RM-77-51

INTERNATIONAL TRADE POLICIES IN MODELS OF BARTER EXCHANGE

M.A. Keyzer

October 1977

Preface

The food problem is to a large extent a local one. Accordingly, the starting point in the Food and Agriculture research of IIASA is the modeling of national food and agricultural systems. After having investigated local, national strategies directed towards specific goals (e.g. introducing new technologies, changing the agricultural structure, etc.) a generalization will be possible and conclusions can be drawn concerning the global outcomes of changing agricultural systems. Thus, the global investigation will be based on national models and their interactions.

To reflect these interactions in a model, a methodological research is required which is concerned with the linkage of national models for food and agriculture. This paper is the third of a series on this topic.

Previously on this topic: RM-77-2, Linking National Models of Food and Agriculture: An Introduction, January 1977.

RM-77-19, Analysis of a National Model with Domestic Price Policies and Quota on International Trade, April 1977.

Abstract

This paper is the third in the series on the linkage of national models of food and agriculture.

International agreements are discussed in which given levels of world market prices are aimed at. Buffer stock agreement, various compensatory financing schemes and specific types of cartels are represented in general equilibrium models of barter exchange.

Table of Contents

Introduction

When all the actors on world markets are considered to be price takers the resulting equilibrium prices on these markets are rather incidental outcomes, although policies of the actors such as stock policies will influence world market prices.

In the present paper, international policies which explicitly aim at reaching certain given world market prices are discussed. Various instruments can be used to realize such a policy. Within the framework of a general equilibrium model of barter exchange (no money explicitly introduced), the following schemes are discussed:

1. An international buffer stock agency which tries to keep prices (a) at given levels,
   (b) within given range.

2. An agreement to keep world market prices at given levels by adjusting internal prices (a) for all nations,
   (b) for a subset of nations.
   This agreement also has a compensatory finance interpretation, in which developing nations are compensated against adverse development on the world market.

3. Various other compensatory financing schemes.

Price formation on international commodity markets has become more and more an issue of political debate, the oil crisis and the price boom for agricultural commodities in 1973-74 have stimulated the discussion.

The International Development Strategy adopted by the United Nations for the Second Development Decade [6] emphasizes the improvement of international trade relations in favour of the least developed countries. This was re-emphasized in the U.N. Declaration and Programme of Action on a New International Economic Order. UNCTAD's Committee on Commodities has proposed both a multi-commodity buffer stock and schemes of compensatory financing. All these proposals imply that international agreements should be made by which world market prices could be kept at target levels. How these targets should be fixed is an open issue and a very political one. The welfare implications of different schemes are difficult to evaluate theoretically because of all the distortions already introduced by the national policies. This paper, therefore, limits itself to the presentation of some models in which price targets are given.

Before the international policy schemes are discussed, the main points of the two previous papers will be summarized:

1. A national model is considered as a continuous nondifferentiable excess demand function: $z_j = z_j(p^w)$, where p^w is the vector of world market prices for n-1 agricultural commodities and one (residual) nonagricultural commodity, z_j is the vector of excess demand (demand minus supply) by the j^{th} country.

2. The national excess demand is assumed to satisfy for all prices p^w: $p^w z_j + k_j(p^w) = 0$, where k_j is set in such a way that $\sum_j k_j = 0$; k_j is a continuous function of prices. This means that the balance of trade of the world is in equilibrium in any given period of time and that the balance of trade of a nation has a predetermined disequilibrium, k_j.

3. National excess demand is assumed to be homogenous of degree zero in world market prices.

4. Given 1, 2, and 3, a competitive equilibrium can be shown to exist on the world market, that is a price vector p^{w*} such that world excess demand is nonpositive:

$$\{p^{w*} \mid z \leqq 0 , \quad z = \Sigma\, z(p^w)\} ,$$

which implies that at prices p^{w*}, a feasible allocation exists. The equilibrium may be nonunique.(1)

5. A national model describes how at given world market prices a national excess demand is arrived at. This implies that a <u>unique</u> excess demand should correspond to given world market prices. This sets restrictions on the specification of the national models. It however makes a modular approach

---

(1) The issue of the computation of equilibrium prices on the world market will be discussed in the next forthcoming paper in this series, although some remarks on this topic are made in Appendix 2.

possible in the sense that it makes it feasible to develop independently quite different national models. A national model for a market economy should describe price formation on domestic markets. Under the free trade assumption, this is computationally a simple matter, the question, however, is not so simple when domestic price policies, buffer stocks and quota on international trade are considered. These problems have been discussed in [4], where algorithms have been developed

The Results of [4] are summarized below. The numerical implementations of the algorithms will be reported in a separate paper.

	Free Trade	Domestic Price Policy	Quota	Domestic Price Policy and Quota
One consumer* no production	1	1	3	5,6
More consumers no production	1	2	4	4
One consumer* production	1	2	3	6
More consumers production	1	2	(7)	(7)

1. direct computation;
2. iteration over taxation rate (assumption: no inferior goods);
3. convex programming problem; 4,5 can solve special cases of this;
4. complementary pivoting algorithm. The commodities with quota form a linear expenditure subsystem; several taxation policies are possible;
5. 4 but also valid for generalized C.E.S. utility function;
6. parametric convex programming;
7. only solved for cases with quota on inputs which are not consumer goods.

---

* A Hicksian economy is considered as an economy with one consumer.

1. A MODEL WITH STOCK ADJUSTMENT UNDER STRONG RIGIDITIES

In this paragraph a model will be presented which describes a world market equilibrium in which stock adjustment prevails over price adjustment: as long as the vector of stocks of commodities remains within given bounds no price adjustment takes place. We call this a strong price rigidity. A weak price rigidity is a more general case in which, as long as stocks permit prices are restricted to remain within given bounds; this will be discussed in the following paragraph.

