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Real Options Model for Energy and Ancillary Services Markets

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Abstract

Traditionally, power plant valuation is based only on its energy output. However, there are other products that can increase its value, as a consequence of the liberalization of energy markets, competitive markets for ancillary services are developing throughout the world.

Under this new dynamic environment we introduce a valuation model, which covers the presence of ancillary services markets. We analyze the opportunities of how the energy producer can increase his profit by providing these services. In the application, we evaluate one turbine from a power plant situated in Germany. We perform a comparison of the situation including and excluding the existence of ancillary services markets.

About the Author

At the time of writing this paper, Zuzana Goceliaková was a researcher at the Institute for Advanced Studies, Vienna. For three months during the summer of 2002 she was a participant in IIASA's Young Scientists Summer Program, working in the Forestry Project under the supervision of Michael Obersteiner.

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1 Introduction

Over the last ten years many countries around the world have begun to restructure their electric power industry and others will soon follow. Deregulation of electricity markets has brought a substantial change in the operating environment for producers. Traditionally, electric power companies have operated as vertically integrated and regulated monopolies that generate, transmit and distribute power to the consumers. Given the fixed electricity price producers, the only uncertainty has been the actual demand. The optimal scheduling has been arranged so that the level of the forecasted demand has been met and the operating costs have been minimized.

Moving the electric industry away from its traditionally regulated monopoly structure towards competition among companies is based on the needs that industry becomes more cost saving, and that competition results in lower energy prices, better services, and more customer choices for electric power. The key difference between the regulated environment and competitive markets are the volatile market prices, which means that the electricity producers face more risk. However, the deregulation process has been beneficial for the producers. They now have the flexibility to decide for the more profitable variant, namely either to produce electricity (if the market price is high enough) or to buy it from the market (if the costs of generation are higher than the market price).

As a consequence of the liberalization of energy markets, competitive markets for ancillary services are developing throughout the world. Since there is no efficient way of storing electricity, a continuous balance between generation and the load of electricity must be maintained. Ancillary services are necessary to support the reliable operation of the grid when disturbances occur. The markets for electricity and ancillary services have been established as separate markets in most countries. However, as ancillary services are produced by the same equipment as electricity itself, they are also highly interdependent. One difficulty for the electricity producer is to decide how to formulate bids to maximize profits from both of these markets simultaneously.

In this paper, we present a theoretical valuation model for a power plant. To illustrate the introduced model, we evaluate one turbine from a combined heat and power (CHP) plant situated in Germany. We use German price data (provided by the Leipzig Power Exchange, Internet: www.lpx.de; and RWEnet, Internet: www.rwenet.de).

In the new dynamic environment traditional valuation approaches, such as net present valuation (NPV), are no longer adequate methods for determining the value of generation assets. More elaborated valuation methods have to be used in order to develop the appropriate model. The valuation model introduced in this paper is based on the *real options approach* developed in Tseng and Barz (2002).

Real options are based on the same principle as financial options, that is to have a "real option" means to have the right for a certain period to either choose for or against some strategic decisions.

Real option strategies differ from NPV analysis in their point of view relating to uncertainty, that is instead of "fearing uncertainty and minimizing investment" they seek gains from "uncertainty". The incorporation of a wider range of possible actions into strategic modeling makes the real options approach an applicable tool in investment analysis (Leslie and Michaels, 1997).

There are two kinds of real options that a power plant faces: *operational real options* and *capital investment options*. Operational real options offer the possibility of making short-term decisions concerning the production of electricity. Capital investment options concern long-term decisions, for example investments in production technology, increasing the amount of electricity the power plant can produce, or installing the equipment to control emissions.

Hence, we seek an appropriate model to determine the value of a power plant under the real options it faces. In this paper, two operational real options will be considered. The first covers the *unit commitment decision*, which is an option to produce electricity if the market price is higher than the costs of generation or, if this is not the case, to turn the generator off. The second option, which we will deal with, is the option to choose one of the following alternatives: to generate electricity or to provide ancillary services.

Usually the valuation is only based on electricity production. Until today, the electric industry has paid insufficient attention to ancillary services. As market evidence from California and New England demonstrates (see Griffes *et al.,* 1999), selling ancillary services can be very profitable and should therefore not be ignored in the real option analysis.

The paper is organized as follows. We begin by describing the technical details; different types of ancillary services are explained in Section 2. In Section 3 we discuss the basic consideration regarding ancillary services bidding. A simple numerical example demonstrates the basic idea that providing ancillary services can increase the operational profit. In Section 4 we present the real option valuation model of generating assets, which covers ancillary services. Section 5 contains the description of the solution procedure together with the results of the application. The conclusions as well as further possible research steps are presented in Section 6.

2 Overview of Technical Details

2.1 Ancillary Services

Ancillary services (AS) are a series of services that are *"necessary to support the transmission of energy from generation sources to the consumers and to maintain reliable operations of the transmission system"*.[1] The purpose of ancillary services is to compensate all possible deviations in the power balance that may occur between expected conditions and those

[1] Defined by the Federal Energy Regulatory Commission (FERC) in the USA.

that actually occur. In this paper, we concentrate on a special category of services called *reserve services.* Such services are able to support electricity production in peak times when the load may exceed the expected value, for instance, due to climate influences, e.g., air temperatures below the long-term mean or due to short-term changes in consumption habits.

The deregulation of energy markets has led to the creation of a competitive market for ancillary services. The example of California has shown (see Brien, 1999) that the creation of ancillary services markets is unavoidable — because of the absence of the ancillary services markets and due to high market prices for electricity, producers in California shifted their output entirely from ancillary services to electricity generation and the system ended up without having any ancillary services available.

There is a lot of variety in the structure of ancillary services markets among different restructured systems. Typically a series of ancillary services is defined and a classification is based on the quality and response time of the service. For illustration purposes we introduce the key ancillary services of two markets: California and Germany.

2.1.1 Ancillary Services in California

For a description of ancillary services in California see, e.g., Hirst and Kirby (1997; 1998), Hirst (2000), and Siddiqui *et al.,* (2000).

- *Regulation* is an immediate response service that can adjust output quickly (MW/minute) to moment-to-moment fluctuations in customers' loads.
- *Spinning Reserve* is the use of generating equipment that is online and synchronized with the electrical system and can be fully available to respond to a signal within 10 minutes to provide energy.
- *Non-Spinning Reserve* is similar to spinning reserve, but it does not need to be online and synchronized with the system, although it must respond within 10 minutes.
- *Replacement Reserve* is classified as incremental generation that can be obtained in the next hour to replace spinning and non-spinning reserve used in the current hour.
- *Black Start* is the ability to start up and synchronize the generator to the system without requiring power from the electrical system.
- *Voltage Support* is the use of transmission system equipment to inject or absorb reactive power to maintain voltages on the transmission system within required ranges.

Regulation, spinning, non-spinning, and replacement reserves can be provided by competitive markets. Black start and voltage support are based more on the long-term basis. For instance, voltage support service is expected to be physically applied close to the location, where it has to affect the actual electricity transmission. As a consequence, therefore, it is not possible to create a competitive market for the last two services. Hence, they are not relevant for the purposes of a real option analysis and may continue to stay regulated.

2.1.2 Ancillary Services in Germany

In Germany the competitive market for ancillary services has just been developed. The following four types of reserve services have been defined (DVG, 2000):

- *Primary Control Reserve* is a stabilizing control, operating automatically in the seconds range. All generating units with a nominal capacity greater than 100MW must feature the primary control capability. It is used in both directions in the event of a frequency deviation.
- *Secondary Control Reserve* is a seconds reserve for power control. It may be offered by all of the connected generating units. The positive and negative control directions are awarded separately.
- *Minute Reserve* (*Warm Reserve* or *Tertiary Reserve*) may be offered by all of the connected generating units that are capable of injecting the agreed reserve power into the network within 15 minutes. It is mainly offered by storage stations, pumped-storage stations, gas turbines, and thermal power stations operating at less than full output.
- *Hours Reserve* (*Cold Reserve* or *Stand-by Reserve*) available in thermal power stations, which must be started for this purpose.

