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Modeling and robust design of networks under risk: the case of information infrastructure

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Foreword

The paper analyses networks with interdependent risks or network risks, which are the key issue defining the robustness of infrastructures. Standard risk management methods mainly consider the case of a relatively simple system facing only external (exogenous) sources of risk and uncertainty.

The paper focuses on approaches for dealing with, in general endogenous network risks. In particular, it proposes a stochastic, dynamic model of attitude formation that takes account of individual interactions under uncertainty and networks governing intrinsic dynamic of attitudes and adoption patterns.

Abstract

Study of network risks allows to develop insights into the methods of building robust networks, which are also critical elements of infrastructures that are of a paramount importance for the modern society. In this paper we show how the modern quantitative modeling methodologies can be employed for analysis of network risks and for design of robust networks under uncertainty. This is done on the example of important problem arising in the process of building of the information infrastructure: provision of advanced mobile data services.

We show how portfolio theory developed in the modern finance can be used for design of robust provision network comprising of independent agents. After this the modeling frameworks of Bayesian nets and Markov fields are used for the study of several problems fundamental for the process of service adoption such as the sensitivity of networks, the direction of improvements, and the propagation of user attitudes on social networks.

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Yuri Ermoliev is the Institute Scholar and Senior Researcher with the Integrated Modeling Environment Project. Yuri's scientific interests are modeling of decision-making processes in the presence of risks and uncertainties, stochastic and dynamic systems optimization, optimization on networks, and nonlinear dynamics. He has authored over 200 publications.

His major publications include Stochastic Programming Methods (1976), Stochastic Models in Economics (1979), and Techniques for Stochastic Optimization (1988). Other publications concern the study of path-dependent adaptation processes, pollution control problems, energy and agriculture modeling, reliability theory, and optimization of discontinuous systems, in particular, discrete event systems optimization.

Alexei Gaivoronski is a Professor at Department of Industrial Economics and Technology Management, Norwegian University, Norway. His scientific interests are focused on mathematical methodologies for optimal decisions under risk and uncertainty, spanning the relevant aspects of mathematics, optimization, finance, management science, economics. The main focus of his work is modeling, analysis and optimization of uncertain and stochastic systems and, in particular, stochastic programming.

Alexei has published more than 60 papers on these topics in international journals and book chapters. He collaborates with industry (telecom, finance, energy) both nationally in Norway and internationally in Europe.

Marek Makowski is a leader of IIASA Integrated Modeling Environment Project. His research interests focus on model-based support for solving complex problems, and cover a cluster of areas relevant to the adaptation (whenever possible) or development (when needed) of methodology, algorithms, and software for model-based decision-making support. This includes more specific topics in Operations Research such as: multicriteria problem analysis, large scale optimization, optimization of badly conditioned problems, use of database management systems for complex models, decision analysis and support, user interfaces in decision support systems, effective treatment of uncertainty and risk.

Marek has published over 80 papers and book-chapters, coordinated or led several scientific projects, and has been twice guest editor of the European Journal of Operational Research.

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1 Introduction

This paper is dedicated to a study of the network risks which are the key issues defining the robustness of infrastructures. There are similarities between network risks and catastrophic risks: both have interdependencies in space and time. An appropriate analysis of these two classes of risks requires adaptation, integration, extension and further development of methodologies for quantitative modeling of uncertainty and risks that have emerged during recent decades in such fields as economics and finance, optimization, simulation of stochastic and multiagent systems. For more detailed treatment we have selected two such methodologies: portfolio theory of finance and Bayesian networks, both coupled with optimization approach. The purpose of this paper is to show how these methodological tools can be extended and applied for the study of network risks with the emphasis on information infrastructure.

Besides clarifying the methodological issues, we aim also at creation of integrated modeling decision support environment for analysis of network risks. This environment will enable identification and evaluation of critical bottlenecks inherent in important infrastructures seen as specialized networks and allow to give advice to planning and regulating bodies on robust design and improvement of these infrastructures.

More specifically, we look at the risk adapted performance networks composed of nodes and links of different levels of complexity. The risk adjusted performance of each node can be improved by selecting appropriate control parameters. In addition, performance of each node is affected by uncertainties. These network elements or nodes are designed (or behave) from the point of view of local tradeoff between local performance and risk. This risk can be exogenous to the network as well as endogenous, generated by inappropriate functioning of other network elements. What is important, the overall performance of the network is also affected by risk on the global level. This risk is understood as eventuality that the global performance can differ, sometimes drastically, from the expected network performance. The key issue in the designing of robust network is to

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assure that a local level of risk/performance tradeoff results in desirable risk/performance tradeoff on the global level. This is one of the central issues which we aim to clarify.

Here we briefly discuss three examples of infrastructure which can be described as risk/performance networks.

1. *Energy infrastructure*. Electric power infrastructure can be described as the network with several levels of hierarchy where the nodes correspond to production, distribution and consumption facilities while the arcs represent transmission lines. On the local level each node has production and consumption targets subject to uncertainty and risk manifested as equipment failures, local demand variations, local weather patterns important for hydro and wind generation. On the global level this infrastructure should meet electric power demand of consumers and industries subject to uncertainties and risk of disruptions, prices for fuel and energy, weather, societal attitudes towards certain generation technologies, climate change.

Earlier was the case when this performance/risk tradeoff was much easier to achieve. This was when the public utilities managed generation and transport in almost each country. Since then the electric power industry is deregulated or being deregulated in almost all developed countries. The industry is now composed of many independent actors which decide their production plans according to market conditions. Besides, the totally new actors have entered the field, like energy contracts traders and speculators, and energy exchanges. Yet they have to act in concert if this infrastructure is to fulfill in a robust manner the energy needs of society at large. This is a critical issue as the power shortages in California and price surges in Norway have shown.

What are robust risk management methods which will mitigate these new risks which result from market forces and individual actors' behavior? What is the robust way to assure that the local decisions on risk/performance tradeoff which every actor takes will transform into optimal or even acceptable tradeoff on the global level? What lessons developing countries can learn from the experience of developed countries in this respect? These are the questions which our paper aims to answer.

2. *Gas transport and distribution infrastructure*. Similar issues of networked risk/performance management arise in other types of infrastructure. For example, developing the gas transport and consumption infrastructure in Europe largely follows deregulation patterns of electric power infrastructure according to EU directives, and the same is true about railroad transport.

3. *Information and communication network infrastructure*. It can be described as superposition of several layers of hierarchical networks each one consisting of nodes connected with links. There are also mappings connecting different layers. The network nodes are represented by heterogeneous devices like routers, switches, crossconnects, etc. Each of these devices is equipped by control structures which govern communication flows through the network like communication protocols, routing tables, call admission rules, etc. These control parameters are tuned largely independently in order to meet performance targets of each node. Uncertainty on the node level comes from highly variable communication flows, but also from actions of adjacent nodes. There are also risks of equipment failures, congestion, malicious attacks, link failures which threaten the performance targets.

Each of the nodes is build to achieve admissible tradeoff between performance, costs and risk on the local level. The entire network, however, should satisfy various global performance targets, like a satisfaction of communication and information demand with quality of service guarantees for the user population. Besides, the operation of this infrastructure should be economically attractive for industrial actors which own its different parts. Global sources of risks and uncertainty include both external component (changing usage patterns and global malicious disturbances) and internal component (connected with conflicting interests of different actors).

How this local tradeoff between performance and risk at the node level affects the global tradeoff on the infrastructure level and vice versa? What are the economically sound principles for further robust development and operation of this infrastructure under inherent risk and uncertainty? Where are the bottlenecks which threaten its global performance? These are the questions which require the new methodology of the network risk analysis and robust risk management to which this paper aims to contribute.

Methods for taking the optimal decisions under uncertainty and related issues of risk management have been at the center of methodological development in the last couple of decades and more recently they have met also a considerable and rising industrial interest. One can mention stochastic programming which is a hot topic in operations research community now and it has become an important modeling tool in quantitative finance, energy, telecommunications and other industrial fields. Understanding of importance of risk management in finance resulted in the development of several risk management paradigms and industrial standards which are now being gradually adopted also in other industrial branches. However, these and other methods mainly consider the case of a relatively simple system under control which is facing external sources of risk and uncertainty. The real challenge is to look at the network of such systems and study the effects of risk and uncertainty on its overall performance.

In this paper we look how these general considerations about network risks are translated for the case of information infrastructure. This infrastructure consists of several components which are currently in the different stages of development. In this paper we consider development and deployment of one such component which importance increasing: advanced mobile data services. We show how related risks can be analyzed using and further extending for this case quantitative risk modeling methodologies. In particular, portfolio theory developed in finance is extended for analysis and design of service provision networks (Sections 2-6). After this Bayesian nets and Markov fields models are used in order to predict and analyze the sensitivity of networks, the directions of improvements, and the service adoption patterns, all of which depend on complex interplay of attitudes of different groups of population. The main message is that advanced methodological toolbox is necessary for analysis of network risks, here we consider and develop two components of such toolbox.

2 Cooperative provision of advanced mobile data services

Design of advanced mobile data services to be carried on 3G networks and the networks of further generations is the hot topic in telecommunication industry and academy. This is because the business success of provision of such services will define the business success of the mobile operators and other relevant industrial actors in the near to medium future. In this respect considerable attention is given to design and development of service provision platforms which support a set of tools and basic services that facilitate development, deployment and customization of specialized services by service providers and even non-professional end users.

Deployment and operation of service provision platforms and provision of individual services requires collaboration of different industrial actors who contribute to the common goal with their individual capabilities and expertise. One can think about fixed network operators, mobile operators, providers of different information content, internet providers, software developers and other actors who will join forces to provide a successful service. Provision of a service involves assuming different roles and industrial actors can combine such roles. All this gives a rich picture of service provision environment where a multitude of actors cooperate and compete in order to deliver to customers a wide range of services in a profitable manner.

Understandably, the main research and development effort so far has been concentrated on technological and engineering aspects which enable the provisioning of advanced mobile data services. The history of information technology testifies, however, that the possession of the best technological solution is not necessarily enough to assure the business success of an enterprise. Very important and sometimes neglected aspect is design and evaluation of appropriate business model which would support the service provision. Business models for provision of a service requiring a single actor are pretty well understood, both organizationally and economically. This is the case, for example, of provisioning of the traditional voice service over fixed network. When an actor evaluates the economic feasibility of entering the provision of such service, he can employ quantitative tools developed by investment science, like estimation of the Net Present Value of such project [19]. Usually an actor should choose between several service provisioning projects, each providing return on investment and generating the risky cash flows. Then the portfolio theory [21] suggests the way to balance between return and risk and select the best portfolio of projects taking into account the actor's risk attitudes. The adequate risk management is especially important in a highly volatile telecommunication environment and the industrial standards in this respect are starting to emerge, originating from the financial industry [1]. Industrial projects in high-tech industries are often characterized by considerable uncertainty and at the same time carry different flexibilities. The real options approach [25] allows to take these flexibilities into account while making evaluation of the profitability of the project. Stochastic programming [10], [3], [13], [18] provide the optimization models for adequate treatment of uncertainty in the planning of service provision.

