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On the Costs of Reducing GHG Emissions and its Underlying Uncertainties in the Context of Carbon Trading

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Abstract

This paper considers the problem of trading uncertain emissions under the Kyoto Protocol. We analyze a market structure that encourages the reduction of inventory uncertainty, although this option is not explicitly mentioned in the Protocol. According to the setting, parties to the Protocol are allowed to meet their targets by reducing emissions, buying permits, or reducing relative uncertainty. The goal of the paper is to account for the dependence in reductions of both emissions and uncertainty. Although formally a carbon emissions market may be restrained from the convergence to its least cost solution, a numerical experiment shows that it reaches equilibrium on its own. The necessary conditions for cost-effective solutions have been derived for the case of cost functions modeled with quadratic functions.

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About the Author

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On the Costs of Reducing GHG Emissions and its Underlying Uncertainties in the Context of Carbon Trading

Joanna Horabik

1 Introduction

Aiming to suspend an anthropogenic climate change, the Kyoto Protocol obligates its Parties (developed countries and countries that are undergoing the process of transition to a market economy) to reduce or to limit greenhouse gas (GHG) emissions compared to their base year levels. The Protocol was drawn up in 1997 under the United Nations Framework Convention on Climate Change (UNFCCC, 1998). The emission targets, specified for each Party, need to be achieved within the commitment period of 2008–2012.

According to Article 5 of the Kyoto Protocol, Parties are required to conduct "a national system for the estimation of anthropogenic emissions by sources and removals by sinks". Guidelines for such national systems have been specified by the International Panel on Climate Change (IPCC). However, uncertainty is unavoidable in preparing such large scale inventories.

Greenhouse gas emissions are generally not directly measurable and they are assessed on the basis of (1) emission factors from laboratory data or scientific calculations, and (2) activity data reflecting GHG related activities. Both quantities are uncertain. Uncertainties in emission factors are associated with insufficient knowledge about processes generating emissions. Emission factor uncertainties also arise from lack of relevant measurements and thus inappropriate generalizations. Activity data are generally statistical data. Uncertainties in activity data come from inaccuracies in data collecting systems and lack of relevant investigations (Rypdal and Flugsrud, 2001). According to Rypdal and Flugsrud (2001) and Winiwarter and Rypdal (2001), most activity data and CO_2 emission factors for energy sources are reported with small uncertainty of around 5%. Emission factors for other pollutants are much more uncertain—usually more than 20%. The most uncertain sources identified include: N_2O from agricultural soils, CH_4 from landfills, PFCs and SF_6 from aluminium production, HFCs from product use and N_2O from road traffic (Rypdal and Winiwarter, 2001).

It is anticipated that over the next 10 years inventory uncertainty can be reduced. This can be acquired, for example, through national research programs as well as through improvements in data collecting systems (Lim *et al.*, 1999; Rypdal and Flugsrud, 2001; Rypdal and Winiwarter, 2001).

Uncertainty inherent in national emission reporting becomes crucial in the context of emissions trading. Article 17 of the Kyoto Protocol introduces emission trading in order to facilitate achieving national, agreed-upon reduction targets. The Parties listed in Annex B to the Protocol are allowed to sell their excess emission reductions. Environmental markets are regarded as an attractive policy instrument since they provide potential for cost-efficiency.

The issue of carbon market performance under inventory (reporting) uncertainty has already been addressed in the literature. Godal *et al.* (2003; see also Godal, 2000) incorporated uncertainty into emissions trading in the following way. They assumed that uncertainty has to be added to the emissions when checking the compliance with Kyoto targets. Targets are met by investing in the reduction of emissions or uncertainty or by buying permits. In other words, an incentive-based mechanism was considered to reduce both emissions and uncertainty of reported emissions. The price formation process has been simulated following the scheme of sequential bilateral trade proposed by Ermoliev *et al*. (2000). This scheme offers to analyze the dynamics of an emissions market when bilateral transactions are sequentially made under changing non-equilibrium prices. The advantage of this decentralized optimization procedure is that when bargaining on a permit price, parties do not reveal information on their cost functions. Godal *et al*. (2003) treated the reduction of both emissions and uncertainty of reported emissions as two independent processes. In the equilibrium, the permit price was equal to the marginal costs of both emission reduction and uncertainty reduction. The applied sequential bilateral trading scheme converged to this equilibrium.

In this paper we consider the case of dependence when reducing emissions and uncertainty. To illustrate the approach consider the following example. In the case of estimating CO_2 emissions, the problem is to achieve better activity data to reduce uncertainties; CO_2 emission factors are well known. For estimating, e.g., N_2O emissions this is the opposite. The main source of uncertainty is the emission factor. To reduce this kind of uncertainty, it is recommended to develop standard measurement methods (Lim *et al.*, 1999). In both cases, the reduction of uncertainty would require substantial investments. However, the costs of these two improvement processes need to be modeled in a different manner. One has to distinguish between quantities that can be statistically surveyed only once (activity data) and quantities that can be repeatedly measured experimentally (emission factors). In the case of activity data (CO_2), costs of uncertainty reduction are associated with absolute uncertainty. On the contrary, in the case of emission factors (e.g., N_2O), costs of uncertainty reduction are associated with relative uncertainty. This distinction is significant as emissions trading is a cost-effective problem and here we consider a market-based encouragement to reduce estimates of emission uncertainty.

This paper is structured as follows. Section 2 recalls the scheme of sequential bilateral trading introduced by Ermoliev *et al*. (2000) and includes the case of uncertainty reduction as provided by Godal *et al*. (2003). In Section 3, we introduce relative uncertainty into the described model, i.e., we account for dependence when reducing emissions and uncertainty. The model is checked for convergence properties. Analytically, we show that the system becomes nonlinear and may suffer from non-convexities. In Section 4, we derive conditions for non-convexity in the case of quadratic cost functions. We do this by determining two parameters and finding the range of values necessary for non-convexity to occur. Consequently, the carbon market simulation exercise in Section 5 is performed for cost functions of emission and uncertainty reduction that are quadratic. In the numerical experiment of the bilateral sequential trade we focus on the analysis of both derived parameters. Concluding remarks are given in Section 6.