Primary and secondary reserve are contracted on the basis of long-term contracts and are therefore not interesting from the viewpoint of short-term modeling. In this paper, the analysis mainly focuses on the minute reserve. There are two kinds of minute reserve traded on the market, namely positive and negative. Positive minute reserve (additional generation of electricity) is needed in situations when it is necessary for the system to compensate some losses. Negative reserve is required for consuming the excess electricity out of the grid.

2.2 The Role of the Transmission System Operator

Without the transmission grid, electric power would never reach the consumer. A restructured competitive environment where generation is unbundled from transmission and distribution, has enforced the creation of a new entity — the *Transmission System Operator (TSO)*. The main objective of the system operator is to ensure the reliable operation of the grid and safe transport of electric power. Moreover, the TSO

- is a non-profit corporation;
- has the obligation and therefore the authority to control and, if necessary, to prohibit power transfers and injections if there is a risk of system failure;
- specifies which ancillary services should be provided, when, and by whom;
- is the only entity with sufficient and timely information to decide how much of each service is required;
- sends signals to each generating unit that is providing the service;

- does not own or operate any ancillary services; and
- has a crucial role because it is much more cost effective to provide ancillary services for the aggregate load than for each load separately.

2.2.1 System Operators in Germany

Germany is divided into six TSO areas of responsibility. In the numerical analysis we used data only from the largest, namely the TSO controlled by the RWEnet.

3 Simple Decisions

3.1 Motivation

As a simple example of what the power plant achieves when bidding on the AS market, consider the following simplified demonstration of the problem. A power plant uses a generator with minimum and maximum generating capacities 140MWh and 284MWh,[2] respectively and has the marginal cost of 10€/MWh. Suppose the actual hourly market price is 12€/MWh. When bidding on the energy market only, the maximum hourly profit achieved by the power plant will be $284 \times (12 - 10) = 568$€/MWh.

Now suppose that there is the possibility to also bid on the AS market. The simplest example is to consider just one service, e.g., a spinning reserve. Suppose that the actual market price is 4€/MWh and the marginal cost is 1€/MWh. In this case, the power plant owner may consider running the unit on its minimum generating level (naturally, the unit has to be online for the time when a bid on the ancillary market was made) and thus bidding just 140MWh on the energy market and the rest (144MWh) on the AS market. The profit under this scenario is: $140 \times (12 - 10) + 144 \times (4 - 1) = 712$€/MWh.

The situation is even more interesting when the hourly price for electricity is smaller than the marginal cost for energy generation, say 8€/MWh is the market price. Now, instead of turning off the generator (which would be a natural choice without AS) the power plant can consider a bid of the spinning reserve on the AS market — when running the generator on its minimal capacity (necessary for the provision of spinning reserve) the rest can still be bid on the AS market earning a positive profit of $140 \times (8-10) + 144 \times (4-1) = 152$€/MWh.

3.2 Notation

We proceed in elaborating a simple decision model to show the basic difference between the ancillary service, for which the generator must be online, and the ancillary service, which one can bid on the AS market also when the generator is off.

Denote by:

[2] These parameters correspond to the operational characteristics of the turbine, which is analyzed in Section 5.3.

p^E	...	the market price of electricity (€/MWh).
p^{AS}	...	the market price of AS (€/MWh).
c^E	...	the marginal cost for producing 1 MWh of electricity (€/MWh).
c^{AS}	...	the marginal cost associated with providing 1 MWh of AS (€/MWh).
q^E	...	the amount of capacity bid on the energy market (MWh).
q^{AS}	...	the amount of capacity bid on the AS market (MWh).
$q^{\max}$	...	the maximum generating capacity of the unit (MWh).
$q^{\min}$	...	the minimum generating capacity of the unit (MWh).

3.3 An Online Unit

Consider first the case of the ancillary service for which the generator must be online (e.g., spinning reserve in the Californian market or minute reserve in the German market).

We compare the optimal decision together with the maximum achievable profits in situations including and excluding the presence of the ancillary services markets, respectively. In this simple example, the optimal decision is driven only by the market prices and by the costs of production. Particularly, without the presence of the ancillary services market:

- If $p^E > c^E$, the unit naturally decides for $q^E = q^{\max}$. In this case, the profit is $(q^{\max} - q^{\min})(p^E - c^E)$.
- If $p^E \leq c^E$, then the unit decides to turn off (i.e., $q^E = 0$). This leads to zero profit.

With the option to invest on the AS market:

- If $p^E > c^E$ and $p^E - c^E \geq p^{AS} - c^{AS}$, then it will still be more profitable to sell as much energy as possible only to the energy market, that is $q^E = q^{\max}$ and $q^{AS} = 0$. This means that the profit is the same as in the situation without the presence of the AS market:
$$(q^{\max} - q^{\min})(p^E - c^E).$$
- However, if $p^E > c^E$ and $p^E - c^E < p^{AS} - c^{AS}$, then the unit optimizes its production by producing just the minimum generating capacity ($q^E = q^{\min}$) and provides the remaining available capacity as a reserve: ($q^{AS} = q^{\max} - q^{\min}$). This increases the profit to:
$$(q^{\max} - q^{\min})(p^{AS} - c^{AS}) + q^{\min}(p^E - c^E) > q^{\max}(p^E - c^E).$$
- If $p^E \leq c^E$ and $q^{\min}(c^E - p^E) < (p^{AS} - c^{AS})(q^{\max} - q^{\min})$, then setting $q^E = q^{\min}$ and $q^{AS} = q^{\max} - q^{\min}$ leads to a positive profit from providing AS instead of earning zero profit in the above case without AS:
$$(q^{\max} - q^{\min})(p^{AS} - c^{AS}) + q^{\min}(p^E - c^E) > 0.$$
- On the other hand, if $p^E \leq c^E$ and $q^{\min}(c^E - p^E) \geq (p^{AS} - c^{AS})(q^{\max} - q^{\min})$, then there is no better choice than to switch off the generator, which means $q^E = 0$, $q^{AS} = 0$. Again, this leads to zero profit.

3.4 A Unit that is Possibly Offline

Now consider an ancillary service that does not require a generator to be online for the period when a bid is made (e.g., non-spinning reserve in California). Actually for $p^E > c^E$ the situation is quite similar to the case with the online generator. The only difference here is the case when $p^E > c^E$, and $p^E - c^E < p^{\mathrm{AS}} - c^{\mathrm{AS}}$, e.g., when the provision of AS is more profitable than the generation of electricity. Since now, the generator does not need to be online one can bid all of the capacity on the AS market.

This differs, however, for $p^E \le c^E$, since without the need to be online, the generator can make some profit from the AS market also when it is turned off, whereas the online generator has to produce at least the minimum generating capacity in order to bid on the AS market. This means, without the option to bid on the AS market:

- If $p^E \le c^E$ unit decides to turn off (i.e., $q^E = 0$). Then the profit is zero.

However, with the option on the AS market:

- If $p^E \le c^E$ and $p^{AS} - c^{AS} > 0$, then setting $q^E = 0$ and $q^{\mathrm{AS}} = q^{\max}$ leads to positive profit from providing AS instead of earning zero profit in the above case without the presence of the AS market:

$$q^{\max}(p^{\mathrm{AS}} - c^{\mathrm{AS}}) > 0.$$

- On the other hand, if $p^E \le c^E$ and $p^{AS} - c^{AS} \le 0$, then there is, of course, no better choice than to turn the generator off, which means $q^E = 0$, $q^{\mathrm{AS}} = 0$. In which case the profit is also zero.

3.5 Simple Decision Model

The analysis of the different cases in Sections 3.3 and 3.4 can be formalized by the following profit functions.

