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Influence of Workforce Ageing on Human Capital Formation

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Abstract

This paper addresses the question of how workforce ageing influences human capital formation, human capital deterioration, and future productivity growth. The method presented in this paper focuses on the magnitude of human capital that has been accumulated in an individual. It takes into consideration education, acquiring knowledge and experience, knowledge becoming obsolete or forgotten, as well as the impact of health. The estimated human capital curve (based on the net effect of the various determinants of human capital) has an inverted U-shaped profile. The consequences of workforce ageing can be considered in the short and long term. In the short term, the accumulated human capital could increase if well-educated workers from large cohorts reach the age at which their human capital is the highest. However, in the long term, a decrease in aggregate human capital is expected because the working-age population shrinks. This could also affect economic growth levels in the future.

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Influence of Workforce Ageing on Human Capital Formation

Marcin Stonawski

Introduction

The 21st century is called by demographers the century of population ageing (Lutz et al. 2004). Population projections show that the share of old people will increase dramatically in the future. Population ageing not only brings about demographic consequences, but affects all aspects of society. One of the most significant problems is the ageing of the workforce which generates a wide range of challenges in a social and economic context. The important issue is how the process of ageing influences the accumulation of human capital stock in the workforce population, which is one of the main driving forces of economic productivity growth. The shrinkage of available human capital in the labor market could cause slower or even negative economic growth in the future. The main objective of this work is to assess the influence of workforce ageing on human capital formation. The question arises when human capital resources increase or decrease together with the process of population ageing.

In the following sections, the concept of human capital is presented, followed by a discussion of the various methods of measuring human capital. We then describe our proposed method of human capital measurement and elaborate on the individual human capital profiles. Interaction rates and aggregated human capital are discussed. Using the multi-state demographic population method, we present the population projection for Poland for 2002-2052 and discuss human capital formation in Poland during the same period. The paper ends with concluding remarks.

Human Capital

Although there are many studies on human capital, there are not many definitions of this specific kind of capital. Even the famous author G. Becker does not specify the exact definition of human capital. In his book, *Human Capital. A Theoretical and Empirical Analysis with Special Reference to Education*, he defines it in the context of investment differences between various kinds of capital (Becker 1993).

Human capital could be defined as a capacity for work, including the capacities, skills, personality and capabilities of a person. The specific feature of human capital is inseparability of the owner and indivisibility. In contrast to physical and financial capital, it is not possible to separate a person from his or her knowledge, skills, health or values (Becker 1993), which cannot be bought, sold, etc. Important elements that form human capital are experience, knowledge, health, personality, and skills. In the

Webster's II New Riverside University Dictionary (1984) we find the following definitions of these characteristics:

a) experience – knowledge or skill derived from active participation in events or activities; the totality of such events in the past of an individual;
b) health – physical and mental condition of the individual;
c) individual knowledge – knowledge derived from study, experience, or instruction; the sum or range of what has been perceived, discovered, or learned by an individual;
d) personality – totality of qualities or traits, as of character or behavior, that are peculiar to an individual;
e) skills – collective set of arts, trades, techniques, or abilities, especially those requiring the hands or body.

In the OECD report, *The Well-being of Nations: The Role of Human and Social Capital,* human capital is characterized in the following way: "The knowledge, skills, competencies and attributes embodied in individuals that facilitate the creation of personal, social and economic well-being" (OECD 2001: 18).

Therefore, the issue is how to measure the accumulated stock of human capital on an individual level and on a population level, and which characteristics should be taken into account in order to measure human capital appropriately.

Methods of Measuring Human Capital

Various methods of measuring human capital can be found in the research papers of economists who try to explain the reasons of economic growth, changes in productivity and disproportion in development between countries. In the early works on these topics, human capital was equated with the labor force and counted as a simple sum of individuals. This way of thinking has been changing gradually. Current research on human capital can be divided into three groups: a) output-based or education-based measures; b) cost-based measures; and c) income-based measures.

The most widespread proxies of human capital based on education are adult literacy rates (Azariadis and Drazen 1990; Romer 1990), school enrolment ratios (Barro 1991; Mankiw et al. 1992), educational attainment and average years of schooling (Barro and Sala-i-Martin 1995; Krueger and Lindahl 2000). These proxies are simple and imperfect, but their accessibility to data makes them popular in economic studies.

The cost-based methods take into consideration the costs incurred in the process of human capital formation. Thus, the human capital of an individual equals these costs. Stroombergen et al. (2002) indicate three streams of costs of human capital formation incurred by the

- individual or individual's family, e.g., costs of tuition, foregone income;
- employer, e.g., support of attendance at courses, on-the-job training;
- government, e.g., costs of running the education system.

The drawbacks of this approach are the unavailability of information about all costs, and the problems with human capital comparisons between different times and populations. For example, in a country where the costs are relatively low, an individual with a high level of education has a smaller stock of human capital than a person with a similar educational background in a country with higher costs.

The income-based measures treat human capital as a variable expressed in units of money. These methods assume that human capital is valued by the market. The level of human capital equals the money (wages, earnings) that someone wants to pay for it. Among these methods we find the discounted stream of earnings measure. Accumulated human capital is worth the same as the current value of all lifespan earnings. In this case the market (relations between demand and supply) regulates the earnings of individuals. Employers price human capital, taking into consideration the productivity of employees. Thus, if an individual is more productive, he should earn more, but if his productivity decreases, the earnings should be less. This method might sometimes lead to incorrect conclusions. The relatively larger cohorts could experience a crowd effect on the labor market. The competition between people in such cohorts is stronger and leads to a decrease in earnings. This phenomenon is in accordance with the rules of the market which constitute that the price of goods, whose supply is relatively higher than demand, goes down. In this case, the earnings cannot show the real value of human capital. It is possible that, among cohorts with the same educational structure, some have more human capital than others.

Another issue is that the market is not perfect in the real world, therefore, earnings are not perfectly dependent on the productivity of workers and supply-demand relations. Many factors can influence the level of wages, for example, the bargaining power of pressure groups such as labor unions. In this case, workers who are in the majority and have the strong bargaining power can secure their own financial stability. This leads to the situation where individuals with smaller human capital resources and lower productivity can earn more than people who are better educated, more productive, and whose power is smaller. In the context of population ageing, it can be very problematic to measure human capital using this group of methods because there will be more and more older workers who will have greater bargaining power than younger ones.

Method

We present a method that is based on the magnitude of human capital that has been accumulated in an individual. It belongs to the group of education-based methods, because the base of human capital accumulation is formal education (education during the years of schooling) (Kurkiewicz et al. 1999). But contrary to other methods, it takes into consideration other factors that influence the stock of human capital during a lifespan. This approach seems to be more adequate for studying human capital formation in the context of population ageing because it includes many different factors, but is not connected to earnings. In the method presented here, the following factors are assumed to influence individual human capital (see also Figure 1):

- learning during formal education;
- acquiring knowledge and experience during one’s professional life;
- obsolescence of knowledge;
- health;
- forgetting knowledge.

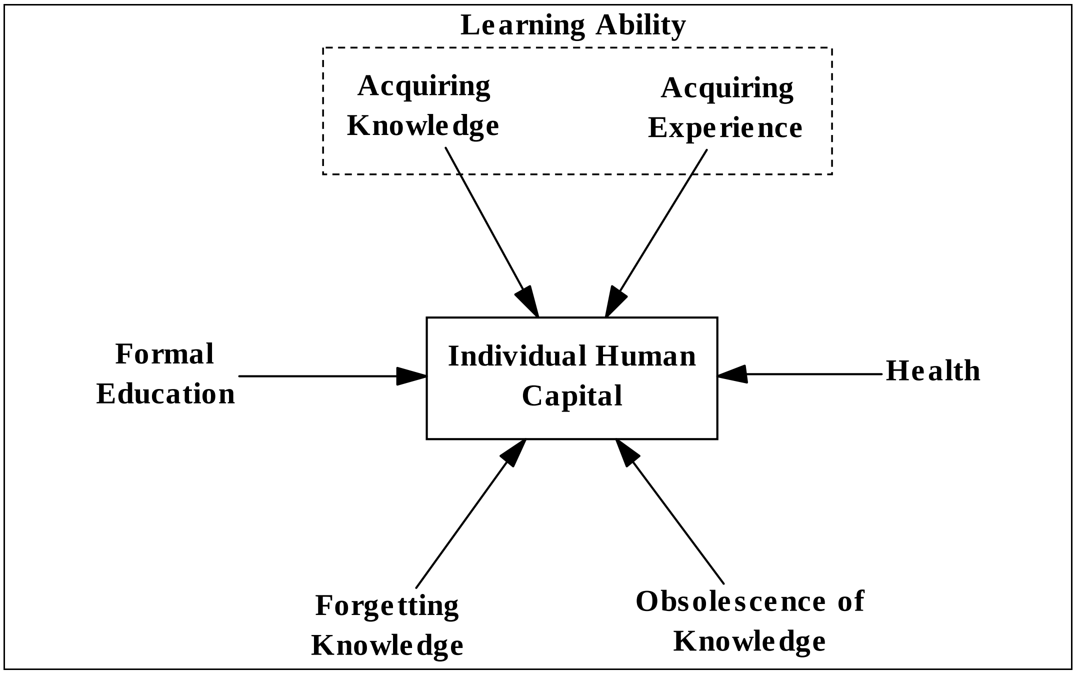


Figure 1. Outline of factors affecting human capital.

