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PLANNING LONG RANGE AGRICULTURAL INVESTMENT PROJECTS: A DYNAMIC LINEAR PROGRAMMING APPROACH

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Preface

Systematic assessment and planning to meet food needs on regional and national levels is of increased interest in many countries. Such planning efforts must recognize the interrelationship existing between technology, resources, economics and other components of the food system.

The long-range modelling of agriculture development has been a point of joint research for IIASA's Food and Agriculture program and System and Decision Sciences area and related to the IIASA-Bulgarian methodological work on the agro-industrial regional project at Silistra.

Related papers have been prepared by C. Csåki and A. Propoi.[1]

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[1] Csåki, C., Dynamic Linear Programming Model for Agricultural Investment and Resources Utilization Policies, RM-77-36, IIASA, Laxenburg, Austria and

Propoi, A., Dynamic Linear Programming Models for Livestock Farms, RM-77-29, IIASA, Laxenburg, Austria.

Abstract

This paper outlines a dynamic linear programming (DLP) model for planning a diversified agri-industrial complex. Six production subsystems are presented: livestock, crops, primary product utilization, processing, inputs and resource capacities. In addition a financial subsystem is described. The final two sections discuss briefly alternative goal functions and some limitations of the DLP model for investment planning.

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Planning Long Range Agricultural Investment Projects: A Dynamic Linear Programming Approach

1. Introduction

Events of the last few years that saw wide fluctuations in world food prices, production and inventories have helped to reemphasize the need for more systematic assessment and planning to meet food needs for an expanding world population. Many countries are now planning and undertaking large scale agricultural investment projects either to increase their food self sufficiency or to expand trade with other countries. While the complexity and size of agricultural projects vary greatly between countries because of the availability of (a) natural resources, (b) capital, (c) labor and management skills, there still may be a common element or framework for considering such planning schemes.

Models of agricultural systems may be formulated using various techniques and with different degrees of detail and sophistication. At the beginning in the early 1960's several versions of linear programming models had been developed for agricultural planning purposes (1), (2). In recent years more advanced programming techniques (e.g. integer, quadratic, stochastic programming) have also been applied (7), (9), (11), (15), (18), and considerable efforts have been devoted to the analysis of agriculture systems by simulation methods (4), (7).

For planning and long range investigations the dynamic (multistage) approach (DLP) seems to offer several advantages (3), (5),(12), (13), (14), (17). The DLP allows us to formulate and derive optimal plans of farm development over extended time periods (say 5-10 to 30 years). To demonstrate the flexibility of the approach we outline a general DLP model for a diversified production-processing crop-livestock complex.

Perennial as well as annual crops are considered. Specifically, the problem is to determine the optimal crop-livestock mix maximizing some specified performance index for a given planning period. Each of the main components of the model will be discussed. We conclude with a discussion of some general problems and limitations of the DLP model.

2. Formulating the DLP Problem

In formulating the DLP problems it is useful to define and consider separately (14).

1) *State equations* of the system distinguishing between *state* (descriptive) and *control* (decision) variable.

2) *Constraints* imposed on these variables.

3) *Planning period* T--the number of periods during which the system is considered and the *length* of each period.

4) *Performance index* (objective function) which quantifies the contribution of each variable to some performance measure or index (e.g. profit, net return, asset value, etc.).

As our purpose is to determine an optimal plan for the whole system we consider separately only state equations and constraints for each subsystem and then specify means for linking the sub-models into a general model with a common performance index and planning horizon.

3. Production Subsystems[1)]

We consider 6 subsystems:

- livestock subsystem
- crop subsystem (perennial and annual crops)
- product utilization subsystem of primary production activities
- processing subsystem
- utilization of purchased inputs
- capacities subsystem.

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1) Irrigation subsystem are not considered explicitly in this paper but may have relevance for the Bulgarian Selistra project. See further (6) and (8) .

3.1. *Livestock Subsystem*

We consider a livestock subsystem consisting of several types of livestock. All animals in accordance with their type (dairy, beef, hogs, etc.) and maturity or age class are divided into I groups.

Let

$x_i(t)\ (i = 1,\ldots,I)$ - the number of animals of type i (dairy calf, dairy heifer, dairy cow, sow, etc.) at year (period) t;

$u_i^+(t)$ - the number of animals of type i purchased at period t;

$u_i^-(t)$ - the number of animals of type i sold at period t;

a_{ij} - the coefficient, which shows what proportion of animals of type j will progress to type i in the succeeding period (i.e. attrition rate $= 1 - a_{ij}$).

Then we can write the state equations for the livestock subsystems as:

$$x_i(t+1) = \sum_{j=1}^{I} a_{ij} x_j(t) + u_i^+(t) - u_i^-(t) \qquad (1)$$

or in matrix form

$$x(t+1) = Ax(t) + u^+(t) - u^-(t) \quad . \qquad (1a)$$

Here $x(t) = \{x_1(t),\ldots,x_I(t)\}$ is the vector of state variables; $u^+(t) = \{u_1^+(t),\ldots,u_I^+(t)\}$ and $u^-(t) = \{u_1^-(t),\ldots,u_I^-(t)\}$ are vectors of control variables.

The state equations (1) can be specified in a more detailed form. Let x_i^a equal the number of animals of type i and group a at period t.

An animal belongs to group a, if its age is τ and $a\Delta \leq \tau \leq (a+1)\Delta$, Δ is given time interval ($i = 1,\ldots,n$; $a = 0,1,\ldots,N-1$; $t = 0,1,\ldots,T-1$).

Vector $x^a(t)$ defines the animals distributions over their type in group a at period t:

$$x^a(t) = \{x_1^a(t),\ldots,x_i^a(t),\ldots,x_n^a(t)\} \quad .$$

Let the reproductive age begin with the group a_1 and end by group a_2. Usually, $a_2 = N-1$. Then the number of animals born (that is, of group 0) at year $t+1$ is equal to

$$x^0(t+1) = \sum_{a=a_1}^{N-1} B(a)x^a(t) \quad , \tag{2}$$

where $B(a)$ is a birth matrix of group a; the element $b_{ij}(a)$ of $B(a)$ shows what number of animals of type i "produced" (born) by one animal of type j and group a.

The transition of animals from group a into group $a+1$ is described by equation

$$x^{a+1}(t+1) = S(a)x^a(t) \tag{3}$$

where the survival matrix $S(a)$ shows what proportion of animal group "a" progresses to group $a+1$ for one time period.

