RR-74-11

THE VALUE OF INFORMATION IN SPECULATIVE MARKETS

Jeffrey F. Jaffe

Robert L. Winkler

August 1974

The Value of Information in Speculative Markets

Jeffrey F. Jaffe* and Robert L. Winkler**

1. Introduction

A number of writers have examined the value of information in economic activity. Among other uses, the literature has has emphasized that information can improve the decisions of the firm, bring together potential buyers and sellers, and increase the trading profits of individuals.[1] This paper considers the effects of information on trading profits and determines optimal trading policies under two types of trading procedures.

Previous research in this area has emphasized that the informed investor should be able to exploit the uninformed investor, that information will be produced for trading purposes (even at a cost), and that this information production is not necessarily Pareto-optimal.[2] Though the present paper does not disagree with the above, we point out that the magnitude of the effects is determined by the type of market involved. In particular, the above effects are minimized in markets that are dominated by speculative traders (i.e. traders who are not interested in changing either their level of risk or their total investment). This occurs essentially because naive speculative investors have a simple

---

*University of Pennsylvania.

**Indiana University; on leave and visiting the International Institute for Applied Systems Analysis, Laxenburg, Austria, from July 1973 to January 1974.

mechanism, previously ignored in the literature, for avoiding exploitation by informed investors. They can merely stop trading in markets where they have been losing money from previous trades.

The above considerations are discussed in Sections 2 and 3, where it is assumed that there is no market maker (specialist) in the economy. However, Section 4 focuses on speculative trading in an economy with a market maker. Under this regime, every investor, no matter how little information he possesses, should attempt to trade with the specialist. The volume of speculative trading, however, depends on the relative ability of the specialist vis-à-vis the investors as well as on other considerations such as commissions. A statistical model is developed in this section to determine optimal policies for trading with a specialist, and practical applications are also discussed.

The first four sections are developed in a framework similar to that of the financial economics literature, because the type of trading discussed is seen most clearly in financial markets. However, the results should be applicable to all areas of economic activity. In Section 5 we illustrate this applicability by considering alternative forms of gambling.

2. A Model for Speculative Trading

Traditionally, stock-market observers have differentiated speculative trading from liquidity trading. Trading on information for the purpose of increasing profits is considered speculative trading. Trading without information for the purpose of changing either the perceived risk of a portfolio or the amount of funds committed to the stock market is considered liquidity trading. Admittedly, it is difficult to separate these two types of trading. For example, if the risk-return opportunity set of an investor is changed when he receives information concerning a security, he may adjust his risk position and his current total investment in the stock market. Though both liquidity and speculative motives are involved in most transactions, for purposes of exposition we present a model that attempts to separate the two motives.

In order to highlight the speculative trading decision, the following model is chosen:

1. The capital market has zero transaction costs and no indivisibilities in the trading of securities.
2. Information cannot be pooled among investors.
3. Each individual is risk-neutral and attempts to maximize expected return. This restriction is added to abstract from trading to change risk.
4. A one-period model is assumed. Each individual brings both cash and claims on securities to a tatonnement auction held at the beginning of the period. Here all consumption-investment decisions are made and securities are exchanged. The value of each security, which is determined by an exogenous stochastic process,

is announced at the end of the period, and the securities are exchanged for consumption at this time. During the period, investors can trade in response to the continuous flow of new information. However, there will be no withdrawals from the market during the period, as investors are not permitted to purchase any consumption goods during the period. This assumption is included to separate liquidity trading, which should occur only at the beginning of the period, from speculative trading, which occurs throughout the entire period.

5. Investors cannot sell short. As pointed out by Fama and Laffer,[3] unlimited buying and short selling by expected value maximizers will result in infinite buying by those who expect a high price for a security and infinite buying by those who expect a high price for a security and infinite short selling by those who expect a low price.

From the fourth assumption, the market for all risky assets clears through a tatonnement bidding process at the outset of the period. Next, assume that later in the period, the equilibrium situation is disturbed by the introduction of new information pertaining to a security. An individual who, after receiving the new information now expects a greater return from this security than from other securities, will buy the security. Similarly, an individual believing that the information implies a smaller return for the security might sell the security.

To formalize the notion of trading during the period, consider a simple situation involving one security and two investors, A and B.[4] The trading mechanism is a sealed-bid procedure whereby the two investors write down forecasts, P_A and P_B, of the price, or value, of the security at the end of the period, and trading occurs at a price mid-way between the two forecasts. If $P_A > P_B$, B sells the security to A at a price of $(P_A + P_B)/2$, and if $P_B > P_A$, A sells to B at that price. The amount of the security that is traded is not important for our purposes, so it is assumed that just one unit (share) of the security changes hands. Alternatively, given the assumption that the investors are risk-neutral, it might be more realistic to assume that the individual selling the security in this trading procedure sells his entire holdings of the security (and would sell more if it were not for the prohibition of short sales).

The investors' forecasts can be written in the form $P_A = P + u_A$ and $P_B = P + u_B$, where P is the actual value of the security at the end of the period and u_A and u_B are error terms. Assuming that the two investors are knowledgeable and experienced enough to avoid systematic errors, suppose that $u = (u_A, u_B)$ has a bivariate normal distribution with $E(u_A) = E(u_B) = 0$, $V(u_A) = \sigma_A^2$, $V(u_B) = \sigma_B^2$, and $Cov(u_A, u_B) = \rho\sigma_A\sigma_B$. Moreover, it is assumed that this distribution is known to both traders.[5] For example, the distribution could be based on considerable past data in the form of forecasts by A and B and the corresponding price observations.

After A determines P_A but before P_B is known, A's prior distribution for P is a normal distribution with mean P_A and variance $\sigma_A{}^2$. After learning the value of P_B, A's posterior distribution for P is a normal distribution with mean P_A^* and variance σ_A^{*2}, where

$$P_A^* = \frac{k(k - \rho)P_A + (1 - \rho k)P_B}{k^2 - 2\rho k + 1} ,$$

and

$$\sigma_A^{*2} = \frac{k^2(1 - \rho^2)\sigma_A{}^2}{k^2 - 2\rho k + 1} ,$$

with $k = \sigma_B/\sigma_A$. The derivation of the posterior distribution is given in Appendix I.