2 Background Literature

Montgomery (1972) proved that emission trading creates an opportunity to reduce pollution at the least total costs for all Parties. The mechanism can be feasible under the assumption that all the transactions are made at the same point in time and at equilibrium prices known to all Parties. This would require complete knowledge of cost functions by a central environmental agency and this fact places the countries in an unfavorable position, when bargaining on a permit price.

Starting from this point, Ermoliev *et al.* (2000) developed a dynamic scheme for trading permits, where permit equilibrium prices are adjusted in consecutive steps. A scheme of sequential bilateral trade was analyzed. At each step, two sources with differing marginal costs meet at random. The idea behind this is that an emitter with relatively high marginal costs of emission reduction is interested in buying permits and, conversely, the low cost source is willing to sell its permits. When a transaction is made the seller reduces its emissions by the same quantity as the purchaser increases its emissions. However, since the permit purchaser exhibits higher marginal costs than his partner then, while exchanging the same quantity of emissions, the permit purchaser decreases its total costs more than the seller increases its own total costs. What follows is that total (aggregated) costs for all participants will be diminished after any single transaction. Next, another couple of parties are picked and the process is repeated. This will go on as long as there are two or more parties with differing marginal costs. It has been demonstrated that this scheme will lead the Parties to the least cost solution, while the information about the cost function of each Party's emission reduction remains un-revealed (for mathematical proof of convergence, see Appendix 1 in Ermoliev *et al.*, 2000).

The issue of uncertainties related to carbon reporting has been introduced into the Kyoto framework by Obersteiner *et al.* (2000) (see also Jonas *et al.*, 1999). It has been considered that apart from managing its emission level a party can also actively reduce its uncertainty level. The idea of a cost function referring to the reduction of uncertainty (c_ε) has been introduced. According to the model set-up this function depends on the level of absolute uncertainty (ε). The general aim of the model is that a country can choose between emission reduction and uncertainty reduction to meet its reduction target under the Kyoto Protocol.

Godal *et al.* (2003) simulate a carbon permit market that considers the reporting of uncertain carbon fluxes. Applying the methodology of sequential bilateral trade, the market approaches equilibrium. The reduction of uncertainty is accounted for in absolute terms. The employed cost function models the costs that are involved in the reduction of uncertainty (referred to by Godal *et al.* as 'unreported' emissions). Below we give an overview of the authors' setting as background to the present study.

Consider a set of Kyoto Parties (numbered i = 1, …, N) that participate in a permit market. Each of them faces a two-step optimization problem. First, they choose between emissions and uncertainty abatement for a given amount of permits. Similarly to Obersteiner *et al.* (2000), two separate cost functions are considered:

$c_{F,i}(F_i)$ —total costs for Party i of keeping emission on the level F_i;

$c_{\varepsilon,i}(\varepsilon_i)$—total costs for Party i of keeping uncertainty on the level ε_i.

Additionally, let us denote:

K_i—emissions of Party i in compliance with its Kyoto target; y_i—the number of emission permits handled by Party i. The value may be positive (purchaser of permits) or negative (supplier of permits).

F_i—emissions of Party i.

ε_i—absolute uncertainty of Party i.

The optimization task for an individual Party is formulated as follows:

$$f_i^{IND}(y_i) = \min_{F_i,\varepsilon_i}\left[c_{F,i}(F_i) + c_{\varepsilon,i}(\varepsilon_i)\right] . \quad (1)$$

Such that $F_i + \varepsilon_i \le K_i + y_i$ for $i = 1,...,N$. (2)

Both cost functions $c_{F,i}(F_i)$ and $c_{\varepsilon,i}(\varepsilon_i)$ are assumed to be positive, decreasing and convex in F_i and ε_i, respectively. The convexity of the function f_i^{IND} (y_i) is assured as it is the minimum of the sum of two convex functions subject to a linear constraint. There is one solution to equations (1) and (2). Setting up the Lagrangian, it can be found that in the cost minimum solution the marginal costs of reducing uncertainty and emissions will be equal, indicating permit shadow price (λ_i). [The shadow price is the increase in the amount that the objective function would increase if the constraint (K_i) were relaxed by one unit and can thus be interpreted as the willingness of Party i to pay for an additional unit of emission permit y_i.] For a given number of permits y_i, the permit shadow price λ_i is equal to the marginal costs of reducing emissions or uncertainty multiplied by –1:

$$\lambda_i = -\left(f_i^{IND}(y_i)\right)' = -c_{F,i}\left(F_i^*\right)' = -c_{\varepsilon,i}\left(\varepsilon_i^*\right)' \quad (3)$$

where F_i^* and ε_i^* denote the optimal level of emissions and absolute uncertainty, respectively, and the apostrophes marginal costs. This optimization step is performed independently by each Party. As long as the market will not be in equilibrium, shadow prices λ_i will differ between seller and buyer reflecting the potential for trade (see also Figure 4).

The second optimization problem involves finding the permit distribution among the Parties that would equalize shadow prices among all participants, assuring the least cost solution for all of them. The aggregate cost of reaching the Kyoto agreement is defined as the sum over all individual costs:

$$\min_{y_i} \sum_{i=1}^{N} f_i^{IND}(y_i) \ , \tag{4}$$

$$\text{s.t.} \ \sum_{i=1}^{N} y_i = 0 \ . \tag{5}$$

Convexity of the individual cost functions $f_i^{IND}(y_i)$ assures also convexity of the aggregate cost function, thus achieving the global least-cost solution. The applied sequential bilateral trading scheme is proved to converge to this equilibrium.

