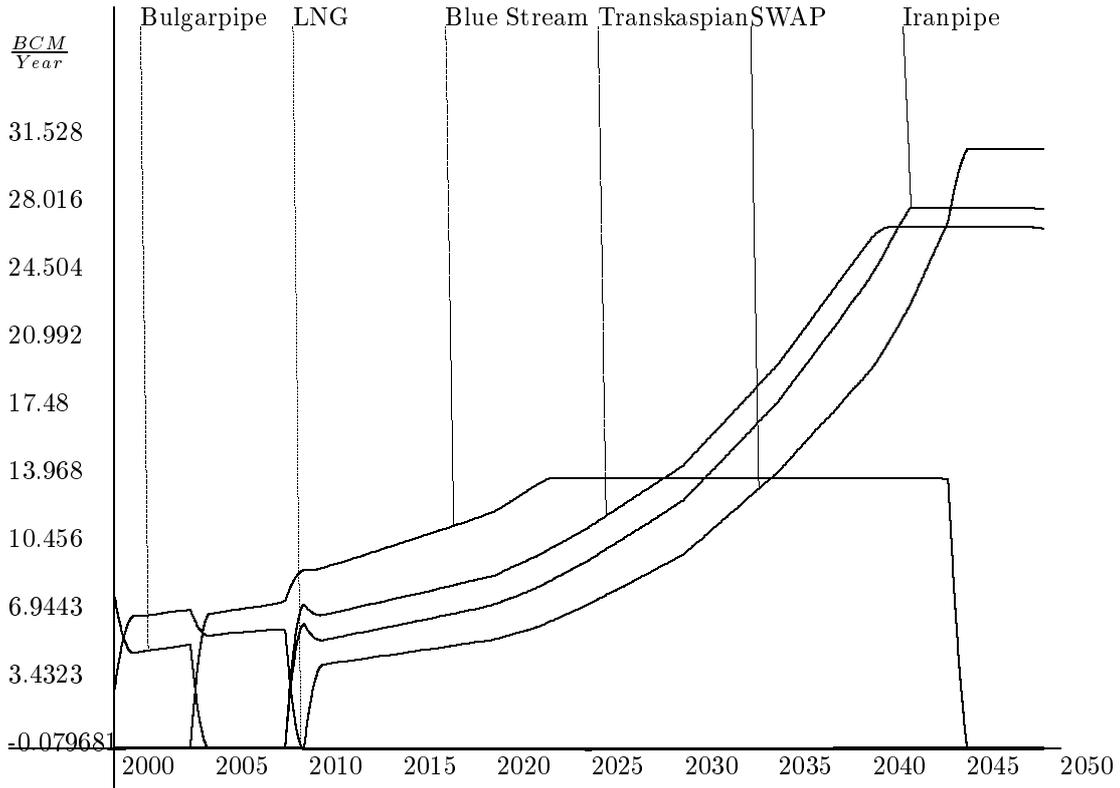


Figure 20: Case A, Scenario 6: Supply by competitive pipelines under optimization of “Transcaspian” maximal capacity.

“Blue Stream” delivery time	2004
“Transcaspian” delivery time	2009
“Ekarum” delivery time	2010
“Iranpipe” delivery time	2009
Price elasticity $e_p$	-0.7
GDP elasticity $e_y$	1.25
Demand scale $A$	400
“Blue Stream” maximal capacity	14 bcm/year
“Transcaspian” maximal capacity	optimization

Optimization of “Transcaspian” maximal capacity is conducted for the case when its construction is completed in 2009. If construction of pipeline is completed in 2009 the optimal maximal capacity would be 27 bcm/year instead of the proposed 31 bcm/year. The results of optimization procedure for this scenarios is shown on Figure 20.

Let us stress that in these simulations the level of investments for each competitor is fixed and only maximal capacity of pipelines is optimized, and thus delivery times may be not optimal.

### 5.7 Case B: Price Elasticity $-0.3$

We consider the case with gas price elasticity equal to  $-0.3$ . This level of elasticity might be realistic for developing countries in early stages of development of the gas market. Simulations show that for the scenario with the price elasticity of  $-0.3$  instead of  $-0.7$  the gas price oscillates near the exogenously given LNG price (see Figure 21). This behavior of