First the model will be written out, then an existence proof for the equilibrium will be presented, some implications of the model will be discussed and finally some possible applications will be discussed. In loose terms, the question is does a price vector on the world market exist, such that:

- the aggregate excess demand after stock adjustment is nonpositive;
- the constraints on stocks are satisfied;
- the equilibrium prices only deviate from target prices when constraints on stocks are effective and do so in the intuitively expected sense;
- the a priori stock commitment expressed in target prices will not be over drawn and will actually be met at least as long as not all the minimum constraints on stocks are effective?

1.1 The model

a) Aggregate net demand functions

Consider:

$$z = z(p^w, t)$$

which satisfies

$$p^w z = -t \quad .$$

z is the net demand (= aggregate endowments - aggregate demand), where the demand is defined excluding demand by a stock agency; p^w is the current world market price; t is the income committed for the stock holding activity. It is assumed that these commitments are distributed over consumers (countries) in such a way that they are smaller than the wealth: $t^j < p^w y^j + k_j$

(y^j endowment vector, k_j credit received ; $\Sigma k_j = 0$). As only world market prices will be discussed here we further drop the superscript w. The same distributional problems and opportunities come up as were discussed in [4, §1.3.2].

The aggregate net demand correspondence is assumed to be represented by continuous functions (the restriction is only needed for computational reasons). The functions are assumed to be bounded below.

b) Policy Equations

$i = y - x - s = -(z+s)$	final stock
$y = y^g + y^p$	definition
$\bar{p}(y^g + \phi y^p - i) \leq 0$ $\rho\bar{p}(y^g + \phi y^p - i) = 0$	a priori commitment
$i\ min \leq i \leq i\ max$	boundaries on final stocks
$p = \rho\bar{p} + \mu - \nu$	definition
$t = \mu\ smin - \nu\ smax + \rho\bar{p}(y^g + \phi y^p)$	effective commitment
$\Sigma_h(\rho\bar{p}_h + \mu_h + \nu_h) = 1$	

c) Market equilibrium conditions

$s = \max(o, -(imax + z))$	excess supply
$s, \mu, \nu, p, i \geq 0$	
$\mu(i - imin) = 0$	
$\nu(i - imax) = 0$	
$ps = 0$	

d) Assumptions on constraints

$$imin \geq 0$$
$$\bar{p}\ imin \leq \bar{p}(y^g + \phi y^p) \leq \bar{p}\ imax$$
$$\Sigma\bar{p}_h = 1$$

From the stock adjustment follows

$$p\ i = t$$

y is the vector of endowments, s is the free disposal or excess supply, i is the final stock, x is the demand by the nations.

y^g is thought to be an external stock e.g. a stock owned by an international agency before exchange. We may assume $y_t^g = i_{t-1}$. [1)]

y^p is the total endowment owned by the nations themselves.

The nations commit themselves to make available for the stock a certain maximum amount of wealth $\bar{p}(y^g + \phi y^p)$ expressed in target prices, $\bar{p}$. The stock agency is allowed to deviate from this commitment only to finance final stocks (when actual prices deviate from target prices). Note that the commitment is a scalor. The models, therefore, do not imply that a stock needs to be carried over in kind by the agency although this may be one way to interpret the model. The commitment is expressed as an international income transfer, not as a domestic tax.

Note also that the model is not implying that the stock policy can fully accomodate differences between supply and demand, price adjustments may be needed.

1.2 Existence proof

The proof of the existence of the equilibrium proceeds along the same lines as the proof for domestic equilibrium in [4,sec.1]. It is again only a slight extension of Debreu's proof (see appendix [1]). The proof will only be presented in shorthand. The details are comparable to those in [4, section1]. Walras Law for this case is:

$$pi = t \quad .$$

Substituting the price definition and the rule for effective commitment one gets:

$$(\rho\bar{p} + \mu - \nu)\ i = \mu.imin - \nu.imax + \rho.\bar{p}(y^g + \phi y^p) \quad .$$

---

1) In this interpretation all the stocks are physically held by the agency and $\phi\bar{p}y^p$ is a new commitment which is used to buy new stocks for the agency. The variable ϕ may therefore become zero once the agency has built up sufficient stocks.

Consider the following linear programme:

$$\max J = \mu(imin - i) + \nu(i - imax) - \rho\bar{p}(i - y^g + \phi y^p) \quad .$$

$$\left\| \begin{array}{ll} S.T. & \Sigma(\mu_i + \nu_i + \rho\bar{p}_i) = 1 \\ \text{and} & \mu,\ \nu,\ \rho \geqq 0 \\ \text{and} & p = \rho\bar{p} + \mu - \nu \geqq 0 \\ \text{and} & p\ s = 0 \end{array} \right.$$

$$i,s \quad \text{given}$$

Define p as the set of constraints of the linear programme.

The linear programme defines a mapping which maps p into itself and is upper semicontinuous; p is a compact, closed, convex set; the mapping $p \to I\ (p)$ is upper semicontinuous to that there will be by Kakutani's theorem a fixed point $(s^*, i^*, \mu^*, \nu^*, \rho^*)$ in which $\mu^*\ (imin - i^*) + \nu^*\ (i^* - imax) - \rho^*\bar{p}(i^* - (y^g + \phi y^p)) = 0$ by Walras' Law.

