# Discrete Decision Problems with Large Number of Criteria

**Wierzbicki, A.P., Granat, J. and Makowski, M.**

**Interim Report** **IR-07-025**

# Discrete Decision Problems with Large Number of Criteria

*Andrzej P. Wierzbicki (a.wierzbicki@itl.waw.pl)*
*Janusz Granat  (j.granat@itl.waw.pl)*
*Marek Makowski (marek@iiasa.ac.at)*

**Approved by**

# Foreword

Practically all important decisions involve analysis of several (or even many), typically conflicting, criteria. Analysis of trade-offs between criteria is difficult because such trade-offs for most problems are practically impossible to be defined a-priori even by analysts experienced in Multi-Criteria Analysis (MCA). Therefore the trade-offs emerge during an interactive MCA which actually supports a learning process about the trade-offs. However, in some situations an objective MCA is desired, i.e., the trade-offs are elicited rather from the data than from preferences of actual decision-makers. Therefore effective MCA methods are important for actual support of decision-making processes, especially those related to policy-making.

IIASA has been developing novel methods for MCA since mid 1970s, and successfully applying them to many practical problems in various areas of applications. However, there are new practical problems for which the existing MCA methods (developed not only at IIASA but also in many centers all over the world) are not satisfactory. In particular, discrete decision problems with a large number of criteria (as compared to the number of alternatives) demand new methods. For example, MCA analysis of future energy technologies involves about 40 criteria and over 30 discrete alternatives; a careful requirement analysis of this application has proven that none of the existing MCA methods is suitable for effective support of the corresponding MCA problem.

The paper introduces new methods for MCA of discrete alternatives together with several associated concepts, including objective versus subjective decision selection, compensatory versus non-compensatory criteria, preservation of Pareto-nondominance in hierarchical aggregations. The methods are illustrated by practical examples.

# Abstract

Discrete decision problems with a large number of criteria (as compared to the number of alternatives) present specific difficulties, for example, most of decision alternatives tend to be Pareto-nondominated, some criteria might have binary character, etc. For these reasons, classical methods such as weighted sum aggregation or full utility elicitation are barely applicable for such problems. Methods that might be applied include reference point approaches, particularly if appropriately modified, equitable aggregation approaches and special hierarchical aggregation schemes.

The paper presents descriptions and necessary modifications of such methods, together with associated concepts of objective versus subjective decision selection, compensatory versus non-compensatory criteria, preservation of Pareto-nondominance in hierarchical aggregation, etc. Examples show that methods effective in most difficult cases are based on reference point approaches combined with equitable aggregation and objective decision selection.

**Keywords:** discrete decisions, large numbers of criteria, reference point approaches, equitable aggregation, objective decision selection

# Acknowledgments

# About the Authors

Andrzej Piotr Wierzbicki is research professor at the National Institute of Telecommunications, Warsaw, Poland. His current interests include vector optimization, multiple criteria and game theory approaches, parallelization of optimization algorithms, diverse aspects of negotiation and decision support, diverse issues of information society and knowledge civilization, rational theory of intuition, theories of knowledge creation and management, philosophy of technology. Recently, in cooperation with the School of Knowledge Science of Japan Advanced Institute of Science and Technology, he proposed diverse models and a synthesis of contemporary microtheories on knowledge creation.

Andrzej authored over 200 publications, including 14 books (7 monographs, 7 editorship of joint international publications), over 80 articles in scientific journals, over 100 papers at conferences; author of 3 patents granted and industrially applied.

Janusz Granat is a leader of a Division of Advanced Information Technology at the National Institute of Telecommunications. He also gives lectures on decision support systems and management information systems at the Warsaw University of Technology. His scientific interests include techno-economic analysis of the telecommunications network, broadband development, multi-criteria optimization, modeling, decision support systems and data mining. He has been involved in various industrial and scientific projects e.g., parallel computation, data warehousing and decision support systems for telecommunication industry, building data mining models for marketing departments.

Marek Makowski is a leader of IIASA Integrated Modeling Environment Project. His research interests focus on model-based support for solving complex problems, which incorporates three interlinked areas. First, integration of interdisciplinary knowledge and its representation by mathematical models. Second, creation of knowledge by comprehensive model analysis, including multicriteria methods. Third, tailoring the modeling process to meet the needs of decision-making processes. Thus Marek's research interests cover a cluster of areas relevant to the adaptation (whenever possible) or development (when needed) of methodology, algorithms, and software for model-based decision-making support. This includes more specific topics in Operations Research (OR) such as: multicriteria problem analysis, large scale optimization, optimization of badly conditioned problems, use of database management systems for complex models, decision analysis and support, user interfaces in decision support systems, effective treatment of uncertainty and risk.

Marek has published over 130 papers and book-chapters, coordinated or led several scientific projects, and has been twice guest editor of the European Journal of Operational Research.

# Contents

# List of Tables

# List of Figures

# Discrete Decision Problems with Large Number of Criteria

*Andrzej P. Wierzbicki*[*] *(a.wierzbicki@itl.waw.pl)*
*Janusz Granat*[* **] *(j.granat@itl.waw.pl)*
*Marek Makowski*[**] *(marek@iiasa.ac.at)*

## 1    Motivation and scope of the paper

There are many decision problems that involve very large numbers of criteria - for example, facility location problems with very large numbers of customers, dynamic optimization problems with objective functions in every time interval treated as separate criteria, some formulations of ranking technologies of energy production, and many others. The problem of large numbers of criteria was addressed before - e.g., as a problem of multiobjective trajectory optimization, see (Wierzbicki 1977, 1988), or a large number of criteria in engineering design, see (Nakayama 1994); these approaches deal however with continuous decision variables, therefore the corresponding decision problems have infinite number of alternative solutions.

On the other hand, if the decisions are discrete and we select between a given number of alternatives (or classify, rank them, etc.; these are quite different problems, but we shall discuss these distinctions later), then the situation changes substantially. In some applications (for example, if the criteria are selected in a discourse, by a political process) the number of criteria might even exceed, sometimes significantly, the number of alternatives. This has several important consequences:

- First, almost all alternatives tend to be Pareto-nondominated: for most alternative decisions with a large number of criteria we can usually find several such criteria that cannot be improved by other alternatives without worsening other criteria values.

- Second, almost all alternatives tend to correspond to the best values of some criteria (this implies Pareto-nondomination, hence it is actually a stronger property than the first one): each criterion has its best value at some alternative and only for very degenerated or artificial examples these best values would be concentrated at a very small number of alternatives.

- Third, almost all alternatives tend to correspond to the worst values of some (obviously other than above) criteria, by a similar argument. This is especially the case, if the criteria have binary character - their values are either very large, or very low.

---

[*]National Institute of Telecommunications, Szachowa 1, 04-894 Warsaw, Poland.
[**]Integrated Modeling Environment Project, IIASA.

- Fourth, if all alternatives are Pareto-nondominated, then the worst values of criteria define the nadir point (since the nadir point can be equivalently defined as the collection of worst values, restricted to non-dominated alternatives - see, e.g., Ehrgott and Tenfelde-Podehl 2000). This is related to the difficulty that, when we want to distinguish the worst alternative; we might have several alternatives corresponding to the worst values, each for several criteria.

Therefore, discrete decision problems with large numbers of criteria deserve special attention and might require special analysis and solution methods. We review in this paper some methods that might be applicable and some classical or popular methods that are definitely not applicable for such a case. Between methods that are not applicable for diverse reasons, we mention classical methods based on weighted sum aggregation or on nonlinear utility identification. We discuss the use of weighting factors understood as scaling factors, or as classical preferential weighting coefficients, or as importance factors, utilized differently than weighting coefficients. Between applicable methods, we distinguish *reference point approach*, possibly accompanied by an objectifying definition of aspiration and reservation levels; *equitable aggregation* or lexicographic minimax approach, either based on Lorentz ordering of criteria or on ordered weighted averaging (OWA); and *hierarchical aggregation of criteria* (applicable, however, only under rather restrictive assumptions). We compare these approaches on some examples and derive conclusions.

## 2   Basic formulations and assumptions

We assume that we have a decision problem with $n$ criteria, indexed by $i = 1, \ldots, n$ (also denoted by $i \in I$), and $m$ alternative decisions called also alternatives, indexed by $j = 1, \ldots, m$ or $j = A, B, \ldots, H$ (also denoted by $j \in J$). The corresponding criteria values are denoted by $q_{ij}$; we assume that all are maximized or converted to maximized variables. The maximal values

$$\max_{j \in J} q_{ij} = q_i^{up}$$

are called upper bounds for criteria, and are equivalent to the components of so-called ideal or utopia point $\boldsymbol{q}^{uto} = \boldsymbol{q}^{up} = (q_1^{up}, \ldots, q_i^{up}, \ldots, q_n^{up})$ - except for cases when they were established a priori as a measurement scale, see further comments. The minimal values

$$\min_{j \in J} q_{ij} = q_i^{lo}$$

are called lower bounds and, generally, are not equivalent[1] to the components of so called nadir point $\boldsymbol{q}^{nad} \geq \boldsymbol{q}^{lo} = (q_1^{lo}, \ldots, q_i^{lo}, \ldots, q_n^{lo})$.

An alternative $j \in J$ is Pareto optimal (Pareto-nondominated or shortly nondominated, also called efficient), if there is no other alternative $j \in J$ that dominates $j^*$, that is, if we denote $\boldsymbol{q}_j = (q_{1j}, \ldots, q_{ij}, \ldots, q_{nj})$, there is no $j \in J$ such that $\boldsymbol{q}_j \geq \boldsymbol{q}_{j*}, \boldsymbol{q}_j \neq \boldsymbol{q}_{j*}$.

While there is an extensive literature how to select the best alternative (usually between nondominated ones) or to rank or classify all alternatives in response to the preferences of a decision maker, this literature usually makes several tacit assumptions:

---

[1] The nadir point $\boldsymbol{q}^{nad}$ is defined similarly as the lower bound point $\boldsymbol{q}^{lo}$, but with the minimization restricted to the Pareto optimal or efficient or nondominated alternatives.

1. A standard and usually undisputed assumption is that there is a *decision maker* (DM) that *does not mind to reveal her/his preferences* - either *a priori*, before the computer system proposes her/his supposedly best decision (in this case, we should actually not speak about *decision support*, only about *decision automation*), or *interactively*, exchanging information with a computerized decision support system (in this case, truly supporting decisions). In group decision making, it is often assumed that the group members do not mind discussing their preferences. However, highly political decision makers might intuitively (using their experience in political negotiations) refuse to discuss their preferences, and do not have time for a long interaction with the decision support system. There are also many rational reasons why a decision maker might want to obtain an advice on the best decision or ranking of decisions that is *as objective as possible*, thus independent from her/his preferences, particularly if the final decision will be highly political, or there is actually a large group of decision makers or *stakeholders* in the decision situation.

2. Another standard and usually undisputed assumption is that there is an *analyst* that knows well decision theory and practice, interacts with decision makers on the correct definition and modeling of the decision situation, thus influences e.g., the choice of criteria, further programs or fine-tunes the decision support system, etc. (even if the role of the analyst might be hidden just by an assumed approach used for constructing the decision support system). However, the role of an analyst is essential even if it should not be dominant; for example, the choice of criteria might be a result of a political process, and even if the analyst would know the extensive literature how to select criteria reasonably from decision theoretical point of view, she/he has just to accept even unreasonable criteria.

In further discussions, we assume that there are decision makers and analysts, but their roles should be interpreted more broadly than in standard approaches.

