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Stable Mutual Tracking Block for a Real Dynamical Object and a Virtual Model-Leader

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Contents

Abstract

The work is devoted to the problem of the stochastic stable mutual tracking of motions of the real dynamical x-object and some virtual computer z-model under the dynamical and informational disturbances. The elaborated algorithms are applied to some typical differential games on minimax and maximin for the positional quality index of the control process. Results of computer simulation of model problems are presented.

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Stochastic Stable Mutual Tracking Block for a Real Dynamical Object and a Virtual Model-Leader

Andrew N. Krasovskii

1. Introduction

This report is devoted to a problem of stochastic stable mutual tracking of motions of a real dynamical x-object and some virtual computer simulated z-model-leader under dynamical and informational disturbances. It will be sown how the elaborated block of mutual tracking can be applied to some control problems, namely, to solving optimal control problems with ensured results.

The investigation is based on approaches, methods and constructions from the theory of optimal control, the theory of tracking and observation and the theory of stochastic processes.

The statements of the problems considered here and methods for their solution are based on the mathematical formalization developed in Ekaterinburg at the Ural State University (USU) and the Institute of Mathematics and Mechanics (IMM) of the Ural Branch of the Russian Academy of Science, first of all in works by Academicians N.N.Krasovskii and A.I.Subbotin, and their collaborators.

The main problem considered here consists in the development of a control block of mutual stable stochastic tracking of motions of a real x-object and some computer (virtual) z-model-leader. The control dynamical system (the x-object) is described by the ordinary differential equation:

$$\dot{x} = A(t)x + f(t,u,v) + h_{din}(t), \quad 0 \le t \le T, \tag{1}$$

here x is an n-dimensional phase vector, u is a vector of control, and v is a vector of disturbances satisfying the constraints

$$u \in \mathrm{P}, \quad v \in \mathrm{Q}. \tag{2}$$

P and Q are fixed sets,

$$\mathrm{P} = \{ u^{[1]}, \ldots, u^{[M]} \}, |u^{[i]}| \le \tilde{M},$$

$$\mathrm{Q} = \{ v^{[1]}, \ldots, v^{[N]} \}, |v^{[i]}| \le \tilde{N}. \tag{3}$$

In (1) $t_0 = 0$ and T are given instants (the beginning and the end of the control process), $A(t)$ and $f(t,u,v)$ are functions piecewise continuous in time t, $h_{din}(t)$ is an arbitrary random bounded dynamical disturbance, $E\{h_{din}(t)\} \le \delta_{din}$, $E\{\ldots\}$ stands for the mathematical expectation.

We consider the case where the so-called saddle point condition [2]

$$\max_{v\in Q}\min_{u\in P} < l, f(t,u,v) > = \min_{u\in P}\max_{v\in Q} < l, f(t,u,v) >,\ \forall\ l \in R^n \tag{4}$$

is not valid for the function $f(t,u,v)$. So, in the considered case for function $f(t,u,v)$ in (1) there exists a vector $l^* \in R^n$ (or a point $t\in[0,T]$) such that

$$\max_{v\in Q}\min_{u\in P} < l^*, f(t,u,v) > \neq \min_{u\in P}\max_{v\in Q} < l^*, f(t,u,v) >. \tag{5}$$

Here $< l^*, f(t,u,v) >$ denotes the scalar product.

It is known that in this case it is effective to use stochastic algorithms of control for the construction of the control actions for the x-object and to form the motion of the x-object coupled with a suitable z-model-leader.

We use the well-known discrete feedback control scheme. Namely, on the time interval [0,T] we fix an arbitrary partition

$$\Delta\{t_k\} = \{t_0 = 0,\ t_1,\dots,t_k < t_{k+1},\dots,t_K = T\}, \tag{6}$$

and below consider the x-object described by the finite difference equation:

$$x(t_{k+1}) = x(t_k) + (A(t_k)x(t_k) + f(t_k,u,v) + h_{din}(t_k))\Delta t, \tag{7}$$

where $\Delta t = t_{k+1} - t_k$, $t_k \in \Delta\{t_k\}$ (6). The actions $u = u[t] \in$ P, $v = v[t] \in$ Q, $t\in[t_k, t_{k+1})$ are defined through random tests described below.

