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Interim Report IR-08-007

Assessment of the Market Development Trajectory for Optimal Timing of Technological Innovation

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April, 2008

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Abstract

A dynamic model of investment process for a technology innovator in a market environment is designed. The model is focused on three interrelated decision making problems for an innovator: (1) identification of the econometric trends and calibration of the model parameters; (2) optimization of the commercialization time; (3) optimal control design of the investment policy. A stochastic model based on different types of probabilistic distribution for description of the price formation mechanism is realized in the part of identification of technological trajectories of the market. It has been proven that the extremum of the profit function coincide with the points of intersection of two functions, one of which is the market distribution function that describes the market price formation mechanism and the other is the marginal costs of the project of technology innovation. The model is calibrated basing on the econometric data analysis for the CANON firm provided by the Tokyo Institute of Technology and realized in the illustrative software.

Acknowledgments

The research was supported by the SIMOT Program of the Japanese Ministry of Education, Science and Technology.

It was also supported by the Russian Fund for Basic Research, Grants 05-01-00601, 05-01-08034, the Russian Fund for Humanities RFH 05-02-02118a, and by the Program for the Sponsorship of Leading Scientific Schools, Grant NSCH-8512.2006.1.

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Introduction

The research is devoted to the analysis of a dynamic model of investment process for a technology innovator in a market environment. The model construction includes elements of the economic theory of growth and optimal allocation of resources (see Arrow, 1985; Cellini, Lambertini, Leitman, 2005; Intrilgator, 1971; Kryazhimskii, Watanabe, 2004). Application of the theory of economic growth to modeling of financial flows in investment planning seems to be quite adequate since they catch the main growth and decline trends which can be calibrated basing on the standard econometric software. One of the main control parameters of the model is the stopping time of the process. This parameter is introduced analogously to the model of optimal timing (see Barzel, 1968; Tarasyev, Watanabe, 2001). This second element of the model plays the key role in the decision making process due to the fact that the optimal time can distinguish investment scenarios depending on the current market conditions.

In the model three main interacting objectives of the innovator are in focus. These three tasks can be formulated as: (i) assessment of the market potential innovation on the basis of econometric data, (ii) selection of the possible innovation scenario and optimization of the commercialization time, (iii) optimal design of the investment policy. The main feature of the model is in its dynamic setting: all three problems are considered as the time evolved processes. At each moment of time the innovator can make a decision on the new innovation scenario, optimal time of innovation and optimal investment level in the feedback interaction based on information about the current econometric characteristics of its own technology stock, the market technology stock and the market technology rate. The problem is to find a policy strategy for assessing the potential market innovation, choosing a scenario, optimizing the commercialization time and the investment level.

At each level of the model the peculiar optimization problem is solved. Constructions of the mathematical theory of optimal processes (see Pontryagin, Boltyanskii, Gamkrelidze, Mishchenko, 1962) are used for optimal design of the investment policy. For solving the problem of competition of the innovator in the market environment we

apply methods of the game theory (see Krasovskii, Krasovskii, 1995; Schelling, 1980; Subbotin, 1995). Analysis of the market potential innovation is based on the econometric models of innovation processes (see Griliches, 1984; Watanabe, Lei, 2007). Using the Pontryagin maximum principle we construct analytically the optimal investment plan, optimal technological trajectory and the cost function. The Pontryagin maximum principle in the considered problem can be interpreted as the method of characteristics of the dynamic programming approach for construction of the value function. The obtained formulas constitute the basis for analysis and solution of the problem of choosing optimal commercialization time. Appropriateness of application of methods of optimal control theory is confirmed by results of computer simulations on the basis of the real data which show that the synthetic model trajectories fit well to actual trends of financial flows of investment scenarios.
A sensitive part of the model is the stochastic description of the market behavior. This block is based on different types of probabilistic distribution for simulation of the price formation mechanism and identification of the technological trajectories of the market.

The solution to the problem of construction of optimal investment policy is based on the analysis of the properties of the profit function and its dependence on stopping time of the process. This stopping time is called the commercialization time of the innovation process. The profit function is calculated as the discounted balance between benefits of innovation and investment costs. It has been proven that the extremum of the profit function coincide with the points of intersection of two functions one of which is the market distribution function that describes the market price formation mechanism and the other is the marginal costs of the project of technology innovation.

The model parameters are identified on the basis of the econometric data analysis for the CANON firm provided by the Tokyo Institute of Technology. For this data it is shown that the unique stable point of profit maximum for all states of the technology trajectory of the innovator exists. These results select the unique innovation scenario for the CANON firm and prescribe the sustainable tracking of this scenario.

1. Dynamic Model of Innovation Strategy

We consider the dynamical model of innovation strategy for an innovating firm. The model focuses on three interacting objectives of decision-making: (i) dynamical modeling and econometric analysis of the market of new technology; (ii) selection of the innovation scenario with optimization of the innovator's commercialization time; (iii) optimal control design of the investment policy.

In the problem (iii) of the optimal investment we assume that the current technology stock $x(t)$ is subject to the growth dynamics with the time-delay and obsolescence effects

$$\dot{x}(t) = -\sigma \cdot x(t) + r_a^{\gamma}(t) \qquad (1.1)$$

Here parameter $\sigma > 0$ is coefficient of technology obsolescence, the control parameter $r_a(t)$ is the index of R&D investment, parameter γ, $0 < \gamma < 1$ is the time-delay exponential coefficient. Note that dynamics (1.1) describes the energetic behavior of the innovator since the controlled investment $r_a(\cdot)$ directly influences the technology rate $\dot{x}$. The homogeneous part of equation (1.1) is the Maltus' law for diminishing processes.

The innovator starting the innovation process at time t_0 from the initial level x_0 of the technology stock $x(t)$ should reach at the commercialization time t_a the level x_a, $x_a > x_0$ which is necessary for launching commercialization. In this investment process the innovator is minimizing its expenditures

$$J(t_0, x_0, t_a, x_a, r_a(\cdot), \gamma, \lambda, \sigma) = \int_{t_0}^{t_a} e^{-\lambda s} r_a(s) ds \qquad (1.2)$$

$$r_a = r_a(s) = r_a(s, t_0, x_0, t_a, x_a, \gamma, \lambda, \sigma)$$

here parameter $\lambda > 0$ is a constant rate of discount, and functional (1.2) is the net present value of the innovation.

The dynamic optimization problem with dynamics (1.1) and the functional of expenditures (1.2) can be treated in the framework of optimal control theory (see Pontryagin, Boltyanskii, Gamkrelidze, Mishchenko, 1962; Arrow, 1985).

Assume that the problem (iii) is solved. Denote by the symbol $r_a^0 = r_a^0(s)$ the optimal investment intensity, and by the symbol $x^0 = x^0(s)$ the corresponding scenario of the technology growth. Substituting the optimal intensity into the functional (2.1) one can calculate the optimal total investment

$$w(t_0, x_0, t_a, x_a, \gamma, \lambda, \sigma) = \int_{t_0}^{t_a} e^{-\lambda s} r_a^0(s) ds \qquad (1.3)$$

Fixing in relation (1.3) parameters $t_a, x_a, \gamma, \lambda, \sigma$ and varying initial positions $(t_0, x_0) = (t, x)$ one can consider the series of value functions (optimal result functions)

$$(t, x) \to w(t, x, t_a, x_a, \gamma, \lambda, \sigma) \qquad (1.4)$$

parameterized by variables $t_a, x_a, \gamma, \lambda, \sigma$. In the problem (ii) of selecting the innovation scenario we will be interested in the dependence of the series $w(\cdot)$ (1.4) with respect to the commercialization time t_a and consider this time as the basic parameter of optimization.

The stochastic model for the description of dynamics of the market is considered in problem (i). The probability of the presence of new agents on the market at the current time t is defined by the distribution function $F(t)$. This function is being constructed on the basis of analysis of econometric parameters of the market. The sensitivity analysis of the considered functions of parameters allows modeling the possible distribution functions of the market. Then one can forecast possible technological trajectories of the market and solve the decision-making problems for the innovator.

We consider some distribution functions that are well known from the theory of econometrics and statistics and fit the statistical data on the price parameters and sales of the market. These distribution functions are defined by parameters that can be economically interpreted and have some basic numerical characteristics.

Using standard software for econometric and statistical data analysis (SPSS13, STATISTICA6), we identify the parameters of distribution functions from the real statistical data on the market of considered innovation technology. The results of analysis of real data provided by the Department of Industrial Engineering and Management of Tokyo Institute of Technology show that the following distribution functions fit quite well to the data and can be used for description of the price formation mechanism of the market: distribution with δ-function, exponential distribution, logistic and bi-logistic curves, Johnson-Schumacher distribution, Weibull distribution:

Distribution with δ-function	$F(x) = \begin{cases} 0, & -\infty < x \le x_0 \\ 1, & x_0 < x < +\infty \end{cases}$
Exponential distribution	$F(x) = c + e^{(b_0 + b_1 x)}$
Logistic curve	$F(x) = \dfrac{b_1}{1 + b_2\, e^{-b_3 x}}$
Johnson-Schumacher distribution	$F(x) = b_1 \exp\left(\dfrac{-b_2}{(x + b_3)} \right)$
Weibull distribution	$F(x) = b_1 - b_2 \exp(-b_3 x^{b_4})$

Further, to model the market technology trajectories of the exponential growth we apply "heavy" dynamics, which describes the inert behavior of the market environment

$$\dot{y}(t) = -\sigma \cdot y(t) + r_b(t) = -\sigma \cdot y(t) + z(t) y(t) \tag{1.5}$$

$$\dot{z}(t) = v(t)\,,\ |v(t)| \le v_0$$

Here parameter $y(t)$ stands for the average market technology stock, parameter $r_b(t)$ denotes the average market investment, and variable $z(t) = r_b(t) / y(t)$ is the market R&D intensity.