1) The commodities can now be divided into two groups:

- group 1 with $s_h > 0 \to p_h = 0$
- group 2 with $s = 0$

We first note that by construction

$$imin_1 \leqq i_1^* = imax_1$$

Rewriting the goal function into two component yields:

$$J_1^* = J_1^* + J_2^*$$

where

$$J_1^* = (\mu_1^* - \nu_1^*)\ imax_1 + \mu_1^*\ (imin_1 - imax_1) + \rho^*\bar{p}_1\ (y_1^g + \phi y_1^p)$$

$$J_2^* = \mu_2^*\ (imin_2 - i_2^*) + \nu_2^*(i_2^* - imax_2) - \rho^*\bar{p}_2(i^* - (y_2^g + \phi y_2^p)$$

Considering J_1^* we may remark that because $imin_1 \leqq imax_1$, by the maximum properties $\mu_1^* = 0$.

2) Setting $\rho = 0$ and $\nu = 0$ we get

$$0 \geqq \mu_2\ (imin_2 - i_2^*), \quad \mu_2 \mid \nu = 0,\ \rho = 0;\ \mu\ \nu\ \rho\ \varepsilon\ p$$

(p is the set of constraints of the linear programme),

therefore

$$imin_2 \leqq i_2^*;$$

we know by construction that

$$i_2^* \leqq imax_2 \quad .$$

3) Setting $\mu_2 = \nu_2 = 0$

and substituting

$$\nu_1 = \rho\bar{p}_1$$

we get

$$0 \geqq \rho\ \bar{p}_1((y_1^g + \phi y_1^p) - i_1) + \rho\bar{p}_2\ ((y_2^g + \phi y_2^p) - i_2^*)$$

so that

$$0 \geqq \rho\ \bar{p}\ ((y^g + \phi y^p) - i^*)$$

This means that J^* is a sum of nonpositive terms so that individual terms are zero because $J^* = 0$
Therefore we may summarize:

$$\begin{array}{llll} \mu^* & (i^* - imin) = 0 & & imin \leqq i^* \\ \nu^* & (i^* - imax) = 0 & & i^* \leqq i^*max \\ \rho^*\bar{p} & ((y^g + \phi y^p) - i^*) = 0 & & \bar{p}((y^g + \phi y^p) - i^* \leqq 0 \end{array}$$

so that the equilibrium conditions of the model are satisfied.

1.3 Interpretations of the model

1. A straightforward interpretation of the model is to assume that there exists an international buffer stock agency, which in the first years of its creation operates with direct contributions in wealth or in kind ($\phi .\ \rho\bar{p}y^p$) from the participating nations and later on operates with its own stocks y^g. The profits or losses of the operations are however carried by the nations.

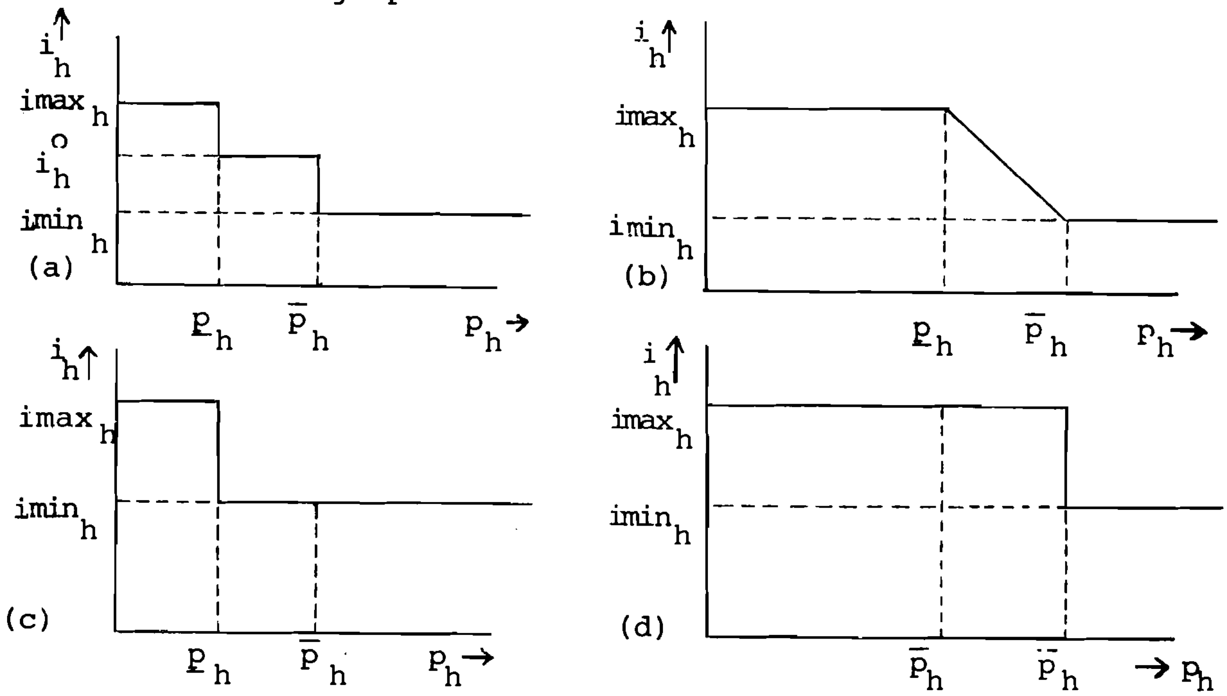
2. A slightly different interpretation suggests that a cartel is formed which operates the bufferstock, assuming that the other countries remain pricetakers.

3. A third interpretation is that prices have an inherent rigidity: the price target is just last year's price. The rigidity is part of the market itself. One could call this interpretation a disequilibrium interpretation.

These three interpretations (altruistic, oligopolistic and structural) can be given to most models in this paper.

