RM-77-29

DYNAMIC LINEAR PROGRAMMING MODELS FOR LIVESTOCK FARMS

A. Propoi

June 1977

PREFACE

The problems of the best allocation of limited resources for different agricultural activities have attracted the attention of many researchers. These problems can be treated by linear programming (LP).

In recent years however it was understood that better results could be achieved when time aspects of resource allocation are taken into account. Thus we come to the planning problems of the growth of agricultural farms, and the problems become dynamical (multistage) linear programming ones (DLP). It should also be noted that the larger the scale of agricultural activities being considered, the greater the economic effect that can be obtained. So, these problems are basically large-scale.

In this introductory paper simple DLP models for livestock breeding with feed production are considered. This approach may be used for the optimal planning of cattle, pig, and sheep breeding farms, poultry farming, optimal control of fish breeding, fur farming, etc. Similar problems also arise when planning the migration of wild animals or suppression of pests is necessary.

SUMMARY

This paper considers the dynamic linear programming model for multi-species livestock farming with a feed production subsystem.

The problem is to determine the optimal livestock mix with the projected growth rate and corresponding development of feed production in order to obtain the maximum profit for the given planning horizon.

As examples the planning model for a dairy farm and the control problem of age/size structure of a biological population are given.

Dynamic Linear Programming Models for Livestock Farms

Introduction

In this introductory paper planning and control models for large livestock farms are considered. These models allow to elaborate optimal plan of farms development for long-range period (5 - 10 - 30 years) or to design a control system for the farms in stationary regime (production scheduling). Such kind of models were considered, for example in [1 - 6].

The models are formalized as dynamic linear programming (DLP) problems. DLP is a next stage of linear programming (LP) development and is aimed for solution of large-scale optimization problems [7].

In the paper the general DLP model for multi-species livestock farm with forage production subsystem is considered (section 1). As particular cases of this model, the planning model for a dairy farm (section 2) and control model of age structure and size of a biological population (section 3) are given.

1. Multi-species livestock farm

We consider here the planning problem for a large livestock farm with several species of animals. The problem is to determine the optimal livestock mix with projected growth rate in order to obtain the maximal profit for the given planning period (T years). The livestock subsystem is considered together with forage producing subsystem.

<u>Livestock subsystem</u>. We consider a livestock subsystem consisting of n species of animals. All animals in accordance with their type i and age τ are divided into N groups.

Let

$x_i^a(t)$ be the number of animals of type i and group a at stage (i.e. year) t.
$(i = 1,...,n;\ a = 0,1,...,N-1;\ t = 0,1,...,T-1)$.

An animal belongs to group a, if its age is τ, and $a\Delta \leqslant \tau < (a+1)\Delta$, Δ is given time interval (days, months, years).

Vector $x^a(t)$ defines the animals' distribution over their type (sex) in group a at stage t:

$$x^a(t) = \{x_1^a(t),\ ...,\ x_i^a(t),\ ...,\ x_n^a(t)\} \quad .$$

Let the reproductive age begin with the group a_1 and end by group a_2. Usually, $a_2 = N - 1$. Then the number of animals born (that is, of group 0) at year $t + 1$ is equal to

$$x^0(t+1) = \sum_{a=a_1}^{N-1} P(a)x^a(t) \quad , \tag{1}$$

where $P(a)$ is a birth matrix of group a; the element $p_{ij}(a)$ of $P(a)$ shows the number of animals of type i "produced" (born) by one animal of type j and group a.

For example, let the farm keep two kinds of animals: cows and sows of one producing group a, and during each year each cow has one calf, while each sow has ten pigs born (with approximately equal distribution over sex). Then the equations (1) can be written as

$$\begin{bmatrix} x_1^0(t+1) \\ x_2^0(t+1) \\ x_3^0(t+1) \\ x_4^0(t+1) \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 5 & 0 \end{bmatrix} \begin{bmatrix} x_1^a(t) \\ x_2^a(t) \\ x_3^a(t) \\ x_4^a(t) \end{bmatrix}$$

where $x_1^0(t+1)$ is the number of heifers born at year $t+1$;
$x_2^0(t+1)$ is the number of bulls born;
$x_3^0(t+1)$ is the number of pigs (female) born;
$x_4^0(t+1)$ is the number of pigs (male) born;
$x_i^a(t)$ $(i = 1,2,3,4)$ is the number of animals of type i and group a at year t.

