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MULTILEVEL STRUCTURES FOR
ON-LINE DYNAMIC CONTROL

By

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Abstract

The paper presents basic ideas of three of the possible structures of hierarchical control for a dynamic system. All these structures are closed-loop, that is, they make use of feedback in form of the state measured in the controlled system. Local decision units and a supremal unit (coordinator) exist in each of the hierarchical structures, but they differ in the tasks assigned to each level. The price coordination method allows for relative autonomy of the local units, which are being asked to solve appropriate short-horizon, dynamic problems in a repetitive way. In the other two structures the local problems are non-dynamical, but would be much less natural for a human decision maker.

Multilevel Structures For On-Line Dynamic Control

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1. Introduction

Structures of on-line dynamic control using decomposition present difficulties unknown to static systems. The problem lies in the use of feedback from the system in operation. In steady-state control it could be enough to use feedback in form of measured inputs or outputs of the system elements and to provide for an extremum of a current or "instantaneous" performance index (Ref. 2,5,6,17). The dynamic optimization needs considering at time t the future behavior of the system, that is to consider an "optimization horizon". Since the future behavior depends on both control and the initial state, we cannot determine optimal control input unless we know the present state of the system. It means that if we wish to have a control structure with feedback from the reality this feedback must contain information on the state $x(t)$, (see Ref. 1,3, 7,13).

We should not be misled by a possibility to obtain dynamic optimization solution by iterations on the real system, for example determining the best price trajectories $\hat{p}(t)$, $t \in (0,T)$, using a search procedure. Iterations could apply only to consecutive runs of a batch process (Ref. 9,15). A certain class of processes only would allow such an optimization and even in that case we would still be interested in having a feedback control structure in the course of a single run of the process.

As opposed to on-line control problems as they are considered in this paper, there exists an excellent coverage of dynamic optimization methods using decomposition (see Ref. 8, 10,12,14,16). The two things should not be confused.

2. Dynamic price coordination

In this section one of possible structures for dynamic optimal control using both decomposition and feedback is presented (Ref. 7). Its distinctive feature is the use of prices on inputs and on outputs of the system elements with the aim to achieve coordination of the local decisions.

(i) The global problem

Assume the optimal control problem of interconnected system to be as follows

$$\text{(1)} \quad \text{minimize } Q = \sum_{i=1}^{N} \int_0^{t_f} f_{oi}(x_i, m_i, u_i)\,dt$$

subject to

$$\dot{x}_i = f_i(x_i, m_i, u_i), \quad i = 1,\dots,N \quad \text{(state equations)}$$

$$y_i = g_i(x_i, m_i, u_i), \quad i = 1,\dots,N \quad \text{(output equations)}$$

$$u = Hy \quad \text{(interconnections)}$$

with $x(0)$ given, $x(t_f)$ free or specified.

(ii) Decomposition

Consider that in solving the problem we incorporate the interaction equation into the following Lagrangian:

$$L = \sum_{i=1}^{N} \int_0^{t_f} f_{oi}(x_i, m_i, u_i)\,dt + \int_0^{t_f} \langle p, u - Hy \rangle\, dt$$

where $\langle p, u - Hy \rangle$ means $\sum_{j=1}^{\dim u} p_j (u-Hy)_j$.

Assume the solution to the global problem using this Lagrangian has been found and it has provided for

$\hat{x}_i$, $i = 1,\ldots,N$ - optimal state trajectories

$\hat{m}_i$, $i = 1,\ldots,N$ - optimal controls

$\hat{u}_i$, $i = 1,\ldots,N$ - optimal inputs

$\hat{y}_i$, $i = 1,\ldots,N$ - optimal outputs

$\hat{p}$ - solving value of Lagrangian multipliers.

Note that now our Lagrangian can be split into additive parts, thus allowing to form a kind of local problems:

$$\text{(2)} \quad \text{minimize } Q_i = \int_0^{t_f} [f_{oi}(x_i,m_i,u_i) + \langle \hat{p}_i,u_i \rangle - \langle \hat{q}_i,y_i \rangle]dt$$

where

$$y_i = g_i(x_i,m_i,u_i)$$

and optimization is subject to

$$\dot{x}_i = f_i(x_i,m_i,u_i)$$

$x_i(o)$ given, $x_i(t_f)$ free or specified as in the original problem.

In the local problem the price vector $\hat{p}_i$ is an appropriate part of $\hat{p}$ and $\hat{q}_i$ is also given by $\hat{p}$ as

$$\hat{q}_i = \sum_{i=1}^{N} H_{ji}^T \hat{p}_i \quad .$$

Notice that we have put optimal value of price vector $\hat{p}$ into the local problems, which means we have solved the global problem before. Thanks to it the solutions of local problems will be strictly optimal. There is little sense, however, in solving the local problems if the global was solved before, because the global solution would provide not only $\hat{p}$ but also $\hat{x},\hat{m}$ for the whole system.