# 3   Why classical approaches are not applicable in this case

We discuss here two classes of methods taught usually - for historical reasons - as "the basic approach" to multiple criteria decision making. The first of them is the weighted sum aggregation of criteria: determining by diverse approaches, between which the AHP (Saaty 1982) is one of the most developed, weighting coefficients $\omega_i$ for all $i \in I$, with the additional requirement on the scaling of weighting coefficients that

$$\sum_{i \epsilon I} \omega_i = 1,$$

and then using them to aggregate all criteria by a weighted sum:

$$\sigma_j^{sum} = \sum_{i \epsilon I} \omega_i q_{ij} \tag{1}$$

We use the aggregated values $\sigma_j^{sum}$ to select the best alternative (maximizing $\sigma_j^{sum}$ over $j \in J$) or to rank alternatives (ordering them from the largest to the lowest value of $\sigma_j^{sum}$). Such an aggregation might be sometimes necessary, but it has several limitations, particularly for large numbers of criteria. The most serious between them are the following:

1. The weighted sum is based on a tacit (unstated) assumption that a compensatory trade-off analysis is applicable to all criteria: a worsening of the value of one criterion might be compensated (at the rates constant for the whole ranges of criteria values) by the improvement of the value of another one. While often encountered in economic applications, this compensatory character of criteria is usually not encountered in interdisciplinary applications.

2. Changes of weighting coefficients in interactive decision processes with more than two criteria often lead to counter-intuitive changes of criteria values (Nakayama 1995); with large numbers of criteria, this might result in totally unpredictable changes of results.

3. The linear aggregation of preferences expressed by the weighted sum tends to promote decisions with unbalanced criteria, as illustrated by the Korhonen paradox quoted below; in order to accommodate the natural human preference for balanced solutions, a nonlinear aggregation is necessary.

The Korhonen paradox[2] can be illustrated by the following example. Suppose we select a product and consider two criteria: quality and cost, while using an assessment scale 0 - 10 points for both criteria (0 points for cost means very expensive, 10 points means very cheap products). Suppose we have three alternative decisions:
• alternative A has 10 points for quality, 0 points for cost,
• alternative B has 0 points for quality, 10 points for cost,
• alternative C has 4.5 points for quality and 4.5 points for cost.

It is easy to prove that when using a weighted sum for ranking the alternatives, alternative C will be never ranked first - no matter what weighting coefficients we use. Thus, weighted sum indeed tends to promote decisions with unbalanced criteria values; in order to obtain a balanced solution (the first rank for alternative product C), we have either to use additional constraints or a nonlinear aggregation scheme.

Educated that weighting coefficients methods are basic, the legislators in Poland introduced a public tender law. This law requires that any institution preparing a tender using public money should publish beforehand all criteria of ranking the offers and all weighting coefficients used to aggregate the criteria. This legal innovation backfired: while the law was intended to make public tenders more transparent and accountable, the practical outcome was opposite because of effects similar to the Korhonen paradox. Organizers of the tenders soon discovered that they are forced either to select the offer that is cheapest and worst in quality or the best in quality but most expensive one. In order to counteract, they either limited the solution space drastically by diverse side constraints (which is difficult but consistent with the spirit of the law) or added additional poorly defined criteria such as the degree of satisfaction (which is simple and legal but fully inconsistent with the spirit of the law, since it makes the tender less transparent and opens hidden door for graft).

The example of counter-intuitive effects of changing weighting coefficients given by Nakayama is simple: suppose $n = 3$ and the criteria values for many alternatives are

---

[2]Not published by Pekka Korhonen in print, but discussed many times at conferences, the original formulation of this Paradox was not quite acceptable for feminists: it involved criteria such as sex-appeal and ability to cook when selecting a partner of life.

densely (or continuously) spread over the positive part of the surface of a sphere,[3] $q_1^2 + q_2^2 + q_3^2 = 1$. Suppose we select first $w_1 = w_2 = w_3 = 1/3$, which results in the best alternative with criteria values $q_1 = q_2 = q_3 = 0.5770$. Suppose we want next to increase the values of $q_1$ strongly and of $q_2$ slightly, while agreeing to decrease $q_3$; what modifications of weighting coefficients would do the job? If we choose $w_1 = 0.55$, $w_2 = 0.35$ and $w_3 = 0.1$, the result will be a strong increase of $q_1 = 0.8338$ accompanied by a decrease of both $q_2 = 0.5306$ and $q_3 = 0.1516$; in order to increase $q_1$ strongly and $q_2$ slightly we must increase $w_2$ almost as strongly as $w_1$. If we have more criteria, it might be sometimes very difficult to choose a change of weighting coefficients resulting in a desired change of criteria values.

Both such theoretical examples and recent practical experience presented above show that we should be very careful when using weighted sum aggregation. In short summary, a linear weighted sum aggregation is simple mathematically but too simplistic in representing typical human preferences that are usually nonlinear; using this simplistic approach resulted in practice in adverse and unforeseen side-effects. Thus, we should rather look for nonlinear approximations of the preferences of decision makers. There are many highly developed methods of the elicitation of nonlinear utility or value functions, see e.g. (Keeney and Raiffa 1972), (Keeney 1992). However, these classical methods might be also not applicable for decisions involving political processes and large numbers of criteria, because of several reasons:

1. As noted above, politically minded decision makers might be adverse to a disclosure and detailed specifications of their preferences;

2. Such elicitations of utility or value functions require a large number of pairwise comparisons of alternatives, done in the form of questions addressed to the decision maker and her/his answers; this number is nonlinearly growing with the number of criteria, thus such elicitations are not practical for problems with large numbers of criteria.

For these and other reasons, we should further look for more ad hoc and rough nonlinear approximations of preferences of decision makers, which do not require much time nor a detailed specification or identification of preferences.

# 4 Reference point approaches, objective ranking and equitable aggregations

The large disparity between the opposite ends of the spectrum of preference elicitation - full value or utility identification versus a weighted sum approach - indicates the need for a middle-ground approach, simple enough and easily adaptable but not too simplistic.

---

[3]The fact that we use a nonlinear function - the equation of the surface of a sphere - in this example is not essential: we could as well approximate the sphere by a regular polyhedron with sufficiently many facets and thus represent this example as a linear multiobjective optimization problem.

## 4.1   Reference point approaches

The first type of such rough approximation of decision maker preferences is provided by reference point approaches. In these approaches, we note that:

1. The preferences of decision maker can be approximated using several degrees of specificity, and the reference point approaches assume that this specification should be as general as possible, since a more detailed specification violates the sovereign right of a decision maker to change her/his mind.

2. The most general specification of preferences contains a selection of outcomes of a model of decision situation that are chosen by the decision maker (or analyst) to measure the quality of decisions, called criteria (quality measures, quality indicators) or sometimes objectives (values of objective functions) and denoted here by $q_i$, $i \in I$. This specification is accompanied by defining a partial order in the space of criteria - simply asking the decision maker which criteria should be maximized and which minimized, while another option, stabilizing some criteria around given reference levels, is also possible in reference point approaches, see (Wierzbicki et al. 2000). Here we consider the simplest case when all criteria are maximized.[4]

3. The second level of specificity in reference point approaches is assumed to consist of specification of *reference points* - generally, *desired levels of criteria*. These reference points might be interval-type, double, including *reservation levels*, denoted here by $r_i$ (worst levels of criteria values still acceptable for the decision maker) and *aspiration levels* $a_i$ (levels of criteria values that should be achieved according to the decision maker). Specification of reference levels is treated as an alternative to trade-off or weighting coefficient information that leads usually to linear representation of preferences and unbalanced decisions as discussed below, although some reference point approaches - see, e.g., (Nakayama 1994), (Granat, Makowski 2000), (Ruiz et al. 2007) - combine reference levels with trade-off information.

4. While the detailed specification of preferences might include full or gradual identification of utility or value functions, as shortly indicated above, this is avoided in reference point approaches that stress *learning instead of value identification* - according to the reference point philosophy, the decision maker should learn during the interaction with a decision support system, hence her/his preferences might change in the decision making process and she/he has full, sovereign right or even necessity to be inconsistent.

5. Thus, instead of a nonlinear value function, reference point approaches approximate the preferences of the decision maker by a nonlinear *achievement function which is an ad hoc, easily adaptable nonlinear approximation of the value function of decision maker consistent with the information contained in criteria specification, their partial order and the position of reference point (or points) between the lower and upper bounds for criteria.* As opposed to goal programming, similar in

---

[4]This assumption is made only in order to simplify the presentation of diverse methods of decision analysis; but all these methods can be easily adapted to analysis of problems with criteria of different types, including minimized, maximized, achieving a given target value, following a specified trajectory; see e.g., (Makowski 1994).

approach to reference point methods but using distance concepts instead of achievement functions, the latter functions preserve strict monotonicity with respect to the partial order in criteria space.

6. The particular form of this nonlinear approximation of value function is determined essentially by max-min terms that favor solutions with balanced deviations from reference points and express the Rawlsian principle of justice (concentrating the attention on worst off members of society or on issues worst provided for, see (Rawls 1971); these terms are slightly corrected by regularizing terms, resulting in nondomination (Pareto optimality) of alternatives that maximize achievement functions. It can be shown (Wierzbicki 1986) that such achievement functions have the property of full controllability, independently of convexity assumptions. This means that, also for discrete decision problems, any nondominated (Pareto optimal) alternative can be selected by the decision maker by modifying reference points and maximizing the achievement function; this provides for the *full sovereignty of the decision maker*, see e.g., (Wierzbicki et al., 2000).

While there are many variants of reference point approaches, see (Makowski, 1994), (Miettinen, 1999), (Granat, Makowski, 2000), (Ruiz et al. 2007), we concentrate here on a reference point approach that requires the specification of interval-type reference, that is, two reference levels (aspiration and reservation) for each criterion. After this specification, the approach uses a nonlinear aggregation of criteria by an achievement function that is performed in two steps.

- We first compute achievements for each individual criterion or satisfaction with its values by transforming it (monotonically and piece-wise linearly) e.g., in the case of maximized criteria as shown in Eq. (2) below. For problems with a continuous (nonempty interior) set of options, for an easy transformation to a linear programming problem, such a function needs additional specific parameters selected to assure the concavity of this function, see (Wierzbicki et al. 2000). In a discrete decision problem, however, we do not necessarily need concavity and can choose these coefficients to have a reasonable interpretation of the values of the *partial (or individual) achievement function*. Since the range of [0; 10] points is often used for eliciting expert opinions about subjectively evaluated criteria or achievements, we adopted this range in Eq. (2) below for the values of a partial achievement function $\sigma_i(q_i, a_i, r_i)$:

$$\sigma_i(q_i, a_i, r_i) = \begin{cases} \alpha(q_i - q_i^{lo})/(r_i - q_i^{lo}), & \text{for} \quad q_i^{lo} \leq q_i < r_i \\ \alpha + (\beta - \alpha)(q_i - r_i)/(a_i - r_i), & \text{for} \quad r_i \leq q_i < a_i \quad (2) \\ \beta + (10 - \beta)(q_i - a_i)/(q_i^{up} - a_i), & \text{for} \quad a_i \leq q_i \leq q_i^{up} \end{cases}$$

The parameters $\alpha$ and $\beta$, $0 < \alpha < \beta < 10$, in this case denote correspondingly the values of the partial achievement function for $q_i = r_i$ and $q_i = a_i$. The value $\sigma_{ij} = \sigma_i(q_{ij}, a_i, r_i)$ of this achievement function for a given alternative $j \in J$ represents the satisfaction level with $q_{ij}$ (i.e., the $i$-th criterion value for $j$-th alternative). Thus, the above transformation assigns satisfaction levels from 0 to $\alpha$ (say, $\alpha = 3$) for criterion values between $q_i^{lo}$ and $r_i$, from $\alpha$ to $\beta$ (say, $\beta = 7$) for criterion values between $r_i$ and $a_i$, from $\beta$ to 10 for values between $a_i$ and $q_i^{up}$.