Business models for cooperative service provision involving different constellations of actors are studied to much lesser extent. The understanding of their importance has lead to some qualitative analysis in [11], [17], but the quantitative analysis similar to what exists for the single actor case remains a challenge. The methods mentioned above are all developed to be used by a single actor engaged in the selection and risk management of his portfolio of industrial projects. The influence of other actors is present only implicitly on the stage of estimation of the future cash flows. This is not enough for adequate analysis of collaborative service provision. Suppose, for example, that a service provider delivers a service to a population of users and receives a revenue for this delivery. If a service is composed from modules and enablers provided by different actors then this service provider has to decide about the revenue division between the actors which will make it attractive to them to participate in the service provision. This revenue sharing decision together with a concept of what is attractive to other actors he should explicitly incorporate in the evaluation of profitability of this project.

Here we contribute to the adaptation and further development of the methods of evaluation and risk management of business models and industrial projects for the case of the collaborative service provision. We look at the actors engaging in a service provision as making a decision about the composition of their portfolio of services to which they are going to contribute. They do this independently following the risk management framework of portfolio theory. The pricing and revenue sharing schemes induce the actors to contribute the right amount of provision capacity to participation in the service provision. We develop a two tier modeling framework which results in selection of pricing and revenue sharing in the optimal way. This is done by utilizing the approach of stochastic optimization with bilevel structure [2], combining it with portfolio theory.

3 Simplified model of the service portfolio

In this Section we are going to develop a quantitative description of the service provisioning model involving several actors having as the background the environment presented in the previous Section.

3.1 Description of services

The composition of a service can be quite complex, especially if we take into account that various components of that picture can be services themselves and subject to further disaggregation. For the purposes of clarity we are going to start from a simpler description which still possess the main features of the provision environment important for business modeling. Namely, two levels of the service composition will be considered here as shown in example in Figure 1.

In this case the service environment is composed from two types of services. The first type is comprised from services with structure and provision we are interested in and which we are going to consider in some detail. They can be provided in the context of a service platform and therefore they will be referred to as *platform* services. There will be also *3rd party* services whose structure is of no concern to our modeling purposes. They are present in the model for the purposes of the adequate modeling of the environment in which the provisioning of the platform services happens. Let us now consider the model of provisioning of platform services.

The main building blocks of the platform services are service *enablers* indexed by $i = 1 : N$ and *services* indexed by $j = 1 : M$. Enablers are measured in units relevant for their description, like bandwidth, content volume, etc. The relation between enablers and services is described by coefficients λ_{ij} which measure the amount of enabler i necessary for provision of the unit amount of service j. Thus, a service j can be described by vector

$$\lambda_j = (\lambda_{1j}, \ldots, \lambda_{Nj}) \tag{1}$$

This description is obtained from analysis of the usage scenarios described in the Section 2. A service j generates a revenue v_j per unit of service. This quantity depends on the service pricing which in its turn depends on the user behavior and market structure. For the moment let us assume that v_j is the random variable with known distribution, later in the Section 4.3 we shall describe this revenue in more detail. This distribution can be recovered from the expert estimates and from simulation models which would explore the structure of user preferences and market features. The random variables v_j can be correlated due to the service substitution, macroeconomic phenomena and other causes.

Services can be provided by different constellations of actors. In this paper we consider one such constellation where the actors are the enterprises which have the capability to provide service enablers assuming different *roles*, they are indexed by $k = 1 : K$. Actors may choose to join forces to provide a service. Contribution of a given actor consists of taking responsibility for provision of one or more enablers of the service. Sometimes these actors will be referred to as *enabler providers*. There will be an actor who provides the service aggregation functionality and organizes the overall service delivery to the end users, this actor will be referred to as a *service provider*. This actor can provide the whole bundle of platform services and he will decide which services to include in this bundle. Often he will collect the revenue from the end users and distribute it among the enabler providers.

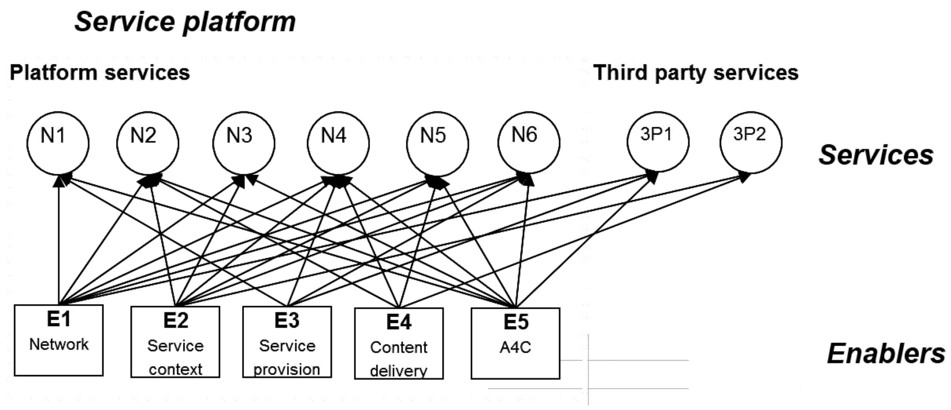

Figure 1: Service provision for business person on the move.

Example 1 Service provision for business person on the move. (see Figure 1).

This is a simplified yet realistic example of service provision which was developed on the basis of the project results of the EU project SPICE and NFR project ISIS. The addressed terminal here is a smart mobile phone used by a business person on the move.

We consider here the services which run on the service platform and third party services which partially compete with them, being accessible from the same terminal. We *have simplified this example to just total of six native services, two third party services and five enablers (from hundreds of services and dozens of enablers, distributed in several service platforms) available in this service platform. However, services in this platform correspond well to the business offer of a typical service provider. Service bundles have been defined in accordance to the market segments, corresponding customer classes, user behavior, requirements and various subscription schemes. More specifically, we consider the following services.*

Native services of the platform:

1. N1 - Messaging;

2. N2 - Audio conferencing;

3. N3 - Video conferencing;

4. N4 - Location based services.

5. N5 - News.

6. N6 - Point of Interest service.

Third party services.

1. 3P1 - Third party Information service

2. 3P2 - Third party News service

The following business actors collaborate in providing the mobile service bundle to the users

1. E1 - Network provider – *providing the network access.*

2. E2 - Context provider – *service context retrieval and management.*

3. E3 - Service provider – *responsible for service provision.*

4. E4 - Content provider – *content retrieval and management.*

5. E5 - Provider of A4C *(authentication, authorization, auditing, accounting and charging) enabler. This actor will often coincide with the service provider, but one can envisage also the cases when it will be a distinct actor.*

Besides, there are one or more providers of the third party services which are in partial competition with the platform services.

This example will be treated in some detail in Section 6.

The objective of an enabler provider is to select a portfolio of services to which this actor will make a contribution. This decision is made on the grounds of balance between projected profit from enabler provision balanced against the risk of variations in demand and service acceptance among the prospective users of services. In order to quantify this decision process it is necessary to use a simplified profit model for an actor.

It is assumed that the revenue v_j generated by a unit of service j is distributed among the actors who participate in the creation of service. There can be different schemes for such subdivision. It is assumed here that this distribution is performed using a vector of revenue shares

$$\gamma_j = (\gamma_{1j}, .., \gamma_{Nj}) \, , \; \gamma = (\gamma_{11}, .., \gamma_{N1}, ..., \gamma_{1M}, .., \gamma_{NM})$$

such that an actor which contributes with the enabler i receives the revenue $\gamma_{ij} v_j$. Determination of these revenue sharing coefficients is one of the objectives of the design of the business model for service provision.

Besides platform services the actors can supply enablers also to the 3rd party services. The structure of these services is not specified and it is assumed that they are fully described by the revenue v_{ij} generated by provision of the unit of enabler i to 3rd party service j, $j = M+1, ..., \bar{M}$.

3.2 Profit model of an actor

Let us consider the situation when all the actors have already developed the capacities for provision of enablers. Thus, for the time being the investment process necessary for creation and expansion of these capacities is not the part of our model, however, it will be considered at the later stages. For this reason at this stage it is enough to consider only variable costs due to the operation of capacities and provision of enablers. Alternatively, one can assume that the cost structure includes both the operational and discounted portion of the investment costs for enabler development, recalculated down to the enabler and the service instances.

For further formulation of the actor's profit model let us introduce the following notations.

c_{ik} - unit provision costs for enabler i by actor k;

W_{ik} - provision capability of enabler i of actor k;

x_{ijk} - the portion of provision capability for enabler i of actor k dedicated to participation in provision of service j.

Now the revenue of actor k obtained from contribution to provision of the platform service j can be expressed as follows. The quantity $x_{ijk}W_{ik}$ will be the volume of provision of enabler i dedicated by actor k to service j. Assuming that the required quantity of other enablers is available, this will result in the volume of service j in which the actor k participates to be $x_{ijk}W_{ik}/\lambda_{ij}$. The total revenue from this service will be $v_j x_{ijk}W_{ki}/\lambda_{ij}$ and the part of the revenue which goes to actor k will be $v_j x_{ijk}W_{ik}\gamma_{ij}/\lambda_{ij}$.

For the 3rd party service the revenue will be $v_{ij}x_{ijk}W_{ik}$.

The total costs incurred by actor k for the provision of enabler i to service j will be $x_{ijk}c_{ik}W_{ik}$.

In order to simplify the following discussion let us assume now that the actor k participates in the provision of service j by contributing only one enabler $i = i(k, j)$ or assuming only one role. Taking the profit π_k to be the difference between the revenue and costs, the profit of the actor k can be expressed as follows:

$$\begin{aligned}\pi_k &= \sum_{j=1}^{M}\left(v_j x_{ijk}W_{ik}\frac{\gamma_{ij}}{\lambda_{ij}} - x_{ijk}c_{ik}W_{ik}\right) + \sum_{j=M+1}^{\bar{M}}\left(v_{ij}x_{ijk}W_{ik} - x_{ijk}c_{ik}W_{ik}\right)\\ &= \sum_{j=1}^{M} x_{ijk}W_{ik}c_{ik}\left(\frac{v_j\gamma_{ij}}{c_{ik}\lambda_{ij}} - 1\right) + \sum_{j=M+1}^{\bar{M}} x_{ijk}W_{ik}c_{ik}\left(\frac{v_{ij}}{c_{ik}} - 1\right)\end{aligned}$$

In the expression above index i depends on the values of indices j and k. Now let us assume that the actor k assumes only one role which consists in the provision of enabler i to different services which require this enabler. Thus, we consider a generic actor whose role is to provide enabler i to different services. Then we can simplify notations by taking $x_{ijk} = x_{ij}$, $W_{ik} = W_i$, $c_{ik} = c_i$, $\pi_k = \pi_i$. In this case the profit will be

$$\pi_i = W_i c_i\left(\sum_{j=1}^{M} x_{ij}\left(\frac{v_j\gamma_{ij}}{c_i\lambda_{ij}} - 1\right) + \sum_{j=M+1}^{\bar{M}} x_{ij}\left(\frac{v_{ij}}{c_i} - 1\right)\right)$$