3 Introducing Relative Uncertainty into the Emissions Trading Scheme

Below we aim at investigating the consequences for the carbon market resulting from introducing a cost function that measures the reduction of uncertainty in relative terms. We argue that this approach is more suitable to analyze the costs of reducing uncertainty involved in the inventory of non-CO_2 greenhouse gas emissions (e.g., N_2O and CH_4) and possibly also their aggregate in combination with CO_2. In practice, assessing the relative uncertainty of emission factors is difficult, not to mention assessing the costs involved in the reduction of it. The uncertainty of emission factors depends mainly on the emission source and can be associated with the general knowledge of the process. As already mentioned, the uncertainty of, e.g., CO_2 emission factors for energy related sources is around 5%, for N_2O from agricultural sources it is up to 100%, and for N_2O from combustion it is up to 200% (95% confidence level). Experts derive this kind of information, e.g., from physical constraints on how large the variation may be, and also comparing different emission assessment results (Rypdal and Winiwarter, 2001).

We assume that costs $c_{R,i}(R_i)$ incur to reduce relative uncertainty R_i, which is given by $R_i = \varepsilon_i / F_i$. The sum of these costs and the emission reduction costs $c_{F,i}(F_i)$ contribute to the total abatement costs incurred by a party. However, operating under the Kyoto Protocol requires expressing uncertainty in absolute terms: Emission level F_i plus (absolute) uncertainty, $\varepsilon_i = F_i \cdot R_i$, shall not exceed the agreed-upon Kyoto target K_i increased/decreased by a specific amount of permits y_i (see boundary condition (7)).

The first step is to find the optimal level of emission and uncertainty for a single Party given a certain amount of permits y_i. A Party's optimization problem is stated as follows:

$$f_i^{DEP}(y_i) = \min_{F_i, R_i} z_i(F_i, R_i) \ , \tag{6}$$

such that $F_i + F_i \cdot R_i \leq K_i + y_i$ for $i = 1,...,N$, (7)

where $z_i(F_i, R_i) = c_{F,i}(F_i) + c_{R,i}(R_i)$.

The approach can be also viewed to reflect the dependence between the reduction of emissions and their underlying uncertainty.[1] In inequality (7) the uncertainty component ε_i on the left side is composed of two factors: F_i and R_i, each of which refer to another cost function: $c_{F,i}(F_i)$ and $c_{R,i}(R_i)$, respectively. This formulation is opposite to the optimization task (1), which we called the case of independent uncertainty and emission reduction (with $f_i^{IND}(y_i)$). It should be stressed that in our analysis we consider only the case of dependence between the reduction of emissions and their underlying uncertainty. An approach combining the two approaches would perhaps be more suitable to reflect nonlinearities. Nonlinearities also arise when one considers temporal detection of emission changes as discussed by Jonas and Nilsson (2001, pp. 34–35).

It is assumed that the two cost functions $c_{F,i}(F_i)$ and $c_{R,i}(R_i)$ exhibit usual economic properties: they are positive, decreasing, convex and continuously differentiable. We consider the level of emissions F_i and the level of relative uncertainty R_i as being positive, which reflects reality. We also assume that $K_i + y_i > 0$, i.e., we assume that countries do not sell more permits y_i than their Kyoto endowments K_i. The Lagrange function is:

$$L(F_i, R_i, \lambda_i) = c_{F,i} + c_{R,i} + \lambda_i \cdot (F_i + F_i \cdot R_i - K_i - y_i) , \tag{8}$$

and the first order conditions become:

$$\begin{aligned} \frac{\partial L}{\partial F_i} &= c'_{F,i} + \lambda_i + \lambda_i \cdot R_i = 0 \\ \frac{\partial L}{\partial R_i} &= c'_{R,i} + \lambda_i \cdot F_i = 0 \\ \frac{\partial L}{\partial \lambda_i} &= F_i + F_i \cdot R_i - K_i - y_i = 0 \quad . \end{aligned} \tag{9}$$

Here, it is assumed (first-order approach) that relative uncertainty does not change in the case of an emissions change, i.e., that the cost function $c_{R,i}(R_i)$ is independent of the emissions level F. The Lagrange multiplier λ_i, interpreted as the permit shadow price, is equal to:

$$\lambda_i = \frac{-c'_{F,i}(F_i^*)}{1 + R_i^*} = \frac{-c'_{R,i}(R_i^*)}{F_i^*} , \tag{10}$$

where F_i^* and R_i^* represent the optimal levels of emissions and relative uncertainty, respectively.

[1] This is the reason for the superscript [DEP] in $f_i^{DEP}(y_i)$ in formula (6).

In other words, in the cost-minimum solution the ratio between the marginal costs $\frac{c'_{F,i}}{c'_{R,i}}$ depends on the level of optimal emissions F_i^* and optimal relative uncertainty R_i^*. For comparison, in the case of independent emission and uncertainty reduction, the ratio of marginal costs was equal to 1.

In our set-up $f_i^{DEP}(y_i)$ is a minimum of two convex functions subject to a constraint which is nonlinear with respect to the two variables F_i and R_i (because $\varepsilon_i = F_i \cdot R_i$). In order to learn more about the minimum of the Lagrange function, the second derivative of the function $z_i(F_i, R_i)$ has to be analyzed.

Considering constraint (7) for the case of equality (in our approach there is no reason for parties to 'over-comply' with their Kyoto targets), we express the goal function to be dependent on F_i by substituting (7) in (6):

$$z_i(F_i) = c_{F,i}(F_i) + c_{R,i}\left(\frac{K_i + y_i - F_i}{F_i}\right) \quad . \tag{11}$$

Then the second derivative becomes:

$$\left.\frac{\partial^2 z_i}{\partial F_i^2}\right|_{\frac{\partial z_i}{\partial F_i}=0} = c''_{F,i} + \left(\frac{c'_{F,i}}{c'_{R,i}}\right)^2 \cdot c''_{R,i} + 2\sqrt{\frac{c'_{F,i}}{c'_{R,i}}} \cdot \frac{c'_{F,i}}{\sqrt{K_i + y_i}} \quad . \tag{12}$$

The first two components on the right side of equation (12) are positive as a result of $c_{F,i}$ and $c_{R,i}$ being convex. The third component is negative for the same reason, i.e., $c'_{F,i} \leq 0$. Thus, depending on the values of the three components, the second derivative may become negative for the arguments for which $\frac{\partial z_i}{\partial F_i} = 0$.