- After this transformation of all criteria values, we might use then the following form of the overall achievement function:

$$\sigma(\boldsymbol{q}, \boldsymbol{a}, \boldsymbol{r}) = \min_{i \in I} \sigma_i(q_i, a_i, r_i) + \epsilon/n \sum_{i \epsilon I} \sigma_i(q_i, a_i, r_i) \qquad (3)$$

where:

$\boldsymbol{q} = (q_1, \ldots, q_i, \ldots, q_n)$ is the vector of criteria values,

$\alpha = (\alpha_1, \ldots, \alpha_i, \ldots, \alpha_n)$ and $\boldsymbol{r} = (r_1, \ldots, r_i, \ldots, r_n)$ are the vectors of aspiration and reservation levels, respectively,

$\epsilon > 0$ is a small regularizing coefficient.

The achievement values $\sigma_j = \sigma(\boldsymbol{q}_j, \boldsymbol{a}, \boldsymbol{r})$ for all $j \in J$ can be used either to select the best alternative, or to order the options in an overall ranking list or classification list, starting with the highest achievement value.

The formulae (2), (3) do not express the only form of an achievement function; there are many possible forms of such functions as shown in (Wierzbicki et al. 2000). All of them, however, have an important property of partial order approximation: their level sets approximate closely the positive cone defining the partial order in criteria space (see Wierzbicki 1986). As indicated above, the achievement function has also a very important theoretical property of controllability, not possessed by utility functions nor by weighted sums: for sufficiently small values of $\epsilon$, given any point $\boldsymbol{q}^*$ in criteria space that is ($\epsilon$-properly) Pareto-nondominated[5] and corresponds to some alternative decision (such as the alternative C in the Korhonen paradox), we can always choose such reference levels - in fact, it suffices to set aspiration levels equal to the components of $\boldsymbol{q}^*$ - that the maximum of the achievement function (3) is attained precisely at this point. Conversely, if $\epsilon > 0$, all maxima of achievement function (3) correspond to Pareto-nondominated alternatives - because of the monotonicity of this function with respect to the partial order in the criteria space, similarly as in the case of utility functions and weighted sums, but not in the case of a distance norm used in goal programming, since the norm is not monotone when passing zero. As noted above, precisely the controllability property results in a fully sovereign control of the decision support system by the user.

## 4.2 Objective selection and ranking; importance coefficients

We already indicated that the decision maker in diverse situations might be reluctant to specify her/his preferences and would rather consider then a most objective selection of the best alternative or ranking or classification of all alternative decisions - while reserving the right to correct the selection or classification in her/his final decision; such are, in fact,

---

[5]By an $\epsilon$-properly Pareto-nondominated alternative we understand a Pareto-nondominated alternative with trade-off coefficients bounded by the number $1 + 1/\epsilon$, see Wierzbicki et.al. (2000). The property that any $\epsilon$-properly Pareto-nondominated alternative can be selected as the best by maximizing an achievement function is called the controllability property and is much stronger then the efficiency property (that any maximum of a function, which is strictly monotone with respect to the partial order, is Pareto-nondominated). The controllability property is possessed by functions such as (3) that are not only strictly monotone with respect to the partial order, but also have level sets approximating the positive cone that defines the partial order. This property does not depend on convexity assumptions, see Wierzbicki (1986).

the most frequent situations in any political decision making. Thus, *objective selection or even ranking* is often required in practical decision making.

However, most decision theory is concentrated on *subjective selection and ranking*, where *subjective* does not mean *intuitive or a-rational*, but rational although concentrated on the preferences of the decision maker. This has diverse historical and also some deep philosophical reasons; we add here some comments on them. The industrial era episteme - sometimes called not quite precisely positivism or scientism - valued objectivity; today we know that absolute objectivity does not exist, see e.g., (Wierzbicki 2005, 2007). The destruction of this episteme started early, e.g., since Werner Heisenberg (Heisenberg 1927) has shown that not only a measurement depends on a theory and on instruments, but also the very fact of measurement distorts the measured variable. This was followed by diverse philosophical debates, summarized, e.g., by Van Orman Quine (Quine 1953) who has shown that the logical empiricism (neo-positivism) is logically inconsistent itself, that all human knowledge "is a man-made fabric that impinges on existence only along the edges". This means that there is no absolute objectivity; however, this was quite differently interpreted by hard sciences and by technology, which nevertheless tried to remain as objective as possible, and by social sciences which, in some cases, went much further to maintain that all knowledge is subjective - results from a discourse, is constructed, negotiated, relativist, depends on power and money, see, e.g., (Latour 1987). This has led to a general divergence of the episteme - the way of constructing and justifying knowledge - of the three different cultural spheres (of hard and natural sciences, of technology, and of social sciences and humanities), see (Wierzbicki 2005). Full objectivity is obviously - after Heisenberg and Quine - not attainable; *but in many situations we must try to be as much objective as possible*. This concerns not only technology that cannot advance without trying to be objective and, in fact, pursues Popperian falsificationism (Popper 1972) in everyday practice when submitting technological artifacts to destructive tests in order to increase their reliability - while post-modern social sciences ridicule falsificationism as an utopian description how science develops. However, objectivity is needed also - as we show here in further examples - in management. For an individual decision maker, this might mean that she/he needs some independent reasons for selection or ranking, such as a dean cannot rank the laboratories in her/his school fully subjectively, must have some reasonable, objective grounds that can be explained to entire faculty. For a selection or ranking that expresses the preferences of a group, diverse methods of aggregating group preferences might be considered; but they must be accepted as fair - thus objective in the sense of inter-subjective fairness - by the group.

However, it is not obvious how to define the grounds of an objective selection or ranking. In multiple criteria optimization, one of similar issues was to propose *compromise solutions*, see, e.g., (Zeleny 1974); however, such solutions might depend too strongly on the assumed metric of the distance from the utopia or ideal point. (Wierzbicki 2006) proposes to define objective selection and ranking as dependent only on a given set of data, agreed upon to be relevant for the decision situation and independent of any more detailed specification of personal preferences than that given by defining criteria and the partial order in criteria space. The specification of criteria and their partial order (whether to minimize, or maximize them) can be also easily be agreed upon, be objective in the sense of inter-subjective fairness.

It is also not obvious how an objective selection and ranking might be achieved, be-

cause almost all the tradition of aggregation of multiple criteria concentrated on rational subjective aggregation of preferences and thus subjective selection and ranking. While we could try, in the sense of inter-subjective fairness, identify group utility functions or group weighting coefficients, both these concepts are too abstract to be reasonably debated by an average group (imagine a stakeholder meeting trying to define their aggregate utility function under uncertainty). Thus, neither of these approaches is easily adaptable for rational objective selection or ranking. The approach that can be easily adapted for rational objective selection and ranking, also classification, is reference point approach as described above, because reference levels needed in this approach can be either defined subjectively by the decision maker, or established statistically - as objectively as possible - from the given data set.

Now, the question is: how to define aspiration and reservation levels in order to obtain rational objective ranking? Several ways were listed in (Granat et al. 2006): neutral, statistical, voting; we shall concentrate here on statistical determination. A statistical determination of reference levels concerns values $q_i^{av}$ that would be used as basic reference levels, a modification of these values to obtain aspiration levels $a_i$, and another modification of these values to obtain reservation levels $r_i$; these might be defined (for the case of maximization of criteria) as follows:

$$q_i^{av} = \sum_{j \in J} q_{ij}/m; \qquad r_i = 0.5(q_i^{lo} + q_i^{av}); \qquad a_i = 0.5(q_i^{up} + q_i^{av}) \qquad (4)$$

Recall that $m$ is just the number of alternative decision options, hence $q_i^{av}$ is just an average (for all alternatives) criterion value; the aspiration and reservation levels are just averages of these averages and the lower and upper bounds, respectively. However, as shown by examples presented later, there are no essential reasons why we should limit the averaging to the set of alternative options ranked; we could use as well a larger set of data in order to define more adequate (say, historically meaningful) averages, or a smaller set - e.g., only the Pareto-nondominated alternatives. Even if the decision maker might require first advice that is as objective as possible, she/he might also want to have some interaction with the computerized decision system. This interaction might be organized in different ways. Classically, when supporting an analysis of Pareto-nondominated solutions, we used (see, e.g., Granat and Makowski 2000) changes of aspiration and reservation points through a graphical interface; this interface, called ISAAP, was used in many and diverse applications and found very effective. Weighting coefficients might be also used, but allowing for diverse interpretation of their meaning. (Ruiz et al. 2007) analyzes in detail the relations of weighting coefficients to changes of reference points, while taking into account two functions of weighting factors: the scaling function and the preferential function (the former one converts all criteria ranges to a common, say, percentage scale, the latter expresses the preferences of the decision maker). Many versions of scaling and preferential properties of diverse reference point approaches are analyzed in that paper, but interval-type reference point approaches - found by us the most effective in diverse applications - are not analyzed in detail there.

Here we propose another interpretation of weighting factors - we suggest using them as *importance factors* that influence the shape of achievement functions rather than as weighting coefficients that are used directly to modify criteria values by multiplication. Thus, we limit the use of classical weighting coefficients to their scaling role. However,

the decision maker can express her/his wishes by specifying importance factors for criteria, e.g., using the scale *very low importance - low importance - normal importance - high importance - very high importance*. Afterwards, the partial achievement functions are computed as in Eq. (2), but with parameters $\alpha$ and $\beta$ depending on importance factors, e.g., as specified by the following table:

| importance factor | very low | low | normal | high | very high |
|---|---|---|---|---|---|
| $\alpha$ | 5 | 4 | 3 | 2 | 1 |
| $\beta$ | 9 | 8 | 7 | 6 | 5 |

Table 1: Parameter values for partial achievement functions depending on importance factors.

This modification of achievement values might seem counter-intuitive: we *increase the achievement for criteria with lower importance*. This is consistent, however, with the form of the overall achievement function as given in Eq. (3): its value is determined mostly by the worst partial achievements, hence more important are criteria which have lower achievements. We propose here a linear scale of modifying achievement values, even if (Lootsma 1996) indicated that weighting coefficients should be spaced nonlinearly (e.g., using an exponential scale) when they are implied by importance factors. Our experience with modifying aspiration and reservation levels also shows their strongly nonlinear impact on the selection of solutions when they are becoming close to the upper or lower bounds for criteria. We expect similar nonlinear effect of changing the parameters $\alpha$ and $\beta$; obviously, the ranges of changes suggested in Table 1 might be modified if desired.

Thus, we are ready to propose one basic version of an objectified reference point approach for discrete decision alternatives with very large numbers of criteria and possibly political character of the decision process. Here are our advises for the analyst:

1. Accept the criteria and their character (which to maximize, which to minimize) proposed by decision maker(s), but insist on a reasonable definition of their upper and lower bounds.[6]

2. Gather (the evaluation of) all criteria values for all alternative decisions. In the case that some criteria have to be assessed by expert opinions, organize an objectifying process for these assessments (e.g., voting on these assessments as if judging ski-jumping, with deleting extreme assessments or even with using median score,[7] allowing for a dispute and a repeated vote in cases of divergent assessments).