Dividing the profit by the total costs $W_i c_i$ we obtain the return r_i on investment by a generic actor which assumes the role of provision of enabler i to services which require this enabler.

$$r_i = \sum_{j=1}^{M} x_{ij} \left(\frac{v_j \gamma_{ij}}{c_i \lambda_{ij}} - 1 \right) + \sum_{j=M+1}^{\bar{M}} x_{ij} \left(\frac{v_{ij}}{c_i} - 1 \right) \tag{2}$$

3.3 Service portfolio: financial perspective

The profit representation (2) allows us to look at the enabler provision from the point of view of financial portfolio theory [21]. The actor with the role to provide the enabler i has to choose the set of services to which provide this enabler from all the possible available services requiring this enabler. In other words, he has to select his service portfolio. This portfolio is defined by shares x_{ij} of his provision capability,

$$x_i = (x_{i1}, ..., x_{i\bar{M}})$$

Return coefficients associated with his participation in each platform service are expressed as

$$r_{ij} = \frac{v_j \gamma_{ij}}{c_i \lambda_{ij}} - 1, \; j = 1 : M \tag{3}$$

and for the 3rd party services these coefficients are

$$r_{ij} = \frac{v_{ij}}{c_i} - 1, \; j = M + 1 : \bar{M}. \tag{4}$$

These coefficients depend on the random variables which are mostly the revenue per unit of service v_j and the revenue per component provision v_{ij}. Randomness here is due to the uncertainty in demand and the user acceptance of service. However, both enabler provision costs c_i and even enabler shares λ_{ij} also will be random variables due to uncertainty inherent in the service usage patterns and the evolution of costs. Besides, the costs c_i often will be the estimates of the provision costs of enabler provider i made by some other actor. Such estimates are inherently imprecise and are better described by random variables similarly to how it was done in [2]. The expected return coefficients are

$$\mu_{ij} = \gamma_{ij} \mathbb{E} \frac{v_j}{c_i \lambda_{ij}} - 1, \; j = 1 : M, \; \mu_{ij} = \mathbb{E} \frac{v_{ij}}{c_i} - 1, \; j = M + 1 : \bar{M} \tag{5}$$

and expected return $\bar{r}_i(x_i)$ of service portfolio is

$$\bar{r}_i(x_i) = \sum_{j=1}^{\bar{M}} \mu_{ij} x_{ij} = \sum_{j=1}^{M} x_{ij} \left(\gamma_{ij} \mathbb{E} \frac{v_j}{c_i \lambda_{ij}} - 1 \right) + \sum_{j=M+1}^{\bar{M}} x_{ij} \left(\mathbb{E} \frac{v_{ij}}{c_i} - 1 \right) \tag{6}$$

However, the realized return can differ substantially from the expected return due to uncertainty discussed above. This introduces the risk $R(x_i)$ for an actor which assumes the enabler provision role. Financial theory traditionally measures this risk as the variance of portfolio return [21]. Recently several different risk measures were introduced and, in particular Value at Risk (VaR) and its many modifications. The VaR has attained the level of industrial standard in the financial risk management [1]. In this section the variance and the standard deviation of the return will be used as the risk measure because the correct selection of the risk measure in the context of collaborative service provision is outside the scope of this paper and it will be addressed by us in the subsequent papers. What is important here is that the consideration of the risk measures allow an actor to estimate the probability and size of his future losses. Thus, we take

$$R(x_i) = \mathrm{StDev}(r_i(x_i)) = \mathrm{StDev}\left(\sum_{j=1}^{\bar{M}} r_{ij} x_{ij}\right) \tag{7}$$

where return coefficients r_{ij} are taken from (3),(4).

Portfolio theory looks at the portfolio selection as the trade-off between risk and return. Its application to our problem of service portfolio consists of the following steps.

1. *Construction of efficient frontier.* Some average return target η is fixed. The risk of service portfolio is minimized with constraint on this return target. The risk minimization problem looks as follows.

$$\min_x \ \mathrm{StDev}^2\left(\sum_{j=1}^{\bar{M}} r_{ij} x_{ij}\right) \tag{8}$$

$$\sum_{j=1}^{\bar{M}} \mu_{ij} x_{ij} = \eta \tag{9}$$

$$\sum_{j=1}^{\bar{M}} x_{ij} = 1, \ x_{ij} \geq 0 \tag{10}$$

Solution of this problem for all admissible values of target return η will provide the set of service portfolios which are the reasonable candidates for selection by actor who provides the enabler j. They constitute the *efficient frontier* of the set of all possible service portfolios. This concept is illustrated in Figure 2.

Each service portfolio x can be characterized by pair (risk,return) defined by (7) and (6) respectively. Therefore it can be represented as a point in the risk-return space depicted in Figure 2. The set of such points for all possible portfolios describes all existing relations between risk and return and is called *the feasible set*. Which of possible service portfolios an actor should choose? It depends on the objectives which an actor pursues. Here we assume that an actor's decision depends on return and risk only. Namely, an actor will seek the highest possible return among equally risky alternatives and he will seek the lowest possible risk among equally profitable alternatives. This is a simplification because in reality the actors can be driven by other considerations, like increase of market share, revenue, regulatory constraints, etc. However, the consideration of only risk and return provides with the reasonable starting point for analysis of business models. More complex cases can be taken into account in a similar manner by introducing additional constraints on the feasible set or by modifying the concept of performance. For example, suppose that an actor has three objectives: return and market share to maximize and the risk to minimize. Then the market share and the return can be integrated in one performance measure by assigning weights to these objectives. The weights will measure the relative importance of return and market share to the actor. The composite performance measure is obtained by computing the weighted sum of the original objectives. The risk is defined as the variation of this composite measure. This composite performance measure is used in Figure 2 instead of return.

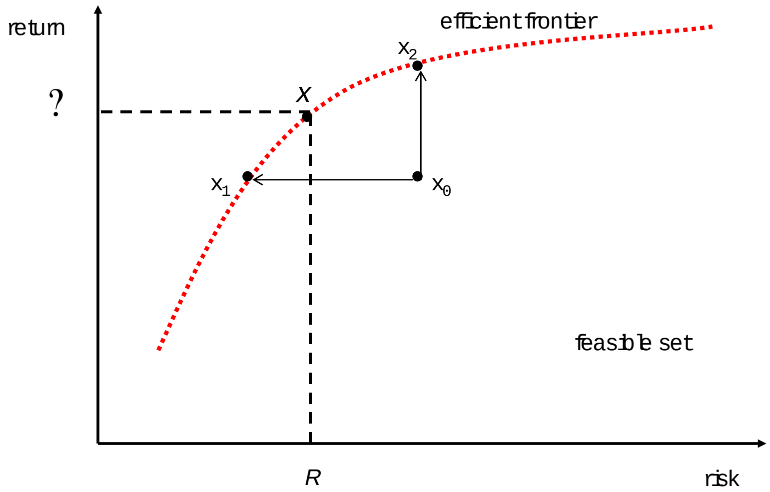

Figure 2: Selection of service portfolio

Considering the Figure 2 it becomes clear that some of the service portfolios should be preferred to others. For example, let us consider portfolio x_0 to which corresponds the point in risk-return space inside the feasible set, as in Figure 2. It is clear that portfolio x_2 should be preferred to x_0 by an agent who makes his decision on the basis of return and risk. This is because portfolio x_2 has the same risk as portfolio x_0 and larger return. Similarly, portfolio x_1 also should be preferred to x_0 because it provides the same return with the smaller risk. Thus, portfolio x_0 is dominated by both portfolios x_1 and x_2 and should not be taken in consideration. The actor whose decisions are guided by risk and return should consider only nondominated portfolios which constitute *efficient frontier*, depicted by dotted curve in Figure 2. This efficient frontier can be computed by solving the problem (8)-(10) for different values of η. This efficient frontier can be viewed also as the solution of the two objective optimization problem of minimizing risk and maximizing return under the constraints (10).

The above outlined approach to the multicriteria analysis is suitable for problems having a small number of independent criteria that have compensatory character, i.e., for which it is possible to define a composite performance measure using weights, and to compute the *efficient frontier* by a parametric optimization approach. However, the weight-bases approaches have several limitations discussed e.g., in [20]. Therefore one should consider a truly multicriteria analysis approach, see e.g., [26], and a modular tool (such as ISAAP described in [16]) that supports interactive analysis of trade-offs between conflicting objectives.

2. *Selection of the target service portfolio*. The previous step resulted in the selection of much smaller set of efficient service portfolios from the set of all possible service portfolios. These portfolios form the efficient frontier in the risk-return space. An actor selects his target service portfolio from this efficient set by choosing the trade-off between risk and return. One way to achieve this trade-off is to consider the largest risk an actor is willing to take. Suppose that the value of such risk is R (see Figure 2). Then the actor should choose the portfolio x on efficient frontier with this value of risk. Suppose that this service portfolio yields return η. No other portfolio yields better return without increasing the risk. If an actor is not satisfied with return η this means that he should increase his risk tolerance or look for opportunities to participate in the service provision not yet described in this model. Or, such actor should seek more advantageous revenue sharing scheme.

From these considerations it is clear that all important opportunities of participation in service provision should be included in this model. For example, suppose that an actor assumes the role of content provision and can contribute his content to advanced mobile data service and at the same time this content can be contributed to, say, traditional newspaper. Both opportunities should be included in the model with the traditional service being modeled as a 3rd party service.

4 Modeling of collaborative service provision

In the previous Section we highlighted the importance of having the adequate forecasts of the cash flows generated by services in order to quantitatively evaluate the economic future of the service and business models which support the service provision. Due to uncertainty inherent in the user response and technological development any such forecast should be given in terms of random variables which assign probabilities to different scenarios of user response and possible evolution of other uncertain parameters. The forecasts should take into account the mutual influence of services which result in correlation between cash flows generated by different services.

Such description allows to look at the providers of different service enablers as actors which independently select the service portfolios having their targets described in terms of return on investment and risk tolerance. However, a service can become a reality only if the participation in its provision will be consistent with these individual targets. This means that all actors which cover the roles indispensable for provision of a particular service should have this service in their efficient service portfolio. In other words, the service portfolios of the relevant actors should be *compatible*. There are several items which affect the risk/return characteristics of a service portfolio and decide whether a particular service will be present in it. One is the cash flow generated by a service j, another is the revenue sharing scheme γ_j. Besides, the enabler provision capacities, industrial risk/return standards, market prices, all play a role in making service portfolios compatible. In this Section we are going to characterize the properties which facilitate the service portfolio compatibility and develop a model for selection of the revenue sharing scheme.