The market dynamics with the small acceleration describes the exponential growth of the market technology stock $y(t)$. The small variations of the second derivative $\ddot{y}(t)$ of the market technology stock describe the small variations $\dot{z}(t)$ of R&D intensity.Let us introduce the benefit function $d(\cdot)$ of commercialization of the new technology as the total average present value of revenues. Denote by the symbol S_a the usual amount of sales of innovator; by the symbol S_b - bonus sales of innovator; by the symbol $f(\tau)$ - the density of distribution function that describes the probability of presence of all technology agents on the market at time τ. Let time t_a denote the beginning of investment process. Let us fix time $s \geq t_a$ and denote random variable that describes the bonus sales at time s by the symbol $\xi_b(\tau, s)$

$$\xi_b(\tau, s) = \begin{cases} 0, & \tau < s \\ S_b, & \tau \geq s \end{cases} \tag{1.6}$$

The expectancy of random bonus sales $\xi_b(\tau,s)$ (average expected sales) at time s is defined by the following formula (1.7)

$$\mathrm{E}\xi_b(\cdot, s) = \int_{-\infty}^{+\infty} \xi_b(\tau, s) f(\tau) d\tau = S_b \int_{s}^{+\infty} f(\tau) d\tau = S_b(\int_{-\infty}^{+\infty} f(\tau) d\tau - \int_{-\infty}^{s} f(\tau) d\tau) = S_b(1 - F(s))$$

Here function $F(s)$ stands for the probability distribution function describing presence of technological competitors on the market.

We assume that sales are subject to exponential growth with the rate μ of discounted stream of innovation (see Barzel, 1968). The coefficient of discount λ is chosen on the level of average values of the internal rate of return of innovation. The rate μ of discounted stream of innovation and the constant rate λ of discount are connected by inequalities $0 < \mu < \lambda$.

The benefit function $d(\cdot)$ of innovation is the total revenues S_a estimated on the usual level of sales. The expected bonus sales ξ_b are described by the money-flow discounted to the initial time t_a

$$d = d(t_a, S_a, S_b, \lambda, \mu, F(\cdot)) = \int_{t_a}^{\infty} (S_a + S_b(1 - F(s)))e^{-(\lambda - \mu)s} ds. \tag{1.8}$$

Let us introduce the profit function $R(\cdot)$ of the innovation (the net present value of innovation) as the balance of the benefit function $d(\cdot)$ and the optimal investment expenditures $w(\cdot)$

$$R(t, x, t_a, x_a, S_a, S_b, \gamma, \lambda, \mu, \sigma, F(\cdot)) = d(t_a, S_a, S_b, \lambda, \mu, F(\cdot)) - w(t, x, t_a, x_a, \gamma, \lambda, \sigma). \tag{1.9}$$

The key problem of the innovator is to maximize its profit R in the dynamical investment process. The optimal solution essentially depends on the distribution function $F(\cdot)$ of the market commercialization. Identifying dynamically the possible distribution functions of the market, the innovator can choose the possible scenarios of optimal investment policy which correspond to the profit function $R(\cdot)$.

Combining all three levels of the model: (i) identification of the market trajectories, (ii) scenarios selection, and (iii) feedback optimization of the investment level, we obtain the dynamic design of the optimal innovation strategy.

2. Dynamic Optimality Principles and Investment Synthesis

Let us consider the first problem of optimal control design for the investment level. To reach this objective we are dealing with the investment dynamics (1.1) of the innovator and its expenditure functional (1.2). Introducing notations

$$u(t) = r_a^{\gamma}(t)\,,\ t_0 \le t \le t_a\,,\ 0 < \gamma < 1\,, \tag{2.1}$$

we obtain the optimal control problem with the linear dynamics for the growth of the technology stock $x(t)$

$$\dot{x}(t) = -\sigma x(t) + u(t)\,, \tag{2.2}$$

and the exponential expenditure functional

$$J(t_0, x_0, t_a, x_a, u(\cdot), \gamma, \lambda, \sigma) = \int_{t_0}^{t_a} e^{-\lambda s} u^{\alpha}(s) ds\,, \tag{2.3}$$

$$\alpha = \frac{1}{\gamma} > 1\,,\ \ u = u(s) = u(s, t_0, x_0, t_a, x_a, \gamma, \lambda, \sigma)\,.$$

The problem is to find the optimal investment level $u^0(\cdot)$ and the corresponding trajectory $x^0(\cdot)$ of the technology stock subject to dynamics (2.2) for minimizing the expenditure functional (2.3).

As an example, let us consider the new variable

$$w(t) = \int_{t_0}^{t} e^{-\lambda s} u^{\alpha}(s) ds \tag{2.4}$$

for the accumulated effective R&D investment and substitute the problem with integral functional (2.2), (2.3) by the terminal optimal control problem

$$\begin{aligned} &\dot{x}(t) = -\sigma x(t) + u(t) \\ &\dot{w}(t) = e^{-\lambda t} u^{\alpha}(t) \end{aligned} \tag{2.5}$$

with the following boundary conditions

$$x(t_0) = x_0\,,\;\; x(t_a) = x_a\,,\;\; w(t_0) = w_0\,, \tag{2.6}$$

$$t_a > t_0 \geq 0\,,\;\; x_a > x_0 \geq 0\,,\;\; w_0 \geq 0\,.$$

For dynamics (2.5) it is necessary to minimize the terminal boundary value of coordinate $w(t)$ at time t_a

$$w(t_a) \to \min_{(u(\cdot),x(\cdot),w(\cdot))}\,, \tag{2.7}$$

or equivalently to maximize the terminal boundary value of negative coordinate $-w(t)$ at time t_a

$$-w(t_a) \to \max_{(u(\cdot),x(\cdot),w(\cdot))}\,. \tag{2.8}$$

We solve the problem of optimal investment (2.5), (2.8) using Pontryagin's maximum principle (see Pontryagin, Boltyanskii, Gamkrelidze, Mishchenko, 1962). We find the optimal investment process $t \to (u^0(t), x^0(t), w^0(t))$ as the planned scenario, starting from the initial position (t_0, x_0, w_0). We then synthesize the equivalent optimal feedback procedure $u = u(t,x)$ which react in the interactive regime on the current position (t,x) of the technology stock and generate the same optimal trajectory $t \to x^0(t)$. Finally, we calculate the optimal accumulated R&D investment $w(\cdot)$ as the function of the problem's parameters $t_0, x_0, t_a, x_a, \alpha, \lambda, \sigma$. Function $w(\cdot)$ is called the value function of the optimal control problem (2.8).

One can calculate the expression for the optimal investment plan

$$u^0 = u^0(s, t_0, x_0, t_a, x_a, \alpha, \lambda, \sigma) = \frac{(x_a e^{(t_a - s)\sigma} - x_0 e^{-(s-t_0)\sigma})\rho}{(e^{(t_a - s)\rho} - e^{-(s-t_0)\rho})}\,. \tag{2.9}$$

Remark 2.1. *The optimal investment plan $u^0(s)$ (2.9) is the exponential growing function of time s on the time interval $[t_0, t_a]$ with the growth rate $(\lambda + \sigma)/(\alpha - 1)$.*

3. Sensitivity Analysis of Optimal Investment Plan

Let us examine the sensitivity of the optimal investment plan $u^0(\cdot)$ (2.9) with respect to parameters α, λ, σ. One can prove the following results.

Proposition 3.1. *For the range of time s*

$$s \in [t_0, (t_0 + t_a)/2] \tag{3.1}$$

the level of the optimal plan $u^0(s)$ (2.9) decreases to zero, while the discount parameter λ grows to infinity, or parameter α declines to unit.

If time s is located in the second half of time interval $[t_0, t_a]$

$$s \in ((t_0 + t_a)/2, t_a), \tag{3.2}$$

then the level of the optimal plan $u^0(s)$ (2.9) first grows and then declines to zero, while the discount parameter λ grows to infinity, or parameter α declines to unit.

For time $s \in [(t_0 + t_a)/2, t_a)$ the level of optimal plan $u^0(s)$ (2.9) first grows and then decreases to zero, while the obsolescence parameter σ grows to infinity. For time $s \in [t_0, (t_0 + t_a)/2)$ there are two alternatives for the level of the optimal plan $u^0(s)$ (2.9) depending on the values of parameters $t_0 < t_a$, $x_0 < x_a$ u $\alpha > 1$, $\lambda > 0$: it can strictly decline to zero, or it can first grow and then decline to zero, while the obsolescence parameter σ grows to infinity.

At the final moment of time

$$s = t_a$$

the level of the optimal plan $u^0(s)$ (2.9) grows to infinity, while the discount parameter λ grows to infinity, or the obsolescence parameter σ grows to infinity, or parameter α declines to unit.

Remark 3.1. *Proposition 3.1 means that the optimal investment plan $u^0(s)$ (2.9) asymptotically has an impulse character: for the discount parameter $\lambda > 0$, or the obsolescence parameter $\sigma > 0$ tending to infinity, or the delay parameter $\alpha > 1$ tending to unit, the optimal investment level $u^0(s)$ (2.9) converges to zero for times $t_0 \le s < t_a$ and it converges to infinity for $s = t_a$.*

The properties of solution indicated in proposition 3.1. are shown in Fig. 3.1 and Fig. 3.2.

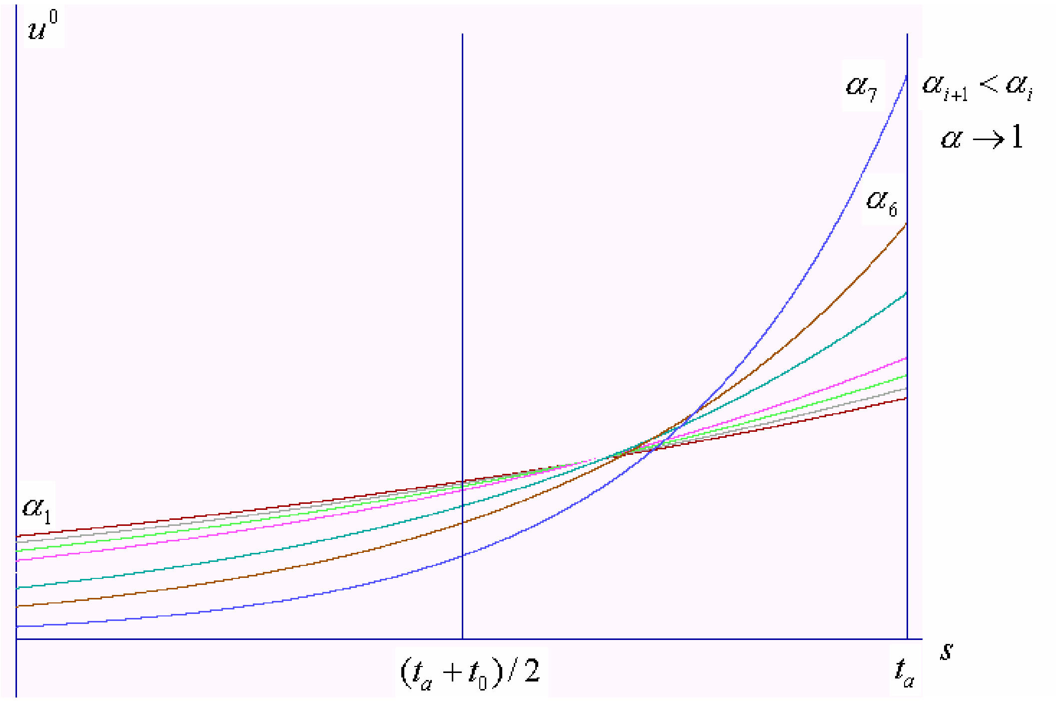

Fig. 3.1. Sensitivity analysis of the optimal investment plan with respect to parameter $\alpha = 1/\gamma$ of time-delay of investments.

