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The GAINS Optimization Module as of 1 February 2007

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Abstract

This document describes the optimization framework of the GAINS model for Europe. The approach is compared to the approach used in the RAINS model and a detailed description of the objective function, the constraints and the impact functions is given. Finally a comparison of individual single pollutant cost curves generated from the RAINS model and with the optimization module of GAINS is given to illustrate the consistency of the two approaches for single pollutant measures.

Key words: optimization, GAINS model, air pollution

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1 Introduction

This document describes the optimization module of the GAINS model, in particular the mathematical formulation of the optimization problem(s) that can be solved. As a reader we had in mind the technical expert who is familiar with the structure of the RAINS model. A recent description of the RAINS model can be found at

<http://www.iiasa.ac.at/rains/review/review-full.pdf>

We have attempted to stay concise but comprehensive in the coverage of the structure of the model. On the other hand, it is not the purpose of this documentation to reproduce what has been published in the GAINS 1.0 documentation (IIASA interim reports IR-05-053 (on CO₂), IR-05-054 (on CH₄), IR-05-055 (on N₂O), and IR-05-056 (on F-Gases) cf. [5]-[8]:

<http://www.iiasa.ac.at/rains/gains/documentation.html>

Before we enter the world of the GAINS model let us briefly recapitulate the optimization approach that was used in the RAINS model.

1.1 Optimization approach in RAINS

The optimization approach of the RAINS as it was used in the Clean Air For Europe (CAFE) Programme of the European Commission is described in detail in [1]. Here we only briefly summarize this approach.

In RAINS the objective function that is to be minimized is the cost of air pollution control, given that certain environmental constraints are to be met. The total air pollution control costs is the sum over air pollution control costs for five different pollutants (SO₂, NO_x, PM_{2.5}, NH₃ and VOC) and the sum over EU member states. In RAINS multi-pollutant measures, such as the Euro standards for mobile sources, are explicitly excluded from the optimization: in policy applications they are treated separately on a scenario basis.

Single pollutant reduction technologies are represented by their removal efficiency, abatement potential and a unit cost, which can be used to calculate marginal abatement costs. Having sorted pollutant-specific control technologies according to their marginal cost, it is possible to generate marginal abatement cost curves for each pollutant and country. These marginal abatement cost curves form the basic cost input data for the optimization procedure. The decision variables in the RAINS optimization are the segments on the marginal abatement cost curves, representing the use of specific technologies. In

order to speed up the computations, the marginal abatement cost curves were smoothed out where possible.

Impact indicators in RAINS, such as Years of life lost (YOLL), cumulative exceedance of critical loads and SOMO35, are represented as linear functions in the emissions of the relevant pollutants. The coefficients in the linearized relationships are calculated using the EMEP atmospheric transport model, and they were calibrated to be valid in a range likely to contain the then unknown Thematic Strategy scenario.

The optimization procedure thus consists of the following steps:

- set target values for the environmental impacts
- minimize costs for achieving these targets
- read off the optimal emission levels, costs and control strategies for all pollutants and countries.

Since the objective function and all constraints are linear functions in the decision variables, the problem can be solved using linear programming (LP) methods. The optimization problem was formulated with GAMS and solved using the CPLEX solver.

1.2 GAINS approach to optimization

In contrast to the single-pollutant cost curve approach used in RAINS, the optimization module of GAINS uses an explicit representation of technologies. While in RAINS the decision variables of the cost optimization are the segments of (independent) cost curves for each pollutant based on a fixed energy projection, in GAINS the decision variables are the activity levels of individual technologies themselves.

The advantages of the GAINS approach are fourfold:

- Multi-pollutant technologies are represented adequately in this approach. Multi-pollutant emission control technologies, such as those meeting the various Euro-standards for road vehicles, can be cost-effective in a multi-pollutant multi-objective regulatory framework, even though as single pollutant control technologies they may be not. Thus, while in a cost curve approach multi-pollutant technologies often do not appear to be cost effective, in the GAINS optimization these technologies are appraised on the basis their efficiency to meet (potentially) several environmental objectives simultaneously.
- GAINS allows for (limited) changes in the underlying energy system, primarily as possible measures to reduce greenhouse gas emissions. With each change in the energy system, however, the quantitative potential for air pollution control technologies may change. Thus, in RAINS, the corresponding cost curve would need to be recalculated for each change in the energy system. Using an explicit technology representation in the GAINS optimization avoids such a cumbersome procedure, as the model ‘sees’ the available technologies and their potentials for their application at every stage.
- The GAINS approach fully integrates air pollution control and greenhouse gas mitigation measures so that both aspects of emission control can be addressed simultaneously. In contrast, the two issues have been addressed sequentially with RAINS. With GAINS the economic efficiency and environmental effectiveness can thus be increased.

- Emission control costs are directly associated with technologies, rather than with pollutants. For single pollutant technologies this difference is spurious, but for multipollutant technologies and for activity changes that are important greenhouse gas mitigation options it is often inappropriate to attribute costs to the reduction of a single pollutant. With the technology approach of GAINS no such allocation is needed, nor is it always possible. Another important consequence of the technology representation in GAINS is the extension of the concept of maximum technically feasible reductions (MTFR). While in the RAINS approach the point of MTFR on a single pollutant cost curve was determined by the maximum application of end-of-pipe technologies, in GAINS further reductions can be achieved by changing the underlying activities, e.g., the energy mix for a given sub-sector. Thus, for example, a switch from coal to gas or to a renewable fuel will reduce emissions of particles below a level that could be achieved with filter technologies. Though a particular fuel switch may not be cost-effective as a control measure for a single pollutant, it is important to take this additional potential for reduction into account when air pollution targets are discussed, particularly in a carbon constrained setting.

2 Formal Approach

The GAINS optimization module answers the question: how can a given set of environmental targets across Europe be achieved most cost-effectively, and how much does it cost? A solution to the first question is given in the form of an energy mix and set of emission control measures for each country and sector involved. The answer to the second question is given by the total control and fuel substitution cost at appropriate levels of aggregations. The optimization is formulated as a Linear Programming problem, i.e., all equations, definitions and constraints are linear in the decision variables. This allows us to use very fast solvers that are commercially available.

2.1 Dimensions

In the following it will be useful to recall some of the structure of the GAINS model. The general structure of the GAINS model is identical to that of the RAINS model [4] so that for readers familiar with the RAINS structure this section will only introduce some useful notation.