4.1 Service provision capacities

According to (1) a platform service j is described by vector λ_j of the service enablers. Let us denote by I_j the set of enablers which are present in the service description in nonzero quantities:

$$I_j = \{i : \lambda_{ij} > 0\}$$

For each enabler $i \in I_j$ an actor should be found who is willing to take a role of provision of this enabler. This means that the position j in service portfolio of generic actor who provides enabler $i \in I_j$ should be nonnegative: $x_{ij} > 0$. The value of this position allows to estimate the enabler provision capacity which an actor should possess. Indeed, x_{ij} is a fraction of provision capacity which an actor is going to dedicate to provision of enabler i to service j. Therefore λ_{ij}/x_{ij} is the capacity necessary to provide a unit of service j. Suppose that $B_j^{\min}$ is the minimal volume of provision of service j which makes such provision viable, and B_j is the target volume of service provision for a generic constellation of actors which is going to provide this service. Then we have the following constraints on the service provision capacities of actors:

$$W_i x_{ij} \geq \lambda_{ij} B_j^{\min},\ i \in I_j \tag{11}$$

if the provision of service j will be viable at all and

$$W_i x_{ij} \geq \lambda_{ij} B_j,\ i \in I_j \tag{12}$$

if only one actor with provision capability of enabler i is desirable in the constellation which provides service j. These constraints can help to make decisions about the nature of the actors which should be encouraged to participate in the provision of different services. For example, some enablers of some services will be provided by established actors with large provision capacity. In such cases the share x_{ij} of capacity dedicated to service j can be small. In other cases the service enablers will be provided by startups with relatively small capacity. In such cases the share x_{ij} should be large or even equal to 1. These shares implicitly depend on the revenue sharing scheme γ_j through the solution of problem (8)-(10) and in the latter case it may be beneficial on the initial stages of service penetration to encourage startups by appropriate adjustment of the revenue sharing scheme.

The constraints (11)-(12) can be also looked at as the constraints on the composition of service portfolio. Suppose that $W_i^{\max}$ is the maximal desirable component provision capacity which an actor providing enabler i should possess. Then the smallest share x_{ij} dedicated to service j should be

$$x_{ij} \geq \lambda_{ij} \frac{B_j^{\min}}{W_i^{\max}},\ i \in I_j \tag{13}$$

4.2 Risk/return industrial expectations

Provision of advanced mobile data services will involve different actors coming from different backgrounds and industries. There will be many startups, but there will be also established actors from other industrial branches. One example is the content provision where the same content can be provided to newspapers, internet and mobile terminals.

Such actors will have the attitudes towards admissible and/or desirable returns on investment and rewards which taking risk should bring. Often such attitudes will be influenced by industrial standards and expectations inherited from their previous activity. One way to express these expectations is to include all generic projects, in which an actor can be involved in his traditional business, as services in the set of all considered 3rd party services in this model. This is especially useful approach if the revenues from the traditional activities will influence and will be influenced by the revenues from the mobile services under consideration. Another possibility is to account for these expectations explicitly. This can be done by introducing the connection between the expected return $\bar{r}_i(x_i)$ and risk $R(x_i)$ from (6),(7) as follows:

$$\bar{r}_i(x_i) \geq a_i + b_i R(x_i) \tag{14}$$

where a_i will be the return on investment associated with traditional activity while $b_i R(x_i)$ will be the risk premium associated with the participation in provision of advanced mobile data services. The coefficients a_i and b_i will depend also on individual characteristics of an actor like size, market position.

Beside this an actor will have the risk tolerance expressed in terms of the upper bound on risk which he is willing to take irrespective of return:

$$R(x_i) \leq \bar{R} \tag{15}$$

The upper bound on admissible risk $\bar{R}$ will again depend on the characteristics of a particular actor. To put it simply, this is the maximal loss an actor can afford during the time period under consideration.

4.3 Pricing

The revenue per unit of service v_j together with the service composition λ_j and the revenue sharing scheme γ_j defines the unit price p_i of enabler i :

$$p_i = \frac{v_j \gamma_{ij}}{\lambda_{ij}}$$

This is a random variable since the revenue is also random. Therefore the expected price $\bar{p}_i = \mathbb{E} p_i$ will be

$$\bar{p}_i = \frac{\gamma_{ij}}{\lambda_{ij}} \mathbb{E} v_j$$

An actor providing the enabler i may have the target p_i^* for the price of his product and the tolerances Δ^+ and Δ^- within which he is willing to accept a different price. These targets can result from the market prices in established industries, internal market studies, internal cost estimates. This will lead to the following constraints

$$p_i^* - \Delta^- \leq \frac{\gamma_{ij}}{\lambda_{ij}} \mathbb{E} v_j \leq p_i^* + \Delta^+ \tag{16}$$

This constraint should be taken into account while considering the revenue sharing schemes.

4.4 Revenue sharing schemes

Now let us look at the problem of selecting the revenue sharing coefficients γ_j which would be compatible with the concerted provision of a platform service. Summarizing the discussion present in Sections 3, 4.1 and 4.2 we obtain that the actor which supplies enabler i will select portfolio of services $x_i = (x_{i1}, ..., x_{i\bar{M}})$ by solving the following problem

$$\max_{x_i} \bar{r}_i(x_i, \gamma_j) \tag{17}$$

subject to constraints

$$\sum_{j=1}^{\bar{M}} x_{ij} = 1, \; x_{ij} \geq 0 \tag{18}$$

$$\bar{r}_i(x_i, \gamma_j) \geq a_i + b_i R(x_i, \gamma_j) \tag{19}$$

$$R(x_i, \gamma_j) \leq \bar{R} \tag{20}$$

where $\bar{r}_i(x_i, \gamma_j)$ is the expected return of the actor on his expenditure and $R(x_i, \gamma_j)$ is the risk defined in (6) and (7) respectively. We emphasize here the dependence of risk and return on the revenue sharing scheme γ_j. Solution of this problem will give service portfolios $x_i(\gamma_j)$ for all generic actors providing enabler i for the platform services $j = 1 : M$. These service portfolios will depend on the revenue sharing schemes γ_j. Let us now concentrate on a particular service with index j. In order that a provision of this service becomes possible it is necessary that all actors which provide the necessary enablers to this service will include it in their service portfolios in desirable proportions. This means that

$$x_i(\gamma_j) \in X_j \text{ for all } i \in I_j \tag{21}$$

where the set X_j can be defined, for example, by constraints (13). Constraints (21) define the feasible set of the revenue sharing coefficients and if these constraints are not satisfied then the service will not come into being.

Suppose now that the enabler number 1 of service j is a service aggregation enabler which is provided by an actor which bears overall responsibility for the functioning of service and receives the revenue stream from the end users. His responsibility includes also the division of the revenue stream between the participating actors and the selection for this purpose of the revenue sharing coefficients γ_j. He should select these coefficients in such a way that the constraints (21) are satisfied. Between all such revenue sharing coefficients he would select ones which would maximize his return. This can be formulated as the following optimization problem.

$$\max_{\gamma_j} \bar{r}_1(x_1(\gamma_j), \gamma_j) \tag{22}$$

subject to constraints

$$x_i(\gamma_j) \in X_j \text{ for all } i \in I_j \tag{23}$$

$$\gamma_j \in \Gamma_j \tag{24}$$

where the set Γ_j can be defined, for example, by constraints (16). Even simpler, this actor may wish to maximize his revenue share

$$\max_{\gamma_j} \gamma_{1j} \tag{25}$$

under constraints (23)-(24). Observe that the feasible set of this optimization problem depends on the solution of the other actor's optimization problems (17)-(20) similar to how it depends in optimization problems with bilevel structure.

5 The properties of models and implementation issues

In Section 4.4 we have presented two models for strategic assessment of collaborative provision of mobile data services. These models possess quite complicated structure, although we have made a few simplifying assumptions during their development. They can be looked at as a special type of stochastic optimization problems with bilevel structure [2], where the lower level is composed from the problems of individual component providers (17)-(20) while the upper level contains the problem of service provider (22)-(24) . Stochasticity comes from uncertainty inherent in the information about the characteristics of advanced data services and the user response to them. So far we have adopted a relatively simple treatment of uncertainty substituting the random variables by their expected values in some cases, while in the other cases the special structure of the problem allowed to limit the modeling to the expected values and covariance matrix. This can be viewed as a special type of the deterministic equivalent of stochastic programming problems, a technique widely used in stochastic programming (see [3] for more discussion on different types of deterministic equivalents). More detailed description of this uncertainty can be introduced in these models similar to how it was done in [2]. Different bilevel optimization problems have drawn considerable attention recently, see [5], [6], [22], [24].

Such problems provide quite a challenge to current numerical optimization procedures. While many theoretical issues are understood reasonably well, the solution techniques have not yet reached the off-shelf commercial codes like CPLEX available for linear and some nonlinear programming problems. The main challenge here is that the upper level problems can be highly nonlinear and nonconvex with multiple local minima. Therefore substantial implementation work is needed which would exploit the structure of the problems. Still, our aim here is to create a set of decision support tools for evaluation of business models, where the computational complexities should be hidden from the end user. We have found that this aim can be achieved by combination of customized implementation with the use of general purpose mathematical modeling systems and commercial software. The general architecture of the system under development is shown in Figure 3.

The system is composed of four components: data and user interface, a set of service models, a set of mathematical models and a library of solvers.

Data and user interface is implemented in Excel due to its familiarity to potential users. Its purpose is to provide an easy tool for storing and changing the data describing the service and customer properties, for presentation of results of business modeling

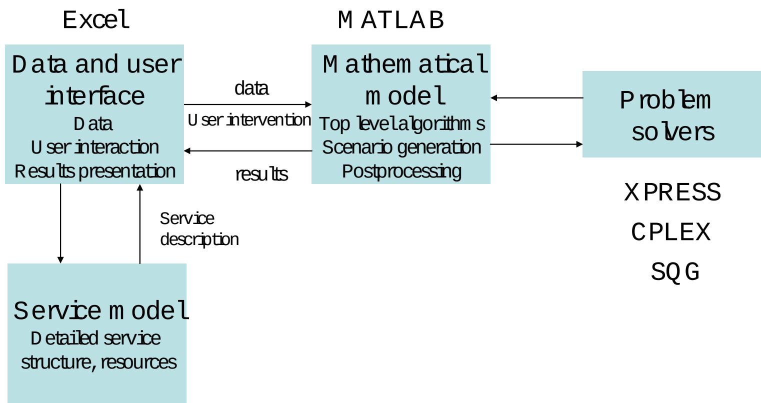

Figure 3: Architecture of decision support system for evaluation of business models of service provision

and for providing the capability to the system user to ask what-if questions pertaining to different scenarios. For example, the efficient frontier from Figure 2 is presented to user through this component.

Service model provides the capability to perform detailed modeling of advanced data services. It is implemented in specialized modeling language which has the necessary features for describing communication sequences. This model provides the aggregated description of services composition λ_j from (1).

Mathematical model implements the quantitative description of the business decision process of collaborative service provision from previous Sections. It imports data from data interface and implements the top level structures and algorithms necessary for representation and solution of models (17)-(20),(22)-(24). The custom algorithms for analysis and solution of these models are also implemented in MATLAB. This component is also responsible for calling external software for solving subproblems where standard approaches and commercial software are available. For example (8)-(10) is a quadratic programming problem which can be solved by many solvers, among them by CPLEX and MATLAB optimization toolbox.