We propose to measure the human capital accumulated in an individual in the following way:

$$h_{as}=h_s \cdot \alpha_{as} \cdot \beta_{as} \cdot \delta_{as} \cdot \gamma_{as} \quad (1)$$

where:

h_{as} – human capital accumulated in a person at age *a* and education level *s*;

h_s – human capital accumulated during formal education;

α_{as} – rate of acquirement of knowledge and skills during professional life;

β_{as} – rate of obsolescence of knowledge;

δ_{as} – rate of forgetting knowledge;

γ_{as} – rate of an individual's health condition.

Thus, the stock of human capital is a product of five components which will be discussed in the subsections below.

In this method, the unit of measurement of human capital as introduced by Polachek and Siebert (1993) is Ed. According to their definition: "Eds represent skill units – degrees, qualifications, on-job experience – acquired by individuals throughout life" (Polacheck and Siebert 1993: 20). It is assumed that 1 Ed equals one year of formal education. Thus, in the method presented in this paper, all changes (increases or decreases) in human capital stock during a lifespan are related to the amount of human capital accumulated during one year of education.

Formal education

The main source of an individual's human capital is the education obtained during the first stage of a lifespan. During this period, a person acquires new knowledge and skills from the simple and basic skills of reading and writing in primary school to more precise and complicated issues which become the basis for one's professional life, as well as learning about social behavior and life in general. The importance of education during this period for the accumulation of human capital has been emphasized by many researchers (Mincer 1981; Schultz 1960). Becker wrote that "education and training is the most important investment in human capital" (Becker 1993: 17).

The method presented in this paper takes into consideration the factor of acquiring skills and knowledge during formal education. As mentioned earlier, the measurement unit of human capital is Ed, which equals one year of formal education. For example, if an individual has been studying for eight years in primary school and four years in secondary school, he would accumulate 12 Eds. This simple calculation assumes that all years of education add the same amount of Eds to the accumulated human capital. However, it is difficult to estimate which stage of formal education is more important. We cannot say that one year of tertiary education is more valuable than one year of primary simply because people with a higher education earn more than less-educated people. It is impossible to start at a higher level without the basic skills and knowledge acquired from an elementary education. Thus, we treat every year of education as an investment; this gives an output of 1 Ed to human capital.

Acquiring new knowledge and experience during one's professional life

The completion of formal education does not finish the process of increasing the human capital stock. This process lasts during the whole lifespan of an individual; only the pace of acquisition changes. During one's professional life two factors play major roles in the accumulation of human capital. First, an individual can acquire new knowledge through life-long learning activities, such as taking part in courses and lectures that are connected with the professional career or by broadening a particular knowledge of personal, non-professional interests, etc. All of these activities add human capital to the individual stock. The second factor is experience, which is acquired during the permanent performance of actions, duties and work. Experience improves personal efficiency, competence and productivity (Prskawetz and Lindh 2006).

The acquisition of individual human capital through learning and gathering experience depends on one's mental abilities (Park et al. 2002; Skirbekk 2004). These abilities can be divided into two groups: the fluid abilities which decrease during the life cycle, and the crystallized abilities which remain at the same level until the older ages. Park et al. (2002) have found that a continuous, regular decline occurs in the speed of processing, the working memory, and the short- and long-term memory during the lifespan (as of the age of 20) (see Figure 2). Only verbal knowledge increases across the period of life. These functions are responsible for the ability to learn which is used as a proxy for the changes in abilities to acquire experience and learn new knowledge during one's professional life.

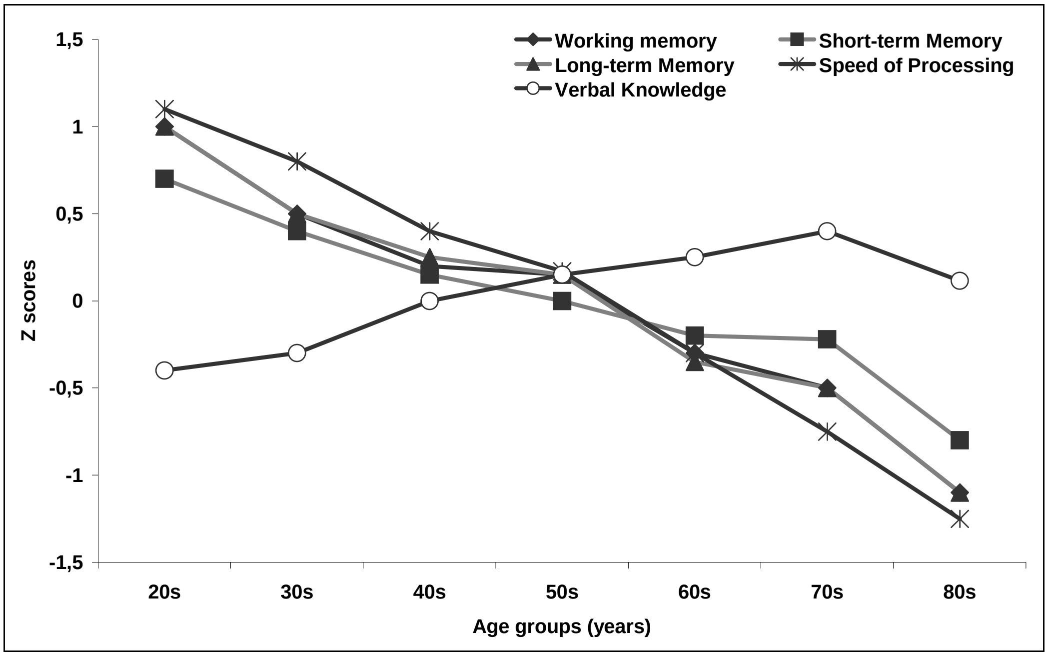


Figure 2. Lifespan performance measures. A composite view of the aforementioned measures. Source: Park et al. (2002).

To measure the changes in human capital acquisition caused by the ability to learn, we use the linear function based on Park et al.'s results. It is assumed that the decrease in the ability to learn takes place between the ages of 25 and 60. Before 25, the ability is stable on the highest level. We assume that after the age of 60, it is stable on the minimal level. Thus, the pace of human capital decreases linearly between the ages of 25 and 60.

The accumulation of additional human capital units (Eds) begins immediately after the end of formal education. In the first year after completion of formal education, an acquisition rate of human capital (*r*) is the highest and remains stable to the age of 25. The acquisition rates are the same for all persons. It is assumed that an annual increase of human capital (ΔC) cannot exceed 1 Ed. This is very important because otherwise an individual would prefer to leave education earlier and start working. In this case, a person could accumulate more human capital annually than during formal education and receive additional earnings. Thus, the annual acquisition rate is counted as:

$$\Delta C < 1$$

$$h_s \cdot r < 1$$

$$r < \frac{1}{h_s}$$

where

h_s – stock of human capital accumulated during formal education,

r – annual acquisition rate of human capital during professional life.

Individuals with a tertiary education have the highest stock of human capital. In order to count the acquisition rate we use the stock for this group. We assumed that they accumulate 16 Eds which equals 16 years of schooling. Therefore, the acquisition rate is

$$16 \cdot r < 1$$

$$r < 0.0625 \cdot$$

Hence, the assumption is that a maximal annual acquisition rate equals 6.24 percent. This value is the same for all education levels.

To establish the minimal value of the acquisition rate, we assume that everyone has the ability to learn until the end of their lives. They can obtain new knowledge and acquire experience, but this process is not effective and does not bring significant growth in human capital for people of older ages. Thus, the minimal annual acquisition rate is established at the level of 1 percent. This level is achieved by people at the age of 60 and is stable until the end of their lives (see Figure 3).