From this posterior distribution and the assumption that trading of one unit of the security occurs at a price of $(P_A + P_B)/2$, A's expected return from the trade, as determined <u>after</u> P_A and P_B are known, is simply $P_A^* - \left[(P_A + P_B)/2\right]$ if $P_A > P_B$ (in which case B sells the security to A) and $\left[(P_A + P_B)/2\right] - P_A^*$ if $P_A < P_B$ (in which case A sells the security to B). Thus, A's expected return is

$$\pi_A = \begin{cases} \dfrac{k(k - \rho)P_A + (1 - \rho k)P_B}{k^2 - 2\rho k + 1} - \dfrac{P_A + P_B}{2} & \text{if } P_A > P_B , \\[2ex] \dfrac{P_A + P_B}{2} - \dfrac{k(k - \rho)P_A + (1 - \rho k)P_B}{k^2 - 2\rho k + 1} & \text{if } P_A < P_B , \end{cases}$$

which simplifies to

$$\pi_A = \begin{cases} \dfrac{(k - 1)(k + 1)(P_A - P_B)}{2(k^2 - 2\rho k + 1)} & \text{if } P_A > P_B \;, \\[2ex] \dfrac{(k - 1)(k + 1)(P_B - P_A)}{2(k^2 - 2\rho k + 1)} & \text{if } P_A < P_B \;. \end{cases}$$

Obviously, $(k + 1) > 0$ (since $k = \sigma_B/\sigma_A > 0$), $(P_A - P_B) > 0$ if $P_A > P_B$, and $(P_B - P_A) > 0$ if $P_A < P_B$. The term in the denominator, $k^2 - 2\rho k + 1$, is nonnegative if $\rho \leq (k^2 + 1)/2k$. But $(k^2 + 1)/2k \geq 1$, with equality holding only when $k = 1$. Thus, $k^2 - 2\rho k + 1 > 0$ except when $\rho = k = 1$, the uninteresting case in which there is no trading since $P_A \equiv P_B$. As a result, all of the factors of π_A are strictly positive except for $k - 1$, implying that the sign of $k - 1$ is the sign of π_A:

$$\pi_A \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \qquad \text{iff} \qquad k - 1 \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \;.$$

Recalling that $k = \sigma_B/\sigma_A$, we see that A's expected return from the trade is positive if $\sigma_A < \sigma_B$ and negative if $\sigma_A > \sigma_B$. Moreover, since A and B start with the same joint distribution for (P_A, P_B) and since the trading procedure is a zero-sum game,[6] $\pi_B = -\pi_A$, where π_B represents B's expected return from the trade. Therefore, the investor with the smaller standard deviation of forecast error has a positive expected return, whereas the other investor has a negative expected return.

The preceding analysis assumes that the expected error of each investor's forecast is zero. Suppose that $E(u_A) = v$, so that A's forecasts have a systematic error of magnitude v in addition to random error. If everything else in the model is unchanged, the effect of this systematic error is to change A's posterior mean to $P_A^* - [k^2(1 - \rho^2)v/(k^2 - 2\rho k + 1)]$, where P_A^* is the posterior mean when the expected error of A is calibrated to zero. The difference in A's expected return (uncalibrated expected return minus calibrated expected return) is $-\delta v$ when $P_A > P_B$ and δv when $P_A < P_B$, where $\delta = k^2(1 - \rho^2)/(k^2 - 2\rho k + 1)$. Before seeing P_A and P_B, however, the distribution of $P_A - P_B$ is normal with mean v and variance ${\sigma_A}^2 + {\sigma_B}^2 - 2\rho\sigma_A\sigma_B$. Transforming to the standard normal distribution, $\Pr(P_A > P_B) = \Pr\left[z > -v/({\sigma_A}^2 + {\sigma_B}^2 - 2\rho\sigma_A\sigma_B)^{1/2}\right] = \gamma$ can be found, implying that before (P_A, P_B) is observed, the uncalibrated expected return for A is $-\delta v\gamma + \delta v(1 - \gamma) = \delta v(1 - 2\gamma)$. But δ is strictly positive and $v(1 - 2\gamma) < 0$ if $v \neq 0$, since $v > 0$ implies $.5 < \gamma < 1$ and $v < 0$ implies $0 < \gamma < .5$. Therefore, the difference in expected returns is negative, indicating that it is better for A to be calibrated than to be uncalibrated. Calibrating is advantageous to an uncalibrated investor, and a calibrated investor diminishes his expected return by intentionally uncalibrating, or hedging.

In the sealed-bid trading procedure, then, a calibrated investor with a small forecast error variance has an advantage over an uncalibrated investor with a larger error variance. Moreover, the essential nature of this result generalizes to a sealed-bid trading procedure with n traders, where $n > 2$. This is illustrated in Appendix II for the special case in which the errors of the different forecasters are independent.

As new information continually appears in the market, investors will continually repeat the trading procedure. Because of differences in either the amount of information obtained or the inferences drawn from the information, some individuals will earn trading profits while others will suffer trading losses. Rational investors who continue to earn less by trading than they would have earned by a buy-and-hold strategy will eventually stop trading. Other investors, who were previously trading successfully, may now find that they are suffering trading losses as a result of trading only with superior forecasters, and they will eventually desist from trading also. The process will be repeated until only the most successful trader desires to trade.

It is possible that rational investors with optimistic expectations of their future trading performance will continue to trade in spite of initial trading losses. This could be caused by misperceptions of the distribution of forecast errors.[7] However, our model specifies a continuous flow of in information with no transaction costs. Each individual will trade continuously and the results of the infinite number of trades implied by this process should "swamp" the initial prior judgments. Thus, in a rational world where individuals form reliable judgments concerning their trading proficiency over time, there should be no speculative trading.[8]

The situation could be complicated by assuming that some individuals are adept at trading in certain industries or with respect to certain information. In this case, individuals would form expectations of their trading proficiency for individual categories of trading. Again, only the most proficient will remain in each category, and no trading will result. It may appear paradoxical that an investor who possesses information can expect to earn a higher return by ignoring the information than by trading on it. However, the fact that two investors are able to trade indicates that they differ in their forecasts. In trading with an individual who is a better forecaster (because of different information or because of other reasons), an investor is at a disadvantage, as the model presented in this section shows.