For the x-object (7) we consider the z-model-leader :

$$z(t_{k+1}) = z(t_k) + (A(t_k)z(t_k) + \tilde{f}_{pq}(t_k))\Delta t, \tag{8}$$

where

$$\tilde{f}_{pq}(t_k) = \sum_{i=1}^{M}\sum_{j=1}^{N} f(t_k, u^{[i]}, v^{[j]}) p_i q_j. \tag{9}$$

In (8), (9) $u^{[i]} \in$ P, $v^{[j]} \in$ Q and the numbers p_i and q_j satisfy the conditions

$$\sum_{i=1}^{M} p_i = 1,\quad p_i \geq 0,\qquad \sum_{j=1}^{N} q_j = 1,\quad q_j \geq 0. \tag{10}$$

The numbers p_i and q_j are connected with the probabilities that define the random choice of the actions u and v for the x-object (7).

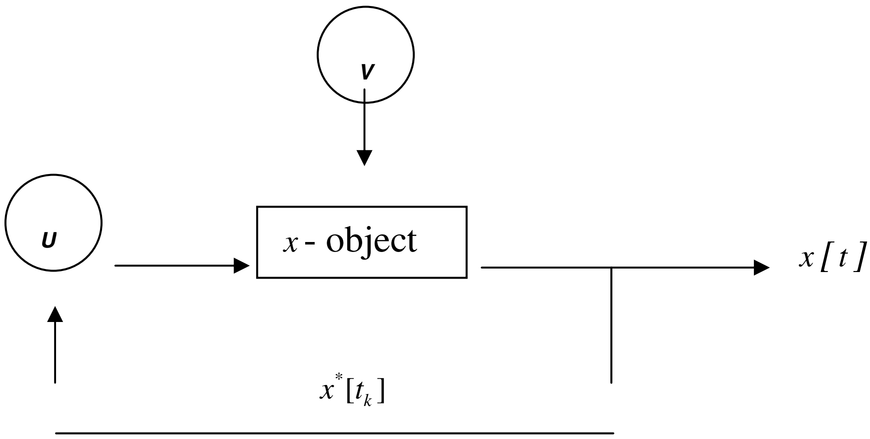
The main problem discussed in this report is to construct and to justify a control algorithm (using some stochastic mechanism) that provides the stable mutual tracking of the motions of the x-object (7) and z-model-leader (8).

Note that we will also consider the case where position of the x-object (7) is estimated with some informational error $\Delta_{\inf}$ such that at each time moment $t_k \in \Delta\{t_k\}$ (6), $k = 0, \dots, K$, we know only the distorted position $\{t_k, x^*[t_k]\}$, where

$$x^*[t_k] = x[t_k] + \Delta_{\inf}[t_k], \tag{11}$$

and $\Delta_{\inf}[t_k]$ is a random restricted vector.

We use the following well-known positional feedback scheme of control:



$$x^*[t_k] = x[t_k] + \Delta_{\inf}[t_k] \text{ - } \textit{informational image}$$

Let us note that these two values: the dynamical disturbance h_{din} in the system (1) and the informational error $\Delta_{\inf}$ in the scheme are the original elements of this work.

2. Construction of the control actions for x-object and z-model

At first let us describe probability tests that define the random choice of the action u = $u[t_k] \in$ P, $t \in [t_k, t_{k+1})$ for x-object.

As the ideal case, we accept that at the moment t_k one can make an instant probability test on choosing a vector $u[t] \in$ P.

This test is defined by the suitable probabilities { p_i^0 }, i.e.

$$P(u[t_k] = u^{[i]}) = p_i^0, \qquad i = 1, \ ..., M \ , \tag{12}$$

where the symbol P(…) denotes the probability.
Let us choose the probabilities p_i^0; $p_i^0 \geq 0$, $\sum p_i^0 = 1$, from the following extremal minimax shift [3] condition:

$$\min_p \max_q < l^*[t_k], \sum_{i=1}^{M} \sum_{j=1}^{N} f(t_k, u^{[i]}, v^{[j]}) p_i q_j > =$$
$$= < l^*[t_k], \sum_{i=1}^{M} \sum_{j=1}^{N} f(t_k, u^{[i]}, v^{[j]}) p_i^0 q_j^* \tag{13}$$

under the conditions

$$\sum_{i=1}^{M} p_i = 1, \quad p_i \geq 0, \qquad \sum_{j=1}^{N} q_j = 1, \quad q_j \geq 0.$$

In (13) we have

$$l^*[t_k] = x^*[t_k] - z[t_k] \tag{14}$$

Secondly, let the "control action" $q^0[t_k]$ for z-model (this model is formed by including into the control loop of the computer simulations) is chosen from the extremal maximin shift condition

$$\max_q \min_p < l^*[t_k], \sum_{i=1}^{M} \sum_{j=1}^{N} f(t_k, u^{[i]}, v^{[j]}) p_i q_j > =$$

$$= < l^*[t_k], \sum_{i=1}^{M} \sum_{j=1}^{N} f(t_k, u^{[i]}, v^{[j]}) p_i^* q_j^0 . \qquad (15)$$

Probabilities $\{q_j\}$ that define the stochastic actions (disturbances) $v[t_k]$ on x-object, and "actions" $\{p_i\}$ for z-model can be arbitrary: $\sum_{j=1}^{N} q_j = 1$, $q_j \geq 0$, $\sum_{i=1}^{M} p_i = 1$, $p_i \geq 0$.