Similarly, we derive the goal function $z_i(R_i)$ and the corresponding second derivative:

$$z_i(R_i) = c_{F,i}\left(\frac{K_i + y_i}{1 + R_i}\right) + c_{R,i}(R_i) \quad , \tag{13}$$

$$\left.\frac{\partial^2 z_i}{\partial R_i^2}\right|_{\frac{\partial z_i}{\partial R_i}=0} = c''_{R,i} + \left(\frac{c'_{R,i}}{c'_{F,i}}\right)^2 \cdot c''_{F,i} + 2\sqrt{\frac{c'_{R,i}}{c'_{F,i}}} \cdot \frac{c'_{R,i}}{\sqrt{K_i + y_i}} \quad . \tag{14}$$

Equivalent considerations as above indicate that the second derivative (14) can also be negative. As a consequence, the optimization problem (6), (7) may appear to be non-convex and several local minima may exist. A non-convex cost function $f_i^{DEP}(y_i)$ would also imply a non-convex aggregate cost function in the second optimization step (confer equations (4) and (5)).

For the permit market, this would mean that the achieved solution may not be the least cost one. The market may be locked in a local minimum. As a consequence, to reach the global minimum a central agency would be required that knew the cost functions for all countries. The feasibility of sequential bilateral trade to reach market equilibrium would thus become questionable.

4 Non-convexity Problem in the Case of Quadratic Cost Functions

In this section we investigate the possibility for the carbon permit market to reach local minimum conditions. Whether or not such a situation will occur will depend on the proportion of the parameters that we employ in modeling our emissions trading scheme.

To reflect the reduction of emissions, consider a convex (downside) cost function, which is equal to $c_{F,i}(F_i) = b_i \cdot (F_i - a_i)^2$ for $F_i \in [0; a_i]$ and 0 for $F_i > a_i$. The emissions level $F_i = a_i$ shall reflect baseline emissions (also called 'business-as-usual' (BAU) level). These involve no costs as emission regulations are absent.

We model the cost function for uncertainty reduction in the same way: $c_{R,i}(R_i) = d_i \cdot (R_i - R_{0,i})^2$ for $R_i \in [0; R_{0,i}]$ and $c_{R,i}(R_i) = 0$ for $R_i > R_{0,i}$. As above, $R_{0,i}$ indicates the 'baseline' level for relative uncertainty level (i.e., no costs incurred).

Consider the $z_i(F_i)$ according to equation (11) with $c_{F,i}(F_i)$ and $c_{R,i}(R_i)$ as characterized above. We require the first derivative of $z_i(F_i)$ to be equal zero:

$$F_i^4 - a_i \cdot F_i^3 + \frac{d_i}{b_i} \cdot (1 + R_{0,i}) \cdot (K_i + y_i) \cdot F_i - \frac{d_i}{b_i} \cdot (K_i + y_i)^2 = 0 . \qquad (15)$$

Normalizing F_i upon a_i, i.e., $\frac{F_i}{a_i} = x_i$, and denoting for convenience two non-dimensional parameters,

$$\alpha_i = \frac{d_i}{a_i^3 b_i}(1 + R_{0,i})(K_i + y_i)$$
$$\gamma_i = \frac{K_i + y_i}{a_i(1 + R_{0,i})} \qquad (16)$$

we obtain:

$$x_i^3(x_i - 1) + \alpha_i(x_i - \gamma_i) = 0 , \qquad (17)$$

where $x_i > 0$, $\alpha_i > 0, \gamma_i > 0$. We get a 4th degree equation involving the two parameters α_i and γ_i. We now determine the (necessary) conditions (i.e., parameter values) for a two-minima solution. Below we provide the answer without solving equation (17) analytically, which can be written as a difference:

$$\alpha_i(x_i-\gamma_i)-x_i^3(1-x_i)=0\,. \qquad (18)$$

As it becomes clear below, we can consider the expression $\alpha_i(x_i-\gamma_i)$ as the tangent line to the function $x_i^{\,3}(1-x_i)$, which allows determining the parameters α_i and γ_i. However, the crucial point of the analysis is to evaluate the tangent to the function $x_i^{\,3}(1-x_i)$ so that we can draw conclusions about its minima. The standard analysis of $x_i^{\,3}(1-x_i)$ shows that the function possesses two points of inflection, namely, $x_i=0$ and $x_i=\frac{1}{2}$. Because of $x_i > 0$, we focus on $x_i=\frac{1}{2}$. For $x_i<\frac{1}{2}$, the expression $x_i^{\,3}(1-x_i)$ is concave, for $x_i>\frac{1}{2}$ it is convex (see Figure 1). This results in a tangent line at $x_i=\frac{1}{2}$ that must lie above $x_i^{\,3}(1-x_i)$ for $x_i>\frac{1}{2}$, and below $x_i^{\,3}(1-x_i)$ for $x_i<\frac{1}{2}$. This fact is sufficient to elaborate further on equation (18). In order to determine the tangent line of $x_i^{\,3}(1-x_i)$ at $x_i=\frac{1}{2}$, we calculate its ordinate at $x_i=\frac{1}{2}$, i.e., $x_i^{\,3}(1-x_i)\Big|_{x_i=\frac{1}{2}}=\frac{1}{16}$, and its slope at this point, i.e., $\left[x^3(1-x_i)\right]\Big|_{x_i=\frac{1}{2}}=\frac{1}{4}$. One straight line exists, whose slope is $\frac{1}{4}$ and which passes through $\left(\frac{1}{2},\frac{1}{16}\right)$. Thus, we obtain $\alpha_i(x_i-\gamma_i)=\frac{1}{4}x_i-\frac{1}{16}$, which results in $\alpha_i = \gamma_i = 0.25$ (see Figure 1).

For $\alpha_i=0.25$ and $\gamma_i=0.25$ equation (18) exhibits only one positive solution in the specified domain (i.e., $x_i=\frac{1}{2}$). Equation (18) can exhibit more than one solution in the domain only for both $\alpha_i<0.25$ and $\gamma_i<0.25$ which, however, constitutes a necessary, not a sufficient, condition.