3. Compute the averages of criteria values, the statistically objective reservation and aspiration points as in Eq. (4). Assuming the neutrality of all criteria ($\alpha = 3$ and $\beta = 7$ for all criteria) and using the achievement functions as defined by Eq. (2),

---

[6]It is not reasonable to set upper and lower bounds equal to infinity and minus infinity, respectively; or to their arbitrary approximations e.g., by $10^{10}$; consider for example the problem of nuclear waste: it will most probably not last say $10^{10}$ years, because even after 50 years we might devise quite new ways of its disposal, say, sending it with space vehicles into the Sun.

[7]Median score is equivalent to repeated deletion of extreme assessments, until only one or two assessments remain (taking the average between the two in the latter case).

(3), compute achievement factors $\sigma_j$ for all alternatives and order alternatives in a decreasing fashion of these factors (say, randomly if $\sigma_j = \sigma_j$' for some j and j'; we shall suggest in the next Section a way of improving such ordering). Use this ordering either for a suggested (objective and neutral) selection of the best alternative, or a classification of alternatives (say, into projects accepted and rejected), or an objective and neutral ranking.

4. Discuss with decision maker(s) the suggested objective and neutral outcome. If she/he wants to modify it, suggest a definition of importance factors for every criterion, and use them to define (e.g., as proposed in Table 1) the values of parameters $\alpha$ and $\beta$ for the corresponding criterion. Then, re-compute achievement factors $\sigma_j$ as defined by Eq. (2), (3) with $\alpha$ and $\beta$ individualized for every criterion; use these factors for ordering, selecting, classifying or ranking the alternatives. If there are many decision makers, they might either vote on importance factors for every criterion using an objectifying process as in point 2 above, or investigate first individually the impact of changing importance factors, then debate the resulting issues and prepare a vote on final importance factors.

## 4.3 Equitable aggregation and lexicographic minimax approach

The decision process described above needs to be modified in case when two or many alternatives have the same achievement factors $\sigma_j$, particularly if these factors are close to zero: when there are many criteria, many alternatives might have one or several criteria close to their lower bound, thus $\sigma_j$ might be close to zero. In such cases, instead of increasing $\epsilon$ and letting actually the sum in Eq. (3) determine the aggregated achievement, it is better to set $\epsilon = 0$ or close to 0 and use one of a group of approaches related to equitable aggregation, see (Ogryczak 2006), (Kostreva et al. 2003). In such approaches we consider first the minimal, worst individual criterion achievement $\sigma_j$ computed as in Eq. (2) with $\epsilon = 0$; if, however, two (or more) alternatives $j$ and $j'$ have the same worst achievement value, we order them according to the second worst individual criterion achievement, and so on. This approach is called *lexicographic minimax*; however, there are several equivalent or almost equivalent formulations of this approach that can be conveniently applied or modified for the problem considered here, including the questions of selection, ranking and classification.

An important equivalent formulation to the lexicographic minimax is a *cumulative ordering map* leading to a *Lorentz curve*. Ordering map $\theta$ is such a rearrangement of indices $i$ of criteria and their partial achievements $\sigma_i$ that $\theta_1 \leq \theta_2 \leq \ldots \leq \theta_i \leq \theta_{i+l} \leq \ldots \leq \theta_n$, while each $\theta_i$ corresponds to the value of some partial achievement $\sigma_i$. Cumulative ordering map and Lorentz curve correspond to the arrangement of consecutive sums of $\theta_i$.

$$\theta_i^{sum} = \sum_{1 \leq k \leq i} \theta_k, \qquad L_i = \theta_i^{sum}/i, \qquad i \in I \qquad (5)$$

Thus, Lorentz curve $L_i$ is the graph of the dependence of $\theta^{sum}/i$ on $i$ (actually, assuming a scaling of both axes to obtain intervals between 0 and 1, but this normalization is not necessary in our application). The lexicographic minimax is equivalent to the selection of best alternative according to the highest first nonzero value of the Lorentz curve.

However, we can as well take into account other values of achievement functions by ordering the alternative decisions $j$ according to a Lorentz quantile, i.e., the Lorentz value $L_{ij}$ (meaning the value of Lorentz curve for alternative $j$) for some larger $i$ for example $i = [n/3]$ or $i = [n/2]$, where $[\cdot]$ denotes the whole part of a number. This results in an equitable aggregation possibly different than the lexicographic minimax, but preserving equitability: preference for alternatives with a distribution of criteria that is as close to a uniform one as possible. Another possibility is to use ordered weighted averaging (OWA), see e.g. (Yager 1988) and (Kostreva et al. 2003). We average the ordering map (or the cumulative ordering map, which is equivalent up to a transformation of weighting coefficients) using a sequence of weighting coefficients decreasing with $i$, e.g., $\omega_i = 2^{-i}$. Note that

$$\sum_{1 \le i \le n} \omega_i \approx 1$$

for sufficiently large $n$. Then we use the OWA achievements to order all alternatives:

$$\sigma_j^{owa} = \sum_{1 \le i \le n} 2^{-i} \theta_{ij} \qquad (6)$$

where $\theta_{ij}$ denotes the value of ordering map on position $i$ for alternative $j$. Such an ordering of alternatives also preserves equitability.

Thus, we can propose two alternative modifications of the decision procedure outlined in the previous section. Instead of using the overall achievement function (3) in points 3. and 4. of the procedure, we use the partial achievement indices $\sigma_{ij}^{sum}$ defined as in (2) for alternatives $j$, then use either

- the cumulative ordering maps $\theta_{ij}^{sum}$ and Lorentz curves $L_{ij}$ defined as in (5) with $i = [n/3]$ or $i = [n/2]$, or
- the OWA achievements $\sigma_j^{owa}$ defined as in (6) for ordering of alternatives $j \in J$, the ordering implies then either the selection of the best alternative, or ranking, or a classification.

## 5 Hierarchical aggregation of criteria

Both weighted sum approaches and value and utility elicitation methods devoted much attention to hierarchical aggregation of criteria. Criteria are arranged in a hierarchical (inverted tree-like) structure: criteria relevant for a given aspect are considered as a group. For example, very often a group of criteria might indicate economic aspects, another group - environmental aspects, yet another group - social aspects. Groups can be subdivided further into subgroups, as, for example, economic aspects into financial part and resource-related part; but we omit this complication here in order to simplify presentation. If either weighting or importance coefficients are specified in an inverted tree-like structure, consistency of their normalization becomes crucial (e.g., if we start with the root, the top of the inverted tree and assign the weighting coefficients, then they should sum up to 1 in each group and subgroup, while the resulting weighting coefficients should correspond to products of coefficients along each branch of the tree). We have a set of alternatives $A$ and a set $J$ of indexes $j \in J$, which identify alternatives. On each hierarchy level we can distinguish groups of criteria

$$\boldsymbol{L_l} = \{L_{l,1}, \ldots, L_{l,g_l}\},$$

where $g_l$ - is a number of groups on $l$-th level of the hierarchy. As before, we denote the value of a quality indicator $q_i$ for the decision option $j$ by $q_{ij}$; its upper bound by $q_i^{up}$, and its lower bound by $q_i^{lo}$. It is also assumed that all criteria or quality indicators are scaled down to a relative scale by the transformation:[8]

$$q_{ij}^r = (q_{ij} - q_i^{lo})/(q_i^{up} - q_i^{lo}) \tag{7}$$

We assume that such a transformation is performed and will not later indicate the upper index $r$, stipulating that all further values of quality indicators are measured in a common, relative percentage scale. Moreover, we assume that such a transformation is performed once, before the analysis, and later small changes of the problem - e.g., deleting one of the alternative decisions in consideration - are not followed by the change of measurements scales. This is consistent with the historical practice of measurement systems: we do not change measurement units every time new data arrive - if we did, we should be prepared for diverse inconsistencies resulting not from the method of analysis, but from the inconsistency of measurements.

## 5.1   Compensatory versus non-compensatory criteria

Apart from many technical and theoretical drawbacks of using weighting coefficients in multiple criteria analysis as discussed above, we should be aware that all criteria (and the corresponding weighting coefficients) can be divided into two essentially distinct types:

1. *Compensatory criteria* are such that a given improvement - increase of the value of one quality indicator by a given number in the relative percentage scale - can be rationally substantiated to compensate a deterioration of another quality indicator - its decrease by a unit or by one percent in the relative scale. The (inverse) ratio of these changes can be used as a basis for determining the *rationally substantiated weighting coefficients*, called also *compensatory weighting coefficients*. A basic example of such compensatory criteria is given by two financial quality indicators (both of minimized type): operational costs and investment costs. If we know what is the cost of a banking credit, then we can rationally substantiate the trade-off, thus a weighting coefficient converting the investment costs into an addition to operating costs.[9] The popularity of the use of weighting coefficients is based on an (erroneous) economic intuition that all criteria are of the compensatory character.

2. *Non-compensatory criteria* are such that no rational substantiation exists for defining weighting coefficients. A basic example of non-compensatory criteria are costs and loss of human life; many decision-makers and/or analysts refuse on principal grounds to give a value for such a compensation, even if some economists try

---

[8]Moreover, it is consistent with measurement theory, see Barzilai (2004) - who points out that all utility and value theory in this respect is not necessarily consistent with measurement theory. We must also stress that the practice of measurement systems requires stable definitions of measurement units: once we defined the meter as 1: 40 000 000 part of a meridian, we obtained a certain standard exemplar every time the meridian is measured and turns out to be slightly different than 40 000 000 meters.

[9]Two criteria might be compensatory even if they are positively linearly correlated, e.g., one criterion is a sum including the other criterion; we must admit only possibly negative compensatory weighting coefficients for such cases.

to define such values for insurance firms. Weighting coefficients defined for non-compensatory criteria are not substantiated rationally, they are a result of an estimation of trade-offs for a given decision situation, while this trade-offs might change with the context, thus this estimation is often subjective or intuitive. As such, they do not define coefficients that might be objectively used in a weighted sum, they estimate only local relative importance of criteria.

It is important to understand well the nature of *noncompensatory weighting coefficients* interpreted as importance factors related to non-compensatory criteria. Comparing relative importance of (local) increases of criteria is not equivalent to the assumption that a weighted sum might be applied for aggregation of the criteria, for many reasons (only one of the reasons being that a local approximation is usually not good for the whole ranges of criteria values). For the example of the cost of saving human lives, we might submit to the argument that there is simply no more money to be spent on a rescue program, or debate the argument by showing that a small increase of the expenditure can save a considerable number of human lives. Thus we might agree to treat the economic aspect as a constraint, while never accepting the conclusion that saving one more human life is preferentially equivalent to a given sum of money, thus never treating such two criteria as compensatory.

This is one of the reasons that we proposed above to use importance coefficients in a different way than weighting coefficients. This basic distinction is also important when considering a hierarchical structure of criteria. An aggregation of quality indicators belonging to a group of criteria might be performed by a weighted sum only if all criteria in this group are of compensatory type. For non-compensatory criteria, other ways of aggregation - such as an achievement function based on the reference point method - must be used.