The GAINS Europe model covers 42 land-based regions in Europe, most of them individual countries and four subnational regions in the European part of Russia. Moreover, there are currently five sea regions represented in the model. For simplicity only, in this document we may refer to these 47 regions as ‘countries’. We use the index $i \in I$ to denote the set of emitter countries, and in circumstances in which it is necessary to draw the distinction between emitter and receptor countries, we denote the receptor countries by an index $k \in K$. With GAINS it is possible to include all of the regions, or subsets of regions in the optimization. For the optimization the flexibility is twofold:

- It is possible to select a subset $I_0 \subset I$ of emitter countries i on which the optimization operates, i.e., whose emissions can be changed by changing the country’s control strategy and activity data. For all other countries included in I but not in I_0 all activity data and control measures are fixed at the baseline level. This allows us to study the different implications of whether a policy is applied, e.g., only in EU27, or also beyond.

- It is also possible to select a subset $K_0 \subset K$ of receptor countries that are included in the impact calculation. By defining K_0 independently of I_0 we are able to calculate, e.g., the value of the YOLL function (see below) in EU27 only, but taking into account emissions from all 47 regions.

GAINS covers a number of sectors, and each sector may be associated with a number of different activities. Hence, in GAINS activity data are structured by sector-activity combinations. For example, in the sector ‘industrial boilers’ the associated activities are the various fuels that are used in industrial boilers, i.e., coal, oil, etc. Activities may be further subdivided, e.g., hard coal (grade 1), hard coal (grade 2), etc. The sectors covered by GAINS are indexed by $s \in S$, and likewise the set of activities is indexed by $f \in F$.

In many circumstances it is useful to consider certain subsets of sectors or activities. For example, we define the subset $F_{i,s}$ as the set of activities in country i that are occurring in sector s . This set is clearly only a subset of F , the set of all activities, since not all activities are associated with each sector. Note that the activities actually occurring may be different in different countries. For example, in some countries heavy fuel oil is used as a fuel in the power plant sector, whereas in others it is not. Hence the sets $F_{i,s}$ can be different for different countries.

In the GAINS optimization certain sector-activity combinations (s, f) may be substituted by others (s', f') , for example the use of coal can be reduced in favour of an increase in the use of renewable sources. Yet, not all activities can be substituted by others, and those that can be replaced can only be replaced by certain others. In fact, it is useful to define the set of all sector-activity combinations (s', f') that can replace a given sector-activity combination (s, f) and to denote this set by $A_{i,s,f}$. Thus, the allowed transitions depend on (s, f) and may depend on the country i .

The set of pollutants $p \in P$ in GAINS covers both the traditional air pollutants (SO_2 , NO_x , $\text{PM}_{2.5}$, NH_3 and VOC) as well as the greenhouse gases CO_2 , CH_4 , N_2O and FGAS (a GWP-weighted average of HFCs, PFCs, SF_6).

Emissions of pollutants can be controlled with control technologies $t \in T$, but not every technology controls every pollutant. Rather, for a given pollutant p , the set of technologies that controls this pollutant is denoted by $T_p \subset T$, and conversely, for a given technology t it is useful to define the set of pollutants P_t that are controlled by that technology. In the set of technologies T we have also included pollutant-specific ‘no-control’ technologies NOC_p , for example ‘ NOC_{NOX} ’. In this way any activity, whether controlled or uncontrolled is associated with a technology. The significance of this provision will become clearer in due course.

It will be very useful to define the set of technologies that can be applied in sector s to activity f , and to denote it by $T_{s,f}$ (NB: this set does not depend on the country-index i). Also, we will make use of the set $T_{s,f,p}$, the set of technologies t that are applicable to the sector-activity combination (s, f) and control pollutant p (NB: this set includes the ‘technology’ no-control, NOC_p). Finally, not every sector-activity combination is associated with each pollutant; hence it is helpful to define the set $P_{s,f}$ of pollutants that are associated with the activity-sector combination (s, f) .

2.2 Decision variables and Emissions

GAINS uses two sets of decision variables, which will be explained in detail in the following:

- **Technology-specific activity data.** These variables describe the level of the activity f in sector s and country i that is controlled by technology t . We denote

these variables by $x_{i,s,f,t}$. Naturally, these variables can only take non-negative values and the following has to hold: $f \in F_{i,s}$ and also $t \in T_{i,s,f}$. Thus,

$$0 \leq x_{i,s,f,t}, \quad \forall i \in I, \forall s \in S, f \in F_{i,s}, t \in T_{i,s,f} \quad (1)$$

- **Activity substitution variables.** In GAINS certain activities may be substituted by others. For example the use of coal can be reduced in the power plant sector in favour of an increased use of gas or a renewable source. These transitions or substitutions are described by variables $y_{i,s,f,s',f'}$ where the sector-activity combination (s, f) is replaced by the combination (s', f') .¹ The values of $y_{i,s,f,s',f'}$ have to be non-negative, and it has to be remembered that the set of allowed substitutions $(s, f) \rightarrow (s', f')$ is restricted and may be country-specific. We denote this set by $A_{i,s,f}$.

$$0 \leq y_{i,s,f,s',f'} \quad \forall i, (s', f') \in A_{i,s,f} \quad (2)$$

Since we actually often have fuel substitutions in mind we will refer to the y 's as 'fuel substitutions' even though more general activity substitutions can be conceived.

There are a number of variables that can be derived from these two sets of decision variables. Among these are the activity data that can be linked to the activity data in the GAINS/RAINS database, the application rates of technologies, the country emissions and the end-of-pipe control costs, as well as others. In the following we shall describe some of these derived variables.

- **Activity data.** The technology-specific activity data $x_{i,s,f,t}$ describe the extent to which a certain control technology is applied in a given sector and country to a given activity, but it does not tell us what the total level of activity is. For, example the value for $x_{i,s,f,t}$ in a certain country may be 10 PJ for $s = \text{PP_NEW}$, and $f = \text{HC1}$ and $t = \text{RFGD}$. The total use of HC1 can only be inferred by summing over all 'appropriate' technologies. Since RFGD is an SO2 control technologies we have to sum over all SO2 control technologies (including the 'no-SO2-control technology' NOC_SO2) in order to recover the total use of HC1 in PP_NEW. This can be generalized. Let us define:

$$xp_{i,s,f} = \sum_{t \in T_{s,f,p}} x_{i,s,f,t} \quad (3)$$

This is the pollutant-specific activity data, which by itself may not be an intuitive concept. Its significance becomes apparent shortly. Note that, mathematically for different pollutants the $xp_{i,s,f}$ are independent, i.e., they may be different. However, since $xp_{i,s,f}$ represents the total activity level, independently of the pollutant p under consideration, the $xp_{i,s,f}$ have to be the same for all pollutants:

$$xa_{i,s,f} = xp_{i,s,f}, \quad \forall p \in P_{s,f}, i \in I, s \in S, f \in F_{i,s} \quad (4)$$

Eq. (4) defines the activity data for the sector-activity combination (s, f) in country i and it is used in GAINS as a constraint to ensure consistency of the activity data across pollutants.