Library of solvers contains solvers for linear and nonlinear programming problems and some specialized solvers for stochastic programming problems like SQG [12].

The system depicted in Figure 3 is now in advanced stages of development, in particular the service model component and some mathematical models of service provisioning were implemented in Matlab [2]. The next Section describes some of the results of one case study performed using this system.

6 Case study

This case study deals with the analysis of the service provider centric business model for provision of the platform bundle of services to a business person on the move who uses his smart mobile phone to access this service offer. The setting of this case study is described in Example 1 introduced in Section 3. The Edition 1 of the prototype of the decision support system implementing models from this paper was used for the analysis of this case study. This edition includes the models described in Sections 3 and 4.

Considerable data preparation effort was made for this case study. First of all, we have developed the service composition matrix, showing which enabler participates in which services. This relation between different enablers and services is shown in Figure 1. We have obtained also an average estimate of the service usage (in service instances) in a period of interest, and prices per service instance. These data we have estimated by averaging various service composition and business scenarios. On the basis of technical and economic analysis we have obtained the cost estimates and the correlation matrix showing the correlation between the usage of services and the variance of service usage.

Suppose that the service provider is using our DSS for performing the feasibility study for provision of this bundle of business services similarly to the discussion at the end of Section 3.3. There are many different what-if questions of interest to the prospective service provider to which this DSS can provide the answers. Let us provide an example of this analysis. Suppose that the service provider feels that the success of the whole enterprise depends critically on the quality and offer of specialized content which can be obtained for his services by engaging prospective content providers (enabler E4 from Figure 1). He wants to get insight into the properties of the content providers which may be interested in collaboration with him and in the chances that his service offer in this respect will stand against the competition of the 3rd party services. One way to do this is to look how the service portfolios and risk/profit preferences of prospective partners will depend on correlation and relative pricing of his offer against the offer of competition. Figures 4 and 5 provide examples of answers which our modeling system can deliver.

Figures 4 and 5 show how the characteristics and attitudes of the content providers towards the service platform depend on the alternatives which the competition can offer to them. Figure 4 shows risk/profit efficient frontiers similar to frontier presented in Figure 2 while Figure 5 depicts the percentage share of the content provision capability of the content providers dedicated to the service platform. In other words, the Figure 5 shows the market share of the service platform in the market for this specific type of content provision dependent on the risk tolerance of the content providers. The competing offer is described by the average price per unit of content and by how the actual price can differ from the average price dependent on the future market conditions, as measured by the price variance.

The figures present three scenarios. In all three scenarios the competition tries to undercut the service platform by offering about 15% higher average price to content providers for their services. The three scenarios differ by how strong the competition is, that is by its capability to maintain the price consistently higher under the changing market conditions. In scenario 1 shown by the thick solid lines the competition is strong and has its price variance about two times smaller compared with the platform offer. In scenario 2 depicted by the thick dashed lines the competition is about as strong as the platform offer and has the similar price variance. In scenario 3 shown with thin solid lines

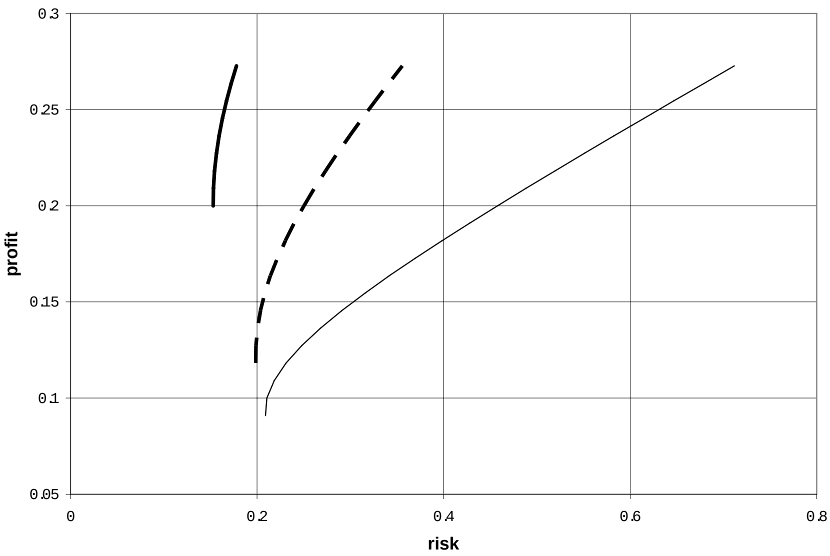

Figure 4: Dependence of risk/return preferences of content providers on the strength of competition

the competition is weaker than the platform and has about twice higher variability of its offer to content providers than the platform.

The results show that in scenario 1 with strong competition only economically weak content providers with small tolerance towards losses will be interested in the collaboration with the platform. Often this will correspond to small firms or even individuals who can not sustain large losses. For such entities participation in the platform means additional security and insurance against losses in the case when the strong competing offer will prove to be deceitful in reality. Even then, the interest of such firms drops sharply when their risk tolerances grow even by a small amount.

Scenario 3 corresponds to the opposite case when the platform faces aggressive but economically relatively weak competition. Its weakness manifests itself in large variability of its price offer to the content providers despite the 15% higher average price. In this case the market share of the platform services is much higher and the platform manages to attract also strong actors with higher capacities to sustain losses. Also the market share drops slower with the increase of the loss tolerance of the agents. Scenario 2 corresponds to the intermediate case when the competition is about as strong as the platform and has about the same capability to maintain its price offer to the service providers.

Similar patterns arise when the variability of the revenue steam of content providers is due not to the changes in the unit price of content but due to the variability of usage frequencies of this content. Having these predictions, the platform service provider can now realistically weight his own strength and weaknesses, invest more effort into market research and decide under which market conditions, with what kind of partners and with what kind of competition he can successfully operate the platform.

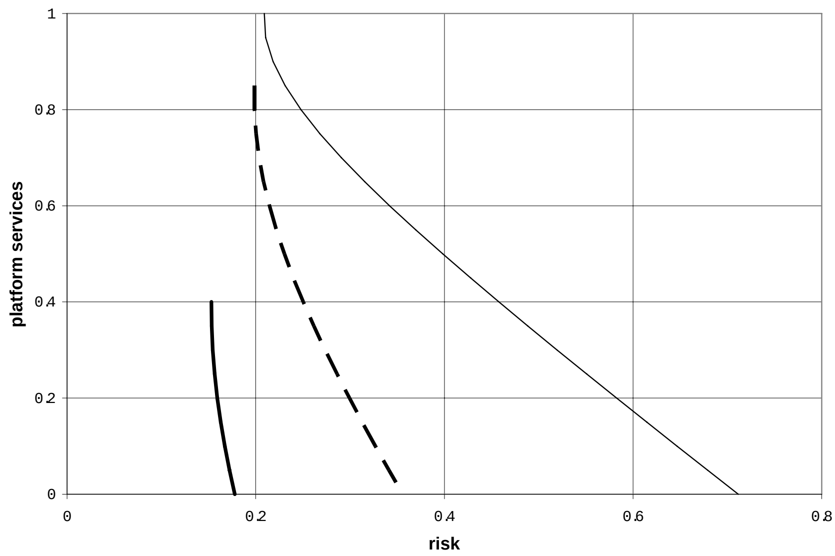

Figure 5: Share of platform services in the service portfolios of content providers

7 Modeling issues

In the previous Sections we have analyzed the network risks which arise in the process of collaborative provision of advanced mobile data services which will form ever more important part of information infrastructure. In this Section we look at another aspect of the same situation which is the process of service adoption by a heterogeneous population of users. The process of service adoption is of fundamental importance to the successful development of information infrastructure because by its very nature the value of this infrastructure for a given user grows with the amount of users already covered by its different components. Adopting the language of microeconomics one can say that the elements of information infrastructure exhibit strong externalities. Modeling of these externalities and related risks requires tools and approaches for quantitative modeling of attitudes. In this and subsequent Sections we look at one such possible methodologies based on Bayesian nets.

We formulate a stochastic, dynamic model of attitude formation that takes special account of individual interactions and networks governing intrinsic dynamics of attitudes. The model accounts also explicitly for various external factors such as new information, stimulus, events, actions or some sort of social pressure. If different sets of external factors are activated at different times, the system may show more or less complex dynamics, in particular, it may lead to different alternative attractors. This we distinguish between two types of influences: (1) the influence of the attitudes of others, and (2) the influence of information about external factors.

According to our model different individuals may receive different information. This information with subjective judgments is transformed through chains of communications to other individuals. Attitudes change in a probabilistic manner depending on the attitudes of others and information about the external factors. Individuals are socially linked to others by relationships mediated through a series of intertwining interactions and resulting in a highly diverse social network. This complexity can be modeled by the recent notion of Bayesian networks or more general notion of Markov fields. This notion is natural generalization of Markov chains to dynamic and spatial processes, whose domains are not necessarily linearly ordered.

7.1 Simplified model: direct and indirect interdependencies

In order to demonstrate the dynamics of attitude change, we can begin with representing the public in groups with similar attitudes. Most empirical attitude research structures the sampled population into cohorts, possibly by age, sex, income, profession, ethnicity, profession, geographic location, political party affiliation, etc. We assume that population may be divided into ”similar-attitude” groups such that an individual has a higher probability of sharing the same attitudes with others in the similar-attitude group than with individuals in other groups. Individuals communicate mainly with individuals within their group, but also with individuals in other groups. Figure 6 shows a simplified illustration of possible interactions between five similar-attitude groups that individuals of each group have the same attitudes. The groups are represented by nodes, and interactions are represented by arrows. Thus there is a link from group 3 to group 1, meaning that group 3 has an influence (positive or negative) on group 1. There are also links from group 1 to group 2 and from group 2 to 3. The example can be generalized to N groups, where the arrow from node i to node j indicates a link from i to j.

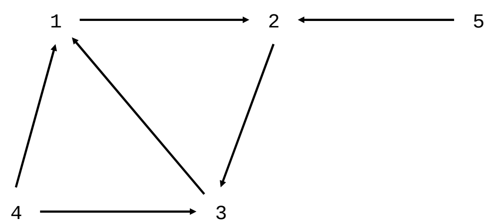

Figure 6: Graph of direct relationships

The direct links between nodes can be represented by the adjacency matrix shown in Fig. 7. An element of this $N \times N$ matrix indicates the position and possibly strength of direct dependency links from i to j.

nodes	1	2	3	4	5
1	0	1	0	0	0
2	0	0	1	0	0
3	1	0	0	0	0
4	1	0	1	0	0
5	0	1	0	0	0

Figure 7. The incidence matrix of the graph from Figure 6

In addition to direct dependencies, individuals are also indirectly influenced by one another by chains of communications. If A is the adjacency matrix, then an element (i, j) of the matrix $A^2 = A \times A$ represents the number of sequential dependency paths of length 2 involving an intermediate group from i to j and in general A^l indicates the number of sequential dependency paths of length l with $l - 1$ intermediate groups from i to j. Thus, the entries of the matrix

$$A + A^2 + ... + A^l$$

represent the number of all possible direct and indirect paths of no more length than l.

