

## Interim Report

IR-04-041

---

### The SEDIM Model: Version 0.1

Warren Sanderson ([wsanderson@notes.cc.sunysb.edu](mailto:wsanderson@notes.cc.sunysb.edu))

---

#### Approved by

Wolfgang Lutz ([lutz@iiasa.ac.at](mailto:lutz@iiasa.ac.at))  
Leader, World Population Program

August 27, 2004

## Contents

1. Introduction .....	1
2. The Model Equations .....	3
2.1. The determinants of $L(t)$ .....	4
2.2. The determinants of $K(t)$ .....	6
2.3. Technology .....	11
2.4. Some thought experiments .....	12
3. Illustrative Examples for Egypt.....	14
4. Concluding Thoughts .....	16
5. References .....	16
Appendix: SEDIM, Version 0.1 .....	20

## **Abstract**

This paper presents the SEDIM (Simple Economic Demographic Interaction Model) Model. The model illustrates how economic, demographic, and technological factors can be integrated into a rich, but still relatively simple framework that predicts rates of economic growth. SEDIM is designed to show the effects of demographic changes, including changes in the education composition of the population on economic performance. The model includes age- and labor market experience-based productivity rates, policy-related changes in retirement ages, old age support from intergenerational transfers, public pension systems, private capital accumulation, and endogenous technological change. The model is designed for studying out of equilibrium dynamics. SEDIM can be used to investigate historical patterns of economic development as well as for making scenarios for the future. The advantage of the model is that it can be relatively easily parameterized for a variety of countries so that it can be used to discuss the determinants of economic growth in a consistent comparative framework. It is roughly parameterized for Egypt and a few forecasts for Egypt's economic growth are shown.

## **About the Author**

Dr. Warren Sanderson is from the Departments of Economics and History at the State University of New York at Stony Brook, USA.

# The SEDIM Model: Version 0.1

Warren Sanderson

## 1. Introduction

Within the last decade, there has been remarkable progress in assessing the contribution of demography to economic growth. The long and rather sterile debate on the effects of population growth has been replaced with a more nuanced view. Empirical studies have now shown that population age structure does have an effect on economic growth (see, for example, Bloom et al. 2000; Bloom and Williamson 1998; Kelley and Schmidt 2003). Demography does not solely produce our economic destiny, but it is certainly one of the factors influencing economic growth. While empirical studies have made it clear that age structure matters, theoretical studies, such as Bommier and Lee (2003), have been bringing more realistic age structured behavior into economic models. In addition, Lutz et al. (2004) introduced the concept of “population balance” and showed that the age structures associated with both rapid population growth and rapid population aging could result in lower welfare than an age structure associated with more moderate demographic change. As a result of this work, the study of the demographic effects on economic growth has, once again, become intellectually exciting.

Pinpointing the age structure of the population as the main route through which demography affects the economy is hardly new. Coale and Hoover (1958) already did this nearly half a century ago. They argued that rapid population growth changes the age structure of the population in such a way that the youth dependency rate increases, leading to a diversion of capital from presumably more productive uses such as construction to presumably less productive uses such as education. The problem with these sorts of theoretical arguments is that they could be quite misleading. Having more children may increase the costs of education. Another way of saying the same thing is that in societies with more children, more money is invested in children. If investments in the human capital of children are very productive, a reallocation of resources from other uses to investments in education could have a large positive payoff in the long run.

The task of this paper is to provide a framework in which theory and empirical studies on economic-demographic interactions can be brought together in a simple enough way that each can aid in the understanding of the other. This last proviso is important. There are many empirical models in which age structured populations enter as a determinant of economic growth. One of the earliest of these is the World3 model that was the basis of the widely read book “The Limits to Growth” (Meadows et al. 1972). A recent contribution is MacKellar et al. (2004). A number of these have been reviewed elsewhere (see, for example, Sanderson 1980, 1994). But these models were

not designed to look into a wide variety of histories of economic-demographic interactions in order to find common features. Normally, they were created to analyze the conditions specific to a given country or region.

Designing a unifying framework for the understanding of a wide variety of historical experiences and for making forecasts on the basis of that understanding implies a particular set of constraints. First, the model has to be applicable in situations that are out of equilibrium (both in an economic sense and a demographic sense), because those are the only histories that we have observed. Second, the model has to incorporate the main factors influencing economic growth. Recent studies of economic growth, including the most recent ones on economic-demographic interactions, find that institutions and policies are important. Thus, the model needs some way to represent the two of them. Third, the model structure has to be general enough that it does not require *ad hoc* adjustments from country to country (parameter values, of course, will differ by country). Thus, SEDIM needs to be as applicable to countries with rapid population growth as it is to populations with rapid population aging.

Creating a framework for understanding, on the other hand, has dimensions in which it is liberating. The model presented here is not a model designed for parameter estimation. Such a model can only include variables that have been measured across a wide variety of countries. Our framework does not have to be so narrowly constrained. For example, the SEDIM model incorporates age-specific levels of education. For many countries, these data do not exist. Age-specific levels of education in SEDIM can, however, be combined into aggregated measures of educational attainment that can often be compared with observed information. In SEDIM there is no need to utilize variables only if they are available for all countries of interest.

SEDIM is not designed to be a balanced model. It is intended to be a model in which demographic influences on economic growth are highlighted. Therefore, it has much more detail about population than other factors influencing economic growth. For example, SEDIM considers the age structure of the population, but does not incorporate the age structure of the capital stock. SEDIM also explicitly takes the age at which people retire into account, but not the age at which capital is taken out of service.

The main contribution of the SEDIM model is in its simplifications of complex relationships. This inevitably involves a certain amount of apparent messiness, because it is easy to think of more correct and more complex formulations than the ones used here. The true test of the model is whether it captures enough about the effects of demographic change on economic growth to enhance our understanding of the process. With the current model, we are not yet in a position to carry out this test.

In this paper, version 0.1 of the SEDIM model is presented. This way of denoting the model clearly indicates two things. First, this is an extremely early version of the model. Hopefully, more refined versions will follow. Second, even by the most generous of standards, SEDIM version 0.1 is seriously incomplete. In its present form, SEDIM is a closed economy (and closed demography) model. No history of economic-demographic interactions in the last half century can seriously be written which omits foreign trade and capital flows. Nevertheless, it is still useful to document the model in its present form, if only to record one step on the way to a more complete and satisfactory formulation.

Section 2 of this paper contains the equations of SEDIM 0.1. Section 3 contains an application to Egypt and the final section contains concluding comments. The Appendix contains the computer code.

## 2. The Model Equations

The SEDIM model has four elements: (1) endogenous variables, (2) scenario variables, (3) initial conditions, and (4) parameters and switches. The endogenous variables are the ones that we are most interested in studying. They include real gross domestic product (GDP), the growth of real GDP, savings, investment, and technological change. Scenario variables are those variables that express a vision of how the drivers of the endogenous variables will change over time. There are two kinds of scenario variables, those that are exogenous and those that are functions of other scenario variables, initial conditions, and parameters. Scenario variables influence endogenous variables, but endogenous variables never influence them. For example, the number of people at each age is a scenario variable. It is influenced by fertility and mortality rates (themselves scenario variables) and by the initial age distribution of the population. The population age structure influences the rate of GDP growth, but the rate of GDP growth does not influence the age structure.

Initial conditions all have a time reference of year zero. When we are considering real countries or regions, the initial conditions need to be set so as to be as close as possible to their actual values. When we are studying hypothetical situations, the initial conditions become more like scenario variables, because they can be set at different values to study various phenomena. Parameters reflect the structure of economic-demographic interactions. Again, when we are dealing with specific countries, the parameters need to be as realistic as possible. They are not matters of choice. In general, parameters do not change over time. In SEDIM 0.1 there are no time-varying parameters, although in principle time-varying parameters are permitted.

In the equations below, we denote endogenous variables with italicized upper case letters. Scenario variables are always written with upper case letters in Times New Roman font. Initial conditions are distinguished by writing them as referring only to time zero. Parameters and switches are denoted by lower case letters. Most often the parameters are given using Greek letters.

There are two sorts of equations in the model, equations with endogenous variables on their left-hand sides and equations with scenario variables on their left-hand sides. The former are the core equations of the model. Changing them changes the fundamental structure of the model. The latter can easily be changed because they simply state different visions of the future.

Output in the model is specified using a Cobb-Douglas production function:

$$Y(t) = A(t) \cdot L(t)^\alpha \cdot K(t)^{1-\alpha}, \quad (1)$$

where  $Y(t)$  is GDP at time  $t$ ,  $A(t)$  is the level of productivity at time  $t$ ,  $L(t)$  is the effective labor input in time  $t$ ,  $K(t)$  is the capital stock in time  $t$ ,  $\alpha$  is the share of GDP at time  $t$  that is paid to labor, and  $1-\alpha$  is the share of GDP at time  $t$  that is paid to owners of capital. This specification is standard in simple models of economic growth.

Demography can only affect economic growth by changing the three variables on the right-hand side of Eq. (1). We use this distinction to guide our presentation of the model. In Section 2.1, we present the determinants of  $L(t)$ . In Section 2.2, we present the determinants of  $K(t)$ , and in Section 2.3 the determinants of  $A(t)$ .

## 2.1. The determinants of $L(t)$

$L(t)$  is a scenario variable. SEDIM 0.1 contains a population projection module that determines the future age structure of the population based on scenarios for fertility and mortality.  $L(t)$  also depends on education, the productivity of workers, labor force participation rates and the ages of entry and exit from the labor market. All of these, however, are also scenario variables. SEDIM 0.1 assumes that education, total fertility rates, labor force participation rates, and ages of entry and exit from the labor market of scenario variables change only on a cohort basis. This is an important simplification. These variables are interrelated among themselves. For example, total fertility rates, labor force participation rates and the ages of entry and exit from the labor market could all depend on education.

Labor input at time  $t$  is the sum over the working ages of the total number of efficiency units of labor employed in the economy. We write:

$$L(t) = \sum_{a=alfe}^{alfx} [\text{POP}(a, t) \cdot \text{EU}(a, t) \cdot \text{LFPR}(a, t)], \quad (2)$$

where  $L(t)$  is labor input in year  $t$ ,  $\text{POP}(a, t)$  is the population at age  $a$  in year  $t$ ,  $\text{EU}$  is the number of efficiency units embodied in each worker of age  $a$  in year  $t$ ,  $\text{LFPR}$  is the labor force participation rate of people of age  $a$  in year  $t$ ,  $alfe$  is the earliest age at labor force entry and  $alfx$  is the latest age at labor force exit. In SEDIM 0.1, labor force participation rates and the ages of labor force entry and exit are fixed. Allowing for variable labor force participation rates, and ages at labor force entry and exit are items that are on the priority list for SEDIM 0.2.

Human capital is a central concern of SEDIM 0.1. People are treated as being heterogeneous with respect to their labor market productivity. We represent this by stipulating that people with different levels of education and different numbers of years of labor market experience contribute different numbers of efficiency units to the aggregate labor input.

Eq. (2) has no subscript for gender. SEDIM 0.1 is a one-sex model. This simplifies matters a great deal, but the matter goes deeper than this. Savings plays an important role in any economic growth model and savings is normally studied in the context of households not individuals. SEDIM 0.1 does not include a translation from individuals to households. This would be far too cumbersome. Nevertheless, it is more plausible here to aggregate the sexes than it would be to study the income and savings paths of individuals.

In SEDIM 0.1, the number of efficiency units per worker depends on the worker's education and the worker's years of labor market experience. We have implemented this using a functional form that is similar to those used in the study of wage rates:

$$\begin{aligned} \ln(EU(a, t)) = & b1 + b2 \cdot EDUC(a, t) + b3 \cdot EDUC(a, t)^2 + b4 \cdot YLME(a, t) + \\ & b5 \cdot YLME(a, t)^2 + b6 \cdot EDUC(a, t) \cdot YLME(a, t) + \\ & b7 \cdot EDUC(a, t)^2 \cdot YLME(a, t) + b8 \cdot EDUC(a, t) \cdot YLME(a, t)^2 + \\ & b9 \cdot EDUC(a, t)^2 \cdot YLME(a, t)^2 \end{aligned} \quad (3)$$

where  $EU(a,t)$  is the number of efficiency units per worker of age  $a$  in year  $t$ ,  $EDUC(a,t)$  is the education of workers of age  $a$  in year  $t$  measured in terms of the number of years of education,  $YLME$  is the years of labor market experience of workers of age  $a$  in year  $t$ . The nine parameters  $b1$  through  $b9$  were determined by fitting Eq. (3) to nine values of  $EU(a,t)$  that were plausible for Egypt. There are many ways of specifying the relationship between productivity, education, and labor market experience. Eq. (3) gives reasonable results, but other formulations can easily be substituted for it.