¹Often, but not always $s = s'$.

- **Application rates/Control strategies.** Having defined the activity data it is possible to derive the application rates $q_{i,s,f,t}$ of individual technologies (the set of application rates of all relevant control technologies is referred to as a 'control strategy') as:

$$q_{i,s,f,t} = \frac{x_{i,s,f,t}}{x_{a_{i,s,f}}}, \quad \forall i \in I, s \in S, f \in F_{i,s}, t \in T_{s,f} \quad (5)$$

so that $0\% \leq q_{i,s,f,t} \leq 100\%$.

- **Emissions of pollutant p in country i .** It is relatively easy to calculate the emissions of pollutant p in country i from the decision variables, i.e., the technology-specific activity data:

$$\text{emissions}_{i,p} = \left(\sum_{s \in S} \sum_{f \in F_{i,s}} \sum_{t \in T_{s,f,p}} \text{EF}_{i,s,f,t,p}^{\text{abated}} \cdot x_{i,s,f,t} \right) + \text{constant_emissions}_{i,p} \quad (6)$$

where the abated emission factor $\text{EF}_{i,s,f,t,p}^{\text{abated}}$ is calculated in standard GAINS/RAINS fashion as

$$\text{EF}_{i,s,f,t,p}^{\text{abated}} = \text{EF}_{i,s,f,p} \cdot (1 - \text{remeff}_{i,s,f,t,p}) \quad (7)$$

where in turn $\text{EF}_{i,s,f,p}$ is the unabated emission factor of pollutant p associated with the sector-activity combination $(s, f,)$ in country i , and $\text{remeff}_{i,s,f,t,p}$ is the removal efficiency for pollutant p associated with technology t .

In the definition Eq. (6) the second term, $\text{constant_emissions}_{i,p}$, refers to emissions from sectors for which there are currently no control technologies defined in the GAINS model. They are kept constant during the optimization and hence are not modeled on the basis of the decision variables $x_{i,s,f,t}$. Examples for this include CO₂ process emissions from cement production and PM emissions from road abrasion.

2.3 Objective function

The objective function (OF) in GAINS is the function that is minimized in the optimization procedure:

$$\text{OF} = \text{EoP_cost} + \text{FSW_cost} + \text{Ceq_revenues} \quad (8)$$

The objective function in GAINS has the following components:

- **End-of-pipe control costs EoP_cost.** Each control technology in GAINS that reduces an emission factor without changing the underlying activity we may call 'End-of-pipe'-technology. This will include technologies such as flue gas desulphurization (FDG) and selected catalytic reduction (SCR), but also packages such as a the EURO standards in the vehicle sector, as well as the package 'BAN' that is a simple ban of an activity (such as uncontrolled burning of agricultural residues). Each technology t is associated with a unit cost $ucx_{i,s,f,t}$, where we set the cost of the no-control option to zero. Thus the first term in (8) is

$$\text{EoP_cost} = \sum_{i \in I} \sum_{s \in S} \sum_{f \in F_{i,s}} \sum_{t \in T_{s,f}} ucx_{i,s,f,t} \cdot x_{i,s,f,t} \quad (9)$$

Note that there is no sum over pollutants but only over technologies. In this way we do not double count the costs for multi-pollutant technologies. On the other hand, it is not possible to associate the cost of a multi-pollutant technology with a particular pollutant without making an arbitrary choice.

- **Fuel substitution costs.** GAINS offers the option to replace certain given baseline activities by others (e.g., coal by gas) in response to a set of environmental targets (or simply as a more cost-effective energy scenario). These fuel substitutions are associated with unit costs $ucy_{i,s,f,s',f'}$ (how much does it cost to replace one unit of coal with the *equivalent* amount of gas in the power plant sector?), so that the total cost for fuel substitutions is

$$\text{FSW_cost} = \sum_{i \in I} \sum_{s \in S} \sum_{f \in F_{i,s}} \sum_{(s',f') \in A_{i,s,f}} ucy_{i,s,f,s',f'} \cdot y_{i,s,f,s',f'} \quad (10)$$

The unit costs $ucy_{i,s,f,s',f'}$ for the substitution variables are calculated from activity unit costs taking into account potential efficiency gains from the substitution.

- **Climate ”Penalty” Term.** In addition to the intuitive terms described above, we further add a term to the objective function that will force the model to react to a non-zero exogenous carbon price in a climate constrained world. Here we describe the use and rationale of such approach. GHG reduction targets specified as percentage reductions relative to a baseline or base year are a commonplace. Such targets can be implemented in GAINS by imposing a cap on the corresponding emission function.

$$\text{GHG-emissions}_i \leq \text{GHG-emission-cap}_i \quad (11)$$

either for individual countries i or across a whole region I . It is well known from the theory of linear optimization that the shadow price of such a cap constraint can be interpreted as the shadow price of GHG reductions, i.e., the CO₂-equivalent carbon price, which is the result of the optimization. It is also well known that there exists a dual formulation of the carbon constraint, in which the carbon price is exogenous and the emission reduction is the endogenous result of the optimization. The dual approach requires to replace the GHG emission constraint by the following term to the objective function

$$\text{Ceq-revenues} = \text{CO2eq-price} \cdot \text{GHG-emissions} \quad (12)$$

where CO2eq-price is the exogenous CO₂ equivalent price that one may want to impose, and

$$\text{GHG-emissions} = \sum_{i \in I} \sum_{p \in P} \text{GWP}(p) \cdot \text{emissions}_{i,p} \quad (13)$$

are the total greenhouse gas emissions in the region I . Here $\text{GWP}(p)$ is the global warming potential for pollutant p (in GAINS currently non-zero only for greenhouse gases). By adding the term (12) to the objective function the model behaves as if a carbon tax was imposed with the value of ‘CO2eq-price’. In the absence of other constraints, the optimization will ensure that all possible GHG mitigation options available in the model are taken that can be implemented at a cost lower than the exogenously given ‘CO2eq-price’.

Cost results are typically given as costs over the baseline costs, and these represent the additional costs for achieving the targets under the the baseline assumptions that represent current planning:

$$\Delta_{\text{OF}} = \text{OF} - \text{OF}^{\text{BL}} \quad (14)$$

where OF^{BL} represents the baseline scenario costs.

2.4 Environmental Impacts

In GAINS we study four different environmental impacts of the five air pollutants (SO₂, NO_x, NH₃, PM_{2.5}, VOC).