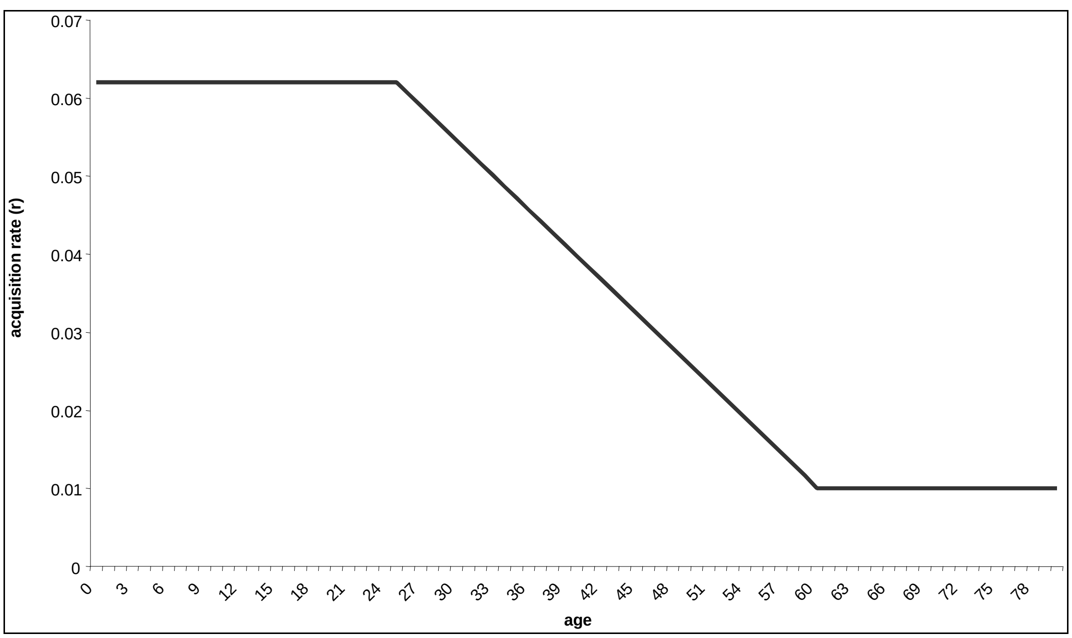


Figure 3. Human capital acquisition during a lifespan. Source: Author's own calculations.

Despite the assumption that the curve of learning ability is the same for all levels of education, there are differences in the mental capabilities of people with different educational backgrounds. These differences can be noticed in the results of studies on human intelligence. Kaufman and Lichtenberger (2002) show the results of psychological tests run by many researchers, such as Matarazzo (1972), Jensen (1980), and Reynolds (1987), who found significant differences in intelligence between people with different educational and occupational accomplishments (see Figure 4). The mean IQ level for people with tertiary education is around 110-120, which is about three times higher than for people without education or with only primary education (performing simple carpentry, mowing lawns). In this paper, the differences in mental capabilities are expressed by the human capital stock accumulated during formal education. For example, an individual with tertiary education has 16 Eds and his acquisition of capital in the first year of working life (ΔC_{16}) equals 0.99 Eds (16 x 0.624), while a person with primary education with 8 Eds of starting capital has an increase of 0.49 Eds. The stages of education and the starting human capital stock are presented in Table 1.

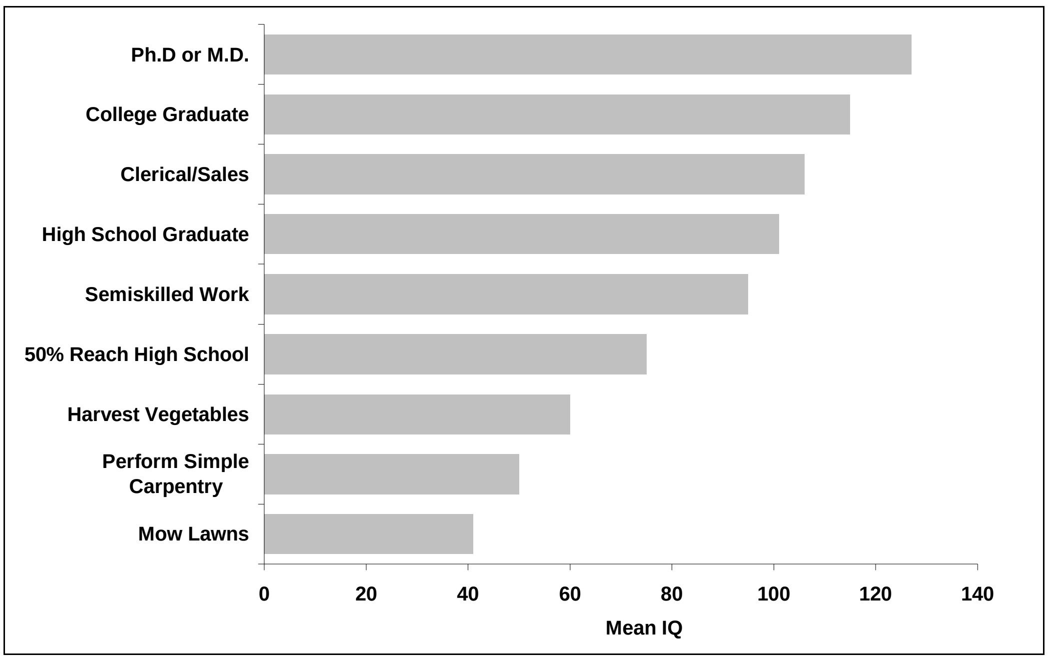


Figure 4. Mean adult IQs (results from the Wechsler IQ test) that correspond to different educational and occupational accomplishments. Source: Kaufman and Lichtenberger (2002).

Table 1. Human capital accumulated during formal education by level of education. Source: Author's own calculations.

Level of education (s)	*Human capital (in Eds)*
1 - Primary not completed	4
2 - Primary	8
3 - Basic vocational	11
4 - Secondary	13
5 - Post-secondary	14
6 - Tertiary	16

The process of accumulation, as mentioned earlier, begins immediately after the completion of formal education. Thus, the function of human capital acquisition takes into consideration the length of time spent in the working life (*i*). In the time range between the first year of professional life and the age of 25, the value of the function is at the same level as in the first year (1).

$$i \in \langle 1, E_{ls} \rangle , \qquad \Delta C_{is} = \Delta C_{1s} \tag{2}$$

where:

i – duration of a working life,

s – education level of a person,

E_{ls} – year of working life at the age of 25 [number of years spent in professional life to the age of 25],

ΔC_{is} – annual acquisition of human capital during the working period,

ΔC_{1s} – acquisition of human capital during the first year of working life [the rate equals 6.24 percent].

Annual acquisition decreases between the ages of 25 and 60, according to the linear function in the following form:

$$i \in \langle E_{ls}+1, E_{hs} \rangle , \quad \Delta C_{is} = \frac{\Delta C_{1s} - \Delta C_{zs}}{(E_{ls}+1) - E_{hs}} i + \left[\Delta C_{zs} - \frac{(\Delta C_{1s} - \Delta C_{zs}) E_{hs}}{(E_{ls}+1) - E_{hs}} \right] \tag{3}$$

where

E_{hs} – year of working life at the age of 60 [number of years spent on professional life to the age of 60],

ΔC_{zs} – accumulation of human capital during the year E_{hs} of working period [the rate equals one percent].

After the age of 60 to the end of life, the function is constant at the level achieved at the age of 60 (Eq. 3).

$$i \in \langle E_{hs}+1, \infty) , \qquad \Delta C_{is} = \Delta C_{zs} \tag{4}$$

We have acquisition rates for a particular year of working life, but it can be useful to present acquisition rates for a particular individual at age *a* and education level *s*. The accumulated human capital for a person at age *a* and education level *s* is:

$$h^*_{as} = h_s + \sum_{i=1}^{E_{as}} \Delta C_{is} \tag{5}$$

where

h^*_{as} – amount of accumulated human capital that includes Eds from acquirement of skills and experience,

E_{as} – year of working life of an individual at age *a* and education level *s*,

h_s – initial amount of human capital accumulated during the period of formal education.