## 5.2   Approaches to hierarchical criteria aggregation

Following this basic distinction, we shall distinguish the following approaches to criteria aggregation in a hierarchical structure of criteria (while we discuss here only two hierarchical levels, additional lower levels can be similarly considered):

1. *Compensatory aggregation on lower level*, *non-compensatory analysis on upper level*. In this case we assume that all criteria are compensatory within their groups and thus weighting coefficients and weighted sums can be used to aggregate criteria in each group. This results in aggregated group quality indicators or aggregated criteria:

$$q_C = \sum_{i \in C} w_i q_i \qquad C = \{L_{l,1}, \ldots, L_{l,g_l}\} \tag{8}$$

where $w_i$ are compensatory weighting coefficients. We stress that these coefficients correspond to relative percentage scales of criteria changes, thus are also dimension-free and should sum up to unity,

$$\sum_{i \in C} w_i = 1, \qquad C = \{L_{l,1}, \ldots, L_{l,g_l}\}.$$

The values $q_C$ of these aggregated criteria for all alternatives $j \in J$ can be used to order them (select the best one, classify them, form a group ranking list, etc.). It

must be stressed that this list orders the alternative decisions only from the point of view of the given group of criteria, e.g., from the perspective of environmental criteria. On the other hand, it is difficult to substantiate the assumption that the aggregate criteria $q_C$ will be also compensatory (for example, environmentalists would never agree to consider an aggregated environmental quality indicator as compensatory to an economic quality indicator). Therefore, we assume that a non-compensatory multiple criteria analysis is needed on the upper level, which is possible when applying the reference point approach. For this purpose, the upper and lower bounds for all aggregated criteria $q_C^{up}$ and $q_C^{lo}$ for all $C = \{L_{l,1}, \ldots, L_{l,g_l}\}$ are first determined. Then we define reference point for each aggregated criterion.

These data help to define partial achievement function for each aggregated criterion $\sigma_C(q_C, a_C, r_{C^`})$ computed as in Eq. (2) for a given alternative $j \in J$ signifies the satisfaction level with the aggregated quality indicator for this alternative decision (say, expressed in the assessment scale 0 - 10); thus, they can be further aggregated as in Eq. (3) or by equitable aggregation methods as described in Section 4. Aspiration and reservation levels $a_C, r_C$ can be either defined statistically as in Eq. (4) in order to be as objective as possible, or defined and changed subjectively by the decision maker in order to control the multiple criteria analysis of the problem.

2. *Non-compensatory aggregation both on lower and on upper level.* If there are non-compensatory criteria in a group on lower level, the use of weighting coefficients for aggregating them is most doubtful. But we can always use the reference point approach also on the lower level and treat the resulting value of an achievement function as the aggregated (though somewhat transformed) criterion or quality indicator $q$ for the group. In this case, instead of Eq. (8), we use:

$$\sigma(\boldsymbol{q}, \boldsymbol{a}, \boldsymbol{r}) = \min_{i \in C} \sigma_C(q_C, a_C, r_C) + \frac{\epsilon}{|C|} \sum_{C=L_{l,1},\ldots,L_{l,g_l}} \sigma_C(q_C, a_C, r_C), \qquad (9)$$

or its relative percentage value after establishing its upper and lower bounds $q_C^{up}$ and $q_C^{lo}$. If we use statistically defined aspiration and reservation levels for all criteria, computed as in Eq. (4), then the resulting values of aggregated quality indicators are defined in a sense objectively.

The analysis on the upper level can be then performed in the same way as in the previous case, because we do not assume compensatory character of the aggregated quality indicators. Using statistically defined aspiration and reservation levels also on the upper level, we obtain an objective overall ordering list of all alternatives $j \in J$. This list depends on parameters derived from data which do not reflect detailed preferences of the decision maker. Sometimes, however, the decision maker wants to take into account also weighting coefficients, but treated differently than in the case of compensatory aggregation.

3. *Non-compensatory aggregation with weighting coefficients treated as importance factors.* If the criteria are non-compensatory but weighting coefficients are given and are interpreted as importance factors, we can use them in a modification of achievement values $\alpha$ and $\beta$ for reservation and aspiration levels as suggested in Table 1. Further analysis can be performed similarly as in the case above.

## 5.3  Pareto-nondominance in the hierarchical aggregation of criteria

When analyzing a multiple criteria decision problem, or making a ranking list of options, we normally require that the solutions considered or option ranked as first are Pareto-nondominated, that they cannot be improved in one criterion without deteriorating other criteria. Observe, however, that such a property could be lost in a hierarchical aggregation of criteria.

Suppose we consider a group of criteria, e.g., environmental quality indicators, and select some alternative decisions as Pareto-nondominated with respect to these criteria, or select top alternatives that are Pareto-nondominated. If we add another group of criteria, or even a single criterion, such as the cost of introducing new technologies, a new set of Pareto-nondominated alternatives emerge. The alternatives that are Pareto-nondominated with respect to the original group of criteria will remain Pareto-nondominated (but, in a sense, on a boundary of the new set of Pareto-nondominated options). However, many new Pareto-nondominated alternatives might emerge for the new set of criteria (including added ones); and, in some of them, *the components of the original group of criteria will have values that are not Pareto-nondominated with respect to the original group of criteria.* This property is obvious when we consider optimizing one criterion first and then adding a second criterion: some Pareto-nondominated solutions might be not optimal for either of the single criteria. However, this obvious property has important consequences for a hierarchical aggregation of criteria.

Suppose we have a large number of criteria and optimize them on a single level, including all of them in the definition of Pareto optimality. The larger is the number of criteria, the more Pareto-nondominated alternatives are possible (since each new criterion might provide for additional nondominated alternatives). If there is not a very large number of alternatives and a large number of criteria, there is a significant chance that all options are Pareto-nondominated. Suppose we aggregate groups of a large number of criteria into aggregate group criteria. This means actually two effects. First, we add new criteria. However, the added criteria are of a special type, because they are usually monotone with respect to the partial order in the space of original criteria, thus their addition might not add new Pareto optimal solutions. Second, in a sense we subtract many criteria, since we confine the analysis to the space of aggregated criteria - and the subtraction of criteria might obviously remove from the list of Pareto-nondominated points many alternatives that were Pareto-nondominated in the original space of all criteria on the lower level.

However, a converse question is following: *in a hierarchical aggregation of criteria, what are the conditions that the alternatives which are Pareto-nondominated in the space of aggregated criteria remain also Pareto-nondominated in the space of all criteria on the lower level?* For example, the balanced alternative that presents a compromise between aggregated criteria, as selected at the top of the overall ordering list based on the values of the overall achievement function with statistically defined aspiration and reservation levels, is obviously Pareto optimal with respect to aggregated criteria; but when is it also Pareto optimal with respect to all original criteria on the lower level? This issue has been actually studied in diverse formulations in the history of decision analysis, but we recall here the answer as specified by the following theorem, see also (Granat et al. 2006):

**Theorem 1**. In a hierarchical aggregation of criteria, suppose that the functions used to aggregate criteria in groups on the lower level (such as the functions (8) or (9) above) are

strictly monotone with respect to the partial orders defining the multiple criteria problems on lower level. Then any decision option that is Pareto-nondominated in the space of aggregated criteria is also Pareto-nondominated in the original space of all lower level criteria (with respect to the overall partial order induced by the partial orders for all groups of criteria).

**Proof.** Assume the converse. Then an alternative decision $j^*$ which is Pareto-nondominated in the space of aggregated criteria is at the same time Pareto-dominated in the original space of all lower level criteria. This means that we can find another alternative decision $j$ that improves some original criteria without deteriorating remaining ones. Since the aggregating functions are strictly monotone, at least one of them will have a larger value at this other option $j$ than at $j^*$ without decreasing the values of other aggregating functions. This means, however, that the original option $j^*$ is also Pareto-dominated in the space of aggregated criteria, which contradicts the assumption.

In the following examples, where the number of criteria is large as compared to the number of options, this property does not play an important role because most options are anyway Pareto optimal.

# 6   Examples

We present here two examples, both corresponding to practical problems but severely simplified and distorted for confidentiality reasons. The first example is more illustrative, shows that even a small number of criteria (4) can be relatively large when compared to the number of alternatives (6); we shall use it also to illustrate the concepts of objective ranking or classification, of equitable aggregation and hierarchical aggregation on a very simple case. The second example is more demanding, with a small number of decision alternatives (4) but a larger number of criteria (15) which results in all the difficulties of problems with large numbers of criteria.

The first example concerns international business management. Suppose an international corporation consists of six divisions $A, B, \ldots, F$ characterized by diverse data items, such as name, location, number of employees etc. However, suppose that the CEO of this corporation is really interested in ranking or classification of these divisions taking into account the following attributes used as criteria:

1. profit (in % of revenue),

2. market share (m.share, in % of supplying a specific market sector, e.g., global market for a type of products specific for this division),

3. internal collaboration (i.trade, in % of revenue coming from supplying other divisions of the corporation), and

4. local social image (l.s.i., meaning public relations and the perception of this division - e.g., of its friendliness to local environment - in the society where it is located, evaluated on a scale 0-100 points).

All these criteria are maximized, i.e., improve when increased. An example of decision table of this type is shown in Table 2, while Pareto-nondominated divisions are distinguished by mark *.

| Division | Name | Location | Employees | profit | m.share | i.trade | l.s.i |
|----------|---------|----------|-----------|--------|---------|---------|-------|
| A | Alpha | USA | 250 | 11 | 8 | 10 | 40 |
| B* | Beta | Brasilia | 750 | 23 | 40 | 34 | 60 |
| C* | Gamma | China | 450 | 16 | 50 | 45 | 70 |
| D* | Delta | Dubai | 150 | 35 | 20 | 20 | 44 |
| E* | Epsilon | C.Europe | 350 | 18 | 30 | 20 | 80 |
| F | Fi | France | 220 | 12 | 8 | 9 | 30 |

Table 2: Data for an example on international business management.

The CEO obviously could propose an intuitive, subjective ranking of these divisions - and this ranking might be even better than a rational one resulting from the table above, if the CEO knows all these divisions in minute detail. However, when preparing a discussion with her/his stockholders, (s)he might prefer to ask a consulting firm for an objective ranking.

Thus, we first illustrate the issue of objective ranking and statistical determination of reservation and aspiration levels. The principle that all criteria improve when increasing is easy to agree upon; similarly, the stockholders would easily accept the principle that the details of ranking should be determined mostly by the data contained in Table 1 (on page 11), and not by any personal preferences. The question how to statistically define reservations and aspirations is actually technical, but interesting for illustration. There are no essential reasons why we should limit the averaging to the set of alternatives ranked; we could use as well a larger set of data in order to define more adequate (say, historically meaningful) averages, or a smaller set - for example, only the Pareto-nondominated alternatives denoted by * in Table 2 - in order to define, say, more demanding averages and aspirations. For the data from Table 1, we can thus present two variants of objective ranking: A - based on averages of data from this table; B - based on averages from Pareto optimal options - see Table 3. We use here the achievement function from Eq. (3) with $\epsilon = 0.4$ ($n = 4$).

We do not observe any changes of ranking and classification when shifting from average A to more demanding B aspirations and reservations; this is confirmed by other applications and shows that objective ranking gives rather robust results. Generally, we might expect rank reversals, although usually not very significant, when shifting to more demanding aspirations; this is, however, a natural phenomenon: average aspirations favor standard though good solutions, truly interesting solutions result from demanding aspirations. Note that we did not change the estimates of the lower and upper bounds and thus measurement ranges when averaging over Pareto-nondominated solutions; although the lower bounds for Pareto-nondominated alternatives (so called nadir point) are in this case different than the lower bounds for all alternatives, a change of ranges would mean a change of measurement units and should be avoided. In further analysis of this example, we shall not compute the more demanding averages, since they give in general rather similar results.