Evidently, these equations break down into two independent set of equations.

The transition of animals from group a into group $a+1$ is described by equation

$$x^{a+1}(t+1) = S(a)x^a(t) \tag{2}$$

Where the survival matrix $S(a)$ shows what part of animal group a progresses to group $a+1$ for one year.

If, for example, $\Delta = 1$ year and group a suffers an attrition rate of r_i^a $(0 \leq r_i^a \leq 1)$ each year, then the equation (2) can be written as

$$\begin{bmatrix} x_1^{a+1}(t+1) \\ \cdot \\ \cdot \\ \cdot \\ x_n^{a+1}(t+1) \end{bmatrix} = \begin{bmatrix} (1-r_1^a) & & & 0 \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ 0 & & & (1-r_n^a) \end{bmatrix} \begin{bmatrix} x_1^a(t) \\ \cdot \\ \cdot \\ \cdot \\ x_n^a(t) \end{bmatrix}$$

It should be noted that the attrition rate r_i^a may express not only the death rate but the effect of certain breeding policy (e.g. culling of cows at a given age).

Let us introduce a vector

$$x(t) = \{x_i^a(t)\} \quad (i = 1, \ldots, n; \quad a = 0, 1, \ldots, N-1).$$

Then equations (1) and (2) can be combined

$$x(t+1) = G\,x(t) \qquad (t = 0, 1, \ldots, T-1) \tag{3}$$

where

$$G = \begin{bmatrix} 0 & 0 & \cdots & P(a_1) & \cdots & P(N-1) \\ S(0) & 0 & \cdots & 0 & \cdots & 0 \\ 0 & S(1) & & & & \cdot \\ & & \ddots & & & \\ \cdot & & & S(a_1) & & \cdot \\ \cdot & & & & \ddots & \cdot \\ \cdot & & & & & \cdot \\ 0 & \cdot & \cdots & \cdots & \cdot & S(N-1) \end{bmatrix} \tag{4}$$

G is the growth matrix [8].

Let us also introduce vectors

$$u(t) = \{u_i^a(t)\} \quad \text{and} \quad v(t) = \{v_i^a(t)\},$$

where $u_i^a(t)\,[(v_i^a(t)]$ is the number of animals of type i and group a, purchased (sold) at stage t.

Then the dynamic of type and age distribution of animals will be described by equation

$$x(t+1) = Gx(t) + u(t) - v(t) \tag{5}$$

with given initial distribution

$$x(0) = x^o \tag{6}$$

and constraints

$$x(t) \geq 0, \quad u(t) \geq 0, \quad v(t) \geq 0 \ . \tag{7}$$

Schematically the equations (5) with the matrix (4) are given on figure 1.

Along with evident constraints (7) it is necessary to take into account constraints associated with the care and feeding of animals.

In rather general form they can be written as

$$Fx(t) \leq f(t) \tag{8}$$

where the component $f_k(t)$ of vector

$$f(t) = \{(f_1(t), \ldots, f_k(t), \ldots, f_m(t)\}$$

determines the available quantity of the k-th resource; the element f_{ki}^a of matrix F shows the per unit consumption of the k-th resource by animals of type i and group a.

However, the livestock farms usually have their own forage production. In this case it is necessary to introduce equations which describe the development of forage production.

Crop subsystem

Let

$z_k(t)$ be the quantity of the k-th resource (corn, hay, etc.) at stage (year) t ($k = 1,\dots,m$);

$w_k(t)$ be the quantity of the k-th resource purchased during year t;

$y_k(t)$ be the number of hectares for producing (planting) of the k-th crop resource;

a_k be the crop capacity of one hectare for the k-th forage resource.

Then the forage (resource) production will be defined by the term:

$$a_k y_k(t) \quad \text{or} \quad Ay(t)$$

where A is a diagonal matrix with elements a_k on the main diagonal ($k = 1,\dots,m$).

If several types k ($k = 1,\dots,m$) of forage can be produced on different lots j ($j = 1,\dots,J$), then it should be introduced:

$y_{kj}(t)$ is the number of hectares of lot j ($j = 1,\dots,J$) used for production of the k-th resource ($k = 1,\dots,m$) at year t; and

a_{kj} is the crop capacity for k-th resource of the j-th lot.