To make the thing practical let us try to shorten the local horizons and to use feedback there.

(iii) Short horizon for local problems

Let us shorten the horizon from t_f to t_f', so that

(2) becomes

$$\text{(3)} \quad \text{minimize } Q_i = \int_0^{t_f'} [f_{oi}(x_i,m_i,u_i) + \langle \hat{p}_i,u_i \rangle - \langle \hat{q}_i,y_i \rangle]dt$$

with $x_i(o)$ given as before, but the target state taken from the global long-horizon solution, $x_i(t_f') = \hat{x}_i(t_f')$.

For the local problem (3) we must of course supply the price vectors $\hat{p}_i,\hat{q}_i$. It may be reasonable to use also $\hat{u}_i$ from the global solution, that is the "predicted" input value.

(iv) The use of feedback at local level

The short horizon formulation (3) will pay off if we will have to repeat the solving of (3) many times as opposed to solving the global problem once only. Consult now Figure 1, where the principle of the proposed control structure is presented.

Feedback at the local level consists in solving the short-horizon local problems at some intervals $T_1 < t_f'$ and using the actual value of measured state $x_{*i}(kT_1)$ as new initial value for each repetition of the optimization problem.

This brings a new quality; we now have a truly on-line control structure and can expect, in appropriate cases, to get results better than those dependent on the models only.

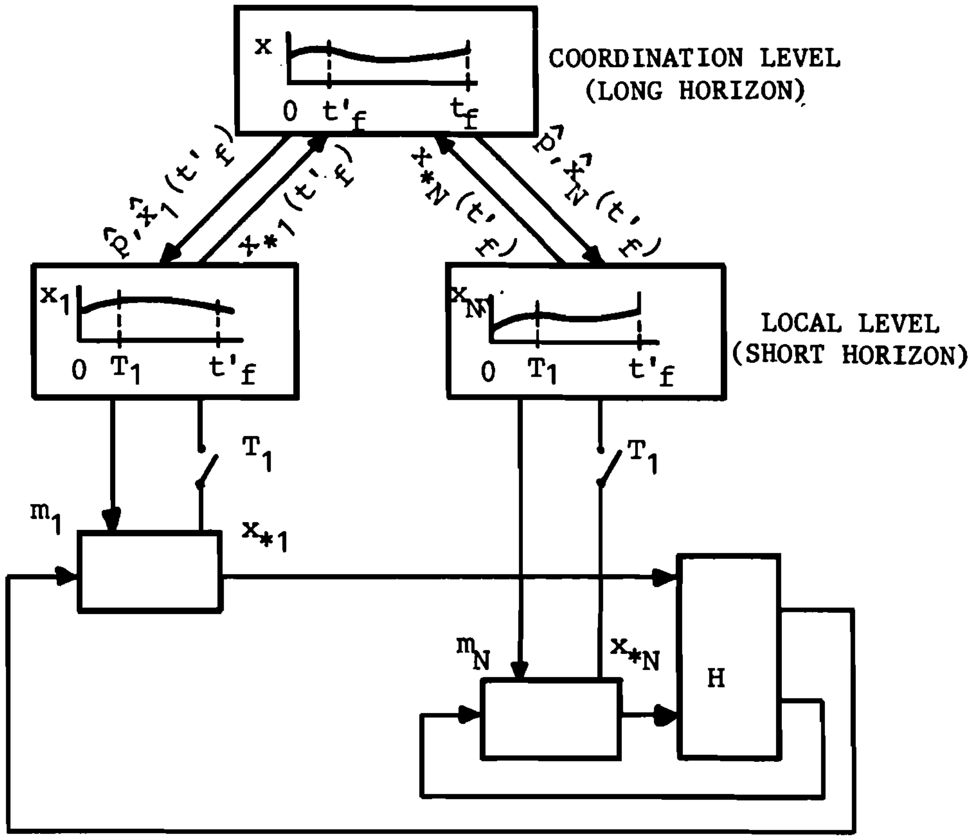

Fig. 1 Structure of on-line dynamic price coordination.

What we do is more exactly as follows: at $t = 0$ solve $\max Q_i$ for $[0,t_f']$ with $x_i(0)$, apply control $\hat{m}_i$ to the real system for $[0,T_1]$, at $t = T_1$ solve $\max Q_i$ for $[T_1,t_f']$ with $x_i(T_1) = x_{*i}(T_1)$ as measured, apply control $\hat{m}_i$ to the real system for $[T_1, 2T_1]$, etc. etc.

Note that we now have a gain from both decomposition and shortening the horizon. The often repeated local problems are low-dimension and short-horizon.