If, for example, $\Delta = 1$ year and group a suffers an attrition rate of $\alpha_i^a (0 \leq \alpha_i^a \leq 1)$ each year, then the equation (3) can be written as

$$\begin{bmatrix} x_1^{a+1}(t+1) \\ \cdot \\ \cdot \\ \cdot \\ x_n^{a+1}(t+1) \end{bmatrix} = \begin{bmatrix} (1-\alpha_1^a) & & & 0 \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ 0 & & & (1-\alpha_n^a) \end{bmatrix} \begin{bmatrix} x_1^a(t) \\ \cdot \\ \cdot \\ \cdot \\ x_n^a(t) \end{bmatrix}$$

Let us introduce a vector

$$x(t) = \{x_i^a(t)\} \qquad (i = 1,\ldots,n;\ a = 0,1,\ldots,N-1) \quad .$$

Then equations (2) and (3) can be combined

$$x(t+1) = Ax(t) \qquad (t = 0,1,\ldots,T-1) \tag{4}$$

where

$$A = \begin{bmatrix} 0 & 0 & \cdots & B(a_1) & \cdots & B(N-1) \\ S(0) & 0 & \cdots & 0 & \cdots & 0 \\ 0 & S(1) & \ddots & & & \\ \vdots & & \ddots & & & \vdots \\ \vdots & & & S(a_1) & \ddots & \vdots \\ \vdots & & & & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & S(N-1) \end{bmatrix}$$

A is the growth matrix.

If we again introduce control vectors:

$$u^+(t) = \{u_i^{a+}(t)\} \quad \text{and} \quad u^-(t) = \{u_i^{a-}(t)\}$$

we again come to state equation of the same general form shown in (1) above:

$$x(t+1) = Ax(t) + u^+(t) - u^-(t) \quad .$$

One additional point should be noted. Attrition rates a_{ii} is usually divided into two terms:

$$a_{ii} = a_{ii}^r + a_{ii}^b \quad ,$$

where a_{ii}^r is the real attrition rates due to accidental death of animals, and the coefficient a_{ii}^b expresses the ratio of animals purposely removed from the subsystem due to breeding or culling policy; a_{ii}^b is a parameter of the system.

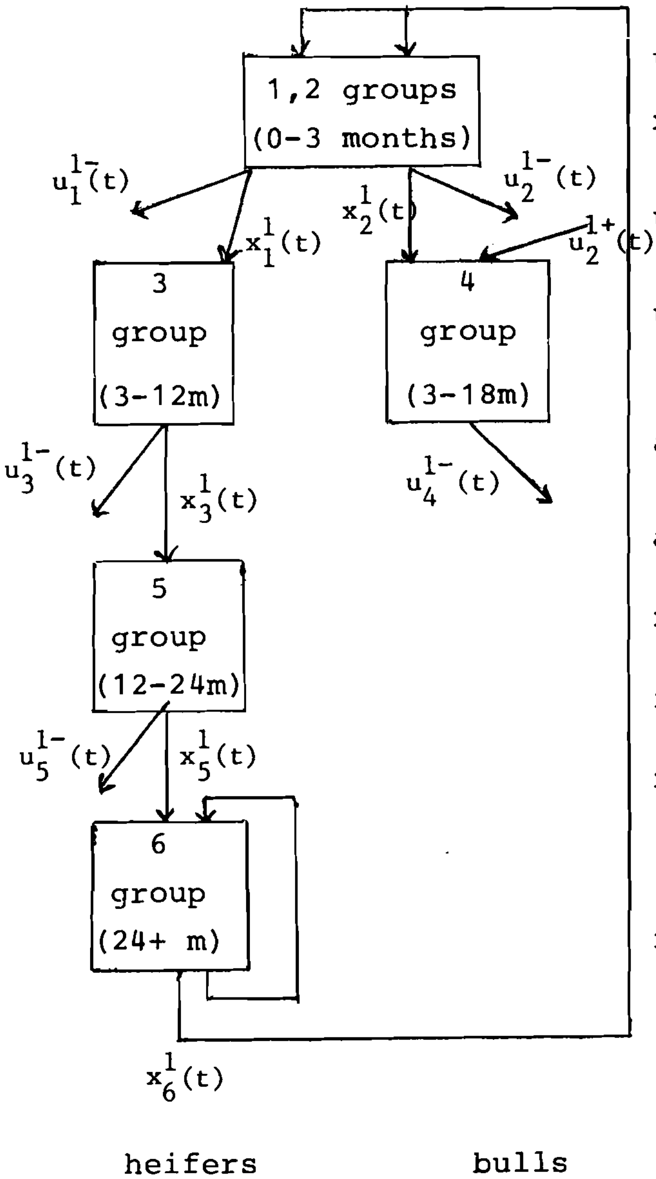
Another way of introducing the breeding or culling policy is to divide control vector $u^-(t)$ into two parts

$$u_i^-(t) = u_{is}^-(t) + u_{ib}^-(t) \quad ,$$

where $u_{ib}^-(t)$ is the number of animals of type i removed at period t from the subsystem for breeding or culling purposes, and $u_{is}^-(t)$ represents intentional selling.

For purposes of illustration, we present some livestock subsystems in diagramatic flows with appropriate state equation at specific periods.

3.1.1. Cattle subsystem (dual purpose dairy cattle)



t = 1 year (time unit)

$x_i^1(t)$ - the number of cattle of group i at year t

$u_i^{1-}(t)$ - the number of cattle of group i sold at year t

$u_i^{1+}(t)$ - the number of cattle of group i purchased at year t.

