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Intergenerational Transfers as a Link Between Overlapping Generations and Ramsey Models

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Abstract

This paper develops a continuous-time model in which intergenerational transfers, different from bequest, serve as a link between the overlapping generations (OLG) model and Ramsey-type model. We show that this theoretical framework covers a range of possible dynamics including the model of perpetual youth and Ramsey model as the two boundary cases. We also prove that, except for the Ramsey case, the new model retains qualitative features of an OLG model: competitive equilibrium can be inefficient, Ricardian debt neutrality may be violated, and asset bubbles can exist.

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Key words: growth models; overlapping generations; intergenerational transfers; asset bubbles; Ricardian neutrality.

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Introduction

Overlapping generations (OLG) models are often used as an alternative to Ramsey-type representative agent models while dealing with sustainability and other intergenerational welfare issues [1]–[3]. Unlike Ramsey-type models, OLG models have an explicit demographic structure to account for various life-cycle aspects. However, particular differences in dynamic properties between the two model types depend significantly on those structural assumptions [2]. For example it was shown in [4] using a discrete-time model that if a bequest motive exists then the behavior of the OLG model is similar to the Ramsey model.

The Yaari-Blanchard-Weil-Buiter model of perpetual youth [5]–[8] is a continuous-time version of an OLG model with constant, possibly infinite, life expectancy and birth rate. Though the framework does not account for the age heterogeneity explicitly, at each moment in time the YBWB model represents cumulative effect of agents of different age and hence different assets and consumption patterns. Due to that the YBWB model exhibits a number of qualitative effects associated with life-cycle models, e.g. inefficiency of competitive equilibrium, violation of debt neutrality and existence of asset bubbles.

In [3] the OLG approach was modified to account for age heterogeneity by grouping several generations into dynasties. This article introduce links between generations in a different way. We extend the YBWB model to include intergenerational transfers and show that the new model covers a range of possible dynamics including YBWB and Ramsey models as the two boundary cases. We also show that, except for the parameter value that corresponds to the Ramsey case, the new model retains qualitative features of YBWB model. This yields an analytical framework that links YBWB and Ramsey models in a simple parametric way.

1 Model setup

1.1 Individual consumers

It is argued in [8, 9] that key differences in the dynamics of the YBWB model compared to the Ramsey model are produced not by finite horizons [6, 10] but by the economic disconnectedness between the cohorts (which eliminates the representative agent). Therefore, we assume for simplicity that life expectancy is infinite. We also assume zero productivity growth. Both non zero death rate and productivity growth can be introduced in a straightforward way [8].

Though the setup is better explained if one considers cohorts instead of individuals, where new cohorts appear for example through migration (see [9]), we keep the term agents as a more usual metaphor.

Denote by $a(s,t)$ and $c(s,t)$ assets and consumption at the instant t of an agent born at time $s \le t$. Under the usual assumption of logarithmic felicity, the agent maximizes his lifetime utility

$$\int_t^{\infty} \ln c(s,\zeta) e^{-\rho(\zeta - t)}\, d\zeta, \qquad \rho > 0, \tag{1}$$

subject to the instantaneous budget constraint

$$\frac{\partial a(s,t)}{\partial t} = (r(t) - \mu) a(s,t) + w(t) - \tau(t) - c(s,t). \tag{2}$$

Here $r(t)$, $w(t)$ and $\tau(t)$ are interest rate, wages and lump-sum taxes, respectively, and μ controls the share of the aggregated per capita wealth donated at birth (see the next section for details). The standard non-Ponzi game condition is also assumed:

$$\lim_{l\to\infty} a(s,l) e^{-(\bar r(t,l) - \mu)(l - t)} = 0. \tag{3}$$

where $\bar r$ is the average interest rate defined as:

$$\bar r(s,t) = \frac{1}{(t-s)} \int_s^t r(\zeta)\, d\zeta.$$

Solution to the maximization problem (1)–(3) is given by

$$c(s,l) = c(s,t) e^{(\bar r(t,l) - \mu - \rho)(l-t)}. \tag{4}$$

Integrating (2) forward with the terminal boundary condition (3), and substituting for c from (4), we get the formula for the consumption as a share of current nonhuman and human wealth:

$$c(s,t) = \rho\left(a(s,t) + h(t)\right), \tag{5}$$

where

$$h(t) = \int_t^{\infty} (w(\zeta) - \tau(\zeta)) e^{-(r-\mu)(\zeta - t)}\, d\zeta$$

is the human capital. The latter implies

$$\dot h = (r(t) - \mu) h - w + \tau. \tag{6}$$

1.2 Aggregation

Let n be the instantaneous birth rate, which in the absence of deaths is the same as the growth rate. Normalize the population by $L(0) = 1$, so that $L(t) = e^{nt}$. Define the aggregate variable $V(t)$ corresponding to the individual variable $v(s,t)$ as follows:

$$V(t) = \int_{-\infty}^{t} v(s,t)\, dL(s) = n \int_{-\infty}^{t} v(s,t) e^{ns}\, ds. \tag{7}$$