3. Mutual tracking in the combined process {x-object, z-model-leader}

The following Theorem holds:

Theorem. Under the described above choice of the random actions $u^0[t_k]$ for x-object and "actions" $q^0[t_k]$ for the z-model, for any chosen beforehand numbers $V^* > 0$ and $0 < \beta < 1$ there exist sufficiently small numbers $\delta_0 > 0$, $\delta_{\inf} > 0$, $\delta_{din} > 0$, and $\delta > 0$ such that the following inequality holds:

$$P(V(t, \; l[t]) \leq V^*, \; \forall \;\; t \in [0, \; T]) \geq 1 - \beta, \qquad (16)$$

if $l[0] \leq \delta_0$, $E\{|l[t] - l^*[t]| \; | l[t]\} \leq \delta_{\inf}$ for any admissible $l[t] = x[t] - z[t]$, $E\{|h_{din}(t)|\} \leq \delta_{\inf}$, $t \in [0, \; T]$, and $\Delta t = t_{k+1} - t_k \leq \delta$. Here

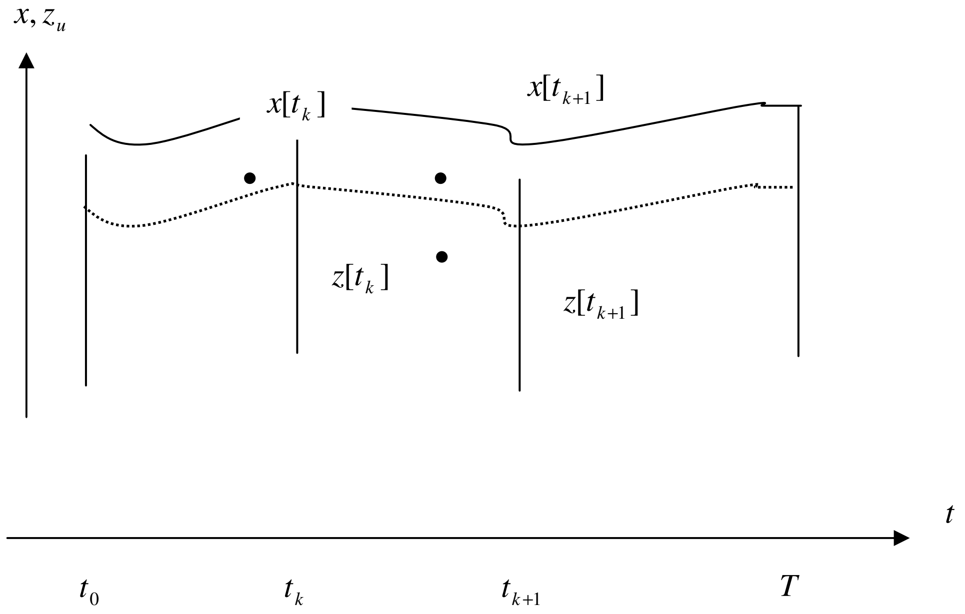
$$V(t, x[t], z[t]) = |x[t] - z[t]|^2 \; e^{-2\lambda t}. \qquad (17)$$

This Theorem has the following informal sense:

If we choose at each time moment t_k, $k = 0, \; ..., K$ the control action $u[t_k] \in P$ (3) as a result of random test with the probability $\{p^0\}$ from (13), i.e.

$$(u[t_k] = u^{[i]} \in P) = p_i^0, \qquad (18)$$

and choose for the z-model the collection $\{q_j^0\}$ (15), then for each admissible $v[t] \in Q$, $t_k \leq t \leq t_{k+1}$ for the x-object and collection $\{p_i\}$ for the z-model the motions of the x-object : $x[t]$, $0 \leq t \leq \mathrm{T}$ and of the z-model leader: $z[t]$, $0 \leq t \leq \mathrm{T}$ will be close to each other in the sense (17) on the whole time interval [0,T] with the probability arbitrarily close to one.