a_{ij} - retension rates

$$a_6^1 x_6^1(t) = 0.5x_1^1(t) + 0.5x_2^1(t)$$

$$x_3^1(t) = a_{31}x_1^1(t) - u_1^{1-}(t)$$

$$x_4^1(t) = a_{42}x_2^1(t) - u_2^{1-}(t) + u_2^{1+}(t)$$

$$x_5^1(t+1) = a_{53}x_3^1(t) - u_3^{1-}(t)$$

$$0 = a_4 x_4^1(t) - u_4^{1-}(t)$$

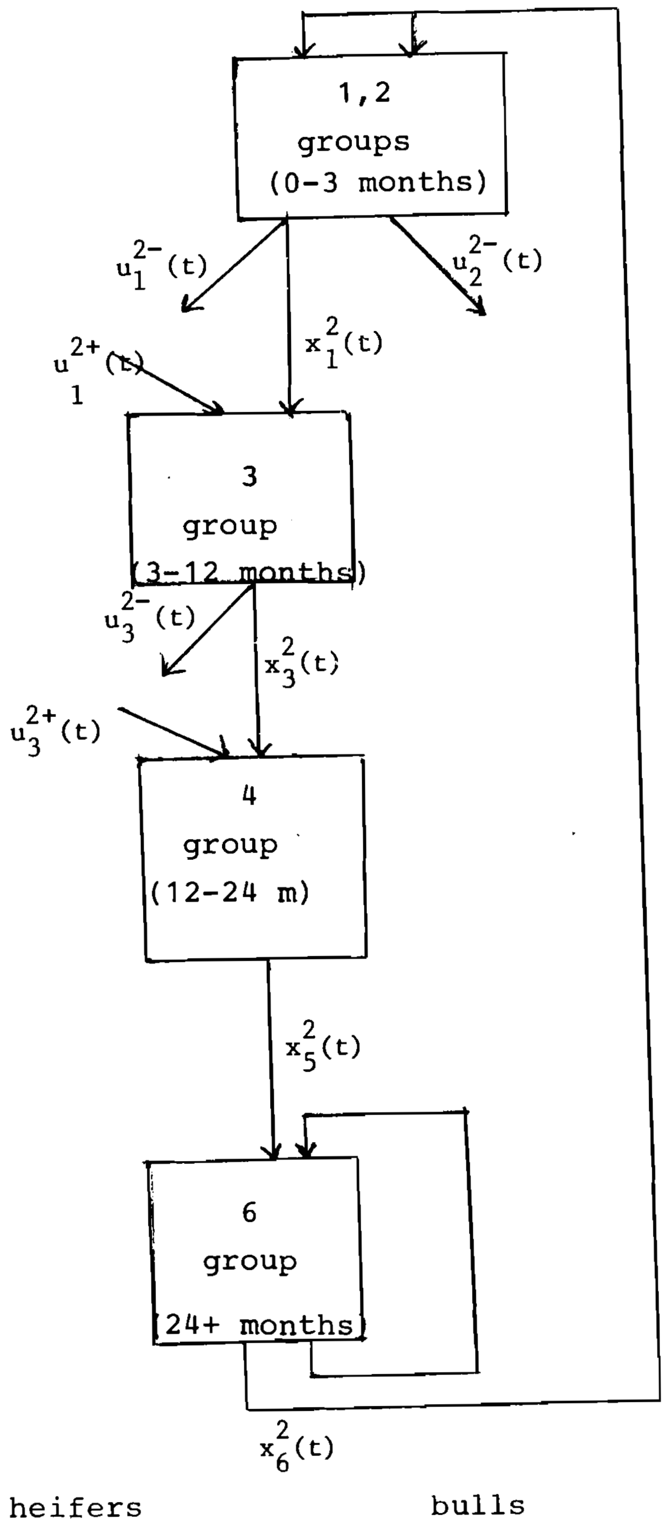
$$x_6^1(t+1) = a_{66}x_6^1(t) + a_{65}x_5^1(t) - u_5^{1-}(t)$$

State variables: $x(t) = \{x_1^1(t), \ldots, x_6^1(t)\}$

Control variables: $u^{1-}(t) = \{u_1^{1-}(t), u_2^{1-}(t), u_3^{1-}(t), u_4^{1-}(t), u_5^{1-}(t)\}$

$u^{1+}(t) = u_2^{1+}(t)\}$

3.1.2 Cattle subsystem (dairy only)



$t = 1$ year (time unit)

$$a_6^2 x_6^2(t) = 0.5x_1^2(t) + 0.5x_2^2(t)$$

$$x_3^2(t) = a_{31}x_1^2(t) - u_1^{2-}(t) + u_1^{2+}(t)$$

$$0 = a_{42}x_2^2(t) - u_2^{2-}(t)$$

$$x_5^2(t+1) = a_{53}^2 x_3^2(t) - u_3^{2+}(t) + u_3^{2+}(t)$$

$$x_6^2(t+1) = a_{66}^2 x_6^2(t) + a_{65}^2 x_5^2(t)$$

Control variables:

$\{u_1^{2-}(t), u_2^{2-}(t), u_3^{2-}(t)\} = u^{2-}(t)$

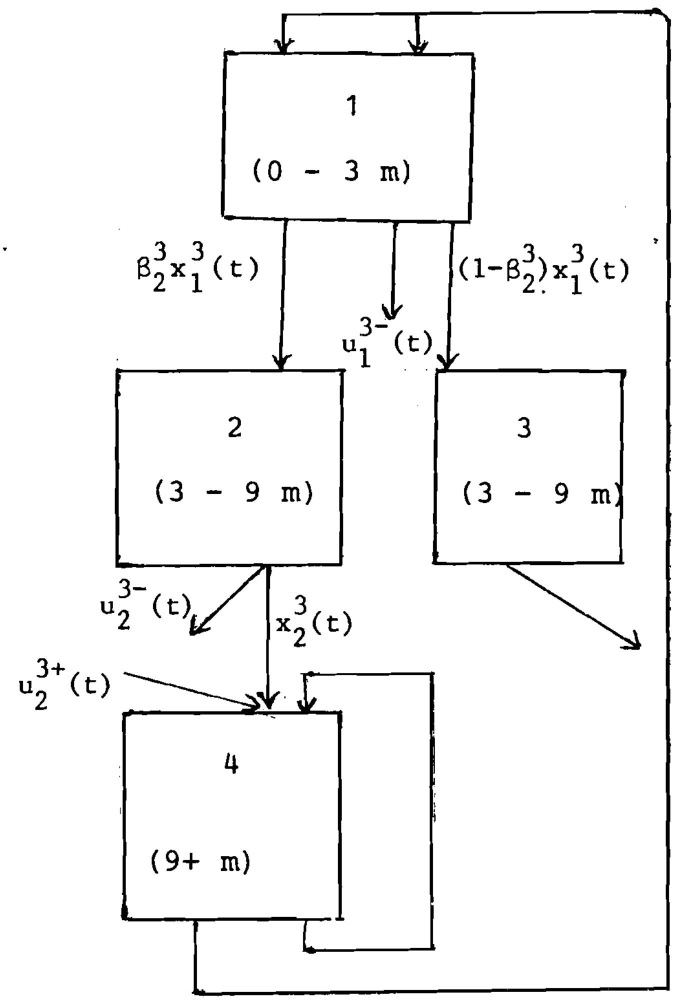
$\{u_1^{2+}(t), u_3^{2+}(t)\} = u^{2+}(t)$

State variables;

$\{x_1^2(t), x_2^2(t), x_3^2(t), x_5^2(t), x_6^2(t)\} = x^2(t)$

3.1.3. Pig-breeding subsystem

$t = 3$ months (time unit)



$$a^3_{41} x^3_4(t) = x^3_1(t)$$

$$x^3_2(t+1) = a^3_{21}[x^3_1(t) - u^{3-}_1(t)]\beta^3$$

$$0 = x^3_3(t+1) = a^3_{31}[x^3_1(t) - u^{3-}_1(t)](1-\beta^3_2)$$

$$x^3_4(t+1) = a^3_{44}x^3_4(t) + a^3_{42}x^3_2(t) + u^{3+}_2(t) - u^{3-}_2(t)$$

State variables

$$\{x^3_1(t), x^3_2(t), x^3_3(t), x^3_4(t)\} = x^3(t)$$

Control variables

$$\{u^{3-}_1(t), u^{3-}_2(t)\} = u^{3-}(t)$$

$$\{v^{3+}_2(t)\} = v^{3+}(t)$$

3.2. *Crop Producing Subsystem.* The crop producing subsystem includes both perennial and annual crops. First we consider perennial crops. (See (6) for discussion of special problems of perennial crop system.)