SEDIM 0.1 takes the distribution of the population in the base year by education as one of its initial conditions. The subsequent evolution of the educational attainment of cohorts is determined in the scenario.

The population projection part of the model is designed to be able to make forecasts for a diverse set of countries. A standard pattern of age-specific fertility rates is used and applied to cohort total fertility rates. In SEDIM 0.1, the pattern assumed by the U.S. Census Bureau of Egypt in 2050 was used. Cohort total fertility rates are assumed to be a function of cohort education levels. Thus, the mean age at childbearing in each calendar year depends on the speed with which education is changing and fertility tempo effects as well as quantum effects are incorporated in the model. An intriguing alternative to the use of a fixed standard pattern of age-specific fertility rates is to incorporate some of the ideas in Schmertmann (2003), but this too must await further versions of the model.

A logistic function was used to relate cohort fertility to cohort education:

$$CTFR(c) = \frac{(uctfr - lctfr) \cdot \exp(d1 - d2 \cdot EDUC(c))}{1 + \exp(d1 - d2 \cdot EDUC(c))} + lctfr, \quad (4)$$

where  $CTFR(c)$  is the cohort total fertility rate for people born into cohort  $c$ ,  $uctfr$  is the upper asymptote of the cohort total fertility rate,  $lctfr$  is the lower asymptote of the cohort total fertility rate,  $EDUC(c)$  is the education of people in birth cohort  $c$ , and  $d1$  and  $d2$  are parameters. If the cohort total fertility rate is to decline with increases in education the sign of  $d2$  must be positive. The parameters  $uctfr$ ,  $lctfr$ ,  $d1$ , and  $d2$  were fit to Egyptian historical education and total fertility rate patterns. Because cohort total fertility rates are scenario variables, there are many possibilities of how to relate them to levels of education.

In contrast to fertility, mortality rates are specified on a period basis. In order to fit age-specific mortality rates to the wide variety of conditions around the world, two types of mortality were considered, early mortality, which in SEDIM 0.1 refers to ages 0 through 24 and senescent mortality that pertains to mortality at ages 25 and above. Nowadays almost all early mortality takes place before the normal age of school entrance and there is no significant mortality between the normal age at school entrance and 24. The probability of surviving to one's 25<sup>th</sup> birthday is a scenario variable. For simplicity, is applied to the number of births in each year. This distorts the number of

people in the population below the age of six somewhat, but this distortion is irrelevant for the measurement of economic growth.

Deaths at age 25 and above are assumed to follow a normal distribution with means and standard deviations that are scenario variables. The mean of that normal distribution minus 25 is the value of life expectancy at 25 if no further improvements in mortality conditions occurred. Using just those three parameters, the probability of surviving to age 25, the mean and the standard deviation of the age at death (at age 25 and above), SEDIM 0.1 can closely approximate almost all of the mortality conditions in the world today, except in those countries with high rates of HIV/AIDS prevalence. If SEDIM were to be applied to one of those countries, an alternative specification of mortality rates would have to be used (see Sanderson 2004, for example). Currently, SEDIM does not incorporate education-specific survival rates, although this would not be hard to do. In the Egyptian example, the mean of the age distribution of senescent deaths increases by two years per decade and the standard deviation is fixed. The initial mean of the age at death distribution in the Egyptian example is consistent with Egypt's observed life expectancy.

## 2.2. The determinants of $K(t)$

The evolution of the capital stock forms the most complex portion of the SEDIM 0.1 model. Capital is the only asset in the model. It changes because of two forces, investment and depreciation. Changes in the capital stock are expressed in the following equation:

$$K(t+1) = K(t) - \delta \cdot K(t) + IG(t), \quad (5)$$

where  $K(t)$  is the capital stock in year  $t$ ,  $\delta$  is the depreciation rate, and  $IG(t)$  is gross investment in year  $t$ . Throughout when we speak about investment we mean gross investment (the  $IG(t)$  in Eq. (5)). Investment can never be negative, so that a country's capital stock can never fall faster than  $\delta$  percent per year. The capital stock at the beginning of year 0,  $K(0)$  is an initial condition of the model. Since SEDIM 0.1 is a closed economy model and since there is only one asset in the model, all savings are invested.

There are two types of savers in the model, lifecycle savers and non-lifecycle savers, and three types of capital holders. We call them: (1) people, (2) corporations, and (3) government, although these names imply a sharper distinction than we really have in mind. Formally, we write:

$$K(t) = PK(t) + CK(t) + GK(t), \quad (6)$$

where  $PK(t)$  is the capital stock held by lifecycle savers at the beginning of period  $t$ ,  $CK(t)$  is the capital held by private non-lifecycle savers at the beginning of period  $t$ , and  $GK(t)$  is the stock of capital held by the government at the beginning of period  $t$ . The values of  $PK(0)$ ,  $CK(0)$ , and  $GK(0)$  are initial conditions.

People save when their incomes are relatively high in order to smooth consumption, especially to provide themselves with income during their old age when they are no longer able to work. There are three main ways old age support is provided:

(1) through intrafamily transfers, (2) through private savings, and (3) through public pension plans. SEDIM 0.1 incorporates all three of these in the simplest possible form.

Let us consider first the case where there is no private savings (perhaps because of the insecurity of the financial sector) and no government pensions. There old age support comes from families. In SEDIM 0.1, there is a social old age support ratio that links the pre-retirement standard of living to the post-retirement standard of living. Children bear the burden of providing that traditional level of support. SEDIM 0.1 does not link parents with their children, but instead views workers as having a generalized obligation to provide for the support of the elderly. In essence, the amount of consumption that the workers have to forego in order to support the elderly is the same in this case as in a pay-as-you go retirement system. Of course, obligations on individual children are different under a system of intrafamily transfers for the support of the elderly than under a pay-as-you-go pension system, but in the aggregate, when there are no private savings, the effects on economic growth of the two systems of old age support are the same. Mixed systems of old age support are common and SEDIM can be made to simulate transitions from situations where one form of old age support dominates to situations where others are more important.

People's saving behavior is forward looking. In each year, they consider their asset holdings and their expected future incomes including public pensions and intergenerational transfers and decide on a pattern of expected savings and consumption that smoothes their consumption levels over their planning periods.

We begin our more formal discussion of lifecycle savings with the determination of the gross return to capital and of the wage rates received by people of differing ages and education levels. The return to capital is computed from:

$$R(t) = \frac{(1 - \alpha) \cdot Y(t)}{K(t)}, \quad (7)$$

which simply restates in equation form that capital's share of output  $Y(t)$  is  $(1 - \alpha)$ .

Age- and education-specific relative wage rates are determined by the same kind of specification found in Eq. (3). Indeed, if the same parameters are used in both, then wages would be proportional to productivities. This was the assumption that was used in the Egypt example below. However, in some countries entitlement to income depends more on seniority than productivity. In this case, the nine parameters would have to be different, but the mathematics is exactly the same except relative wages rather than productivities are on the left-hand side of the equation. We have left the distinction between relative wages and relative productivities to a later version of the model.

The wage rate per efficiency unit of labor can be expressed:

$$W_{eu}(t) = \frac{\alpha \cdot Y(t)}{L(t)}, \quad (8)$$

where  $L(t)$  comes from Eq. (2) above. The wage rate of a particular person of age  $a$  in year  $t$  is:

$$W(a, t) = W_{eu}(t) \cdot EU(a, t) \quad (9)$$

There are a number of important things concerning the wage rate equation that are worth noting here. First, the wage rate of an individual depends on the person's education and labor market experience, because those factors influence the  $EU(a,t)$ . Second, the wage rate also depends on macro-level variables such as the level of technology and the size of the capital stock, because both of those influence  $Y(t)$ . Third, holding age-specific levels of education fixed, the wage rate also depends on the age structure of the labor force, because the age structure influences  $L(t)$ . In the savings specification, people make forecasts of their future wage rates based on two things: (1) an understanding of how their relative productivities (their  $EU(a,t)$ ) will evolve as they grow older and a forecast of how fast wages in general (the  $W_{eu}(t)$ ) will grow.

People do not get to spend all of their labor income. Some of it is spent supporting their elderly parents, either directly through intergenerational transfers or indirectly through state pension systems. Some of it is spent supporting their children, and particularly on their children's education. We denote the proportion of wage income spent on children and the elderly in year  $t$  as  $TAX(t)$ , although it is important to keep in mind that much of the money spent on children and elderly is not collected by the government. We will discuss  $TAX(t)$  in more detail later in this subsection.

Forecasted disposable wages, then, can be written:

$$\overline{W_d(a,t)} = \overline{W_{eu}(t)} \cdot EU(a,t) \cdot (1 - \overline{TAX(t)}), \quad (10)$$

where the bars over a variable indicate that they are forecasted, the year  $t$  indicates some year in the future, the subscript in the variable  $\overline{W_d(a,t)}$  indicates that it is computed after allowance for contributions to children and parents, and the  $EU(a,t)$  are considered to be known. In the Egyptian example below, the predicted value of  $TAX(t)$  is assumed to be the current value and  $\overline{W_{eu}(t)}$  is predicted using the average growth in wages (in efficiency units) over the previous five years.

In making savings decisions, people have to forecast not only their wages while they work, but the amount of pension income they will receive from the government and how much they will receive in terms of intergenerational transfers. To see how this forecasting works in SEDIM 0.1, consider people who plan on retiring at some time in the future. We call the age at retirement for people born in year  $c$ ,  $alfx(c)$ , which may or may not vary with the characteristics of the people born in that year. In the last year of working life, people born in year  $c$  have a forecasted disposable income of

$$\begin{aligned} \overline{W_d(alfx(c), c + alfx(c))} &= \overline{W_{eu}(c + alfx(c))} \cdot EU(alfx, c + alfx(c)) \cdot (1 - \overline{TAX(c + alfx(c))}) \\ &= \overline{W_{d,r}(c)} \end{aligned}, \quad (11)$$

and for simplicity this is denoted  $\overline{W_{d,r}(c)}$ .

In retirement, people in cohort  $c$  expect to obtain, either from their children or from the government, some fixed proportion of  $\overline{W_{d,r}(c)}$ , adjusted for average wage growth. We call this fixed proportion the replacement ratio,  $RR$ , and it is an important scenario variable. For people who have not yet retired, their forecasted retirement income intergenerational transfers and public pensions is:

$$\overline{TR(a,t)} = \overline{W_{d,r}(c)} \cdot RR \cdot \frac{\overline{W_{eu}(t)}}{\overline{W_{eu}(c + alfx(c))}}, \quad (12)$$

where  $\overline{TR(a,t)}$  is the predicted transfer income that a retired person of age  $a$  in year  $t$  is expected to receive,  $\overline{W_{d,r}(c)}$  is the predicted wage income of the person in the last year of his/her working life after transfers to other generations,  $RR$  is the replacement ratio, and  $\frac{\overline{W_{eu}(t)}}{\overline{W_{eu}(c + alfx(c))}}$  is the expected growth in wages from the year of retirement to time  $t$ , when the transfer income is to be received. For people who have already retired, everything in Eq. (12) is already known except for the forecasted future level of wages,  $\overline{W_{eu}(t)}$ .

At each age from labor market entry onward, people make a decision on how much to spend and how much to save based on their previously acquired capital stock (determined by their past savings), their expected future disposable incomes while working on their transfer incomes when retired, and the time horizon of their decision-making. The time horizon for different generations is different because life expectancy changes. In SEDIM 0.1, we assume that people know the current mean and standard deviation of the age distribution of deaths from senescent mortality and have a planning horizon such that covers the lower  $ph$  (for planning horizon) percent of that distribution. For example, let  $ph$  be 10 percent and let  $\text{prob}(a^*)$  be the probability that the person survives to age  $a^*$  or beyond, given the current mean and standard deviation of the age at death distribution. The end of the planning horizon for the person then is that value of  $a^*$  such that  $\text{prob}(a^*)$  is equal to 10 percent. The planning horizon changes from year to year even for people of a specific cohort. It changes because life expectancy changes and it changes because the person did not die in the past year and therefore is facing a different conditional age distribution of dying than was faced in the previous year. We call the final year considered in a person's savings decision  $T(c,t)$  and it is also a scenario variable.