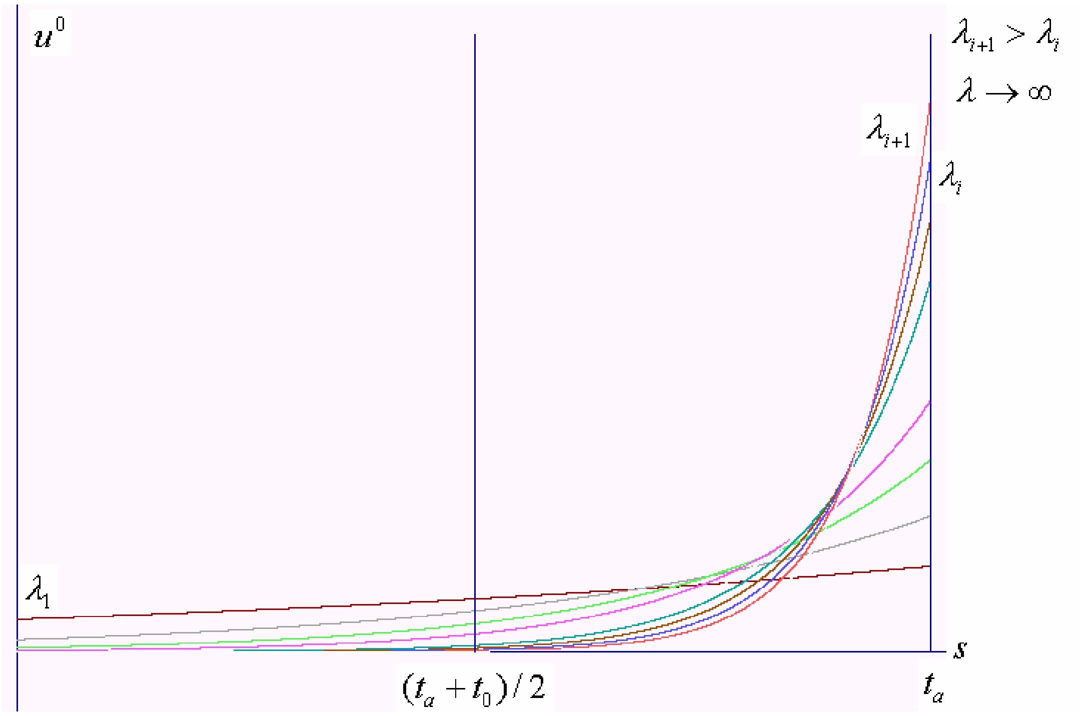

Fig 3.2. Sensitivity analysis of the optimal investment plan with respect to the discount coefficient λ.

4. Optimal Technological Trajectories

In this section we analyze properties of optimal technological trajectories. Substituting the optimal control plan $u^0(\cdot)$ (2.9) into the Cauchy formula (2.5) for technological trajectories $x(\cdot)$ we obtain the optimal technological trajectory

$$x^0(s) = e^{-\sigma(s-t_0)}\left(x_0 + \frac{(e^{\sigma(t_a-t_0)}x_a - x_0)(e^{\rho(s-t_0)}-1)}{(e^{\rho(t_a-t_0)}-1)}\right). \tag{4.1}$$

Let us indicate properties of the optimal technological trajectory $x^0(\cdot)$ (4.1). We begin with indicating boundaries for its values. It is possible to prove the following statements.

Proposition 4.1. *The values of the optimal technological trajectory $x^0(\cdot)$ (4.1) are restricted by boundaries*

$$0 \le x^0(s) \le x_a, \quad t_0 \le s \le t_a. \tag{4.2}$$

Proposition 4.2. *The monotonicity condition with respect to commercialization time t_a is valid for the optimal technological trajectories $x^0(\cdot)$*

$$x^0(s,t'_a) > x^0(s,t''_a), \quad t'_a < t''_a, \quad t_0 < s \le \min\{t'_a, t''_a\} \tag{4.3}$$

Monotonicity condition (4.3) means that optimal technological trajectories for different commercialization times t_a don't intersect each other and thus form the field of characteristics.

Proposition 4.3. *At the commercialization t_a the rate $\dot{x}^0(t_a)$ of the technological trajectory $x^0(\cdot)$ is positive.*

At the initial time t_0 the rate $\dot{x}^0(t_0)$ of the technological trajectory $x^0(\cdot)$ could be positive and negative. Two scenarios depending on the sign of the function $f(t_a) = \dfrac{(e^{\sigma(t_a-t_0)}x_a - x_0)}{(e^{\rho(t_a-t_0)}-1)}$ with time t_0 as a parameter are possible.

The growth scenario

$$f(t_a) > \frac{\sigma}{\rho}, \ \dot{x}_0(t_0) > 0 \tag{4.4}$$

takes place for small innovation times $(t_a - t_0)$.

The scenario with recession

$$f(t_a) \le \frac{\sigma}{\rho}, \ \dot{x}_0(t_0) < 0 \tag{4.5}$$

corresponds to the optimal technological trajectories $x^0(\cdot)$ *first decreasing and then converging to the final level* $x_a > x_0$.

The peculiarities of investment trajectories given in Propositions 4.1-4.3 are indicated in Fig. 4.1.

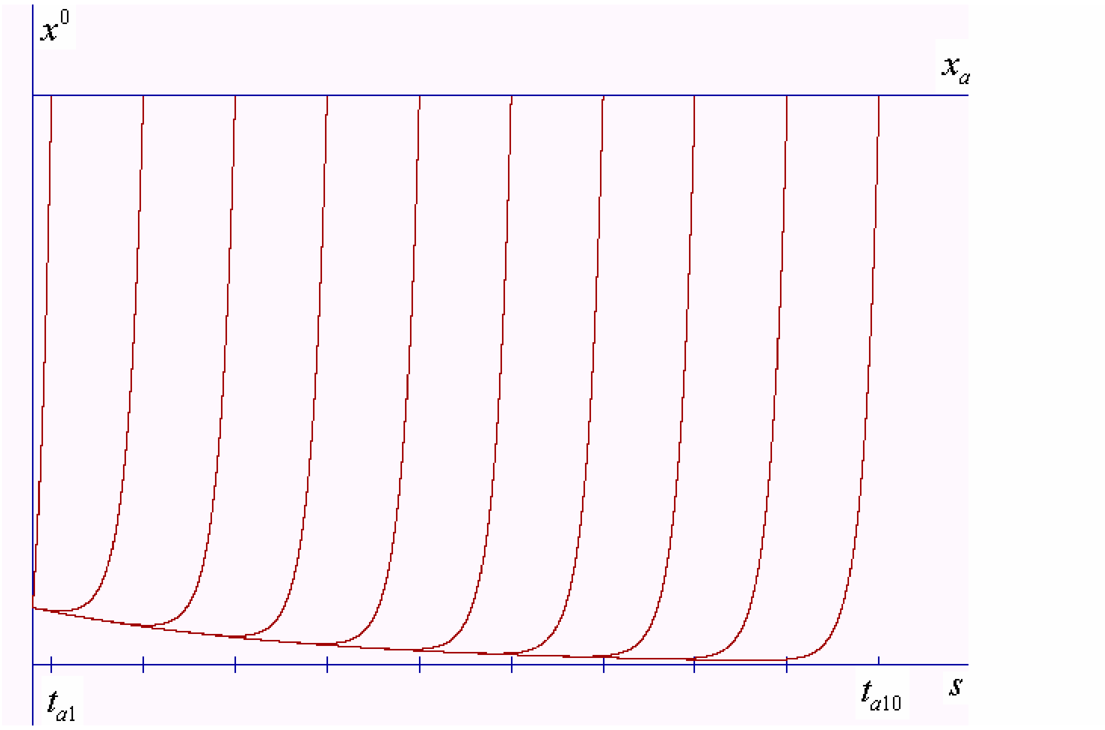

Fig. 4.1. Sensitivity analysis of the optimal technological trajectories with respect to the commercialization time t_a.

5. The Value Function and Optimal Feedback for Technological Dynamics

Let us pass now to the analysis of the value function $(t,x) \to w(t,x)$, $(t,x)=(t_0,x_0)$

$$w = w(t,x,t_a,x_a,\alpha,\lambda,\sigma) = \int_t^{t_a} e^{-\lambda s}(u^0(s))^\alpha\, ds = \tag{5.1}$$

$$K^\alpha(t,x,t_a,x_a,\alpha,\lambda,\sigma) = \int_t^{t_a} e^{-\lambda s} e^{\frac{\alpha(\lambda+\sigma)}{(\alpha-1)}}\, ds =$$

$$\left(\frac{(\alpha\sigma+\lambda)}{(\alpha-1)}\right)^{(\alpha-1)} \frac{e^{-\lambda t}(e^{\sigma(t_a-t)}x_a-x)^{\alpha}}{(e^{\frac{(\alpha\sigma+\lambda)}{(\alpha-1)}(t_a-t)}-1)^{(\alpha-1)}} = \rho^{(\alpha-1)} \frac{e^{-\lambda t}(e^{\sigma(t_a-t)}x_a-x)^{\alpha}}{(e^{\rho(t_a-t)}-1)^{(\alpha-1)}} =$$

$$\rho^{(\alpha-1)} \frac{e^{-\lambda t_a}(x_a - xe^{-\sigma(t_a-t)})^{\alpha}}{(1-e^{-\rho(t_a-t)})^{(\alpha-1)}}, \; \rho = \frac{(\alpha\sigma+\lambda)}{(\alpha-1)}.$$

Let us indicate properties of the value function $w(\cdot)$ with respect to the optimization parameter – the commercialization time t_a. One can prove the following results.