A Unit without the AS Bidding Option

$$f(q^E, p^E, c^E) = \begin{cases} q^E(p^E - c^E) & \text{if the unit is on} \\ 0 & \text{if the unit is off} \end{cases}$$

subject to:

$$q^{\min} \le q^E \le q^{\max} \quad \text{if the unit is on}$$

$$q^E = 0 \quad \text{if the unit is off.}$$

An Online Unit

$$f(q^E, q^{\mathrm{AS}}, p^E, p^{\mathrm{AS}}, c^E, c^{\mathrm{AS}}) = \begin{cases} q^{\mathrm{AS}}(p^{\mathrm{AS}} - c^{\mathrm{AS}}) + q^E(p^E - c^E) & \text{if the unit is on} \\ 0 & \text{if the unit is off} \end{cases}$$

subject to:

$$q^{\min} \le q^E \le q^{\max},\ 0 \le q^{\mathrm{AS}} \le q^{\max} - q^E \quad \text{if the unit is on}$$

$$q^E = 0,\ q^{\mathrm{AS}} = 0 \quad \text{if the unit is off.}$$

A Unit that is Possibly Offline

$$f(q^E, q^{\text{AS}}, p^E, p^{\text{AS}}, c^E, c^{\text{AS}}) = \begin{cases} q^{\text{AS}}(p^{\text{AS}} - c^{\text{AS}}) + q^E(p^E - c^E) & \text{if the unit is on} \\ q^{\text{AS}}(p^{\text{AS}} - c^{\text{AS}}) & \text{if the unit is off} \end{cases}$$

subject to:

$$q^{\min} \leq q^E \leq q^{\max} \quad \text{if the unit is on}$$

$$q^E = 0 \quad \text{if the unit is off}$$

and

$$0 \leq q^{\text{AS}} \leq q^{\max} - q^E.$$

However, in this model the following real world constraints have been omitted for simplicity:

- To turn the unit on/off, one has to consider additional costs. These costs may depend on the time period the unit has spent in the particular state.
- The costs for producing the energy ($q^E c^E$) can be a more general function, in particular it can depend on the parameter describing the price of the fuel used for producing electricity.
- When the generator owner bids on the AS market, he has to be prepared to respond to the "call" for AS, that is to respond to a signal to activate the AS. In this case, the unit that is possibly offline has to be turned on. Moreover, the generator can then expect an additional profit from producing the called energy (according to the amount of energy actually called). Moreover, such a "call" is a stochastic event.
- There are technical conditions on the generating unit connected with the way it produces electricity, e.g., when the unit is on (or off, respectively), it can be turned off only after the pre-defined amount of time expires.

4 Real Options Model

In this section, a "real world" valuation model for a power plant with an AS bidding option will be described. We extend the real option approach developed in Tseng and Barz (2002). Two real options will be incorporated into the model — the unit commitment decision and the option to invest on the AS market.

4.1 Notation

Denote by

Variables:

p_t^F ... the market price of fuel at time t.

p_t^E ... the market price of electricity at time t.

p_t^{AS} ... the market price of AS at time t.

q_t^E ... the amount of electricity generated at time t (excluding electricity generated on contingency).

q_t^{AS} ... the amount of capacity bid on the AS market at time t.

q_t^{CAS} ... incremental amount of electricity[3] called on contingency at time t.

u_t ... decision variable that indicates the unit commitment decision made at time t (the value $u_t = 1$ represents the decision to be on at time $t+1$, whereas $u_t = 0$ represents the decision to be off at time $t+1$).

x_t ... state variable that indicates how long the unit is in on mode ($x_t > 0$) or off mode ($x_t < 0$), respectively.[4]

Functions:

$C(.)$, $C^{\mathrm{AS}}(.)$ ... the cost function.

$S_u(x_t)$ ... the start-up cost.

$S_d(x_t)$ ... the shut-down cost.

Constants:

$q^{\max}$ ... the maximum generating capacity of the unit.

$q^{\min}$ ... the minimum generating capacity of the unit.

$t^{\mathrm{on}}, t^{\mathrm{off}}$ ... minimum up/down time of the generator.

t^{cold} ... number of periods leading to the completely cooled generator, if left in the off-state.

τ ... unit start-up time.

ν ... unit shut-down time.

T ... the time period that is considered.

We will assume that the fuel price and the incremental amount of called on electricity contingency are fully and perfectly known at the time the bids are made. For simplicity, we assume the fuel price to be constant.

For further analysis, it would be convenient to denote the proportion of the incremental amount of called on electricity contingency with respect to the amount bid on the AS market as α_t, that is:

$$q_t^{\mathrm{CAS}} = \alpha_t q_t^{\mathrm{AS}}.$$

[2]The incremental energy q_t^{CAS} has to be injected into the system at time t, that is the TSO has to signal its use in time $t-1$.

[3]We use $x_t = 0$ to indicate that the unit is starting up or shutting down at time t, hence it is unable to respond to the signal or to produce electricity until the mode actually changes.

In simulations (see Section 5) we will distinguish two cases with respect to α_t — the constant proportion ($\alpha_t = \alpha$) and the random proportion.

4.2 Optimal Scheduling on the Electricity Market

First, consider the situation without the option to bid on the AS market. The aim is to find the optimal schedule for the operating unit over the entire planning period that leads to maximum profit.

4.3 The Profit Function

In the case when only a bid on the energy market can be made, the profit function looks like the following:

$$J_0 = \max_{u_t, q_t^E} E \sum_{t=0}^{T} \left[f(x_t, q_t^E, p_t^E, p_t^F) - S_u(x_t)u_t - S_d(x_t)(1-u_t) \right] \tag{1}$$

where

$$f(x_t, q_t^E, p_t^E, p_t^F) = \left(p_t^E q_t^E - C(q_t^E, p_t^F) \right) \mathrm{I}_{\{x_t>0\}}$$

and $\mathrm{I}_{\{.\}}$ denotes an indicator function:

$$\mathrm{I}_{\{x>0\}} = \begin{cases} 1, & \text{if } x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Shutdown costs $S_d(x_t)$ are assumed to be constant, that is:

$$S_d(x_t) = \begin{cases} s_d, & \text{if } x_t > 0 \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

for some suitable constant s_d.

Further, we assume that the start-up cost S_u depends on the amount of time that the generator has already spent in the off-state. For such an assumption, we use the following representation:

$$S_u(x_t) = \begin{cases} b_1 \left(1 - e^{x_t/\gamma}\right) + b_2, & \text{if } -t^{\text{cold}} \le x_t < -t^{\text{off}} \\ b_1 + b_2, & \text{if } x_t < -t_{\text{cold}} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where b_1, b_2, and γ are given constants. This means that for $x_t < -x_t^{\text{cold}}$ we treat the cooling effect in time as negligible.

4.4 Feasible Bids (Technical Constraints)

Naturally, there are technical constraints that a unit has to fulfill. This section deals with these constraints.

Minimum Up/Down Constraints. These constraints state that the unit commitment decision can be made only if the power plant has already been turned on (or off, respectively) for at least the minimum up (or down) time of the generator:

$$u_t = \begin{cases} 1, & \text{if } 1 \leq x_t < t^{\text{on}} \\ 0, & \text{if } -t^{\text{off}} < x_t \leq -1 \\ u_{t-1}, & \text{if } x_t = 0 \\ 0 \text{ or } 1, & \text{otherwise.} \end{cases} \tag{4}$$

State Transition Constraints. At any time, each unit can only be in one of the following modes: online, offline or “changing”. The last mode describes the situation when the unit is in a commitment/decommitment decision lead time, i.e., the state of the unit is changing from online to offline or vice-versa.

The rules for determining the value of the state variable are quoted here depending on the previous state and the unit commitment decisions:

$$x_t = \begin{cases} \min(t^{\text{on}}, x_{t-1}+1), & \text{if } 0 < x_{t-1} \text{ and } u_{t-1} = 1, \\ -1, & \text{if } x_{t-\nu} = t^{\text{on}} \text{ and } u_{t-\nu} = 0 \\ \max(-t^{\text{cold}}, x_{t-1}-1) & \text{if } x_{t-1} < 0 \text{ and } u_{t-1} = 0, \\ 1, & \text{if } x_{t-\tau} \leq -t^{\text{off}} \text{ and } u_{t-\tau} = 1 \\ 0, & \text{otherwise.}^{5} \end{cases} \tag{5}$$

Unit Capacity Constraints. When the unit is active, the amount of generated electricity has to comply with the range $[q^{\min}, q^{\max}]$ that is:

$$q^{\min}\,\mathrm{I}_{\{x_t>0\}} \leq q_t^E \leq q^{\max}\,\mathrm{I}_{\{x_t>0\}}\,. \tag{6}$$

4.5 The Online AS

How does the profit function change in the AS option case? In this section we introduce a real option model in which one ancillary service is considered. Namely, we take into account the service that can be provided only if the unit is online (e.g., the minute reserve or the spinning reserve, respectively).