- **Years of Life Lost (YOLL).** In GAINS the loss of life expectancy (for the population above 30 years of age) is represented as a sum of two terms

$$\text{YOLL}^{\text{tot.}}(K) = \sum_{k \in K} \text{YOLL}_k + \sum_{k \in K} \text{YOLL}_k^{\text{CD}} \quad (15)$$

the first reflecting the population-weighted PM_{2.5} concentration at the national scale, the second representing the 'City-Delta' contribution [2]. In GAINS we can consider various receptor regions K , for instance $K = \text{EU25}$, $K = \text{EU27}$, $K = (\text{EU27} + \text{Norway} + \text{Switzerland})$, etc. More explicitly,

$$\text{YOLL}_k = C_k \cdot \text{POP30}_k \cdot \text{PM2.5}_{k,\text{pop-w}} \quad (16)$$

where C_k a (receptor-)country-specific parameter that can be derived from the Cox Proportional Hazards Model, taking into account the changes in life expectancy for each cohort [3]. The parameter POP30_k is the population above 30 years of age in (receptor-)country k , and the population-weighted PM_{2.5} concentration is given by

$$\text{PM2.5}_{k,\text{pop-w}} = \text{pPM}_{k,\text{pop-w}} + \text{sPM}_{k,\text{pop-w}} + \text{aPM}_{k,\text{pop-w}} + \text{nPM}_{k,\text{pop-w}} + k_{k,\text{pop-w}} \quad (17)$$

Here the individual terms are

$$\text{pPM}_{k,\text{pop-w}} = \sum_{i \in I} \pi_{i,k} \cdot \text{emissions}_{i,\text{PM}} \quad (18)$$

$$\text{sPM}_{k,\text{pop-w}} = \sum_{i \in I} \sigma_{i,k} \cdot \text{emissions}_{i,\text{SO}_2} \quad (19)$$

$$\text{aPM}_{k,\text{pop-w}} = \sum_{i \in I} \alpha_{i,k} \cdot \text{emissions}_{i,\text{NH}_3} \quad (20)$$

$$\text{nPM}_{k,\text{pop-w}} = \sum_{i \in I} \nu_{i,k} \cdot \text{emissions}_{i,\text{NO}_x} \quad (21)$$

The constant $k_{k,\text{pop-w}}$ is used to calibrate the linear approximation and includes also the mineral component of PM_{2.5}. The City-Delta contribution to the YOLL function (15) is given by

$$\text{YOLL}_k^{\text{CD}} = C_k \cdot \text{POP30}_k^U \cdot \text{PM2.5}_k^{\text{CD}} \quad (22)$$

where POP30_k^U is the urban population above 30 years of age in country k , and the City-Delta contribution to the population weighted PM_{2.5} concentration is

$$\text{PM2.5}_k^{\text{CD}} = \sum_{i \in I} \sum_{\text{SNAP1}} \delta_{i,k} \cdot T_{i,k}^{\text{SNAP1}} \cdot \text{emissions}_{i,\text{PM,SNAP1}} \quad (23)$$

Here the sum runs over all SNAP1 sectors, and 'emissions _{$i,\text{PM,SNAP1}$} ' are the primary PM_{2.5} emissions by SNAP1 sector in country i and $T_{i,k}^{\text{SNAP1}}$ is the transfer of primary PM_{2.5} from i to k . In fact, for the City Delta only the local contribution is relevant, and this is ensured in this formulation by using the Kronecker delta $\delta_{i,k}$ which is equal to 1 for $i = k$ (emitter = receptor region), and zero otherwise.

- **Acidification.** The impact indicator used is the *average accumulated exceedance*. In GAINS this is a function that is piece-wise linear in the emissions and it is useful to define this in terms of the maximum of linear functions:

$$\text{acid}_k = \max_{\alpha}(\text{acid}_k^{\alpha}) \quad (24)$$

where

$$\begin{aligned} \text{acid}_k^{\alpha} = & \sum_{i \in I} T_{i,k}^{\alpha,N,ac} \cdot \text{emissions}_{i,\text{NO}_x} + \sum_{i \in I} T_{i,k}^{\alpha,A,ac} \cdot \text{emissions}_{i,\text{NH}_3} \\ & + \sum_{i \in I} T_{i,k}^{\alpha,S,ac} \cdot \text{emissions}_{i,\text{SO}_2} + k_k^{\alpha,ac} \end{aligned} \quad (25)$$

where $T_{i,k}^{\alpha,N,ac}$, $T_{i,k}^{\alpha,A,ac}$, and $T_{i,k}^{\alpha,S,ac}$ are coefficients for NO_x , NH_3 and SO_2 , respectively, and $k_k^{\alpha,ac}$ are constants that are used to calibrate the linear approximation.²

- **Eutrophication.** The impact indicator used is the *average accumulated exceedance*. In GAINS this is a function that is piece-wise linear in the emissions and it is useful to define this in terms of the maximum of linear functions:

$$\text{eutr}_k = \max_{\alpha}(\text{eutr}_k^{\alpha}) \quad (26)$$

where

$$\text{eutr}_k^{\alpha} = \sum_{i \in I} T_{i,k}^{\alpha,N,ec} \cdot \text{emissions}_{i,\text{NO}_x} + \sum_{i \in I} T_{i,k}^{\alpha,A,ec} \cdot \text{emissions}_{i,\text{NH}_3} + k_k^{\alpha,ec} \quad (27)$$

where $T_{i,k}^{\alpha,N,ec}$ and $T_{i,k}^{\alpha,A,ec}$ are coefficients for NO_x and NH_3 , respectively, and $k_k^{\alpha,ec}$ are constants that are used to calibrate the linear approximation.³

- **Ground level ozone.** The impact indicator used is *SOMO35*. SOMO35 is calculated as the sum of the daily eight-hour maximum ozone concentrations in excess of a 35 ppb threshold, integrated over the full year. In linearized form

$$\text{SOMO35}_k = \sum_{i \in I} T_{i,k}^{N,O} \cdot \text{emissions}_{i,\text{NO}_x} + \sum_{i \in I} T_{i,k}^{V,O} \cdot \text{emissions}_{i,\text{VOC}} + k_k^o \quad (28)$$

where $T_{i,k}^{N,O}$ and $T_{i,k}^{V,O}$ are coefficients for NO_x and VOC, respectively, and k_k^o are constants that are used to calibrate the linear approximation between SOMO35 and emissions.

2.5 Constraints

In this section we describe the constraints used for the GAINS optimization.

²For the first set of scenario runs performed for the revision of the National Emissions Ceilings directive two base case scenarios were used for deriving linearized relationships between emissions and exceedances, i.e. $\alpha \in \{1, 2\}$.