2. A MODEL WITH STOCK ADJUSTMENT UNDER WEAK PRICE RIGIDITIES

When the stock (agency) has to defend a price band instead of a price level many different scenario's may be thought of for the behaviour within the price band. This can be illustrated in a few graphs



ad (a) The case can be described as "stay where you are". This concept has a clear interpretation if one thinks of an agency with a given endowment which operates on the market only when the limit of a price band is reached.

ad (b) In this case the stock agency performs a linear interpolation to determine its behaviour. The agent reacts on all price deviations.

ad (c), (d) These cases essentially show the same behaviour as cases of strong rigidity.

We shall in the following consider an arbitrary behavioural function $\bar{i}_h = \bar{i}_h(p)$ defined for $\underline{p} \leq p \leq \bar{p}$ which satisfies over its range of definition the budget constraint: $p\bar{i} = p(y^g + \phi y^p)$.

Within the price band the stock agency is just an actor like any other with an excess demand (which is assumed to satisfy standard continuity assumptions).

It can be considered as the aggregate of behaviour of many (e.g. national) actors. As a matter of fact the only reason for extracting this behaviour from the national models is to depict the behaviour at the band limits and outside of it. Only there do international agreements really need to be made.

2.1 The model:

The model is essentially the same as the one presented in the previous paragraph, so that it will only briefly be described.

a) Aggregate excess demand

$z = z(p,t)$ which satisfies $pz = -t$.

b) Policy

$i = y - x - s$	final stock
$= -(z+S)$	
$q((y^g + \phi y^p) - i) = 0$	a priori commitment[1]
$imin \leq i \leq imax$	boundaries on stocks
$p = q + \mu - \nu$	definition
$t = \mu . imin - \nu . imax + q\ (y^g + \phi y^p)$	effective commitment
$\varepsilon\ \underline{p} \leq q \leq \varepsilon\ \bar{p}$	target prices
$\bar{i} = \bar{i}\ (p)$ which satisfies	
$p\ \bar{i} = p(y^g + \phi y^p)$	target stock within band
$(\varepsilon\bar{p}_h - q_h)\ (i_h - \bar{i}_h) = 0$	upper switch for target stock

1) One might conceptualize this by assuming that the agency is endowed with the physical stock: $y^g + \phi y^p$.

$(\varepsilon \underline{p}_h - q_h)\ (i_h - \bar{i}_h) = 0$ lower switch for target stock

$0 \leqq \varepsilon \leqq 1, \quad \Sigma(q_h + \mu_h + \nu_h) = 1$ constraints

$\mu, \nu \geqq 0$ constraints

c) Market equilibrium conditions

$s = \max\ (0, -\ (imax + z)$

$p, i, s \geqq 0$

$\mu\ (i - imin) = 0$

$\nu\ (i - imax) = 0$

$p\ s = 0$

d) Assumptions on the constraints

$\underline{p} \leqq \bar{p}$

$\Sigma \underline{p}_i \leqq 1$

$\Sigma \bar{p}_i \geqq 1$

$0 \leqq imin \leqq y^g + \phi y^p \leqq imax$.

2.2 Existence proof

The existence proof proceeds along the same lines as in 1.2: one tries to formulate a linear programme, such that in the fixed point of the mapping of prices and quantitites into itself, the goal function is zero by Walras' Law; then use the maximum property of the goal function to show that in the fixed point the equilibrium conditions are satisfied.
Walras' Law is again: $pi = t$.
This may be rewritten as

$$\mu\ (imin - i) + \nu\ (i - imax) - q\ (i - (y^g + \phi y^p)) = 0$$

The side conditions of the linear programme are the constraints on the variables in the model, the switches and the target prices.

The linear programme is:

$$J = \max\ (\mu\ (imin - i) + \nu\ (i - imax) - q(i - y^g + \phi y^p))),$$

$$\text{S.T.}\quad \left|\begin{array}{l} \varepsilon \underline{p} \leqq q \leqq \varepsilon\ \bar{p} \\ (\varepsilon.\bar{p}_h - q_h)\ (i_h - \bar{i}_h) = 0 \\ (\varepsilon.\underline{p}_h - q_h)\ (i_h - \bar{i}_h) = 0 \\ 0 \leqq \varepsilon \leqq 1 \\ \Sigma(q_h + \mu_h + \nu_h) = 1 \\ q + \mu - \nu \geqq 0 \\ \mu,\ \nu \geqq 0 \\ \underline{p}\ s\ = 0 \end{array}\right.$$

The side conditions form a nonempty, compact convex set K, $k = \{\varepsilon, q, \mu, \nu\}$ (intersection of compact convex sets). As the proof proceeds exactly as in 1.2, we directly state that the maximum properties of the goal function in the fixed point permit the conclusions:

$$imin \leqq i^* \leqq imax,$$

$$q^*(i^* - (y^g + \phi y^p)) = 0$$

$$\mu^*(i - imin) = 0$$

$$\nu^*(i - imax) = 0$$

so that the equilibrium conditions of the model are satisfied.

The model allows for the same interpretations as the model with strong price rigidities.

Different assumptions are possible for the formulation of the rigidity (e.g. tying certain prices to a general price index) for this the reader is referred to Drèze [2]. Note that under weak price rigidities the notion of a priori commitment asks for a physical interpretation as an endowment.