Proposition 5.1. *For the fixed parameters α, λ, σ, initial condition (t,x) and the commercialization technology level x_a, $x_a > x$ the value function $w(\cdot)$ (5.1) has the following properties as function $t_a \to w(t_a)$, $w(t_a) = w(t,x,t_a,x_a,\alpha,\lambda,\sigma)$ of the commercialization time t_a:*
it converges to infinity when the commercialization time t_a tend to the initial time t

$$w(t_a) \to +\infty, \; t_a \downarrow t;$$

it decreases to zero with the exponential rate $-\lambda$ when the commercialization time t_a converges to infinity

$$w(t_a) \to 0, \; t_a \to +\infty, \; \lim_{t_a \to +\infty} e^{\lambda t_a} w(t_a) = w_a < +\infty.$$

Remark 5.1 *The optimal investment feedback is quite clear: if the current technology stock $x = x(t)$ does not yet reach the commercialization level x_a, $x < x_a$, then the optimal R&D investment level u^0 increases proportionally to the difference $(e^{\sigma(t_a-t)}x_a - x)$ with the intensification coefficient $\rho/(e^{\rho(t_a-t)}-1)$. This coefficient rapidly increases when time t approaches the commercialization time t_a and enforces the innovator to reach the commercialization technology level $x(t) \uparrow x_a$ with the optimal expenditure.*

The typical trends of the value function are shown in Fig. 5.1.

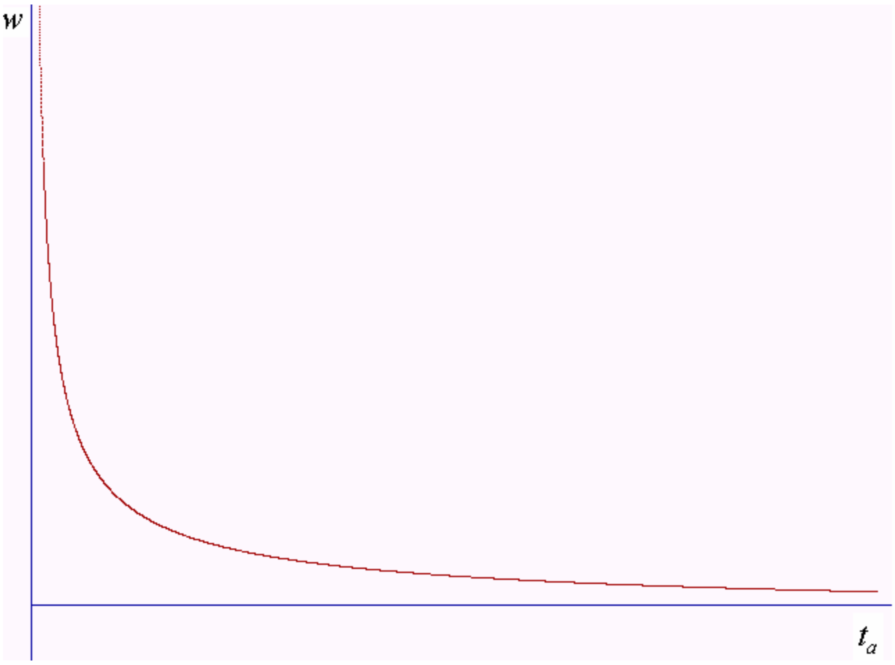

Fig. 5.1. The graph of dependence of the cost function on the commercialization time.

6. Selection of Optimal Scenario and Commercialization Time

Let us introduce the profit function of innovation. It is reasonable to use the usual structure of profit from innovation as a balance between benefit from commercialization of new technologies and expenditure for creating new technologies. The benefit from commercialization of a new technology can be expressed by the amount of sales of goods in which this technology is embedded (see Barzel, 1968). Let us assume that the innovator has the usual amount of sales S_a. In the case when the innovator has the leading position on the market, he can obtain bonus sales S_b. The competitive activities on the innovation market are presented by the density distribution $f(\tau)$ that describes the probability of presence of agents on the market at time τ. Let us fix time $s \geq t_a$ and introduce the random variable for the bonus sales of the innovator as follows

$$\xi_b(\tau, s) = \begin{cases} 0, & \tau < s \\ S_b, & \tau \geq s \end{cases} \tag{6.1}$$

The expectancy of random bonus sales $\xi_b(\tau, s)$ (average expected sales) at time s is defined by the following construction

$$\mathrm{E}\xi_b(\cdot, s) = \int_{-\infty}^{+\infty} \xi_b(\tau, s) f(\tau) d\tau = S_b(1 - F(s)). \tag{6.2}$$

Here function $F(s)$ stands for the probability distribution function describing presence of technological competitors on the market.

The benefit function $d(\cdot)$ of innovation is the total revenues S_a estimated at the usual level of sales. The expected bonus sales ξ_b are described by the money-flow discounted to the initial time t_a

$$d = d(t_a, S_a, S_b, \lambda, \mu, F(\cdot)) = \int_{t_a}^{\infty} (S_a + S_b(1 - F(s)))e^{-(\lambda-\mu)s} ds\,. \tag{6.3}$$

Let us introduce the profit function $R(\cdot)$ of the innovation (the net present value of innovation) as the balance of the benefit function $d(\cdot)$ and the optimal investment expenditures $w(\cdot)$ (6.4)

$$R(t, x, t_a, x_a, S_a, S_b, \gamma, \lambda, \mu, \sigma, F(\cdot)) = d(t_a, S_a, S_b, \lambda, \mu, F(\cdot)) - w(t, x, t_a, x_a, \gamma, \lambda, \sigma)$$

$$\int_{t_a}^{\infty} (S_a + S_b(1 - F(s)))e^{-(\lambda-\mu)s} ds - \rho^{(\alpha-1)} \frac{e^{-\lambda t_a}(x_a - xe^{-\sigma(t_a - t)})^{\alpha}}{(1 - e^{-\rho(t_a - t)})^{(\alpha-1)}}, \ \rho = \frac{(\alpha\sigma + \lambda)}{(\alpha - 1)}\,.$$

The key problem of the innovator is to maximize its profit R in the dynamical investment process. Let us look for the maximum point of profit function R by considering the following equation

$$\frac{\partial R}{\partial t_a} = 0\,. \tag{6.5}$$

Resolving equation (6.5) with respect to the commercialization time t_a as parameter we find the moments of time t_a maximizing the profit function $R(t_a)$. Let us calculate the first derivative of the profit function $R(t_a)$ in time t_a

$$\frac{\partial R}{\partial t_a} = \frac{\partial d}{\partial t_a} - \frac{\partial w}{\partial t_a} = -(S_a + S_b(1 - F(t_a)))e^{-(\lambda-\mu)t_a} - \frac{\partial w}{\partial t_a}\,. \tag{6.6}$$

Substituting this derivative (6.6) into an equation (6.5) we obtain the following equation

$$F(t_a) = 1 + \frac{S_a}{S_b} + \frac{\partial w}{\partial t_a} \frac{e^{(\lambda-\mu)t_a}}{S_b}\,. \tag{6.7}$$

Equation (6.7) means that the maximum profit function R attains at the points of intersection of two functions. One of these functions is the market distribution $F(t_a)$ that describes the market price formation mechanism and another one is the scaled marginal costs $\frac{\partial w(t_a)}{\partial t_a}$ of the project of technology innovation.

Let us examine the properties of the first derivative $\frac{\partial w(t_a)}{\partial t_a}$ of the value function $w(t_a)$

$$\frac{\partial w(t_a)}{\partial t_a} = \rho^{(\alpha-1)}(e^{-\lambda t_a}(\frac{\alpha x\sigma e^{-\sigma(t_a-t)}(x_a - xe^{-\sigma(t_a-t)})^{(\alpha-1)}(1-e^{-\rho(t_a-t)})^{(\alpha-1)}}{(1-e^{-\rho(t_a-t)})^{2(\alpha-1)}} -$$

$$\frac{(\alpha-1)\rho e^{-\rho(t_a-t)}(1-e^{-\rho(t_a-t)})^{(\alpha-2)}(x_a - xe^{-\sigma(t_a-t)})^{\alpha}}{(1-e^{-\rho(t_a-t)})^{2(\alpha-1)}}) - \tag{6.8}$$

$$-\lambda e^{-\lambda t_a}\frac{(x_a - xe^{-\sigma(t_a-t)})^{\alpha}}{(1-e^{-\rho(t_a-t)})^{(\alpha-1)}}).$$

Proposition 6.1. *For the fixed parameters α, λ, σ, initial condition (t,x) and the commercialization technology level x_a, $x_a > x$ the first derivative of the value function with respect to time t_a has the following properties as function $t_a \to \frac{\partial w(t_a)}{\partial t_a}$ of the commercialization time t_a:*

it decreases to infinity when the commercialization time t_a tends to the initial time t

$$\frac{\partial w(t_a)}{\partial t_a} \to -\infty, \ t_a \downarrow t;$$

it decreases to zero when the commercialization time t_a tends to infinity

$$\frac{\partial w(t_a)}{\partial t_a} \to 0, \ t_a \to +\infty.$$

Proof. Let us denote by symbols a and b the following functions:

$$a = (x_a - xe^{-\sigma(t_a-t)}), \ xe^{-\sigma(t_a-t)} = x_a - a \tag{6.9}$$

$$b = (1-e^{-\rho(t_a-t)}), \ e^{-\rho(t_a-t)} = 1-b$$

Substituting a and b (6.9) into expressions (5.1 and 6.8) we obtain the following constructions for the value function $w(t_a)$ and its derivative $\frac{\partial w(t_a)}{\partial t_a}$:

$$w(t_a) = \rho^{(\alpha-1)}e^{-\lambda t_a}\frac{(x_a - xe^{-\sigma(t_a-t)})^{\alpha}}{(1-e^{-\rho(t_a-t)})^{(\alpha-1)}} = \rho^{(\alpha-1)}e^{-\lambda t_a}\frac{a^{\alpha}}{b^{(\alpha-1)}}, \tag{6.10}$$

$$\frac{dw(t_a)}{dt_a} = \rho^{(\alpha-1)}e^{-\lambda t_a}(\frac{\alpha\sigma(x_a-a)a^{(\alpha-1)}b^{(\alpha-1)} - (\alpha-1)\rho(1-b)a^{\alpha}b^{(\alpha-2)}}{b^{2(\alpha-1)}} - \lambda\frac{a^{\alpha}}{b^{(\alpha-1)}}) =$$

$$\rho^{(\alpha-1)}e^{-\lambda t_a}\,\frac{a^{\alpha}}{b^{(\alpha-1)}}(\alpha\sigma x_a a^{-1}-(\alpha-1)\rho b^{-1}+(\alpha-1)\rho-\alpha\sigma-\lambda)\,. \qquad (6.11)$$