The parameter γ_i is well-suited for a straightforward, intuitive interpretation. It reflects the ratio between (i) the Kyoto emissions target plus the traded emission permits [K_i+y_i] and (ii) the BAU emissions level plus its absolute uncertainty [$a_i\cdot(1+R_{0,i})$]. Because of $\gamma_i=\frac{K_i}{a_i}\frac{1+T_i}{1+R_i}\approx\frac{K_i}{a_i}(1+T_i-R_i)$, where $\frac{K_i}{a_i}$ reflects the ratio of agreed Kyoto to BAU emissions and $T_i=\frac{y_i}{K_i}$ the ratio of traded emission permits to Kyoto emissions, the condition $\gamma_i<0.25$ approximately calls for about or more than a 75% reduction from the BAU emissions level. To gain further insight into the range of values for the parameters α_i and γ_i, we present the results of a simulation exercise in the next section.

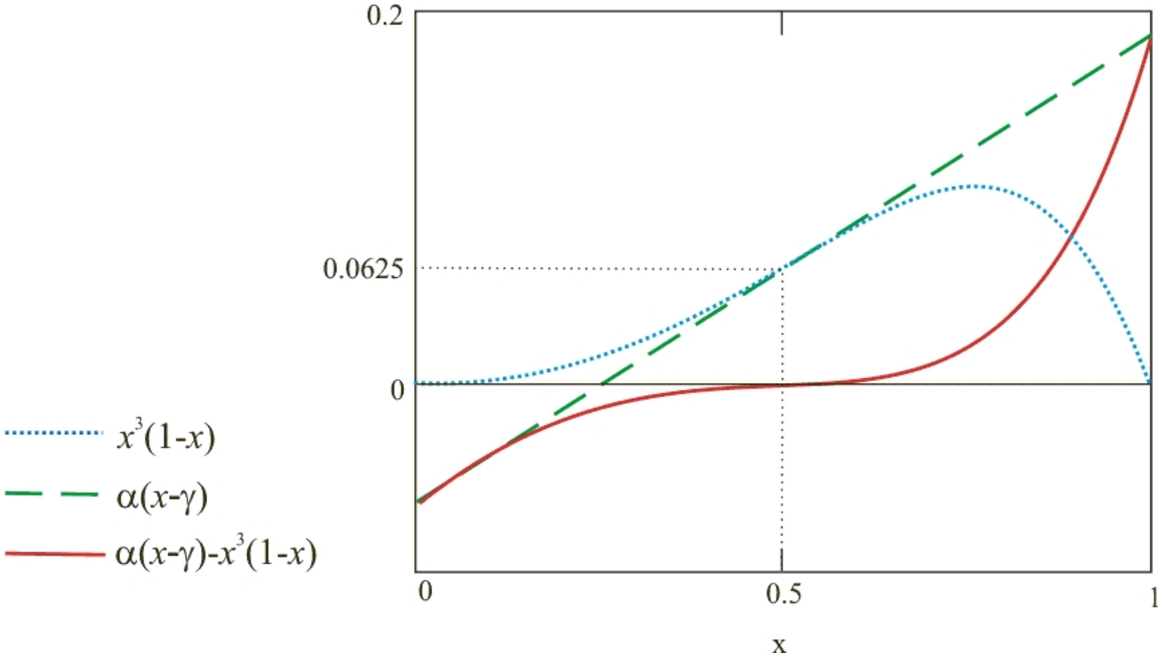


Figure 1: First derivative of the goal function $z_i(F_i)$ as well as its individual components expressed by means of a non-dimensional variable $x_i = \frac{F_i}{a_i}$ and parameters α_i, γ_i ($\alpha_i = \gamma_i = 0.25$).

5 Numerical Exercise

In the numerical part, we analyze the parameters α_i and γ_i for a simulation of the Kyoto market. Each parameter depends on the amount of permits held by a party. This implies that we need to trace the whole process of convergence—starting from the initial state and finishing at equilibrium. We use the sequential bilateral trading scheme to solve the optimization task of permit allocation on the market with the option to reduce emissions or relative uncertainty or both. We do not elaborate on the details of the convergence procedure itself as this kind of analysis has been already provided by Godal *et al.* (2003; see also Godal, 2000). However, we explain what is different in our specific simulation case. Generally, we focus on the afore-mentioned parameters which indicate whether the market can be locked in a local solution.

5.1 Data

We employ data that stem from emission abatement cost functions, derived with the help of the MERGE model and were provided by Godal and Klaassen (2003). The cost functions have been estimated for five Kyoto regions: the US; OECD Europe (OECDE); Japan; Canada, Australia and New Zealand combined (CANZ); and Eastern Europe and Former Soviet Union combined (EEFSU). In our case, linear marginal cost functions were fitted to the data. The parameters for the emission and uncertainty reduction cost functions and other key figures are shown in Table 1. Data on the initial level of uncertainty R_0 are based on Godal (2000) and Rypdal and Winiwarter (2001). The applied version of the MERGE model accounts for energy related CO_2 emissions. Monetary units refer to US dollars as of 1997.

Table 1: Kyoto target and cost function parameters for emissions and uncertainty reductions with reference to 2010. Sources: Godal and Klaassen (2003) for K_i, a_i, b_i of all Kyoto regions except CANZ; Godal (2000) and Rypdal and Winiwarter (2001) for $R_{0,i}$, $R_{0,i}$ of CANZ.

	Kyoto target	**Initial emissions (BAU)**	**Cost function parameter**	**Initial uncertainty**
Variable	K_i	a_i	b_i	$R_{0,i}$
Unit	MtC/yr	MtC/yr	MUS$/(MtC/yr)^2	1
US	1251	1820.3	0.2755	0.13
OECDE	860	1038.0	0.9065	0.20
Japan	258	350.0	2.4665	0.15
CANZ	215	312.7	1.1080	0.20
EEFSU	1314	898.6	0.7845	0.30
Total	***3898***	***4419.6***		

Note: The values are slightly different to those of the UNFCCC database.