In this case we have

$$Ay(t) = \left\{ \sum_{j=1}^{J} a_{kj} y_{kj}(t) \right\}_k$$

with constraints

$$\sum_{k=1}^{m} y_{kj}(t) \leq y_j$$

where y_j is the total area of the j-th lot.

The forage (resource) consumption is defined by the term

$$Bx(t) \quad ,$$

where element b_{kj}^{a} of matrix $B = \{b_{kj}^{a}\}$ shows the per unit consumption of the k-th resource of animals of type j and group a (cf. (8)).

Thus the equation describing the material balance of resources is given by

$$z(t+1) \quad = \quad z(t) + Ay(t) - Bx(t) + w(t) \tag{9}$$

with initial condition

$$z(0) \quad = \quad z^{0} \tag{9a}$$

and constraints

$$z(t) \geq 0 \;, \quad y(t) \geq 0 \;, \quad w(t) \geq 0 \quad . \tag{10}$$

If some types of resource are both purchased and sold, then it is necessary to introduce the term

$$w(t) \quad = \quad w^{+}(t) - w^{-}(t) \quad , \tag{10a}$$

where $w^{+}(t) \geq 0$ is the vector of resources purchased at year t and $w^{-}(t) \geq 0$ is the vector of resources sold at year t. In the case of (10a) the sign of vector w(t) is not predetermined.

Total area of all lots cannot exceed Y:

$$\sum_{j} y_{j}(t) \quad \leq \quad Y \tag{11}$$

Other constraints on variables are also possible. For example, the quantity of $z_k(t)$ can be limited by stock capacity of the farm:

$$z_k(t) \leq \bar{z}_k(t) \tag{12}$$

where $\bar{z}_k(t)$ is given.

If there is no possibility (or necessity) to stock the k-th resource at all, then the equation (9) is reduced to condition

$$a_k y_k(t) - \sum_{a,i} b_{ki}^a x_i^a(t) + w_k(t) = 0 \tag{12a}$$

Equations (5) and (9) are <u>state equations</u>, which describe the development of the systems in time.

For state equations (5), (9) we shall single out:

the <u>state</u> variables: $x(t)$ and $z(t)$;
the <u>control</u> variables: $u(t)$, $v(t)$ and $y(t)$, $w(t)$.

Choosing the controls $\{u(t), w(t), y(t), w(t)\}$ one can compute through (5) and (9) for both initial states x^o and z^o the corresponding state trajectories $\{x(t)\}$ and $\{z(t)\}$.

Each control and its associated trajectory determines the value of performance index of the system. In the case considered it is a profit, which can be obtained during the total planning period. Thus the problem is to obtain maximal profit for the planning period T.

The following function may be chosen for measure of effectiveness:

$$J(u,v,y,w) = \sum_{t=0}^{T-1} \left[\left(\alpha(t), x(t)\right) + \left(\beta(t), v(t)\right) \right] -$$

$$- \sum_{t=0}^{T-1} \left[\left(\gamma(t), z(t)\right) + \left(\delta(t), w(t)\right) + \left(\theta(t), u(t)\right) + \left(\rho(t), y(t)\right) \right] \tag{13}$$

where

$\alpha(t) = \{\alpha_i^a(t)\}$ is the per unit revenue from animals of type i and group a in year t after deduction of the cost of care and other expenses (except feed-producing expenses);

$\beta(t) = \{\beta_i^a(t)\}$ is the return per animal of type i and group a, sold in year t;

$\gamma(t) = \{\gamma_k(t)\}$ is the cost of storing a unit of the k-th resource during year t;

$\delta(t) = \{\delta_k(t)\}$ is the expenses per unit of the k-th resource purchased at year t;

$\theta(t) = \{\theta_i^a(t)\}$ is the expenses per animal of type i and group a purchased at year t;

$\rho(t) = \{\rho_k(t)\}$ is the expenses of growing one hectare of the k-th type of forage at year t.

Finally, the problem can be formulated as follows.

Problem 1 *To find controls* $\{u(t), v(t), y(t), w(t)\}$, *satisfying the state equations (5), (9) with the initial states (6), (9a) and constraints (7), (10), (11), (12), which maximize the performance index (13).*

Various modifications and versions of Problem 1 are possible. Two particular cases of Problem 1 are considered below.