We assume that lifecycle savers save only for the purpose of smoothing consumption. They have no bequest motive for saving.<sup>1</sup> This implies that at the end of their planning horizon they should have zero capital left. This zero capital condition makes it easy for us to compute the smoothed consumption path of people, because the discounted value of the initial capital stock held by people and the discounted value of future predicted incomes has to equal the discounted value of consumption, where the discount rate is the current rate of return to capital after accounting for depreciation. In less technical language, if people are going to have nothing left at the end of their planning horizons then they must consume as much as they earn (including the value of their initial capital stocks).

Smoothed consumption predicted at time  $t$  for a person at age  $a$  can be computed as:

---

<sup>1</sup> People do, however, make *inter vivos* transfers to their children.

$$CONS_s(a,t) = \frac{RN(t)}{[I + RN(t)]^{TR(a,t)} - I} \cdot \prod_{age=a}^{a+TR(a,t)} [I + RN(t)]^{TR(a,t)-age} \cdot INCWT(age,t), \quad (13)$$

where  $CONS_s(a,t)$  is the value of smoothed consumption for a person of age  $a$  on the basis of incomes and rates of return forecasted in period  $t$ .  $RN(t)$  is the net return per unit of capital. It is the  $R(t)$  in Eq. (7) minus the rate of depreciation.  $PK(a,t)$  is the amount of capital that people of age  $a$  have at the beginning of period  $t$ .  $TR(a,t)$  is the number of years remaining in the time horizon of a person who is of age  $a$  in year  $t$ . It is the difference between the last year of the time horizon  $T(t-a,t)$  and current age  $a$ .  $INCWT(a,t^*)$  is the forecasted disposable income from wages or transfers obtained by a person of age  $a$  in year  $t^*$  (but still forecasted in year  $t$ ). When the age refers to a time prior to retirement, the prediction is taken from Eq. (10) and when the age refers to a time at which the person is retired the prediction is taken from Eq. (12).

$PK(a,t)$  is never allowed to be negative. This means that

$$CONS_s(a,t) - INCWT(a,t) \leq PK(a,t) \cdot (I + R(t)) \quad (14)$$

If the inequality in Eq. (14) holds, then

$$PSAVINGS(a,t) = INCWT(a,t) + R(t) \cdot PK(a,t) - CONS_s(a,t), \quad (15)$$

where  $PSAVINGS(a,t)$  is the amount of savings done by a person of age  $a$  at time  $t$ . If the inequality in Eq. (14) is not true, then

$$PSAVINGS(a,t) = -PK(a,t) \cdot (I + R(t)). \quad (16)$$

In this case, people dissave by selling off all their capital stock and consuming all returns to capital that they obtain before all their capital is sold.

The dynamics of personal capital accumulation simply is:

$$PK(a+1,t+1) = PK(a,t) \cdot (I - \delta) + PSAVINGS(a,t). \quad (17)$$

The total amount of capital held by lifecycle savers is determined by summing the age-specific amounts of capital held over all the workers:

$$PK(t) = \sum_{a=alfe}^{\omega} [POP(a,t) \cdot LFPR(a,t) \cdot PK(a,t)], \quad (18)$$

where  $\omega$  is the oldest age ever attained.

In order to make forecasts of their future income streams, workers take into account the amount of money they spend on the education of children, the initial capital endowment they give their children (if any), the amount that they spend on pay-as-you-go pension systems, and the amount that they spend on intergenerational transfers to their elderly parents. We denote the sum of all the money that workers spend on children and the elderly in period  $t$  as  $TFW(t)$ , which stands for transfers from workers. The variable  $TAX(t)$  that appears in Eq. (10) is defined as the ratio of transfers from workers to total labor income:

$$TAX(t) = \frac{TFW(t)}{\alpha \cdot Y(t)}. \quad (19)$$

In each year, some younger people are acquiring capital to support their old age consumption and some older people are selling capital for exactly the same purpose. If, on the whole, people want to sell more capital in a year than they wish to acquire, the corporate sector and the government sector must first buy their capital before investing in new capital.

The dynamics of corporate capital accumulation is:

$$CK(t+1) = CK(t) \cdot (1 - \delta) + CK(t) \cdot R(t) \cdot (1 - \kappa), \quad (20)$$

where  $\kappa$  is the proportion of corporate income that gets consumed. Corporations, or more generally private non-lifecycle savers, receive income from capital. A share of that,  $(1 - \kappa)$  is reinvested and a share,  $\kappa$ , is not. In SEDIM 0.1, income can either be consumed or invested; there is no other choice. So,  $\kappa$  is called the corporate consumption coefficient because it is the fraction of corporate income that is consumed. Broadly speaking  $\kappa$  can be thought of as reflecting the investment climate in a country. If the investment climate is poor,  $\kappa$  is high and a substantial fraction of corporate income is not reinvested. In a model that included capital flows, some of the income of non-lifecycle savers that was not reinvested in the country in which it was earned would naturally find its way into Swiss bank accounts.

Government capital accumulation was in the process of being added to the SEDIM model, version 0.1, when development of the model stopped. Currently, it is not fully implemented, although completing the implementation would not be difficult. The main question in doing so is whether it would add anything interesting to the model. The model does not include government debt. To add it in a natural way would be to include a second asset in the model and this would cause serious complications.

### 2.3. Technology

Technological developments in SEDIM are based on three components: (1) a global technology frontier, (2) a conditional technology frontier, and (3) the convergence of a country from its current level of technology to its conditional frontier.

We define  $A_g(t)$  as the best practice level of technology in year  $t$ . The value of  $A_g(0)$  is an initial condition in the model and the rate of growth of best practice technology is a parameter. Formally, we specify:

$$A_g(t+1) = A_g(t) \cdot (1 + \gamma), \quad (21)$$

where  $\gamma$  is the rate at which best practice technology improves.

Each country has a conditional frontier that is some fraction of the global best practice frontier. The ratio of the conditional frontier to the global frontier is assumed to depend on the average level of education of the workers and on other factors. We specify this relationship using a logistic formulation in which education and the indicator for the other factors are multiplied by one another:

$$\frac{A_c(t)}{A_g(t)} = \frac{\exp(d1 + d2 \cdot EDUCW(t) \cdot OTHER1(t))}{1 + \exp(d1 + d2 \cdot EDUCW(t) \cdot OTHER1(t))}, \quad (22)$$

where  $A_c(t)$  is a country's conditional technological frontier,  $EDUCW(t)$  is the average education level of workers in year  $t$ ,  $OTHER1(t)$  is an index representing how conducive the economic policies and institutions of the country are to obtaining technology from abroad, and  $d1$  and  $d2$  are parameter values. The functional form is important. If  $OTHER1(t)$  is zero, then the ratio of the conditional frontier to the global frontier remains constant regardless of the level of education.  $OTHER1(t)$  would be zero in situations where economic policies and institutions do not allow people with more education to use that education productively in the economy.  $OTHER1(t)$  could also be low in cases where the types of education that students get do not provide the kinds of skills that are needed in the labor market. One interesting implication of Eq. (22) is that differences in education per worker do not matter much when countries are close to the global frontier. In that case, the conditional frontier would already be close to the global frontier and there would not be much gain possible from increasing the level of education more.

Changes in the actual level of productivity depend on how far the current level is from a country's conditional frontier and how fast it catches up to its frontier. We specify that the speed at which a country catches up to its conditional frontier has a logistic shape and depends on the rate of capital formation. We write:

$$A(t+1) = A(t) + \frac{\exp(g1 + g2 \cdot \frac{K(t+1)}{K(t)} - 1)}{1 + \exp(g1 + g2 \cdot \frac{K(t+1)}{K(t)} - 1)} \cdot (ahigh - alow) + alow, \quad (23)$$

$$[A_c(t+1) - A(t)]$$

where  $\frac{K(t+1)}{K(t)} - 1$  is the rate of capital formation, and  $g1$  and  $g2$  are parameters. The parameters *ahigh* and *alow* are the upper and lower asymptotes, respectively, of the catch-up coefficient. It is certainly possible to include a variable reflecting the policy environment and the quality of institutions in Eq. (23) although this has not been done yet.

#### 2.4. Some thought experiments

Before we move on to the illustrative example of Egypt, it is instructive to go through a set of thought experiments. First, let us consider the effects of an increase in the age at retirement in a country where a pay-as-you-go pension system provides a significant fraction of the income of retired people. The initial effect of an unexpected increase in the age at retirement is to increase  $L(t)$ , the number of people working without increasing the capital stock. This lowers the wage per efficiency unit of labor and increases the return to capital. The increase in the number of workers and decrease in

the number of retired people immediately lowers the tax rate that workers face and thus increases their disposable income and, other things being equal, their savings.

Importantly, however, all current workers see that they are going to have their retirement delayed and this affects their savings behavior. While a one-year increase in the age at retirement, for example, produces a small increase in the labor force, it affects the savings behavior of everyone that works in a number of ways. When workers see that they have to save for one fewer year of retirement, they have less of an incentive to save. In addition, the increase in the return to capital means that they have already saved too much (because they saved with an expectation that the rate of return would be lower) and this is another factor reducing their savings. The net result of the increased disposable income and these offsetting effects on savings and on the rate of capital formation is indeterminate.

When personal savings changes it induces a change in the rate of return to capital that, in turn, causes a change in the distribution of capital between lifecycle savers and non-lifecycle savers. This effect could lead to a change in the overall rate of savings in the economy.

The increase in the retirement age affects productivity growth in two ways. First, if the people added to the labor force have an education level that is below the national average, the conditional productivity frontier is reduced relative to the global frontier. Second, if there are induced changes in the rate of capital formation because of changes in savings behavior or because of the change in the distribution of the capital stock, then the speed with which the country catches up to its conditional productivity frontier changes. Changes in productivity feed back to changes in the growth of wages and to changes in the expected growth of wages in the future. The expected growth of wages influences savings behavior and so the loop continues.

This is not the whole story regarding the effects of a change in the age at retirement, because the strengths of the interactions just mentioned depend on the age structure of the population. Population, the economy, and productivity change are all interrelated. It is certainly possible that increasing the age at retirement could reduce the pension burden on future generations while at the same time resulting in lower output growth that makes everyone worse off.

Our second thought example deals with the role of education. Education affects labor productivity, labor force participation, and potentially the ages at labor market entrance and exit. Because education affects the age structure of labor earnings, it affects age-specific savings rates. The average level of education of the labor force also influences the location of the conditional productivity frontier and, through that route, the country's level of productivity and how that productivity changes over time. Education's impact on productivity depends both on how close a country is to the global productivity frontier and on economic policies and institutions. Education affects fertility in SEDIM 0.1 and that has a very long-term influence on the age structure.

Because education is woven deeply into the tapestry of the SEDIM model, it is difficult to disentangle individual threads. For simplicity, consider an increase in enrollment rates this year and forever into the future. The first influence in the model would be that the cost of education would rise and that this increase would have to be paid out of the earnings of workers, reducing the resources that they have for savings and consumption. When the more educated workers enter the labor market, labor input

increases because of their additional skills. The adoption of new technologies will be somewhat easier because the new workers are more educated and the rate of productivity growth will increase. Because productivity is now growing more rapidly, both the return to capital and the return to labor will increase. Increases in the rate of growth of wages, changes in the age-earnings profiles, and increases in the return to capital will influence savings. Changes in savings rates will influence the rate of capital formation and again change the speed of productivity increase.

As time progresses, the long-run effect of age structure change becomes more and more important. The ratio of workers to the population could change in the long run. Tax rates of workers will change as the population ages, leading probably to lower rates of capital formation. Thus, education has different effects in the short run, medium run, and long run. Indeed, it is possible that the signs of the effects differ across these time horizons. It is perhaps for this reason that education effects are sometimes difficult to tease out of the data.