Information of costs associated with the reduction of relative uncertainty is very limited. Therefore, we employ the simplifying assumption that costs of relative uncertainty reduction at any level R_i^1 relative to the initial uncertainty $R_{0,i}$ are dependent on costs of emission reduction according to the formula:

$$\left.\frac{\partial c_{F,i}}{\partial F_i}\right|_{F_i=F_i^1} = \left.\frac{\partial c_{R,i}}{\partial R_i}\right|_{R_i=R_i^1} \cdot \frac{1}{a_i} \tag{19}$$

with $\frac{F_i^1}{a_i} = \frac{R_i^1}{R_{0,i}}$. This formulation stems from the concept employed in (Godal *et al.,* 2003) that the marginal cost of absolute uncertainty reduction $c'_{\varepsilon,i}(\varepsilon_i)$ at any level relative to the initial uncertainty $\varepsilon_{0,i}$ is equal to the marginal cost of emission reduction $c'_{F,i}(F_i)$ at the same percentage of the BAU level, i.e.,

$$\left.\frac{\partial c_{\varepsilon,i}}{\partial \varepsilon_i}\right|_{\varepsilon_i=\varepsilon_i^1} = \left.\frac{\partial c_{F,i}}{\partial F_i}\right|_{F_i=F_i^1} \tag{20}$$

with $\frac{\varepsilon_i^1}{\varepsilon_{0,i}} = \frac{F_i^1}{a_i}$. Godal *et al.* (2003) consider the cost function of absolute uncertainty reduction $c_{\varepsilon,i}(\varepsilon_i)$, which is modeled as a downside function as well:

$c_{\varepsilon,i}(\varepsilon_i) = e_i(\varepsilon_i - \varepsilon_{0,i})^2$ for $\varepsilon_i \in [0; \varepsilon_{0,i}]$ and $c_{\varepsilon,i}(\varepsilon_i) = 0$ for $\varepsilon_i > \varepsilon_{0,i}$. Applying formula (20) they obtain

$$2b_i(F_i^1 - a_i) = 2e_i(\varepsilon_i^1 - \varepsilon_0). \tag{21}$$

Because of $F_i^1 = \frac{a_i \varepsilon_i^1}{\varepsilon_{0,i}}$, we find $e_i = \frac{a_i b_i}{\varepsilon_{0,i}}$.

In our case of relative uncertainty cost function $c_{R,I}$ in equation (19), we assume that when reducing uncertainty we keep the emission level constant, we also assume that we keep it at level a_i. Evaluating equation (19) we finally obtain

$$d_i = \frac{(a_i)^2 \cdot b_i}{R_{0,i}}. \tag{22}$$

5.2 Results

Table 2 presents the results of the first step to optimize equations (6) and (7) before any transaction has been made ($y_i = 0$ MtC/yr). Each region chooses its optimal level of emissions F_i^* and relative uncertainty R_i^*, which together ($F_i^* + F_i^* R_i^*$) contribute to the region's Kyoto target K_i. Shadow prices λ_i vary among market participants showing potential for profitable trades. Japan exhibits the highest shadow price of 527 $/tC. The EEFSU does not need to carry out any abatement in order to comply with the target. Thus, the shadow price is 0 $/tC. Hence emissions and relative uncertainty are equal to its baseline levels a_i and $R_{0,i}$, respectively.

Table 2: Situation before trade, after optimizing at a party level between emissions and uncertainty (optimization results according to equations (6) and (7) with $y_i = 0$).

	Emis-sions	**Relative uncertainty**	**Shadow price**	**Marginal cost of emission reduction**	**Marginal cost of uncertainty reduction**	**Total cost**	α_i	γ_i
Variable	F_i*	Ri*	λ_i	$c'_{F,i}(F_i^*)$	$c'_{R,i}(R_i^*)$	$c_{F,i}(F_i^*) + c_{R,i}(R_i^*)$		
Units	MtC/yr	1	$/tC	$/tC	$	MUS$	1	1
US	1134	0.102	343	-378	-388889	134 818	5.97	0.61
OECDE	738	0.164	466	-543	-344351	87 459	4.97	0.69
Japan	230	0.119	527	-590	-121384	37 119	5.65	0.64
CANZ	185	0.158	243	-282	-45132	18 842	4.13	0.57
EEFSU	898	0.30	0	0	0	0	6.34	1.12
Total						***278 238***		

As mentioned in Section 2, the shadow price λ_i reflects the willingness to pay for an additional permit, i.e., for relaxing the constraint (7) by one unit. The shadow price is different from the marginal abatement costs $c'_{F,i}\left(F_i^*\right)$ and $c'_{R,i}\left(R_i^*\right)$, which reflect the willingness to pay for an additional unit of reduced emissions or relative uncertainty, respectively (confer equation 10). Marginal costs of emission and uncertainty reduction are also depicted in Table 2. Total costs before trading amount to 278 238 MUS$.

Following equation (16), α and γ are calculated for each Kyoto region to check whether they are lower than 0.25. Regarding the parameter d_i in the cost function for uncertainty reduction (confer equation (22)), we can express α_i as:

$$\alpha_i = \frac{\left(1 + R_{0,i}\right)\left(K_i + y_i\right)}{R_{0,i} a_i}, \qquad (23)$$

that is, the ratio between (i) the emissions level ($K_i + y_i$) augmented by the relative uncertainty that refers to the BAU level ($1+R_{0,i}$), and (ii) the uncertain part of BAU emissions ($R_{0,i}\, a_i$). Table 2 shows that before trade α ranges from 4.13 (CANZ) to 6.34 (EEFSU), while γ_i ranges from 0.57 (CANZ) to 1.12 (EEFSU). γ for the EEFSU Kyoto region is greater than 1 because it does not need to strive for any abatement in order to comply with its Kyoto target. In summary, the Kyoto regions exhibit α and γ values before trade that are (considerably) greater than the threshold of 0.25.

Next, we simulate the sequential bilateral trading scheme. At each step two Kyoto regions are picked at random. For an appointed amount of permits exchanged in the step the regions compare their shadow prices λ_i. If they differ, it is mutually beneficial to exchange permits. Both regions end up with new permits y_i and thus they again optimize between emission and relative uncertainty reduction, i.e., the initial optimization step is repeated for the two parties involved in the trading. Then the next two Kyoto regions are randomly selected and the process is repeated. The procedure stops when the shadow prices of all Kyoto participants are equal. This indicates a minimum for the second optimization task. However, if $\alpha_i < 0.25$ and $\gamma_i < 0.25$ for any single participant, the determined minimum can be a local one, and not global.