Let

$y_j(t)$ $(j = 1, \ldots, J)$ - the number of hectares used for perennial crop j at period t; (grape, apricot, alfalfa, etc.) and

$v^+_j(t)$ - the number of hectares, used for new plantings of perennials of type j at year t.

$v_j^-(t)$ - the number of hectares of perennial of type j removed j at year t;

b_{jk} - shows what proportion of lands of type k (i.e. with trees of type k) will progress to type j in one year.

The state equations are then defined as

$$y_j(t+1) = \sum_{k=1}^{J} b_{jk} y_k(t) + v_j^+(t) - v_j^-(t) \qquad (5)$$

or in matrix form

$$y(t+1) = By(t) + v^+(t) - v^-(t) \qquad (5a)$$

where

$$y(t) = \{y_1(t), \ldots, y_J(t)\} \text{ is the state vector}$$

and $v^+(t) = \{v_1^+(t), \ldots, v_J^+(t)\}$, $v^-(t) = \{v_1^-(t), \ldots, v_J^-(t)\}$

are the control vectors.

We can illustrate the state equations for the perennial crop subsystem with an example of apricot production. Consider the following production time periods:

3.2.1. Apricot production subsystem

	Age of trees		
	months	years	
$y_1(t)$	= 0 - 12	0 - 1	
$y_2(t)$	- 12 - 24	1 - 2	
$y_3(t)$	- 24 - 36	2 - 3	
$y_4(t)$	- 36 - 48	3 - 4	
$y_5(t)$	- 48 - ...	4 - ...	producing or mature trees.

The state equation for new plantings is

$$y_1(t+1) = v_1^+(t)$$

and trees in the second year:

$$y_2(t+1) = b_{21}y_1(t)$$

and trees in the third year:

$$y_3(t+1) = b_{32}y_2(t)$$

and trees in the fourth year:

$$y_4(t+1) = b_{43}y_3(t)$$

and trees in the fifth and succeeding years (producing or mature trees)

$$y_5(t+1) = b_{55}y_5(t) + b_{54}y_4(t)$$

with the given b_{jk} $j=1,\dots,5$, $k=1,\dots,5$. In matrix form the state equations are written:

$$y(t+1) = By(t) + hv_1^+(t) \;;$$

where

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ b_{21} & 0 & 0 & 0 & 0 \\ 0 & b_{32} & 0 & 0 & 0 \\ 0 & 0 & b_{43} & 0 & 0 \\ 0 & 0 & 0 & b_{54} & b_{55} \end{bmatrix} \qquad h = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Here we have 5 state variables $y(t) = \{y_1(t),\dots,y_5(t)\}$; one control variable $v_1^+(t)$ and $t = 1$ year.

The system of state variables can be simplified by successive substitution. For example:

$$
\begin{aligned}
y_5(t+1) &= b_{55}y_5(t) + b_{54}b_{43}y_3(t-1) \\
&= b_{55}y_5(t) + b_{54}b_{43}b_{32}y_2(t-2) \\
&= b_{55}y_5(t) + b_{54}b_{43}b_{32}b_{21}y_1(t-3) \\
&= b_{55}y_5(t) + by_1^+(t-4) \quad ,
\end{aligned}
$$

$$\text{where } b = b_{54}\ b_{43}\ b_{32}\ b_{21} \quad .$$

Thus we have one state variable, one time delay and t = 1 year. If we choose time period equals 5 years, then we can eliminate even time delay. The state equation then reduces to:

$$y_5(t+1) = \tilde{b}_5 y_5(t) + \tilde{\tilde{v}}(t), \text{ where}$$

$\tilde{\tilde{v}}(t)$ - the number of planting during 5 year period, and
$\tilde{b}_5$ shows, what proportion of trees, planted during a 5 year period, will be producing.

The state equations in the above form are needed only for perennial crops. For annual crops it is sufficient to introduce the numbers of hectares $\tilde{y}(t)$ used for these crops, which are the control (decision) variables in processing, utilization and other subsystems.

3.3 *Product Utilization Subsystems of Primary Production Activities.*

Outputs of livestock and crop (perennial and annual) subsystems may be processed. We distinguish primary product activities (producing milk, apples, wheat, etc.) and secondary product activities (producing meat, canned fruit, etc.). Then the primary product subsystem is broken down into 3 subsystems (utilization of outputs of livestock, perennial crops and annual crops).

First we consider the product subsystem of primary activities. Let

$z_m^x(t)$ $(m=1,\ldots,M_x)$ - the stock of primary product of type m produced by the livestock subsystem (milk, meat, eggs, etc.)

$z_m^y(t)$ $(m=1,\ldots,M_y)$ = the stock of the product of type m, produced by perennial crop system (apples, plums, etc.)

$\tilde{z}^y_m(t)$ $(m=1,\ldots,\tilde{M}_y)$ - the stock of the product of type m produced by annual crop-subsystem (corn, wheat, vegetables, etc.)

$z_m(t)$ $(m=1,\ldots,M)$ - the stock of the purchased inputs of type m (fertilizers, pesticides, etc.)

The above are state variables.

Similar to the other subsystems, we have buying and selling activities (control variables) for the products subsystem. These are:

$z^{x+}_m(t), z^{x-}_m(t)$

$z^{y+}_m(t);\ z^{y-}_m(t)$

$\tilde{z}^{y+}_m(t);\ \tilde{z}^{y-}_m(t)$

$z^{+}_m(t)$

In addition, we have other control variables:

$\tilde{y}_j(t)$ - the number of hectares for annual crop of type j at period t (corn, wheat, etc.)

$q^x_{mk}(t)$ - the level of activity for processing of the m-th livestock primary product (e.g. milk) into the k-th secondary product (e.g. butter) at period t. $(m = 1,\ldots, M_x,\ k = 1,\ \ldots,K_x)$

$q^y_{mk}(t)$ - $(m = 1,\ldots, M_y,\ k = 1,\ldots, K_y)$ and

$\tilde{q}^y_{mk}(t)$ - $(m = 1,\ldots, \tilde{M}_y,\ k = 1,\ldots, \tilde{K}_y)$ have similar meaning for perennial and annual crops.

Accordingly, we can write the state equations which express the utilization of these products.