The graph on Fig. 6 has a cycle between nodes $1, 2, 3$. The graph on Fig. 8 is acyclic which represents hierarchical structure of opinion formation.

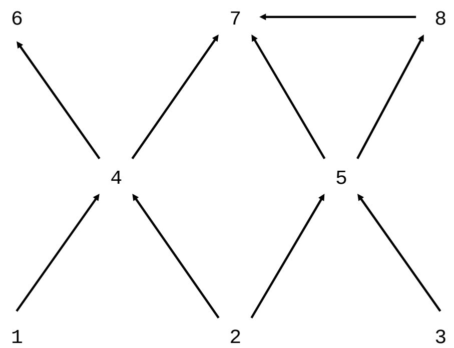

Figure 7: Acyclic graph

Consider now a simple situation of how the links between individuals may affect their attitudes. Suppose that links between five groups of individuals are represented in Figure 6. Figure 9 indicates that groups $1, 2, 3$ have two possible (no-yes) attitudes $j = 0, 1$. This is represented by subnodes 0 or 1 inside of each node $i = 1, 2, 3$. We can also think about $0 - 1$ states of these nodes. In this example we assume the deterministic nature of interactions. Therefore assume the state $j = 1$ is settled down at nodes $i = 1, 2, 3$ at the initial time interval $t = 0$. In general case at time $t = 0$ the state $j = 1$ is accepted only by a fraction of a group with a certain probability

Interdependencies between groups may change their attitudes. Arrows in the graph of Figure 9 indicate that individuals of the group 1 are influenced by group 3 in the sense that it expresses solidarity with group 1 taking the same opinion. Group 2 is antagonistic (in opposition) to group 1. Group 3 has the solidarity with group 2.

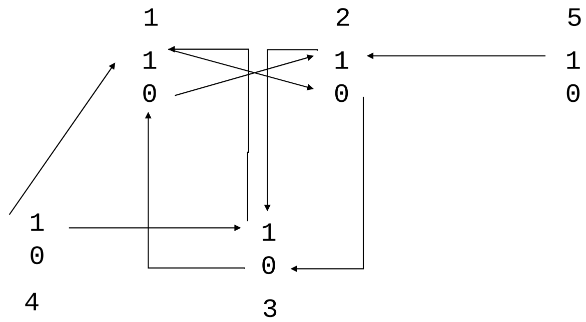

Figure 8: Limit Cycle attractor: waves of opinions

It easy to see a cyclic change of responses in time. When individuals of group 2 learn about the attitude of group 1 (state of node $i = 1$), the group 2 changes the attitude to the opposite. Since the initial state of all nodes $j = 1$, the next state of node 2 is $j = 0$ which triggers changes of states at nodes $3, 1$ to $j = 1$. These changes lead again to state $j = 1$ at nodes $1, 2, 3$ and so on.

Now suppose that group 3 is antagonistic to the group 2 (see Figure 10). If group 2 learns first the response of group 1 it changes the attitude 1 to 0 and so on until attitudes reach values $(1, 0, 1)$ for groups $1, 2, 3$ correspondingly. If group 3 learns first the attitude of group 2, the attitudes are settled down at states $(0, 1, 0)$. Hence, the behavior of the attitudes may display two attractors: $(1, 0, 1)$ and $(0, 1, 0)$.

These simple examples suggest that there may be waves of attitudes. Any opinion survey at a particular time may not represent opinion at a later time. These examples also demonstrate that delays in the learning of attitudes may change the pattern of the overall dynamics towards different attractors. In the model of the next Section we take more general point of view on driving forces of attitude changes. It is assumed that members of a group may react differently at attitudes of other groups. They also may ”hesitate” to react as the opposition or the solidarity.

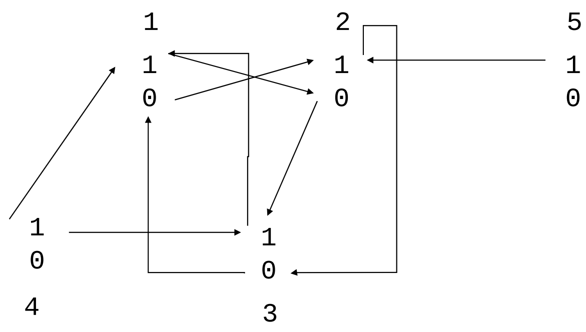

Figure 9: Fixed Point attractors

7.2 Model formulation

In examples of the previous Section individuals of each group have the same attitude. The individuals of a group also react in the same manner at "signals" (attitudes) from other groups. In this Section we undertake more general view. It is assumed that only a fraction p_j^{it} of the group i members have attitude j at time t. Thus in the previous examples there may be fractions p_0^{it}, p_1^{it} of each group $i = 1, 2, 3$ with attitudes 0 and 1 at time t. Of course, $p_0^{it} + p_1^{it} = 1$.

Individuals of a group communicate with different individuals of other groups. Therefore, their attitudes are influenced by random samples of information from adjacent (neighbor) groups. Individuals may form their attitudes at a particular time interval t, on the basis of various rules besides just "solidarity" or "opposition" principles. For example they may follow the majority attitude from a sampled opinions. In general such behavior induces a conditional probability for an individual of group i to take an attitude from the given set of possible attitudes. This probability is conditioned on their current attitudes, the attitudes of adjacent groups and some exogenous variables or external factors.

Let us now formulate the model precisely. To do so, we must represent the driving forces of attitude changes or dependencies between groups as well as dependencies of these relations on external factors. We opted for probabilistic description based on conditional distributions.

The model distinguishes N groups $i = 1, ..., N$ of individuals. The number of the relevant groups depends on the issue under consideration and on the level of detail represented in the model.

Individuals display different attitudes to the given issue, ranging from hostile to very favorable. We assume that there is a finite number M of possible attitudes. The attitudes of individuals of group i are described by the random variable ζ_i which takes values from 1 to M. In other words we assume that individuals from group i statistically follow the same pattern of attitude formation given by the distribution of ζ_i. In this sense, we can say that individuals of group i share approximately the same view. The attitudes of the population are described by random vector $\zeta = (\zeta_1, ..., \zeta_N)$. A fixed value of this vector is denoted by z and the set of all possible attitudes by Z. Let us denote by p_j^{it} the probability that a member of group i assumes the attitude j at time t:

$$p_j^{it} = \mathcal{P}(\zeta_i^t = j)$$

Naturally,

$$\sum_{j=1}^{M} p_j^{it} = 1, \;\; p_j^{it} \geq 0$$

Interactions between individuals are represented as a graph similar to graph of Section 7.1. Groups $i = 1, ..., N$ correspond to nodes of the graph and direct links between individuals of groups are represented by arrows between nodes. Thus there are two sets: nodes $V = 1, ..., N$ and the set of arrows (directed arcs) U. Let us denote this as $G = (V, U)$. If nodes i, j belong to V , $i, j \in V$ and there is an arrow from i to j, $(i, j) \in U$ then i is an adjacent to j node. Define as V_j the set of all adjacent to j nodes and node j. Let z_{V_j} is subvector of the vector of attitudes indexed by V_j. For example, in the case of dependencies indicated by graph in Figure 6 we have $V_1 = \{1, 3, 4\}$, $V_2 = \{1, 2, 5\}$, $V_3 = \{2, 3, 4\}$, $V_4 = \{\emptyset\}$, $V_5 = \{\emptyset\}$ where $\emptyset$ is the symbol of empty set. Then $z_{V_1} = (z_1, z_3, z_4)$, $z_{V_2} = (z_1, z_2, z_5)$.

Individuals change their attitudes depending on current attitudes of their own and adjacent groups, and on a vector x of exogenous external factors or variables. The attitude formation is described by a conditional distribution $H^i(z_i|z_{V_i}, x)$ for each individual of group i: the probability for an individual of group i to have an attitude $z_i = 1, 2, ..., M$ when attitudes of groups $V_i$are $Z_{V_i}$and external factors have values x.

We assume that function H^i can be derived on the basis of appropriate questionnaires answered by a representative member of individuals from each group, as: ”What is your attitude (from 1to M) if attitudes of the adjacent groups are $z_{V_i}$and the ”environment is $x = (x_1, ..., x_n)$?”. Functions H^i may also depend on time interval t, but we skip it in order to simplify the notations. Functions H^i define the dynamics of the attitude change according to the following relation:

$$p_j^{i,t+1} = \sum_{z_{V_i} \in Z} H^i(\zeta_i^{t+1} = j|\zeta_{V_i}^t = z_{V_i}; x^t)\mathcal{P}(\zeta_{V_i}^t = z_{V_i}) \tag{26}$$

The groups i with $V_i = i$ can be identified with ”leaders”, which influence opinions of other individuals but are not influenced themselves.

To define completely the dynamics of the system described by equation (26) it is necessary to fix initial attitudes distributions for $t = 0$. In case when the corresponding graph of direct influences is acyclic, it is enough to define these distributions for nodes i such that $V_i = i$.

Equation (26) together with initial distributions allow us in principle to calculate p_j^{it} for any $t \geq 0$. Of course, for complex graphs it is practically impossible to derive analytical formulas for p_j^{it} as functions of external factors x. The existence of analytical expressions for all p_j^{it} provides an easy tool to analyze implications of changes in x. The next Section is devoted to the analysis of attitudes changes in the case when such a possibility does not exist. The approach is based on the stochastic version of equation (26) dealing directly with random variables ζ_i^t, $i = 1, ..., N$ by using the Monte-Carlo simulation techniques. Instead of H^i, $i = 1, ..., N$ the approach allows also to use myopic rules to generate random changes of ζ^t i.e. the approach allows to analyze cases when functions H^i are given implicitly. One such important case arises in the situations when individuals form their attitudes by asking acquaintances from adjacent groups and use some simple rules based on majority or minority of sampled attitudes.

Let us now formulate some problems which are important in this context.

Problem 1. Evaluation of Attitudes.

The objective here is to predict attitudes of various population groups. As we have seen in the previous Section the attitudes of different groups change in intricate ways and are subject to changes in external factors and direct and indirect dependencies. Direct dependencies involve relatively few adjacent groups, while indirect dependencies and external factors may involve all or almost all population groups. Thus, in Figure 1 group 2 is affected directly only by groups $1, 5$. But there exists a path from 3 to 2, and from 4 to 2. Therefore indirectly individuals of group 2 are affected by all groups. The direct dependencies are much easier to study experimentally through surveys and questionnaires. Suppose that we managed to study the direct dependencies between attitudes of different adjacent population groups. The problem is to predict the public attitudes as the result of complex direct and indirect interactions by using the information about direct dependencies. As was outlined in the previous Section, this problem involves the calculation of all possible direct and indirect paths by using the adjacency matrix (Figure 7). Formally the problem is formulated as follows.

Given conditional distributions $H^i(z_i|z_{V_i}; x)$ for $i = 1 : N$ and the values of x^t for $t = 0, 1, ..., T-1$ find distributions $\mathcal{P}(\zeta_i^T)$, $i = 1 : N$ of random variables ζ_i^T, i.e. the public attitudes at time T.