4. Proof of the main result

Lemma. For the random actions $u^0[t_k]$ of the x-object and "actions" $q^0[t_k]$ of the z-model the following estimation is valid:

$$E\{V(t_{k+1,}l[t_{k+1}]) \mid l[t_k]\} \le V(t_k, l[t_k]) +$$
$$+(C_1\Delta t + C_2\delta_{\inf} + C_3\delta_{din})\Delta t, \quad k = 0,..., K-1, \tag{19}$$

in particular,

$$E\{V(t_{k+1,}l[t_{k+1}])\} \le E\{V(t_k, l[t_k])\} +$$
$$+(C_1\Delta t + C_2\delta_{\inf} + C_3\delta_{din})\Delta t, \quad k = 0,..., K-1, \tag{20}$$

where C_1, C_2, C_3 are positive constants, and $\Delta t = t_{k+1} - t_k$, $\delta_{\inf}$, δ_{din} are sufficiently small positive numbers.

Proof of Lemma. Let at the time moment t_k the admissible random vectors $l[t_k] = x[t_k] - z[t_k]$ and $l^*[t_k] = x^*[t_k] - z[t_k]$ be realized. Then according to dynamics of $x[t]$ and $z[t]$ we have

$$x[t_{k+1}] = x[t_k] +$$
$$+(A(t_k)x[t_k] + f(t_k, u^0[t_k], v[t_k]) + h_{din}(t_k))\Delta t, \tag{21}$$

$$z[t_{k+1}] = z[t_k] + (A(t_k)z[t_k] + \tilde{f}_{pq^0}(t_k))\Delta t, \tag{22}$$

where, as is described above, $u^0[t_k]$ is the result of random choice with probabilities $p^0 = \{p_i^0\}$ that satisfy the minimax condition (13), i.e.

$$P(u^0[t_k] = u^{[i]} \in P\} = p_i^0 \; i = 1,..., M \;; \tag{23}$$

$v[t_k]$ is the result of random choice with some admissible probabilities $q = \{q_j\}$, i.e.

$$P(v[t_k] = v^{[j]} \in Q\} = q_j, \quad j = 1,...,N; \tag{24}$$

$h_{din}(t_k)$ is a random dynamic disturbance independent from $l[t_k]$ and $l^*[t_k]$; $\tilde{f}_{pq}(t_k)$ is defined in (9); p is some admissible "action" for the z-model; q^0 is the "action" that satisfies the maximin condition (15).

From (21), (22) for $l[t_{k+1}] = x[t_{k+1}] - z[t_{k+1}]$ we deduce

$$\begin{aligned}
&\Delta V_{t_k}^{t_{k+1}} = V(t_{k+1}, l[t_{k+1}]) - V(t_k, l[t_k]) = \\
&= e^{-2\lambda t_{k+1}} \left| l[t_{k+1}] \right|^2 - e^{-2\lambda t_k} \left| l[t_k] \right|^2 = \\
&= e^{-2\lambda t_{k+1}} (\left| l[t_{k+1}] \right|^2 - e^{2\lambda \Delta t} \left| l[t_k] \right|^2) = \\
&= e^{-2\lambda t_{k+1}} (| l[t_k] |^2 + 2\langle l[t_k], A(t_k) l[t_k] \rangle \Delta t + \\
&+ \left| A(t_k) l[t_k] \right|^2 \Delta t^2 - e^{2\lambda \Delta t} \left| l[t_k] \right|^2 + \\
&+ \left| \Delta f(t_k) \right|^2 \Delta t^2 + \left| h_{din}(t_k) \right|^2 \Delta t^2 + \\
&+ 2\langle l[t_k], h_{din}(t_k) \rangle \Delta t + 2\langle \Delta f(t_k), h_{din}(t_k) \rangle \Delta t^2 + \\
&+ 2\langle A(t_k) l[t_k], \Delta f(t_k) \rangle \Delta t^2 + 2\langle A(t_k) l(t_k), h_{din}(t_k) \rangle \Delta t^2 + \\
&+ 2\langle l[t_k], \Delta f(t_k) \rangle \Delta t),
\end{aligned} \tag{25}$$

where

$$\Delta f(t_k) = f(t_k, u^0[t_k], v[t_k]) - \tilde{f}_{pq^0}(t_k). \tag{26}$$

Now note that $e^{2\lambda \Delta t} = 1 + 2\lambda \Delta t + 2\lambda^2 \Delta t^2 + o(\Delta t^2)$, where $o(\Delta t^2) > 0$. Remind also that $\lambda > \left\| A(t) \right\|$, $t \in [0, T]$. So, the following relations are valid:

$$\begin{aligned}
&\left| l(t_k) \right|^2 + 2\langle l[t_k], A(t_k) l[t_k] \rangle \Delta t + \\
&+ \left| A(t_k) l[t_k] \right|^2 \Delta t^2 - e^{2\lambda \Delta t} \left| l(t_k) \right|^2 \le \\
&\le \left| l(t_k) \right|^2 + 2\left| l(t_k) \right|^2 \left\| A(t_k) \right\| \Delta t + \left| l(t_k) \right|^2 \left\| A(t_k) \right\|^2 \Delta t^2 - \\
&- \left| l[t_k] \right|^2 - 2\left| l[t_k] \right|^2 \lambda \Delta t - \left| l[t_k] \right|^2 \lambda^2 \Delta t^2 = \\
&= \left| l[t_k] \right|^2 [2(\left\| A(t_k) \right\| - \lambda) \Delta t + (\left\| A(t_k) \right\|^2 - 2\lambda^2) \Delta t^2] \le 0.
\end{aligned} \tag{27}$$

Because of the given above properties of the function $f(t,u,v)$, there exists a number $R>0$, such that $|f(t,u,v)|\le R$, $t\in[0,T]$, $u\in P$, $v\in Q$. Thus, the following inequality holds:

$$|\Delta f(t_k)|\le 2R. \tag{28}$$

Remind also that the considered random values $h_{din}(t_k)$ and $l[t_k]$ satisfy the conditions:

$$|h_{din}(t)|\le H, \qquad E\{|h_{din}(t)|\}\le \delta_{din}, \qquad t\in[0,T], \tag{29}$$

$$l[t_k]\le L, \qquad k=0,...,K. \tag{30}$$

So, from (25) due to (27)-(30) we deduce

$$\Delta V_{t_k}^{t_{k+1}} \le C_1\Delta t^2 +$$

$$+e^{-2\lambda t_{k+1}}(2L|h_{din}(t_k)|\Delta t + 2\langle l[t_k],\Delta f(t_k)\rangle\Delta t), \tag{31}$$

where $C_1 = 4R^2 + H^2 + 4RH + 4\lambda LR + 2\lambda LH$.

The estimation (31) is valid for any realizations of the dynamic disturbances $h_{din}(t_k)$ and for any results of the random trial for the choice of actions $u^0[t_k]$ and $v[t_k]$. Thus, from (31), taking into account the stochastic independence of $h_{din}(t_k)$ from $l[t_k]$ and $l^*[t_k]$, we obtain the estimation:

$$E\left\{\Delta V_{t_k}^{t_{k+1}} \mid l[t_k],l^*[t_k]\right\} \le C_1\Delta t^2 + C_3\delta_{din}\Delta t +$$

$$+2e^{-2\lambda t_{k+1}}\langle l[t_k], E\left\{\Delta f(t_k)\mid l[t_k],l^*[t_k]\right\}\rangle\Delta t, \tag{32}$$

where $C_3 = 2L$. Due to (23), (24) and (26) we have

$$E\left\{\Delta f(t_k)\mid l[t_k],l^*[t_k]\right\} =$$

$$= E\left\{f(t_k,u^0[t_k],v[t_k]) - \tilde f_{pq^0}(t_k)\mid l[t_k],l^*[t_k]\right\} =$$

$$= \tilde f_{p^0q}(t_k) - \tilde f_{pq^0}(t_k). \tag{33}$$

Now, according to the choice of probabilities p^0 (see (13)), "actions" q^0 (see (15)), and because of (28), (33), we deduce

$$\left\langle l[t_k], E\left\{\Delta f(t_k)\mid l[t_k],l^*[t_k]\right\}\right\rangle\Delta t =$$

$$= \left\langle l[t_k]-l^*[t_k], \tilde f_{p^0q}(t_k) - \tilde f_{pq^0}(t_k)\right\rangle\Delta t +$$

$$+\left(\left\langle l^*[t_k], \tilde f_{p^0q}(t_k)\right\rangle - \left\langle l[t_k], \tilde f_{pq^0}(t_k)\right\rangle\right)\Delta t \le$$

$$\le 2R\left|l[t_k]-l^*[t_k]\right|\Delta t. \tag{34}$$

Remind that the random vector $l^*[t_k]$ satisfies the condition: $E\left\{\left|l[t_k]-l^*[t_k]\right| \mid l[t_k]\right\} \le \delta_{\inf}$. Using this and the formula of iterated mathematical expectations, from (22) and (24) we obtain

$$E\left\{\Delta V_{t_k}^{t_{k+1}} \mid l[t_k]\right\} = E\left\{E\left\{\Delta V_{t_k}^{t_{k+1}} \mid l[t_k], l^*[t_k]\right\} \mid l[t_k]\right\} \le$$