Suppose now that the decision maker (the CEO of the corporation) would ask the consulting company that prepares the ranking for taking into account her/his importance coefficients for criteria, classifying $q_1$ and $q_2$ as having normal importance, $q_3$ as much

| Criterion | $q_1$ | $q_2$ | $q_3$ | $q_4$ | | | |
|---|---|---|---|---|---|---|---|
| Upper bound | 35% | 50% | 45% | 80 | | | |
| Lower bound | 11% | 8% | 9% | 30 | | | |
| Reference $A$ (average) | 19.2% | 26% | 23% | 54 | | | |
| Aspiration $A$ | 27.1% | 38% | 34% | 67 | | | |
| Reservation $A$ | 15.1% | 17% | 16% | 42 | | | |
| Reference $B$ (Pareto average) | 23% | 35.0% | 29.7% | 63.5 | | | |
| Aspiration $B$ | 29% | 42.5% | 37.4% | 71.7 | | | |
| Reservation $B$ | 17% | 17% | 19.4% | 46.7 | | | |
| Ranking $A$: Division | $\sigma_1$ | $\sigma_2$ | $\sigma_3$ | $\sigma_4$ | $\sigma$ | Rank | Class |
| A | 0.00 | 0.00 | 0.37 | 2.50 | 0.29 | 5 | III |
| B | 5.63 | 7.50 | 7.00 | 5.88 | 8.23 | 1 | I |
| C | 3.30 | 10.0 | 10.0 | 7.62 | 6.39 | 2 | II |
| D | 10.0 | 3.57 | 3.89 | 3.32 | 5.40 | 4 | II |
| E | 3.97 | 5.48 | 3.89 | 10.0 | 6.30 | 3 | II |
| F | 0.73 | 0.00 | 0.00 | 0.00 | 0.07 | 6 | III |
| Ranking $B$: Division | $\sigma_1$ | $\sigma_2$ | $\sigma_3$ | $\sigma_4$ | $\sigma$ | Rank | Class |
| A | 0.00 | 0.00 | 0.29 | 1.80 | 0.21 | 5 | III |
| B | 5.00 | 6.61 | 6.24 | 5.13 | 7.30 | 1 | I |
| C | 2.50 | 10.0 | 10.0 | 6.73 | 5.42 | 2 | II |
| D | 10.0 | 3.47 | 3.13 | 2.51 | 4.42 | 4 | II |
| E | 3.33 | 5.04 | 3.13 | 10.0 | 5.28 | 3 | II |
| F | 0.50 | 0.00 | 0.00 | 0.00 | 0.05 | 6 | III |

Table 3: An example of objective ranking and classification for the data from the Table 2.

less important and $q_4$ as much more important (e.g., caring much for the local images of the corporation). Suppose we use coefficients $\alpha$ and $\beta$ from Table 1 to express the importance of criteria. The data from Table 3 change then as shown in Table 4.

We see that the change is essential: alternative E, third in ranking when all criteria have the same importance, becomes first in ranking - since it is best in the most important objective $q_4$, while other alternatives have much lower achievement values for this decisive criterion. However, alternatives B and C, formerly first and second in ranking, stay rather close to alternative E and should be considered as practically equivalent, at least, belonging to the same class I.

Suppose now that the analyst (the consulting firm), just in order to check the robustness of the conclusions, decided to apply as well an equitable aggregation by computing the values of a Lorentz quantile for all alternatives and the values of an OWA aggregation. Since the worst alternatives have zero achievement values for many criteria, we might use in this case Lorentz values

$$L_{ij} = \sum_{1 \le k \le i} \theta_k / i$$

where $\theta_i$ are just achievement values $s_i$ reordered in an increasing sequence, for rather large $i = 3$ (if the number of criteria were really large, a lower quantile of the Lorentz curve could be used). The OWA aggregation $\sigma_j^{owa}$ is computed as in Eq. (6). The results

| Criterion | $q_1$ | $q_2$ | $q_3$ | $q_4$ | | | |
|---|---|---|---|---|---|---|---|
| Upper bound | 35% | 50% | 45% | 80 | | | |
| Lower bound | 11% | 8% | 9% | 30 | | | |
| Reference $A$ (average) | 19.2% | 26% | 23% | 54 | | | |
| Aspiration $A$ | 27.1% | 38% | 34% | 67 | | | |
| Reservation $A$ | 15.1% | 17% | 16% | 42 | | | |
| Ranking $A$: Division | $\sigma_1$ | $\sigma_2$ | $\sigma_3$ | $\sigma_4$ | $\sigma$ | Rank | Class |
| A | 0.00 | 0.00 | 0.71 | 0.83 | 0.29 | 5 | III |
| B | 5.63 | 7.50 | 9.00 | 3.88 | 6.48 | 2 | I |
| C | 3.30 | 10.0 | 10.0 | 6.15 | 6.24 | 3 | I |
| D | 10.0 | 3.57 | 5.89 | 1.32 | 3.40 | 4 | II |
| E | 3.97 | 5.48 | 5.89 | 10.0 | 6.50 | 1 | I |
| F | 0.73 | 0.00 | 0.00 | 0.00 | 0.07 | 6 | III |

Table 4: An example of objective ranking and classification with importance coefficients ($\alpha = 3, \beta = 7$ for $q_1$ and $q_2$, $\alpha = 5, \beta = 9$ for $q_3$, $\alpha = 1, \beta = 5$ for $q_4$).

| Criterion | $q_1$ | $q_2$ | $q_3$ | $q_4$ | | | | |
|---|---|---|---|---|---|---|---|---|
| Upper bound | 35% | 50% | 45% | 80 | | | | |
| Lower bound | 11% | 8% | 9% | 30 | | | | |
| Reference $A$ (average) | 19.2% | 26% | 23% | 54 | | | | |
| Aspiration $A$ | 27.1% | 38% | 34% | 67 | | | | |
| Reservation $A$ | 15.1% | 17% | 16% | 42 | | | | |
| Ranking $A$: Division | $\sigma_1$ | $\sigma_2$ | $\sigma_3$ | $\sigma_4$ | $\sigma$ (rank) | $L_3 j$ (rank) | $\sigma^o wa$ (rank) | Class |
| A | 0.00 | 0.00 | 0.71 | 0.83 | 0.29(5) | 0.24(5) | 0.14(5) | III |
| B | 5.63 | 7.50 | 9.00 | 3.88 | 6.48(2) | 5.67(2) | 4.85(2) | I |
| C | 3.30 | 10.0 | 10.0 | 6.15 | 6.24(3) | 6.48(1) | 5.06(1) | I |
| D | 10.0 | 3.57 | 5.89 | 1.32 | 3.40(4) | 2.76(4) | 2.91(4) | II |
| E | 3.97 | 5.48 | 5.89 | 10.0 | 6.50(1) | 5.11(3) | 4.72(3) | I |
| F | 0.73 | 0.00 | 0.00 | 0.00 | 0.07(6) | 0.00(6) | 0.01(6) | III |

Table 5: An example of objective ranking and classification with importance coefficients as in Table 4, but augumented with Lorentz quantiles and OWA aggregation.

of such computations are given in Table 5.

We see that alternatives B, C, E, anyway close in overall achievement, might change between themselves their position in the ranking list, but remain together in class I. This is, however, expected: small changes of parameters or of the method of aggregation can change the detailed position of an alternative in the ranking list, if the differences in overall evaluation between alternatives are small. Thus, it is better to avoid detailed ranking of alternatives (and speak about ordering alternatives instead), while classifying alternatives gives usually robust results. We see that the worst options are also rather robustly classified despite of diverse changes of parameters and aggregation methods.

In some management applications the worst ranked options are the most interesting,

because they indicate the need of a corrective action. Objective ranking was originally motivated by an actual application when evaluating scientific creativity conditions in a Japanese research university, JAIST. The evaluation was based on survey results. The detailed results of the survey were very interesting theoretically, but also very useful for university management, see (Tian et al. 2006), (Wierzbicki and Nakamori 2007). It was found that seven questions ranked as worst practically did not depend on the variants of ranking; thus, the objective ranking gave robust results as to the problems that required most urgent intervention by the university management. The best ranked questions were more changeable, only three of them consistently were ranked among the best ones in diverse ranking variants.

The second example concerns the management of electromagnetic spectrum. This is a highly political issue today: electromagnetic spectrum - with its use for mobile telephony, satellite television, radio, GPS, military, intelligence, radioastronomy etc. - becomes one of the most contested problems and profitable resources today; governments of all countries organize tenders for assigning parts of the spectrum to diverse purposes, institutions and companies. Because of diverse parties interested and involved and highly political pressures exerted, such tenders must rely on specific plans of managing electromagnetic spectrum, prepared beforehand and approved by proper authorities. Typically, there are only a few variants of such plans submitted for selection, but the number of criteria, expressing the interests of diverse parties involved, might be quite high. We present below modified, for confidentiality reasons, data concerning such a problem of selecting a plan of managing electromagnetic spectrum, with 4 alternatives (versions of the plan) and 15 criteria. In this case, all alternatives are Pareto-nondominated; and for all alternatives we can have some criteria with values at the lower bound. This is, as we shall see, an essential difference from the former example.

The data for this example, together with statistics defining objective aspiration and reservation points, partial and overall achievement values (the latter computed with rather large $\epsilon = 0.75$, recall that n = 15) for normal importance of all criteria, Lorentz quantiles $L_{5j}$ (for $i = n/3$), OWA achievement $\sigma_j^{owa}$ computed as in Eq. (6), are given in Table 6. The criteria are arranged in hierarchical groups: 4 criteria of economic character, 3 criteria of environmental character, 8 criteria of socio-political character. Such an arrangement is convenient and even necessary for organization of criteria, however, as we shall see, hierarchical aggregation of criteria should not be used for such a highly political example.

In Table 6., the original values of criteria were converted to a common scale 0 - 100; but the lower bound was determined by a larger number of alternatives, some of which have been deleted from the list, while the measurement scale was not adjusted again (we should not change measurement units, as we commented earlier). The use of overall achievement function (3) is doubtful, since its first (min) term is equal to zero for all alternatives; hence the values of $\sigma_j$ are defined by the second term (which in the function has the regularizing role, therefore its values are relatively small, and similar for each alternative). Therefore the resulting ranking is de-facto defined by the weighted sum of component achievements scaled by $\epsilon$, and it thus sensitive to small changes of data. Fig. 1 shows the values the of overall achievement for each alternative.