4.5.1 The Profit Function

The key characteristic of such a service is that the generator has to be online and synchronized to the grid and has to start to produce additional energy within 15 minutes after the signal. The total profit will be increased by the profit of selling the ancillary service, that is by $p_t^{\text{AS}} q_t^{\text{AS}}$ and by the profit from producing the energy on contingency, that is $p_t^E q_t^{\text{CAS}}$. The associated costs of providing the service must be subtracted, i.e., q_t^{AS}, q_t^{CAS} will enter as new variables into the cost function. Hence, the modified profit function, which covers the possible profit of bidding on the AS market, is as follows:

[4] This indicates that the unit is now in start-up or shut-down period, hence it is unable to produce energy or supply AS.

$$J_0^{\mathrm{AS}} = \max_{u_t, q_t^E, q_t^{\mathrm{AS}}} E \sum_{t=0}^{T} \left[f^{\mathrm{AS}}(x_t, q_t^E, q_t^{\mathrm{AS}}, q_t^{\mathrm{CAS}}, p_t^E, p_t^{\mathrm{AS}}, p_t^F) - S_u(x_t)u_t - S_d(x_t)(1-u_t) \right] \tag{7}$$

where

$$f^{\mathrm{AS}}(x_t, q_t^E, q_t^{\mathrm{AS}}, q_t^{\mathrm{CAS}}, p_t^E, p_t^{\mathrm{AS}}, p_t^F) = \\ = \left(p_t^E (q_t^E + q_t^{\mathrm{CAS}}) + p_t^{\mathrm{AS}} q_t^{\mathrm{AS}} - C^{\mathrm{AS}}(q_t^E, q_t^{\mathrm{AS}}, q_t^{\mathrm{CAS}}, p_t^F) \right) \mathrm{I}_{\{x_t > 0\}}$$

and functions $S_d(x_t)$ and $S_u(x_t)$ are defined by equations (2) and (3) respectively.

4.5.2 Feasible Bids (Technical Constraints)

This section deals with the technical constraints of electricity generation with respect to AS. In the case of AS, it is also important to ask: *When can a bid on the AS market be made?* In the case of the online AS the answer is easy: the unit has to be online at the time period for which the bid has been made, i.e., $I_{\{x_t>0\}} = 0 \Rightarrow q_t^{\mathrm{AS}} = 0$.

Minimum Up/Down Constraints. The minimum up/down constraints for the AS case are identical to the case when no AS option is available, as stated by equation (4).

State Transition Constraints. The state transition constraints for the AS case are identical to the case when no AS option is available, as stated by equation (5).

Unit Capacity Constraints. The unit capacity constraints for the AS case are identical to the case when no AS option is available, as stated by equation (6).

AS Restriction Constraints. It is possible to bid the maximum available reserve capacity, if the generator is on, and none otherwise:

$$0 \leq q_t^{\mathrm{AS}} \leq (q^{\max} - q_t^E)\, \mathrm{I}_{\{x_t>0\}}\,. \tag{8}$$

AS Satisfaction Constraints. The last restriction describes the fact that the TSO cannot request more energy than has been bid on the AS market. We restrict the AS to providing the positive amount of additional energy only:

$$0 \leq q_t^{\mathrm{CAS}} \leq q_t^{\mathrm{AS}}.$$

Equivalently stated:

$$q_t^{\mathrm{CAS}} = \alpha_t q_t^{\mathrm{AS}} \qquad \alpha_t \in [0,1]. \tag{9}$$

4.6 The Cost Function

It is standard (see, e.g., Tseng and Barz, 2002; Hlouskova *et al.,* 2002) to model the cost function associated with running the unit by a quadratic dependence with respect to the amount of electricity to be produced. Hence, for the case when there is no AS bid option, the cost function is defined by:

$$C(q_t^E, p_t^F) = \left(a_0 + a_1 q_t^E + a_2 \left(q_t^E\right)^2 \right) p_t^F\,. \tag{10}$$

We assume that all of the coefficients (a_0, a_1, and a_2) are positive. Note that the component $a_1 q_t^E p_t^F$ is the major component of the cost function and $a_0 p_t^F$ is the cost associated with running the generator with no electricity output and only maintaining the immediate availability of the unit. From $a_2 > 0$ it follows that the cost function is convex.

With this meaning in mind, the cost function changes when introducing the AS bid option as follows :

$$C^{\mathrm{AS}}(q_t^E, q_t^{\mathrm{AS}}, q_t^{\mathrm{CAS}}, p_t^F) = \left(a_0 + a_1(q_t^E + q_t^{\mathrm{CAS}}) + a_2\left(q_t^E + q_t^{\mathrm{CAS}}\right)^2\right) p_t^F. \tag{11}$$

Actually, there are no additional costs (except for perhaps administrative costs which we neglect) associated with bidding on the AS market itself (that is with q_t^{AS}). Only the amount of electricity that will actually be generated on contingency (q_t^{CAS}) is relevant.

5 Numerical Results

5.1 Solution Techniques

The numerical method for finding the optimal solution of the models formulated in the previous section requires integrating the forward-moving Monte Carlo simulation with backward-moving dynamic programming.[6] We use a slight modification of the algorithm described in Tseng and Barz (2002), which has been extended for our purposes. Therefore, we only introduce the basic ideas of the solution procedure.

In order to use the simulation and dynamic programming techniques, we must be able to solve our optimization problem starting at any time point. Therefore, we define

$$J_t(x_t, u_t, p_t^E, p_t^F) = \max_{u_i, q_i^E} E \sum_{i=t}^{T} \left[f(x_i, q_i^E, p_i^E, p_i^F) - S_u(x_i)u_i - S_d(x_i)(1-u_i)\right] \tag{12}$$

$$\begin{aligned} J_t^{\mathrm{AS}}&(x_t, u_t, p_t^E, p_t^{\mathrm{AS}}, p_t^F) = \\ &\max_{u_i, q_i^E, q_i^{\mathrm{AS}}} E \sum_{i=t}^{T} \left[f^{\mathrm{AS}}(x_i, q_i^E, q_i^{\mathrm{AS}}, q_i^{\mathrm{CAS}}, p_i^E, p_i^{\mathrm{AS}}, p_i^F) - \right. \\ &\left. - S_u(x_i)u_i - S_d(x_i)(1-u_i)\right]. \end{aligned}$$

Here, we assume that the prices, states, and decisions at time-point t are known and serve as inputs for J_t and J_t^{AS}. On the other hand, the prices, states, and decisions at time points $(t+1), \ldots, T$ have to be either simulated or determined.

At each time point, two problems have to be solved simultaneously: *the optimal commitment problem* and *the dispatch problem.*

- The commitment decision (that is the decision whether the generating unit should be on or off) is based on the current price and its effect on future prices. The simulation will be used to capture this future effect. This simulation works under the assumption that price processes for electricity and for the ancillary services, respectively, are Markov.

[6] By "moving" we mean moving in time.

- Additionally, once it has been decided that the unit should be on, the optimal dispatch problem has to be solved, that is the optimal production level of electricity and the optimal amount of reserve has to be determined.