One can note that the derivative $\frac{\partial w(t_a)}{\partial t_a}$ (6.11) of the value function $w(t_a)$ can be expressed through the value function $w(t_a)$ (6.10) itself. Returning to the original parameters we obtain the following construction (6.12)

$$\frac{dw(t_a)}{dt_a}=\rho^{(\alpha-1)}e^{-\lambda t_a}\,\frac{a^{\alpha}}{b^{(\alpha-1)}}(\alpha\sigma x_a a^{-1}-(\alpha\sigma+\lambda)b^{-1}+\alpha\sigma+\lambda-\alpha\sigma-\lambda)=$$

$$w(t_a)(\alpha\sigma x_a a^{-1}-(\alpha\sigma+\lambda)b^{-1})=w(t_a)(\frac{\alpha\sigma x_a}{(x_a-xe^{-\sigma(t_a-t)})}-\frac{(\alpha\sigma+\lambda)}{(1-e^{-\rho(t_a-t)})}).$$

Denoted by the symbol $q(t_a)$ the function of commercialization time t_a

$$q(t_a)=\frac{\alpha\cdot\sigma\cdot x_a}{(x_a-x\cdot e^{-\sigma\cdot(\cdot t_a-t)})}-\frac{(\alpha\cdot\sigma+\lambda)}{(1-e^{-\rho\cdot(\cdot t_a-t)})}, \qquad (6.13)$$

we present the derivative of the value function as a result of the multiplication of two functions

$$\frac{\partial w(t_a)}{\partial t_a}=w(t_a)\cdot q(t_a)\,. \qquad (6.14)$$

Let us examine the properties of function $t_a\to q(t_a)$. For convenience we express function $q(t_a)$ (6.13) in the following form:

$$q(t_a)=\frac{\alpha\sigma x_a(1-e^{-\rho(t_a-t)})-(\alpha\sigma+\lambda)(x_a-xe^{-\sigma(t_a-t)})}{(x_a-xe^{-\sigma(t_a-t)})\cdot(1-e^{-\rho(t_a-t)})}= \qquad (6.15)$$

$$\frac{(\alpha\sigma+\lambda)xe^{-\sigma(t_a-t)}-\alpha\sigma x_a e^{-\rho(t_a-t)}-\lambda x_a}{(x_a-xe^{-\sigma(t_a-t)})(1-e^{-\rho(t_a-t)})}.$$

Let us calculate the limits of function $q(t_a)$ (6.15):

$$\lim_{t_a\to t} q(t_a)=\frac{(\alpha\cdot\sigma+\lambda)\cdot(x-x_a)}{(x_a-x)\cdot(1-1)}=-\infty\,, \qquad (6.16)$$

$$\lim_{t_a\to\infty} q(t_a)=\frac{-\lambda\cdot x_a}{x_a}=-\lambda\,.$$

Proposition 5.1 implies that the value function $w(t_a)$ has the following asymptotic behavior

$$\lim_{t_a \to t} w(t_a) = \infty ,$$
$$\lim_{t_a \to \infty} w(t_a) = 0 . \qquad (6.17)$$

Combining expressions (6.16), (6.17) we obtain the necessary relations for limits of derivative $\frac{\partial w(t_a)}{\partial t_a}$

$$\lim_{t_a \to t} \frac{\partial w(t_a)}{\partial t_a} = \lim_{t_a \to t} (w(t_a) q(t_a)) = -\infty ,$$
$$\lim_{t_a \to \infty} \frac{\partial w(t_a)}{\partial t_a} = \lim_{t_a \to \infty} (w(t_a) q(t_a)) = 0 . \qquad (6.18)$$

Proposition 6.2. *There is at least one solution to the equation (6.7).*

Proof. The solution of equation 6.7 coincides with the moments of time t_a at which distribution function $F(t_a)$ of the market meets the function of the scaled marginal costs $\frac{\partial w(t_a)}{\partial t_a}$ in equation 6.7. The probabilistic distribution $F(t_a)$ is continuous on the whole time interval and its values belong to the interval $[0,1]$.
Let us examine the function of the scaled marginal costs $\frac{\partial w(t_a)}{\partial t_a}$ in equation 6.7. Using expressions (6.18) we obtain the following relations for limits of this function

$$\lim_{t_a \to t} \left(1 + \frac{S_a}{S_b} + \frac{\partial w}{\partial t_a} \frac{e^{(\lambda-\mu)t_a}}{S_b} \right) = -\infty ,$$
$$\lim_{t_a \to \infty} \left(1 + \frac{S_a}{S_b} + \frac{\partial w}{\partial t_a} \frac{e^{(\lambda-\mu)t_a}}{S_b} \right) = 1 + \frac{S_a}{S_b} > 1 . \qquad (6.19)$$

Since the function $\frac{\partial w(t_a)}{\partial t_a}$ is continuous and grows asymptotically according to (6.19) the value larger than 1, there exists then at least one point of intersection of this function with the distribution function $F(t_a)$.

The practical consequence of Proposition 6.2 is the algorithm of construction of the innovation scenario. That is, the points of local maximum of profit function obtained as the points of intersection of scaled marginal costs and market distribution function assign different scenarios for the investment process. The optimal strategy of the innovator should be constructed in the following way: Staying at the current position of the investment trajectory (4.1) the innovator estimates the market condition by means of the distribution function (6.7) and forecasts its behavior according to dynamics (1.5). Furthermore, he compares this prediction with his function of the scaled marginal costs (6.6). This comparison extracts the final set of scenarios that are defined by points of intersection. The innovator selects a scenario with the maximum income and follows it according to the optimal investment plan (2.9) until the next moment of decision-making. This procedure is repeated for all positions of decision-making on the investment trajectory.

7. Econometric analysis of the model

Econometric analysis of the models and identification of its parameters is made on the basis of the data provided by the Tokyo Institute of Technology (see [12]). In particular, the innovation process for technology of CANON laser printers has been studied. The data is presented by time series on R&D expenditure, technology stock, sales of printers, and prices of printers measured in money equivalent. Details of the basic results of econometric analysis are given in this section.

7.1. Identification of the coefficient of technology obsolescence

For the econometric identification of the coefficient of technology obsolescence σ in the equation of investment dynamics (1.1) we consider the following model

$$\frac{dx(t)}{dt} - r(t-m) = -\sigma x(t) + \varepsilon_t \,. \tag{7.1}$$

Here symbol $x(t)$ denotes the value of technology stock, symbol $dx(t)/dt$ denotes the rate of technology stock, parameter $r(t-m)$ describes R&D investment in a period of $t-m$, where parameter m assigns the time-delay effect in investment realization, and errors of the model are denoted by symbol ε_t.

Data for variables of the econometric model are given in Table 7.1. By doing calculations it is assumed that the value of the parameter m is equal to three years of delay, $m=3$, according to the average data on electronic industry of Japan. In Table 7.1 and Table 7.3 the R&D expenditure and the technology stock are measured in yen 100 million at 1995 fixed prices.

Table 7.1. Data on delays of R&D investments, technology stock and its rates for the Canon company from 1982-1998.

Years	R-R&D	X - stock	dX/dt	dX/dt-R(t-m)	Y
1982	3,13				
1983	3,99				
1984	5,23	10,521			
1985	6,68	12,324	1,803	-1,327	1,327
1986	7,82	14,625	2,301	-1,689	1,689
1987	8,75	17,632	3,007	-2,223	2,223
1988	10,91	21,681	4,049	-2,631	2,631
1989	12,42	26,91	5,229	-2,591	2,591
1990	13,88	32,924	6,014	-2,736	2,736
1991	15,54	39,466	6,542	-4,368	4,368
1992	16,45	47,729	8,263	-4,157	4,157
1993	17,61	56,952	9,223	-4,657	4,657
1994	21,17	67,015	10,063	-5,477	5,477
1995	23,1	78,065	11,05	-5,4	5,4
1996	28,32	89,284	11,219	-6,391	6,391
1997	32,25	100,907	11,623	-9,547	9,547
1998	35,25	115,321	14,414	-8,686	8,686

For calculations of the model (7.1) the standard program Data Analysis - Regression in Excel has been used. The value of the coefficient of technology obsolescence is identified on the level $\sigma = 0{,}094518$. The high value of determination coefficient R-squared, $R^2 = 0{,}902902$, high value of t-statistics, $t - statistics = 23{,}843209$ and small values of P-value, $P = 4 \cdot 10^{-12}$, indicate the high statistic significance of the model and the high accuracy of econometric calculations.

The detailed report on the results of econometric regression for the coefficient of technology obsolescence σ is given in Table 7.2.

Table 7.2. Results of calculations for the coefficient of technology obsolescence.