2. Planning model for a dairy farm [5]

In this model the cattle are divided into four groups (fig. 2).

The number of milk-producing cows (group 4 cattle) at year t is $x_1^4(t)$.* During each year, each milk-producing cow has one calf, and approximately one half of all calves born will be bulls, the other half being heifers. Consequently,

$$\begin{aligned} x_1^1(t) &= 0.5x_1^4(t) - v_1^1(t) \\ x_2^1(t) &= 0.5x_1^4(t) - v_2^1(t) \end{aligned} \qquad (14)$$

where $v_1^1(t)$, $v_2^1(t)$ are numbers of heifers and bulls sold at birth.

Calves are not sold while they are of group 2. Besides, the progression from group 1 to group 2 is made in the same year. Hence,

$$\begin{aligned} x_1^2(t) &= x_1^1(t) \\ x_2^2(t) &= x_2^1(t) \end{aligned} \qquad (15)$$

Group 2 cattle will become group 3 in the next year and all bulls of that age are to be sold. Hence,

$$\begin{aligned} x_1^3(t+1) &= x_1^2(t) - v_1^3(t) \\ 0 &= x_2^2(t) - v_2^3(t) \end{aligned} \qquad (16)$$

Group 4 suffers an attrition rate of approximately 70% each year and at the same time the group 4 population is enlarged by

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*Here the notations are slightly changed in comparison with [5]. (According to notations of section 1).

the infusion of the previous period group 3 heifers that were kept. Hence,

$$x_1^4(t+1) = x_1^3(t) + 0.7x_1^4(t) \quad . \tag{17}$$

Using (14) to (17) one can write the equations, which describe the cattle subsystem as follows:

$$\begin{aligned}
x_1^3(t+1) &= 0.5x_1^4(t) && -v_1^1(t) - v_1^3(t) \\
x_2^3(t+1) &= 0.5x_1^4(t) && -v_2^1(t) - v_2^3(t); \quad x_2^3(t+1) = 0 \\
x_1^4(t+1) &= 0.7x_1^4(t) + x_1^3(t) \; ;
\end{aligned}$$

or, in matrix form:

$$x(t+1) = Gx(t) - Dv(t) \tag{18}$$

$$x(t) \geq 0 \; , \quad v(t) \geq 0 \; , \quad x_2^3(t+1) = 0 \tag{18a}$$

where

$$x(t) = \begin{bmatrix} x_1^3(t) \\ x_2^3(t) \\ x_1^4(t) \end{bmatrix} ; \quad G = \begin{bmatrix} 0 & 0 & 0.5 \\ 0 & 0 & 0.5 \\ 1 & 0 & 0.7 \end{bmatrix} ;$$

$$D = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} ; \quad v(t) = \begin{bmatrix} v_1^1(t) \\ v_2^1(t) \\ v_1^3(t) \\ v_2^3(t) \end{bmatrix} .$$

Conceptual representation of the dairy farm is given in figure 3 [5].

Now the crop subsystem together with the crop-cattle interaction will be described.

The crop subsystem is described by equations, each of which equates the amount of a certain crop grown during a year t plus the amount available in storage to the amount that will be consumed in that year t plus the amount that is placed in storage for use in subsequent years. Therefore, we have the following equations.

1) For silage:

$$z_1(t+1) = z_1(t) + a_1y_1(t) - [b_{11}^2x_1^2(t) + b_{12}^2x_2^2(t) + + b_{11}^3x_1^3(t) + b_{11}^4x_1^4(t)] \qquad (19)$$

where the coefficients b_{1i}^a $(i = 1,2; \quad a = 2,3,4)$ show the yearly per capita consumption of silage by the various cattle groups of the herd, and the coefficient a_1 indicates that each hectare yields a_1 tons of silage.

The storage for silage is limited:

$$z_1(t) \leq \bar{z}_1(t) \qquad (20)$$

2) For corn:

$$a_2y_2(t) - \sum_{a,i} b_{2i}^a x_i^a(t) = 0 , \qquad (21)$$

where the coefficients a_2 and b_{2i}^a $(a = 1,2,3,4; \ i = 1,2)$ have the same meaning as in (19). It is supposed that there is no corn storage at the farm.