3. PRICE RIGIDITY ON THE WORLD MARKET THROUGH ADJUSTMENT OF ALL NATIONAL POLICIES

Countries influence the world market prices through their national policies. In this paragraph we investigate two world market prices which can be maintained at a given level through adjustment of domestic policies, primarily of domestic prices.[1] It is a generally accepted observation that world market prices of agricultural commodities fluctuate more widely than domestic prices. The rigidity which is discussed here should therefore not be seen as a natural phenomenon but as the result of a concerted action of nations within the framework of something like a "new economic order"; one could think of integrated commodity agreements. Consider the following model:

a) National excess demand functions are

$$z_j = z_j(p, \rho t_j)$$

and satisfy

- $pz_j = \rho t_j$
- and $z_j \leqq k_j$, k_j finite, $k_j > 0$.[2]
- and z_j, monotonous in t_j

ρ is a scaling factor

p is an international price on which countries react

A more precise interpretation of the prices will be given below

---

1) The paper by Drèze [2] can be interpreted as describing this type of policy by allocation of quota to countries. The quota have the property that they are for all participants maxima on imports or exports not minima. This result is, however, obtained by imposing a uniform quota allocation scheme (same quota for all agents) a somewhat artificial scheme in an international context.

2) This can be guaranteed by imposition of an import quota on all national models. The assumption is needed for guaranteeing the existence of t_j satisfying condition b). Note that again t_j is not a domestic tax, but an international transfer of income.

b) Determination of transfers t_j

t_j is determined in such a way that given (p, ρ)

$\rho \cdot \bar{p}^w z_j = 0$ (assuming $y_j > 0$) we get $py_j + \rho t_j > 0$,

because

$\rho \, \bar{p}^w x_j > 0$, $p \neq 0$ implies $px_j > 0$. It is assumed that

t_j is unique.

Furthermore, we define

$t_j = -\infty$ if $\rho = 0$ and $z_j = k$ if $p = 0$

Note that (z_j, t_j) is homogeneous of degree zero in (p,ρ), we therefore may restrict (p,ρ) to:

$$\sum_i p_i + \rho = 1 \quad ; \quad p,\rho \geqq 0$$

c) We consider the following maximization problem at world level (dropping of the subscript j means that a world aggregate is considered):

$$\begin{aligned} \max \quad & pz - \rho\, t \\ \text{S.T.} \quad & \Sigma p_i + \rho = 1 \\ & p,\ \rho \geqq 0 \end{aligned}$$

z,t are given

The mapping (z,t) x (p,ρ) defined by this is uppersemicontinuous and has a fixed point (z^*, t^*, p^*, ρ^*) .

In this fixed point we get

$$p^* z^* = \rho^* t^*$$

$$z^* \leqq 0$$

$$t^* = 0$$

Economic Interpretations

The model described here can be interpreted in several equivalent ways. The main common feature of the interpretations is that the model describes a concerted action of all countries in the world market, an oligopolistic or structural interpretation is not appropriate here.

a) Compensatory financing scheme

The prices p are real world market prices but countries clear their balances of payment with each other according to different prices $\bar{p}^w$ which have been agreed upon. Other compensatory financing schemes will be discussed in § 5 .

b) Adjustment of internal prices

In this interpretation the countries keep on the world market the prices $\bar{p}^w$ as the real equilibrium prices. They do so by creating appropriate differentials with the prices $\bar{p}^w$. This can be done in two ways:

1) adjustment of domestic price policy
2) adjustment of quota

The equilibrium solution of the model shows how much the internal prices have to be adjusted either by a common extra tariff or by a "fair" quota; a fair quota is defined here as a quota constraint which creates equal (shadow) price differentials for all participants. Many non-fair quota allocations could of course realize the same world market prices but this is not considered here. A "fair" quota can imply a minimum on imports or exports, not only a maximum.

4. PRICE POLICIES ON THE WORLD MARKET PURSUED BY A SUBSET OF THE PARTICIPANTS

Consider the following model:

There are two groups of countries

* group 1 takes the world market prices p^w, as given; its excess demand function is

$$z_1 = z_1(p^w)$$

satisfying $p^w z_1 = 0$

we define: if $p^w = 0$ then $\sum_i^n z_{1i} = +\infty$; i = 1, j = n, i is the commodity index.

* group 2 tries to keep world market prices at the level $\bar{p}^w$. In order to do this it adjusts its excess demand (mainly by adaptation of internal prices p_2). It is however only willing/able to do so as long as the excess supply of group 1 remains within the constraints:

$$l \leqq -(z_1 + s_1) \leqq r$$

where s_1 is defined as $s_1 = \max(-(r + z_1), 0)$.

In equilibrium $p^w s_1 = 0$

The excess demand is

$$z_2 = z_2 (p_2, ta), \; z_2 \leqq k, \; k \text{ finite}, \; k > 0$$

satisfying : $p_2 z_2 = ta$; z_2 is assumed to be monotonous in ta

$$ta = \rho t - pw_1$$

where $w_1 = \min(s_1 + z_1, y_2)$

and t is set so that if $\rho > 0$ then $\rho\, \bar{p}^w z_2 = -\mu\, l + \nu r$, else $t = -\infty$

Again it is assumed that t is uniquely defined.

* Define

$$p^w = \rho \bar{p}^w + \mu - \nu \ ; \quad p^w \geqq 0$$

assume if $p_2 = 0$ then $z_2 = k$

*Assumptions on the parameters

$$-y_2 \leqq l \leqq r \leqq y$$

y is the vector of endowments at world level

y_2 is the vector of endowments of the second group

p_2 is the price at which countries within the second group exchange with each other; this is not a domestic price.

$r \geqq 0, \ l \leqq 0$ [1)]

$\bar{p}^w > 0$

The question is now, does an equilibrium exist such that:

1) the world market is cleared;
2) the excess demand of group 1 remains within the prespecified constraints;
3) world market prices deviate from their target level, only if these constraints are effective and do so in the intuitively expected sense;
4) the countries of the "Cartel" (group 2) do not generate a shortage on their balance of trade;
5) the value of excess demand at target prices is zero (this last condition is only a side product of the proof)

1) These conditions guarantee that $\rho \bar{p}^w x_2 = \rho \bar{p}^w y_2 - \mu l + \nu r \geqq 0$ for all prices and strictly positive for $\rho > 0$. Weaker conditions are possible but will not be investigated here. When $\rho \bar{p}^w x_2 > 0$ the positiveness of income at prices p_2 is also guaranteed because $p_2 x_2 = ta + p_2 y_2$, $p_2 \neq 0$.