3. Implications Concerning the Value and Production of Information

Recent models concerning the value of and the production of information[9] imply that information is valuable for trading purposes, in addition to its usefulness in the allocation of investment in real assets. Furthermore, these models suggest that individuals will commit real resources to information production, either for their own private trading activities or for sale to other information traders. Manne apparently approves of this information production, as he states that opportunities for trading on special information can be a successful inducement to the satisfactory performance of business executives. Conversely, Fama-Laffer and Hirshleifer believe that the production of information for trading purposes need not be Pareto-optimal. They state that the production of information for trading purposes employs real resources, while leading to a mere redistribution of assets.

In order to highlight the differences between our results and those of other authors, we define the "full value" of a piece of information pertaining to an individual security as the product of (1) the change in the security's price upon release of the information and (2) the number of shares outstanding. This term corresponds to V, the change in the value of the firm due to information in the Fama-Laffer paper. Fama and Laffer postulate a model of risk-neutral individuals where each individual is willing either to buy or to sell shares in a security at a given price. Here an individual who receives information favorable to a security can buy all of the shares of a security at the market price and subsequently release the information to reap its full value.[10]

In the model of the previous section, we saw that during the period all trading would eventually stop. Should an individual obtain information during the period concerning the value of securities, he would desire to trade on the basis of it. However, no one would be willing to trade with him. As information would be of no value for trading securities in the period, there would be no information production for trading purposes. At least in the case of a purely speculative market, the movement away from Pareto-optimality due to information production shown by Fama and Laffer can be corrected by a curtailing of trading.

As the analysis of Hirshleifer is presented in a time-state preference model, he discusses the trading of actual consumption claims, as opposed to our use of only claims on securities. Since Hirshleifer did not attempt to separate liquidity motives from speculative motives in his model, trading exists in his economy. For example, consumption-investment decisions can be viewed as liquidity decisions, so that consumption claims in different periods may be traded, even by unknowledgeable investors. To see this, imagine a two-period, two-person economy where B only holds claims to certain consumption in period 1 and A only holds claims to uncertain consumption in period 2. Also assume that A is more knowledgeable than B concerning the amount of consumption to be received in period 2. B can expect that, in the normal case, A will trade away fewer period 2 claims when he expects the return in period 2 to be high than when he expects the return in period 2 to be low. In spite of this, B may decide to trade with A in order not to starve in period 2. In a similar fashion, it can be shown that consumption claims contingent on different states in the same period may be traded for liquidity purposes.

It is assumed in our model that each individual brings cash and claims on securities to the auction at the beginning of the period and that each individual must make a consumption-investment decision at that time. Hence, our results parallel those of Hirshleifer at this point, as even unknowledgeable investors may trade at the tatonnement auction. Since trading occurs, information is of value and will be produced. Along the lines suggested by Hirshleifer,[11] we can show that this information production is not necessarily Pareto-optimal. Consider a situation in which all individuals have identical holdings of cash and the market portfolio prior to the tatonnement process, possess identical utility functions, and have homogeneous expectations with regard to the behavior of the market portfolio. Furthermore, suppose that one unit of additional infomation may be purchased at a small positive cost. Though each person might purchase it, no trades will result since all initially held identical portfolios. The information leads to no action, so it is of no value. As our model implies the possibility of trading, information production, and non-optimality, our results from the tatonnement process do not differ from those of Hirshleifer.

Thus, trading and non-optimal information production result from liquidity motives. They would not, however, arise in a purely speculative market such as the one occuring in our model after the auction. Therefore, those models of purely speculative markets which imply trading and information production ignore that the rational action of naive investors in the presence of informed traders is to curtail trading.

Of course, we recognize that liquidity trading exists in the real world, so we do not suggest that our results have predictive content. However, at any given moment, the number of investors desiring to trade due to liquidity motives is probably small, so that even without transaction costs, investors cannot be expected to earn the full value of their information.

4. Speculative Trading with a Specialist

A model where individuals trade securities directly with each other was presented in Section 2. However, in the real world, individuals usually trade securities indirectly through a broker and a specialist. The broker, who is responsible for executing transactions in the investor's name (as well as for providing advice), is compensated by a commission. As orders to transact in a particular security occur irregularly, investors who desire to buy (sell) cannot be certain of locating investors who want to sell (buy) immediately. To insure that individuals can transact promptly, the specialist stands ready to buy shares of the security at a specified price (bid price) and to sell shares of the security at another specified price (ask price). The specialist is compensated by the difference between the ask price and the bid price, which is commonly called the spread.

Though the specialist and broker are compensated in proportion to the trading volume, the specialist is also compensated according to his ability to forecast the future price of a security. The bid and ask prices can be viewed as the specialists' forecasts. Investors who possess information unavailable to the specialist implying a certain rise in the price of the security will purchase the security from the specialist and will subsequently reap gains at the specialists' expense. The model of Section 2 implies that individuals should eventually stop trading for speculative reasons. However, a world where specialists cannot desist from trading with informed investors may yield different results.

To formalize the notion of trading with a specialist, assume the same model presented in Section 2, but let A represent an investor and let B represent the specialist. Instead of a sealed-bid trading procedure, A sees P_B before making a trading decision. A can buy the security at a price of $P_B + T$, sell the security at a price of $P_B - T$, or do nothing. The difference between P_B and the prices at which A can trade represents the spread and/or comission. Obviously T is nonnegative, with $T = 0$ corresponding to the case in which A can buy <u>or</u> sell at P_B, the price set by the specialist (i.e. the specialist's forecast of the value of the security).