3) For haylage:

$$z_3(t+1) = z_3(t) + a_3y_3(t) - b_{31}^4x_1^4(t) \qquad (22)$$

(haylage is consumed only by group 4 cattle), with

$$z_3(t) \leq \bar{z}_3(t) \qquad (23)$$

4) For hay:

$$z_4(t+1) = z_4(t) + a_4 y_4(t) + w_4(t) - b_{41}^1 x_1^1(t) - b_{42}^1 x_2^1(t) - b_{41}^2 x_1^2(t) - b_{42}^2 x_2^2(t) \qquad (24)$$

$$z_4(t) \leq \bar{z}_4(t) \qquad (25)$$

where $w_4(t)$ is the amount of hay purchased in year t.

In matrix form the equations (19), (21), (22), (24) can be rewritten as:

$$z(t+1) = z(t) + Ay(t) - Bx(t) - w(t) \quad , \qquad (26)$$

where

$$z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_4(t) \end{bmatrix} ; \quad A = \begin{bmatrix} a_1 & & & 0 \\ & a_2 & & \\ & & a_3 & \\ 0 & & & a_4 \end{bmatrix} ;$$

$$B = \begin{bmatrix} 0 & 0 & b_{11}^2 & b_{21}^2 & b_{11}^3 & b_{11}^4 \\ b_{21}^1 & b_{22}^1 & b_{21}^2 & b_{22}^2 & b_{21}^3 & b_{21}^4 \\ 0 & 0 & 0 & 0 & 0 & b_{31}^4 \\ b_{41}^1 & b_{42}^1 & b_{41}^2 & b_{42}^2 & 0 & 0 \end{bmatrix} ;$$

$$w(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ w_4(t) \end{bmatrix} .$$

Evidently,

$$z(t) \geq 0 , \quad w(t) \geq 0 , \quad y(t) \geq 0 . \qquad (27)$$

Besides, available capacity of land for cultivation is limited.

$$y_1(t) + y_2(t) + y_3(t) + y_4(t) \leq Y . \qquad (28)$$

The problem is to maximize the total profit during the planning period T (T = 25 years):

$$J = \sum_{t=0}^{T-1} \Big[\alpha_1^4 x_1^4(t) + \left(\beta_1^1 v_1^1(t) + \beta_1^3 v_1^3(t) + \beta_2^1 v_2^1(t) + \beta_2^3 v_2^3(t)\right) -$$

$$- \left(\alpha^1 x_1^1(t) + \alpha^2 x_2^1(t) + \alpha^2 x_1^2(t) + \alpha^2 x_2^2(t) + \alpha^3 x_1^3(t)\right) -$$

$$- \rho_1 y_1(t) + \rho_2 y_2(t) + \rho_3 y_3(t) + \rho_4 y_4(t) -$$

$$- \delta_4 w_4(t) - \left(\gamma_1 z_1(t) + \gamma_2 z_2(t) + \gamma_3 z_3(t) + \gamma_4 z_4(t)\right)\Big] \qquad (29)$$

where

α_1^4 is the revenue from milk of one cow of group 4;

α^1, α^2 and α^3 are the cost of care and other expenses for groups 1, 2 and 3;

the meaning of coefficients β_i^1, β_i^3, ρ_j, γ_j, δ_4 $(i = 1,2;$ $j = 1,2,3,4)$ is similar to those of (13).

Thus, we can formulate the following problem.

Problem 2 *To find controls* $\{v(t), y(t), w(t)\}$ *satisfying the state equations (18), (26) with constraints (18a), (20), (21), (23), (25), (26), which maximize the performance index (29) for given initial states* $x(o) = x^o$, $z(o) = z^o$.

3. The age structure control of species population use [6]

The following problems can be singled out here:

1) determination of optimal stationary structure of the species population;
2) control of the population with given structure;
3) determination of optimal transition of the population to a given new structure.

The solution of the first problem is necessary for long-range use of a species population.

Let the environment be stationary and the age structure of the species population under control be in equilibrium state. Then the change of population distribution in time will be described by equation*

$$x(t + 1) = G(t)x(t) - u(t) \quad (t = 0, 1, \dots) \tag{30}$$

where

$x(t)$ is the population distribution over type of species and age (state-vector of the system);

$u(t)$ is the intensity vector of removing species from the population;

$G(t)$ is the growth matrix.