Our third and last thought experiment concerns the relationship between SEDIM 0.1 and the recent empirical results in Kelley and Schmidt (2003). They find that the rate of growth of output per worker increases as the youth dependency rate decreases, holding everything else including education constant – even in the long run. If we were to try to set up a similar experiment in SEDIM 0.1, we could use two stable populations and assume that age-specific education attainment levels are constant. This set up would give us two equilibrium states. The population with the higher youth dependency rate would have higher fertility and a faster rate of population growth. Will SEDIM 0.1 be consistent with the Kelley and Schmidt (2003) findings or will it suggest that their regression results are being driven by transitory phenomena that would disappear in the long run? Our Egyptian experiments lead us to believe that the latter might well be the case.

### 3. Illustrative Examples for Egypt

SEDIM 0.1 has been roughly calibrated to the situation of Egypt in 2000. We have used the age structure of Egypt by single years of age, Egypt's total fertility rate, Egypt's life expectancy, and the approximate age distribution by education in 2000. So the main dimensions of the demography in the model reflect what has been observed in 2000. Most of the values of the parameters have been set to values that we thought to be reasonable for Egypt, but they and their implications have not yet been checked against data, so they must be taken with the utmost of caution. A few parameters, like the age and labor force entry (25) have not been changed because of time constraints and inelegant computer coding. For now, a more plausible parameterization will have to wait.

Egypt is in the midst of a vast educational transformation and since education is at the heart of the SEDIM model, we focus here on the effects of this educational revolution on Egypt's future economic growth. For this purpose, we have developed three education scenarios for Egypt, all of which emphasize the enormous increase in education that is already in the process of taking place. The *Continuation* scenario assumes that the education revolution in Egypt will continue and that ultimately the average Egyptian worker will have 12 years of schooling, up from somewhere around four years today. The *Less education* scenario assumes that enrollment rates will stop

increasing in 2005. The *More education* scenario portrays a situation in which there are slightly higher completed levels of education attained than in the *Continuation* scenario.

Figure 1 shows the average levels of education per worker from 2001 to 2100 in the three scenarios. The difference between the *Less education* and the *Continuation* scenarios only appear after 2025 because, in SEDIM 0.1, 25 is the assumed age of labor force entry. The increase in the average years of education per worker from 2001 to 2025 shows the momentum built into the age-education structure. Even with constant enrollment rates after 2005, there is still almost a doubling of the average level of education in Egypt during the first half of this century. In the *More education* scenario, the average level of education per worker rises to 13 years by the end of the century. Almost all the differences between the *More education* and *Continuation* scenarios become evident only in the last quarter of the century.

Figure 2 shows the effects on the total fertility rate of the three education scenarios. Again, the differences between the *Continuation* and the *Less Education* scenarios only become evident after 2025. In the *Less education* scenario, where increases in enrollment rates stop abruptly after 2005, we see a decrease in the total fertility rate to a trough around 2.5 and then a slight increase. The increase is due to the sudden ending of what demographers call the “tempo effect.” While fertility is falling, the mean age at childbearing is rising, causing the period total fertility rate seen in Figure 2 to be below its long-run cohort level. When the fall in fertility suddenly stops, as in the *Less education* scenario, there is a clear increase in the total fertility rate back to its long-run level. In the other two scenarios, there is no evident period of rise. In those cases, the fall in fertility happens much more gradually, and the slow decline in the total fertility rate and the slow increase back to its long-run average roughly cancel one another out.

The total fertility rates in the three scenarios virtually stabilize. In the *Less education* scenario, the stabilization is above replacement, so the population in that scenario is constantly growing. In the *Continuation* and the *More Education* scenarios, the total fertility rate converges to levels close to replacement. Population growth in those scenarios slows down to almost nothing by the end of the century. A stable population is not attained because life expectancies continue to increase.

Figure 3 shows the technology indicator,  $A(t)$ . Another name for the technology indicator is the level of productivity and we will use the two terms interchangeably here. Productivity in Egypt in all three scenarios will increase by a factor between four and five by 2050. This, by itself, is a very striking result of past educational efforts. The effects of continuing Egypt’s educational revolution can be clearly seen in the differences of economic productivity at the end of the century in the *Continuation* and the *Less education* scenarios. Productivity increases by around 12 times over the century in the *Less education* case, and around 25 times in the *Continuation* scenario. Other things being equal, the productivity effect would make output per worker twice as high in 2100 in the *Continuation* scenario than in the *Less education* one.

Figure 4 shows the rates of per capita economic growth under the three scenarios. In all of them Egypt makes a transition from a rate of per capita income growth of around one percent per year today to almost six percent per annum in the 2030s. This improvement in economic performance is driven by past investments in education. If the educational revolution suddenly stops now, the per capita rate of

economic growth never exceeds six percent and falls back to around three percent per year. When the educational revolution continues, as in the *Continuation* and the *More education* scenario, Egypt's economy turns into a real tiger with per capita growth rates around nine percent per year in the middle of the century before falling back to around three percent per year by 2100.

Figure 4 gives us some indirect insight into the question of the relationship between age structure and economic growth. The youth dependency rate is considerably higher under the *Less education* scenario than the other two scenarios because of its higher total fertility rates. If we were to look around 2060, it would seem that the higher youth dependency rate would be associated with a considerably smaller rate of per capita economic growth. If we were to look around 2100, we would see that the rates of per capita income growth would be quite similar, even with very different youth dependency rates. Regressions that included many cross-sectional observations analogous to what we see in Figure 4 around 2060 could confuse the effects of transitional dynamics with equilibrium relationships.

#### **4. Concluding Thoughts**

The best perspective from which to view this paper is to treat it as a proof of concept. The question at hand was whether it was possible to build a relatively simple model that incorporated the main interactions between demography, economics, and technology and that could be widely used to understand the evolution of economic growth rates. That question remains unanswered, but this paper provides some evidence that it is indeed in the affirmative. The concept is not yet proven for two reasons. First, the model in its present form is too incomplete. It needs, at a minimum, to add foreign trade and international capital movements. Second, the model needs to be carefully parameterized and tested for a variety of different countries. Such testing could reveal that there are important interrelationships that have been omitted or very inappropriately incorporated into the model.

Nevertheless, SEDIM 0.1 does provide a relatively simple model of demographic, economic, and technological interactions when they are out of equilibrium. The advantage of a model like SEDIM is that it can be relatively easily parameterized for a variety of countries so that we can use it to discuss the determinants of economic growth in a consistent comparative framework.

#### **5. References**

- Bloom, David E., David Canning, and Pia Malaney. 2000. Population dynamics and economic growth in Asia. Pages 257-290 in C.Y. Cyrus Chu and Ronald D. Lee, eds., *Population and Economic Change in East Asia. Population and Development Review*, Supplement to Volume 26. New York: Population Council.
- Bloom, David E. and Jeffrey G. Williamson. 1998. Demographic change and human resource development. Pages 141-197 in *Emerging Asia: Changes and Challenges*. Manila: Asian Development Bank.
- Bommier, Antoine and Ronald D. Lee. 2003. Overlapping generations models with realistic demography. *Journal of Population Economics* 16(1): 135-160.

- Coale, Ansley J. and Edgar Hoover. 1958. *Population Growth and Economic Development in Low-Income Countries*. Princeton, N.J.: Princeton University Press.
- Kelley, Allen C. and Robert M. Schmidt. 2003. Evolution of Recent Economic-Demographic Modeling: A Synthesis. Draft manuscript.
- Lutz, Wolfgang, Warren C. Sanderson, and Brian C. O'Neill. 2004. Conceptualizing population in sustainable development: From "population stabilization" to "population balance". Pages 315-334 in Wolfgang Lutz, Warren C. Sanderson, and Sergei Scherbov, eds., *The End of World Population Growth in the 21<sup>st</sup> Century: New Challenges for Human Capital Formation and Sustainable Development*. London: Earthscan.
- Meadows, Donella H., Dennis L. Meadows, Jorgen Randers, and William W. Behrens III. 1972. *The Limits to Growth*. New York: Potomac Associates.
- MacKellar, Landis, Tatiana Ermolieva, David Horlacher, and Leslie Mayhew. 2004. *The Economic Impact of Population Ageing in Japan*. Cheltenham Glos, U.K.: Edward Elgar.
- Sanderson, Warren C. 2004. Interactions between education and HIV: Demographic examples from Botswana. Pages 227-263 in Wolfgang Lutz, Warren C. Sanderson, and Sergei Scherbov, eds., *The End of World Population Growth in the 21<sup>st</sup> Century: New Challenges for Human Capital Formation and Sustainable Development*. London: Earthscan.
- Sanderson, Warren C. 1994. Simulation models of demographic, economic, and environmental interactions. Pages 33-71 in Wolfgang Lutz, ed., *Population, Development, Environment: Understanding their Interactions in Mauritius*. Berlin: Springer-Verlag.
- Sanderson, Warren C. 1980. *Economic-Demographic Simulation Models: A Review of Their Usefulness for Policy Analysis*. Research Report RR-80-14. Laxenburg, Austria: International Institute for Applied Systems Analysis.
- Schmertmann, Carl. 2003. A system of model fertility schedules with graphically intuitive parameters. *Demographic Research* 9: 5. <http://www.demographic-research.org/>

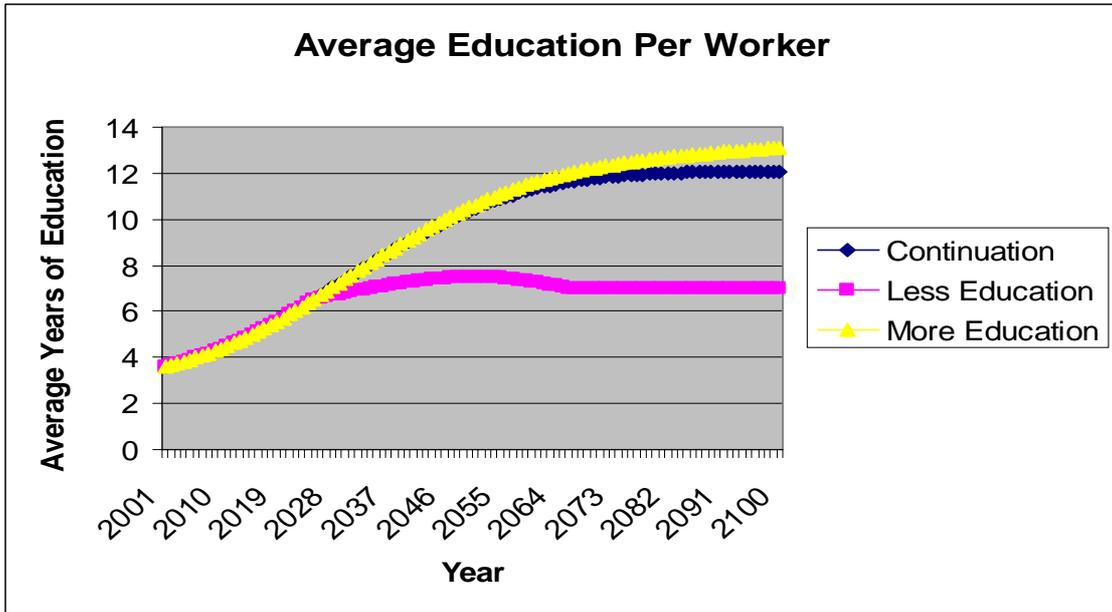


Figure 1. Average levels of education per worker from 2001 to 2100 in the three scenarios.

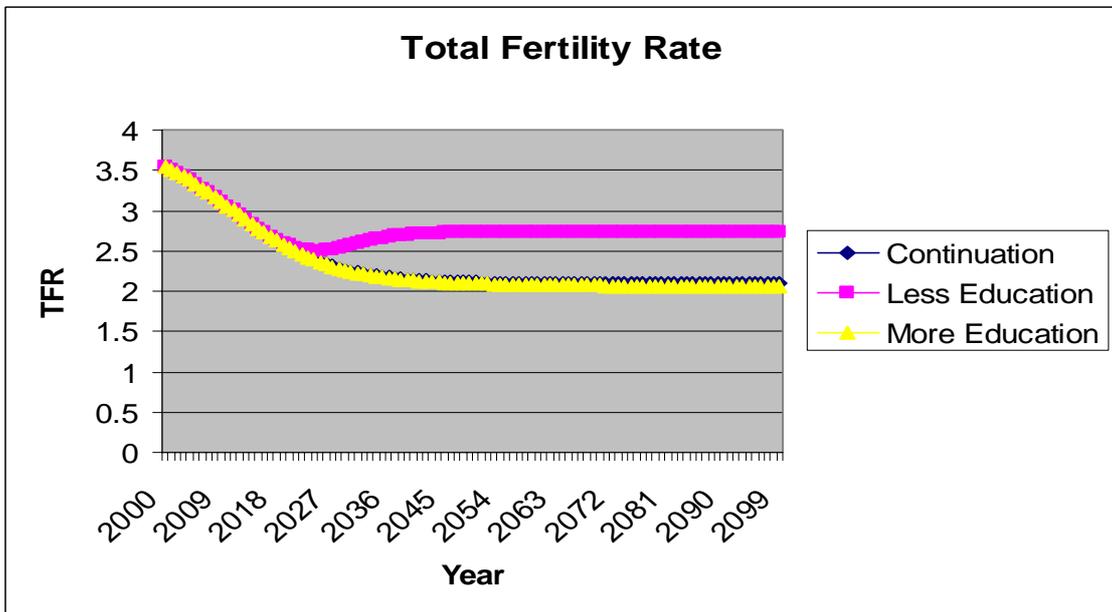


Figure 2. Effects on the total fertility rate of the three education scenarios.