A much more robust classification can be done using the Lorentz quantiles. Fig. 2 shows the graphs of ordered values of partial achievements for all alternatives. Fig. 3 shows the Lorentz curves and Fig. 4 shows the values for Lorentz quantile for each al-

| Criterion $i$ | statistics Alternative | | | A | | B | | C | | D | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $q_i^a v$ | $r_i$ | $a_i$ | $q_iA$ | $\sigma_iA$ | $q_iB$ | $\sigma_iB$ | $q_iC$ | $\sigma_iC$ | $q_iD$ | $\sigma_iD$ |
| Economic | | | | | | | | | | | |
| 1 | 68 | 34 | 84 | 100 | 10.0 | 89 | 7.94 | 85 | 7.19 | 0 | 0.00 |
| 2 | 81 | 40 | 90 | 62 | 4.76 | 100 | 10.0 | 88 | 6.84 | 75 | 5.80 |
| 3 | 30 | 15 | 65 | 100 | 10.0 | 0 | 0.00 | 21 | 3.48 | 0 | 0.00 |
| 4 | 10 | 5 | 55 | 17 | 3.96 | 23 | 4.44 | 0 | 0.00 | 0 | 0.00 |
| Environmental | | | | | | | | | | | |
| 5 | 96 | 48 | 98 | 98 | 7.00 | 100 | 10.0 | 99 | 8.50 | 92 | 6.52 |
| 6 | 94 | 47 | 97 | 96 | 6.92 | 100 | 10.0 | 97 | 7.00 | 86 | 6.12 |
| 7 | 86 | 43 | 93 | 100 | 10.0 | 75 | 5.56 | 93 | 7.00 | 79 | 5.88 |
| Socio-political | | | | | | | | | | | |
| 8 | 96 | 49 | 99 | 96 | 6.76 | 99 | 7.00 | 100 | 10.0 | 99 | 7.00 |
| 9 | 84 | 42 | 92 | 92 | 7.00 | 87 | 6.60 | 88 | 6.68 | 68 | 5.08 |
| 10 | 30 | 15 | 65 | 0 | 0.00 | 18 | 3.24 | 2 | 0.40 | 100 | 10.0 |
| 11 | 74 | 37 | 87 | 0 | 0.00 | 100 | 10.0 | 100 | 10.0 | 100 | 10.0 |
| 12 | 96 | 48 | 98 | 100 | 10.0 | 98 | 7.00 | 98 | 8.00 | 91 | 6.52 |
| 13 | 64 | 32 | 82 | 67 | 5.80 | 48 | 4.28 | 38 | 3.48 | 100 | 10.0 |
| 14 | 72 | 36 | 86 | 0 | 0.00 | 99 | 9.79 | 99 | 9.79 | 92 | 8.29 |
| 15 | 74 | 37 | 87 | 0 | 0.00 | 96 | 9.08 | 100 | 10.0 | 99 | 9.54 |
| Overall achievement $\sigma_j$ (rank) | | | | 4.11 (4) | | 5.24 (1) | | 4.86 (2) | | 4.54 (3) | |
| Lorentz quantile $L_5 j$ (rank) | | | | 0.65 (4) | | 3.50 (1) | | 2.80 (2) | | 2.17 (3) | |
| OWA achievement $\sigma_j^{owa}$ (rank) | | | | 0.30 (4) | | 2.02 (1) | | 1.12 (2) | | 0.69 (3) | |

Table 6: Data and analysis for an example of spectrum management; normal importance of criteria.

ternative. Note that while the alternative (plan) B remains ranked first, Lorentz quantile gives it a more decisive margin - while alternative C and D, by overall achievement function ranked second and third with similar achievement values, by Lorentz quantile gives a more decisive margin.

This order (B-C-D-A) is strongly confirmed by OWA achievement ranking which gives most clear classification. Fig. 5 shows OWA graphs and Fig. 6 graph of OWA achievements.

We shall not repeat this example for changed importance of criteria, since such a change gives rather obvious results. For example, we could improve slightly the position of alternative A (which is good in most economic criteria) if we declared all economic criteria as very important; however, alternative A could not be ranked first by the analyzed approaches even in this case, since it is ranked worst, with zero achievements, on many socio-political criteria.

The situation changes entirely, if we agree to a hierarchical compensatory aggrega-
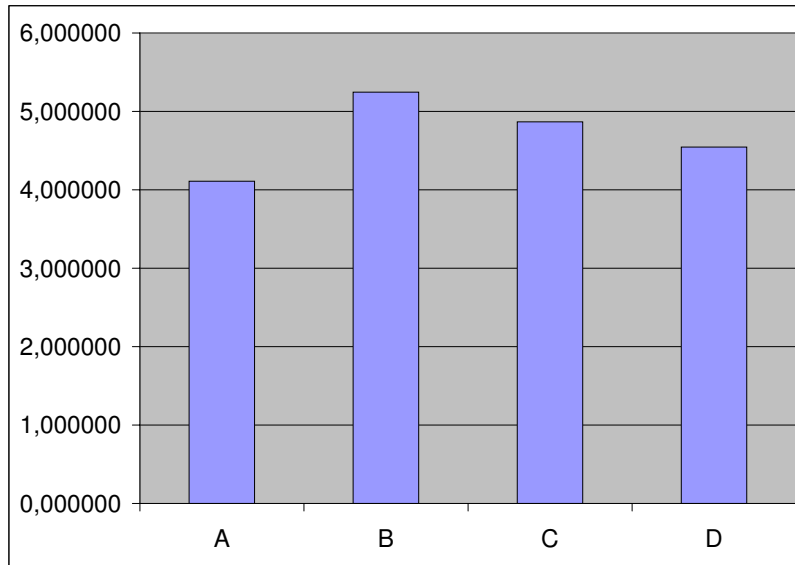
Figure 1: Values of the overall achievement (using the RFP method) for each alternative.
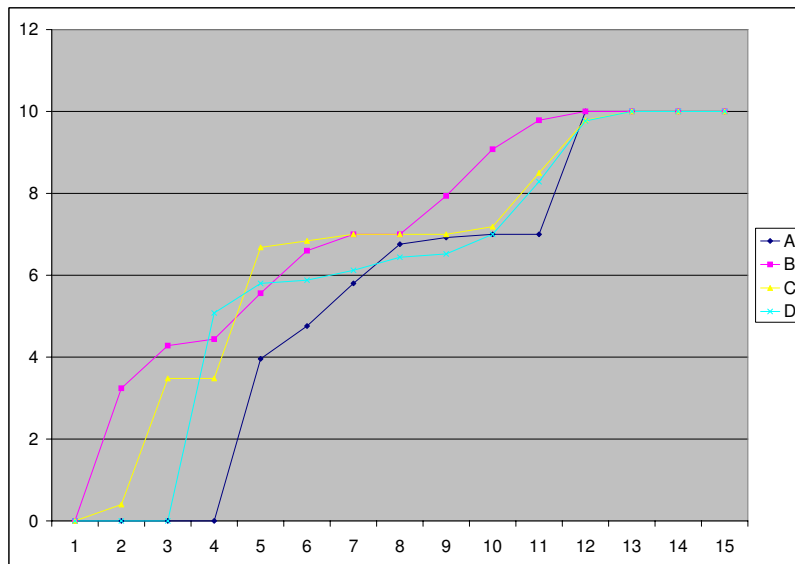


Figure 2: Ordered values of partial achievements for all alternatives.

tion of criteria: if we assume that all criteria in each group are compensatory, we can use weighting coefficients for lower level aggregation. In actual political situation, such an aggregation is not applicable, for many reasons; e.g., socio-political criteria represent interests of diverse political groups that would never agree to treat their criteria as compensatory and consider them jointly. Nevertheless, we present below such aggregation just to illustrate possible dangers related to it. In Table 7, such an aggregation was performed when assuming normal and equal importance, thus equal weights, for all criteria in each group.

Because there are only 3 aggregated criteria, we use here Lorentz quantile L2j. The results of hierarchical aggregation seem to confirm the earlier analysis, although the two
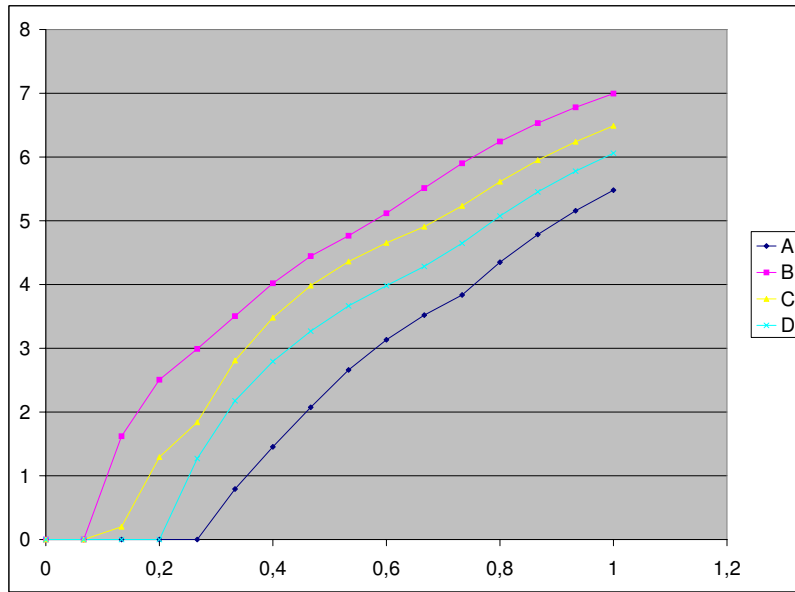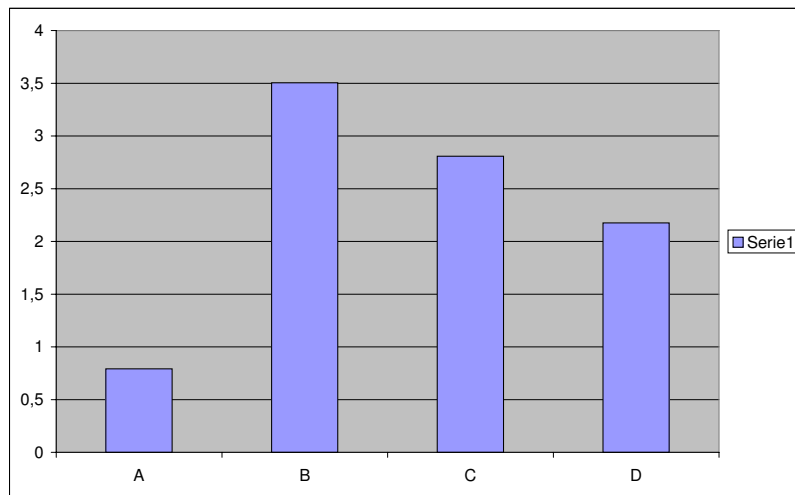
Figure 3: Lorentz curves.



Figure 4: Values of Lorentz quantiles.

last alternatives (D and A) exchange their rank. However, there is a deeper danger in linearly averaging assessment data and preparing a decision based on averages: since human preferences are essentially nonlinear, details important for preferential assessment might get lost in averaging. For example, after averaging we know only that alternative A is slightly worse on socio-political criteria, not that is it is very bad on half of them. Therefore, after averaging, if we declared that economic criteria are very important, the alternative A could be even ranked first - which was impossible with nonlinear assessment of not aggregated data. For this reason, aggregation is a tool of political manipulation used quite often - after a suitable aggregation, data might indicate much better assessment than actually applicable.
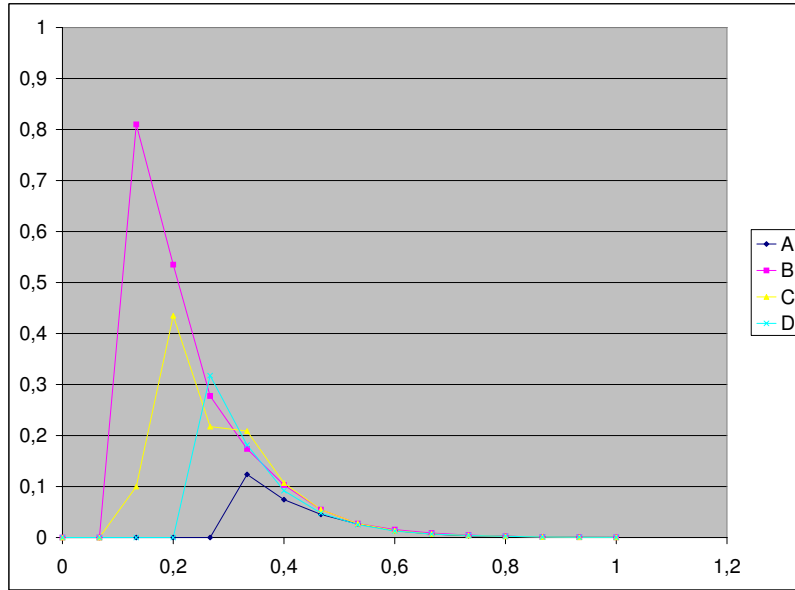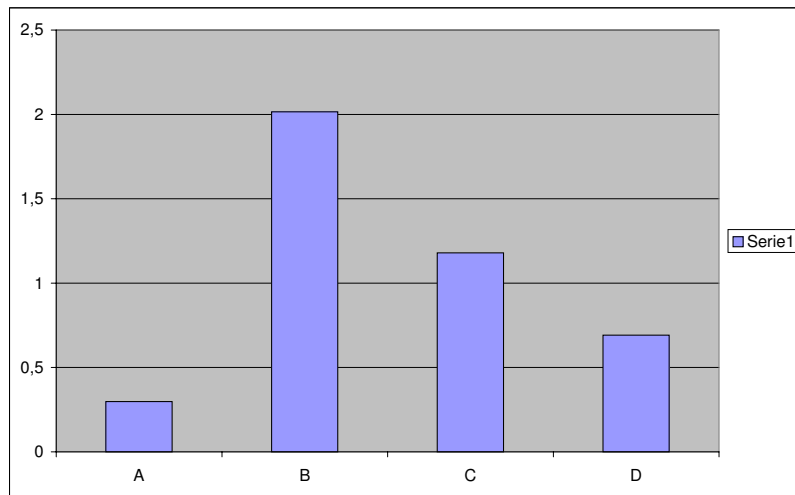
Figure 5: OWA graphs.