The feedback algorithm just indicated would be referred to as repetitive optimization scheme.

We should mention disturbances which act on the real system and were not yet shown explicitly in the formulations. Disturbance prediction would be used while solving (1) and (3), that is the global and the local problems. And it is indeed because of the disturbances which in reality will differ from their prediction that we are inclined to use feedback structure of Figure 1.

(v) The use of feedback in coordination

The feedback introduced so far cannot compensate for the errors done by the coordination level in setting the prices p. Another repetitive feedback can be introduced to overcome this shortage, for example bringing to the coordinator actual values x_{*i} at time t_f', $2\,t_f'$,... and asking the global problem to be resolved for each new initial value. This structure of control is presented in Figure 1.

We should very well note that feeding back the actual values of state achieved makes sense if the models used in computation differ from reality, for example because of disturbances. Otherwise the actual state is exactly equal to what the models have predicted and the feedback information is irrelevant.

A doubt may exist whether the feedback to the coordinator shown in Figure 1 makes sense, because the lower level problems have to achieve $x_i(t_f') = \hat{x}_i(t_f')$ as their goal and already use feedback to secure it. It should be remembered, however, that the model-based target value $\hat{x}_i(t_f')$ is not optimal for the real system and asking the local decision making to achieve exactly $x_{*i}(t_f') = \hat{x}_i(t_f')$ may be not advisable or even not feasible.

The coincidence of feedback to coordination level with times t_f', $2\,t_f'$ is not essential. It might be advisable to use this feedback and do the re-computation of the global problem prior to time t_f', that is more often.

(vi) Static elements in the system

The length of the global problem horizon t_f has to be matched to the slowest system element dynamics and the slowest of the disturbances. The shortened horizon t_f' for the local problems would in fact result from considering repetitive optimization at the coordination level, for example as 1/10 of t_f. It may then happen that the dynamics of a particular system element are fast enough to be neglected in its local optimization problem within the horizon t_f'. This means, in other words, that if we would take $\hat{m}_i, \hat{u}_i$ from the global optimization solution, the optimal state solution $\hat{x}_i$ follows these with negligible effect of element dynamics.

To make this assumption more formal let us consider that the system element has been supplied with first-layer follow-up controls of some appropriately chosen controlled variables c_i (Ref. 4,6). We are then allowed to assume that c_i determines both x_i and m_i of the original element and the optimization problem becomes

$$(4) \quad \text{minimize } Q_i = \int_0^{t_f'} [f_{0i}'(c_i,u_i) + \langle \hat{p}_i,u_i \rangle - \langle \hat{q}_i,y_i \rangle]dt$$

where $f_{0i}'(\cdot)$ is a reformulation of the function f_{oi} due to substituting c_i in place of x_i, m_i.

Note well that although (4) will not be a dynamic problem its results will be time functions. In particular $\hat{c}_i$ will be time-varying control. This is due to time-varying prices $\hat{p}_i, \hat{q}_i$.

Let us repeat the essential assumption under which the dynamic local problem (3) reduces to the static problem (4): the dynamic optimal solutions $\hat{m}_i, \hat{u}_i, \hat{x}_i$ were assumed to be slow.

(vii) The use of simplified models

In the described structure of on-line dynamic coordination we have made no use till now of the possibility of having a simplified model in the global problem which is being solved at the coordination level at times 0, t_f', 2 t_f', etc.

The global problem may be simplified for at least two reasons: the solution of the full problem may be too expensive to be done, and the data on the real system, in particular prediction of disturbances, may be too inaccurate to justify a computation based on the exact model.

Simplification may concern dimension of state vector (introduce aggregated x^c instead of x), dimension of control vector (m^c instead of m) and dimensions of inputs and outputs ($u^c = H^c y^c$ instead of $u = Hy$).

The global problem Lagrangian will now be

$$L = \sum_{i=1}^{N} \int_0^{t_f} f_{oi}^c \ (x_i^c, m_i^c, u_i^c) dt + \int_0^{t_f} \langle p^c, u^c - H^c y^c \rangle \, dt \ .$$

The simplified solution will yield optimal state trajectory $\hat{x}^c = (\hat{x}_1^c, \hat{x}_2^c, \ldots, \hat{x}_N^c)$ and optimal price function $\hat{p}^c$. The linking of those values to the local problems cannot be done directly, because the local problems consider full vectors x_i, u_i and y_i.

We have to change the previous requirement $x_i(t_f') = \hat{x}_i(t_f')$ into a new one

$$\gamma_i[x_i(t_f')] = \hat{x}_i^c(t_f')$$

which incidentally is a more flexible constraint, and we also have to generate a full price vector $\hat{p}$:

$$\hat{p} = R \, \hat{p}^c$$

where R is an appropriate "price proportion matrix". The prices composing the aggregated p^c may be termed "group prices".