From the model in Section 2, A's posterior distribution of P after A observes P_B is a normal distribution with mean P_A^* and variance σ_A^{*2}. The expected return to A from buying the security is $P_A^* - (P_B + T)$, so it is optimal for A to buy when $P_A^* - (P_B + T) > 0$. From Section 2, this is equivalent to

$$\frac{k(k - \rho)P_A + (1 - \rho k)P_B}{k^2 - 2\rho k + 1} - P_B > T \quad ,$$

which simplifies to

$$P_A - P_A > \beta T \qquad \text{if } k - \rho > 0$$

and

$$P_A - P_B < \beta T \qquad \text{if } k - \rho < 0 \quad ,$$

where

$$\beta = \frac{k^2 - 2\rho k + 1}{k(k - \rho)} \quad .$$

If $k - \rho > 0$, so that the correlation between P_A and P_B is smaller than σ_B/σ_A, the expected return to A from buying the security is positive when $P_A - P_B$ is greater than a certain positive multiple of T. This follows from the fact that $\beta > 0$. The larger β is, the larger the price difference that is required before it is advantageous for A to buy the security. The case of $\beta = 1$, of course, corresponds to the situation in which the breakeven value of $P_A - P_B$ is exactly equal to T.

The relationship among β, ρ, and k is as follows when $k - \rho > 0$:

$$\beta \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \qquad \text{iff} \qquad \rho \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \frac{1}{k} \quad .$$

This can be seen by writing β in the form $1 + [(1 - \rho k)/(k^2 - \rho k)]$. The denominator of the second term is strictly positive (since $k > \rho$), so the sign of the second term is simply the sign of $1 - \rho k$. But it might be expected that the specialist is a better forecaster (in the sense of having a smaller error variance) than most investors, in which case $\sigma_B < \sigma_A$, or $k < 1$. If $k < 1$, then $1/k > 1$, so $\rho < 1/k$ by definition, implying that $\beta > 1$. Even if A has a smaller error variance than the specialist, β will still be greater than one unless the correlation is large enough so that $\rho \geq \sigma_A/\sigma_B$. This implies that when A has a smaller variance than the specialist <u>and</u> the correlation is larger than σ_A/σ_B, A can take advantage of his superior forecasting ability and obtain a positive expected return even in some cases where $P_A - P_B < T$. In most situations, however, we would expect to see $\beta > 1$. Moreover, if $k \leq 1$, $d\beta/dk < 0$, implying that β increases as the specialist becomes a better forecaster (in terms of the ratio of error variances) relative to the investor.

If $k - \rho < 0$, then $\beta < 0$, and the optimal buying rule is to buy if $P_A - P_B < \beta T < 0$. This seems to be a strange result; A should buy the security when $P_A - P_B$ is <u>negative</u> enough! Noting that $k < \rho \leq 1$, we see that the specialist has a smaller error variance than A, and ρ is high enough that it is highly likely that P_A is on the same side of P as is P_B but that P_A is further from P. For instance, if $k < 1$ and $\rho = 1$, A <u>knows</u> that P_A is on the wrong side of P_B, so A utilizes this information to <u>buy</u> when $P_A < P_B$. In this situation, $P_A^* = (p_B - kP_A)/(1 - k)$, and $P_A^* - P_B$ and $P_A - P_B$ are of opposite sign.

The situation in which A sells the security is analogous to the buying situation. The expected return to A from selling is positive when

$$P_B - P_A > \beta T \qquad \text{if } k - \rho > 0 \ ,$$

or

$$P_B - P_A < \beta T \qquad \text{if } k - \rho < 0 \ ,$$

Thus, combining the buying and selling situations, A has a positive expected return (i.e. A will trade) whenever

$$|P_A - P_B| > \beta T \qquad \text{if } k - \rho > 0 \ ,$$

or

$$|P_A - P_B| < \beta T \qquad \text{if } k - \rho < 0 \ .$$

If $k - \rho > 0$, what is the probability, calculated before seeing P_A and P_B, that A will trade? This probability is $Pr(|P_A - P_B| > \beta T)$, which by symmetry is equal to $2Pr(P_A - P_B > \beta T)$. Since $P_A - P_B$ is normally distributed with mean zero and variance $\sigma_A{}^2 + \sigma_B{}^2 - 2\rho\sigma_A\sigma_B = \sigma_A{}^2(k^2 - 2\rho k + 1)$, the probability that A will trade can be written in the form $2Pr[z > (k^2 - 2\rho k + 1)^{1/2}Tk^{-1}(k - \rho)^{-1}\sigma_A{}^{-1}]$, where z is a standard normal random variable. Letting τ represent the probability that A will trade and letting $\theta = (k^2 - 2\rho k + 1)^{1/2}Tk^{-1}(k - \rho)^{-1}\sigma_A{}^{-1}$, we see that $\partial\tau/\partial T = (\partial\tau/\partial\theta)(\partial\theta/\partial T)$. But $\partial\tau/\partial\theta < 0$ and $\partial\theta/\partial T > 0$, so that τ is a decreasing function of T, as would be expected. Moreover, $\partial\theta/\partial k < 0$, implying that τ is an increasing function of k. As A's error variance decreases relative to the specialist's error variance, A is more likely to trade. The appearance of σ_A^{-1} in the expression for θ suggests that with all other variables held constant, a smaller σ_A makes trading less likely. But "all other variables held constant" means that k, a function of σ_A^2, is held constant, so that σ_B^2 decreases as σ_A^2decreases. The increased precision of both A and the specialist means that the distribution of $P_A - P_B$ is tighter. But T is being held constant, implying that $P_A - P_B$ is less likely to overcome the spread and/or transaction costs. If σ_B/σ_A is substituted for k in the expression for θ, differentiation yields $\partial\theta/\partial\sigma_A > 0$, which implies that $\partial\tau/\partial\sigma_A < 0$. Therefore, the probability that A will trade is a decreasing function of σ_A^2, as would be expected.