Both α_i and γ_i depend linearly on y_i., the amount of traded permits. In our simulation it is virtually impossible for α_i to become as small as 0.25. Notwithstanding, we analyze the trading procedure to make sure that the conclusions are robust. Figure 2 shows the amount of emission permits y_i traded by each Kyoto participant, while Figure 3 depicts the values that α_i and γ_i take on during the transaction process.

In our case a non-convex solution does not exist. The parameter α_i is not smaller than 3 and γ_i not smaller than 0.45. The market reaches equilibrium on its own (confer Table 3). The permit shadow price settles at 252 \$/tC (see Figure 4). Figure 4 illustrates how shadow prices vary between a randomly chosen pair of buyer and seller during subsequent transactions until equilibrium is reached. As Table 3 shows, the US purchases the most (y_i = 201 MtC/yr), while the EEFSU can sell the most (y_i = -437 MtC/yr). The other Kyoto participant that can sell emission permits is CANZ (5 MtC/yr). Total abatement cost for all participants amount to 166 631 MUS$/yr.

Compared to the situation before trade (278 238 MUS$/yr), this represents a reduction of 40%.

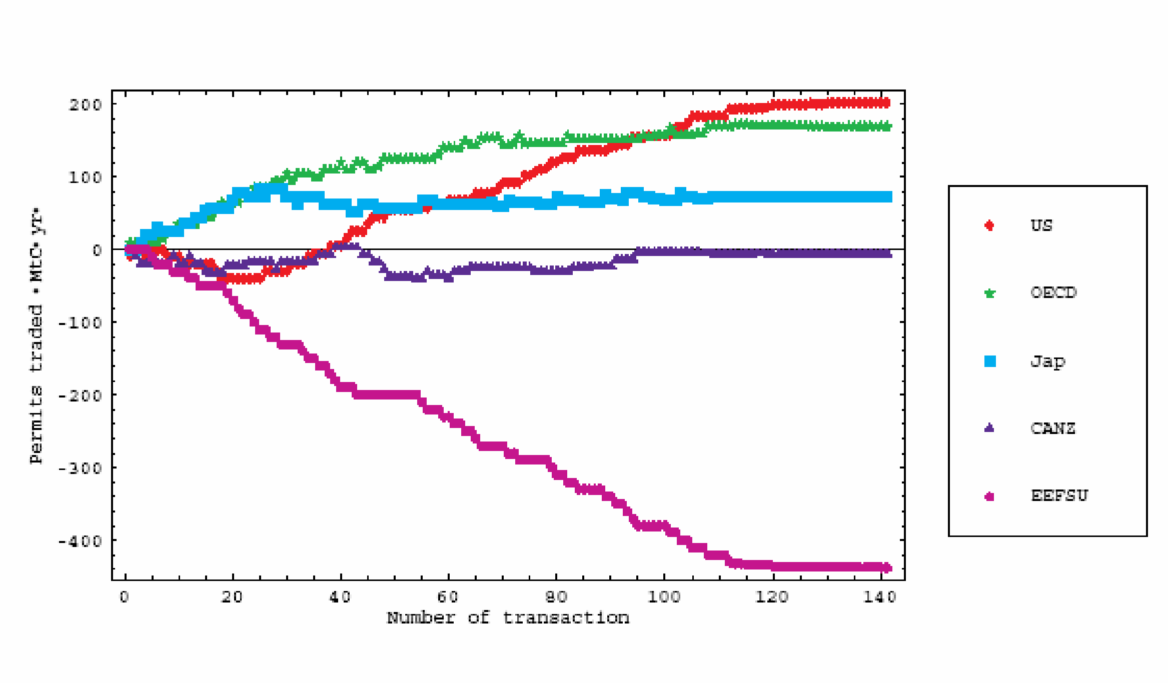
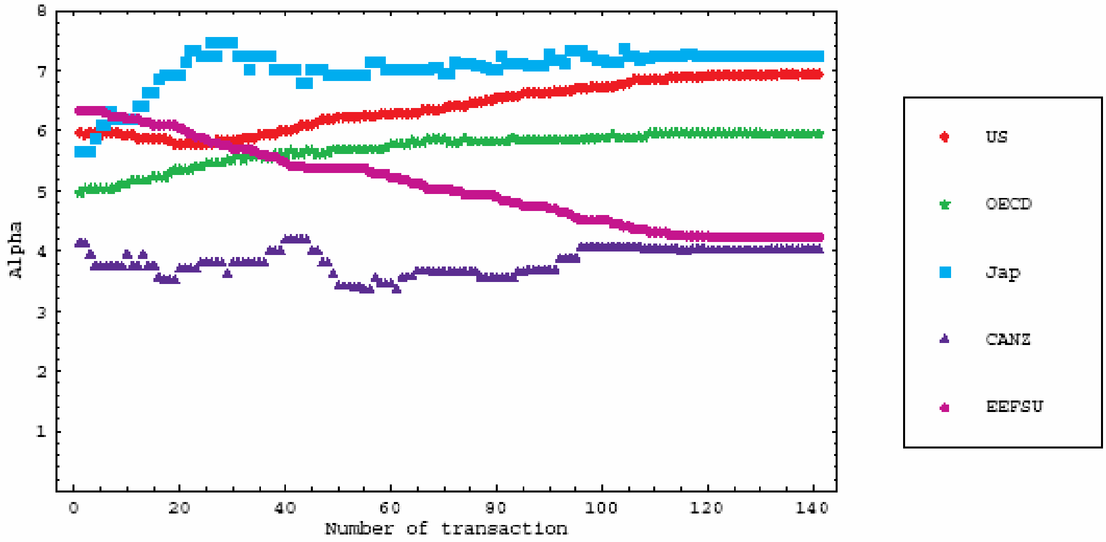


Figure 2: Traded permits y_i for five Kyoto regions. As Kyoto regions are randomly selected to trade permits bilaterally, particular realizations differ. However, they always end up in the same equilibrium (with α_i and $\gamma_i < 0.25$) as specified in Table 3, thus the same amount of traded emission permits y_i.