³For the first set of scenario runs performed for the revision of the National Emissions Ceilings directive two base case scenarios were used for deriving linearized relationships between emissions and exceedances, i.e. $\alpha \in \{1, 2\}$.

2.5.1 Balance Equations

Balance equations ensure the consistency between activity data variables $xa_{i,s,f}$ (cf. Eq(4)) and fuel substitution variables $y_{i,s,f,s',f'}$: if an activity changes relative to the baseline then a transition variable y describing this change takes a corresponding non-zero value. In this way it is ensured that the change is accounted for both for the activity (s, f) that is being replaced, but also for the activity with which it is replaced (s', f') . The consistency is ensured by imposing the following constraints:

- **Energy Balance - Electricity**

$$xa_{i,s,f} - \sum_{(s',f') \in A_{i,s,f}} y_{i,s',f',s,f} \cdot \chi_{i,s',f',s,f}^{(1)} + \sum_{(s',f') \in A_{i,s,f}} y_{i,s,f,s',f'} = xa_{i,s,f}^{\text{BL}} \quad \forall i, \forall s, f \in (\mathbf{E})_i$$

where $xa_{i,s,f}^{\text{BL}}$ is the baseline activity and $\chi_{i,s',f',s,f}^{(1)}$ is the substitution factor, which takes into account the electricity conversion efficiency changes in replacing (s, f) with (s', f') .

- **Energy Balance - Heat**

$$xa_{i,s,f} - \sum_{(s',f') \in A_{i,s,f}} y_{i,s',f',s,f} \cdot \chi_{i,s',f',s,f}^{(2)} + \sum_{(s',f') \in A_{i,s,f}} y_{i,s,f,s',f'} = xa_{i,s,f}^{\text{BL}} \quad \forall i, \forall s, f \in (\mathbf{H})_i$$

where $xa_{i,s,f}^{\text{BL}}$ is the baseline activity and $\chi_{i,s',f',s,f}^{(2)}$ is the substitution factor, which takes into account the heat conversion efficiency changes in replacing (s, f) with (s', f') .

The reason we have to impose these two constraints separately is that in the power sector CHP plants produce both electricity and heat, and it needs to be ensured that for both equivalent amounts as in the baseline are supplied independently.

2.5.2 Technological Constraints

- **Applicability of technologies.** Certain technologies, in particular the best available technology for a given sector-activity combination, may not be applicable beyond a certain limit. This may be due to the fact that, e.g., there is not enough space close to a power plant for housing the additional equipment, etc. This is reflected by imposing an upper limit on the application rate $q_{i,s,f,t}$ of each technology. This maximum application rate $q_{i,s,f,t}^{\text{max}}$ is also referred to as the *applicability* of a technology.

$$q_{i,s,f,t} \leq q_{i,s,f,t}^{\text{max}} \quad \forall i, \forall s, \forall f \in F_{i,s}, \forall t \in T_{s,f} \quad (31)$$

If no information is available, the applicability by default it is set to 100 %.

In the case of NH_3 the situation is a little more complex. Control technologies for NH_3 from livestock can be applied at different stages. In order to keep the number of technology combinations manageable, in GAINS control technologies have been combined into packages. Since it is difficult to consistently define maximum application rates for these packages, we instead define applicabilities for the underlying basic technologies that can be applied at different stages, and constrain the total use of these basic technologies across all packages.

A simple example will illustrate the point: Note that ‘stable adaptation’ (SA) appears in the following table in the first row as a basic or individual technology. It also appears in the last few rows of the table in the technology packages SA and SA_LNA. The package SA means that only SA is applied, whereas SA_LNA stands for a package in which both SA and LNA are applied.

		Basic technologies										
		BF	CS	CS_high	CS_low	LNA	LNA_high	LNA_low	LNF	PM_INC	SA	SUB_U
Technology packages	BF	1	0	0	0	0	0	0	0	0	0	0
	BF_CS	1	1	0	0	0	0	0	0	0	0	0
	BF_CS_LNA	1	1	0	0	1	0	0	0	0	0	0
	BF_LNA	1	0	0	0	1	0	0	0	0	0	0
	BF_LNA_high	1	0	0	0	1	1	0	0	0	0	0
	BF_LNA_low	1	0	0	0	1	0	1	0	0	0	0
	CS_LNA	0	1	0	0	1	0	0	0	0	0	0
	CS_high	0	1	1	0	0	0	0	0	0	0	0
	CS_low	0	1	0	1	0	0	0	0	0	0	0
	LNA_high	0	0	0	0	1	1	0	0	0	0	0
	LNA_low	0	0	0	0	1	0	1	0	0	0	0
	LNF	0	0	0	0	0	0	0	1	0	0	0
	LNF_BF	1	0	0	0	0	0	0	1	0	0	0
	LNF_BF_CS	1	1	0	0	0	0	0	1	0	0	0
	LNF_BF_CS_LNA	1	1	0	0	1	0	0	1	0	0	0
	LNF_BF_LNA	1	0	0	0	1	0	0	1	0	0	0
	LNF_BF_LNA_high	1	0	0	0	1	1	0	1	0	0	0
	LNF_BF_LNA_low	1	0	0	0	1	0	1	1	0	0	0
	LNF_CS	0	1	0	0	0	0	0	1	0	0	0
	LNF_CS_LNA	0	1	0	0	1	0	0	1	0	0	0
	LNF_LNA	0	0	0	0	1	0	0	1	0	0	0
	LNF_LNA_high	0	0	0	0	1	1	0	1	0	0	0
	LNF_LNA_low	0	0	0	0	1	0	1	1	0	0	0
	LNF_SA	0	0	0	0	0	0	0	1	0	1	0
	LNF_SA_LNA	0	0	0	0	1	0	0	1	0	1	0
	PM_INC	0	0	0	0	0	0	0	0	1	0	0
	SA	0	0	0	0	0	0	0	0	0	1	0
	SA_LNA	0	0	0	0	1	0	0	0	0	1	0
	SUB_U	0	0	0	0	0	0	0	0	0	0	1

If q_{SA}^{\max} is the maximum applicability of the basic technology SA, then it has to hold that

$$q_{SA} + q_{SA_LNA} \leq q_{SA}^{\max} \tag{32}$$

i.e. the sum of the application rates of technology packages has to be smaller or equal to the maximum application rate of the individual technology SA. Note that the left hand side of (32) is the sum over all technology packages that contain ‘SA’, i.e., those that have a ‘1’ in the SA column. More generally, for NH₃ control technologies, we impose

$$\sum_{t \in T_{t_0}} q_{i,s,f,t} \leq q_{i,s,f,t_0}^{\max} \tag{33}$$

where t_0 is an individual or basic technology (i.e., appears in the first row in the above table), and T_{t_0} is the set of technology packages that contain the basic technology

t_0 . Elements of the set T_{t_0} are those that are indicated by a ‘1’ in the appropriate column.