Problem 2. Response interpretation.

This problem deals with the interpretation (identification) of public response to a mixture of events and efforts which influence the public. This interpretation is made on the basis of our knowledge which has two components. The first component is the apriori information on direct dependencies deduced from responses in the past. The second component consists of new direct observations of attitudes for some groups to a given new issue. This type of knowledge can be called aposteriori knowledge. The response interpretation deals with the following questions:

- Suppose that we have direct observations on the attitudes of only some selected groups, or we have observations of aggregated response from several groups. How can we recover attitudes of unobserved groups?
- How to use the newly acquired aposteriori knowledge to update our knowledge about direct dependencies between groups?
- Often a public response is the result of mixture of different, sometimes conflicting events and efforts. What is the contribution of each single event to the attitude dynamics?

Formally these problems can be formulated as follows. Let us consider only first problem. Denote by $V_{\mathcal{E}}$ the set of observed groups.

Given conditional distributions $H^i(z_i|z_{V_i}; x)$ for $i = 1 : N$, the values of x^t and distributions $\mathcal{P}(\zeta_i^t)$, $i \in V_{\mathcal{E}}$ for $t = 0, 1, ..., T-1$ find distributions $\mathcal{P}(\zeta_i^t)$ for $i \notin V_{\mathcal{E}}$.

Problem 3. Sensitivity analysis.

Here we want to analyze a sensitivity of attitudes with respect to changes of environmental variable x. Through such an analysis we may find that attitudes are especially resistant to changes in certain directions or in certain positions. For example, in siting a waste processing facility as we mentioned already it is necessary to analyze the choice of its size, decide on the distance between facility and population centers, choose routes of the waste transportation etc. Different population groups react differently on different options. Small changes in critical parameters may affect considerably the public attitudes, while substantial and possibly costly changes in non-critical parameters will not move the public response. The objective of the sensitivity analysis is to identify the critical parameters utilizing the knowledge of the direct dependencies and how these dependencies are affected by changes in environmental parameters.

In terms of our model the sensitivities of public responses is defined in terms of changes in response distribution $\mathcal{P}(\cdot)$ with respect to parameters x. This leads to the following formulation.

Given conditional distributions $H^i(z_i|z_{V_i};x)$ for $i = 1 : N$ and the values of x^t for $t = 0, 1, ..., T-1$ estimate derivatives of distributions $\mathcal{P}(\zeta_i^T)$, $i = 1 : N$ with respect to $x^0, ..., x^T$.

Note that distributions $\mathcal{P}(\cdot)$ depend on x indirectly through conditional distributions H, what leads to a challenging problems analyzed in next Sections.

Problem 4. Social learning.

As it is emphasized in social psychology people receive information from their social environment. A lack of connectivity between them develops clusters of people sharing similar views in a more heterogeneous population. Traditionally it is assumed that a change in thought or behavior is only a reaction to some external factors, stimulus. The proposed model emphasized the existence of intrinsic changes generated by interdependencies between individuals in the absence of any external factor. This dynamics may be perturbed by external factors activated at different times leading to more or less complex patterns of alternative dynamics. How can we learn the variety of alternative attitudinal developments and how we can characterize them? What are optimistic and pessimistic ”scenarios” of such developments? Can attitudes reach vital levels? Answers to these type of questions depend not only on existing links between individuals but also on paths of activated external factors. The main problem is to use the model in order to learn possible alternative scenarios and their outcomes. For example, in the debates on siting a waste processing facility, for example, there is a possibility to change sizes of facilities, their locations, premiums and other compensations in order to change public responses.

The power of a model is its ability to learn patterns of possible responses of the system without time consuming real observations and trial - and - error experiments. In our case the model allows to identify paths of external factors leading to different outcomes, for example decreasing a social tension, or to worst case situation. In order to conduct such analysis we need to introduce a set of ”performance” indicators or ”score” functions distinguishing one trajectory of attitudes from another. For example, if c_{ij} is relative importance of attitude j by individuals of group i, then the cumulative score of a trajectory of attitudes can be expressed by the following score (performance) function:

$$F(x) = \sum_{t=0}^{T} \sum_{i=1}^{N} \sum_{j=1}^{M} c_{ij} p_j^{it}, \tag{27}$$

which implicitly depends on external factors x through conditional probabilities in (26).

The sensitivity analysis can indicate changes in x which lead to increasing of an indicator $F(x)$. Using this information, it is possible to identify, for example, the worst or the best case sets of possible external factors.

The fundamental complexity of such type of problems is due to the probabilistic nature of $F(x)$ and implicit dependencies on variables x. Next Sections discuss tools enabling to deal with involved complexity.

7.3 Bayesian Networks and Markov Fields

The Bayesian Net is a powerful tool which was developed primarily to deal with stochastic problems defined on acyclic graphs (see Figure 8). We show that algorithms developed for Bayesian Nets constitute the building blocks for more general algorithms which can deal also with general graphs.

Bayesian nets are specifically designed for cases when the vector of random parameters ζ can have considerable dimension and/or it is difficult to come up with traditional parametrical model of the joint distribution of random parameters.

1. The cause-effect structure is associated with the vector of random parameters $\zeta = (\zeta_1, ..., \zeta_i, ..., \zeta_N)$. That is, for any parameter ζ_i the set of indices V_i is selected such that the elements of subvector $\{\zeta_j\}_{j \in V_i}$ can be identified with "causes" of ζ_i. Vector ζ_i is changed in time leading to a random path or trajectory $\zeta^t, t = 0, 1, ...$ In the proposed model groups of a population, say in a given region, are represented as nodes of a graph. Direct dependencies between groups are represented by arrows indicating directions of communications. Node i of the graph has different random states ζ_i^t at time t reflecting different attitudes of the group. Thus the stochastic dynamics of attitudes is characterized by a random vector ζ^t with dependent components. The important feature is that changes in a component ζ_i^t are triggered only by its "neighbors" $\zeta_j^t, j \in V_i$, apart from changes in external factors.

Such stochastic processes define Markov random fields. They can be regarded as generalization of Markov processes to situations when the cause effect structure is not necessarily linearly ordered. Let us start the formal exposition by gathering the basic definitions which will be used in next Sections. Consider a directed graph defined as a pair of two sets (V, U): nodes V and connecting them arrows (directed arcs) U. An example of such graph is given in Figure 6 with $V = \{1, 2, ..., 9\}$. For each node $v \in V$ let us define the set of parents

$$c(v) = \{j | (j, v) \in E\} \tag{28}$$

and the set of descendants

$$d(v) = \{j | (v, j) \in E\} \tag{29}$$

Consider a set of random variables $\zeta_V = \{\zeta_v, v \in V\}$ indexed by nodes from V and defined on appropriate probability space $(\Omega, \mathbb{B}, \mathbb{P})$. Suppose that W is an arbitrary subset of V. A Markov random field ζ is characterized by the property

$$\mathbb{P}(\zeta_v|\zeta_{W\cup c(v)}) = \mathbb{P}(\zeta_v|\zeta_{c(v)}) \tag{30}$$

If G is a directed acyclic graph, then (ζ, G) which satisfies (30) is called *Bayesian Network.*

Suppose now that $H(\zeta)$ is the joint distribution of ζ. Then it follows from the definition of the Bayesian Network that

$$H(\zeta) = \prod_{v\in V} H^v(\zeta_v|\zeta_{c(v)}) \tag{31}$$

where $H^v(\zeta_v|\zeta_{c(v)})$ is a conditional distribution function of ζ_v given $\zeta_{c(v)}$.

Coming back to the model of Section 3 we see that it fits in the framework of Bayesian Networks and Markov fields. The essential new feature is the dependence of the conditional distribution functions H^v on the vector of external variables x from a feasible set $X \subseteq R^n$:

$$H^v(\cdot) = H^v(\zeta_v|\zeta_{c(v)}, x) \tag{32}$$

The change of $x \in X$ affects the interactions between groups.

7.4 Sensitivity analysis

The analysis aims to analyze attitudes with respect to changes in external factors x. These changes are characterized by certain indicators or ”score” functions such as (27). A given x defines the random attitudes $\zeta = \zeta(x)$. A performance indicator is a function of ζ and possibly x, e.g., $f(x, \zeta)$.

The first question which can be raised is the following. How can we estimate the expected outcome

$$F(x) = \mathbb{E}f(x, \zeta) \tag{33}$$

at some point x? How sensitive is this value with respect to changes in parameters x? What are the most critical parameters? What is the value of the performance indicator at point $x + \delta x$ where δx is a small perturbation of x? These questions are not the trivial because each estimation of the value of $F(x)$ can be very time consuming taking into account indirect interdependencies of the network.

More precisely, in order to evaluate the sensitivity of $F(x)$ we need to develop algorithms for estimating the value of the gradient of this indicator. That is, for given x we need to compute vector ξ such that

$$\mathbb{E}\xi = F_x(x) = \frac{d}{dx}\mathbb{E}f(x, \zeta)$$

One possibility is to use the finite differences:

$$\xi = \sum_{i=1}^{n} \frac{\hat{F}(x + \Delta e_i) - \hat{F}(x)}{\Delta} e_i$$

where $\hat{F}(x)$ is an estimate of the value of function $F(x)$ at point x. In this case, however, it is necessary to compute at least $n+1$ estimates of the performance measure which may be too demanding computationally.

Let us introduce simple but useful differentiation formulas for the gradient of function $F(x)$, which are used in stochastic optimization.

Theorem 2 *Suppose that*

1. Random variables ζ_v have conditional densities $h(\zeta_v|\zeta_{c(v)},x)$

2. Functions $h(\zeta_v|\zeta_{c(v)},x)$, $f(x,\zeta)$ are differentiable with respect to x uniformly with respect to ζ. Then $F(x)$ defined in (33) is differentiable and

$$F_x(x) = \mathbb{E}\left\{ f_x(x,\zeta) + f(x,\zeta) \sum_{v\in V} \frac{h_x(\zeta_v|\zeta_{c(v)},x)}{h(\zeta_v|\zeta_{c(v)},x)} \right\} \tag{34}$$

Proof.