Thus, the acquisition rates (α_{as}) are counted using the equations (for the calculations, see Appendix 1):

$$i \in \langle 1, E_{ls} \rangle , \qquad \alpha_{as} = 1 + \frac{E_{as}\Delta C_{1s}}{h_s} \tag{6}$$

$$i \in \langle E_{ls}+1, E_{hs} \rangle ,$$

$$\alpha_{as} = 1 + \frac{2E_{ls}\Delta C_{1s} + (E_{as}-E_{ls})\left[2 \cdot \Delta C_{zs} - \frac{(\Delta C_{1s}-\Delta C_{zs})(2E_{hs}-(E_{ls}+1+E_{as}))}{(E_{ls}+1)-E_{hs}}\right]}{2h_s} \tag{7}$$

$$i \in \langle E_{hs}+1, \infty) ,$$

$$\alpha_{as} = 1 + \frac{E_{ls}\Delta C_{1s} + 0.5(E_{hs}-E_{ls})(\Delta C_{1s}+\Delta C_{zs}) + (E_{as}-E_{hs})\Delta C_{zs}}{h_s} \tag{8}$$

where α_{as} is the acquisition rate of human capital for an individual at age *a* and education level *s*. The equation for the accumulated human capital for an individual at age *a* with education level *s* is:

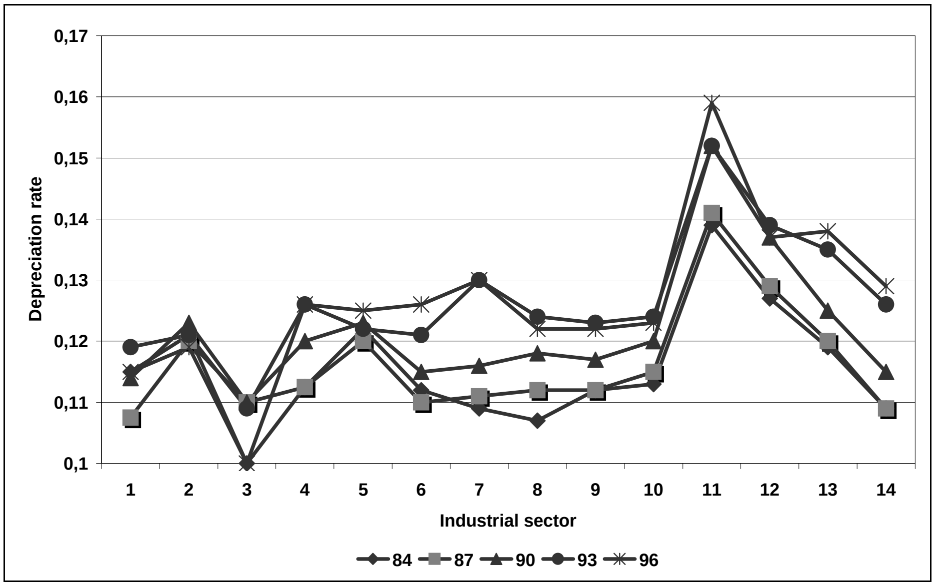
$$h^*_{as} = h_s \cdot \alpha_{as} \tag{9}$$

Obsolescence of knowledge

Acquired skills and knowledge lose their relevance with the passing of time. As civilization develops, parts of one’s knowledge and skills become inapplicable due to technological progress and developments in science. But not all acquired skills and knowledge depreciate over time. For example, during primary education, individuals learn basic skills such as reading and writing, which are the basis for personal development. These skills do not lose their usefulness. The skills required for computer programming are another example. During education a person learns how to program the computer in a particular language. The relevance of this knowledge depends on the technological progress. If this language is not used because new, more useful programs are developed, the acquired knowledge of the old language becomes unimportant and will no longer be used. However, the person retains the basics of programming which are still relevant and can be used as a basis to learn a new language. Nevertheless, the person lost some part of his/her human capital and we should take this into consideration when counting the human capital.

Outlining the speed and course of the depreciation process is problematic. The pace of this process depends on the development of technology and science. It is a positive relation. The faster the development, the faster the pace of obsolescence. Thus, the different knowledge and skills depreciate at different speeds. In this paper we do not take into consideration the differences between an individual's educational background and occupational status. We are conscious that this is a huge simplification because people are not homogeneous in this respect. Individuals with different levels of education have different courses of human capital depreciation – for example, physicians and high-tech engineers – but it is impossible to specify all differences when accounting for the human capital of a population. To assume the course of the obsolescence of knowledge, we based our results on research regarding the depreciation of technological knowledge. The methods of estimating depreciation rates presented here are based mainly on technological change theories (e.g., technology cycle time) (Abernathy and Utterback 1978; Ayres 1994) using patent renewal data (e.g., Bosworth 1978; Pakes and Schankerman 1984) and patent citation data (Park et al. 2006). Park et al. (2006) use data from several industries to analyze the depreciation rates of technology. They find that there is a gradual increase of these rates over time, and within 12 years, the change was about 2-3 percentage points (see Figure 5). Thus, we assumed the linear depreciation of knowledge and treated it as a whole. The pace of depreciation is constant for every unit of human capital.

It is assumed that knowledge gained during primary education (Eds number from 1 to 8) does not become outdated because it represents simple, basic abilities (e.g., reading and writing) which are used during a lifespan. The fastest pace of ageing is assumed for knowledge which is acquired during tertiary education. This knowledge is very precise, particular and dependant upon the development of science and technology. We have to take into consideration that even during tertiary education, the level of detail increases with the advancement of the student. Thus, the rate of human capital depreciation is different for the particular Ed accumulated during formal education. The process of the obsolescence of knowledge begins immediately after the accomplishment of formal education. The pace of depreciation is constant during the working life, but it is different for the particular Ed accumulated during formal education.



Industries:
1 - food and beverages
2 - textiles
3 - wood products
4 - paper, printing and publishing
5 - chemicals
6 - rubber and plastics
7 - non-metallic minerals
8 - basic metals
9 - fabricated metals
10 - machinery
11- electrical, computing and communication
12 - precision
13 - transport equipment
14 - other manufacturing

Figure 5. Changing patterns of depreciation rates across industries in three-year intervals. Source: Park et al. (2006).

Table 2. Annual depreciation of human capital according to the number of Ed. Source: Author's own calculations.

Ed no.	*Annual depreciation*
1-8	0
9	0.0625*(9-8)/100= 0.000625
10	0.00125
11	0.001875
12	0.0025
13	0.003125
14	0.00375
15	0.004375
16	0.005

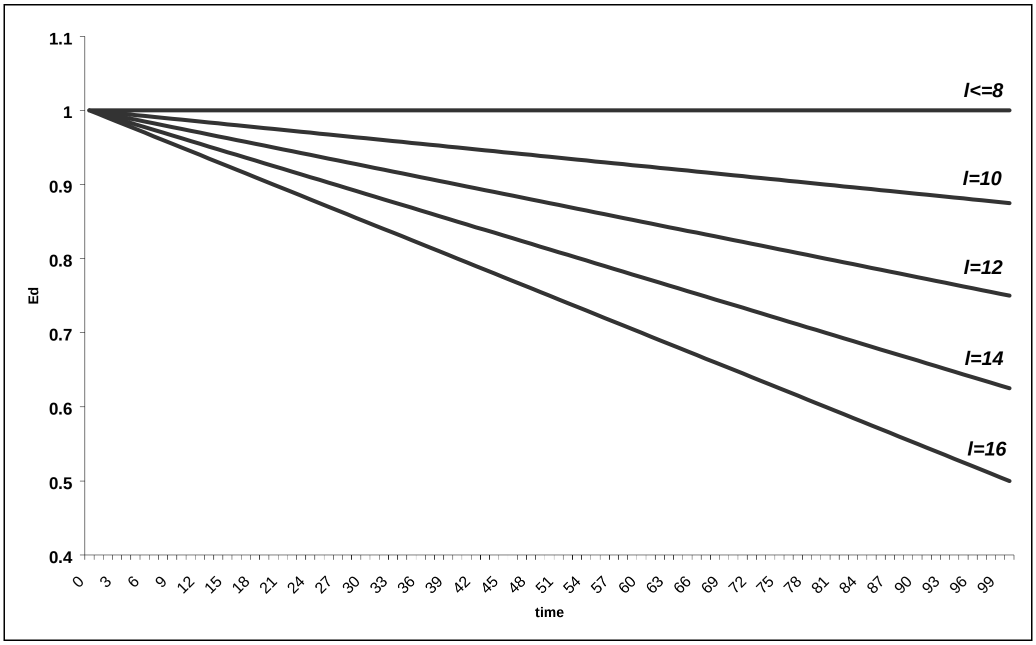


Figure 6. Depreciation of particular units of human capital which were accumulated during formal education. Source: Author's own calculations.

The depreciation of human capital is presented in Table 2 and Figure 6. We assume that the last unit of human capital (accumulated during the last year of tertiary education) decreases by 50 percent over 100 years. The decreases of the other units are proportionally lower. Thus, the annual depreciation for a particular unit of human capital is counted in the following way:

$$\frac{0.0625\cdot(l-8)}{100},$$

where *l* is the number of Ed in order.

The equation for accumulated human capital that considers the influence of the obsolescence of knowledge is:

$$h^*_{as} = 8 + \sum_{l=9}^{h_s} \left[1 - E_{as} \cdot 0.000625(l-8)\right] \tag{10}$$

Using Eq. (9) we can calculate the depreciation rate (β_{as}) for an individual at age *a* and education level *s* (the calculations can be found in Appendix 2):

$$\beta_{as} = 1 - 0.000625 \cdot E_{as} \frac{(h_s - 7)(h_s - 8)}{2h_s} \tag{11}$$

Forgetting knowledge

The next factor of human capital depreciation included in this method is the process of forgetting knowledge. In contrast to the obsolescence of knowledge, which is independent of an individual, forgetting knowledge depends on personal abilities. Forgetting is the decline of memory retention over time. The pace of forgetting is connected to the strength of one's memory. The stronger the memory, the longer an individual can retain the knowledge. In the context of human capital measurement we have to take into consideration the long-term memory and the process of forgetting during the lifespan.