We should note that functions γ_i and matrix R have to be appropriately chosen. The choice may be made by model considerations, but even with the best possible choice optimality of overall solution will be affected, except for some special cases.

(viii) System interconnection through storage elements

The system interconnections in Figure 1 were stiff, that is an output was assumed to be connected to an input in a permanent way. The dynamic problem formulation gives an opportunity to consider another type of interconnection, a "soft" constraint of integral type:

$$\int_{kt_b}^{(k+1)t_b} (u_{ij} - y_{1r})dt = 0$$

which corresponds to taking input u_{ij} from a store, with some output y_{1r} connected to the same store and causing its filling. Asking for integral over $[kt_b,(k+1)t_b]$ to be zero means that supply and drain have to be in balance over each balancing period t_b.

A store may be supplied by several outputs and drained by more than one subsystem input. There may also be many stores, for example for different products. If we assume the same balancing period for all of them the integral constraint becomes

$$\int_{kt_b}^{(t+1)t_b} (Pu_w - My_w)dt = 0$$

where u_w,y_w are parts of u, y connected to the stores (the stiffly interconnected parts will be termed u_s,y_s).

Matrices P,M show the way by which u_w, y_w are connected to various stores. The number of stores is of course $\dim My_w = \dim Pu_w$. A state vector w of the inventories can also be introduced

$$w(kt_b + t) = w(kt_b) + \int_{kt_b}^{kt_b+t} (Pu_w - My_w)d\tau \quad .$$

With both stiff and soft interconnections present in the system, the global problem Lagrangian becomes

$$L = \sum_{i=1}^{N} \int_0^{t_f} f_{oi}(x_i,m_i,u_i)dt + \int_0^{t_f} < p,u_x - Hy_x > dt +$$

$$+ \sum_{k=0}^{k=\frac{t_f}{t_b}-1} < \eta^k, \int_{kt_b}^{(k+1)t_b} (Pu_w - My_w)dt >$$

and we of course continue to consider

$$\dot{x}_i = f_i(x_i,m_i,u_i), \qquad i = 1,\ldots,N$$

$$y_i = g_i,(x_i,m_i,u_i) \qquad i = 1,\ldots,N$$

$$x_i(0) \text{ given, } x_i(t_f) \text{ free or specified, } i \in \overline{1,N} \quad .$$

In comparison with the previous Lagrangian a new term has now appeared, reflecting the new constraint. Note that prices η^k associated with the integral constraint are constant over periods t_b. Note also, that if t_b will tend to zero, the integral constraint gets similar to the stiff one and the stepwise changing η will change continuously, like p does.

With two kinds of interconnections the local problems also change correspondingly and they become

$$(5)\quad \text{minimize } Q_i = \int_0^{t_f} [f_{oi}(x_i,m_i,u_i) + \langle \hat{p}_i,u_{si}\rangle - \langle \hat{q}_i,y_{si}\rangle]dt + \sum_{k=0}^{k=\frac{t_f}{t_b}-1} \langle \hat{\eta}^k, \int_0^{t_b} (P_i u_{wi} - M_i y_{wi})dt \rangle$$

where $y_{si} = g_{si}(x_i,m_i,u_i)$, $y_{wi} = g_{wi}(x_i,m_i,u_i)$ and optimization is subject to

$$\dot{x}_i = f_i(x_i,m_i,u_i)$$

$x_i(0)$ given, $x_i(t_f)$ free or specified.

A new quality has appeared in problem (5) in comparison with (3): the inputs u_{wi} taken from the stores are now free control variables and can be shaped by the local decision maker, who previously had only m_i in his hand. The local decisions will be under the influence of prices $\hat{p}$ and $\hat{\eta}$ $(\hat{\eta}^0,\hat{\eta}^1,\ldots)$, where both $\hat{p}$ and $\hat{\eta}$ have to be set by the solution of the global problem.

The local problem (5) has no practical importance yet; it will make sense when we introduce local feedback and shorten the horizon, like it was in the previous stiff-interconnection case.

We shall omit the details and show it only as a control scheme (see Figure 2).

Thinking about how to improve action of the coordinator we made previously a proposal to feed actual $x_*(t_f')$ to his level. We have now additional state variables, the inventories w. If the price $\hat{\eta}^k$ is wrong, the stores will not balance over $[kt_b,(k+1)t_b]$. It is almost obvious that we can catch-up by influencing the price for the next period η^{k+1} and that we should condition the change on the difference $\hat{w}[(k+1)t_b] - w_*[(k+1)t_b]$, where $w_*(\cdot)$ is a value measured in the real system. This kind of feedback is also shown in Figure 2.