5.1.1 The Optimal Dispatch Problem

From these problems, the optimal dispatch problem is easier to solve since there is a corresponding analytical solution. Without an option to bid on the AS market, one solves the following problem (see equations (1) and (4)):

$$\max_{q_t^E} \left[p_t^E q_t^E - C(q_t^E, p_t^F) \right]$$

subject to

$$q^{\min} \leq q_t^E \leq q^{\max}.$$

Its optimal solution is determined as follows:

$$\hat{q}_t^E = \min\left(q^{\max}, \max\left(q^{\min}, \frac{1}{2a_2}\left(\frac{p_t^E}{p_t^F} - a_1\right)\right)\right). \tag{13}$$

If there is an option to bid on the AS market, one solves the following problem (see equations (7), (4), (8), and (9)):

$$\max_{q_t^E, q_t^{AS}} \left[p_t^E(q_t^E + \alpha_t q_t^{\mathrm{AS}}) + p_t^{\mathrm{AS}} q_t^{\mathrm{AS}} - C^{\mathrm{AS}}(q_t^E, q_t^{\mathrm{AS}}, q_t^{\mathrm{CAS}}, p_t^F) \right]$$

subject to

$$\begin{aligned} q^{\min} \leq q_t^E &\leq q^{\max} \\ 0 \leq q_t^{\mathrm{AS}} &\leq q^{\max} - q_t^E. \end{aligned}$$

This problem has an analytical solution, too. It can be derived using the standard optimization techniques. Given the constraints, the following candidates for the optimal solution must be considered:

$$\begin{aligned}
q_{t,1}^E &= \min\left(q^{\max}, \max\left(q^{\min}, \frac{1}{2a_2(1-\alpha_t)}\left(\frac{p_t^E}{p_t^F} - \frac{p_t^{AS}}{(1-\alpha_t)p_t^F} - (a_1 + 2a_2\alpha_t q^{\max})\right)\right)\right) \\
q_{t,1}^{\mathrm{AS}} &= q^{\max} - q_{t,1}^E \\
\\
q_{t,2}^E &= q^{\min} \\
q_{t,2}^{\mathrm{AS}} &= \min\left(q^{\max} - q^{\min}, \max\left(0, \frac{1}{2a_2\alpha_t}\left(\frac{p_t^{AS}}{\alpha_t p_t^F} + \frac{p_t^E}{p_t^F} - a_1\right) - \frac{q^{\min}}{\alpha_t}\right)\right) \\
\\
q_{t,3}^E &= \min\left(q^{\max}, \max\left(q^{\min}, \frac{1}{2a_2}\left(\frac{p_t^E}{p_t^F} - a_1\right)\right)\right) \\
q_{t,3}^{\mathrm{AS}} &= 0.
\end{aligned} \tag{14}$$

Among these three cases the optimal solution is the pair $(\hat{q}_t^E, \hat{q}_t^{\mathrm{AS}})$, which gives the greatest value of the objective function.

5.1.2 The Optimal Commitment Problem

In the previous section, we presented the analytical solution of the optimal dispatch problem for the case when the unit is on. Naturally, if the unit is off, no electricity can be produced and no AS can be bid (recall that we are only interested in the online unit analysis). The existence of such a solution reduces the complex problem to the optimal decision making commitment: it is sufficient to find the optimal series of decisions with respect to turning the unit on or off (and complying with the technical constraints at the same time). The actual optimal electricity production and AS bids will then be determined by equations (13) and (14), respectively.

The commitment or decommitment decision cannot be made for every state x_t arbitrarily. The power plant owner can only actually make a decision in states $x_t = t^{\text{on}}$ and $x_t \in [-t^{\text{cold}}, -t^{\text{off}}]$. For other states x_t his decision is driven by the constraints (see equation (5)). We denote any of the states t^{on} and $[-t^{\text{cold}}, -t^{\text{off}}]$ as $\hat{x}_t$.

We proceed as follows: Since the commitment/decommitment decision is driven by the current prices p_t^E and p_t^{AS} and their future expectations, we calculate the *critical prices* $\hat{p}_t^E$ and $(\hat{p}_t^E, \hat{p}_t^{AS})$, that is the prices that can change the commitment/decommitment decision of the power plant owner. This is achieved by solving the equations:

$$J_t(\hat{x}_t, u_t = 1, p_t^E, p_t^F) = J_t(\hat{x}_t, u_t = 0, p_t^E, p_t^F) \tag{15}$$

and

$$J_t^{\text{AS}}(\hat{x}_t, u_t = 1, p_t^E, p_t^{\text{AS}}, p_t^F) = J_t^{\text{AS}}(\hat{x}_t, u_t = 0, p_t^E, p_t^{\text{AS}}, p_t^F) \tag{16}$$

in the case without and with the AS bid opportunity, respectively.

The prices and price pairs that satisfy equations (15) and (16) form the so-called *indifference locus*. We compute the indifference loci for each time period t starting at time T and moving backwards.

This is relatively easy without the presence of the AS market. We do this by finding the root of the function:

$$h(y) = J_t(\hat{x}_t, u_t = 1, y, p_t^F) - J_t(\hat{x}_t, u_t = 0, y, p_t^F) = 0.$$

When the AS market is presented, the indifference locus is formed by the price pairs $(\hat{p}_t^E, \hat{p}_t^{AS})$. Theoretically, there are infinitely many price pairs that fit equation (16). In practice, we set the value of the electricity price $\hat{p}_t^E$ from the pre-specified range and find the corresponding ancillary services price as a root of the equation:

$$h^{\text{AS}}(y) = J_t^{\text{AS}}(\hat{x}_t, u_t = 1, \hat{p}_t^E, y, p_t^F) - J_t^{\text{AS}}(\hat{x}_t, u_t = 0, \hat{p}_t^E, y, p_t^F) = 0.$$

In this way, we obtain the sufficiently dense net of indifference locus points. Hence, we change the problem logic from the continuous space to the discrete space and for practical purposes we approximate the continuous indifference loci using the pre-computed price pairs. This reduces the computation complexity.

Assuming that the indifferent locus is known at time t, the optimal value of the decision variable u_t can be easily determined by comparing the observed actual price p_t^E or price pair (p_t^E, p_t^{AS}), respectively, and the appropriate values from the indifference locus (see Tseng and Barz, 2002 for more details).

Input: starting time point t_0, starting state x_{t_0}, commitment decision for the starting time point u_{t_0}, electricity price for the starting time point $p^E_{t_0}$, fuel price for the starting time-point $p^F_{t_0}$.

Constants: number of simulations $n \gg 0$, ending time point $T \geq t_0$.

Step 1: For $i \leftarrow 1$ to n repeat Steps 2–9.

Step 2: Set $J^{(i)} \leftarrow 0$.

Step 3: For $t \leftarrow t_0$ to T repeat Steps 4–9.

Step 4: If $t = t_0$ then set $x^{(i)}_t \leftarrow x^{(i)}_{t_0}$, $u^{(i)}_t \leftarrow u_{t_0}$, $p^{E(i)}_t \leftarrow p^E_{t_0}$, $p^{F(i)}_t \leftarrow p^F_{t_0}$ and go to Step 7. Otherwise go to Step 5.

Step 5: Obtain the prices p^E_t, p^F_t by simulation.

Step 6: Determine $u^{(i)}_t$ using equation (4). If the unit commitment decision can be made, compare (for the corresponding x_t) the current price p^E_t with the critical price on the indifference locus.

Step 7: Determine the optimal production $\hat{q}^{E(i)}_t$ using equation (13).

Step 8: $J^{(i)} \leftarrow J^{(i)} + f\left(x^{(i)}_t, \hat{q}^{E(i)}_t, p^{E(i)}_t, p^{F(i)}_t\right) - C\left(\hat{q}^{E(i)}_t, p^{F(i)}_t\right) - S_u\left(x^{(i)}_t\right) u^{(i)}_t - S_d\left(x^{(i)}_t\right)\left(1 - u^{(i)}_t\right)$.

Step 9: Determine $x^{(i)}_{t+1}$, $x^{(i)}_{t+\nu}$, $x^{(i)}_{t+\tau}$ using equation (5).

Output: Return $\frac{1}{n}\sum_{i=1}^{n} J^{(i)}$, the average value obtained by simulation.

Figure 1: The algorithm for computing J_t assuming all future indifference loci are known.

The algorithm for computing J_t (and J^{AS}_t) is depicted in Figure 1 (and 2). In order to determine the indifference loci associated with time point t using equations (15) and (16), the computations of J_t (or J^{AS}_t) according to our algorithm are necessary. These computations require knowledge of indifference loci, however, only for the time points $(t+1), \ldots, T$. Therefore, it is possible to compute the indifference loci moving backwards in time, starting at T.