The first significant research about memory retention and forgetting was run by Ebbinghaus (1913), who published a book about these topics in 1885. He argues that there exists an exponential nature of forgetting, which can be described as:

$$R = e^{-t/s} \tag{12}$$

where R is memory retention, s is strength of mind, and t is time. Ebbinghaus made a short-term study on the changes in retention during the learning process and the influence of repetition on the forgetting process. The very important finding is that forgetting occurs rapidly at first and more slowly as time goes on (Kahana and Adler 2002). This is still accepted in psychological science, but the function equation is questioned by many scientists (e.g., Wixted 1990; Rubin and Wenzel 1996). Some scientists are in favor of the power function equation, arguing that based on the results of their research, the power law can be applied to forgetting (Kahana and Adler 2002).

One of the scientific papers that examines long-term memory is by Bahrick (1984). He researched the retention (the ability to retain facts and figures in one's memory) of the Spanish language learned at school during a period of 50 years. He found that the highest pace of forgetting knowledge is observed in the first three years after completion of a language course. During this time, an individual loses about 40-50 percent of his/her knowledge (Figure 7). After that period the pace of forgetting is very slow. At the end of the period under consideration, a person still has about 40-50 percent of all knowledge acquired at school.

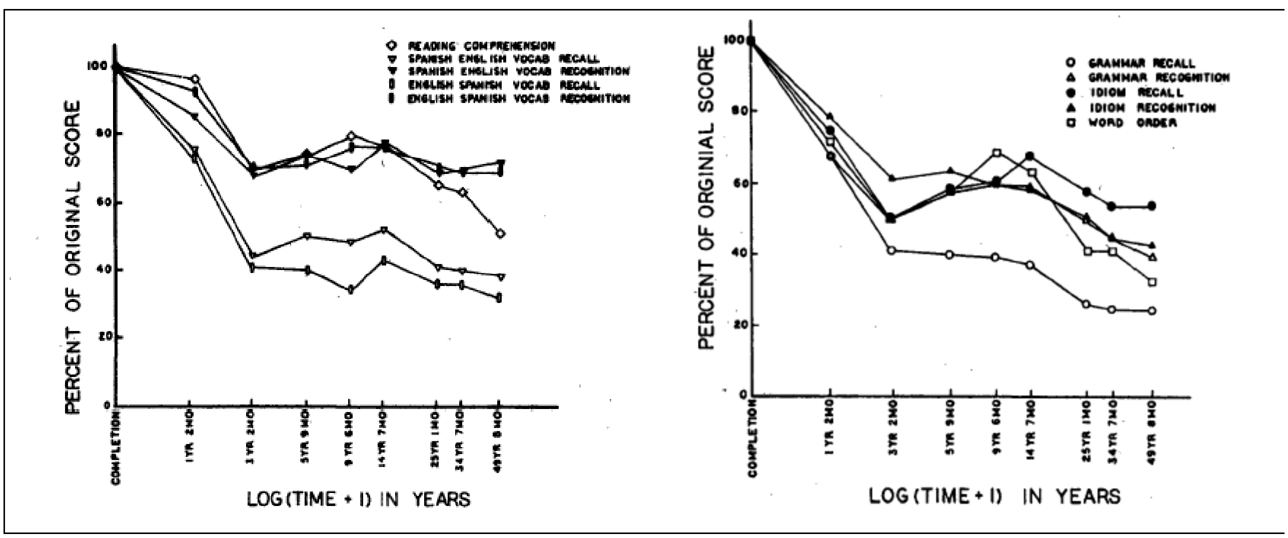


Figure 7. Adjusted retention functions. Source: Bahrick (1984: 10-11).

In the method presented in this paper, we assume that the process of forgetting knowledge starts immediately after the completion of formal education. The depreciation function is the same for all education levels. Taking the results of Bahrick's research into account, we use the memory retention rate (γ) based on the power function:

$$\gamma_{as} = z \cdot \left(E_{as}\right)^{-w} \qquad (13)$$

where *z*, *w* are the parameters. The rate represents the knowledge that is still remembered. Thus, the depreciation rate equals (1- γ_a). Figure 8 shows the memory retention rate during the period of working life.

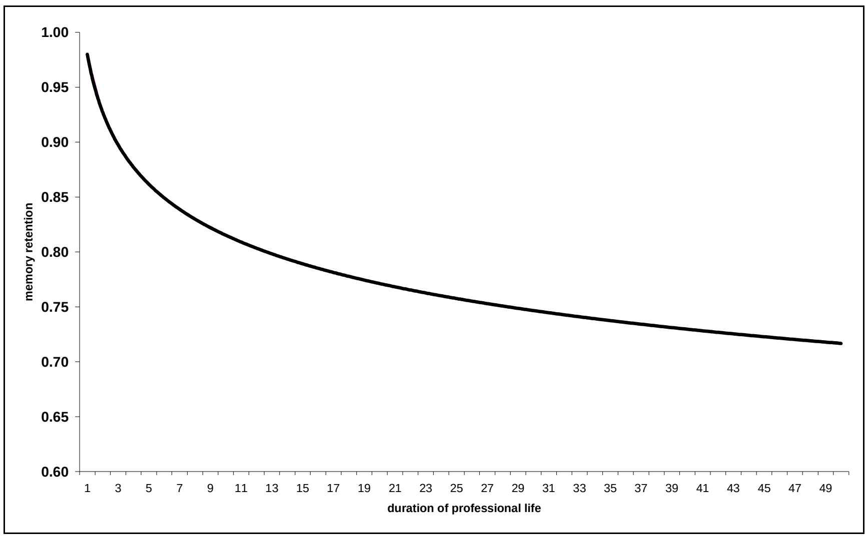


Figure 8. Memory retention rate. Source: Author's own calculations.

In order to calibrate the function, we assume that during a period of 50 years of working life, an individual loses about 30 percent of his/her human capital due to the process of forgetting. We cannot base our findings exactly on the results of Bahrick because we do not take the knowledge repetition process into account (Jaber and Kher 2002). We use the following equation:

$$\gamma_{as} = 0.98 \cdot \left(E_{as}\right)^{-0.08}$$

Health

One of the most important factors of human capital creation is an individual's health. The importance of health is emphasized by Becker (1993) and Mincer (1981). They treat expenditure on health improvement (e.g., medical care) as an investment in human capital. Human capital theory treats the state of health as stock-health capital. This stock depreciates over time at an increasing rate in later ages (Schultz 1981).

In the method presented in this paper, changes in health are a factor of depreciation of human capital. Together with ageing, the condition decreases and because of that, accumulated human capital becomes smaller. A very important factor in determining incentives to invest in human capital and the value of its stock is the life expectancy of a population (Schultz 1981). Thus, we measure the influence of health on human capital formation using information from the period life tables. In our method the intensity of depreciation is proportional to the mortality of a population. It might be better to use the data from health life tables (healthy time, sickness-free time) which take into account the changes in health conditions of a population, but they are not commonly used.

The equation for the rate is:

$$\delta_{as} = \frac{L(a)}{L(a - E_{as})} \tag{14}$$

where

$L(a)$ – stationary population at age a

$L(a - E_{as})$ – stationary population at age $a - E_{as}$

This measure shows how much human capital is still possessed by an individual at age *a*. Thus, the depreciation rate has the form: 1- δ_{as}. It is assumed there are no health differences between people with different levels of education (but of course it is possible to use specific education life tables in our method). The only difference implemented in the measure is the duration of formal education which influences the value of $L(a - E_{as})$. Using Eq. (14) we compare the "health condition" of an individual at age a with the value from the beginning of his/her professional life. The improvements in mortality cause an increase in the health rate δ_{as} and hamper the process of human capital depreciation.

Individual Human Capital Profiles

Using data from Poland we estimate the stock of human capital by age and level of education. We assume six age groups: 15-25, 25-35, 35-45, 45-55, 55-65, and 65 and older. Inside each particular group, people are at the median (e.g., it is assumed that in group 15-25, people are 20 years old). There are also six education groups: primary not completed, primary, basic vocational, secondary, post-secondary and tertiary. We count the human capital accumulated during formal education by taking the duration of schooling on the different levels in 2002 (see Table 1). It is assumed that rates of acquiring experience and learning new knowledge during the professional life (α_{as}), obsolescence of knowledge (β_{as}), forgetting knowledge (γ_{as}) are the same for all populations. There are no differences between populations in this respect, with biological abilities (cognitive abilities) connected to acquiring experience and learning, and forgetting knowledge. The technological progress affects all populations in the same way. The differentiation factors are duration of the formal education and health conditions. In this exercise we use the period life tables for males in 2002.