From (31) we have the following expression for $F(x)$:

$$F(x) = \int f(x,\zeta) \prod_{v\in V} h(\zeta_v|\zeta_{c(v)},x) \prod_{v\in V} d\zeta_v \tag{35}$$

Under assumptions of the theorem we can change the order of differentiation and integration, which yields:

$$F_x(x) = \int \frac{d}{dx}\left(f(x,\zeta) \prod_{v\in V} h(\zeta_v|\zeta_{c(v)},x) \right) \prod_{v\in V} d\zeta_v =$$

$$\int f_x(x,\zeta) \prod_{v\in V} h(\zeta_v|\zeta_{c(v)},x) \prod_{v\in V} d\zeta_v +$$

$$\sum_{w\in V} \int f(x,\zeta) h_x(\zeta_w|\zeta_{c(w)},x)) \prod_{v\in V,\, v\neq w} h(\zeta_v|\zeta_{c(v)},x) \prod_{v\in V} d\zeta_v =$$

$$\int \left(f_x(x,\zeta) + f(x,\zeta) \sum_{v\in V} \frac{h_x(\zeta_v|\zeta_{c(v)},x)}{h(\zeta_v|\zeta_{c(v)},x)} \right) \prod_{v\in V} h(\zeta_v|\zeta_{c(v)},x) \prod_{v\in V} d\zeta_v$$

The proof is completed. ◇

Similar result holds when each of the random variables ζ_v takes finite number of values and instead of conditional densities we have conditional probabilities $P(\zeta_v|\zeta_{c(v)},x)$. If these probabilities are differentiable with respect to x we obtain the following expression for $F_x(x)$:

$$F_x(x) = \mathbb{E}\left\{ f_x(x,\zeta) + f(x,\zeta) \sum_{v\in V} \frac{P_x(\zeta_v|\zeta_{c(v)},x)}{P(\zeta_v|\zeta_{c(v)},x)} \right\} \tag{36}$$

Note that the second terms in expressions (34),(36) can be interpreted as sums of likelihood ratios [15, 23]. The calculation of exact values $F_x(x)$ is possible only in exceptional cases for simple networks.

Let us now present estimation algorithms which exploit the structure of Bayesian network and expressions (34),(36) in order to obtain statistical estimates of $F(x)$ and $F_x(x)$. We shall make reference to discrete case and use (36), the continuous case is treated similarly. The simplest estimation scheme is the following:

$$\xi = \frac{1}{K}\sum_{k=1}^{K}\left(f_x(x,\zeta^k) + f(x,\zeta^k)\sum_{v\in V}\frac{P_x(\zeta_v^k|\zeta_{c(v)}^k,x)}{P(\zeta_v^k|\zeta_{c(v)}^k,x)}\right) \tag{37}$$

where $\zeta^k, \zeta_v^k, \zeta_{c(v)}^k, \quad k = 1 : K, \; K \geq 1$ are independent observations of random vectors $\zeta, \zeta_v, \zeta_{c(v)}$ respectively. Stochastic vector ξ is termed as stochastic gradient. The estimation scheme (37) in the context of Bayesian networks is called also as Logic Sampling. The estimator (37) requires $K \geq 1$ observations ("scenarios") of attitudes ζ. This scenarios can be sequentially generated by using Monte-Carlo simulation techniques. An arbitrary scenario generating cycle $k = 1 : K$ have the following simple steps.

1. *Initialization.* Select a node $v \in V$ such that $c(v)$ is the empty set, $c(v) = \emptyset$. Since the graph G is acyclic such nodes exist and we interpreted them as sources of opinions or leader nodes. Sample the component ζ_v from unconditional distribution $P(\zeta_v|\zeta_{c(v)}, x)$ and denote the result as ζ_v^k. Perform this step for all nodes with empty parent set $c(v)$.

2. *Selection.* Select a not sampled yet node $v \in V$ such that all random variables $\zeta_j \;\; j \in c(v)$ have been sampled already during current scenario generating cycle. Suppose that the observation of ζ_j is $\zeta_j^k, \;\; j \in c(v)$.

3. *Sampling.* Sample random variable ζ_v and obtain an observation ζ_v^k.

Steps 2 and 3 are repeated until all random components of ζ are sampled. In other words until all groups reveal their opinions.

Note that by using observations $\zeta^k, \;\; k = 1 : K, \; K \geq 1$ along with the estimates of the gradient F_x we obtain the estimate of $F(x)$ itself. For example, $F(x)$ may correspond to the probability for ζ to belong to a certain desirable domain (for example, majority of attitudes 1 (yes) to competing attitudes 0 (no)) we obtain the estimates of this probability and its sensitivity ξ.

There may be other sampling schemes known as Evidence Weighting [4] and Gibbs Sampling [14] which are particularly useful for the solution of the response interpretation problem (Problem 2 from Section 7.2). Such schemes use the structure of the network and aposteriori distributions instead of $P(\zeta_v|\zeta_{c(v)}, x)$. The estimates of stochastic gradient obtained above can be used for obtaining the optimal values of parameters using stochastic quasigradient methods like in [7, 8, 9].

7.5 General interdependencies

Let us extend the analysis of the previous Section to the case of cyclic graphs. The exposition follows closely the exposition of the previous Section and therefore we concentrate on the new features only. The most important among these features is the dynamic aspect of attitudes change which in the case of conventional Bayesian Network was reduced to analysis of their propagation from "leaders" to other groups.

Let us fix the time horizon $[0, l]$ during which we study the attitude dynamics and sensitivity estimates. Such estimates will be called l-links estimates. Generally, the variables x may change during this time period: $x = (x^0, ..., x^{l-1})$. For the sake of simplicity we shall derive our estimates for the case when $x^0 = x^1 = ... = x^{l-1}$ denoting this constant vector by x. The general case does not bring any conceptual difficulties and is treated similarly.

Consider again the case when conditional distributions has densities. Consider a performance measure which will be the following generalization of the measure (35):

$$F(x) = \int f(x, \bar{\zeta}^l) h(x, \bar{\zeta}^l) d\bar{\zeta}^l \tag{38}$$

where for $t = 0, ..., l$ we denoted by $h(x, \bar{\zeta}^t)$ the joint density of the random vector $\bar{\zeta}^t = (\zeta^0, ..., \zeta^t)$ with $\zeta^t = (\zeta_1^t, ..., \zeta_N^t)$ and

$$d\bar{\zeta}^l = \prod_{t=1}^{l} \prod_{v \in V} d\zeta_v^t$$

The density of $\bar{\zeta}^t$ is connected with the density of $\bar{\zeta}^{t-1}$ for $t = 1, ..., l$ with the following relation:

$$h(x, \bar{\zeta}^t) = h(x, \bar{\zeta}^{t-1}) \prod_{v \in V} h(\zeta_v^t | \zeta_{c(v)}^{t-1}, x) \tag{39}$$

with the initial distribution $h(x, \bar{\zeta}^0)$ density being simply

$$h(x, \bar{\zeta}^0) = \prod_{v \in V} h(\zeta_v^0) \tag{40}$$

Combining expressions (38)-(40) we obtain the basic formula for the performance measure:

$$F(x) = \int f(x, \bar{\zeta}^l) \prod_{v \in V} h(\zeta_v^0) \prod_{t=1}^{l} \prod_{v \in V} h(\zeta_v^t | \zeta_{c(v)}^{t-1}, x) \prod_{t=1}^{l} \prod_{v \in V} d\zeta_v^t \tag{41}$$

Utilizing this expression similarly to the previous Section we obtain the following result:

Theorem 3 *Suppose that*

1. Random variables ζ_v^t have conditional densities $h(\zeta_v^t | \zeta_{c(v)}^t, x)$ for $t = 1, .., l$ and densities $h(\zeta^0)$ for $t = 0$.

2. Functions $h(x, \zeta_v^t | \zeta_{c(v)}^t)$, $f(x, \bar{\zeta}^l)$ are differentiable with respect to x uniformly with respect to $\bar{\zeta}^l$. Then $F(x)$ defined in (38) is differentiable and

$$F_x(x) = \mathbb{E} \left\{ f_x(x, \bar{\zeta}^l) + f(x, \bar{\zeta}^l) \sum_{t=1}^{l} \sum_{v \in V} \frac{h_x(\zeta_v^t | \zeta_{c(v)}^{t-1}, x)}{h(\zeta_v^t | \zeta_{c(v)}^{t-1}, x)} \right\} \tag{42}$$

As in the previous Section similar result holds when each of the random variables ζ_v^t takes finite number of values and instead of conditional densities we have conditional probabilities $P(x, \zeta_v^t | \zeta_{c(v)}^t, x)$. If these probabilities are differentiable with respect to x we obtain the following expression for $F_x(x)$:

$$F_x(x) = \mathbb{E}\left\{ f_x(x, \bar{\zeta}^l) + f(x, \bar{\zeta}^l) \sum_{t=1}^{l} \sum_{v \in V} \frac{P_x(\zeta_v^t | \zeta_{c(v)}^{t-1}, x)}{P(\zeta_v^t | \zeta_{c(v)}^{t-1}, x)} \right\} \tag{43}$$

The l-stage sensitivity estimate based on logic sampling in the discrete case takes the form:

$$\xi = \frac{1}{K} \sum_{k=1}^{K} \left(f_x(x, \bar{\zeta}^{kl}) + f(x, \bar{\zeta}^{kl}) \sum_{t=1}^{l} \sum_{v \in V} \frac{P_x(\zeta_v^{kt} | \zeta_{c(v)}^{k,t-1}, x)}{P(x, \zeta_v^{kt} | \zeta_{c(v)}^{k,t-1}, x)} \right) \tag{44}$$

where $\bar{\zeta}^{kt}, \zeta_v^{kt}, \zeta_{c(v)}^{kt}, \quad t = 0, 1, ..., l, \quad k = 1 : K, \ K \geq 1$ are independent observations of random vectors $\bar{\zeta}^t, \zeta_v^t, \zeta_{c(v)}^t$ respectively. The vector ξ defines a stochastic gradient of the function $F(x)$ in (38).

Observations ζ^{kt}, $k = 1 : K$ of random vectors ζ^t, $t = 0, 1, ..., l$ can be simulated similar to acyclic graphs of the previous Section. In fact the study of l-stage interdependencies between groups (nodes) on general graphs can be reduced to the study of l-stage sensitivity estimates on acyclic graphs.

This reduction of l-stage sensitivity analysis to acyclic graphs provides a general approach to the study of the general nets with cyclic graphs. Keeping in mind this possibility let us describe the Monte-Carlo simulation procedure for the generation $k = 1 : K$ scenarios ζ, $t = 0, 1, ..., l$ describing l-stage propagation effects of public attitudes.

1. *Initialization* Leader nodes of the equivalent acyclic graph (see Figure 6) correspond to $t = 0$ Therefore for all $v \in V$ sample the values ζ^{kt} of random variables ζ_v^t, $t = 0$.

2. *Selection* Select all nodes $v \in V$ sequentially for $t = 1 : K$.

3. *Sampling* For a sampled node v at step t sample random variables ζ_v^{kt+1} from distribution $P(\zeta_v^{t+1} | \zeta_t^v = \zeta_v^{kt}, x)$. Steps 2, 3 are repeated until all components ζ^{kt} of vectors ζ^t, $t = 0, 1, ..., l$ are sampled.

Scenarios ζ^{kt}, $t = 0, 1, ..., l$, $k = 1 : K$ of public attitudes allow to estimate vector $F_x(x)$ according to (44). As in the previous Section this vector indicates directions of changes in x towards the increase of indicator $F(x)$.

8 Conclusion

In the present paper we set the stage for further development of modeling and decision support tools for analysis of design and deployment of robust information infrastructure using as the case study one component of this infrastructure: advanced mobile data services. Particular methodologies utilized here: portfolio theory and Bayesian nets combined with stochastic optimization have a potential to be utilized also for solution of similar problems arising with other types of infrastructure.

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