Regression Statistics	
Multiple R	0,950211339
R Square	0,902901589
Adjusted R Square	0,825978512
Standard Error	0,78168997
Observations	14

ANOVA	df	SS	MS	F	Significance F
Regression	1	73,86534	73,86534	120,8848	1,2737E-07
Residual	13	7,94351	0,611039		
Total	14	81,80885			

Coefficients	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
X Variable 1	0,094518182	0,003964155	23,843209	4,092E-12	0,085954147	0,103082217

7.2. Identification of the cost-effectiveness coefficient

The cost-effectiveness coefficient γ can be determined from the model (7.2) in which amortization factor σ is identified. To be more precise, the following model is used for econometric identification of the cost-effectiveness coefficient γ

$$\frac{dx(t)}{dt} - \sigma x(t-1) = r^{\gamma}(t) + \varepsilon_t . \qquad (7.2)$$

Here, as above, variable $x(t-1)$ is the current technology stock of innovator in a period of $(t-1)$; variable $dx(t)/dt$ denotes the rate of technology stock; parameter σ is the coefficient of technology obsolescence; parameter $r(t)$ denotes R&D investments, and errors of the model are denoted by symbol ε_t.

Data for model 7.2 is given in Table 7.3.

Table 7.3. Data on logs of R&D investments, technology stock and its rates for the Canon company from 1984-1998.

Years	X-stock	R-R&D	DX	DX-sig*X(t-1)	LN(Y)	LN(R)
1984	10,521					
1985	12,324	6,68	1,803	2,967842	1,087835	1,899118
1986	14,625	7,82	2,301	3,683328	1,303817	2,056685
1987	17,632	8,75	3,007	4,673545	1,541918	2,169054
1988	21,681	10,91	4,049	6,098249	1,808002	2,38968
1989	26,91	12,42	5,229	7,772484	2,05059	2,519308
1990	32,924	13,88	6,014	9,125917	2,211118	2,630449
1991	39,466	15,54	6,542	10,27225	2,329447	2,743417
1992	47,729	16,45	8,263	12,77426	2,547432	2,800325
1993	56,952	17,61	9,223	14,606	2,681432	2,868467
1994	67,015	21,17	10,063	16,39714	2,797107	3,052585
1995	78,065	23,1	11,05	18,42856	2,913902	3,139833
1996	89,284	28,32	11,219	19,65796	2,978482	3,343568
1997	100,907	32,25	11,623	21,16055	3,052138	3,473518
1998	115,321	35,25	14,414	25,31393	3,231355	3,562466

The calculations of model 7.2 give the following results. The value of the cost-effectiveness coefficient is identified on the level $\gamma = 0{,}856144$. The high value of determination coefficient R-squared, $R^2 = 0{,}855223$, high value of t-statistics, $t - statistics = 34{,}58397097$ and small values of P-value, $P = 3{,}5 \cdot 10^{-14}$, indicate the statistic significance of the model and the reasonable accuracy of econometric calculations.

The detailed results of econometric calculations are given in Table 7.4.

Table 7.4. Results of calculations for the cost-effectiveness coefficient.

Regression Statistics	
Multiple R	0,924782665
R Square	0,855222978
Adjusted R Square	0,778299901
Standard Error	0,259905848
Observations	14

ANOVA	df	SS	MS	F	Significance F
Regression	1	5,187465	5,187465	76,79325	1,46187E-06
Residual	13	0,878163	0,067551		
Total	14	6,065628			

Coefficients	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
X Variable 1	0,856143556	0,024755501	34,58397097	3,4946E-14	0,802662556	0,909624556

7.3. Identification of the rate of the discounted stream of innovation

Parameter μ that determines the rate of the discounted stream of innovation is identified on the basis of data on the sales level of CANON laser printers. The exponential model is the adequate construction for description of this economic process

$$\ln \frac{S(t)}{P(t)} = \ln \frac{S_0}{P_0} + \mu\, t + \varepsilon_t \,. \tag{7.3}$$

Here parameter $S(t)$ is the sales time series, variable $P(t)$ describes the price dynamics, parameter t provides the exponential time trend with the growth rate μ, and symbol ε_t denotes errors of the model.

Data for model (7.3) is given in Table 7.5. The prices of printers are measured in yen 10,000 at 1995 fixed prices and the sales of printers are measured in yen 100 million at 1995 fixed prices.

Table 7.5. Data on prices and sales of Canon printers from 1985-1998.

Years	P-prices	S-sales	Y=S/P	LN(Y)	t - time
1985	76,346	302	3,955675	1,375151	0
1986	47,051	529	11,24312	2,419756	1
1987	32,702	883	27,00141	3,295889	2
1988	23,608	1302	55,1508	4,010071	3
1989	18,139	1753	96,64259	4,57102	4
1990	14,517	2084	143,5558	4,966724	5
1991	11,716	2782	237,4531	5,46997	6
1992	9,55	3585	375,3927	5,927973	7
1993	8,234	4025	488,8268	6,192008	8
1994	7,189	4605	640,562	6,462346	9
1995	6,3	5801	920,7937	6,825236	10
1996	5,463	7478	1368,845	7,221723	11
1997	4,995	7914	1584,384	7,367951	12
1998	4,436	9058	2041,93	7,621651	13

The following results have been obtained for the model (7.3). The value of the growth rate of the discounted stream of innovation is identified on level $\mu = 0{,}448931$. The high value of determination coefficient R-squared, $R^2 = 0{,}950008$, high value of t-statistics, $t - statistics = 15{,}100957$ and small values of P-value, $P = 3{,}599 \cdot 10^{-9}$, indicate the statistic significance of the model and the high accuracy of econometric calculations.

The detailed results of econometric analysis for model 7.4 are shown in Table 7.6.

Table 7.6. Results of calculations for the growth rate of the discounted stream of innovation.

Regression Statistics	
Multiple R	0,974683593
R Square	0,950008106
Adjusted R Square	0,945842115
Standard Error	0,448400591
Observations	14

ANOVA	df	SS	MS	F	Significance F
Regression	1	45,85021	45,85021	228,0389	3,59937E-09
Residual	13	2,412757	0,201063		
Total	14	48,26297			

Coefficients	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	2,348193801	0,2273806	10,327151	2,526E-07	1,85277404	2,843613562
X Variable 1	0,448931376	0,02972867	15,100957	3,599E-09	0,38415817	0,513704582

7.4. Identification of the distribution function

The stochastic model is based on different types of distribution functions: distribution with δ-function, exponential distribution, logistic and bi-logistic curves, Johnson-Schumacher distribution, Weibull distribution, All are used for the description of price formation mechanism of the market. Numeric experiments prove that the most fitting distributions for the CANON data are exponential distribution and Johnson-Schumacher distribution. Let us consider the first model in which distribution is determined by the exponential function

$$y(t) = c + e^{(b_0 + b_1 t)} + \varepsilon_t \,. \qquad (7.4)$$

For identification of parameters of the distribution function of the nonlinear exponential model econometric software SPSS SigmaStat 3.0 is used. Data for calculations of the model are in Table 7.7. The sales of printers are measured in yen 100 million at 1995 fixed prices.

Table 7.7. Data on prices of Canon printers from 1985-1998.

Years	y-prices	t-time
1985	76,346	0
1986	47,051	1
1987	32,702	2
1988	23,608	3
1989	18,139	4
1990	14,517	5
1991	11,716	6
1992	9,55	7
1993	8,234	8
1994	7,189	9
1995	6,3	10
1996	5,463	11
1997	4,995	12
1998	4,436	13

The following results have been obtained for the nonlinear exponential model (7.4). The values of parameters of the exponential function are determined as follows: $c = 5{,}841$, $b_0 = 4{,}234$, $b_1 = -0{,}455$. Calculations show that the model is statistically significant. Detailed results of econometric analysis for the model (7.4) are given in Table 7.8.

Table 7.8. Results of SPSS calculations for the parameters of the exponential distribution.

Nonlinear regression

Data source: Data 1 in Canon prices
[Parameters]
c=0,1; b0=0,1; b1=0,1
[Variables]
y=col(1); t=col(2)
[Equation]
f=c+Exp(b0+b1*t)
fit f to y

Results

R = 0,998;	Rsqr = 0,996;	Adj Rsqr = 0,995;	Standard Error of Estimate = 1,384		
	Coefficient	**Std. Error**	**t**	**P**	**VIF**
c	5,841	0,602	9,703	<0,001	2,649
b0	4,234	0,0191	221,267	<0,001	1,524
b1	-0,455	0,0196	-23,180	<0,001	2,533

Analysis of Variance

	DF	**SS**	**MS**	**F**	**P**
Regression	2	5,433,435	2,716,718	1,418,149	<0,001
Residual	11	21,072	1,916		
Total	13	5,454,508	419,578		

Normality Test:	Passed	(P = 0,689)
Constant Variance Test:	Passed	(P = 0,482)
Power of performed test with alpha = 0,050: 1,000		

The graph of fitness of the data approximation on the basis of the nonlinear exponential model is shown in Fig. 7.1.

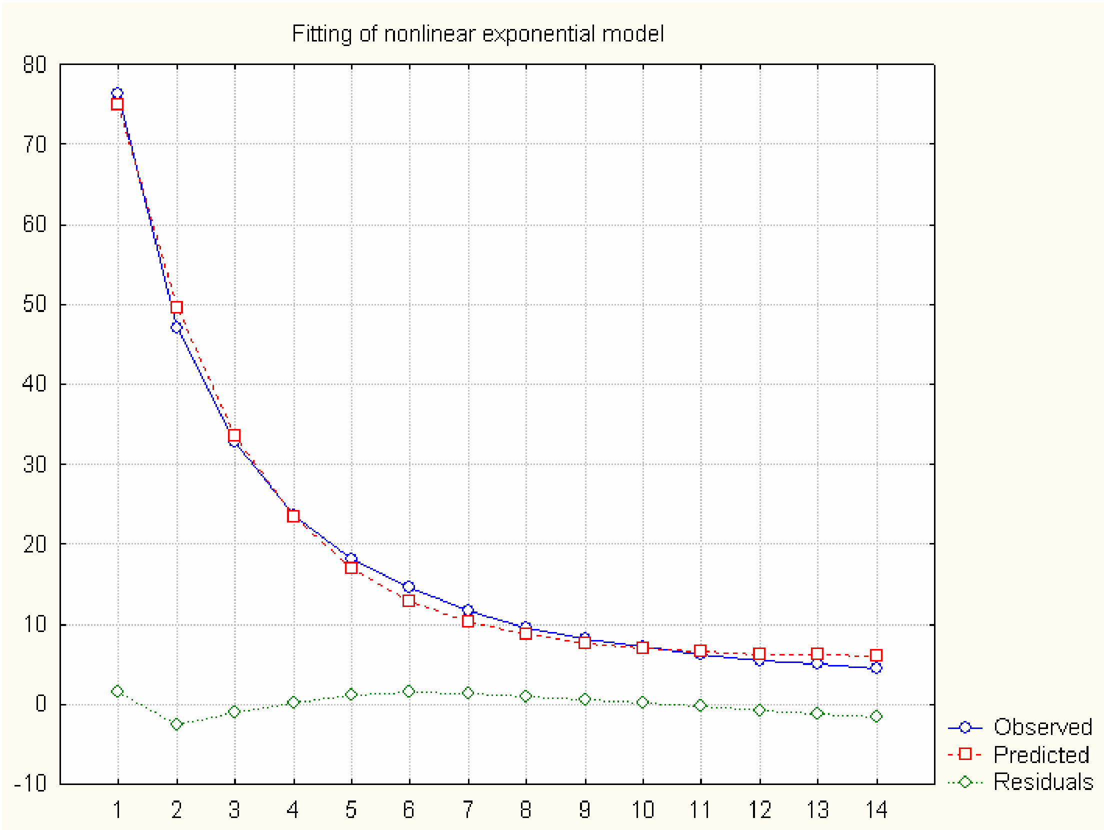

Fig. 7.1. Fitness of the exponential distribution to the real data.