- **Minimum application rates for NH₃ technologies.** For the technologies $t_0 = \text{SA, LNF, BF, LNA, LNA_high and LNA_low}$, we further require that the application rate does not decrease relative to the baseline. Since at this stage we do not allow activities to change in the agriculture sector yet, this can be formalized in terms of activity data:

$$q_{i,s,f,t_0}^{\min} \leq \sum_{t \in T_{t_0}} q_{i,s,f,t} \quad (34)$$

- **Emission standards.** Each activity-sector combination is associated with a control strategy. In the baseline this control strategy implies baseline emission levels for each relevant pollutant for every activity sector combination. In GAINS it is required that for each sector-activity combination the emissions of any pollutant can only decrease, but not increase:

$$\sum_{t \in T_{s,f,p}} \text{EF}_{i,s,f,t,p}^{\text{abated}} \cdot x_{i,s,f,t} \leq \text{IEF}_{i,s,f,p}^{\text{BL}} \cdot xa_{i,s,f} \quad (35)$$

where $\text{IEF}_{i,s,f,p}^{\text{BL}}$ is the ‘implied emission factor’ of pollutant p for the sector-activity combination (s, f) in country i in the baseline (BL)

$$\text{IEF}_{i,s,f,p}^{\text{BL}} = \frac{\sum_{t \in T_{s,f,p}} \text{EF}_{i,s,f,t,p}^{\text{abated}} \cdot x_{i,s,f,t}^{\text{BL}}}{xa_{i,s,f}^{\text{BL}}} \quad (36)$$

There are few exceptions for which (35) does not apply. For example, if a NO_x control technology increases emissions of N₂O or NH₃ (e.g., catalytic converter), the constraint is not applied to N₂O or NH₃.

- **Technology standards.** Certain control technologies, e.g. those resulting from earlier emission control legislation, such as EURO-II must not increase their share in optimized scenarios. In particular this constraint applies to the respective no-control options NOC _{p} for each pollutant p .

$$x_{i,s,f,t} \leq \frac{\text{appl}_{i,s,f,t}^{\text{BL}}}{100} \cdot xa(i, s, f), \quad \forall i, \forall s, f \in F_{i,s}, \forall t \in T_{s,f} \quad (37)$$

- **Technology potentials.** The absolute amount of activity that can be controlled may be bounded either from above or below:

$$x_{i,s,f,t}^{\min} \leq x_{i,s,f,t} \leq x_{i,s,f,t}^{\max}, \quad \forall i, \forall s, f \in F_{i,s}, \forall t \in T_{s,f} \quad (38)$$

With the help of these constraints it is possible to disallow the premature scrapping on recently installed equipment. The default value for $x_{i,s,f,t}^{\min}$ is zero, and for $x_{i,s,f,t}^{\max}$ it is infinity.

Since the RAINS optimization procedure does not allow for an adequate representation of multi-pollutant technologies, such as the EURO standards, these were excluded from the optimization and only considered on the basis of scenarios (such as EURO IV vs EURO V/VI) as background to the cost optimization of the stationary

sources. To reproduce RAINS results in GAINS, the equivalent can be achieved by requiring that

$$x_{i,s,f,t}^{min} = x_{i,s,f,t}^{BL} = x_{i,s,f,t}^{max} \quad , \text{ for mobile sources} \quad (39)$$

so that only for stationary source changes in the control strategy (and activity levels xa) are allowed.

2.5.3 Activity constraints.

- **Resource/scrapping constraints** If an activity level can change in GAINS then it is associated with a corresponding upper and/or lower bound

$$X_{i,s,f}^{min} \leq xa_{i,s,f} \leq X_{i,s,f}^{max} \quad \forall i, \forall s, f \in F_{i,s} \quad (40)$$

For instance, the use of renewables is limited to the economic potentials that are used in GAINS. Similarly, there may be a fossil fuel base level (derived with the help of comprehensive energy models) that must always be maintained in GAINS scenarios, and thus serves as a lower limit on certain activities.

A variation of the resource constraint (40) is the upper bound for a resource across more than one GAINS sector:

$$\sum_{s \in S^*} xa_{i,s,f} \leq X_{i,f}^{max} \quad (41)$$

Currently, such a constraint is used in GAINS for GAS use across the power plant subsectors ($S^* = PP_NEW, PP_EX_OTH$).

- **Bounds on fuel substitutions.** In addition to resource constraints and technological constraints described in (40)-(41) there may also be limitations to individual fuel substitutions.

$$y_{i,s,f,s',f'} \leq \text{MAX}_{y_{i,s,f,s',f'}} \quad (42)$$

This is relevant, e.g., in limiting the potential for co-firing in the power plant and the industry sectors.

2.5.4 Aggregations/Consistency

- **Aggregating power plant types.** In contrast to RAINS, GAINS also distinguishes between different power plant types: there are (1) IGCC plants and (2) non-IGCC plants, and these are further distinguished as (1a) electricity-only producing IGCC plants, (1b) district heat CHP IGCC plants, (1c) industrial CHP IGCC plants, (2a) non-IGCC electricity-only producing plants, (2b) district heat CHP non-IGCC plants, (2c) industrial CHP non-IGCC plants, (2d) district heat only non-IGCC plants. From an emissions perspective, the difference between (1) and (2) is more significant than the difference between (a), (b), (c) and (d), and therefore the emissions are calculated at the level of (1) and (2). Hence we aggregate the activity data to this level to ensure consistency between detailed energy balances and emission calculations. Symbolically,

$$x_{i,IGCC,f,t} = x_{i,(1a),f,t} + x_{i,(1b),f,t} + x_{i,(1c),f,t} \quad (43)$$

$$x_{i,\text{non-IGCC},f,t} = x_{i,(2a),f,t} + x_{i,(2b),f,t} + x_{i,(2c),f,t} + x_{i,(2d),f,t} \quad (44)$$

and compactly we write this is as

$$x_{i,PP(L1),f,t} = \sum_{PP(L2)} x_{i,PP(L2),f,t} \cdot \delta_{PP(L1),PP(L2)} \quad (45)$$

indicating that in the power sector (PP) the sum over activities at the aggregation level 2 (L2) matches the activity at the aggregation level 1 (L1), and $\delta_{PP(L1),PP(L2)}$ links level 1 and level 2 activities (e.g. (2a) belongs to (2) in the above notation, etc.).