3.3.1 *Utilization of outputs of livestock subsystem.*

$$z_m^x(t+1) = z_m^x(t) + \sum_i g_{mi}^x x_i(t) + \sum_i g_{mi}^u u_i^-(t) -$$

$$- \left[\sum_i \alpha_{mi}^x x_i(t) + \sum_j \beta_{mj}^x y_j(t) + \sum_j \tilde{\beta}_{mj}^x \tilde{y}_j(t) + \right.$$

$$\left. + \sum_\kappa \delta_{m\kappa}^x q_{mk}^x(t) \right] + z_m^{x+}(t) - z_m^{x-}(t) \qquad (6)$$

where

g_{mi}^x - the volume of product of type m from a unit of livestock of type i (without withdrawing from the system)

g_{mi}^u - the same as g_{mi}^x but when withdrawing it from the system

α_{mi}^x - the volume of livestock product m consumed by unit of livestock i.

β_{mj}^x, $\tilde{\beta}_{mj}^x$ - the volume of livestock product m (e.g. manure) utilized on one hectare of type j (perennial and annual crops)

$\delta_{m\kappa}^x$ - the utilization of livestock product m for producing one unit of secondary product κ.

In matrix form equations (6) can be rewritten as:

$$z^x(t+1) = z^x(t) + G^x x(t) + G^u u^-(t) - \{\alpha^x x(t) \qquad (6a)$$

$$+ \beta^x y(t) + \tilde{\beta}^x \tilde{y}(t) + [\Delta^x Q^x(t)]\} + z^{x+}(t) - z^{x-}(t)$$

with matrices

$$G^x = \{g_{mi}^x\}; \quad G^u = \{g_{mi}^u\}; \quad \alpha^x = \{\alpha_{mi}^x\}; \quad \beta^x = \{\beta_{mj}^x\}$$

$$\tilde{\beta}^x = \{\tilde{\beta}_{mj}^x\}; \quad \Delta^x = \{\delta_{mk}^x\}; \quad Q^x(t) = \{q_{mk}^x(t)\}$$

$[\Delta^x Q^x]$ is the vector of the "row-by-row" product of the matrices Δ^x and Q^x.

In the above equation it is assumed that all animals $u_i^-(t)$ to be sold are processed before sale. Otherwise it is necessary to divide variables $u_i^-(t)$ into two parts: (1) to be sold and (2) to be processed.

3.3.2 *Utilization of outputs of perennial crop subsystem*

$$z_m^y(t+1) = z_m^y(t) + \sum_j g_{mj}^y y_j(t) - \left[\sum_i \alpha_{mi}^y x_i(t) + \sum_j \beta_{mj}^y y_j(t) + \sum_\kappa \delta_{m\kappa}^y \; q_m^y \; q_{m\kappa}^y(t)\right] + z_m^{y+}(t) - z_m^{y-}(t) \qquad (7)$$

In matrix form

$$z^y(t+1) = z^y(t) + G^y\, y(t) - \{\alpha^y x(t) + \beta^y y(t) + [\Delta^y Q^y(t)]\} + z^{y+}(t) - z^{y-}(t), \qquad (7a)$$

where matrices G^y, α^y, β^y and Δ^y have the same meaning as in (6a).

3.3.3 *Utilization of outputs of annual crop subsystem*

$$\tilde{z}_m^y(t+1) = \tilde{z}_m^y(t) + \sum_j \tilde{g}_{mj}^y \tilde{y}_j(t) - \left[\sum_i \tilde{\alpha}_{mi}^y x_i(t) + \sum_j \tilde{\beta}_{mj}^y \tilde{y}_j(t) + \sum_\kappa \tilde{\delta}_{m\kappa}^y \tilde{q}_{m\kappa}^y(t)\right] + \tilde{z}_m^{y+}(t) - \tilde{z}_m^{y-}(t) \qquad (8)$$

In matrix form

$$\tilde{z}^y(t+1) = \tilde{z}^y(t) + \tilde{G}^y \tilde{y}(t) - \{\tilde{\alpha}^y x(t) + \tilde{\beta}^y \tilde{y}(t) + [\tilde{\Delta}^y \tilde{Q}^y(t)]\} + \tilde{z}^{y+}(t) - \tilde{z}^{y-}(t) \qquad (8a)$$

3.4 *Processing Subsystem*

State variables are defined as:

$s_k^x(t)$ $(k = 1,\ldots,K_s^x)$ is the stock of the product of type m produced by the secondary processing of primary livestock subsystem (cheese, butter, canned meat, bacon, etc.)

$s_k^y(t)$ $(k = 1,\ldots,K_s^y)$ is the stock of the secondary product of type m from perennial crop subsystem (juice, canned fruit, frozen goods)

$\tilde{s}_k^y(t)$ $(k = 1,\ldots,\tilde{K}_s^y)$ is the stock of the secondary product of the type m from annual crops (wheat flour, sugar, etc.)

Selling activities (control variables) are as follows:

$$s_k^{x-}(t),\ s_k^{y-}(t),\ \tilde{s}_k^{y-}(t)$$

Thus state equations can be written as:

$$s_k^x(t+1) = s_k^x(t) + \sum_m d_{mk}^x q_{mk}^x(t) - s_k^{x-}(t) \tag{9}$$

$$s_k^y(t+1) = s_k^y(t) + \sum_m d_{mk}^y q_{mk}^y(t) - s_k^{y-}(t) \tag{10}$$

$$\tilde{s}_k^y(t+1) = \tilde{s}_k^y(t) + \sum_m \tilde{d}_{mk}^y \tilde{q}_{mk}^y(t) - \tilde{s}_k^{y-}(t) \tag{11}$$

Here

d_{mk}^x, d_{mk}^y and d_{mk}^y are the amounts of products of type m required per unit of activity 1, for primary animals, perennials crops and annual crop products, respectively.

In matrix form

$$s^x(t+1) = s^x(t) + [D^x(t)Q^x(t)] - s^{x-}(t) \tag{9a}$$

$$s^y(t+1) = s^y(t) + [D^y(t)Q^y(t)] - s^{y-}(t) \tag{10a}$$

$$\tilde{s}^y(t+1) = \tilde{s}^y(t) + [\tilde{D}^y(t)\tilde{Q}^y(t)] - \tilde{s}^{y-}(t) \tag{11a}$$

3.5 *Utilization of Purchased Inputs*

Let $z_m(t)$ equal the stock of the purchased inputs of type m (m = 1,...M) (fertilizers, pesticides, fuel, etc.)