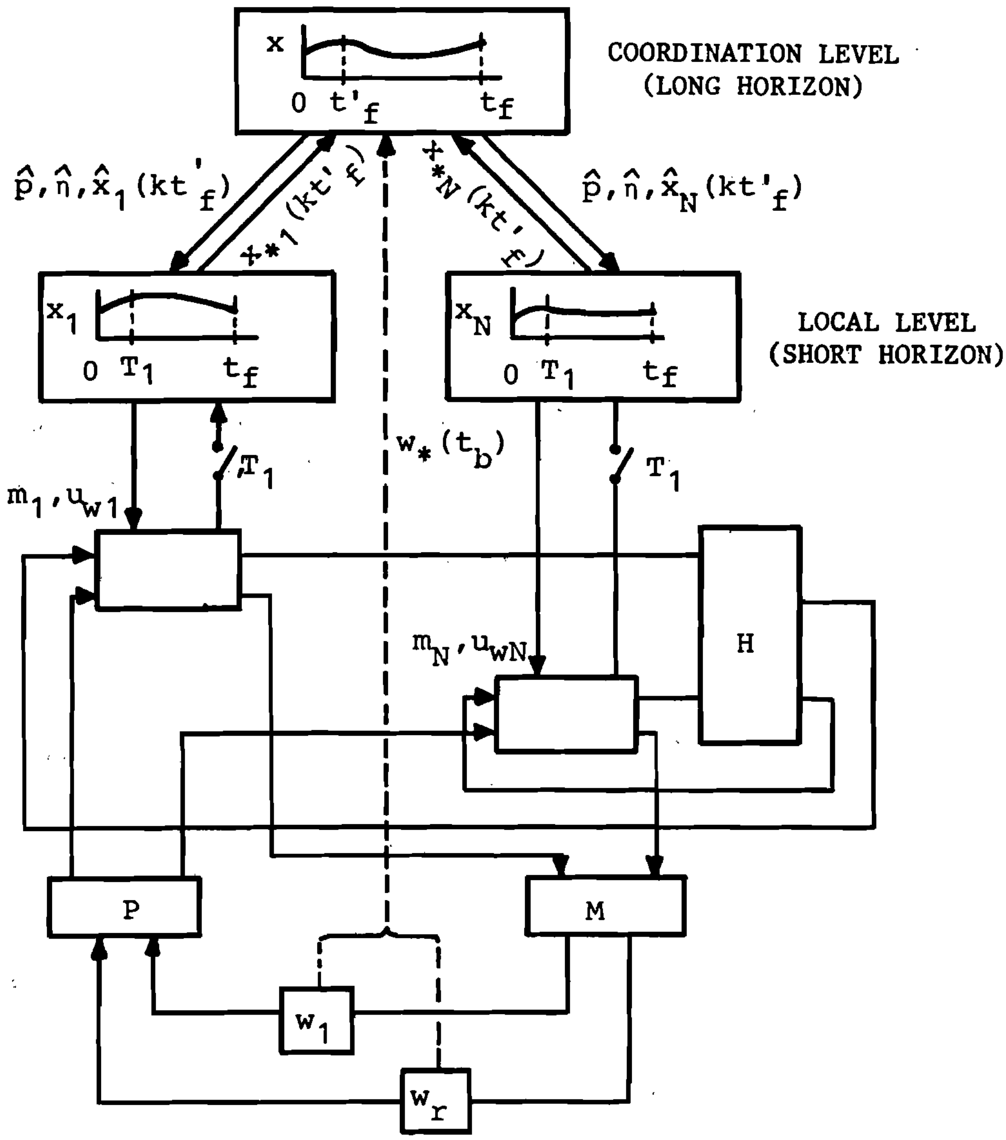

Fig. 2. On-line dynamic price coordination in a system containing stores in the interconnections.

(ix) Conclusions on dynamic price coordination

Time-varying prices have been shown to be a possible coordination instrument in a multilevel structure of on-line control.

Although the local problems may be formulated as short-horizon and each of them has low dimension, the coordination level must solve the global problem for full horizon in order to generate optimal prices and the target states for the local problems. Simplified global model may be used in appropriate cases.

Price coordination structure applies to systems with stiff interconnections and also to interconnections through storage elements.

Analytical solutions of the dynamic problems involved are not needed, therefore we are by no means restricted to linear-quadratic problems.

3. Multilevel control based upon state-feedback concept

The literature on optimal control has paid considerable attention to the structure where the control at time t, that is m(t), would be determined as a given function of current state x(t). Comprehensive solutions exist in this area for the linear system and quadratic performance case, where the feedback function proved to be linear, that is

$$m(t) = R(t)\ x(t)$$

where R(t) is in general a time-varying matrix.

Trying to apply this approach to the complex system we might implement for each local problem

(6) $m_i(t) + R_{ii}(t)\ x_i(t)$

where R_{ii} is one of the diagonal blocks in the matrix R.