- **Boiler type shares are constant.** In GAINS each of the sectors $s^* = PP_NEW, PP_EX_OTH, IN_BO, IN_OC, CON_COMB$ and DOM has a sub-structure reflecting different boiler types (e.g. $s_\beta = PP_NEW1, PP_NEW2, PP_NEW3$), and each subsector makes up a share of the total activity in that sector. In the optimization we assume for simplicity that the shares of these subsectors do not change, i.e.,

$$xa_{i,s_\beta^*,f} = share_{s^*,\beta,f} \cdot xa_{i,s^*,f} \quad (46)$$

where s_β^* are the corresponding subsectors of the sector s^* and $share_{s^*,\beta,f}$ is the share of subsector s_β^* in sectors s^* (for a given fuel f). It follows from this that

$$xa_{i,s^*,f} = \sum_{\beta} xa_{i,s_\beta^*,f} \quad (47)$$

i.e., in the sectors that cover different boiler types the subsector activities add up to the total activity.

- **Aggregation of solid fuels OS1 and OS2.** GAINS distinguished two types of 'Other Solid' fuels, OS1 and OS2. These are further distinguished (e.g., fuel wood (FWD), agricultural waste residues (ARD), etc). Emissions of some of the pollutants are calculated in the domestic sector at the most detailed level of fuel disaggregations (e.g. $PM_{2.5}$ at the level of FWD in the sector DOM_FPLACE), whereas other pollutants are calculated at a more aggregate level (e.g. NO_x at the level of OS1 in the sector DOM). The consistency of the subsector aggregation is already taken care of by the aggregation constraint (46) above, so here we require only in addition the correct aggregation of the other solid fuels:

$$x_{i,DOM,OS1} = \sum_{f \in F_{OS1}} x_{i,DOM,f} \quad (48)$$

where the set F_{OS1} contains all solid fuels in the category OS1 (i.e. FWD, ARD, etc).

For sectors other than the domestic sector the basic activity data of the baseline scenario are given at the level of FWD, ARD, etc, whereas emissions are calculated at the level of OS1 and OS2. Hence we require

$$x_{i,s,OS,t} = \sum_{f \in F_{OS}} x_{i,s,f,t} \quad (49)$$

where $OS \in \{OS1, OS2\}$ and F_{OS} is the corresponding set of fuels that are aggregated to OS.

- **Aggregations of fuel substitutions.** In order to ensure consistency in the power sector not only with regard to the activity levels but also with regard to the fuel substitu

$$y_{i,PP(L1),f,PP(L1)',f'} = \sum_{PP(L2)} \sum_{PP(L2)'} y_{i,PP(L2),f,PP(L2)',f'} \quad (50)$$

in analogy with (45) above, and also the corresponding constraint for the aggregation of solid fuels

$$y_{i,s,f,s',OS} = \sum_{f' \in F_{OS}} y_{i,s,f,s',f'} \quad (51)$$

in analogy with (49).

2.5.5 Environmental Targets

So far we have been describing two classes of constraints that are in place by default: those that ensure consistency across the model, and those whose numerical values represent data that are collected in the GAINS/RAINS databases, such as maximum application rates and resource constraints. Environmental constraints that are presented in this section are used for environmental target setting in policy applications, such as the revision of the NEC Directive.

- **Ceiling for YOLL-indicator.**

$$YOLL^{\text{tot.}}(K) \leq _YOLL_ceiling(K) \quad (52)$$

- **Ceiling for Acidification-indicator.**

$$acid_k \leq acid_ceiling_k \quad (53)$$

- **Ceiling on Eutrophication-indicator.**

$$eutr_k \leq eutr_ceiling_k \quad (54)$$

- **Ceiling for SOMO35-indicator.**

$$SOMO35_k \leq SOMO35_ceiling_k \quad (55)$$

3 Features of the GAINS optimization

3.1 Option: Restriction for mobile sources

As mentioned above it is possible to restrict the optimization to a subset of sectors by excluding others, e.g. by fixing technology-specific activity data $x_{i,s,f,t}$ of all mobile sources to their respective baseline values. In this way only the control measures of stationary sources can be optimized. For sensitivity studies the baseline scenario for mobile sources may then be varied to assess the cost effectiveness of specific (packages) of measures for mobile sources.

3.2 Option: The RAINS mode of GAINS

In the RAINS mode of GAINS we restrict the GAINS model to operate exactly as the RAINS model, i.e. to only optimize the end-of-pipe control measures without allowing any changes in the underlying activity data, i.e. no fuel substitutions are allowed. This means that we impose

$$y_{i,s,f,s',f'} = 0, \quad \forall i, s, f, s', f' \quad (56)$$

Equation (29) then implies that

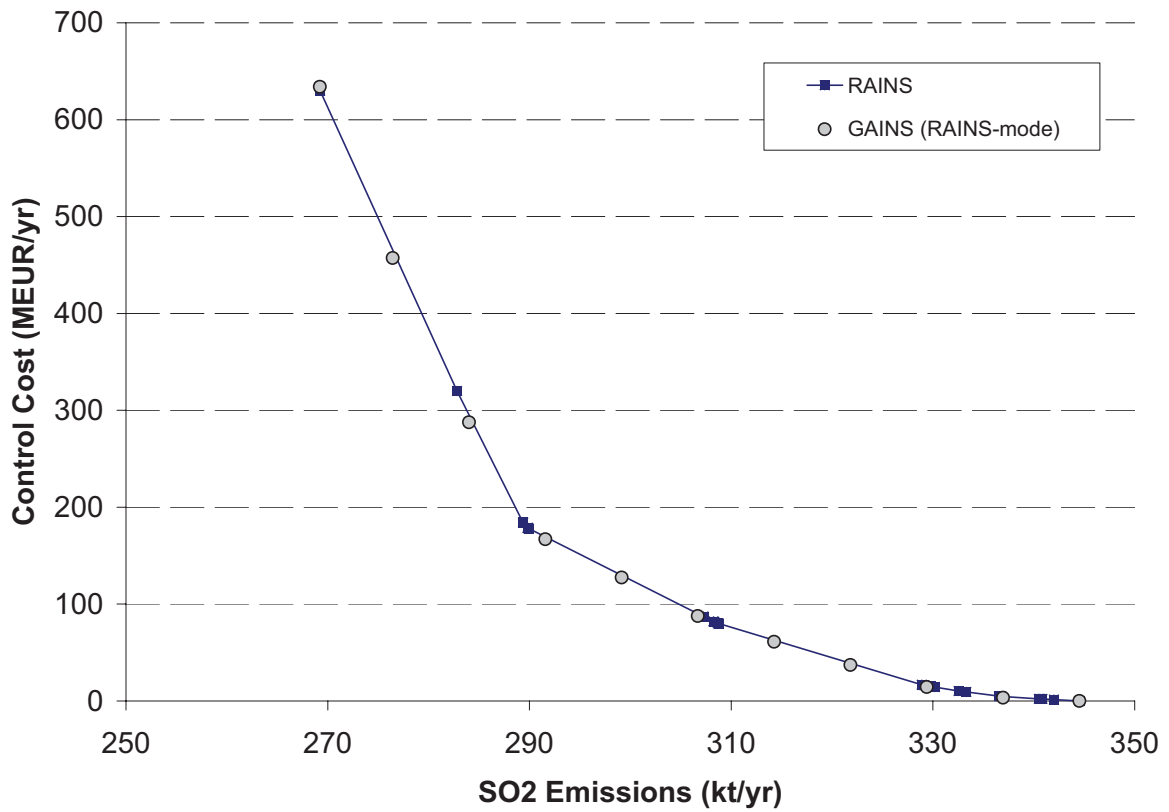
$$xa_{i,s,f} = xa_{i,s,f}^{\text{BL}} \quad (57)$$