Therefore we can write for all stored goods:

$$z_m(t+1) = z_m(t) + z_m^+(t) - \sum_i \alpha_{mi} x_i(t) + \sum_j \beta_{mj} y_j(t) + \sum_j \tilde{\beta}_{mj} \tilde{y}_j(t) - \sum_k \gamma_{mk}^x q_{mk}^x(t) + \sum_k \gamma_{mk}^y q_{mk}^y(t) + \sum_k \tilde{\gamma}_{mk}^y \tilde{q}_{mk}^y(t) \qquad (12)$$

where α_{mi}, β_{mj}, $\tilde{\beta}_{mj}$ represent the use of purchase inputs of type m by unit of livestock, perennial and annual crop subsystems, respectively;

γ_{mk}^x, γ_{mk}^y, $\tilde{\gamma}_{mk}^y$ are the utilization of purchased inputs, of type m per unit of type k activity for processing of animals, perennial and annual crop products, respectively.

In matrix form these equations are written:

$$z(t+1) = z(t) + z^+(t) - \left[\alpha\, x\,(t) + \beta y(t) + \tilde{\beta}\tilde{y}(t)\right] - \left\{\left[\Gamma^x\, Q^x\,(t)\right] + \left[\Gamma^y\, Q^y(t)\right] + \left[\tilde{\Gamma}^y\, \tilde{Q}^y\,(t)\right]\right\} \qquad (12a)$$

For nonstorable goods and services (e.g. electricity) the state equation (12) is replaced by:

$$z_m^+(t) - \left[\sum_i \alpha_{mi} x_i(t) + \sum_j \beta_{mj} y_j(t) + \sum_j \tilde{\beta}_{mj} \tilde{y}_j(t)\right] - \left[\sum_k \gamma_{mk}^x q_k^x + \sum_k \gamma_{mk}^y q_k^y + \sum_k \tilde{\gamma}_{mk} \tilde{q}_k^y\right] = 0 \qquad (13)$$

And matrix form:

$$z^+(t) - \left[\alpha\, x\,(t) + \beta y(t) + \beta\tilde{y}(t)\right] - \left\{\left[\Gamma^x\, Q^x\,(t)\right] + \left[\Gamma^y\, Q^y(t)\right] + \left[\tilde{\Gamma}^y\, \tilde{Q}^y\,(t)\right]\right\} = 0 \qquad (13a)$$

In summary we illustrate the producing subsystems in diagramatic form in Figure 1.

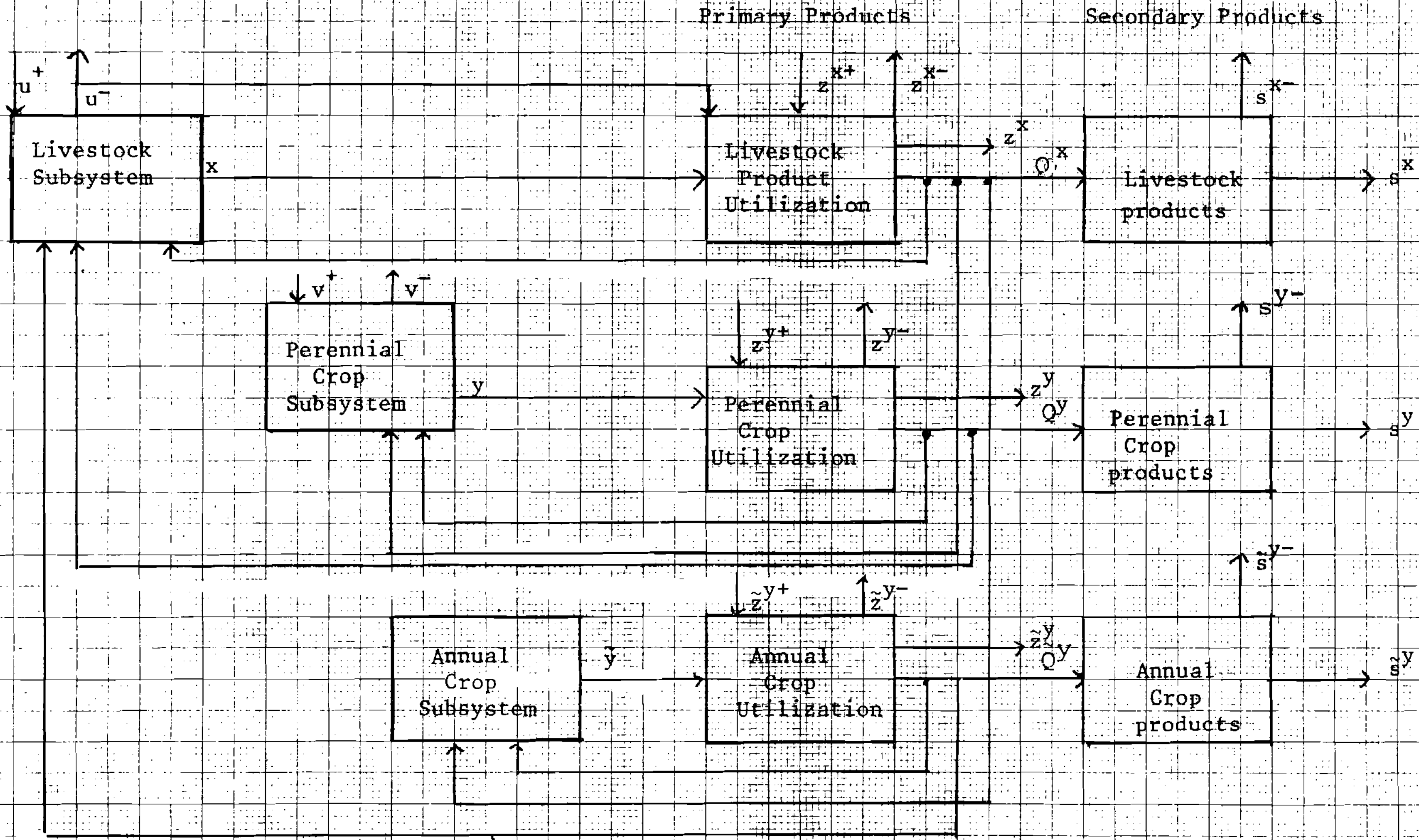


FIG. 1. DIAGRAM OF PRODUCING SUBSYSTEMS

3.6 *Capacities Subsystem.* The capacity of the physical resources (fixed assets) of the system (buildings, machinery, etc.) may change over the planning horizon, due to various investment and disinvestment policies.

Let $k_n(t)$ $(n = 1,...,N)$ - the physical resource capacity of type n (buildings, machinery, storage, etc.) available at the beginning of period t

$w_{nr}(t)$ - the intensity of activity of type r (purchasing of various types of tractors, construction of cow barns, etc.) at period t for increasing the capacities of type n at period t + 1 $(r = 1,...,R)$

$k_n^-(t)$ - the resource capacity of type n removed from the system during period t (e.g., disposal)

d_{nr} - shows, on what amount the capacity of type n will increase when using activity r at unit level for one period

c_n - depreciation rate of asset of type n.