The variables have the evident constraints

$$x(t) \geq 0 , \quad u(t) \geq 0 . \tag{31}$$

It is also necessary to take into account the resource constraints:

$$\sum_t F(t)x(t) \leq f \tag{32}$$

where matrix $F(t) = f_{kj}(t)$ determines the per unit consumption

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* In [6] the continuous model is considered.

of resource k by species j at stage t, the vector $f = \{f_k\}$ represents the available quantity of resource k.

The performance index can be given in the form

$$J = \sum_t \left[\left(\alpha(t), x(t)\right) + \left(\beta(t), u(t)\right) \right] , \qquad (33)$$

where $\alpha(t)$ is the per unit profit $\left(\alpha(t) \geq 0\right)$ or expenses $\left(\alpha(t) \leq 0\right)$ from the species population use; $\beta(t)$ is the per unit profit from removing the species outside of the population.

As a result we obtain the following problem.

Problem 3 *To find control* $\{u(t)\}$ *and trajectory* $\{x(t)\}$, *satisfying state equation (30) with initial state* $x(o) = y^o$, *and constraints (31), (32), which maximize the performance index (33).*

If it is necessary to find an optimal transient process from the given initial state

$$x(o) = x^o$$

to the given terminal state x^T, then the boundary condition

$$x(T) = x^T$$

is added to the constraint of Problem 3.

4. Canonical form of DLP problems

The problems considered above are related to the class of dynamic linear programming problems [7]. One can see that Problems 1 to 3 and their modifications can be reduced to the following canonical form:

Problem 4 *To find a control*

$$u = \{u(o), u(1), \ldots, u(T-1)\}$$

and a trajectory

$$x = \{x(o), x(1), \ldots, x(T)\}$$

satisfying the state equations

$$x(t+1) = A(t)x(t) + B(t)u(t) + s(t) \tag{34}$$

with initial condition

$$x(o) = x^{o} \tag{35}$$

with constraints

$$G(t)x(t) + D(t)u(t) \leq f(t); \quad x(t) \geq 0, \quad u(t) \geq 0 \tag{36}$$

which maximize the performance index

$$J(u) = (\alpha(T), x(T)) + \sum_{t=0}^{T-1} \left[(\alpha(t), x(t)) + (\beta(t), u(t)) \right] \tag{37}$$

Here $x(t) = \{x_1(t), \ldots, x_n(t)\}$ is the state of the system at stage t; $u(t) = \{u_1(t), \ldots, u_r(t)\}$ is the control action at stage t; $f(t) = \{f_1(t), \ldots, f_m(t)\}$ is the given (resource) vector; matrices A(t), B(t), G(t) and D(t) have the corresponding dimensions.

Various modifications and particular cases of Problem 1 are possible [9].

One can consider Problem 4 as an ordinary LP problem with constraints given in the form of equalities (34), (35) and inequalities (36), (37) (see Table 1) and use for its solution the standard LP codes. However, the DLP problems of large dimensions require the development of special DLP methods [7,10].

Conclusion

Some models of planning and control of species population have been considered. These problems can be used for the optimal planning of cattle-breeding, pig-breeding farms, poultry farming, optimal control of fish-breeding, fur farming, etc. Similar problems also arise when planning of migration of wild animals or suppression of pests is necessary.

The first practical examples show that the solution of such DLP problems may yield significant economic effect. Thus, authors of [5] write that the solution of DLP planning problems for the dairy farm with a herd of 1000 head of cattle (using a planning horizon of 25 years) has given a possibility to triple productivity and to increase profit tenfold.

The solution of DLP problem for determining optimal age structure of a herd of cattle [6] increases the revenue up to 5 - 7%.

It should be stressed here that the larger the scale of the livestock farm model that is being considered, the greater the economic effect that can be obtained.

In conclusion, the author has a pleasure to thank C. Csaki for many fruitful comments and suggestions and H. Carter for discussions on the subject.