The above results imply that any investor with some information, no matter how little, can profit from that information by employing a decision rule such as the one arrived at by the model of this section. All investors have a positive probability of trading with the specialist (i.e. a positive probility of encountering a situation with a positive expected return from buying or selling at the prices offered by the specialist). However, the probability of trading is a decreasing function of the investor's variance of error of estimation. The more accurately an individual forecasts the price of a security and the smaller T is, the more likely it is that a trade will occur. Thus, the probability of trading is a function of the degree of information that the investor possesses. For investors with little information, the probability of trading is small (i.e. the number of opportunities for profitable trading is small). On the other hand, for very knowledgeable investors, the number of opportunities for profitable trading may be large. The volume of trading will depend on the forecasting abilities of the investors relative to that of the specialist.

The results of the model of this section differ from those of the model of Section 2, where individuals with large variances of estimation error can expect to lose in trading with individuals with small variances of estimation error. The key difference in the two models, of course, involves the trading procedure. In this section, the specialist is forced to set prices at which any investor can buy and sell the security. An investor can observe these prices and revise his judgments concerning P before deciding whether to trade. Furthermore, he can determine a decision rule such as the following: buy only if $P_A - P_B$ is larger than a certain breakeven point and sell only if $P_B - P_A$ is larger than another breakeven point. In the sealed-bid trading procedure, the investor is not able to see P_B before deciding whether or not to trade. A's decision must be made before P_B is known, and the two forecasts, P_A and P_B, jointly determine the details of the trade.

Because the market maker (specialist) can expect to lose to all rational investors, a purely speculative market with a market maker is unstable. The market maker will let $T \to \infty$ in order to reduce his trading losses unless he is either subsidized in some manner and/or the size of T is restricted. Governmental subsidization is not only possible but is "supposed" to be the practice on organized exchanges.[12]

Liquidity traders, as well as investors who either misperceive their forecasting ability or do not use rules such as those given in this section, should also subsidize the specialist. However, as pointed out in Section 2, if there are liquidity traders or traders who continually misperceive their forecasting ability, trading will exist even without a market maker. Thus, it is not clear that information is of greater value in an economy with a market maker than in an economy without one. Rather, the value of information under a market maker depends on the forecasting ability of the market maker vis-à-vis the investors and on policies regarding spreads, suspension of trading, and so on.

Although the model presented in this section was conceived primarily for theoretical purposes, it should be useful to investors in practice. In order to use the model, it is necessary for an investor to determine the necessary inputs to the model, and the key input is a probability distribution. The spread and/or transaction costs, represented by T, should be known in any given investment situation, and the remaining factors affecting the results of the model are the parameters of the joint distribution of the forecast errors of the investor and the specialist.

The question of determining probability distributions for future security prices is discussed in detail in two papers by Winkler,[13] so to conserve space we will not repeat these discussions here. In general, an investor should utilize any information and means for processing information that are available, implying that a probability distribution for future security prices should typically involve the use of past data, subjective judgments of the investor and any expert that may be consulted, and output from any statistical forecasting models that may be available. In the particular situation posited in this paper, a great deal of relevant past data is available or will become available quickly as the trading process goes on, so an investor's probability distribution should depend heavily on such data. By systematically looking at the past history of his own and the market's errors of estimation, an investor should be able to investigate whether a normal distribution provides a reasonable approximation of the joint distribution of forecast errors. Moreover, the past data can be used to calculate estimates of the parameters of the joint distribution. In practice, an investor might make forecasts at regular intervals, such as intervals of one month. It would then be necessary to investigate these forecasts as well as month-to-month shifts in the market price. It should be mentioned that models concerning equilibrium pricing of risky capital assets, such as those discussed by Sharpe, Lintner, and Black,[14] may be useful in investigating the performance of investors and specialists. In particular, empirical work implementing these models has focused on the "residual" (i.e. the movement in the stock price not accounted for by either the stock's level of risk or the movement in the price level of the entire stock market).[15]

The residual, which is the return above or below the equilibrium return for all assets with a given risk, appears to be a reasonable estimate of the specialist's error of estimation and can easily be estimated for any security where past data are available.

Of course, to the extent that all investors possess the same amount of information, determining a distribution of forecast errors and utilizing the model of this section would seem to be a sterile exercise as far as the practical applicability of the analysis is concerned. There is evidence that few individuals or models possess enough special information to predict stock prices with any degree of accuracy.[16] However, some investors and models have been particularly successful in this regard, so it should be useful to briefly review a few such cases.

First, there is ample evidence that corporate insiders possess and act upon special information.[17] However, each individual insider may possess information on only one or a few securities. In addition, these insiders may receive information at irregular intervals, and it may be of varying quality. Thus, the process may not be stationary, making it more difficult to determine a probability distribution of forecast errors.

Though many investment counselors, mutual funds, and brokers do not appear to possess significant information,[18] evidence suggests that the Value-Line Advisory Service[19] may possess such information. As the performance of many other advisory services has not been examined, it is possible that other services also possess special information. These services investigate many different securities at regular intervals, so that past data could be used to arrive at a distribution of future forecasting errors. At present, many of the services only indicate whether an investor should buy or sell a particular security; a method of forecasting the price of the security would be needed in order to apply our model.

As a final example of successful prediction, a few models of security valuation presented in the financial literature appear to be successful. For example, Jones and Litzenberger[20] have shown that stock prices do not adjust immediately when firms have sizeable increases in accounting earnings. Though Jones and Litzenberger only decide which securities should be bought, a model yielding forecasts of the prices of securites could be developed. Black and Scholes[21] have shown that their theoretical model for pricing options has information content As their model yields a numerical value for options, an empirical implementation could follow easily.

Even if investors do possess special information, they may not use it to full advantage. A model of the type presented here is of value in formulating optimal trading rules so that investors are able to utilize their information in an attempt to increase expected return. For example, the results of Black and Scholes indicate that although proper use of their option pricing equation can yield positive returns with no transaction costs, the equation cannot be used to earn profits net of transaction costs. However, in the calculation of profits after transaction costs, all options were either bought or sold. Using our model, Black and Scholes could have bought only those options with values estimated to be above the market price by at least βT and sold only those options with values estimated to be below the market price by at least βT. By taking a position in fewer securities, their option model might have been profitable even with positive transaction costs.