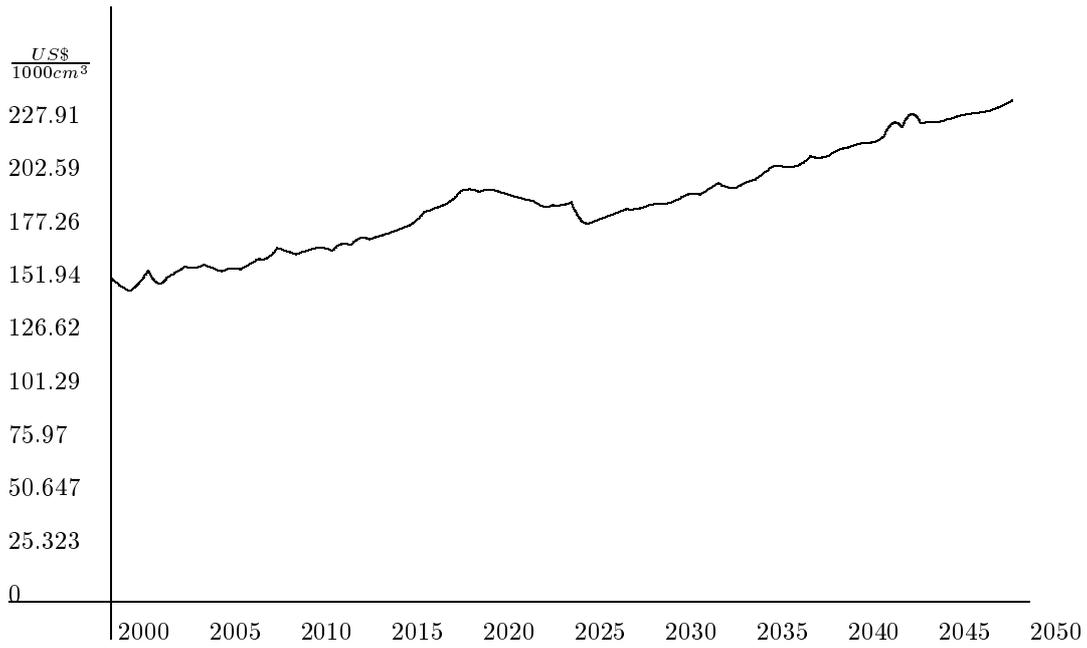


Figure 21: Case B, Scenario 1: Dynamics of gas price.

the gas price can be explained by relatively large value of proportion between gas demand elasticity 1.25 and gas price elasticity  $-0.3$ . In this case the restricted gas supply through pipelines enhances the gas price to the level of LNG world price.

### 5.7.1 Scenario 1: Analysis of Nowadays Plans of Pipeline Projects

First we analyze the nowadays plans of pipeline projects without performing any optimization.

The following data is used for this scenario.

“Blue Stream” delivery time	2003
“Transcaspian” delivery time	2002
“Ekarum” delivery time	2010
“Iranpipe” delivery time	2009
Price elasticity $e_p$	-0.3
GDP elasticity $e_y$	1.25
Demand scale $A$	50

Figure 22 shows that the gas pipelines share the market almost equally. The gas pipelines run at a lower level than the maximal capacity.

At the initial stage of exploitation of pipelines gas deliveries are lower than the maximal capacity and the gas price remains at a high level (see Figure 21). In this period the profits of gas deliveries are accumulated with high rates (see Figure 23).

In this case the payback time of investments depends rather on their amounts than on the level of gas deliveries.

### Case B, Scenario 4: Search of Nash Equilibrium in the Game of Timing

We use the following data in this scenario.