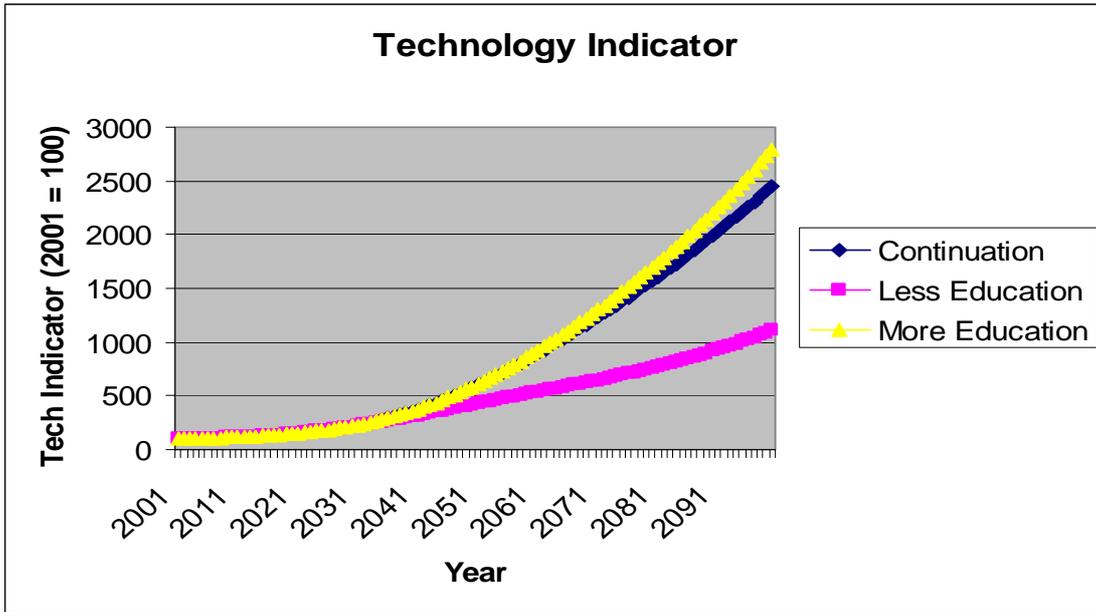


Figure 3. Technology indicator,  $A(t)$ .

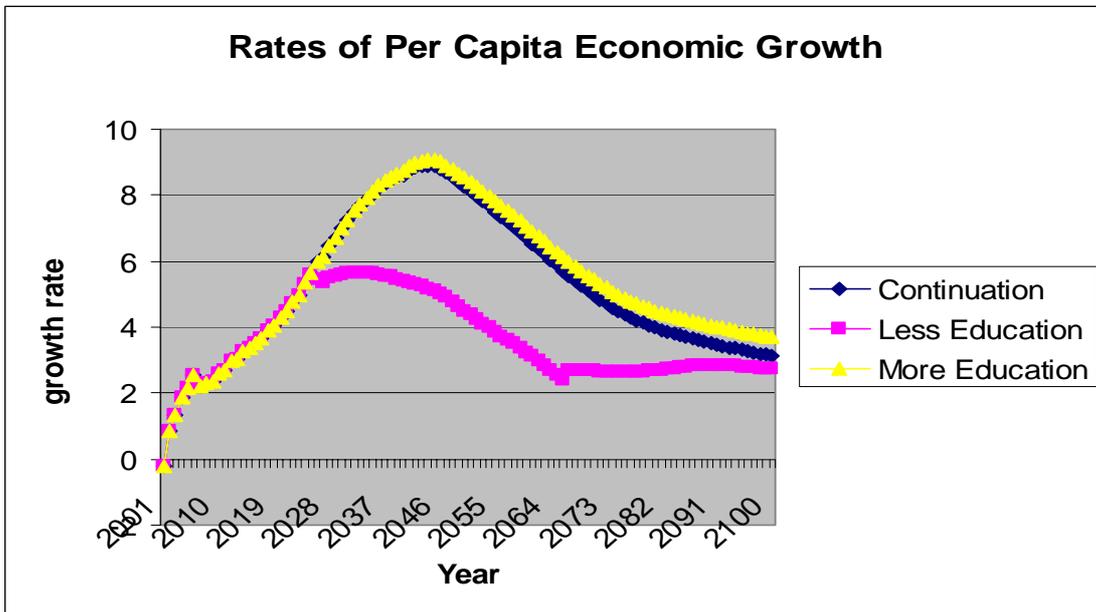


Figure 4. Rates of per capita economic growth under the three scenarios.

## Appendix: SEDIM, Version 0.1

PROGRAMMED IN GAUSS<sup>2</sup>

Please note that this code has not been edited. Historical remnants of code have been commented out in places where I thought they might one day need to be restored or reconsidered. Also commented out is code that was used for debugging. Some code used for checking the output remains active, although it has no bearing on the results. The program uses input files on Egypt's population by single years of age and on Egypt's age-specific fertility rates that are not reproduced here. If you wish to run this program, it would be advisable to contact the author first at:

[wsanderson@notes.cc.sunysb.edu](mailto:wsanderson@notes.cc.sunysb.edu) .

```
/* Model Name: Egypt3b.gau
Version: as of August 17, 2004
This version of the model was used to compute the Egyptian examples
in section 3 of the paper.
Almost all the parameters are set in the beginning portion of the model
*/
```

```
new;
library pgraph;
output file = c:\egypt\output1.out reset;
```

```
fname1 = "c:\\egypt\\dta18p.xls";
fname2 = "c:\\egypt\\dta18d.xls";
```

```
@let us consider cohorts from age 21 to 120@
```

```
@***** PARAMETERS
*****@
```

```
@Basic *****@
eylast = 151;
numyears = 100;
parms = zeros(1,40);
```

```
@Demographic *****@
Births1 = 1000;
format /rd 6,4;
bgrwth = 0.00;
"growth rate of births: " bgrwth;
parms[1,1] = bgrwth;
```

---

<sup>2</sup> The Gauss programming language is a product of Aptech Systems, Inc., Maple Valley, Washington, USA.

```

igrwth = 0.00;
"initial stable growth rate for age structure:" igrwth;
parms[1,2] = igrwth;

leint = 70;
"initial mean age at death:" leint;
parms[1,3] = leint;
leincr = 0.0; @base case dta2 database@
leincr = 0.25; @dta3@
"mean age at death increase per year:" leincr;
parms[1,4] = leincr;
stdle = 12.82;
"standard deviation of age at death:" stdle;
parms[1,5] = stdle;

brsurvrt = 0.94; @birth survival rate - used as a proxy for all mortality of young@

beqtax = 0.00;
"tax on bequests:" beqtax;
parms[1,6] = beqtax;

tfrdev = 1.00000; @no change when equal to 1.0 - otherwise multiple raised to iyear
power@
tfrswth = 1; @switch - takes on values only of 1 or zero; 1= normal
tfr; 0 tfr = tfrconst @
tfrconst = 3.525149; @this is the constant tfr when tfrswth = 0. Otherwise
meaningless @
@Economic *** *****@

AA = 1;
"alternative tech constant in Cobb-Douglas production function:" AA;
parms[1,7] = AA;
alpha = 0.67;
"labors exponent in the Cobb-Douglas production function:" alpha;
parms[1,8] = alpha;

m1 = zeros(9,9);
m1[.,1] = ones(9,1);
m1[.,2] = 26| 41| 66| 26| 46| 66| 26| 56| 66; @ages for 3 levels of education - used in
income computation @
"nine values of ages used in computing income education age profile :" m1[.,2]';
m1[.,3] = m1[.,2].^2;
m1[.,4] = 0| 0| 0| 8| 8| 8| 16| 16| 16; @ levels of education @
"nine values of education levels used in earnings profiles by age and education :"
m1[.,4]';
m1[.,5] = m1[.,4].^2;
m1[.,6] = m1[.,2].*m1[.,4];
m1[.,7] = m1[.,3].*m1[.,4];

```

```

m1[.,8] = m1[.,2].*m1[.,5];
m1[.,9] = m1[.,3].*m1[.,5];

m2 = ones(9,1);
m2[.,1] = 1 | 1.2| 0.8| 1.5| 2.0| 1.6| 2| 4| 3.5;           @wage levels using ages and
education levels above @
"nine values of relative wages used in computing earnings by age and ed :" m2[.,1]';
m2 = ln(m2);

coefs = m2/m1;
inccoef = coefs;
output off;
"coefs" coefs;

atemp = exp(m1*coefs);

"predicted values " atemp;

output on;
govcons = 0.00;
"fraction of gdp consumed by the government:" govcons;
parms[1,9] = govcons;
govcap = 0;
"government capital - not yet implemented :" govcap;
parms[1,10] = govcap;
pastgwth = 1.02;
" past rate of wage growth over the five year prior to beginning of run:" pastgwth;
parms[1,11] = pastgwth;
taxlim = 0.6;
"tax limit - highest proportion of earnings that government can take:" taxlim;
parms[1,12] = taxlim;
penindex = 1;
"switch for pension indexing :" penindex;
parms[1,13] = penindex;
ireprate = 0.0; @pension replacement rate@
"initial pension replacement rate :" ireprate;
parms[1,14] = ireprate;
itf = 0.25.*ones(numyears+1,1);           @intragenerational transfer maximum @
itf = 0.50.*ones(numyears+1,1);           @dta9@
"first year of intergenerational transfer rate :" itf[1,1];
parms[1,15] = itf[1,1];

depr = 0.05;
"capital depreciation rate :" depr;
parms[1,16] = depr;
corpcap = 0;
"initial corporate capital - now superceded :" corpcap;
parms[1,17] = corpcap;

```

```

corpcons = 0.0;          @base case assumption dta2 database@
corpcons = 0.5;          @dta4@

"fraction of corporate earnings that are consumed :" corpcons;
parms[1,18] = corpcons;

w1=zeros(151,1);
@w1[26:66,1] = ones(41,1).*2;@
@this is our initial wealth profile with age@

savlgth = 0.10;
"saving time horizon parm :" savlgth;
parms[1,19] = savlgth;

starttax = 0.05;

@Technology *****@

frontr = seqm(2,1.02,numyears+1);
"starting frontier level:" frontr[1,1];
parms[1,20] = frontr[1,1];
"second year frontier level:" frontr[2,1];
parms[1,21] = frontr[2,1];
icfrontr = 0.8;
"initial fraction of frontier:" icfrontr;
parms[1,22] = icfrontr;
other1 = 0.3; @baseline assumption dta2 @
@other1 = 0.1; @ @dta5 assumption@
"other factors in conditional frontier:" other1;
parms[1,23] = other1;
othfact = ones(numyears+1,1).*other1;
cfrontr = zeros(numyears+1,1);
capital = zeros(numyears+1,1);
capform = 1;
"index for capital formation :" capform;
parms[1,24] = capform;
capformc = 0.04; @alpha1 in change in tech eqn - only operational when capform = 0@
"fixed alpha1 is capform is zero :" capformc;
parms[1,25] = capformc;

techswth = 1;
"technology switch: " techswth;
parms[1,26] = techswth;

kor1 = 4; @kor1 is the initial capital output ratio - it is used to get us off to a
somewhat standardized start@
"initial capital output ratio :" kor1;
parms[1,27] = kor1;