The state equations are then defined as:

$$k_n(t + 1) = c_n k_n(t) + \sum_{r=1}^{R} d_{nr} w_{nr}(t) - k_n^-(t) \qquad (14)$$

or in matrix form

$$k(t + 1) = Ck(t) + [DW(t)] - k^-(t) \qquad (14a)$$

where

$$C = \begin{bmatrix} c_1 & & 0 \\ & \ddots & \\ 0 & & c_n \end{bmatrix} \quad ; \quad D = \{d_{nr}\} \; ; \; W(t) = \{w_{nr}(t)\}$$

$k(t)$ is the state vector,

$W(t)$, $k^-(t)$ are control variables.

If we incorporate time lags our state equations are modified as follows:

$$k_n(t + 1) = c_n k_n(t) + \sum_r d_{nr} w_r(t-\tau_r) - k_n^-(t), \tag{14b}$$

where

τ_r - time for full depreciation of activity r.

The development region may have initial capacities inconsistent with a future desired set (mix) of these capacities. Hence, from a practical view not only the construction of new capacities is necessary to consider also the reconstruction of existing assets. In this case the state equations (14) should be rewritten as follows:

$$k_n(t + 1) = c_n k_n(t) + \sum_r d_{nr} w_{nr}(t) - \sum_s x_{ns}(t) +$$

$$\sum_s \chi_{sn} x_{sn}(t) - k_n^-(t) \tag{14c}$$

Here

$x_{ns}(t)$ $(n,s=1,\ldots,N)$ is the decreasing capacity of type n at step t which at this step began reconstruction into the capacity of type s (for example, the modernization of technology, changing of the type of activity, etc.) We call this process "conversion n — s".

χ_{sn} is the conversion coefficient which shows the increase of the capacity n due to reconstruction of a unit of the capacity s.

Thus the total increase of the capacity n at step t due to conversion from the others capacities will be

$$\sum_s \chi_{sn} x_{sn}(t)$$

and the total decrease of the capacity n at step t due to conversion into the others capacities will be

$$\sum_s x_{ns}(t)$$

Obviously

$$c_n k_n(t) - \sum_s x_{ns}(t) \geq 0$$

for each n.

Usually the process of reconstruction takes more than one step. In this case the above equations become

$$k_n(t + 1) = c_n k_n(t) + \sum_r d_{nr} w_{nr}(t - \tau_r) -$$

$$- \sum_s x_{ns}(t - \tau_{ns}) + \sum_s \chi_{sn} x_{sn}(t - \tau_{sn}) - k_n^-(t) \qquad (14e)$$

where τ_{ns} is the time (number of steps) for conversion n $\longrightarrow$ s . Models of reconstruction in more details are considered in [10].

4. Constraints

Any realistic economic model contains constraints of various types. First, we list those related to the technical requirements of the DLP model. Secondly, we note those related to available resource capacities.

4.1 *Non negativity*

Obviously all variables (both state and control) are nonnegative in the considered case:

State variables:

$$
\begin{aligned}
& x_i(t) \geq 0 && \tilde{z}_m^y(t) \geq 0 \\
& y_j(t) \geq 0 && s_m^x(t) \geq 0 \\
& k_n(t) \geq 0 && s_m^y(t) \geq 0 \\
& z_m^x(t) \geq 0 && \tilde{s}_m^y(t) \geq 0 \\
& z_m^y(t) \geq 0 && z_m(t) \geq 0
\end{aligned}
\tag{15a}
$$

Control variables:

$$
\begin{aligned}
& u_i^+(t),\ u_i^-(t) \geq 0 && s_m^{x-}(t) \geq 0 \\
& v_j^+(t),\ v_j^-(t) \geq 0 && s_m^{y-}(t) \geq 0 \\
& w_n(t),\ k_n^-(t) \geq 0 && \tilde{s}_m^{y-}(t) \geq 0 \\
& z_m^{x+}(t), z_m^{x-}(t) \geq 0 && q_{mk}^x(t),\ q_{mk}^y(t), \\
& z_m^{y+}(t), z_m^{y-}(t) \geq 0 && \qquad \tilde{q}_{mk}^y(t) \geq 0 \\
& \tilde{z}_m^{y+}(t), \tilde{z}_m^{y-}(t) \geq 0 && z_m^+(t) \geq 0 \\
& && \tilde{y}_j(t) \geq 0
\end{aligned}
\tag{15b}
$$

4.2 *Resource Capacities*

The values of resource capacities $k_n(t)$ can be derived from state equations (14). Generally, we can combine from different values of (physical) resources capacities $k_n(t)$, $n = 1.,,,,N.$ (tractors of different types, separate buildings, etc.) the available capacities $K_g(t)$ for a specific g^{-th} operation:

$$K_g(t) = \sum_{n=1}^{N} \mu_{gn} k_n(t) \qquad (g = 1,\ldots,G)$$

where coefficients μ_{gn} show per unit (say, tractor power) capacity for g^{-th} operation.

Frequently, $\mu_{gn} = 1$ for $g = n$ and $\mu_{gn} = 0$ otherwise. In that case we have:

$$K_n(t) = k_n(t)$$

The constraints on available capacities is written as follows:

$$\sum_i \lambda^x_{gi} x_i(t) + \sum_j \lambda^y_{gj} y_j(t) + \sum_j \tilde{\lambda}^y_{gj} \tilde{y}_j(t) +$$

$$+ \sum_m \sum_k \lambda^{q^x}_{gmk}(t) + \sum_m \sum_k \lambda^{q^y}_{gmk} q^y_{mk}(t) + \sum_m \sum_k \tilde{\lambda}^{q^y}_{gmk} \tilde{q}^y_{mk}(t)$$

$$\leq \sum_n \mu_{gn} k_n(t) \qquad (16)$$

It should be noted that the above general equation covers most cases dealing with resource constraints. But in many of the equations most coefficients are zero.

Also to complete the system, certain control variables may need to be constrained by separate inequalities (e.g. available land, disease control, etc.).

For example, storage capacities of all products can be limiting as illustrated by the following:

$$
\begin{aligned}
z_m^x(t) &\le \bar{z}_m^x(t) & s_m^x(t) &\le \bar{s}_m^x(t) \\
z_m^y(t) &\le \bar{z}_m^y(t) & s_m^y(t) &\le \bar{s}_m^y(t) \\
\tilde{z}_m^y(t) &\le \bar{\tilde{z}}_m^y(t) & \tilde{s}_m^y(t) &\le \bar{\tilde{s}}_m^y(t) \\
z_m(t) &\le \bar{z}_m(t) & &
\end{aligned}
\tag{17}
$$

where values of $z_m^x(t)$, $z_m^y(t)$, $\tilde{z}_m^y(t)$, $s_m^x(t)$, $s_m^y(t)$, $\tilde{s}_m^y(t)$ and $z_m(t)$ are derived from the state equations (6) to (12). For those subsystems without storage capacity inequalities (17) should be replaced by constraints of type (13).