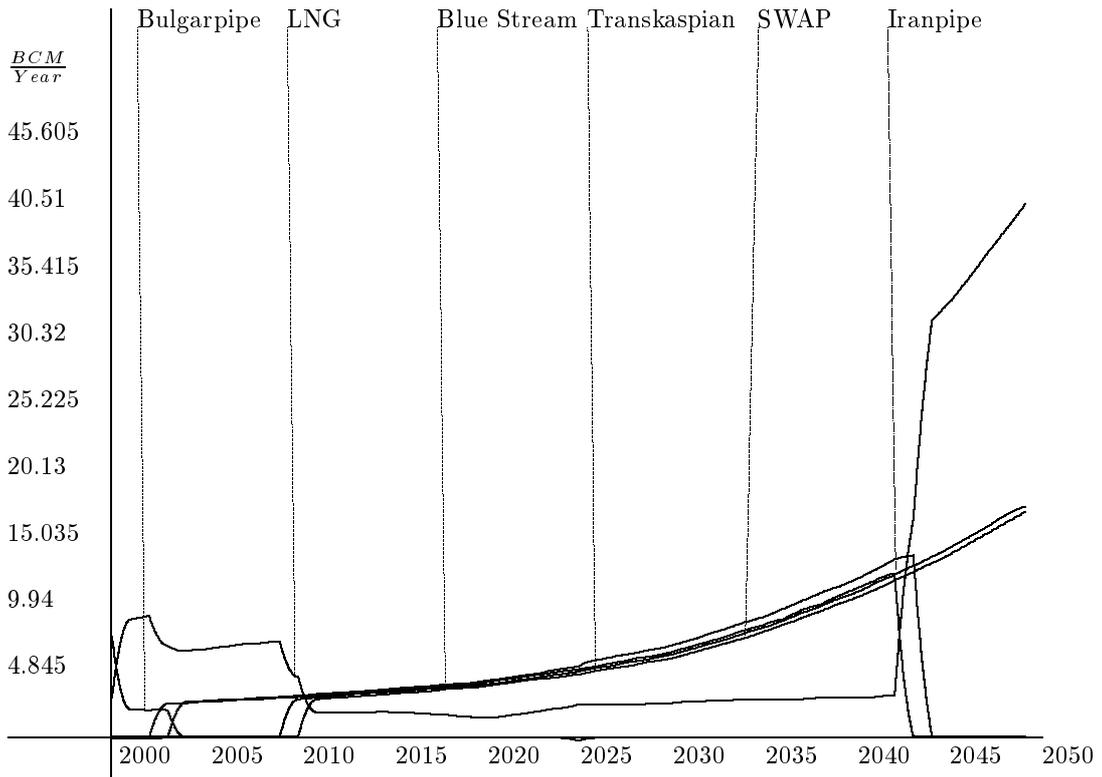


Figure 22: Case B, Scenario 1: Gas supply.

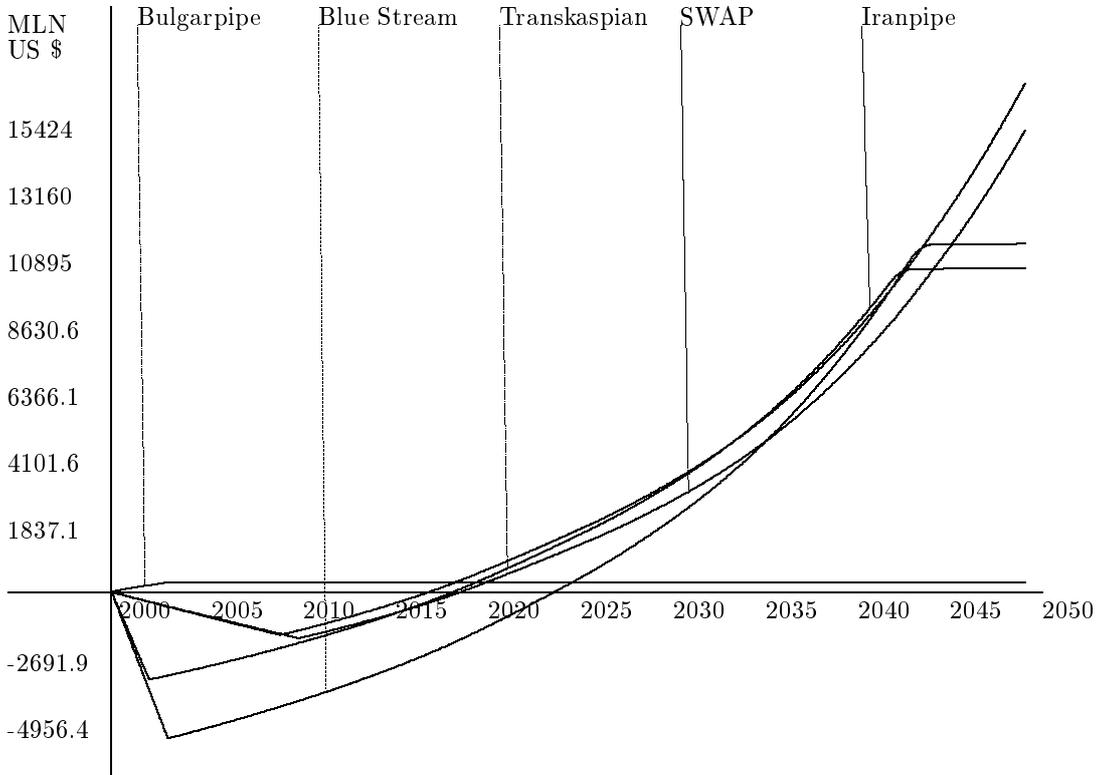


Figure 23: Case B, Scenario 1: Net present values of gas suppliers.