5.2 Modeling Price Processes

In order to perform the algorithms for computing J_t (and J^{AS}_t), the forward simulation of the price processes (for p^E_t and p^{AS}_t) is necessary. (Recall that we do not simulate the price process for fuel price p^F_t since, for simplicity, we assume that the fuel price is constant.) For modeling electricity prices, we consider the hourly data from the Leipzig Power Exchange (LPX) starting on 1 August 2001 until 30 April 2002. Among the different models that describe the electricity price process (see, e.g., Knittel and Roberts, 2001) we follow the analysis of the LPX prices in Cuaresma *et al.*, 2002). The model with the best forecasting performance for the whole time series has the following AR(1) representation:

$$\ln\left(p^E_t\right) = \alpha^E_t + \beta^E \ln\left(p^E_{t-1}\right) + \nu^E_t \tag{17}$$

Input: starting time point t_0, starting state x_{t_0}, commitment decision for the starting time point u_{t_0}, electricity price for the starting time point $p^E_{t_0}$, ancillary services price for the starting time point $p^{\mathrm{AS}}_{t_0}$, fuel price for the starting time point $p^F_{t_0}$.

Constants: number of simulations $n \gg 0$, ending time point $T \geq t_0$, the proportion of the bid AS amount, generated on contingency $\alpha \in \langle 0, 1 \rangle$.

Step 1: For $i \leftarrow 1$ to n repeat Steps 2–9.

Step 2: Set $J^{(i)} \leftarrow 0$.

Step 3: For $t \leftarrow t_0$ to T repeat Steps 4–9.

Step 4: If $t = t_0$ then set $x^{(i)}_t \leftarrow x^{(i)}_{t_0}$, $u^{(i)}_t \leftarrow u_{t_0}$, $p^{E(i)}_t \leftarrow p^E_{t_0}$, $p^{\mathrm{AS}(i)}_t \leftarrow p^{\mathrm{AS}}_{t_0}$, $p^{F(i)}_t \leftarrow p^F_{t_0}$ and go to Step 7. Otherwise go to Step 5.

Step 5: Obtain the prices p^E_t, p^{AS}_t, p^F_t by simulation.

Step 6: Determine $u^{(i)}_t$ using equation (4). If the unit commitment decision can be made, compare (for the corresponding x_t) the current price pair $(p^E_t, p^{\mathrm{AS}}_t)$ with the critical price on the indifference locus.

Step 7: Determine optimal production $(\hat{q}^{E(i)}_t, \hat{q}^{\mathrm{AS}(i)}_t)$ using equation (14).

Step 8: $J^{(i)} \leftarrow J^{(i)} + f^{\mathrm{AS}}\left(x^{(i)}_t, \hat{q}^{E(i)}_t, \hat{q}^{\mathrm{AS}(i)}_t, \alpha\hat{q}^{\mathrm{AS}(i)}_t, p^{E(i)}_t, p^{\mathrm{AS}(i)}_t, p^{F(i)}_t\right) - C^{\mathrm{AS}}\left(\hat{q}^{E(i)}_t, \hat{q}^{\mathrm{AS}(i)}_t, \alpha\hat{q}^{\mathrm{AS}(i)}_t, p^{F(i)}_t\right) - S_u\left(x^{(i)}_t\right) u^{(i)}_t - S_d\left(x^{(i)}_t\right)\left(1 - u^{(i)}_t\right)$.

Step 9: Determine $x^{(i)}_{t+1}$, $x^{(i)}_{t+\nu}$, $x^{(i)}_{t+\tau}$ using equation (5).

Output: Return $\frac{1}{n}\sum_{i=1}^{n} J^{(i)}$, the average value obtained by simulation.

Figure 2: The algorithm for computing J^{AS}_t assuming all future indifference loci are known.

where β^E is assumed to be constant, ν^E_t is a white noise with constant variance $(\sigma^E)^2$ and α^E_t is defined as:

$$\alpha^E_t = \alpha^E + \sum_{i=1}^{24} \alpha^E_{1,i}\, \mathrm{I}_{\,\mathrm{Hour}(t,i)} + \sum_{i=1}^{4} \alpha^E_{2,i}\, \mathrm{I}_{\,\mathrm{Season}(t,i)} + \alpha^E_3\, \mathrm{I}_{\,\mathrm{Weekend}(t)}\,.$$

The predicate $\mathrm{Hour}(t, i)$ is true, if the time point t corresponds to the i-th hour of the day; the predicate $\mathrm{Season}(t, i)$ is true, if the time point t corresponds to the i-th season of the year; and finally the predicate $\mathrm{Weekend}(t)$ is true, if the time point t corresponds to the weekend.

This model captures two important features of electricity price behavior: *mean reversion* and *seasonality*. The estimated parameters of the model are expressed in Table 1.

Table 1: Estimated parameters of the electricity price process.

β^E	0.8873	$\alpha^E_{1,7}$	0.2124	$\alpha^E_{1,15}$	0.0545	$\alpha^E_{1,23}$	0.1289
α^E	0.2349	$\alpha^E_{1,8}$	0.3609	$\alpha^E_{1,16}$	0.0868	$\alpha^E_{1,24}$	0
$\alpha^E_{1,1}$	0.0442	$\alpha^E_{1,9}$	0.2959	$\alpha^E_{1,17}$	0.1695	$\alpha^E_{2,1}$	−0.0232
$\alpha^E_{1,2}$	−0.0588	$\alpha^E_{1,10}$	0.2598	$\alpha^E_{1,18}$	0.3233	$\alpha^E_{2,2}$	−0.0386
$\alpha^E_{1,3}$	0	$\alpha^E_{1,11}$	0.2747	$\alpha^E_{1,19}$	0.2175	$\alpha^E_{2,3}$	0
$\alpha^E_{1,4}$	0	$\alpha^E_{1,12}$	0.3499	$\alpha^E_{1,20}$	0.0895	$\alpha^E_{2,4}$	0
$\alpha^E_{1,5}$	0.0795	$\alpha^E_{1,13}$	0	$\alpha^E_{1,21}$	0.0746	α^E_3	−0.0494
$\alpha^E_{1,6}$	0.1773	$\alpha^E_{1,14}$	0.0818	$\alpha^E_{1,22}$	0	$(\sigma^E)^2$	0.0378

For modeling the reserve price process, we consider the data of positive minute reserve (provided by the RWE grid operator) starting on 1 August 2001 until 30 April 2002. Such reserve is traded in five blocks per day, namely, the following blocks of hours have been stated: 1–4, 5–8, 9–16, 17–20, 21–24.

The selection of the appropriate model for estimating the reserve prices is not straightforward. As the corresponding market has only been open for one year, there are no time series studies of the market prices available at the moment. Nevertheless, bearing in mind the purpose of using this model for simulation (especially as our algorithm requires the simulated process to be Markov), we considered the following representation:

$$\ln\left(p_t^{\mathrm{AS}}\right) = \alpha_t^{\mathrm{AS}} + \beta^{\mathrm{AS}} \ln\left(p_{t-1}^{\mathrm{AS}}\right) + \nu_t^{\mathrm{AS}} \tag{18}$$

where β^{AS} is assumed to be constant and ν_t^{AS} is a white noise with constant variance $\left(\sigma^{\mathrm{AS}}\right)^2$. Since the minutes reserve is traded in blocks, t refers to the block-time in this case.

Again, we consider the seasonal and weekend effect of the block, hence α_t^{AS} is the time varying mean defined as:

$$\alpha_t^{AS} = \alpha^{AS} + \sum_{i=1}^{5} \alpha_{1,i}^{AS}\, \mathrm{I}_{\,\mathrm{Block}(t,i)} + \sum_{i=1}^{4} \alpha_{2,i}^{\mathrm{AS}}\, \mathrm{I}_{\,\mathrm{Season}(t,i)} + \alpha_3^{\mathrm{AS}}\, \mathrm{I}_{\,\mathrm{Weekend}(t)}\,.$$

The predicate Block(t,i) is true, if the time point t corresponds to the i-th block of the day. The meaning of predicate Season(t,i) and Weekend(t) is identical to the electricity price process case. The estimated parameters of the AS price process are listed in Table 2.