Taking all of these factors into account, we estimate six education-specific human capital profiles (see Figure 9). The results suggest that human capital follows an inverted U-shaped profile where the significant decreases of the stock are found after the age of 45. It is independent of the education level. But there are differences between the profiles: The better the education, the steeper the curve. The very flat profile is found for people with primary not-completed education. The steeper profile is observed for tertiary education. The peak of human capital stock is achieved in the age group 45-55.

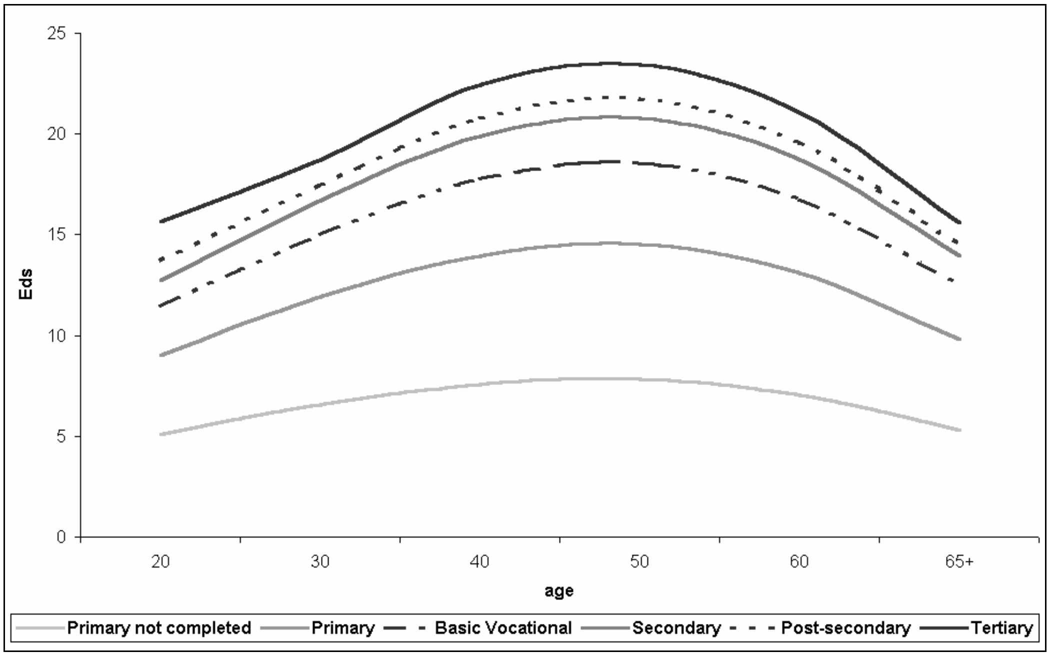


Figure 9. Education-specific human capital profiles. Source: Author's own calculations.

In the context of population ageing, it is very important to understand how improvements in the health conditions of a society and the faster pace of progress in science and technology could influence human capital formation. In the method presented in this paper, the development in health conditions is shown through an increase in the health rate δ_{as} (see Eq. 14).

We present three levels of life expectancy at birth which could show the progress of health conditions (life expectancy equals 60, 70 and 80 years). Using the human capital profile for tertiary education, this phenomenon can be seen in Figure 10. An increase in life expectancy significantly affects the human capital profile. There are changes in the shape of the curve and in the level of human capital for all age groups. The curve becomes more flat as the decrease in human capital at older ages slows down. The levels of human capital are the highest when life expectancy equals 80 years. For example, there is a growth of about 32 percent in the age group 55-65. The most important finding is that the peak of the human capital profile changes with the progress in life expectancy. The peak shifts to the older age groups.

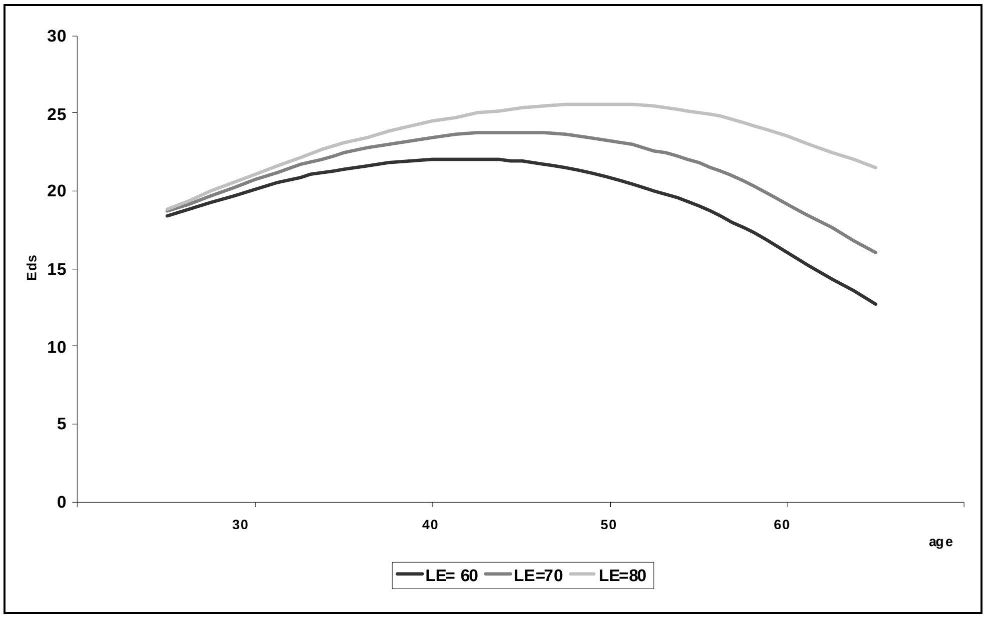


Figure 10. Human capital profiles for tertiary education by level of life expectancy. Source: Author's own calculations.

Another important factor is the influence of technological and scientific progress on society. The acceleration of progress gives rise to the faster pace of the obsolescence of knowledge. Thus, the human capital stock decreases faster under the ceteris paribus assumption. In the method presented in this paper, we assume that the latest accumulated unit of human capital decreases by 50 percent during 100 years, but we also test other scenarios that assume a more rapid pace of knowledge obsolescence. The following scenarios were prepared:

I. The latest accumulated unit of human capital decreases by 50 percent during the next 100 years of an individual's life.
II. The unit decreases by 80 percent during the next 100 years.
III. The unit equals 0 after 100 years; and then
IV. The unit decreases by 80 percent during the next 50 years.

The results are presented in Figures 11 and 12. We assume that the human capital stock of people with low education does not depreciate because their knowledge is basic. Thus, we can focus only on the secondary and tertiary education levels. There is a very weak effect of the progress of acceleration on the human capital profiles of individuals with a medium education level (see Figure 11). People with a higher education could lose more human capital with accumulated stock during the acceleration, because part of their knowledge is very specific and depreciates quickly. The effects are connected to the speed of progress: The faster the progress, the higher the losses of human capital. The overall influence is rather moderate. Changes are visible particularly in the older age groups (see Figure 12).

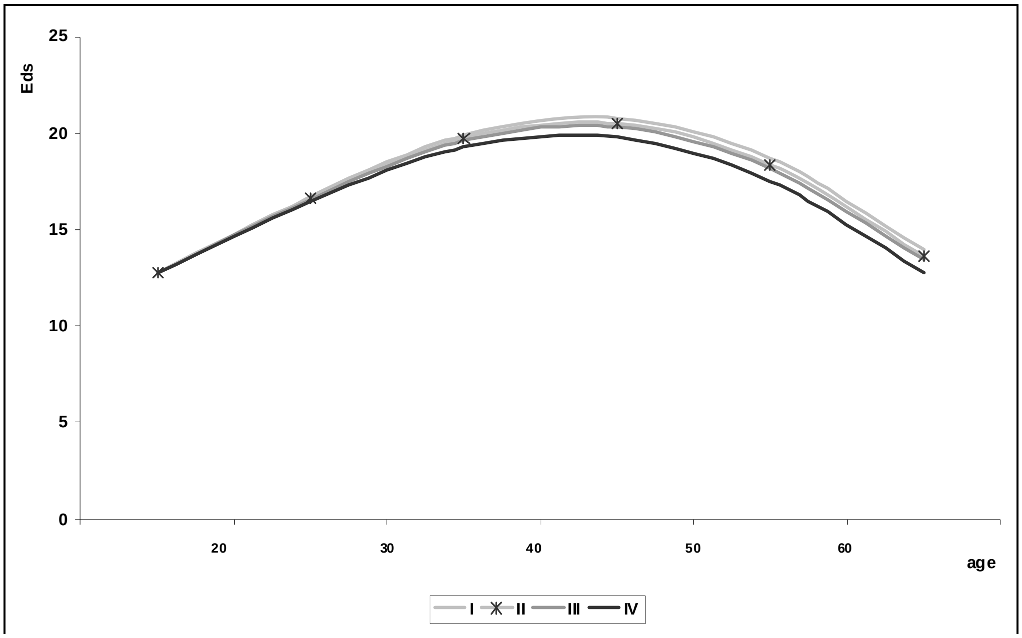


Figure 11. Human capital profiles for levels of secondary education by the speed of the obsolescence of knowledge. Source: Author's own calculations.