Figure 6: Achievement values for the OWA method.

# 7   Conclusions

This paper presents some aspects of multiple criteria analysis of discrete decision problems with large numbers of criteria, including a critique of classical methods. It proposes modifications of reference point approaches, statistical determination of reservation and aspiration levels, treatment of weighting coefficients as importance factors, equitable aggregation approaches, hierarchical aggregation of criteria - all motivated by practical needs resulting from the specificity of problems with large numbers of criteria, but introducing, in our opinion, some important distinctions and concepts. To these belong:

1. The modification of reference point approaches with treatment of *importance factors* as achievement values for reservation and aspiration levels.

| Alternative | Statistics | | | A | | B | | C | | D | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Criterion $i$ | $q_i^{av}$ | $r_i$ | $a_i$ | $q_{iA}$ | $\sigma_{iA}$ | $q_{iB}$ | $\sigma_{iB}$ | $q_{iC}$ | $\sigma_{iC}$ | $q_{iD}$ | $\sigma_{iD}$ |
| Economic | | | | | | | | | | | |
| 1 | | | | 100 | | 89 | | 85 | | 0 | 0.00 |
| 2 | | | | 62 | | 100 | | 88 | | 75 | |
| 3 | | | | 100 | | 0 | | 21 | | 0 | |
| 4 | | | | 17 | | 23 | 0 | | 0 | | |
| Aggregated economic | 48 | 24 | 74 | 69 | 6.60 | 53 | 5.32 | 49 | 5.00 | 19 | 2.37 |
| Environmental | | | | | | | | | | | |
| 5 | | | | 98 | | 100 | | 99 | | 92 | |
| 6 | | | | 96 | | 100 | | 97 | | 86 | |
| 7 | | | | 100 | | 75 | | 93 | | 79 | |
| Aggregated environmental | 92 | 46 | 96 | 98 | 8.50 | 92 | 6.68 | 96 | 7.00 | 86 | 6.20 |
| Socio-political | | | | | | | | | | | |
| 8 | | | | 96 | | 99 | | 100 | | 99 | |
| 9 | | | | 92 | | 87 | | 88 | | 68 | |
| 10 | | | | 0 | | 18 | | 2 | | 100 | |
| 11 | | | | 0 | | 100 | | 100 | | 100 | |
| 12 | | | | 100 | | 98 | | 98 | | 91 | |
| 13 | | | | 67 | | 48 | | 38 | | 100 | |
| 14 | | | | 0 | | 99 | | 99 | | 92 | |
| 15 | | | | 0 | | 96 | | 100 | | 99 | |
| Aggregated socio-political | 92 | 46 | 96 | 98 | 8.50 | 92 | 6.68 | 96 | 7.00 | 86 | 6.20 |
| | 76 | 38 | 88 | 44 | 3.48 | 80 | 6.36 | 78 | 6.20 | 93 | 8.25 |
| Overall achievement $\sigma_j$ (rank) | | | | 4.41 (3) | | 6.24 (1) | | 5.91 (2) | | 3.21 (4) | |
| Lorentz quantile $L_5j$ (rank) | | | | 5.04 (3) | | 5.84 (1) | | 5.60 (2) | | 4.28 (3) | |
| OWA achievement $\sigma_j^{owa}$ (rank) | | | | 4.45 (3) | | 5.06 (1) | | 4.92 (2) | | 3.77 (4) | |

Table 7: Data and analysis for an example of spectrum management; hierarchical aggregation assuming compensatory character and equal importance of all criteria in a group.

2. The use of statistical determination of reservation and aspiration levels for obtaining decision selection, classification or ranking based on data thus *as objective as possible.*

3. The modification of *equitable aggregation approaches combined with objective ranking (statistical determination of reservation and aspiration levels).*

4. In the problem of hierarchical aggregation of criteria, the distinction between *compensatory and non-compensatory* groups of criteria, depending on the possibility of

rational substantiation of the values of weighting coefficients.

5. The distinction of three approaches to a hierarchical aggregation of criteria in multiple criteria decision analysis:
   - *Compensatory aggregation on lower level, non-compensatory analysis on upper level;*
   - *Non-compensatory aggregation both on lower and on upper level;*
   - *Non-compensatory aggregation with weighting coefficients treated as importance factors.*

6. The discussion and a theorem on the preservation of Pareto-nondominance after hierarchical aggregation with strictly monotone aggregating functions.

7. Two realistic (even if distorted for confidentiality reasons) examples of problems with relatively large numbers of criteria. The examples indicate also that while a hierarchical organization of criteria is an important and even necessary tool for discrete decision problems with large numbers of criteria, hierarchical aggregation of criteria has rather limited applicability. In particular, compensatory aggregation on lower levels might hide important details and allow political manipulation.

# References

.

Barzilai J. (2004) A New Theory of Measurement (Correcting the Classical Errors): Preference Function Modeling. Proceedings of MCDM 2004 (17th International Conference on Multiple Criteria Decision Making, Whistler, Canada), Simon Fraser University

Ehrgott M., Tenfelde-Podehl, D. (2000) Nadir Values: Computation and Use in Compromise Programming. Universitt Kaiserslautern Fachbereich Mathematik

Granat J., Makowski M. (2000) Interactive Specification and Analysis of Aspiration-Based Preferences. European Journal of Operational Research 122: 469-485

Granat J., Makowski M., Wierzbicki A.P. (2006) Hierarchical Reference Approach to Multicriteria Analysis of Discrete Alternatives. CSM'06: 20th Workshop on Methodologies and Tools for Complex System Modeling and Integrated Policy Assessment. IIASA, Laxenburg, Austria

Heisenberg W. (1927) Uber den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. Zeitschrift fr Physik 43: 172-198

Keeney R. (1992) Value Focused Thinking, a Path to Creative Decision Making. Harvard University Press, Cambridge, Mass.

Keeney R., Raiffa H. (1976) Decisions with Multiple Objectives: Preferences and Value Tradeoffs. J. Wiley and Sons, New York

Kostreva M.M., Ogryczak W., Wierzbicki A. (2004) Equitable Aggregations and Multiple Criteria Analysis. European Journal of Operational Research 158: 362-377

Latour B. (1987) Science in action. Milton Keynes: Open University Press Lootsma F.A. (1996) A Model of the Relative Importance of the Criteria in the Multiplicative AHP and SMART. European Journal of Operational Research 94: 467-476

Makowski M. (1994) Methodology and a Modular Tool for Multiple Criteria Analysis of LP Models. WP-94-102, International Institute for Applied Systems Analysis (IIASA), Laxemburg, Austria

Miettinen K. (1999) Nonlinear Multiobjective Optimization. Kluwer Academic Publishers, Boston

Nakayama H. (1995) Aspiration Level Approach to Multi-Objective Programming and its Applications.

Pardalos P.M., Siskos Y., Zopounidis C. (eds) Advances in Multicriteria Analysis, pp. 147-174, Kluwer Academic Publishers, Dordrecht

Ogryczak W. (2006) On multicriteria Optimization with Fair Aggregation of Individual Achievements. CSM'06: 20th Workshop on Methodologies and Tools for Complex System Modelling and Integrated Policy Assessment, International Institute for Applied Systems Analysis (IIASA), Laxemburg, Austria

Pawlak Z. (1991) Rough Sets - Theoretical Aspects of Reasoning about Data. Kluwer Academic Publishers, Dordrecht

Popper K.R. (1972) Objective Knowledge. Oxford University Press, Oxford Quine W.V. (1953) Two Dogmas of Empiricism. In Benacerraf, P. and Putnam, H.: Philosophy of Mathematics, Prentice-Hall, Englewood Cliffs

Rawls J. (1971) A Theory of Justice. Belknap Press, Cambridge, Mass

Ruiz F., Luque M., Cabello J.M. (2007) The weight factor in reference point procedures: from objective normalization to subjective information. European Journal of Operational Research, in print.

Saaty T. (1982) Decision Making for Leaders: the Analytical Hierarchy Process for Decisions in a Complex World. Lifetime Learning Publications, Belmont, Ca

Tian J., Wierzbicki A.P., Ren H., Nakamori, Y. (2006) A Study on Knowledge Creation Support in a Japanese Research Institute. Proceeding of First International Conference on Knowledge Science, Engineering and Management (KSEM06, pp. 405-417, Springer Verlag, Berlin Heidelberg

Wierzbicki A.P. (1977) Basic Properties of Scalarizing Functionals for Multiobjective Optimization. Mathematische Operationsforschung und Statistik - Optimization 8: 55-60

Wierzbicki A.P. (1986) On the Completeness and Constructiveness of Parametric Characterizations to Vector Optimization Problems. OR Spektrum 8: 73-87

Wierzbicki A.P. (1991) Dynamic Aspects of Multi-Objective Optimization. In Lewandowski A. and Volkovich V. (eds): Multiobjective Problems of Mathematical Programming. Springer-Verlag, Berlin

Wierzbicki A.P., Makowski M., Wessels J. (2000) Model-Based Decision Support Methodology with Environmental Applications. Kluwer, Dordrecht

Wierzbicki A.P. (2005) Technology and Change: The Role of Technology in Knowledge Civilization. Ith World Congress of IFSR, Kobe

Wierzbicki A.P. (2006) Reference Point Approaches and Objective Ranking. Schloss Dagstuhl Seminar 06501: Practical Approaches to Multi-Objective Optimization, Dagstuhl, Germany

Wierzbicki A.P., Nakamori Y. (eds) (2007) Creative Environments: Creativity Support for the Knowledge Civilization Age. Springer Verlag, Berlin-Heidelberg

Yager R.R. (1988) On Ordered Weighted Averaging Aggregation Operators in Multicriteria Decision Making. IEEE Transactions on Systems, Man and Cybernetics 18:

183-190

Zeleny M. (1974) A Concept of Compromise Solutions and the Method of Displaced Ideal. Computers and Operations Research 1: 479-496