5. Financial Subsystem

This subsystem summarizes the financial results of the activities described by the other subsystems largely in physical terms. Because of the wide range of possible solutions of such calculations, according to different economic and accounting systems followed, we describe only general elements of the financial subsystem which are important. The specific accounting procedure and management organization will dictate the exact form of the equation and constraints upon the system.

5.1 *Return in Period t*

$$
\begin{aligned}
&\sum_i p_i u_i^-(t) + \sum_j p_j v_j^-(t) + \sum_n p_n k_n^-(t) + \sum_m p_m^x z_m^{x-}(t) + \sum_m p_m^y z_m^{y-}(t) \\
&+ \sum_m \tilde{p}_m^y \tilde{z}_m^{y-}(t) + \sum_m p_m^{q^x} s_m^{x-}(t) + \sum_m p_m^{q^y} s_m^{y-} + \sum_m \tilde{p}_m^{\tilde{q}^{\tilde{y}-}} \tilde{s}_m^{\tilde{y}-} = f^r(t)
\end{aligned}
\tag{18}
$$

$f^r(t)$ is the total amount of return in period t.

p_i, p_j, etc. are the prices or appropriate indicators.

5.2 *Expenditures*

$$\sum_m p_m^x z_m^{x+}(t) + \sum_m p_m^y z_m^{y+}(t) + \sum_m \tilde{p}_m^{\tilde{y}} \tilde{z}_m^{\tilde{y}+}(t) + \sum p_m^+ z_m(t) +$$

$$+ \sum_i \sum_g p_g^c \lambda_{gi}^x x_i(t) + \sum_j \sum_g p_g^c \lambda_{gj}^y y_j(t) + \qquad (19)$$

$$+ \sum_j \sum_g p_g^c \tilde{\lambda}_{gj}^y \tilde{y}_j(t) + \sum_m \sum_k \sum_g p_g^c \lambda_{gmk}^{q^y} q_{mk}^y(t) + \sum_m \sum_k \sum_g p_g^c \tilde{\lambda}_{gmk}^{q^y} \tilde{q}_{mk}^y(t)$$

$$= f^e(t)$$

$f^e(t)$ is the amount of expenditures in period t.
p_g^c is the expenses on g^{-th} resource usage, including depreciation

5.3 *Money Balance*

$$z_p(t + 1) = z_p(t) + f^r(t) - f^e(t) \qquad (20)$$

$$z_p(t) \gtrless 0$$

$z_p(t)$ is the income generated by the system

5.4 *Investments*

$$\sum_i p_i u_i^+(t) + \sum_j p_j v_j^+(t) + \sum_n \sum_r p_{nr} w_{nr}(t) = f^i(t) \qquad (21)$$

$f^i(t)$ is the amount invested in period t.
The investments may be restricted.

$$f^i(t) \leq z_p(t), \qquad (21a)$$

or

$$f^i(t) \leq z_p(t) + \bar{f}^i(t) + f^d(t), \qquad (21b)$$

$\bar{f}^i(t)$ is the exogenously given upper limit of investment funds available from external sources.

5.5 *Fixed Capital*

$$z_c(t + 1) = z_c(t) + f^i(t)$$

$z_c(t)$ is the net value of fixed assets

6. Objective Function and Planning Horizon

Multiperiod or dynamic linear programming models generally assume a finite time horizon, therefore requiring consideration of the appropriate goal functions, discounting procedures and specification of terminal conditions (and or values for the fixed assets). However, for the latter problem of appropriately valuing terminal "fixed" assets we can note that theoretically their value is determined by the present value of earnings beyond the terminal date. Hence, the implicit consideration of an infinite horizon cannot be avoided. One alternative is to explicitly consider the problem in an infinite horizon framework by specifying that the activities entering the solution in the final time period specified in the model continue indefinitely; the objective function values for terminal period activities are thus the present value of the earning stream of that activity from that point to infinity (see (6)).

The question of the appropriated objective or goal function becomes more complex as we move from a single period model to one of multiple periods. The question is open as to what the decision maker should or does maximise in the longer run and the constraints under which such maximization takes place. For example the Lutzes (8) suggest four possibilities:

> "First, the entrepeneur may find the present value of the future gross revenue stream (v) and the present value of the future cost stream (c) by capitalizing at the interest rate ruling in the market, and maximize the difference (v-c) between these present values. Secondly, he may maximize the present value of the future revenue stream (again formed by capitalizing at the given market rate of interest) divided by the present value, similarly calculated, of the future cost stream, i.e., he may maximize v/c. Thirdly, he may maximize the "internal rate of return" on the capital sum invested. Fourthly, he may maximize the rate of return on his own capital, which is assumed to be a given amount and may be smaller than the total sum invested whenever part of the latter is financed out of borrowed funds."

Hirshleifer emphasizes that while no rule is universal, the present value rule is correct in a wide variety of cases.[9] However, Solomon draws attention to the maximization of wealth or net present worth as an operating objective for financial management.[16] Perhaps we can conclude this discussion only by giving a partial list of objective functions that have found some use in investment analysis.

1. Maximization of the present value of future consumption
2. Maximization of the present value of future return (profits) both (a) in the situation where profits are withdrawn at the end of each accounting period and (b) in the situation where profits are reinvested as they eventuate
3. Maximization of the discounted cash flow
4. Maximization of the present value of future cash flows
5. Maximization of terminal net worth.

For example for the problem being discussed in this paper the following objective functions may be considered;

$$\text{Max} \sum_{t=1}^{T} \omega(t)\; z_p(t)$$

$\omega(t)$ is the discount coefficient

or

$$\text{Max } z_c(T) + z_p(T)$$

$z_c(T)$ is the fixed capital in the terminal year T.

7. Some Limitations of DLP Approach

The DLP model assumes constant prices of inputs and outputs, that is linearity is assumed. If output prices were a function of output which well may be the case in large scale projects, then the model should be reformulated as a nonlinear programming model(18). In practice, appropriate sensitivity analysis by parametric programming techniques often allow good approximations to the non-linear solutions while retaining the computational efficiency of linear programming.

Another objection to DLP is that it is a deterministic approach to a problem with many stochastic elements. Here again alternative techniques may be conceptually superior (e.g. quadratic programming, stochastic programming) but operational problems are more formidable because of massive data requirements. Furthermore, it can be argued that some of the annual stochastic variations may be relatively minor compared to the more critical sources of uncertainty in models of long planning horizons (changes in the general level of prices, yields, and the variables due to technological change and general economic conditions).

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