<u>Existence of equilibrium</u>

Summation of the different balance equations yields the goal function for the linear programme needed for the proof.

$$J = p^{w}(z_1 + s_1) + \rho\bar{p}^{w}\, z_2 + \mu l - \nu r - \rho t + p_2(w_1 + z_2)$$

The constraint set K is:

$$\sum_i (\mu_i + \nu_i + p_i) + \rho = 1$$

$$p^{w} = \rho\bar{p}^{w} + \mu - \nu$$

$$p_2, \mu, \nu, p^{w}, \rho \geqq 0$$

$$p^{w}\, s_1 = 0$$

The linear programme can be defined as

$$\max J = J(\rho, \mu, \nu, p_2)$$

$$\rho, \mu, \nu, p_2 \,\varepsilon\, K$$

The proof that the fixed point is indeed the equilibrium proceeds, as usual in several stages.

1) $z_1^* + s_1^* \leqq -1$

The goal function in the fixed point may be written as

$$J^* = \mu^*(z_1^* + s_1^* + 1) - \nu^*(z_1^* + s_1^* + r) + \rho^*\bar{p}^{w}(z_1 + s_1^* + z_2^*)$$

$$+ p_2^*\,(w_1^* + z_2^*) - \rho^* t^*$$

$$J^* = 0$$

Because J^* is a maximum we may write, setting

$\rho = 0$ and $p_2 = \nu = 0$

$$0 \geqq \mu(z_1^* + s_1^* + 1), \ \forall \mu | \mu \,\varepsilon\, K \text{ and } \rho = 0,\ p_2 = \nu = 0 \ ;$$

therefore in the fixed point:

$$s_1^* + z_1^* \leqq -1$$

Note that this implies

$$w1^* = z_1^* + s_1^* \quad \text{because} \quad -y_2 \leqq 1 \ .$$

Therefore in the following only $z_1^* + s_1^*$ will be used

Define $z^* = z_1^* + s_1^* + z_2^*$.

2) $z^* \leqq 0$

Proof:

Setting $\rho = 0$ and $\mu = \nu = 0$ we get

$0 \geqq J = p_2 z^*$, $\forall p \mid p_2 \in K$ and $\rho = 0$, $\mu = \nu = 0$

so that $z^* \leqq 0$

3) $z_1^* + s_1^* + r \geqq 0$

This holds by the construction of s_1^*

4) Complementarity conditions

The goal function can be rewritten as

$$J^* = \mu^*(z_1^* + s_1^* + 1) - \nu^*(z_1^* + s_1^* + r) + \rho^* \bar{p}^w z^* + + p_2^* z^* - \rho^* t^* = 0$$

All components are known to be less or equal zero, they are, therefore, equal to zero.

Moreover, we know that $\mu^* \nu^* = 0$ so that $\mu^* s_1^* = 0$

as a consequence we may write

a) $\mu^*(z_1^* + 1) = 0$ $\quad$ $z_1^* + 1 \leqq 0$

b) $\nu^*(z_1^* + s_1^* + r) = 0$ $\quad$ $z_1^* + s_1^* + r \geqq 0$

c) $\bar{p}^w z^* = 0$ $\quad$ $z^* \leqq 0$

5) $p^{w*} z_2^* \leqq 0$

Proof: Knowing that

$p^{w*} z_1^* = 0$,

$p^{w*} s_1^* = 0$,

and $z_1^* + s_1^* + z_2^* \leqq 0$

we get $p^{w*}(z_1^* + s_1^* + z_2^*) = p^{w*} z_2^* \leqq 0$

Interpretations of the model

The model allows for an oligopolistic and an "altruistic" interpretation and shows clearly how "altruistic" institutions might be misused for oligopolistic purposes.

Note that the larger the endowments of group 2 are in relation to those of group 1 the easier it is for group 2 to set the level of world market prices. As long as the constraints 1, r are ineffective the actual clearing is done by group 2 only.

5. VARIOUS COMPENSATORY FINANCING SCHEMES

5.1 Target Prices

We consider a compensatory financing scheme for one subset of countries supported by another subset.
Here we distinguish three groups of countries

1) Countries having a compensatory finance scheme
2) Countries supporting this by limited income transfers
3) Non-participating countries

p is the actual world market price, $\bar{p}$ the target price for group 1. The countries of the second group have the following balance of payments equation:

$$pz_2 = - t \text{ with } tmin \leqq t \leqq tmax$$

where t is the support to countries of group 1.

Countries of group 1 receive the (possibly negative!) transfer. The balance of payments equation is

$$pz_1 = t$$

Where t is set in such a way that

$\bar{p}z_1 = \alpha$ in which $\bar{p}$ is the target price

and α is set such that:

$$\alpha = \alpha_1 - \alpha_2$$

$$\alpha_1 (t - tmax) = 0$$

α_2 (tmin - t) = 0 (constaints on t, α)

tmin $\leqq$ t $\leqq$ tmax

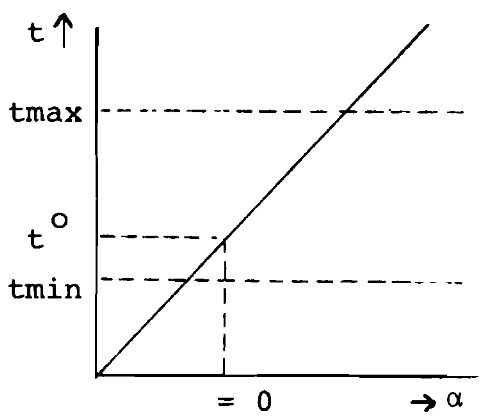
α_1, α_2, $\geqq$ 0

Under the traditional assumption that z_1 is a monotonously increasing nonsaturing function of t, one can at given p, $\bar{p}$, tmax, tmin, solve for (t, α) such that the constraints on it are satisfied. This can be done in a two-step fashion