The second nonlinear model is considered by Johnson-Schumacher function

$$y(t) = b_1\, e^{(-b_2\,/(t+b_3))} + \varepsilon_t\,. \tag{7.5}$$

For identification of parameters of the Johnson-Schumacher function of the nonlinear model (7.5) econometric software SPSS SigmaStat 3.0 is used. The following results are obtained for model 7.5. Values of parameters of the Johnson-Schumacher function are determined as: $b_1 = 0{,}6484$, $b_2 = -43{,}7363$, $b_3 = 9{,}1766$. Calculations show that the model is statistically significant. Detailed results of the econometric analysis for model 7.5 are given in Table 7.9.

Table 7.9. Results of SPSS calculations for the parameters of the Johnson-Schumacher distribution.

Data source: Data 1 in Canon prices
[Parameters]
b1=0,6; b2=-44; b3=9
[Variables]
y=col(1); t=col(2)
[Equation]
f=b1*Exp((-b2)/(t+b3))
fit f to y

Results

R = 1,000;	Rsqr = 1;	Adj Rsqr =1,000;	Standard Error of Estimate = 0,270		
	Coefficient	**Std. Error**	**t**	**P**	**VIF**
b1	0,647	0,0555	11,668	<0,001	1,077,080
b2	-43,774	1,975	-22,163	<0,001	5,741,627
b3	9,181	0,254	36,178	<0,001	1,932,042

Analysis of Variance

	DF	**SS**	**MS**	**F**	**P**
Regression	2	5,453,706	2,726,853	37,420,328	<0,001
Residual	11	0,802	0,0729		
Total	13	5,454,508	419,578		

Normality Test:	Passed	(P = 0,510)
Constant Variance Test:	Passed	(P = 0,173)
Power of performed test with alpha = 0,050: 1,000		

The graph of fitness of the data approximation on the basis of the nonlinear Johnson-Schumacher model is shown in Fig. 7.2.

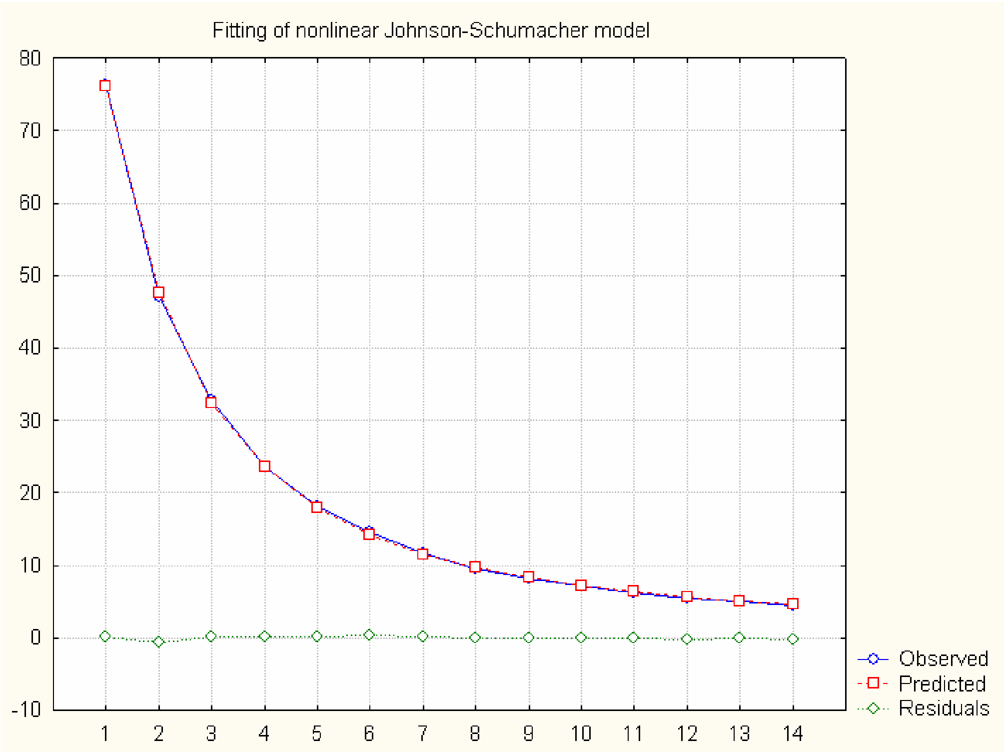

Fig. 7.2. Fitness of the Johnson-Schuhmacher distribution to the real data.

8. Simulation of the model on optimal strategies

Numeric experiments are carried out for all blocks of the model. Identified parameters of the model have been used in these experiments. Results of these experiments for three cases are given below. The aim of numerical experiments is to demonstrate that the proposed algorithm has a universal character allowing us to find the optimal commercialization time for distribution functions of different types. Another aim is to show that solutions of the algorithm qualitatively depend on shapes of distribution functions.

In the first experiment the simulation of the model is performed, in which the density distribution function that describes price formation mechanism is defined as δ-function. Such a probability density function describes the instantaneous change of the price on the innovation product upon appearance of principal competitors on the market at time $t_b = 1.54$.

$$F(t_a) = \begin{cases} 0, & t_0 \le t_a < t_b \\ 1, & t_b \le t_a < +\infty \end{cases}$$

The results of modeling are shown in Fig. 8.1. They distinctively show that the stepwise distribution function has exactly three intersections with the marginal costs function. One point of intersection corresponds to the local minimum of the profit function, and two points of intersection correspond to points of the local maximum, one of which is the global maximum. These points of local maximum at times $t_1^m = 1.483$ and $t_2^m = 1.627$ correspond to two possible investment scenarios, one of which is the fast scenario and the other one being the slow scenario. The problem for the innovator is to determine these scenarios at each current position of the investment trajectory and to make a decision on the selection of the more preferable scenario between these two scenarios. It is assumed that one can switch from one scenario to another one depending on the information about the market dynamics.

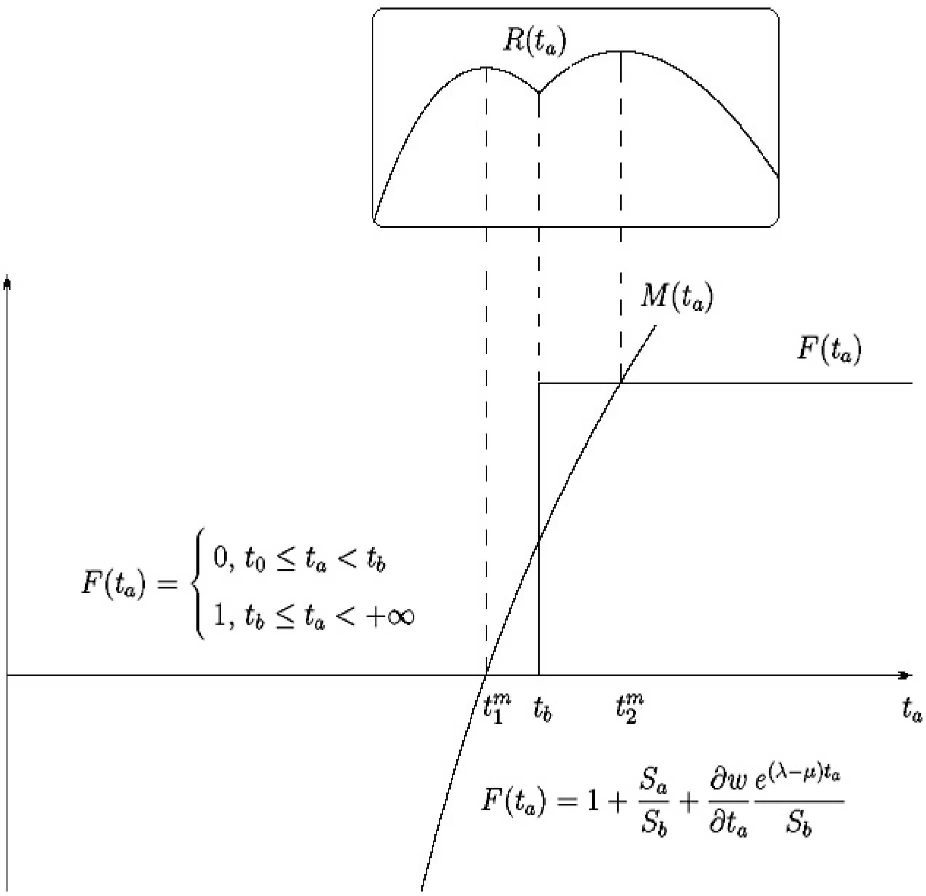

Fig. 8.1. Double-humped curve of the profit function for the Heaviside step function of probability distribution.

In the second experiment, the exponential distribution function has been chosen for the description of the market dynamics.

$$F(t_a) = \begin{cases} 0, & t_0 \le t_a < t_b \\ 1 - e^{\beta(t_a - t_b)}, & t_b \le t_a < +\infty \end{cases}$$

The parameters of this distribution function have been using CANON data $\beta = -0.455$, $t_b = 1.34$. The results of the modeling are shown in Fig. 8.2. In this figure one can see that in the case of using real data in the model the distribution function has only one point of intersection with the marginal costs function at time $t^m = 1.49$. This point corresponds to the global maximum of the profit function. It determines the unique investment scenario. Experiments show that such a situation is stable in the sense that there exists the unique investment scenario for any position of decision-making on the investment trajectory. Parameters of this scenario can vary depending on the market dynamics, but the qualitative behavior of the solution is stationary in the sense that the investment scenario is unique.