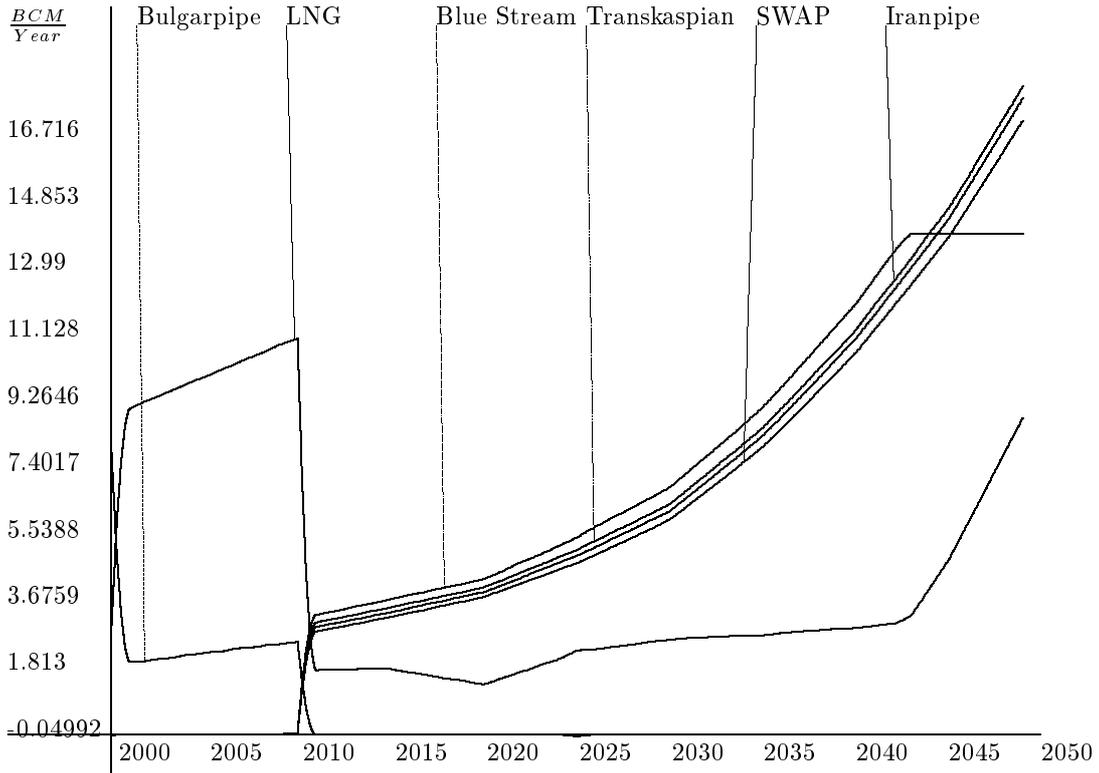


Figure 24: Case B, Scenario 4: Supply by competitive pipelines.

“Blue Stream” delivery time	optimization
“Transcaspian” delivery time	optimization
“Ekarum” delivery time	optimization
“Iranpipe” delivery time	optimization
Price elasticity $e_p$	-0.3
GDP elasticity $e_y$	1.25
Demand scale $A$	50

Optimization of investments in a scenario with price elasticity of gas demand  $e_p = -0.3$  gives the following results. It appears (see Figures 24, 25) that for all suppliers it is optimal to start gas supply around 2009. Therefore the coordinates of Nash equilibrium in the game of timing are approximately equal.

In this scenario the competitors share the gas market equally. At the moment when a new gas pipeline supplier enters the market, the LNG deliveries decrease but do not disappear completely as it happens in the game scenario with price elasticity  $e_p = -0.7$ . This fact can be explained by relatively high level of the market gas price in this scenario.

## 5.8 Sensitivity Analysis with respect to Discount Rate

The discount rate is introduced in the model to calculate net present values of gas pipeline projects. One can estimate the confidence interval for the discount rate, simulate the model for different values from this interval and make sensitivity analysis. The results of sensitivity analysis can be summarized as follows.

- As the discount rate increases, the optimal delivery time for gas pipelines slightly decreases, But after some critical value it starts to increase and for large discount