In particular,

$$H(t) = \int_{-\infty}^{t} h(t)\, dL(s) = h(t)L(t).$$

Assume that a newly born agent is donated with the initial asset proportional to the aggregated assets divided by the number of the newly born:

$$a(s,s) = \mu(A(s)/nL(s)) = \mu \int_{-\infty}^{s} a(\zeta, s)e^{n(\zeta-s)}\, d\zeta. \tag{8}$$

Applying (7) to (2), (5) and (6) for aggregate assets, consumption, and human wealth, we get

$$C = \rho(A + H), \tag{9}$$
$$\dot{A} = r(t)A + W - T - C, \tag{10}$$
$$\dot{H} = (r(t) - \mu + n)H - W + T. \tag{11}$$

or

$$\dot{A} = r(t)A + W - T - C,$$
$$\dot{C} = (r(t) - \mu + n - \rho)C - \rho(n - \mu)A.$$

In per capita variables, the latter can be rewritten as

$$\dot{a} = (r(t) - n)a + w - \tau - c, \tag{12}$$
$$\dot{c} = (r(t) - \mu - \rho)c - \rho(n - \mu)a. \tag{13}$$

The term μa which is absent in (13), unlike in (5), is compensated by the donation at birth (8), and thus does not affect per capita aggregated assets. Note that for $\mu = 0$ we get the same equations as in the standard YBWB model (cf. [8]), while for $\mu = n$ the structure of equations is the same as in the Ramsey model.

2 Government and market

2.1 Open economy

In the absence of governmental spending, the government's budget equation in per capita variables reads:

$$\dot{b} = (r(t) - n)b - \tau.$$

Let the production is equal to an exogenous endowment on labor: $y = w$ (output is non-produced and non-storable). If the only asset is governmental liabilities then $a = b$, and also $y = c$.

In the equilibrium consumption is at its constant level, $c = e$, so that from (13) we have:

$$\dot{c} = (r(t) - \mu - \rho)e - \rho(n - \mu)b = 0. \tag{14}$$

Nonnegativity of e implies that

$$r(t) \geq \rho + \mu, \tag{15}$$

and $\mu \leq n$. On the other hand, since individual consumption (5) is nonnegative for all t, we also have $h(t) \geq 0$. This yields $b \leq c/\rho$. Substituting the latter inequality in (14), we obtain

$$r(t) \leq \rho + n. \tag{16}$$

Following [8, 9], we consider now several possibilities for assets.

2.1.1 Nonmonetary economy

Let there be no valuable assets: $b = 0$. Then due to the inequality (15) we conclude

Proposition 1. *The steady state can be dynamic inefficient, i.e.* $r(t) < n$, *if* $\rho + \mu < n$.

Note that this is impossible in the Ramsey case: $\mu = n$.

2.1.2 Monetary economy

Consider now $\tau = -\sigma b$. Then at an equilibrium, where intrinsically useless asset b (money) has real value, we have

$$\dot{b} = (r(t) - n + \sigma)b = 0.$$

Existence of the equilibrium requires $r(t) = n - \sigma = r^*$. Together with the condition $b > 0$ this implies

Proposition 2. *Asset bubbles appear if* $-\rho < \sigma < -\rho + (n - \mu)$.

As before a valid monetary equilibrium cannot exist when $\mu = n$.

2.1.3 Public debt

Relabelling b as the public debt, Ricardian neutrality means the consumption path $c(t)$ is independent of $b(t)$ and $\tau(t)$. From (5), (6) and (12), we derive

Proposition 3. *Debt neutrality holds if and only if the growth rates for assets* $b(t)$ *and human capital* $h(t)$ *coincide, i.e.* $\mu = n$.

2.2 Closed economy

Let k denote capital-labor ratio, and $f(k)$ be a neoclassical production function. Existence of fully competitive market yields the interest rate and wages be equal the marginal productivity of capital and labor: $r = f'(k)$ and $w = f(k) - kf'(k)$. Assume depreciation on capital is $\delta > 0$, and all private assets are invested into production (shares): $a = k$. If we also let $\tau = 0$ then

$$\dot{k} = f(k) - (n + \delta)k - c, \tag{17}$$

$$\dot{c} = [f'(k) - (\mu + \rho)]\, c - \rho(n - \mu)\, k. \tag{18}$$

It is easy to see that, due to the second term in the righthand side of the latter equation, both capital and consumption in the equilibrium are less or equal than in the Ramsey model.

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