$$\le C_1 \Delta t^2 + C_3 \delta_{din} \Delta t + C_2 \delta_{\inf} \Delta t, \tag{35}$$

where $C_4 = 4R$. From (35) we conclude

$$E\left\{V(t_{k+1}, l[t_{k+1}]) \mid l[t_k]\right\} = E\left\{V(t_k, l[t_k]) + \Delta V_{t_k}^{t_{k+1}} \mid l[t_k]\right\} =$$

$$= V(t_k, l[t_k]) + E\left\{\Delta V_{t_k}^{t_{k+1}} \mid l[t_k]\right\} \le$$

$$\le V(t_k, l[t_k]) + (C_1 \Delta t + C_2 \delta_{\inf} + C_3 \delta_{din}) \Delta t.$$

The proof of Lemma is complete.

Proof of Theorem. Take the numbers $0 < V_* < V^*$, such that for any possible realization $l[t,\omega] = x[t,\omega] - z[t,\omega]$ the variation of the vector $C_4 = 4R$ satisfies the inequality

$$\left|V(\tau^*, l[\tau^*, \omega]) - V(\tau_*, l[\tau_*, \omega])\right| < V^* - V_* \tag{36}$$

for all $\tau_* \in [t_k, t_{k+1}]$, $\tau^* \in (\tau_*, t_{k+1}]$, $k = 0, ..., K$.

Consider the induction on t_k from $k = 0$ to $k = K$.

For the first step from $t_0 = 0$ to $t_1 = \Delta t$ according to results of Lemma and because of relations $V(0, l[0, \omega]) = \left|l[0]\right|^2$, $\left|l[0]\right| \le \delta_0$, $\Delta t \le \delta$, we have the estimation:

$$E_\Omega\left\{V(t_1, l[t_1, \omega])\right\} \le \delta_0^2 + (C_1 \delta + C_2 \delta_{\inf} + C_3 \delta_{din}) \Delta t. \tag{37}$$

Here $E_\Omega\{...\}$ is the mathematical expectation, i.e. averaging, with respect to all possible elementary events $\omega \in \Omega$. Select a subset $\Omega_1 \subset \Omega$ for which the following inequality is valid:

$$V(t_1, l[t_1, \omega]) \le V_*, \qquad \omega \in \Omega_1. \tag{38}$$

Then, using Chebyshev's inequality and according to (37), we deduce

$$P_1 = P(\Omega) - P(\Omega_1) \le (\delta_0^2 + (C_1 \delta + C_2 \delta_{\inf} + C_3 \delta_{din}) \Delta t) / V_*. \tag{39}$$

Inequalities (38) and (39), if we take into account (36), means that the probability of all realizations $l[t,\omega]$ that for all $t \in [t_0, t_1]$ remain in the domain $V(t,l) \le V^*$, satisfies the inequality

$$P\left(V(t, l[t, \omega]) \le V^*, \forall t \in [t_0, t_1]\right) \ge 1 - P_1 \ge$$

$$\ge 1 - (\delta_0^2 + (C_1 \delta + C_2 \delta_{\inf} + C_3 \delta_{din}) t_1) / V_*. \tag{40}$$

Now by induction, suppose that for the time moment $t_k < T$ we have the inequality:

$$P\left(V(t,l[t,\omega]) \le V^*, \forall t \in [t_0, t_k]\right) \ge$$

$$\ge 1 - (\delta_0^2 + (C_1\delta + C_2\delta_{\inf} + C_3\delta_{din})t_k)/V_* . \qquad (41)$$

Let Ω_k be the set of all elementary events a for which inequality (41) and the following inequality hold

$$V(t_k, l[t_k,\omega]) \le V_*, \qquad \omega \in \Omega_k . \qquad (42)$$

Note that here, by analogy with Lemma, we can use the estimation

$$E_{\Omega_k}\left\{V(t_{k+1}, l[t_{k+1},\omega])\right\} \le E_{\Omega_k}\left\{V(t_k, l[t_k,\omega])\right\} +$$

$$+(C_1\Delta t + C_2\delta_{\inf} + C_3\delta_{din})\Delta t . \qquad (43)$$

Select the subset $\Omega_{k+1} \subset \Omega_k$ such that the following inequality holds:

$$V(t_{k+1}, l[t_{k+1},\omega]) \le V_*, \qquad \omega \in \Omega_{k+1} . \qquad (44)$$