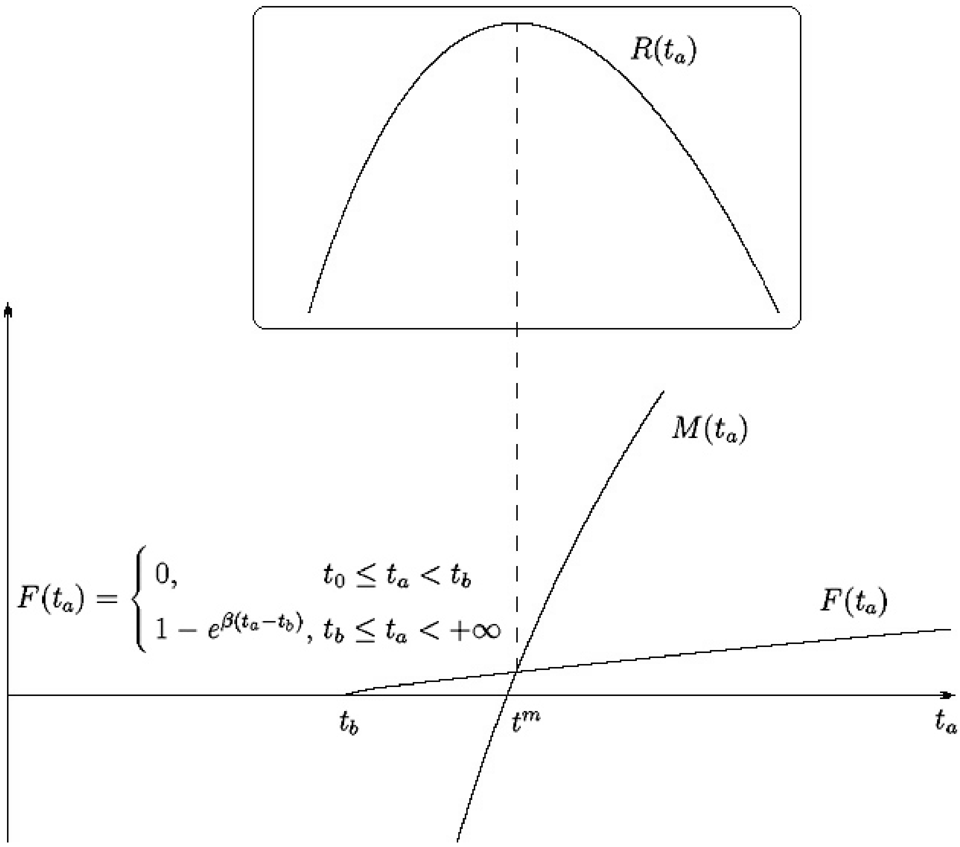

Fig. 8.2. The graph of the profit function with the unique maximum.

In the third numeric experiment the bi-logistic curve has been considered for description of the distribution function. It corresponds to the probability density function with two modes, which describe the more representative positions of competitors on the market. The results of this experiment are shown in Fig. 8.3. In this case the distribution function has three points of intersection with the marginal costs function. One of these points corresponds to local minimum, while the other two correspond to points of local maximum of the profit function. The problem for the innovator is to select his own “niche” in the market. To be more precise, the innovator should deviate from the point of local minimum that corresponds to time of appearance of principal competitors on the market and then to shift to the point of global maximum that is chosen between two points of local maximum.

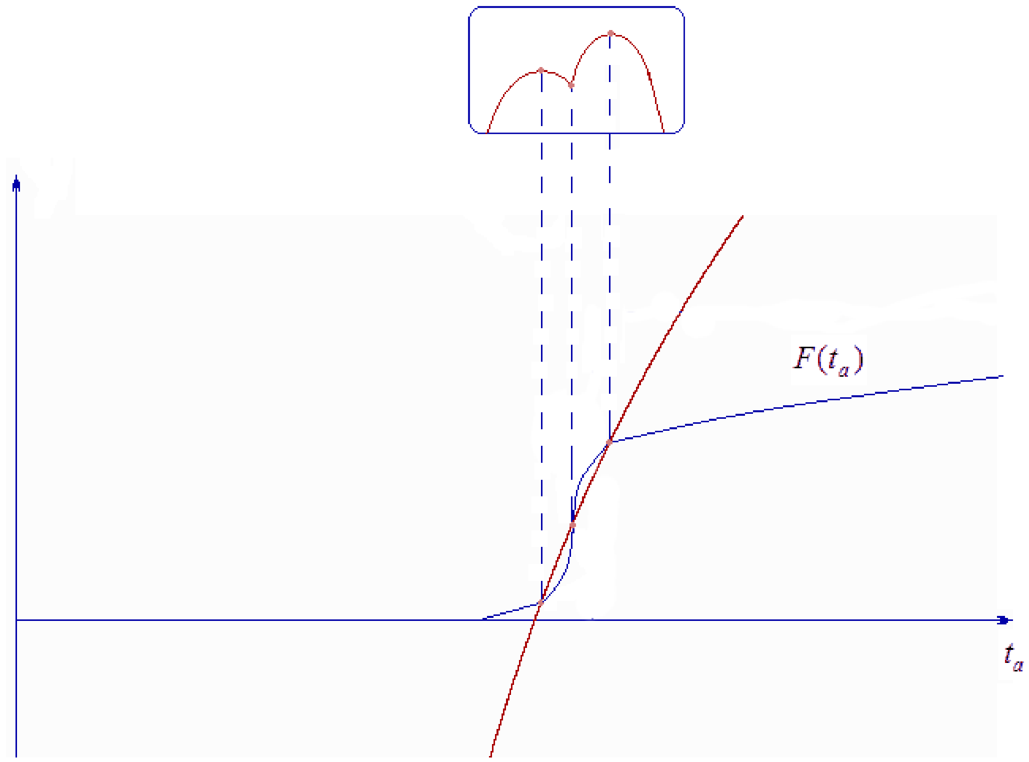

Fig. 8.3. Niche searching in the market.

These three experiments demonstrate that the proposed algorithm for the construction of the optimal investment plan has a universal character with respect to variations of probability distribution functions describing the market dynamics. That is, for three essentially different distributions the algorithm selects the optimal commercialization time in a robust way as points of intersection of the market distribution function and marginal costs. On the other hand, these three experiments show that depending on the shape of the market distribution function different qualitative cases for these points of intersection are possible: the number of points of intersection determines different investment scenarios.

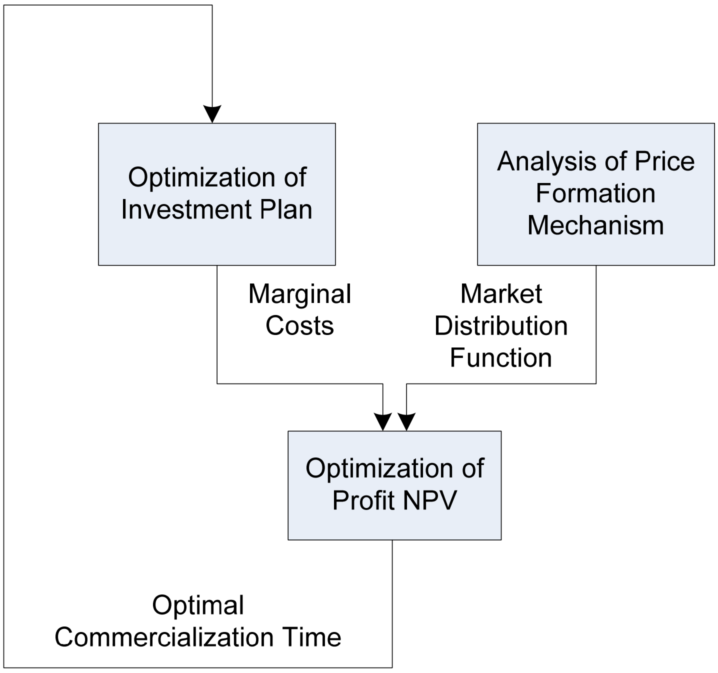

Fig. 8.4. Scheme of decision making on investment strategy.

The application of the proposed algorithm to strategic decisions of investors can be described in brief as follows: from the endogenous block of "Optimization of Investment Plan" one can obtain the "Marginal Costs", and, in parallel, from the exogenous block "Analysis of Price Formation Mechanism" one can estimate the "Market Distribution Function"; using this information one can optimize the "Profit NPV" by intersecting the marginal costs and the market distribution function; as a result, the "Optimal Commercialization Time" is obtained and passed to the block "Optimization of Investment Plan" closing the feedback loop of the endogenous scheme. It is worth noting that the proposed scheme is constructed on the feedback principle and responds to the current situation in the market and current position of the investment plan, and, hence, the procedure of decision making can be gradually updated. The general scheme of the proposed algorithm of investment strategy is depicted in Fig. 8.4.

It is worth noting, that the model has a block structure and in the present version the blocks are adjusted to the case study of the innovation process of Canon laser printers. In principle, one can modify the model blocks in such a way that they fit to the data of various high-tech sectors.

9. Conclusions

In the paper a stochastic version of an investment dynamic model is elaborated for a process of technology innovation. Three interrelated decision making problems for an innovator are: econometric identification of trends of the market; optimal selection of the commercialization time; construction of the optimal investment strategy on the feedback principle. Stochastic modeling of the price formation mechanism constitutes the basic element of the proposed algorithm for identification of market technological trajectories. A general method is proposed for construction of the optimal commercialization time and investment scheduling in reply to price trends of the market. The model is constructed under the assumption that the basic parameters such as the discount rate, the rate of discounted stream of innovation, and the level of bonus sales, are fixed at the constant level. Besides that, it is assumed that the market distribution function can be identified uniquely. In the future research one can focus on the game statement of the problem of investment optimization when the basic parameters of the model and the procedure of selection of the market distribution function are considered as a counterpart.

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