The result of such local controls, although all state of the system is measured and used, is not optimal. Note that for $m_i(t)$ we would rather have to use

$$m_i(t) = R_i(t)x(t)$$

that is we should make $m_i(t)$ dependent on the whole state $x(t)$.

We can compensate for the error commited in (6) by adding a computed correction signal

$$(7) \quad m_i(t) = R_{ii}(t)x_i(t) + \hat{v}_i(t) \quad .$$

The exact way to get $\hat{v}_i(t)$ would be to generate it continuously basing upon the whole $x(t)$. This would, however, be equivalent to implementing state feedback for the whole system directly, with no advantage in having separated the local problems.

Exactness has to be sacrificed. With this in mind we may propose various solutions, for example (see also Figure 3).

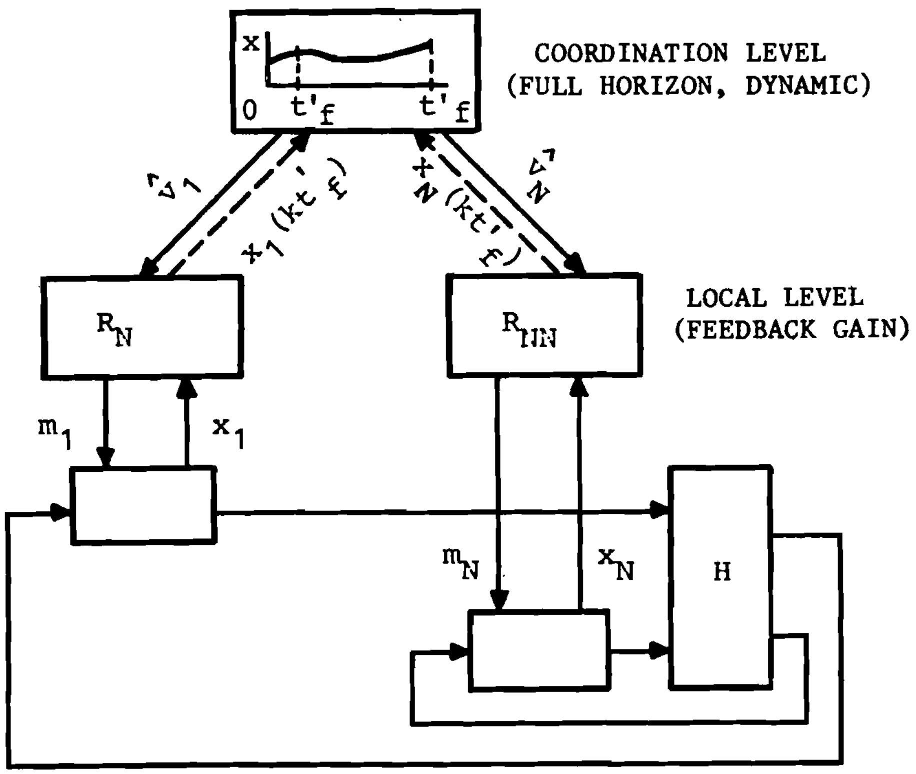

Fig. 3. Dynamic multilevel control based on feedback gain concept.

(i) $\hat{v}_i$ will be generated at $t = 0$ for the whole optimization horizon t_f (open-loop compensation);

(ii) $\hat{v}_i$ will be generated at $t = 0$ as before but will be recomputed at $t = t_f' < t_f$, using actual $x(t_f')$, etc. (repetitive compensation);

(iii) $\hat{v}_i$ will not be generated at all, but we implement instead in the local problems

(8) $m_i(t) = \tilde{R}_{ii}(t) x_i(t)$

where $\tilde{R}_{ii}$ is adjusted so as to approach optimality. This structure may be referred to as decentralized control. We could think of re-adjusting $\tilde{R}_{ii}$ at some time intervals, which could be looked upon as adaptation. This adaptation would present a way of on-line coordination of the local decisions.

It may be worthwhile to mention that local decision making based upon (6), (7) or (8) makes more sense for a mechanistic implementation than for a hierarchy of human operators, where the previous approach based on "maximization of local performance subject to imposed prices" seems to be more adequate, to what really happens in the system.

We should also remember that the feedback gain solutions to optimization problems are available for a restricted class of these problems only.

4. Structures using conjugate variables

It is conceivable to base on-line dynamic control upon maximization of the current value of the Hamiltonian, thus making a direct use of the Maximum Principle.