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Variables											
u(0)	x(1)	...	x(t)	u(t)	x(t+1)	...	x(T-1)	u(T-1)	x(T)	Constraints	Right-hand side constants
-B(0)	I									=	$s(0) + A(0)x^0$
D(0)		⋱								≤	$f(0) + G(0)x^0$
											⋮
			-A(t)	-B(t)	I					=	s(t)
			G(t)	D(t)		⋱				≤	f(t)
											⋮
							-A(T-1)	-B(T-1)	I	=	s(T-1)
							G(T-1)	D(T-1)		≤	f(T-1)
Performance Index Constants										Max	
b(0)	a(0)	...	a(t)	b(t)	a(t+1)	...	a(T-1)	b(T-1)	a(T)		

Table 1

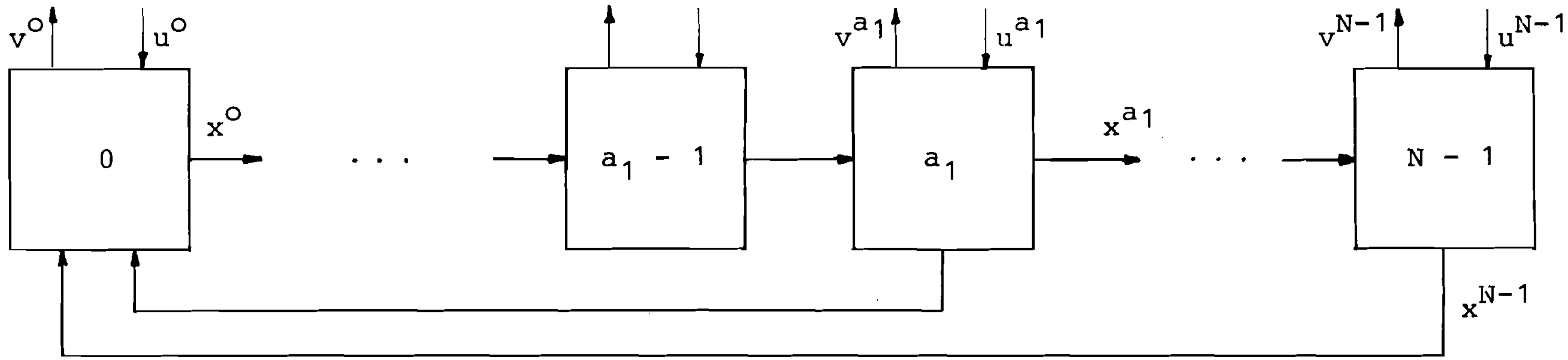

Fig. 1

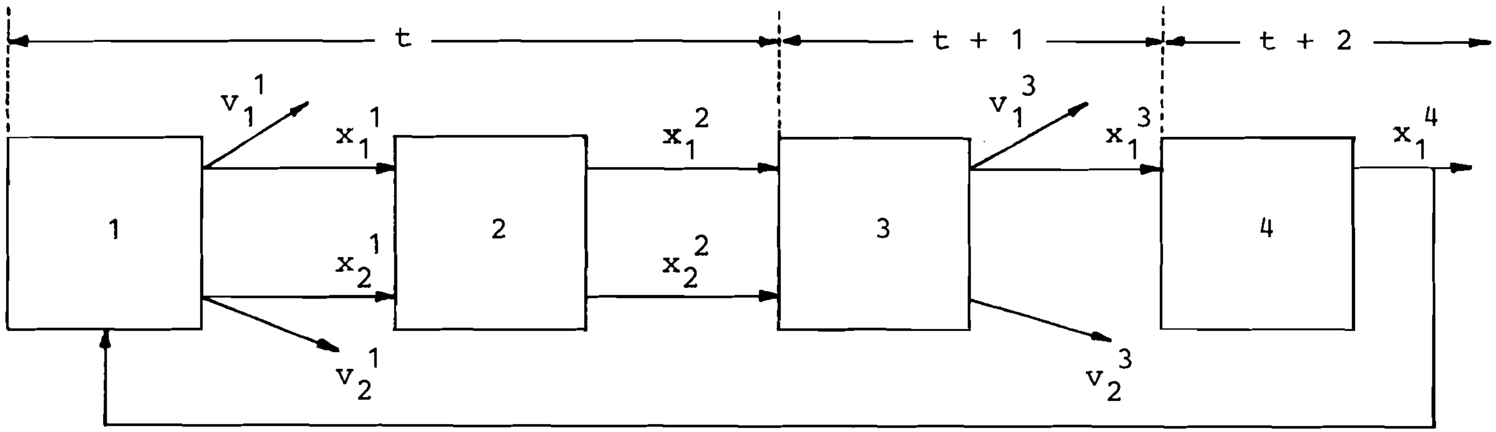

Fig. 2

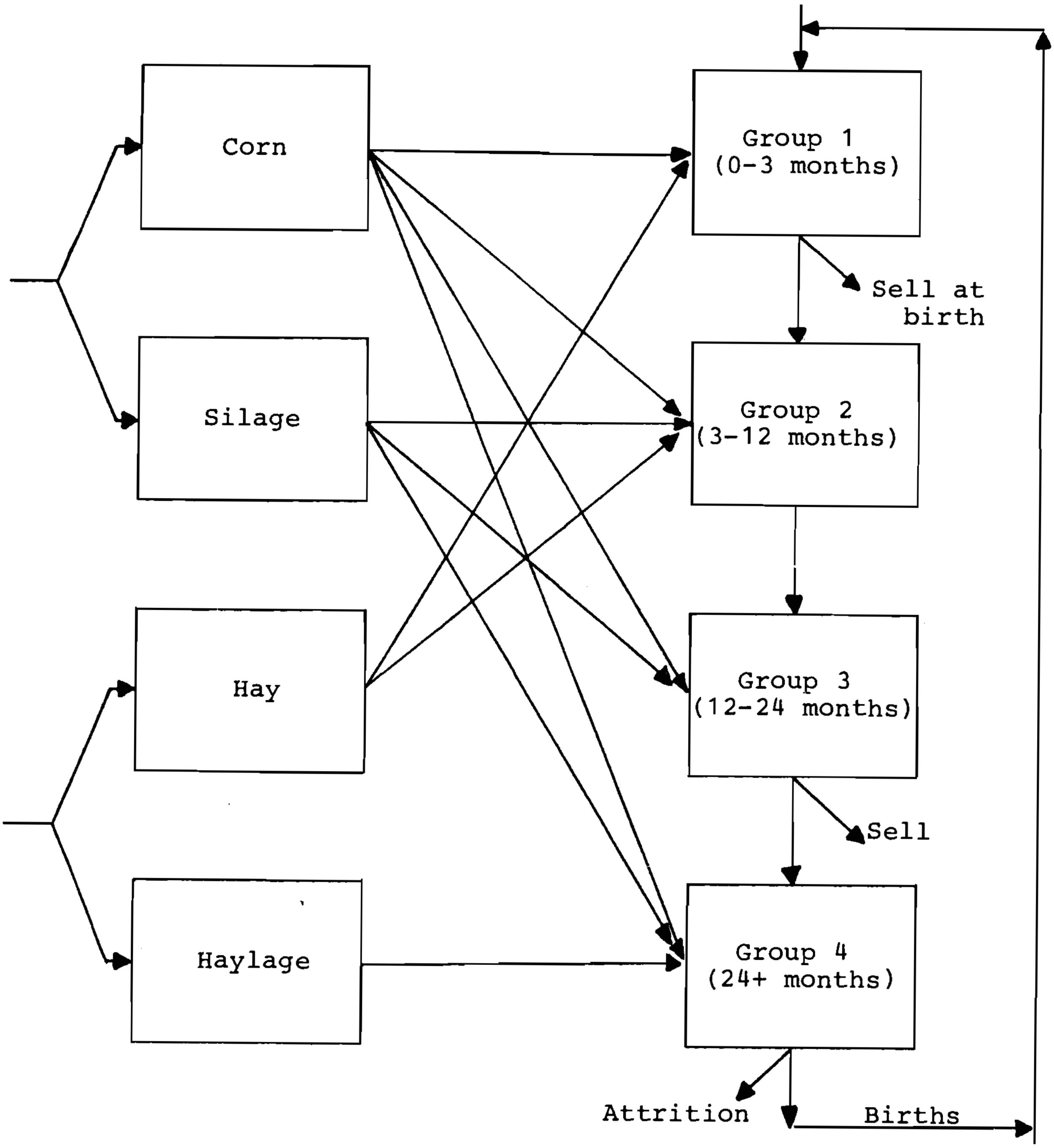
Corn
Silage
Hay
Haylage
Group 1
(0-3 months)
Sell at
birth
Group 2
(3-12 months)
Group 3
(12-24 months)
Sell
Group 4
(24+ months)
Attrition
Births

Figure 3