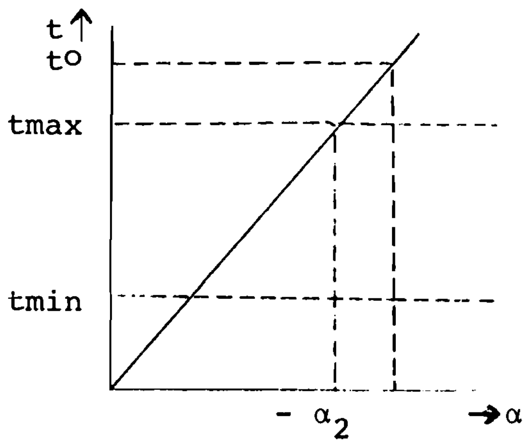
1) Compute the value t^o of t corresponding to α = 0
2) if t^o > tmax compute the value of α corresponding to t = tmax
3) else if t^o < tmax compute the value of α corresponding to t = tmin.

A diagram will show that the constraints are satisfied.

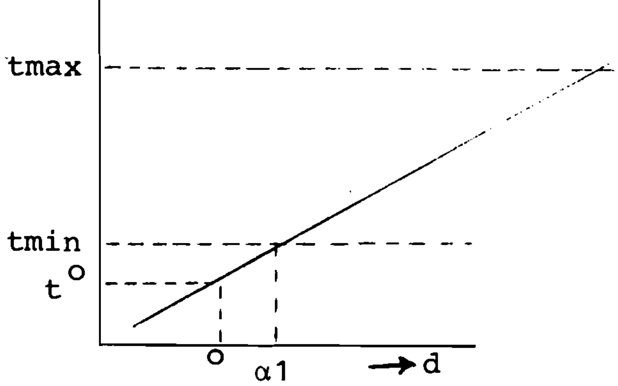
case 1 tmin $\leqq$ t^o $\leqq$ tmax



case 2 t^o > tmax



case 3 t^o < tmin



As can be seen from the diagrams, in case 1 $\alpha = 0$, in case 2 $\alpha < 0$ and in case 3 $\alpha > 0$. The resulting aggregate excess demand mapping $p \to z(p)$ is still uppersemicontinuous and satisfying $pz = 0$ so that at world market level the computational procedure remains the same as without compensatory financing. The procedure described here is admittedly somewhat unclean from the mathematical point of view. It however has the advantage that the dimensions of the world market equilibrium problem itself are not increased by the policy introduced. Moreover the computation of domestic equilibrium at unchanged world market prices is in general relatively straightforward and only affects countries of the first group so that the computational cost of the second iteration is very limited, while the necessary assumption of monoticity in t does not seem very restrictive for practical purposes. Note that if taxation limits are effective, a scheme for stabilizing income distribution within group 1 is still possible (the situation in which group 2 does not contain any country is of course a special case of this).

5.2 Target income

We again have three groups of countries supported, supporting and nonparticipating.

Suppose group 1 has a target income: k

Its balance of trade equation is again

$$p\,x_1 = py_1 + t$$

with $py_1 + t = k$ so that at given world market prices p the aid by group 2 is $k - py_1$.

As long as no limits are set on t the computation of aggregate excess demand is straightforward. Limits on t only restrict k but do not change the computations.

5.3 Target quantities

When output y is subject to random disturbances compensatory finance might can be such that the national income of the country is stabilized at its value for "normal" output levels we denote this by y^* .

For group 1 we get: $px = py^{*}$

$$t = p(y^{*} - y)$$

t may again be subjected to inequality constraints. As noted before, if all countries are in group 1 only distribution and not the level of income can be subjected to a compensatory financing scheme.

5. 4 Lagged compensation

If the compensating transfers have a lag of one period or more, the transfers are predetermined in the model. One should realize, however, that even when the actual payments are lagged countries might already in the current period adapt their spending to the compensation scheme.

APPENDIX 1

Debreu's Excess Demand Theorem [1]

Consider the set of excess demand function $z = z(p)$, which satisfies $p\, z(p) \leqq 0$. Does this problem have a solution $z \leqq 0$? Let p be the set of normalized prices. This is clearly a compact convex set. Denote by Z the set of all $z(p)$ for $p \in P$ [Z is the union of the sets $Z(p)$]. If Z is not convex, we replace it by any compact convex set containing Z, which we denote by Z'.

$Z(p)$ need not be compact. However the equilibrium necessarily lies within the subset of attainable states which is by assumption bounded (no infinite production). The set Z' can therefore be considered as the convex closure of a compact set Z which contains as a subset the set of attainable states. The excess demand correspondence can be defined as;

$$Z_i(p) \equiv \min \left[k_i \;, X_i(p) - Y_i(p)\right]$$

$$k_i > 0$$

where the mapping $p \to Z$, clearly is U.S.C. (uppersemicontinuous) if both $p \to X$, and $p \to Y$ are U.S.C., k_i being a constant (continuous function of U.S.C. function is U.S.C.). We know, by assumption that $X_i - Y_i$ is bounded below.

Therefore, the set Z is bounded and the set Z' is convex compact. By construction an equilibrium in Z' will be within Z. In short the a priori information needed is that around the equilibrium point Z_i is bounded below and above, that Z is homogeneous of degree zero in p and that the correspondence $p \to Z(p)$ is U.S.C.