5. Summary and Discussion

We have developed models of two types of trading procedures and we have investigated the possibility of speculative trading under these procedures. The first model involves a sealed-bid trading procedure in which investors trade directly with other investors. In a stationary world where investors can accumulate information concerning their forecasting ability, the amount of speculative trading will diminish until there will eventually be no speculative trading. This result has implications, discussed in Section 3, concerning the value and production of information.

The second model presented in this paper incorporates the notion of trading with a specialist, who is forced to set prices at which any investor can buy and sell a security. When investors trade with the specialist instead of directly with each other, speculative trading will not cease. The volume of such trading, however, will depend on the ability of investors (vis-à-vis the specialist) to forecast the price of a security. The model provedes optimal trading rules for an investor and can be used to determine the probability that an investor will be able to trade profitably in a given situation.

The models discussed here are purposely simple so that the main points are not obscured by unnecessary details. Potential extensions include the simultaneous consideration of several securities, the inclusion of nonlinear utility functions for money on the part of investors, and the development of multiperiod models that take into account the potential effect of future decisions on a current decision.[22] This list is by no means exhaustive, and it is clear that such extensions would complicate matters and would make the model more difficult to solve. However, such extensions would not be expected to change materially the essential nature of the results obtained in this paper.

Though the models developed herein have been placed in a financial setting, they have implications for all areas of economic activity where assets or claims are traded. For instance, an analogy can easily be drawn between investing in securities and gambling on sporting events. Some forms of gambling on sporting events are similar to the sealed-bid trading procedure of the model presented in Section 2 (e.g. pari-mutuel betting, such as encountered at a racetrack).

Other forms of gambling on sporting events are similar to the procedure of trading with a specialist as in the model of Section 4 (e.g. betting on football games with a bookie, who sets the point spread at which bets are placed).

In pari-mutuel betting at a racetrack, the final odds, which can be thought of as a price, are determined for each race by the actions of the bettors. Furthermore, the odds are computed after all bets have been placed, so that are not known for certain in advance. Therefore, the bettors who are better forecasters of the outcomes of races than other bettors can expect to earn a positive return at the expense of other bettors. (It is more difficult to earn a positive return in this context than in the context of the model of Section 2, however, for the track takes its own cut, or commission, before the total amount wagered is divided among the successful bettors.) Under the assumptions of Section 2, unsuccessful bettors would stop betting, and eventually no individuals would be willing to place bets. In the race-track situation, however, nonlinear utility functions for money and a positive increment of utility due simply to the recreational value of being at the racetrack and to the "joy" of gambling explain in part why racetracks do not all go out of business.

The model of Section 4 can be illustrated in terms of gambling on sporting events by considering betting on football games with a bookie. The bookie is analogous to the specialist, for he sets the point spreads, which can be thought of as prices, and bettors can choose to bet on either side of a given point spread. The bookie is compensated by a percentage taken from winning bets and/or by winning ties. (A tie occurs when the actual point spread is identical to the bookie's point spread.) This compensation is comparable to the specialist's spread and/or commission. Also as in the case of the specialist, the bookie's compensation is related in part to his ability to forecast, although in this case the outcomes of football games rather than the values of securities are the object of the forecasts. Ideally, a bookie would like to set a point spread so that equal amounts are wagered on either side of the point spread. In this way, the bookie is not gambling. No matter what the actual outcome of the game is, half of the bettors are, in essence, betting against the other half, and the bookie's "return" is a percentage of the gains of the winning bettors and/or the possibility of keeping all money bet on either side if a tie occurs.

If the bookie sets a point spread and a majority of bettors prefer one particular side of the point spread, the usual procedure is to adjust the point spread in order to "even off" the bets, much as a specialist might adjust his ask price and bid price in response to investors' actions. A bookie who winds up with more bets on one side of the point spread than on the other side becomes a gambler, winning if the majority of bettors turns out to be wrong and losing if they turn out to be right.

Of course, bettors who are able to forecast the outcomes of football games accurately can, by following decision rules such as those generated by the model of Section 4, obtain a positive expected return. The investor who knows the relevant distribution of forecast errors can make a forecast on the basis of his judgments concerning the game and then revise the forecast after seeing the bookie's point spread. If the revised point spread is far enough from the bookie's point spread to overcome the effect of the bookie's percentage of winnings and/or policy of winning ties, the investor should bet.

In the case of investing in securities, the specialist can offset losses to information traders by profiting from liquidity traders. As in the pari-mutuel situation, nonlinear utility and a positive utility for gambling per se may cause bettors to behave differently than the model of Section 4 would indicate. Thus, there may be a class of bettors who would continually lose to bookies. (Similarly, there may be a class of investors who would continually lose to specialists.) This makes the entire situation more attractive to the bookie, overriding the possibility of losing to a few very knowledgeable bettors.[23]

Appendix I. Derivation of A's Posterior Distribution

Before P_B is known, A's prior distribution for P is a normal distribution with mean P_A and variance σ_A^2,

$$f(P|P_A) \propto \exp\{-(P - P_A)^2/2\sigma_A^2\} \quad .$$

The likelihood function is proportional to $f(P_B|P,P_A)$, which is a normal distribution with mean $P + \rho k(P_A - P)$ and variance $k^2\sigma_A^2(1 - \rho^2)$,

$$f(P_B|P,P_A) \propto \exp\{-[P_B - \{P + \rho k(P_A - P)\}]^2/2k^2\sigma_A^2(1 - \rho^2)\} \quad .$$

Thus, the posterior distribution is

$$f(P|P_A,P_B) \propto f(P|P_A)\ f(P_B|P,P_A)$$

$$\propto \exp - \frac{k^2(1 - \rho^2)(P - P_A)^2 + [P_B - \rho k P_A - (1 - \rho k)P]^2}{2k^2\sigma_A^2(1 - \rho^2)}$$

$$\propto \exp - \frac{P^2[k^2(1 - \rho^2) + (1 - \rho k)^2] - 2P[k^2(1 - \rho^2)P_A + (1 - \rho k)(P_B - \rho k P_A)]}{2k^2\sigma_A^2(1 - \rho^2)} \quad .$$

Completing the square and simplifying,

$$f(P|P_A,P_B) \propto \exp\{-(P - P_A^*)^2/2\sigma_A^{*2}\} \quad ,$$

where

$$P_A^* = \frac{k(k - \rho)P_A + (1 - \rho k)P_B}{k^2 - 2\rho k + 1}$$

and

$$\sigma_A^{*2} = \frac{k^2(1 - \rho^2)\sigma_A{}^2}{k^2 - 2\rho k + 1} \quad .$$

Hence, A's posterior distribution for P is a normal distribution with mean P_A^* and variance σ_A^{*2}.