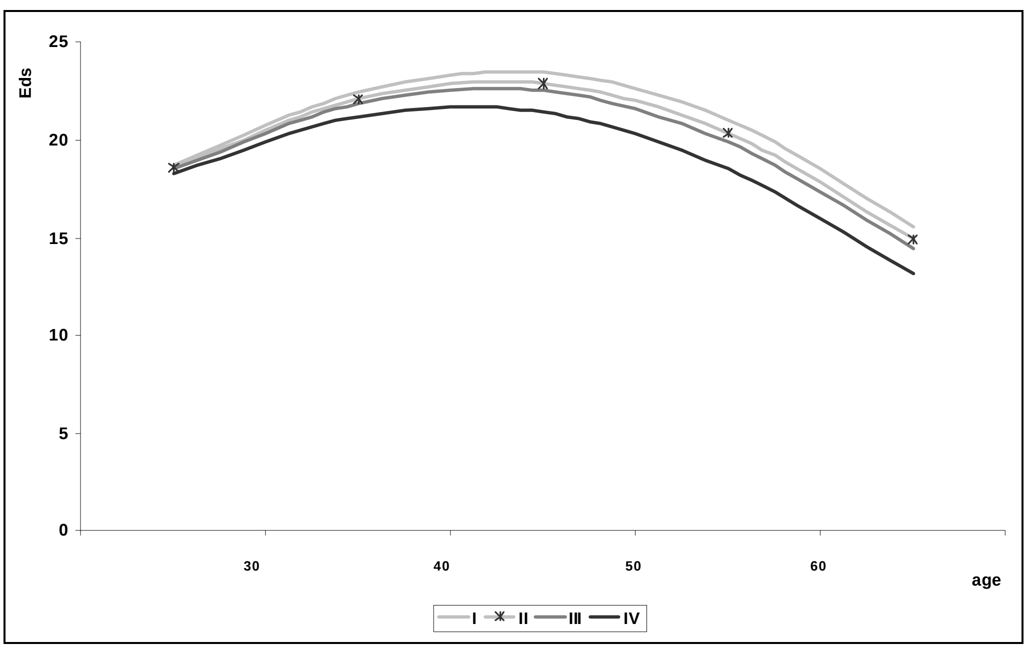


Figure 12. Human capital profiles for levels of tertiary education by speed of the obsolescence of knowledge. Source: Author's own calculations.

Interaction Rates and Aggregated Human Capital

When estimating human capital for a population treated as a whole, it is necessary to take into account the social aspect of an individual. A particular person is not isolated from the other members of a society. The source of additional human capital stock of a population could come from the interactions between people. In the process of human capital formation, the effect of synergy can exist (Fuller 1975; Haskell 1972). Thus, the stock of human capital of persons acting together is greater than a simple sum of individual human capital. This concept is similar to the social capital term that is used mainly by sociologists and economists. Coleman (1988) sets apart three forms of capital:

- Physical capital, created by changes in materials to form tools that facilitate production;
- Human capital, created by changes in persons that bring about skills and capabilities that make them able to act in new ways; and
- Social capital, that comes about through changes in the relations among persons who facilitate action. It exists in the relations among persons.

The manifestations of social capital are, among others, authority relations, relations of trust, norms, and ties between groups. Many scientists emphasize the influence of social capital on economic activity (Ben-Porath 1930; Baker 1984; Putnam et al. 1993). For example, Knack and Keefer (1997) examine the relation between social capital and economic performance using two very important variables of social capital: trust and civic norms. They have found that trust and civic cooperation have significant impacts on aggregate economic activity.

In this paper we use the concept of social capital in a different way. We do not separate human and social capital, but we notice the “product” of the interactions between persons as additional human capital on the level of population.

The synergy effect of human interaction is very hard to measure. We use the interaction rate proposed by Kurkiewicz et al. (1999), which takes into account only simple pairs of interactions. The main assumptions of this measure are:

a) Interactions exist only inside the population;
b) The value of the interactions is not dependent on age and sex but rather on the level of education. It equals the arithmetic mean of the input of the groups taking part in the interaction;
c) The interaction rate is counted as single interactions, all with all; and
d) The interaction of people with tertiary education doubles the number of human capital. This influence is proportionally lower for individuals with lower education.

The interactions are measure according to following equation:

$$I_r = 1 + \frac{\sum_{i=1}^{6}\sum_{i=1}^{6} b_{ij}(h_i + h_j)}{36 \sum_{i=1}^{6}\sum_{i=1}^{6} b_{ij}} \qquad (15)$$

where

$$b_{ij} = \begin{cases} P_i P_j & i \neq j \\ 0{,}5 \cdot P_i (P_i - 1) & i = j \end{cases}$$

P_i and P_j – number of people with education level *i* or *j*

b_{ij} – number of available interactions of people with education *i* and *j*.

In order to measure aggregated human capital we use the equation

$$HC = I_r \sum_{a=1}^{n} \sum_{s=1}^{n} h_{as} p_{as} \qquad (16)$$

where p_{as} is the number of people in age group *a* on education level *s*.

Population Projection for Poland, 2002-2052

In order to project changes in human capital formation in Poland, we prepare a population projection by age, sex and education. The most important issue is to include educational differentials for mortality and fertility. We use the multi-state demographic population method where individual states are levels of education. The education system has been changed in Poland, but the levels from 2002 are used in the projection. An individual may choose one of the six stages of education presented in Figure 13.

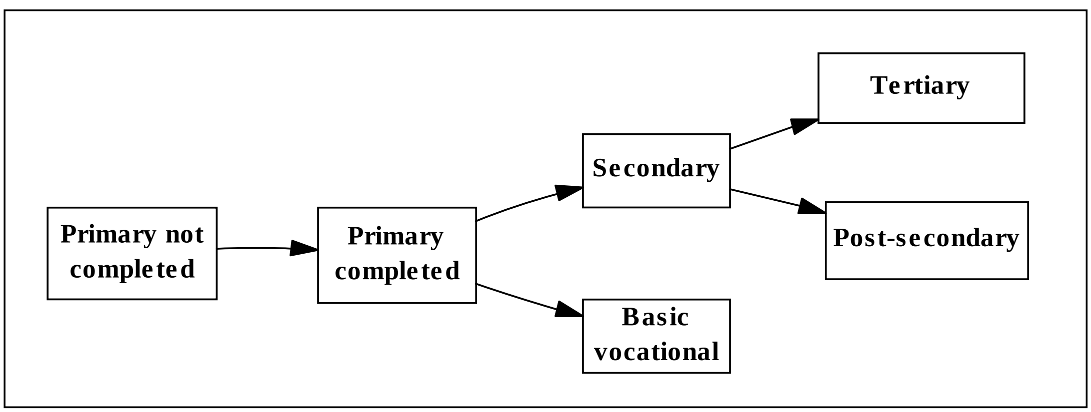


Figure 13. Outline of the six levels of education.

The transition rates between states are calculated using the method based on the *education attainment progression ratios* (EAPR) first introduced by Lutz et al. (2007). In this paper only one scenario of the projection is presented. In this scenario we assume that the pace of fertility changes within education groups following the UN projection (medium variant) for the Polish population. Thus, the relative differences in age-specific fertility rates between education groups are stable at the 2002 level during the period 2002-2052. The pace of change is the same for all education-age-specific fertility rates. It is assumed that life expectancy for all education groups is stable at the 2002 level (see Table 3).

Table 3. Life expectancy at birth by level of education. Source: Author's own calculations using data from the Central Statistical Office in Poland.

Level of Education	*Males*	*Females*
Tertiary	74.5	76.9
Post-secondary	72.6	75.7
Secondary	70.7	74.5
Basic Vocational	66.2	72.5
Primary Completed	64.5	72
Primary Not Completed	67.2	71

The flow of migration is also at the 2002 level. The scenario is very optimistic about fertility because it is based on the UN assumption that the total fertility rate (TFR) will increase and reach a level of 1.85 at the end of the period under consideration. However, it is pessimistic about life expectancy because it does not predict the progress of the lifespan duration. Thus, the scenario assumes a slower pace of population ageing than could happen in the real world in the future.