These conditions have been assumed to hold in the present paper.

Now define the set $S(z)$ as follows:

$$S(z) = [p \mid pz \text{ is a maximum for } z \in Z',\ p \in P] \quad .$$

That is, we choose an arbitrary excess demand vector from the set of all excess demand vectors which are attainable at some prices, then find the price vector for which the value of this excess demand is maximized. It is important to note that the price vector is any price vector, not necessarily the particular p which is associated with z through the mapping $p \longrightarrow Z(p)$

Clearly $z \longrightarrow S(z)$ is a mapping from Z' into a subset of P. Since Z is convex we know this mapping to be upper semicontinuous. S(z) is a convex set since it is the intersection of the hyperplane $[y|yz = \max pz]$ with P.

Consider the set P x Z', that is the set consisting of normalized price vectors paired with excess demand vectors. If we take some point p, z in P x Z', then Z(p) associates a set of excess demand vectors with p, and S(z) associates a set of price vectors with z. In other words, the mapping $p, z \longrightarrow Z(p), S(z)$ maps a point in P x Z' into a subset of P x Z'.

We have shown the mapping $z \longrightarrow S(z)$ to be upper semicontinuous, and $p \longrightarrow Z(p)$ has been assumed to have the same property so that the combined mapping is upper semicontinuous also. We have shown that S(z) is convex and Z'(p) has been assumed convex, so that S(z) x Z'(p) is convex.

Thus, we have an upper semicontinuous mapping $p, z \longrightarrow Z(p) S(z)$ from the compact convex at P x Z' into a convex subset of itself. These are the conditions for invoking the Kakutani Fixed Point Theorem. The theorem states that there exists some p^*, ε P, $z^* \varepsilon Z'$ which is a fixed point, that is, for which $p^* \varepsilon S(z^*)$ and $z^* \varepsilon Z(p^*)$.

From the construction of S(z), $p^* \varepsilon S(z^*)$ implies that, for all $p \varepsilon P$,

$$pz^* \leqq p^* z^*$$

Using the weak budget condition it follows that since $z^* \varepsilon Z(p^*)$,

$$p^* z^* \leqq 0$$

Thus,

$$pz^* \leqq 0, \quad \text{for all } p \in P \quad .$$

Clearly the last inequality is satisfied for all $p \in P$ only if

$$z^* \leqq 0 \quad ,$$

thus proving the theorem. One important feature of this proof is that it does not require p and z to have the same dimension.

The other important feature of this proof for our purpose is that $S(z) = [p|\max pz \text{ for } z \in Z' , p \in P']$ represents the solution of a linear programme.

$$\max p\ z$$
$$\text{S.T. } \Sigma p_i = 1$$
$$p_i \geqq 0 \quad .$$

APPENDIX 2

A Note on the Computation of the Equilibrium

The existence proofs for the equilibrium proceeded along the lines of the proof of Debreu's excess demand theorem, for all models in this paper. We summarize the main features:

1. $y = y(x)$ is a continuous function satisfying $xy = 0$;

2. x is restricted to a set K_y which , at given K, is bounded, compact convex and in the positive orthant

3. Consider the following linear program

   max x y
   S.T. $x \in K_y$ y given

   This program was used in the proofs.

4. Consider now $C_y = \{x : y = f(x), \quad x \in R_+^n\}$. Define $K = \{K_y : C_y \subset K_y \quad , \quad y \in R_+^n\}$. This set is not necessarily convex. However, if the variable y does not enter the constraint set, the set K coincides with K_y .

   In the previous proofs the variable y entered only in two instances:

   as $ps = 0$ §1,2,3,4
   and as $(\varepsilon \bar{p} - q)(i - \bar{i}) = 0$ in §2.2

   in the first case, the restriction may be removed if we know that in equilibrium $p > 0$;

   in the second case, the restriction plays a central role and cannot be removed, the remarks on computation therefore do not apply to §2.2 ; ;

5. Consider the programs:

(a) min $\Sigma \bar{x}_i \max (y_i(x), 0)$

S.T. $x \in K$

where $\bar{x}_i$ is a positive weight,

(b) min $\max (y_i(x))$

S.T. $x \in K$

Both programs (and many others) have the same global solution which is at the same time the equilibrium. We shall use an algorithm [5] which can be shown to converge to a local minimum of the optimization problem but not to a global minimum.[1)] The computation can actually be performed in a space of a smaller dimension that the dimension of the vector y. This is to be discussed in a separate paper.

---

1) This algorithm needs the assumption that $y = y(x)$ is a continuous but not necessarily differentiable function.

REFERENCES

[1] Debreu, G. (1959), *Theory of Value*, Cowles Foundation Monograph 17.

[2] Drèze, J. (1975), Existence of an Exchange Equilibrium Under Price Rigidities, *International Economic Review*, (June).

[3] Keyzer, M.A. (1977), *Linking National Models of Food and Agriculture: An Introduction*, RM-77-2, International Institute for Applied Systems Analysis, Laxenburg, Austria.

[4] Keyzer, M.A. (1977), *Analysis of a National Model with Domestic Price Policies and Quota on International Trade*, RM-77-19, International Institute for Applied Systems Analysis, Laxenburg, Austria.

[5] Mifflin, R., (1977), *An Algorithm for Constrained Optimization with Semismooth Functions*. RR-77-03, International Institute for Applied Systems Analysis, Laxenburg, Austria. To appear in *Mathematics of Operation Research*, (1977)

[6] United Nations, (1971), *Resolutions Adopted by the General Assembly*, Official Records, Twenty-fifth Session, Supplement No. 28, New York.