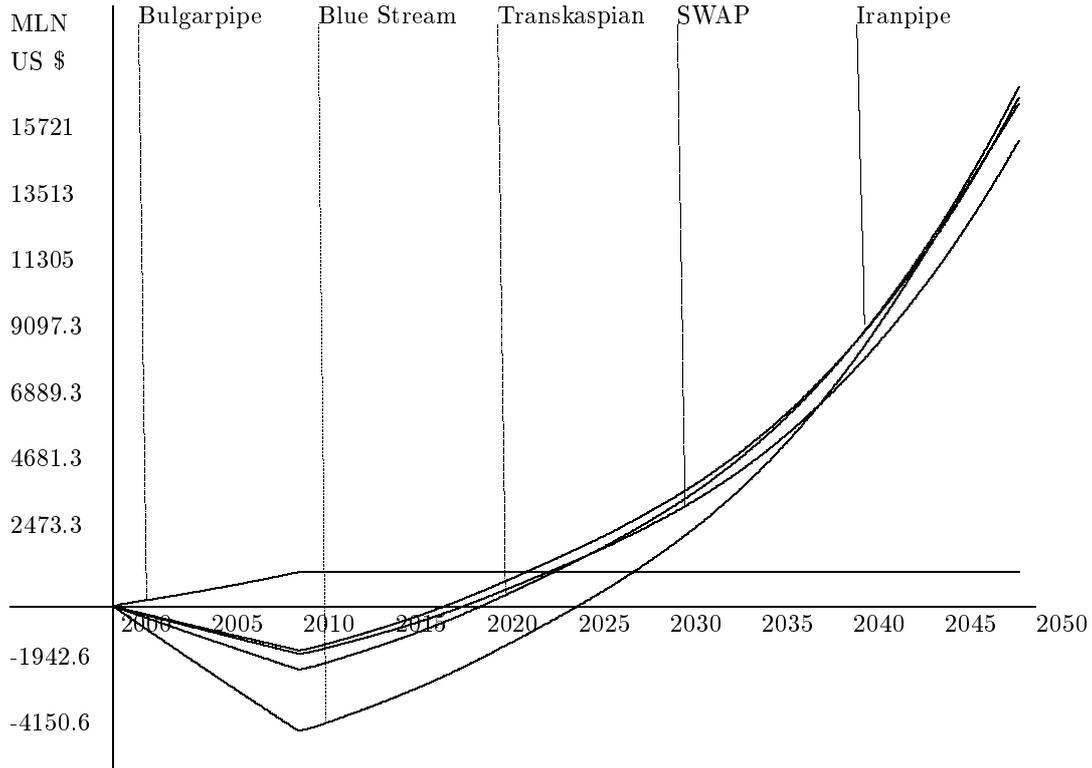


Figure 25: Case B, Scenario 3: Net present values of competitors.

rates tends to infinity since pipeline projects are unprofitable.

- Sensitivity analysis for “Blue Stream” project demonstrates robustness of solutions: optimal delivery time, optimal supply, net present values, with respect to change of discount rate.
- Simulations for “Transcaspian” pipeline with different discount rates show that values of optimal solution change twice quicker than the same values of “Blue Stream” and it is necessary to be more careful in analysis of effectiveness of “Transcaspian” project.

## 6 Conclusions

The aim of this paper was to describe a new, game-theoretical model for analysis of investment plans into international gas transmission lines and to apply this model to the existing and planned gas pipelines to Turkey. The proposed model consists of three levels of analysis. At the first level, current plans of gas pipeline projects are examined and net present values of investment plans are estimated. In the second stage, varying the delivery time for the fixed gas pipeline project, we solve the problem of maximization of the net present value of investment plans under condition that for competitive pipeline projects the delivery time is fixed. At the third level, recursive algorithms are implemented for searching Nash equilibria by controlling the level of gas supply and delivery time. That recursive algorithm is based on the best reply concept: at each step of the algorithm all players maximize their profits taking into account the reaction of other players on their

decisions on optimal timing of gas supply. We prove the existence of a Nash equilibrium at the level of gas supply and outline the algorithm for finding equilibrium and determining the market gas price. We also provide an approach to search Nash equilibrium of commercialization times for gas pipeline projects. This algorithm is also based on the best reply investment policies implemented by competitors.

The method is applied to study the gas market in Turkey and analyzes planned gas pipeline projects from Russia, Turkmenistan and Iran to Turkey. It appears that if the current plans of the proposed gas pipeline projects will be realized then the optimal gas deliveries for almost all pipelines except “Blue Stream” from Russia will not reach the maximal capacities. In the game scenarios for finding Nash equilibria of commercialization times, it appears to be optimal for the Russian Gazprom company to build its “Blue Stream” pipeline according to current plans but Turkmenistan should delay the operation start of “Transcaspian” pipeline until 2009. It also turns out to be optimal to run gas deliveries through the “Transcaspian ” pipeline at a level less than full capacity during a considerable period of time. It means that the planned capacity of “Transcaspian” pipeline is too high and can be reduced. The results turn out to be very sensitive to changes in the price elasticity of gas demand and less sensitive to changes in the discount rate.

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### **Units**

mmcf: million cubic feet  
bcm: billion cubic meters  
bcf: billion cubic feet  
1 bcm = 35.31466672 bcf  
tcf: trillion cubic feet