5.3 The Parameters of the Turbine

As an application, we evaluate a combined heat and power plant situated in Germany. More precisely, we consider just one turbine and its operational characteristics are listed in Table 3.[7]

[7]Source: BEWAG, Berlin, Germany, which is gratefully acknowledged for providing the parameters of one of their turbines.

Table 2: Estimated parameters of the AS price process.

β^{AS}	0.1159	$\alpha_{1,4}^{\text{AS}}$	0.7104	$\alpha_{2,4}^{\text{AS}}$	0.2967
α^{AS}	0.7389	$\alpha_{1,5}^{\text{AS}}$	0	α_{3}^{AS}	−0.5265
$\alpha_{1,1}^{\text{AS}}$	−0.1513	$\alpha_{2,1}^{\text{AS}}$	0.1525	$(\sigma^{\text{AS}})^2$	0.11
$\alpha_{1,2}^{\text{AS}}$	0	$\alpha_{2,2}^{\text{AS}}$	−0.1470		
$\alpha_{1,3}^{\text{AS}}$	1.7123	$\alpha_{2,3}^{\text{AS}}$	0		

Table 3: The operational parameters of the turbine.

$q^{\min}$	$q^{\max}$	t^{on}	t^{off}	t^{cold}	τ	ν	b_1	b_2	γ	s_d
140 MWh	284 MWh	4h	4h	4h	1h	1h	1900	720	2	220€

The fuel for this turbine is coal. We assume a constant coal price of $5.67E/MMBtu$. The cost function has the following quadratic representation:

$$a_0 = 78.8 \qquad a_1 = 1.98 \qquad a_2 = 0.00111.$$

Therefore,

$$C(q_t^E, p_t^F) = \Big(78.8 + 1.98 q_t^E + 0.00111 \left(q_t^E\right)^2\Big) p_t^F$$
$$C^{\text{AS}}(q_t^E, q_t^{\text{AS}}, q_t^{\text{CAS}}, p_t^F) = \Big(78.8 + 1.98 \left(q_t^E + q_t^{\text{CAS}}\right) + 0.00111 \left(q_t^E + q_t^{\text{CAS}}\right)^2\Big) p_t^F.$$

However, we have to deal with the amount of electricity called on contingency (q_t^{CAS}) or equivalently with the parameter α_t in expression:

$$q_t^{\text{CAS}} = \alpha_t q_t^{\text{AS}}.$$

Since there are currently no real data available for the estimation of q_t^{CAS} or α_t, we consider the following two situations in the numerical analysis of the model:

- Parameter α_t is assumed to be constant (i.e., $\alpha_t = \alpha$). In our simulations we use the value $\alpha = 10\%$.
- Parameter α_t will be generated at random. Since we cannot estimate the real data for α_t, we handle this randomness merely as a numerical experiment. With a probability of 75% we take $\alpha_t = 0$. With a probability of 25% we choose α_t to be a random number from the uniform distribution (the uniformity is taken with respect to the interval $[0, 1]$). This choice corresponds to the real situation (although the numerical values may differ). Once a certain amount of generation capacity has been sold as a reserve, the unit must be prepared to respond to the "call" from the TSO. The TSO will require additional energy when unpredictable disturbances occur in the grid. The required amount is also unpredictable.

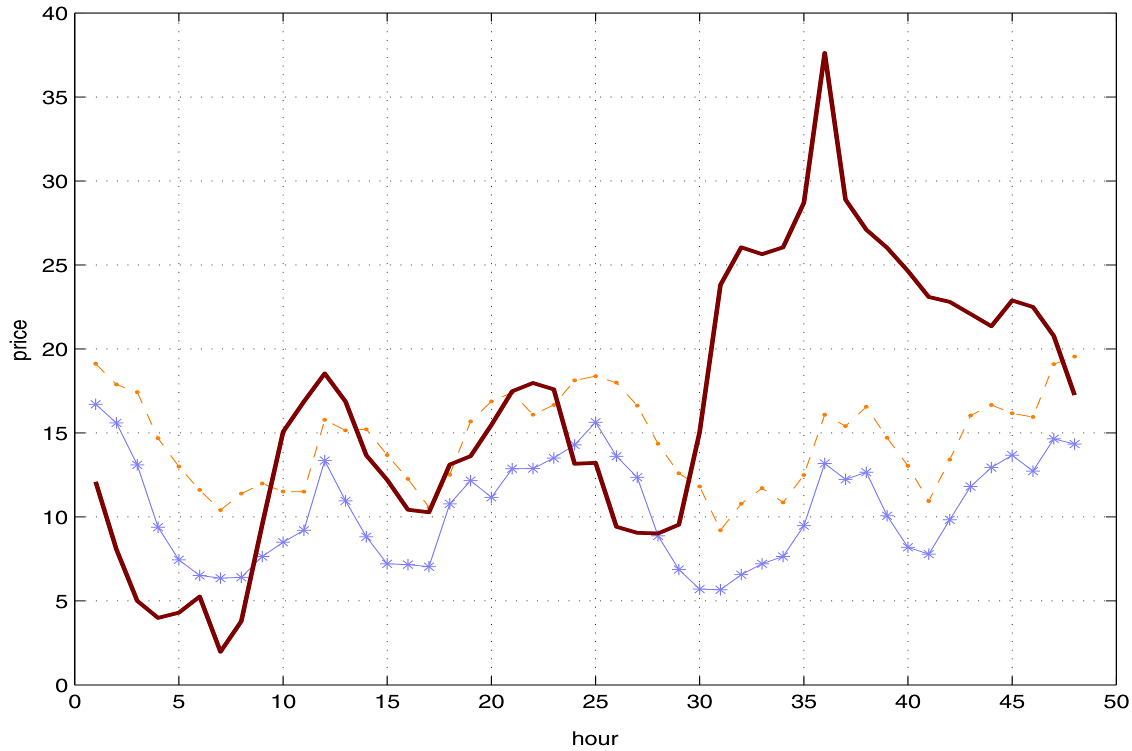

Figure 3: Optimal commitment rules for 28–29 April 2002. The dashed line (with symbol ·) is the turn-on barrier, the solid line (with symbol $*$) is the turn-off barrier and the solid bold line depicts the actual electricity prices.

5.4 Electricity as the Only Output

We start our numerical analysis by studying a situation when electricity is regarded as the only output of the turbine. This is the situation without the possibility of bidding on the AS market. We follow the analysis introduced in Hlouskova *et al.* (2002). In the next section we refer to this analysis and compare it to the two output models introduced in Section 4. The model in which electricity is regarded as the only output, is actually a special case of a two-output model. It is equivalent to the situation when the ancillary service price is zero.

First of all, the indifference loci have to be calculated (see the description in Section 5.1.2). In our case, there are two loci: the first corresponds to $x_t = t^{\text{on}} = 4$ and the second corresponds to $x_t = -t^{\text{off}} = -t^{\text{cold}} = -4$. The first is called a *down locus* or a *turn-off barrier* and the second is called an *up locus* or a *turn-on barrier*. Hence, two curves (the turn-on and the turn-off barriers) have been obtained by simulation, which are depicted in Figure 3, together with the electricity spot prices for 28–29 April 2002.

Following the description in Section 5.1.2, the turn-on barrier indicates that if the spot price is above the curve, and the turbine has already been off for at least its minimum off-time, then it is optimal to turn the turbine on. Similarly, when the spot price is below the turn-off barrier and the turbine has already been on for at least its minimum on-time then the optimal decision is to turn the unit off.

Once the loci have been calculated, following the optimal commitment rules, an ex-

Table 4: Optimal decision rules — electricity as the only output.