Table 3: With reference to Figure 2: equilibrium after sequential bilateral trade.

	Emission	**Relative uncertainty**	**Shadow price**	**Permits traded**	**Total cost**	α_i	γ_i
Variable	F_i*	R_i*	λ_i	y_i	$c_{F,i}\left(F_i^*\right)+c_{R,i}\left(R_i^*\right)$		
Units	MtC/yr	1	$/tC	MtC/yr	MUS$	1	1
US	1313	0.106	252	201	74 803	6.93	0.71
OECDE	873	0.177	252	169	26 881	5.95	0.83
Japan	292	0.131	252	72	8 946	7.23	0.82
CANZ	180	0.157	252	-5	20 250	4.03	0.56
EEFSU	696	0.258	252	-437	35 749	4.23	0.75
Total				***0***	***166 631***		



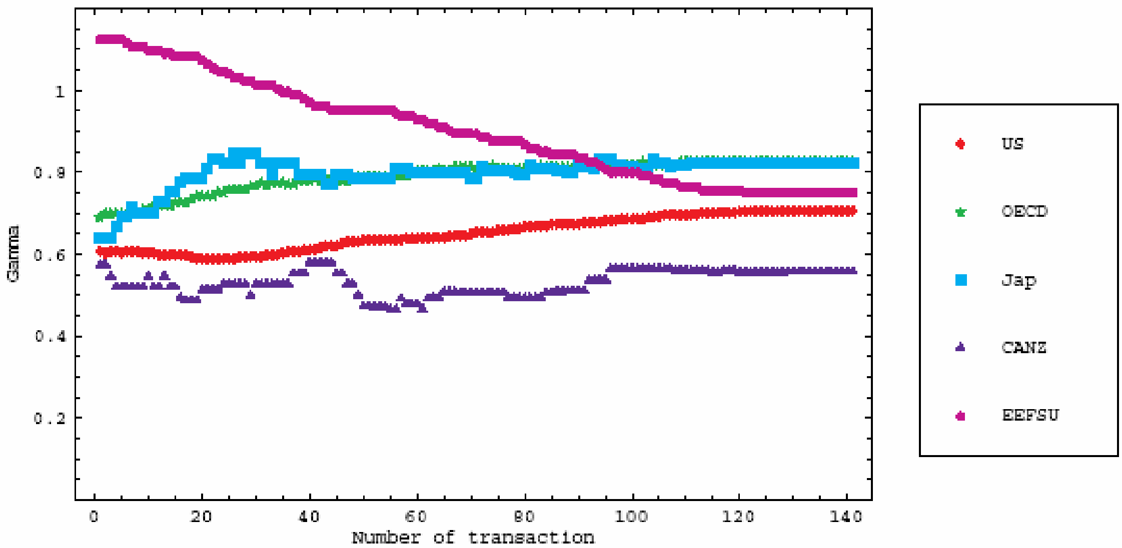


Figure 3: With reference to Figure 2: The values that the parameters α_i and γ_i take on during the transaction process.

Figure 4: Convergence of permit shadow prices at 252 $/tC for a randomly chosen pair of buyer and seller of emission permits.

It is noted that the analysis exhibits two major potential drawbacks: (1) due to the lack of data, combining countries to aggregated Kyoto regions may be too simplistic; the same is true of how costs for reducing relative uncertainty are modeled, and (2) including the US as one of the Kyoto participants, which can be considered unlikely in the foreseeable future, pushes permit prices in our simulation.

5 Concluding Remarks

Emission inventories show what should be considered under emissions trading, which is the starting point for this study. The goal of this study is to consider a market-based mechanism that encourages the reduction of inventory uncertainty taking into account the formal dependence between emission and uncertainty reduction. The rules governing the reduction of the two are the following: emissions plus (absolute) uncertainty have to be equal to the Kyoto target plus the net amount of permits. The Kyoto target can be met by buying permits or reducing emissions or reducing uncertainty in relative terms.

The most important analytical finding is that, under the outlined framework, the market may not reach the least-cost solution on its own. Instead, local minimum solutions may prevail as a consequence of non-convexities when introducing the notion of a cost function for relative uncertainty reduction. Trapping under local minimum conditions leads to an undesirable situation as a central agency with perfect knowledge about the participants' cost curves would be required.

We derive the necessary conditions for non-convexity if we assume quadratic abatement cost functions. We conclude that the Kyoto carbon market would not suffer from non-convexity. The applied procedure of sequential bilateral trade converges to the market equilibrium. The carbon market is simulated for five Kyoto regions and it should be considered as illustrative. The aggregated Kyoto target for all regions amounts to 3 898 MtC/yr. Meeting this target without trade, that is, only by considering the reduction of emissions and relative uncertainty, would cost 278 238 MUS$/yr. As a result of trading, these costs can be decreased by about 40% to 166 631 MUS$/yr.

It should be stressed that the necessary conditions for non-convexity ($\alpha < 0.25$ and $\gamma < 0.25$) are only valid for the case considered in this study, that is, the use of quadratic cost functions for the reduction of emissions and uncertainty. However, the numerical exercise exhibits values for α and γ that are far above the threshold of 0.25 (especially α being up to 7.3). It can be expected that such favorable conditions also exist when applying more general (non-quadratic) approximations for the cost functions.

To account for all Kyoto gases more appropriately, an approach that considers the calculation of costs to reduce uncertainty in relative as well as absolute terms appears to be the next step ahead. This study analyzes market convergence properties for the first case which, however, considers the formal dependence that exists between the reduction of emissions and uncertainty.

Also, the results of Godal *et al.* (2003), who simulated costs for the case of reducing uncertainty in absolute terms, should not be compared with the results provided in this study, which considers the case of reducing uncertainty in relative terms. This comparison would not be appropriate because of the different data applied. More importantly, in both studies cost functions of uncertainty reduction have been parameterized in a simplified way.

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