Appendix II. A Sealed-Bid Procedure with n Traders and Independent Forecast Errors

Assume that investor i's forecast can be written in the form $P_i = P + u_i$ and that the joint distribution of $u_1,\ldots,u_n$ is normal with $E(u_i) = 0$, $V(u_i) = \sigma_i^2$, and $Cov(u_i,u_j) = 0$ for all $i \neq j$. The distribution of P after $P_1,\ldots,P_n$ are known is a normal distribution with mean $P^* = \sum_i (P_i/\sigma_i^2)/\sum_i (1/\sigma_i^2)$ and variance $\sigma^{*2} = 1/\sum_i (1/\sigma_i^2)$. No subscripts are needed on P^* and σ^{*2} because all investors have the same posterior distribution.

In the sealed-bid trading procedure, assume that trading occurs at M, the median of the n forecasts, so that an equal number of investors buy and sell. Investor j buys (sells) one unit of the security if $P_j > (<) M$. Investor j's expected return, then, is $P^* - M$ if $P_j > M$ and $M - P^*$ if $P_j < M$. This expected return can be written as follows:

$$\pi_j = \begin{cases} \sum_i (\sigma^{*2}/\sigma_i^2)P_i - M & \text{if } P_j > M \ , \\ M - \sum_i (\sigma^{*2}/\sigma_i^2)P_i & \text{if } P_j < M \ . \end{cases}$$

Letting $h_i = 1/\sigma_i^2$ represent the precision of investor i's forecast,

$$\pi_j = \begin{cases} \sum_i (h_i/\sum_k h_k)P_i - M & \text{if } P_j > M \ , \\ M - \sum_i (h_i/\sum h_k)P_i & \text{if } P_j < M \ . \end{cases}$$

To examine the effect of changes in a forecaster's precision, consider

$$\frac{\partial \pi_j}{\partial h_j} = \begin{cases} \left[\sum_i h_i(P_j - P_i)\right]/\sum_i h_i^2 & \text{if } P_j > M, \\ \left[\sum_i h_i(P_i - P_j)\right]/\sum_i h_i^2 & \text{if } P_j < M \ . \end{cases}$$

The sign of $\partial\pi_j/\partial h_j$ depends on the particular combination of forecasts and precisions. For example, if for some i, P_i is considerably higher or lower than the other forecasts and h_i is greater than the other precisions, then investor i has a strong influence on π_j and on the sign of $\partial\pi_j/\partial h_j$. However, given the joint distribution of forecast errors, $E[P_j - P_i | P_j > M] > 0$ and $E[P_i - P_j | P_j < M] > 0$ for all $i \neq j$. Therefore, before $P_1, P_2, \ldots, P_n$ are actually observed, $E[\partial\pi_j/\partial h_j] > 0$, implying that for any forecaster, greater precision (i.e. smaller forecast error variance) implies an increased expected return from trading.

Footnotes

[1]For analyses of the use of information for operating decisions of the firm, see Gonedes [12] and Alchian [1]. For analyses of the use of information to bring buyers and sellers together, see Stigler [28] and Stigler [29]. For analyses of the use of information for trading purposes, see Manne [21], Fama and Laffer [9], and Hirschleifer [41].

[2]These points are developed in both Fama and Laffer [9] and Hirschleifer [41].

[3]Fama and Laffer [9].

[4]As noted later in this section, the essential nature of our results generalizes to the case of n investors. For simplicity, we develop the model for the special case of n = 2.

[5]This assumption if useful for purposes of exposition but not necessary for our analysis. The implications of this model concerning trading are based on the ex post reaction of investors to trading successes or failures rather than on their ex ante calculation of expected returns, so knowledge of the distribution is not crucial.

[6]Trading during the period will not influence the value of P, which is determined by an exogenous stochastic process. Thus, the trading procedure can be viewed as a zero-sum game where, over time, some individuals gain by trading on the basis of new information and other individuals lose by such trading.

[7]It could also be caused by nonstationarity in the process generating forecast errors. For example, an investor may improve in terms of forecasting ability over time. Given the continuous trading procedure, the market would quickly adjust to such nonstationarity, so the cessation of trading would only be briefly interrupted. Only very pervasive and extreme nonstationarity would alter the general nature of the results presented in this section.

[8]Of course, in practice a combination of conditions such as misperceptions of forecasting ability, nonstationarity, a large number of traders, the inclusion of new traders over time, and slow response to evidence regarding trading ability may imply that no equilibrium will be reached and that trading will continue to occur.

[9]See Fama and Laffer [9], Hirschleifer [14], and Manne [21].

[10] Fama and Laffer [9, p. 293] point out that information is only of value in their model when it increases the stock price, as short selling is prohibited.

[11] Hirschleifer [14]. This paragraph only presents a rudimentary form of Hirschleifer's argument.

[12] The securities and Exchange Commission expects specialists to buy in falling markets and sell in rising markets in order to stabilize stock prices (i.e. to maintain an "orderly" market). This procedure should result in losses which are to be paid for out of the specialist's salary, which in turn is paid by the stock exchange. However, specialists do not appear to use this stabilizing strategy in general. (See Neiderhoffer and Osborne [23] for an explanation of this phenomenon.)

[13] Winkler [31] and Winkler [32].

[14] Sharpe [26], Lintner [19], and Black [4].

[15] See Fama [8] for a review of empirical work using residual analysis. Specific studies using this concept are Fama, Fisher, Jensen, and Roll [10], Ball and Brown [2], Scholes [25], Mandelker [22], Jaffe [16], and Ibbotson [15].