Time t	Dec. u_t	State x_t	Prod. $\hat{q}_t^E$	Cum. Profit	Time t	Dec. u_t	State x_t	Prod. $\hat{q}_t^E$	Cum. Profit
1	0	−3	0	0	25	0	−1	0	2239
2	0	−4	0	0	26	0	−2	0	2239
3	0	−4	0	0	27	0	−3	0	2239
4	0	−4	0	0	28	0	−4	0	2239
5	0	−4	0	0	29	0	−4	0	2239
6	0	−4	0	0	30	1	−4	0	2239
7	0	−4	0	0	31	1	1	284	4862
8	0	−4	0	0	32	1	2	284	8117
9	0	−4	0	0	33	1	3	284	11256
10	1	−4	0	0	34	1	4	284	14511
11	1	1	284	648	35	1	4	284	18519
12	1	2	284	1771	36	1	4	284	25061
13	1	3	284	2419	37	1	4	284	29113
14	1	4	194.1	2210	38	1	4	284	32676
15	1	4	140	1777	39	1	4	284	35923
16	1	4	140	1095	40	1	4	284	38778
17	1	4	140	393	41	1	4	284	41196
18	1	4	150.4	88.3	42	1	4	284	43528
19	1	4	190.1	−131	43	1	4	284	45656
20	1	4	284	120	44	1	4	284	47577
21	1	4	284	941	45	1	4	284	49934
22	1	4	284	1902	46	1	4	284	52182
23	1	4	284	2758	47	1	4	284	53941
24	0	4	153.6	2239	48	1	4	284	54697

pected profit of the turbine can be computed. The optimal policy together with cumulative profit calculation for 28–29 April 2002 is shown in Table 4. The data are shown on an hourly basis (hour 1–hour 48), describing the optimal values (rounded) for the decision variable u_t, for the state variable x_t, the optimal electricity production $\hat{q}_t^E$, and the cumulative profit obtained (in €). For the initial setup, we assume that the turbine has already been off for three hours. This means that starting at hour 2, the turbine can be turned on, since this situation complies with the minimum off-time constraint of the turbine. In fact, as illustrated in Figure 3, it is not optimal to turn the turbine on before hour 10, when the spot price for the first time rises above the turn-on barrier. The spot price stays above the turn-off barrier for the rest of the day and falls below at hour 24. At that time, as Table 4 shows, it is optimal to turn the unit off and stay off during the night. Again at 6 a.m. the next day (hour 30), it is optimal to turn the turbine on.

5.5 Two Outputs: Electricity and Reserve

Now consider the simulations including the option to bid on the AS market. We again evaluate the turbine over 48 hours, using data from 28–29 April 2002. The indifference loci must also be calculated in this case (see the description in Section 5.1.2), this time using an approximation. Due to the higher dimension of the problem than before, we are not able to depict the turn-on and turn-off barrier similarly to Figure 3. Nonetheless, the optimal decisions are shown in Tables 5 and 6 for constant and random α_t, respectively. The columns correspond to the respective time point t (hour 1–hour 48), the optimal values for the decision variable u_t, for the state variable x_t, the optimal electricity production $\hat{q}_t^E$ and AS bid $\hat{q}_t^{\mathrm{AS}}$, and the cumulative profit obtained. Additionally, the generated α_t value is shown in Table 6.

When compared to the analysis of the previous section, the cumulative profit has increased in the presence of the AS market, as expected. Moreover, one can observe the changes in decision making of the power plant owner: if more advantageous, the owner naturally bids on the AS market. The power plant is even turned on earlier (hours 10 and 30) than in the case without AS (hours 11, 31).

6 Conclusion

In this paper we presented the real options representation of the valuation model for electricity producers. Two real options were considered, namely (1) unit commitment decision, and (2) ancillary services provision.

The application of the model on the real data has confirmed previous observations from other markets. Participation in ancillary services markets has led to an increase in the operating profit. This fact demonstrates that the ancillary services markets can provide significant revenues to electricity producers and cannot be ignored. Therefore, an understanding of these markets is crucial when applying the real option analysis methodology.

The main advantage of the real options approach is its flexibility of including a wide range of options — our model can easily be extended to other types of ancillary services, which are handled by the market. For instance, it could cover the negative minute reserve traded on the German market. Alterations of the model constraints, due to other operational real options and new requirements, is another direction of future research.

Table 5: Optimal decision rules in the case of electricity and AS markets ($\alpha = 10\%$).

Time t	Dec. u_t	State x_t	Prod. $\hat{q}_t^E/\hat{q}_t^{\mathrm{AS}}$	Cum. profit	Time t	Dec. u_t	State x_t	Prod. $\hat{q}_t^E/\hat{q}_t^{\mathrm{AS}}$	Cum. profit
1	0	−3	0/0	0	25	0	−1	0/0	4545
2	0	−4	0/0	0	26	0	−2	0/0	4545
3	0	−4	0/0	0	27	0	−3	0/0	4545
4	0	−4	0/0	0	28	0	−4	0/0	4545
5	0	−4	0/0	0	29	1	−4	0/0	4545
6	0	−4	0/0	0	30	1	1	140/144	5997
7	0	−4	0/0	0	31	1	2	140/144	8800
8	0	−4	0/0	0	32	1	3	280/0	12056
9	1	−4	0/0	0	33	1	4	280/0	15195
10	1	1	140/144	338	34	1	4	280/0	18450
11	1	2	234/50	999	35	1	4	280/0	22458
12	1	3	284/0	2122	36	1	4	280/0	28999
13	1	4	234/50	2783	37	1	4	280/0	33061
14	1	4	140/144	2905	38	1	4	280/0	36615
15	1	4	140/144	2801	39	1	4	280/0	39862
16	1	4	140/144	2423	40	1	4	280/0	42717
17	1	4	140/144	1921	41	1	4	280/0	45135
18	1	4	140/144	1857	42	1	4	280/0	47467
19	1	4	140/144	1871	43	1	4	280/0	49595
20	1	4	179/105	2178	44	1	4	280/0	51516
21	1	4	284/0	3000	45	1	4	280/0	53874
22	1	4	284/0	3960	46	1	4	280/0	56121
23	1	4	284/0	4816	47	1	4	280/0	57880
24	0	4	140/144	4545	48	1	4	280/0	58636

Table 6: Optimal decision rules in case of electricity and AS markets (α_t random).

Time t	Prop. α_t	Dec. u_t	State x_t	Prod. $\hat{q}_t^E/\hat{q}_t^{AS}$	Cum. profit	Time t	Prop. α_t	Dec. u_t	State x_t	Prod. $\hat{q}_t^E/\hat{q}_t^{AS}$	Cum. profit
1	0.62	0	−3	0/0	0	25	0.50	0	−1	0/0	4723
2	0.04	0	−4	0/0	0	26	0	0	−2	0/0	4723
3	0	0	−4	0/0	0	27	0.55	0	−3	0/0	4723
4	0	0	−4	0/0	0	28	0	0	−4	0/0	4723
5	0	0	−4	0/0	0	29	0.70	1	−4	0/0	4723
6	0	0	−4	0/0	0	30	0.09	1	1	140/144	6173
7	0.87	0	−4	0/0	0	31	0.24	1	2	140/144	9197
8	0	0	−4	0/0	0	32	0	1	3	284/0	12452
9	0	1	−4	0/0	0	33	0.74	1	4	140/144	16324
10	0	1	1	140/144	310	34	0	1	4	284/0	19579
11	0	1	2	260/24	961	35	0	1	4	284/0	23587
12	0.99	1	3	140/144	2425	36	0	1	4	284/0	30128
13	0	1	4	260/24	3077	37	0.12	1	4	284/0	34190
14	0	1	4	140/144	3191	38	0.87	1	4	140/144	38666
15	0.95	1	4	140/144	2875	39	0	1	4	284/0	41913
16	0	1	4	140/144	2534	40	0.023	1	4	284/0	44768
17	0.70	1	4	140/0	1832	41	0	1	4	284/0	47186
18	0	1	4	140/144	1768	42	0	1	4	284/0	49518
19	0	1	4	140/144	1773	43	0	1	4	284/0	51646
20	0.30	1	4	140/144	2134	44	0	1	4	284/0	53567
21	0	1	4	284/0	2956	45	0	1	4	284/0	55925
22	0	1	4	284/0	3916	46	0	1	4	284/0	58172
23	0.93	1	4	140/144	4995	47	0	1	4	284/0	59931
24	0	0	4	140/144	4723	48	0.03	1	4	284/0	60687

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