so that the individual $x_{i,s,f,t}$ may change, but their sum $xa_{i,s,f}$ may not.

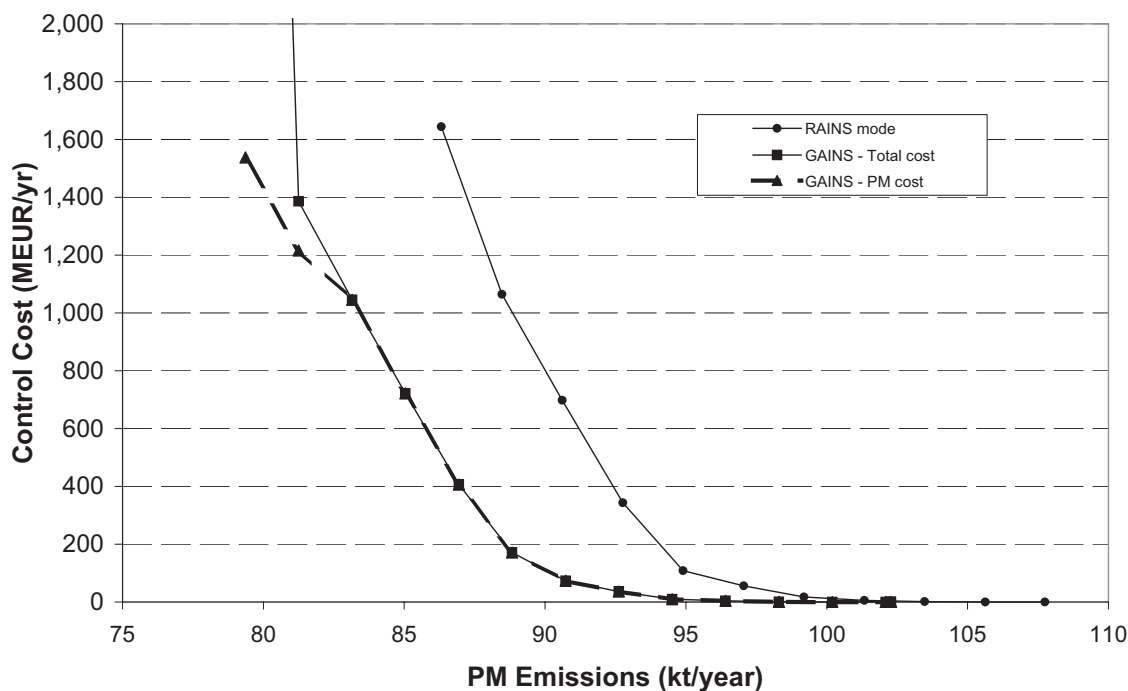
In the RAINS mode of GAINS other equations and constraints simplify. For example, the fuel switch cost term in the objective function vanishes, so that – in the absence of a carbon price – the objective function reduces to the sum over the end-of-pipe control measure costs for air pollution technologies. Also, with vanishing y 's the aggregation constraints for the y 's become trivial.

3.3 Cost Curves in GAINS

GAINS does not produce nor use single pollutant cost curves in the optimization. However, single pollutant cost curves can be constructed by GAINS, if so desired. In the RAINS mode, the GAINS model is allowed to use all add-on technologies for air pollution control like in the RAINS model, but fuel substitutions or efficiency improvement options are suppressed, i.e., are not available. Ignoring multi-pollutant technologies for the time being, the GAINS model in RAINS mode exactly reproduces the results of the original RAINS optimization approach. The next figure shows the validation of the RAINS-mode operation of GAINS for a RAINS SO₂ cost curve for a single country.



The curve connects bold squares that represent individual control technologies in the RAINS model. The curve is generated by ordering the individual control measures according to their marginal cost, taking into account maximum application rates. Each bullet is generated with the GAINS model by imposing an emission ceiling and optimizing for costs. It can be seen that the points calculated by GAINS all lie on the RAINS cost curve. In contrast, when the restrictions on fuel substitutions and efficiency improvements are lifted and the GAINS model is allowed to use all available options, the full 'GAINS-mode' reveals a larger potential for emission reductions. In the following figure, the thin line with bullets illustrates the single pollutant cost curve that is obtained with the GAINS model in RAINS mode.



The curve begins at around 108 kt PM_{2.5} per year and ends at around 86 kt PM_{2.5} per year, which represents the maximum technically feasible reductions scenario generated with the RAINS model ('MRR' scenario). Results emerging from the full GAINS mode are indicated by the thin line with squares. This curve ends at around 79 kt PM_{2.5} per year with costs of around 7 billion /yr (this point is actually off the diagram). These costs include the change in the total system costs, i.e., costs of all fuel substitution options taken to achieve an emission level of 79 kt PM_{2.5} per year. If, however, only those costs are taken into account that are explicitly connected with PM_{2.5} end-of-pipe technologies, then the resulting costs in the MTFR scenario at 79 kt PM_{2.5} per year is lower than 1.6 billion /yr, which is even below the level of the MTFR calculated in the RAINS mode (more than 1.6 billion /yr). This is easily understood if one takes into account that the energy systems in the MTFR situations of the two cost curves are different: the bulleted line is constructed from a baseline scenario, whereas the endpoint of the second and third curves result from a scenario with less use of solid fuels which means that there is less absolute amount of capacities that need to be controlled, which in turn implies smaller amounts of money spent on control equipment (dotted line with triangles).

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