```

```

kdist1 = 0.5; @proportion of initial capital stock in the hands of the corporate sector -
temporary changed with gov cap is introduced @
"initial capital distribution to c sector :" kdist1;
parms[1,28] = kdist1;

```

```

@Education *****@

```

```

sttchrte = 40;
"student teacher ratio :" sttchrte;
parms[1,29] = sttchrte;
up1 = 12.1;
"max years of education :" up1;
parms[1,30] = up1;
low1 = 3;
"minimum years of education :" low1;
parms[1,31] = low1;
today1 = 10;
"today's births level of ed :" today1;
parms[1,32] = today1;
up2 = 12.0;
"ed level reached ysup2 years in future :" up2;
parms[1,33] = up2;
ysup2 = 30;
"years in future where previous leve is reached :" ysup2;
parms[1,34] = ysup2;

```

```

edswth1 = 0; @only one switch should be on@
edswth2 = 0; @constant future education, edcnst2 @
edswth3 = 1; @education changing according to edcnst3 @
edcnst2 = 7;
edcnst3 = 1.0015;

```

```

@characteristics of teachers @

```

```

teacha = 30;
"age of teachers :" teacha;
parms[1,35] = teacha;
teache = 12;
"education of teachers :" teache;
parms[1,36] = teache;
output off;

```

```

@*****INITIALIZATION
*****@

```

```

Deaths1 = zeros(eylast,numyears+1);
Tbirths = zeros(numyears+1,1);
Tdeaths = zeros(numyears+1,1);

```

```

Tpop = zeros(numyears+1,1);
bequest = zeros(numyears+1,1);

startcap = zeros(numyears+1,1);

inccoef = zeros(9,1);
pastwge = ones(numyears+7,1);

foregwth = ones(numyears+1,1);
atech = ones(numyears+1,1);
atechfr = ones(numyears+1,1);
avedwkr = zeros(numyears+1,1);

educ1 = zeros(eylast,numyears+1);
edcost = zeros(numyears+1,1);
pencost = zeros(numyears+1, 1);
pencost2 = zeros(numyears+1, 1);

reprate2 = zeros(numyears+1,1);

taxcorp = zeros(numyears+1,1);
taxcrte = ones(numyears+1,1).*0.00;
taxpers = zeros(numyears+1,1);
taxprte = zeros(numyears+1,1);
taxprte2 = zeros(numyears+1,1);
after1 = zeros(eylast,numyears+1);

output1 = zeros(numyears+1,50);

matrix1 = zeros(151,151);
for i (1,151,1);
    for j (1,151,1);
        matrix1[i,j]=j-i;
    endfor;
endfor;

lexp = ones(eylast,numyears+1);
poptot = zeros(eylast,numyears+1);
@lexp = lexp.*seqa(70,0.2,numyears+1)';@
lexp = lexp.*seqa(leint,leincr,numyears+1)';
@"lexp " lexp[1,1:50];@
@varlexp = 2582 - 26.7.*lexp;@
@variance of life expect from Tulja 2004 working paper on the web@

Tinit = zeros(151,1);
temp1 = zeros(151,1);

```

```

@temp1 = ones(25,1)|cdfnc((seqa(26,1,126)-lexp[1,1].*ones(126,1))./stdle[1,1]);@
temp1 = ones(25,1)|cdfnc((seqa(26,1,126)-(lexp[26:151,1]))./stdle);
temp1[1,1] = 1;
sr1 = temp1[2:151,1]/temp1[1:150,1];
sr2 = cumprodc(sr1);

@
pop1 = births1.*(1|sr2[1:150,1]);
a = seqm(1,(1-igrwth),151);
pop1 = pop1.*a;
@
load pop1 = "c:\\egypt\\pop1";
@
this loads the single years of age population for Egypt for 2000 - from US Census
Bureau put into single
years of age in program c:\\egypt\\egypt1.gau
@
load asfrstd = "c:\\egypt\\asfrstd";

"initial population " pop1;
poptot[.,1]= pop1;
Tpop[1,1] = sumc(poptot[.,1]);

Tinit =(minindc((temp1.*ones(151,151))./temp1' - savlgth.*ones(151,151))^2));

Tinit;

@Tinit is the last age considered for income smoothing
@
@***** RELATIONSHIP BETWEEN EDUCATION
AND TFR *****@

TFRLOW = 2.0;
TFRHI = 5.0;
TFR12 = 2.1;
TFR6 = 3.0;

tfrconst = (ln((tfr6 - tfrlow)./(tfrhi-tfr6)))/(ln((tfr12 - tfrlow)./(tfrhi-tfr12)));
tfrm = zeros(2,2);
tfrm[.,1] = ones(2,1);
tfrm[1,2] = 6;
tfrm[2,2] = 12;
tfrcoef = tfrconst/tfrm;
"tfr coefficients are " tfrcoef;
tfrtest2 = exp(tfrcoef[1,1].*ones(17,1) + tfrcoef[2,1].*seqa(1,1,17));
tfrtest = ((tfrhi-tfrlow).*tfrtest2./(1+tfrtest2)) + tfrlow;

"tfrtest ";

```

tfrtest;

@\*\*\*\*\* SETTING UP AGE-, EDUCATION-SPECIFIC  
LABOR EARNINGS PROFILES \*\*\*\*\*@

@\*\*\*\*\*  
\*\*\*\*\*  
INCORPORATING TIME VARYING LEVELS OF EDUCATION  
\*\*\*\*\*  
\*\*\*\*\*@

a = ln((up2-low1)/(up1-up2));  
b = ln((today1-low1)/(up1-today1));

x = (a-b)/ysup2;  
tstar = b.\*ysup2/(b-a);  
"the two parameters of the education specification x and tstar";  
x~tstar;

t=seqa(-150,1,152+numyears);  
c = exp(x.\*t-x.\*tstar.\*ones(152+numyears,1));  
ed1 = (up1 - low1).\*(c./(1+c)) + low1;  
educ1[.,1] = rev(ed1[1:151,1]);  
ed1[152:252,1] = ed1[152:252,1].\*edswth1 + edswth2.\*edcnst2  
+edswth3.\*ed1[152:252,1].\*edcnst3.^(seqa(1,1,101));

@cost of education@

numed = zeros(25,numyears+1);  
edfract = zeros(25,1);  
b = educ1[7:25,1] - seqa(0,1,19);  
edfract[7:25,1] = (b .ge 1) + b.\* ((b .ge 0) .and (b .lt 1));  
numed[1:25,1] = edfract.\*poptot[1:25,1];  
@\*\*\*\*\* END OF EDUCATION SPECIFICATION  
\*\*\*\*\*@

@\*\*\*\*\*INCOME BY AGE AND EDUCATION  
\*\*\*\*\*@

incage = zeros(151,1);

a = seqa(26,1,41);  
m3 = zeros(66,9);  
m3[26:66,1] = ones(41,1);  
m3[26:66,2] = a;  
m3[26:66,3] = m3[26:66,2].^2;  
m3[26:66,4] = educ1[26:66,1];

```

m3[26:66,5] = m3[26:66,4].^2;
m3[26:66,6] = m3[26:66,2].*m3[26:66,4];
m3[26:66,7] = m3[26:66,3].*m3[26:66,4];
m3[26:66,8] = m3[26:66,2].*m3[26:66,5];
m3[26:66,9] = m3[26:66,3].*m3[26:66,5];

incage[26:66,1] = exp(m3[26:66,.]*coefs);

@ this is our age-, education-specific earnings profile @
labour1 = incage;
@for now labour1 is labor input by age@

@***** END OF EARNINGS
PROFILE BY AGE AND EDUCATION *****@

tlabor = sumc(labour1[26:66,1].*pop1[26:66,1]);
@ "the total labor force is " tlabor; @

@*****PENSION INCOME
*****@

a = 66.*ones(85,1);
m4 = zeros(151,9);
m4[67:151,1] = ones(85,1);
m4[67:151,2] = a;
m4[67:151,3] = m4[67:151,2].^2;
m4[67:151,4] = educ1[67:151,1];
m4[67:151,5] = m4[67:151,4].^2;
m4[67:151,6] = m4[67:151,2].*m4[67:151,4];
m4[67:151,7] = m4[67:151,3].*m4[67:151,4];
m4[67:151,8] = m4[67:151,2].*m4[67:151,5];
m4[67:151,9] = m4[67:151,3].*m4[67:151,5];
pincage = zeros(151,1);
pincage[67:151,1] = exp(m4[67:151,.]*coefs);

incage[67:151,1] = pincage[67:151,1];
@*****
*****@

@***** TECHNOLOGY
*****@

avedwkr[1,1] = sumc(educ1[26:66,1].*poptot[26:66,1])./sumc(pop1[26:66,1]);

y1= 0.10|0.98;
m5 = zeros(2,2);
m5[.,1] = -1.*ones(2,1);
m5[.,2] = ln(y1./(1-y1));

```

```

x1 = 5|16;
b5 = x1/m5;
aa6 = b5[1,1]/b5[2,1];
"a6 is " aa6;
bb6 = 1./b5[2,1];
"b6 is " bb6;
xz = aa6 + bb6 .* avedwkr[1,1] .* othfact[1,1];
b7 = exp(xz);
frontfac = b7./(1+b7);
"average education of workers " avedwkr[1,1];
"coeff " aa6 bb6;
"fraction of frontier value " frontfac;
cfrontr[1,1] = frontfac.*frontr[1,1];
"cfrontr is " cfrontr[1,1];
atech[1,1] = icfrontr.*cfrontr[1,1];
"atech is " atech[1,1];
output1[1,41] = atech[1,1];

```

@Parameters relating capital formation to alpha1 coefficient in technology formulation@

```

alphLOW = 0.0;
alphHI = 0.10;
alph10 = 0.06;
alphn04 = 0.04;

```

```

alphcnst = (ln((alphn04 - alphlow)/(alphhi-alphn04)))/(ln((alph10 - alphlow)/(alphhi-
alph10)));
alpm = zeros(2,2);
alpm[:,1] = ones(2,1);
alpm[1,2] = 0.02;
alpm[2,2] = 0.12;
alphcoef = alphcnst/alpm;
"alpha1 coefficients are " alphcoef;
alphtst2 = exp(alphcoef[1,1].*ones(40,1) + alphcoef[2,1].*seqa(0,0.05,40));
alphtest = ((alphhi-alphlow).*alphtst2./(1+alphtst2)) + alphlow;
"alphtest ";
(seqa(0,0.05,40))~alphtest;

```

@\*\*\*\*\* INITIAL CAPITAL STOCK AND DISTRIBUTION\*\*\*\*\*@

```

atech[1,1] = atech[1,1].*techswth + AA.*(1-techswth);

```

```

Wbar = ((kor1.*atech[1,1])^(1./alpha)).*tlabor;
corpcap = kdist1.*Wbar;
privcap = (1-kdist1).*Wbar;
w1[26:151,1] = privcap.*ones(126,1)./(sumc(pop1[26:151,1]));

@a = seqa(1,1,126);@
@
privcap = sumc(w1[26:151,1].*pop1[26:151,1]);
Wbar= privcap+corpcap;
@
@Wbar is the initial amount of capital. It will go into the first period's production
function@
@"Total capital stock, private capital and corporate capital is " Wbar privcap
corpcap;@

out1 = atech[1,1].*(tlabor^alpha).*(Wbar^(1-alpha));
@out1 = A.*(tlabor^alpha).*(Wbar^(1-alpha));@

@"output is " out1;@
output1[1,1] = out1;
output1[1,2] = out1./sumc(pop1);
output1[1,3] = out1./sumc(pop1[26:66,1]);
output1[1,4] = tlabor;
output1[1,5] = Wbar;
output1[1,6] = privcap;
output1[1,7] = corpcap;
output1[1,8] = sumc(w1);
output1[1,9] = sumc(pop1);

r = (1 - alpha) .* out1 ./Wbar;
output1[1,12] = r;
wage = alpha .* out1 ./ tlabor;
output1[1,13] = wage;
@"r and w are " r wage;@
pastwge[6,1] = wage;
a = seqa(-5,1,5);
pastwge[1:5,1] = wage.*(pastgwth).^a;
foregwth[1,1] = pastgwth;
output1[1,14] = foregwth[1,1];

@***** EDUCATION COST
*****@

@cost of education@

numed = zeros(25,numyears+1);
edfract = zeros(25,1);