[16] Fama [8].

[17] See Glass [11], Rogoff [24], Lorie and Neiderhoffer [20], Jaffe [16], and Scholes [25].

[18] See Cragg and Malkiel [6], Elton and Gruber [7], and Jensen [17].

[19] See Shelton [27], Hausman [13], and Black [4].

[20] Jones and Litzenberger [18].

[21] Black and Scholes [5].

[22] See Winkler and Barry [33].

[23] Empirical evidence (see Winkler [30]) suggests that market point spreads determined by bookies are quite accurate relative to point spreads determined by sportswriters and naive bettors. Such bettors probably bet more often than the model of Section 4 implies that they should for speculative purposes.

References

[1] Alchian, Armen. "Information Costs, Pricing and Resource Unemployment," in E.S. Phelps et al., eds., Microeconomic Foundations of Employment and Inflation Theory, New York, 1970.

[2] Ball, Ray and Phillip Brown. "An Empirical Evaluation of Accounting Income Numbers," Journal of Accounting Research, 6 (1968), 159-178.

[3] Black, Fischer. "Yes, Virginia, There is Hope: Tests of the Value Line Ranking System," unpublished manuscript University of Chicago, 1971.

[4] Black, Fischer. "Capital Market Equilibrium with Restricted Borrowing," Journal of Business, 45 (July 1972), 444-455.

[5] Black, Fischer, and Myron Scholes. "The Valuation of Option Contracts and a Test of Market Efficiency," Journal of Finance, 27 (May 1972), 399-417.

[6] Cragg, John, and Burton Malkiel. "The Consensus and Accuracy of Some Predictions of the Growth of Corporate Earnings," Journal of Finance, 23 (March 1968), 67-84.

[7] Elton, Edwin J. and Martin J. Gruber. "Earnings Estimates and the Accuracy of Expectational Data," Management Science, 18 (April 1972), 409-424.

[8] Fama, Eugene F. "Efficient Capital Markets: A Review of Theory and Empirical Work," Journal of Finance, 25 (May 1970), 383-417.

[9] Fama, Eugene F. and Arthur B. Laffer. "Information and Capital Markets," Journal of Business, 44 (July 1971), 289-298.

[10] Fama, Eugene F., Lawrence Fisher, Michael Jensen, and Richard Roll. "The Adjustment of Stock Prices to New Information," International Economic Review, 10 (February 1969), 1-21.

[11] Glass, Gary A. "Extensive Insider Accumulation as an Indicator of Near Term Stock Price Performance," unpublished doctoral dissertation, Ohio State University, 1963.

[12] Gonedes, Nicholas J. "Information-Production and Capital Market Equilibrium," Chicago: University of Chigago, Center for Mathematical Studies in Business and Economics, Report No. 7256, November 1972.

[13] Hausman, Warren H. "A Note on 'The Value Line Contest: A Test of the Predictability of Stock Price Changes,'" Journal of Business, 42 (July 1969), 317-320.

[14] Hirschleifer, Jack. "The Private and Social Value of Information and the Reward to Inventive Activity," American Economic Review, 61 (September 1971), 561-574.

[15] Ibbotson, Roger. "Price Performance of Common Stock New Issues," unpublished manuscript, University of Chicago, March 1973.

[16] Jaffe, Jeffrey F. "Special Information, Security Regulation and Insider Trading," unpublished doctoral dissertation, University of Chigago, 1972.

[17] Jensen, Michael. "The Performance of Mutual Funds in the Period 1945-61," Journal of Finance, 23 (May 1968), 389-416.

[18] Jones, Charles, and Robert Litzenberger. "Quarterly Earnings Reports and Intermediate Stock Price Trends," Journal of Finance, 25 (March 1970), 143-149.

[19] Lintner, John. "The Valuation of Risky Assets and the Selection of Risky Investments in Stock Portfolios And Capital Budgets," Review of Economics and Statistics, 47 (February 1965), 13-37.

[20] Lorie, James, and Victor Neiderhoffer. "Predictive and Statistical Properties of Insider Trading," Journal of Law and Economics, 11 (1969), 35-51.

[21] Manne, Henry A. Insider Trading and the Stock Market, New York, Free Press, 1966.

[22] Mandelker, Gershon. "The Effect of Mergers on Stock Prices," unpublished manuscript, University of Chicago, May 1973.

[23] Neiderhoffer, Victor, and M.F.M. Osborne. "Market Making and Reversal on the Stock Exchange," Journal of the American Statistical Association, 61 (December 1966), 897-916.

[24] Rogoff, Donald L. "The Forecasting Properties of Insiders' Transactions," unpublished doctoral dissertation, Michigan State University, 1964.

[25] Scholes, Myron S. "A Test of the Competitive Hypothesis: The Market for New Issues and Secondary Offerings," unpublished doctoral dissertation, University of Chigago, 1969.

[26] Sharpe, William F. "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," Journal of Finance, 19 (September 1964), 425-442.

[27] Shelton, John P. "The Value Line Contest: A Test of the Predictability of Stock Price Changes," Journal of Business, 40 (July 1967), 251-269.

[28] Stigler, George. "The Economics of Information," Journal of Political Economy, 69 (June 1961), 213-225.

[29] Stigler, George. "Information in the Labor Market," Journal of Political Economy, 70 (October 1962), 94-105.

[30] Winkler, Robert L. "Probabilistic Prediction: Some Experimental Results," Journal of the American Statistical Association, 66 (December 1971), 675-685.

[31] Winkler, Robert L. "The Assessment of Probability Distributions for Future Security Prices," in J.L. Bicksler, ed., Methodology in Finance-Investments, Lexington, Mass.: Lexington Books, D.C. Heath, 1972, 129-148.

[32] Winkler, Robert L. "Bayesian Models for Forecasting Future Security Prices," Journal of Financial and Quantitative Analysis, 8 (1973), in press.

[33] Winkler, Robert L., and Christopher B. Barry. "A Bayesian Model for Portfolio Selection and Revision," unpublished manuscript, Indiana University, March 1973.