From (41)-(44), using again the Chebyshev's inequality and (36), we obtain the inequality

$$P\left(V(t,l[t,\omega]) \le V^*, \forall t \in [t_0, t_{k+1}]\right) \ge$$

$$\ge 1 - (\delta_0^2 + (C_1\delta + C_2\delta_{\inf} + C_3\delta_{din})t_{k+1})/V_* . \qquad (45)$$

According to the method of mathematical induction we conclude that the inequality (41) holds for any $k = 0,...,K$.
From (41) for $k = K$ we obtain

$$P\left(V(t,l[t,\omega]) \le V^*, \forall t \in [t_0, T]\right) \ge$$

$$\ge 1 - (\delta_0^2 + (C_1\delta + C_2\delta_{\inf} + C_3\delta_{din})T)/V_* \ . \qquad (46)$$

The inequality (16) follows from (46) if we take δ_0, δ, $\delta_{\inf}$, and δ_{din}, such that $\delta_0^2 + (C_1\delta + C_2\delta_{\inf} + C_3\delta_{din})T \le \beta V_*$.
The proof of Theorem is complete.

5. Model problem - 1

The following example illustrates the considered algorithm of mutual tracking of x and z-motions.

The controlled system has the form

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = f(t,u,v) + h_{din}$$

$$t_0 = 0 \le t \le T = 4$$

$$u \in P = \{ u^{[1]} = -1, \ u^{[2]} = 1 \}$$

$$\in Q = \{ v^{[1]} = -1, \ v^{[2]} = 1 \}$$

$$f(t,u,v) = \begin{cases} 0.5u + (u+v)^2 + v, \ t \in [t_0, \frac{T}{4}], \ t \in [\frac{2T}{4}, \frac{3T}{4}] \\ u + (u+v)^2 + 0.5v, \ t \in [\frac{T}{4}, \frac{2T}{4}], \ t \in [\frac{3t}{4}, T]. \end{cases}$$

Using the considered stochastic algorithm of control under the values of parameters of x-object and z-model $x_1(0)$=-1.0, $x_2(0)$=1.0, $z_1(0)$=-0.95, $z_2(0)$=1.05,

$\Delta t = t_{k+1} - t_k = \delta = 0.01$, $E\{ | \Delta_{\inf} | \} \le \delta_{\inf} = 0.01$, $E\{ | h_{din}(t) | \} \le \delta_{din} = 0.01$ we obtain the results of computer simulation for the motions of x-object (solid line) and z-model (dashed line) presented at figure 1. At this figure the phase portrait of the motion of x-object and z-model is depicted.

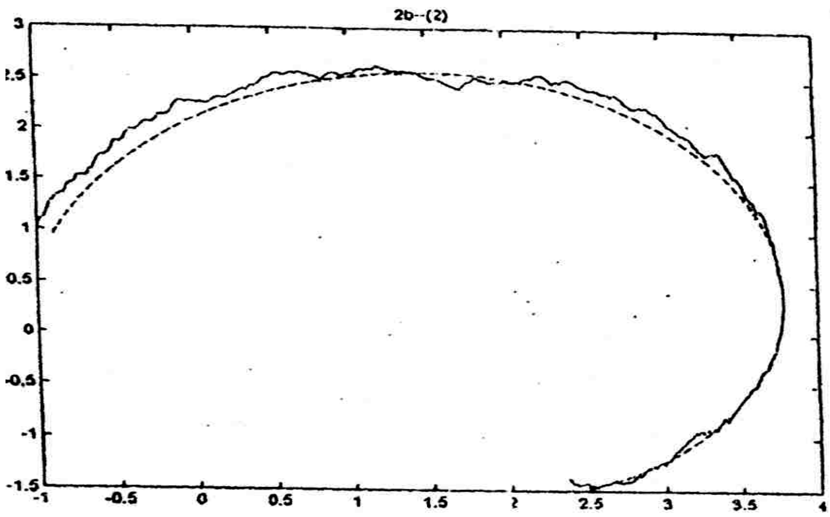


Figure 1

6. Optimal control problem

Of course these constructed above methods of tracking will not be so important if we consider them alone without some informal problem.

So, in the next section we apply the elaborated universal algorithm (or block) of control that was constructed in the first section for solution of some specific problems. At first, we apply the elaborated block of mutual tracking to the solution of some simple problems – tracking along the arbitrary curves, then for the solutions of problem with optimal ensured results for the given positional quality index γ. Then we apply it to the solution of antagonistic differential games of two players, using the formalizations of the theory of differential games developed at USU and IMM in Ekaterinburg.

The following model problems and its simulations illustrate the connections of the constructed block of mutual tracking with solutions of these concrete problems. In this case we use some sub-program that calculate the control actions for z-models-leaders. For example, we use the well studied now method of extremal shift on the accompanying points [3], designed in Ekaterinburg.