```

```

b = educ1[7:25,1] - seqa(0,1,19);
edfract[7:25,1] = (b .ge 1) + b.* ((b .ge 0) .and (b .lt 1));
numed[1:25,1] = edfract.*poptot[1:25,1];

teachv =
1~teacha~(teacha^2)~teache~(teache^2)~(teacha.*teache)~((teacha^2).*teache)~(teacha
.*(teache^2))~((teacha^2).*(teache^2));
teachi = exp(teachv*coefs).*wage; @teachers' income @

edcost[1,1] = teachi.*sumc(numed[.,1])./sttchrte; @ed cost is teachers' income *
number of students education / stud-teacher ratio@
output1[1,15] = edcost[1,1];
@***** AFTER TAX INCOME
*****@

@** PENSION COST *****@
startcap[1,1] = starttax.*alpha.*out1;
output1[1,45]=startcap[1,1];
reprate = maxc(ireprate|itf[1,1]);
reprate2[1,1] = reprate;
pencost[1,1] = sumc(pincage[67:151,1].*poptot[67:151,1].*wage.*reprate);
pencost2[1,1] = pencost[1,1];
taxcorp[1,1] = taxcrte[1,1].*corpcap.*r;
taxpers[1,1] = pencost[1,1]+edcost[1,1]+govcons.*out1-taxcorp[1,1] + startcap[1,1];
taxprte[1,1] = taxpers[1,1]/(alpha.*out1);
if (taxprte[1,1] .gt taxlim .and pencost[1,1] .gt 0);
    pencost2[1,1] = maxc((taxlim.*alpha.*out1-edcost[1,1]-
govcons.*out1+taxcorp[1,1])|0);
    reprate2[1,1] = reprate.*(pencost2[1,1]./pencost[1,1]) ;
    taxprte[1,1] = taxlim;
endif;
if (taxprte[1,1] .gt taxlim .and pencost[1,1] .le 0);
    "NOT ENOUGH REVENUE EVEN WITH NO PENSION COST ";
    "taxlim taxprte taxpers edcost govcons taxcorp out1 labor income";
    taxlim taxprte[1,1] taxpers[1,1] edcost[1,1] govcons taxcorp[1,1] out1
alpha.*out1;
    "edfract ";
    edfract;
    stop;
endif;
output1[1,16] = reprate2[1,1];
output1[1,17] = taxprte[1,1];
output1[1,18] = pencost2[1,1];
after1[26:66,1] = incage[26:66,1].*wage.*(1-taxprte[1,1]);
after1[67:151,1] = pincage[67:151,1].*wage.*(1-taxprte[1,1]).*reprate2[1,1];
"ex ante replacement rate " reprate;
"ex post replacement rate " reprate2[1,1];

```

```

"output " out1;
"pension cost " pencost[1,1];
"education cost " edcost[1,1];
"corporate tax " taxcorp[1,1];
"personal tax " taxpers[1,1];
"personal tax rate " taxprr[1,1];
"teachers' income " teachi;
"student years of education " sumc(numed[.,1]);

"after tax earnings by age ";
after1[26:151,1];

@ r is the return to capital in machine units@

@initial guess of price of capital@

for iyear (1,numyears,1);
    "the year is " iyear;
    pk = 1;
    xr = educ1[.,iyear];
@
    "w1 entering procedure ";
    w1';
@
    {save1,cons1} = sav2(pk,w1,after1[.,iyear],pop1,educ1[.,iyear],iyear);
@
    "consumption in iyear " iyear;
    cons1[26:75,1];
    "savings in year " iyear;
    save1[26:75,1];
@
    @" net savings is " save1;@
@
    "savings and consumption and wealth and income by age ";
    seqa(26,1,126)~save1[26:151,1]~cons1[26:151,1]~w1[26:151,1]~after1[26:151,
iyear];
@
    @"incage is ";
    incage;@
    sns = sumc(save1.*pop1)+startcap[iyear,1];
    output1[iyear,19] = sns;
    cns = sumc(cons1.*pop1);
    "sum of net savings and consumption" sns cns;
    output1[iyear,20] = cns;
    output1[iyear,21] = sns./cns;

```

```

output1[iyear,35] = alpha.*out1;
output1[iyear,36] = (1-alpha).*out1;
@
"iyear privcap depr sns ";
iyear privcap depr sns output1[iyear+1,10];
@
grcpint = corpcap.*r.*(1-corpcons);
output1[iyear,22] = grcpint;
output1[iyear,23] = corpcap.*r;
if (-sns .gt grcpint);
    save1 = save1.*(grcpint./(-sns));
    sns = sumc(save1.*pop1);
    "sum of net savings adjusted
    ***** WATCH OUT HERE ***** ";
    sns;
endif;
output1[iyear,24] = sns;
output1[iyear,37] = sns./output1[iyear,35];
output1[iyear+1,10] = privcap.*(1-depr) + sns;
corpcap = corpcap.*(1-depr)+corpcap.*r.*(1-corpcons);
output1[iyear+1,7] = corpcap;
@stdle = lexp[26,iyear+1].*12.8./70;@
stdle = 12.8;          @standard deviation of life expectancy fixed on 8/4/04 @

temp1 = ones(25,1)|cdfnc((seqa(26,1,126)-(lexp[26:151,iyear+1]))./stdle);
temp1[1,1] = 1;
Tinit =(minindc((temp1.*ones(151,151))./temp1' - 0.10.*ones(151,151))^2));

sr1 = temp1[2:151,1]./temp1[1:150,1];
pop2 = pop1;
@"population structure " pop1;@
pop1[2:151,1] = pop1[1:150,1].*(sr1);
Deaths1[1:151,iyear] = pop2[1:151,1]-(pop1[2:151,1]|0);

@ BIRTHS @
@pop1[1,1]=Births1.*(1+bgrwth)^iyear;@
aed = educ1[16:50,iyear];
"rows and columns ";
rows(aed) cols(aed);
tfrtest3 = exp(tfrcoef[1,1].*ones(35,1) + tfrcoef[2,1].*aed);
tfrtest = ((tfrhi-tfrlow).*tfrtest3./(1+tfrtest3)) + tfrlow;
tfrtest = tfrswth.*tfrtest + (1-tfrswth).*tfrct.*ones(35,1);
@
"tfrs ";
tfrtest;
@

```

```

asfr4 = (tfrtest.*asfrstd./2).*(tfrdev^iyear);
output1[iyear,44] = sumc(asfr4).*2;
Births1 = sumc(asfr4.*pop2[16:50,1]).*brsurvrt;
Tdeaths[iyear,1]=sumc(Deaths1[.,iyear])+sumc(asfr4.*pop2[16:50,1]).*(1-
brsurvrt);

pop1[1,1] = Births1;
Tbirths[iyear,1] = pop1[1,1];

output1[iyear,25] = Tdeaths[iyear,1];
output1[iyear,26] = Tbirths[iyear,1];
poptot[.,iyear+1] = pop1;
Tpop[iyear+1,1] = sumc(poptot[.,iyear+1]);
output1[iyear+1,27]=Tpop[iyear+1,1];
output1[iyear+1,28] = sumc(pop1[26:66,1]);
output1[iyear+1,29] = sumc(pop1[67:151,1]);
output1[iyear+1,30] = sumc(pop1[1:25,1]);
output1[iyear+1,31] = output1[iyear+1,29]/output1[iyear+1,28];
output1[iyear+1,32] = output1[iyear+1,30]/output1[iyear+1,28];

@"population in year " iyear;
pop1;@
@seqa(1,1,85)~save1~incage[1:85,1].*wage~r.*w1[1:85,1]~cons1~w1[1:85,1];
@

"wealth lost w1[151,1] save1[151,1] cons[151,1]" w1[151,1] save1[151,1]
cons1[151,1];
w1[2:151,1] = w1[1:150,1] .* (1-depr) + save1[1:150,1];
w1[1,1] = startcap[iyear,1]/pop1[26,1];
@

ee = sumc(w1 .lt -0.0001);
if (ee .gt 0); "problem in year " iyear; w1; endif;
@

privcap = sumc(w1[1:151,1].*pop1[1:151,1]);
output1[iyear+1,33]= privcap;
bequest[iyear+1,1]=output1[iyear+1,10]-privcap;
output1[iyear+1,34] = bequest[iyear+1,1];
@

"hi there 1 ";
w1[1:151,1]';
@

if privcap .gt 0;
    w1[1:151,1] = w1[1:151,1].*(1+(bequest[iyear+1,1].*(1-
beqtax)/privcap));
else;

```

```

        w1[1:151,1] = zeros(151,1);
    endif;

@
"hi there 1 ";
    w1[1:151,1]';
@

    Wbar= privcap+corpcap;
    @Wbar is the initial amount of capital. It will go into the first period's
production function@
    @"Total capital stock, private capital and corporate capital is " Wbar privcap
corpcap;@

    educ1[.,iyear+1] = rev(ed1[(iyear+1):(iyear+151),1]);
    output1[iyear+1,38] = educ1[1,iyear+1];

    a = seqa(26,1,41);
    m3 = zeros(66,9);
    m3[26:66,1] = ones(41,1);
    m3[26:66,2] = a;
    m3[26:66,3] = m3[26:66,2].^2;
    m3[26:66,4] = educ1[26:66,iyear+1];
    m3[26:66,5] = m3[26:66,4].^2;
    m3[26:66,6] = m3[26:66,2].*m3[26:66,4];
    m3[26:66,7] = m3[26:66,3].*m3[26:66,4];
    m3[26:66,8] = m3[26:66,2].*m3[26:66,5];
    m3[26:66,9] = m3[26:66,3].*m3[26:66,5];
    incage[26:66,1] = exp(m3[26:66,.]*coefs);

    a = 66.*ones(85,1);
    m4 = zeros(151,9);
    m4[67:151,1] = ones(85,1);
    m4[67:151,2] = a;
    m4[67:151,3] = m4[67:151,2].^2;
    m4[67:151,4] = educ1[67:151,iyear+1];
    m4[67:151,5] = m4[67:151,4].^2;
    m4[67:151,6] = m4[67:151,2].*m4[67:151,4];
    m4[67:151,7] = m4[67:151,3].*m4[67:151,4];
    m4[67:151,8] = m4[67:151,2].*m4[67:151,5];
    m4[67:151,9] = m4[67:151,3].*m4[67:151,5];
    pincage = zeros(151,1);
    pincage[67:151,1] = exp(m4[67:151,.]*coefs);

    incage[67:151,1] = pincage[67:151,1];
    labour1 = incage;
    @for now labour1 is labor input by age@

```

```

tlabor = sumc(labour1[26:66,1].*pop1[26:66,1]);
output1[iyear+1,4] = tlabor;
@***** TECHNOLOGY
***** @

output1[iyear+1,5] = Wbar;
output1[iyear+1,11] = (output1[iyear+1,5]./output1[iyear,5])-1; @rate of growth
of capital stock@
avedwkr[iyear+1,1] =
sumc(educ1[26:66,iyear+1].*poptot[26:66,iyear+1])./sumc(poptot[26:66,iyear+1]);
output1[iyear+1,39] = avedwkr[iyear+1,1];
xz = aa6 + bb6 .* avedwkr[iyear+1,1] .* othfact[iyear+1,1];
b7 = exp(xz);
frontfac = b7./(1+b7);
cfrontr[iyear+1,1] = frontfac.*frontr[iyear+1,1];
output1[iyear+1,40] = cfrontr[iyear+1,1];
@
v = 0|0.1;
dum1 = dummy(output1[iyear+1,11],v);
alpha1 = dum1[1,1].*0.0 + dum1[1,2].*output1[iyear+1,11] + dum1[1,3].*0.1;
@
alpltst2 = exp(alphcoef[1,1] + alphcoef[2,1].*output1[iyear+1,11]);
alpha1 = ((alphhi-alphlow).*alpltst2./(1+alpltst2)) + alphlow;
alpha1 = capform.*alpha1 + (1-capform).*capformc;
output1[iyear+1,41] = alpha1;
@alpha1 = 0.1;@

atech[iyear+1,1] = atech[iyear,1] + alpha1.*(cfrontr[iyear+1,1]-atech[iyear,1]);

@"the total labor force is " tlabor;@
AA = 1;
alpha = 0.67;
@out1 = A.*(tlabor^alpha).*(Wbar^(1-alpha));@
atech[iyear+1,1] = atech[iyear+1,1].*techswth + AA.*(1-techswth);

output1[iyear+1,42] = atech[iyear+1,1];
output1[iyear+1,43] = 100.*((atech[iyear+1,1]./atech[iyear,1]) - 1);

out1 = atech[iyear+1,1].*(tlabor^alpha).*(Wbar^(1-alpha));
output1[iyear+1,1] = out1;
output1[iyear+1,2] = out1./sumc(pop1);
output1[iyear+1,3] = out1./sumc(pop1[26:66,1]);
output1[iyear+1,4] = tlabor;
output1[iyear+1,5] = Wbar;
output1[iyear+1,6] = privcap;