For the complex system described as (1) at the beginning of this paper the Hamiltonian would be

$$\mathcal{H} = \sum_{i=1}^{N} f_{0i}(x_i, m_i, u_i) + \langle \psi, f(x,m,u) \rangle \quad .$$

The interconnection equation

$$u - Hy = u - Hg(x,m,u) = 0$$

provides for u to be a function of (x,m) in the interconnected system

$$u = \phi(x,m) \quad .$$

Therefore

$$\mathscr{H} = -\sum_{i=1}^{N} f_{oi}(x_i, m_i, \phi_i(x,m) + < \psi, f[x,m,\phi(x,m)] > \quad .$$

Assume the global problem has been solved (model-based) using the Hamiltonian and hence the optimal trajectories of conjugate variables $\hat{\psi}$ are known.

We are going to use the values of $\hat{\psi}$ in local problems.

First let us note that having $\hat{\psi}$ we could re-determine optimal control by performing at the current time t

$$\text{(9)} \quad \text{maximize } \mathscr{H} = -\sum_{i=1}^{N} f_{oi}(x_i, m_i, \phi_i[x_i(x,m)] + $$

$$+ < \hat{\psi}, f[x,m,\phi(s,m)] >$$

where the problem is an "instantaneous maximization" and needs no consideration of final state and future disturbances. This information was of course used while solving the global problem and determining $\hat{\psi}$.

For the (9) to be performed we need the actual value of state x. We could obtain it by simulating system behavior starting from the time t_1 when initial condition $x(t_1)$ was given, that is by using equation

$$\dot{x} = f[x,m,\phi(x,m)]$$

with $x(t_1)$ given and $m = \hat{m}$ known for $[t_1,t]$ from the previous solutions of (9).

We could also know x(t) by measuring it in the real system (note that a discussion of model-reality differences would be necessary).

Problem (9) is static optimization, not a dynamic one. We would now like to divide it into subproblems. It can be done if we come back to treating $u - Hy = 0$ as a side condition and solve (9) by using the Lagrangian

$$(10) \quad L = - \sum_{i=1}^{N} f_{oi}(x_i,m_i,u_i) + \langle \hat{\psi}, f(x,m,u) \rangle + \langle p, u - Hy \rangle$$

where $y = g(x,m,u)$.

Before we get any further with this Lagrangian and its decomposition let us note the difference with respect to dynamic price coordination presented so extensively before. We have had there

$$L = \int_0^{t_f} \sum_{i=1}^{N} f_{oi}(x_i,m_i,u_i)dt + \int_0^{t_f} \langle p, u - Hy \rangle dt$$

subject to

$$\dot{x}_i = f_i(x_i,m_i,u_i), \qquad i = 1,\dots,N \quad .$$

It was a dynamic problem.

In the present case there are no integrals in $L(\cdot)$ and the dynamics are taken care of by the values of conjugate variables $\hat{\psi}$. The differential equations of the system are needed only to compute the current value of x in our new, "instantaneous"

Lagrangian. No future disturbances are to be known, no optimization horizon considered - all these are imbedded in $\hat{\psi}$.

Assume we have solved problem (10), using system model i.e., by computation and we have the current optimal value of price p, that is $\hat{p}(t)$. We can then form the following static local problems to be solved at time t

$$(11) \quad \underset{m_i u_i}{\text{maximize}}\ L_i = - f_{oi}(x_i,m_i,u_i) + \langle \hat{\psi}_i, f_i(x_i,m_i,u_i) \rangle + \langle \hat{p}_i, u_i \rangle - \langle \hat{q}_i, y_i \rangle \ .$$

These goals could be used in a structure of decentralized control, see Figure 4. The local decision makers are asked here to maximize $L_i(\cdot)$ in a model-based fashion and apply control $\hat{m}_i$ to the system elements. Current value x_i is needed in performing the task. The coordination level would supply $\hat{\psi}_i$ and the prices $\hat{p}_i, \hat{q}_i$ for the local problem. They would be different for each t.

Note that there is no hill-climbing search on the system itself.

Figure 4 would first imply that the local model-based problems are solved immediately with no lag or delay. We can therefore assume, conceptually, that the local decision making is nothing else but implementation of a state feedback loop, relating control $\hat{m}_i(t)$ to the measured $x_i(t)$. In an appropriate case we could think of solving problem (10) analytically with $\hat{\psi}_i$, $\hat{p}$ as parameters and the result would exactly be the feedback formula (the feedback decision rule).

If analytical solution of (10) is not the case we have to implement a numerical algorithm of optimization and some time will be needed to perform it. A discrete version of our control would have to be considered.