In [5] there were prepared a programming toolbox in the MATLAB system. In the toolbox we can change the components of the controlled system (1) $A(t)$, $f(t, u, v)$, $\Delta_{\inf}$, h_{din}.

7. Model problem - 2

We consider the model problems for the following 2-dimensional controlled system:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \mathrm{a}(t)u + \mathrm{b}(t)\,(u+v)^2 + \mathrm{c}(t)v + h_{din}$$

where

$$t_0 = 0 \le t \le T = 4$$

$$u \in P = \{\, u^{[1]} = -1,\ u^{[2]} = 1 \}$$

$$v \in Q = \{\, v^{[1]} = -1,\ v^{[2]} = 1 \}$$

$$\mathrm{a}(t) = \begin{cases} 4, & 0 \le t \le 2 \\ 0, & 2 \le t \le 4 \end{cases}$$

$$\mathrm{b}(t) = \frac{1}{2}$$

$$\mathrm{c}(t) = \begin{cases} 0, & 0 \le t \le 3 \\ 2, & 3 \le t \le 4 . \end{cases}$$

The quality index is defined by the formula

$$\gamma = \max\ \{|x_1[t^{[1]}=1]|,\ |x_1[t^{[2]}=T=4]|\}.$$

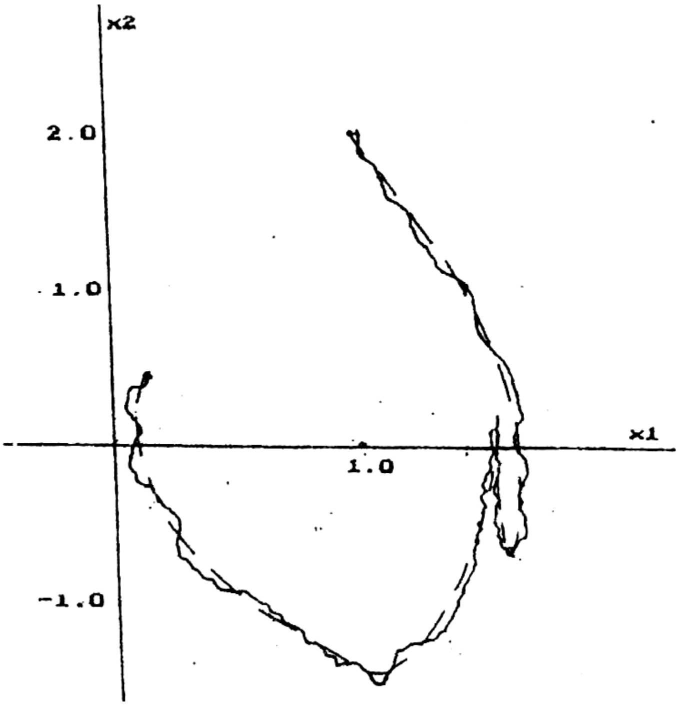


Figure 2

Using the considered stochastic algorithm of control for the x-object and z-model with the values of parameters $x_1(0)$=1.0, $x_2(0)$=2.0, $z_1(0)$=0.95, $z_2(0)$=2.05, $\Delta t = t_{k+1} - t_k = \delta = 0.01$, $E\{\| \Delta_{\inf} \|\} \le \delta_{\inf} = 0.01$, $E\{\| h_{din}(t) \|\} \le \delta_{din} = 0.01$, we obtain the results of computer simulation for the motions of x-object (solid line) and z-model (dashed line) presented at Figure 2. At this figure the phase portrait of the motion of x-object and z-model is depicted. The optimal guaranteed result defined in [3] for the initial position $\{t_0 = 0, x_1(0)$=1.0, $x_2(0)$=2.0} is equal ρ^0(0, x[0])=2.0. The value of the quality index $\gamma(x[t], 0 \le t \le 4) = 1.67$.

8. Summary

In this report the following results are obtained:

1. The block of the mutual stable stochastic tracking for the nonlinear real x-object and its suitable computer (virtual) z-model-leader is constructed in the discrete in time t feedback positional control scheme. This stable stochastic tracking is based on the extremal minimax and maximin conditions. The dynamical system has a random dynamical disturbance in the right hand side of the differential equation. In the positional feedback control scheme the information image has a random information error. It is shown that the closeness between the motions of the object and the model is ensured with the probability that is arbitrary close to one. The corresponding lemmas and theorems are proved.
2. Using the constructed block of mutual tracking some concrete optimal control and game-control problems are solved.

9. References

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