```

```

output1[iyear+1,7] = corpcap;
output1[iyear+1,8] = sumc(w1);
output1[iyear+1,9] = sumc(pop1);

poptot[.,iyear+1]=pop1;
@"output is " out1;@
r = (1 - alpha) .* out1 ./Wbar;
wage = alpha .* out1 ./tlabor;
output1[iyear+1,12] = r;
output1[iyear+1,13] = wage;
pastwge[iyear+6,1]=wage;
foregwth[iyear+1,1] = (pastwge[iyear+6,1]./pastwge[iyear+1])^0.2;
output1[iyear+1,14] = foregwth[iyear+1,1];
@pencost[iyear+1,1] =
sumc(pincage[67:151,1].*poptot[67:151,iyear+1].*wage.*reprate);@

@***** EDUCATION COST
*****@

@cost of education@
@
numed = zeros(25,numyears+1);
edfract = zeros(25,1);
@
b = educ1[7:25,iyear+1] - seqa(0,1,19);
edfract[7:25,1] = (b .ge 1) + b.* ((b .ge 0) .and (b .lt 1));
numed[1:25,iyear+1] = edfract.*poptot[1:25,iyear+1];

@characteristics of teachers @
@
teacha = 30;
teache = 12;
teachv =
1~teacha~(teacha^2)~teache~(teache^2)~(teacha.*teache)~((teacha^2).*teache)~(teacha
.*(teache^2))~((teacha^2).*(teache^2));
teachi = exp(teachv*coefs).*wage; @ @teachers' income @

edcost[iyear+1,1] = teachi.*sumc(numed[.,iyear+1])./sttchrte; @ed cost is teachers'
income * number of students education / stud-teacher ratio@
output1[iyear+1,15] = edcost[iyear+1,1];
@***** AFTER TAX INCOME
*****@

@** PENSION COST *****@

@
pencost[iyear+1,1] = sumc(pincage[67:151,1].*poptot[67:151,iyear+1].*wage.*reprate);
taxcorp[iyear+1,1] = taxcrte[iyear+1,1].*corpcap.*r;

```

```

taxpers[iyear+1,1] = pencost[iyear+1,1]+edcost[iyear+1,1]+govcons.*out1-
taxcorp[iyear+1,1]-bequest[iyear+1,1].*beqtax + startcap[iyear+1,1];
taxprte[iyear+1,1] = taxpers[iyear+1,1]/(alpha.*out1);
after1[26:66,iyear+1] = incage[26:66,1].*wage.*(1-taxprte[iyear+1,1]);
after1[67:151,iyear+1] = incage[67:151,1].*wage;
@
startcap[iyear+1,1] = starttax.*alpha.*out1;
output1[iyear+1,45] = startcap[iyear+1,1];
reprate = maxc(ireprate|itf[iyear,1]);
reprate2[iyear+1,1] = reprate;
pencost[iyear+1,1] = sumc(pincage[67:151,1].*poptot[67:151,iyear+1].*wage.*reprate);
pencost2[iyear+1,1] = pencost[iyear+1,1];
taxcorp[iyear+1,1] = taxcrte[iyear+1,1].*corpcap.*r;
taxpers[iyear+1,1] = pencost[iyear+1,1]+edcost[iyear+1,1]+govcons.*out1-
taxcorp[iyear+1,1]-bequest[iyear+1,1].*beqtax + startcap[iyear+1,1];
taxprte[iyear+1,1] = taxpers[iyear+1,1]/(alpha.*out1);

if (taxprte[iyear+1,1] .gt taxlim .and pencost[iyear+1,1] .gt 0);
    pencost2[iyear+1,1] = maxc((taxlim.*alpha.*out1-edcost[iyear+1,1]-
govcons.*out1+taxcorp[iyear+1,1])|0);
    reprate2[iyear+1,1] = reprate.*(pencost2[iyear+1,1]/pencost[iyear+1,1]);
    taxprte[iyear+1,1] = taxlim;
endif;
if (taxprte[iyear+1,1] .gt taxlim .and pencost[iyear+1,1] .le 0);
    "NOT ENOUGH REVENUE EVEN WITH NO PENSION COST ";
    "taxlim taxprte taxpers edcost govcons taxcorp out1 labor income";
    taxlim taxprte[iyear+1,1] taxpers[iyear+1,1] edcost[iyear+1,1] govcons
taxcorp[iyear+1,1] out1 alpha.*out1;
    "edfract ";
    edfract;

    stop;
endif;
output1[iyear+1,16] = reprate2[iyear+1,1];
output1[iyear+1,17] = taxprte[iyear+1,1];
output1[iyear+1,18] = pencost2[iyear+1,1];

after1[26:66,iyear+1] = incage[26:66,1].*wage.*(1-taxprte[iyear+1,1]);
after1[67:151,iyear+1] =pincage[67:151,1].*wage.*(1-
taxprte[iyear+1,1]).*reprate2[iyear+1,1];
"ex ante replacement rate " reprate;
"ex post replacement rate " reprate2[iyear+1,1];

```

```

    "r, Wage, Capital, Output per capita, Output per worker, Population, Priv
    capital, Corp capital. Retired pop share ed cost tech" ;
    r wage Wbar out1./sumc(pop1) out1./sumc(pop1[26:66,1]) sumc(pop1) privcap
    corpcap sumc(pop1[67:151,1]).sumc(pop1) edcost[iyear,1] atech[iyear,1];

```

```

endfor; @loop for iyear@
"components of output";
@output1[.,1]~output1[.,4]~output1[.,5]~output1[.,6]~output1[.,7];@
output1[.,1 4 5 6 7 8 9];
"pension cost";
pencost;
a=seqa(1,1,numyears+1);
ytics(0,300000,5000,2);
xy(a,pencost);
"population births deaths ";
tpop~tbirths~tdeaths;
"private capital w/o adjustment for deaths and w. adjustment ";
output1[., 6 10];
"technology ";
atech;
"personal capital stock ";
output1[.,10];
"capital per person ";
w1;
@"population ";
seqa(1,1,151)~poptot[.,1:4];@
outgwth = (output1[2:numyears+1,1].output1[1:numyears,1])-ones(numyears,1);
outgwth;
ytics(-2,22,1,2);
xlabel("years");
ylabel("rate of gdp growth");
title("SEDIM - rate of population growth = 1%");
_plwidth = 5;
_pltype = 6;
_pstype = 12;
xy(seqa(1,1,numyears),outgwth.*100);

```

```

output off;

```

```

namelist = 0;
y = export(parms,fname1,namelist);
y = export(seqa(1,1,101)~output1,fname2,namelist);
"y equals " y;
"average growth of output percapita 1st 50 years 2nd 50 years and all 100 years" ;

```

```

100.*(((output1[51,2]/output1[1,2]).^0.02)-1)
100.*(((output1[101,2]/output1[51,2]).^0.02)-1)
100.*(((output1[101,2]/output1[1,2]).^0.01)-1);

"average growth of output per worker 1st 50 years 2nd 50 years and all 100 years" ;
100.*(((output1[51,3]/output1[1,3]).^0.02)-1)
100.*(((output1[101,3]/output1[51,3]).^0.02)-1)
100.*(((output1[101,3]/output1[1,3]).^0.01)-1);
output file = c:\scengen\output1.out off;

proc (2) = sav2 (pk,w1,wincage,pop1,educ3,iyear);
    local s1, iage,dinc, i,cons1,n,j , jage , nterm , t1, t2,rstar, aa,bb,exatinc
,m1,m2,m3,m4, temp1,fgth , a1 , wincage2 , m5, temp2 ;
    cons1 = zeros(151,1);
    m1 = ones(151,151);
    m2 = ones(151,151);
    m3 = zeros(151,151);
    m5 = zeros(151,151);
    m1 = m1.*((seqa(1,1,66))~66.*ones(1,85));
    m2 = m2.*educ3;
    m3 =
exp(inccoef[1,1]+inccoef[2,1].*m1+inccoef[3,1].*m1.*m1+inccoef[4,1].*m2+inccoef[5
,1].*m2.*m2+inccoef[6,1].*m1.*m2+
inccoef[7,1].*m1.*m1.*m2+
inccoef[8,1].*m1.*m2.*m2+inccoef[9,1].*m1.*m1.*m2.*m2);
    m3[.67:151] = m3[.66].*ones(151,85).*reprate2[iyear,1];

    @a1 = foregwth[iyear,1].^seqa(1,1,151);@
@
"foregwth ";
foregwth[iyear,1];
m5 = foregwth[iyear,1].^matrix1;

"matrix1 in part is ";
matrix1[26,26:36];
matrix1[65,65:75];
matrix1[66,66:76];
matrix1[67,67:77];
matrix1[141,141:151];
matrix1[151,141:151];

"m5 in part is ";
m5[26,26:36];
m5[65,65:75];
m5[66,66:76];
m5[67,67:77];
m5[141,141:151];

```

```

m5[151,141:151];

"the first m4 ";
m4[26,26:36];
m4[65,65:75];
m4[66,66:76];
m4[67,67:77];
m4[141,141:151];
m4[151,141:151];
@
@m4 = (a1.*ones(151,151)./(a1')).*m3;@
m4 = (foregwth[iyear,1].^matrix1).*m3;
temp1 = diag(m4).*(diag(m4) .gt 0) + (diag(m4) .le 0);

m4 = m4./temp1;

wincage2 = wincage.*m4;
@
wincage2[:,67:151] = wincage2[:,67:151];

"m3 ";
m3[26,26:36];
m3[65,65:75];
m3[66,66:76];
m3[67,67:77];
m3[141,141:151];
m3[151,141:151];
"m4 ";
m4[26,26:36];
m4[65,65:75];
m4[66,66:76];
m4[67,67:77];
m4[141,141:151];
m4[151,141:151];
"wincage ";
wincage[26 65 66 67 141 151,1];

"wincage2 ";
wincage2[26,26:151];
wincage2[65,65:75];
wincage2[66,66:76];
wincage2[67,67:77];
wincage2[141,141:151];
wincage2[151,141:151];
@
s1 = zeros(151,1);
aa = zeros(151,3);

```

```

rstar = 1 + (r./pk) - depr;

for iage (26,151,1);
    bb = seqa(Tinit[iage,1]-iage+1,-1,Tinit[iage,1]-iage+2);
@
    "w1 is "; iage w1[iage,1];
@
    aa[iage-1,1]=pk.*w1[iage,1];
    aa[iage:Tinit[iage,1],1] = wincage2[iage,iage:Tinit[iage,1]]';

    aa[iage-1:Tinit[iage,1],2]=rstar.^bb;
@
    "aa is ";
    aa[iage-1:Tinit[iage,1],1];
    temp1 = sumc(aa[iage-1:Tinit[iage,1],1].*aa[iage-1:Tinit[iage,1],2]);
    temp2 = sumc(aa[iage:Tinit[iage,1],2]);

    "iage and temps " iage temp1 temp2;
@
    cons1[iage,1] = sumc(aa[iage-1:Tinit[iage,1],1].*aa[iage-
1:Tinit[iage,1],2])./sumc(aa[iage:Tinit[iage,1],2]);
    s1[iage,1] = maxc((-pk.*w1[iage,1])|(wincage[iage,1]+r.*w1[iage,1]-
cons1[iage,1]));
endfor; @ end of loop over initial ages@
@
    "consumption and income paths ";
    "iage 26 ";
    cons1[26,1];
    wincage2[26,26:75];

    xy(seqa(50,1,102),(cons1[50,1].*ones(102,1))~wincage2[50,50:151]');
@
    retp (s1[1:151,1], cons1[1:151,1]);
endp;

end;

```