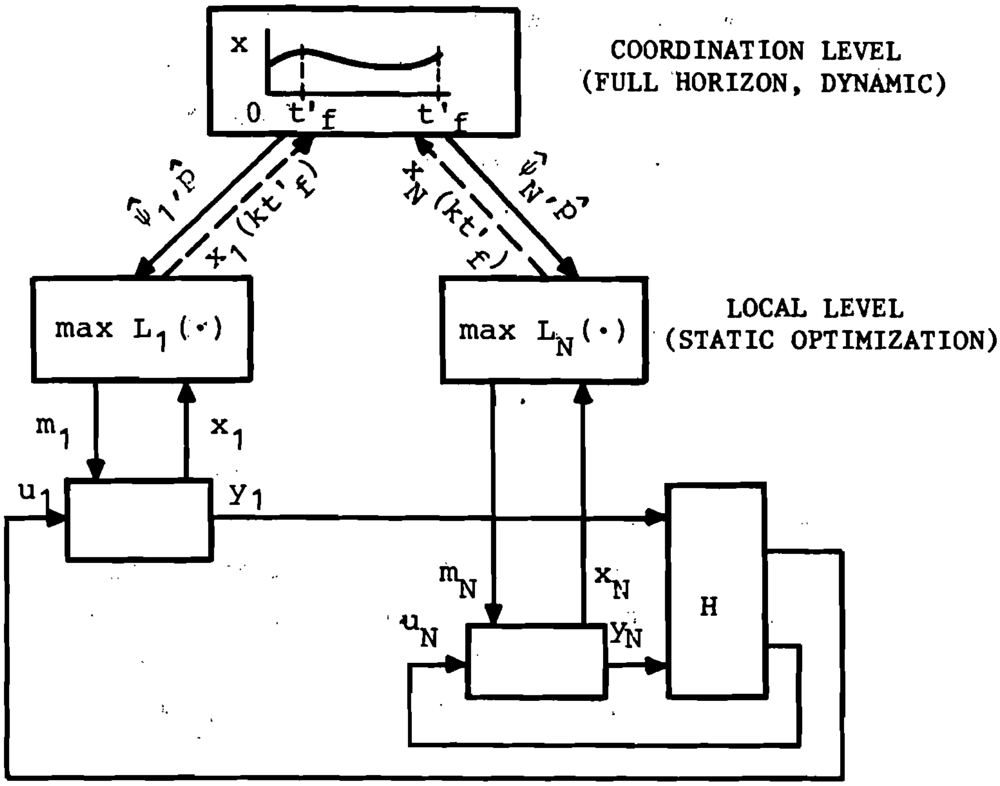

Fig. 4. Dynamic multilevel control using conjugate variables

Now let us think about feedback to the coordinator. We might decide to let him know the state of the system at some time intervals t_f', that is $x(kt_f')$. On this he could base his solution $\hat{\psi}$ for all $t \geqslant kt_f'$ and also the prices $\hat{p}$ for the next interval $[kt_f', (k+1)t_f']$. This policy would be very similar to what we have proposed in the "dynamic price coordination".

It might be worthwhile to make again some comparisons between dynamic price coordination and the structure using both prices and conjugate variables.

In the "maximum principle" structure the local problems are static. The local goals are slightly less natural, as they involve $< \hat{\psi}_i, \dot{x}_i >$ that is the "worth of the trend". This would be difficult to explain economically and hence difficult to implement in a human decision making hierarchy. As the problem is static, no target state is prescribed.

Note that both these cases avoid to prescribe a state trajectory. It is felt that in the dynamic control this kind of direct coordination would be difficult to perform if model-reality differences are assumed.

5. Conclusions

There are several possibilities to structure a dynamic multilevel control system, using feedback from the real system elements in the course of system operation. It is not possible at this stage to evaluate advantages and drawbacks of the alternatives. It may be easily predicted that if the mathematical models used do not differ from reality, all structures would give the same result - fully optimal control. The clue is what will happen if models are inadequate. Quantitative indications are essentially missing in this area.

Another feature of the structures would concern their use in a human decision making hierarchy. In that case it is quite essential what will be the local decision problem, confined to an individual decision maker. He may feel uncomfortable for example, if asked to implement only a feedback decision rule (as it happens in the "state feedback" structure), or to account for the worth of the trend $< \hat{\psi}_i, \dot{x}_i >$ in his own calculations, as it is required in the structure using conjugate variables. For the human decision maker, the structure with price coordination seems to be most natural.

Table 1 shows a comparison of structures discussed in this paper.

Table 1. Comparison of dynamic coordination structures.

SYSTEM TYPE	COORDINATOR	LOCAL PROBLEMS	LOCAL GOALS
DYNAMIC PRICE COORDINATION	solves global problem, sets prices $\hat{p}$ and targets $\hat{x}_i$	dynamic optimization	maximize performance, achieve target state
STATE-FEEDBACK CONCEPT	solves global problem, supplies compensation signal $\hat{v}_i$	state feedback decision rule	no goal
USING CONJUGATE VARIABLES	solves global problem, sets prices $\hat{p}$ and conjugate variables $\hat{\psi}_i$	static optimization	maximize performance inclusive of $< \hat{\psi}_i, \dot{x}_i >$

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