The data for the "starting" population comes from the Central Statistical Office in Poland. The data for the population structure by age, sex and education in 2002 comes from the National Population Census 2002.

The age pyramid for 2002 (see Figure 14) shows that progress has been made in the educational development of Poland. Among the age group 65+ a share of the people with low education (primary not-completed and primary) is more than 50 percent (54 percent for males and 71 percent for females). In the younger age groups the share of people with low education is significantly lower. Among medium age groups the people with a medium level of education predominate. Another finding is that more women have tertiary education than men. For example, in the population aged 25-29, the share of females with a higher education is 25 percent, whereas for males it is only 17 percent.

During the next 50 years there will be significant changes in the population structure (see Figures 15 and 16). The share of people 65+ will increase from 12.7 percent in 2002 to 18 percent in 2027 and 23.3 percent in 2052. The population will decrease from 38 million to about 30 million. At the same time, the working population (aged 15-64) will decrease by 8 million to the level of 18 million. Thus, the shrinkage of population and population ageing is projected. The level of education will increase. In 2052 the share of the population with a tertiary education will be about 25 percent for males and 38 percent for females.

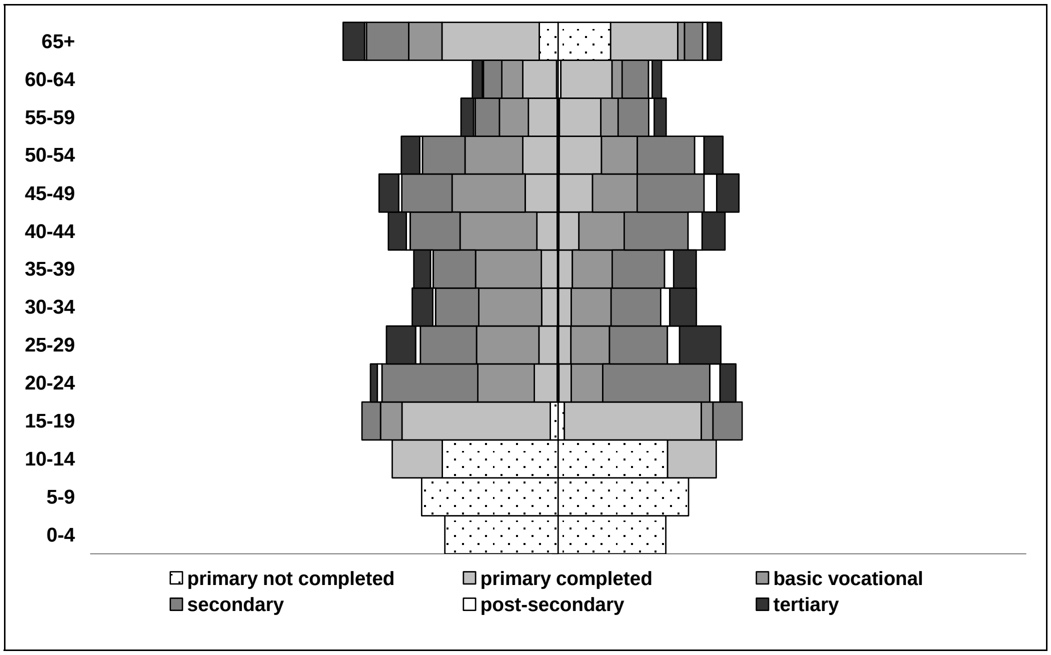


Figure 14. Age and education pyramid for Poland in 2002. Source of data: The Central Statistical Office in Poland, www.stat.gov.pl.

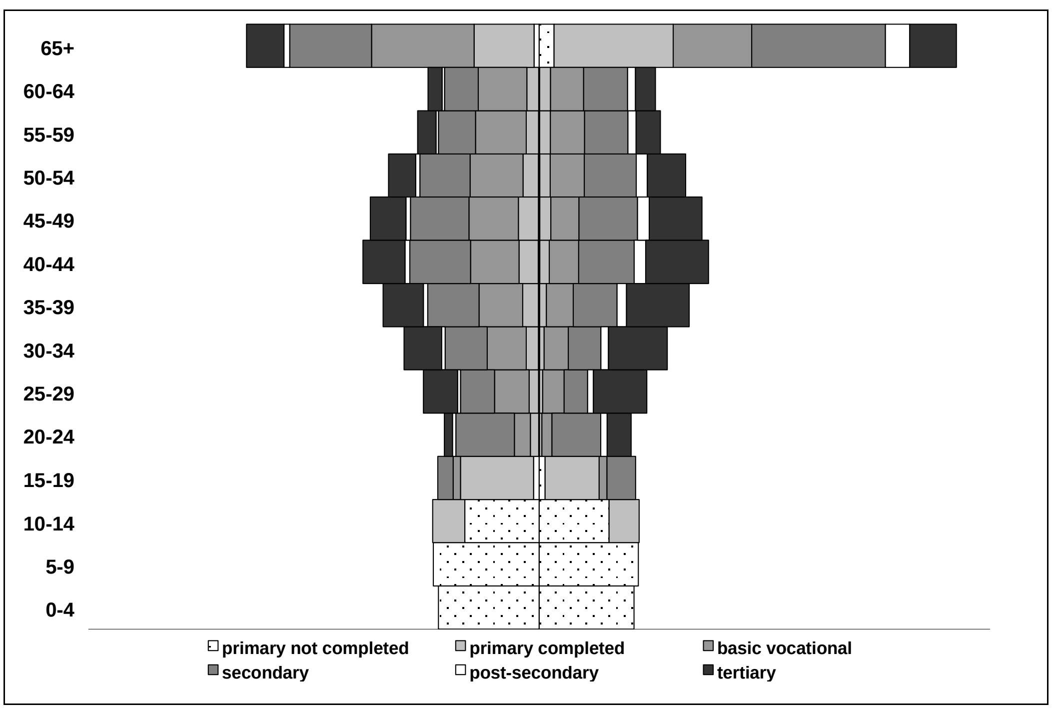


Figure 15. Age and education pyramid for Poland in 2027. Source: Author's own calculations.

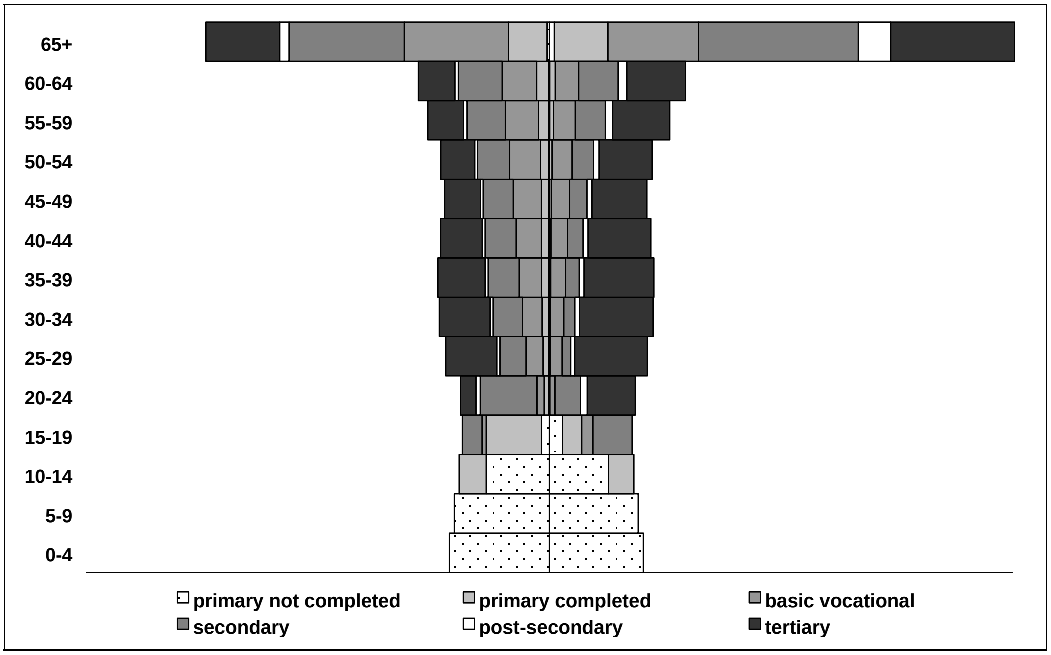


Figure 16. Age and education pyramid for Poland in 2052. Source: Author's own calculations.

Human Capital in Poland in 2002-2052

Using data from the population projection and education-specific human capital profiles, we calculated the human capital stock for the Polish population at ages 15-64 in 2002-2052. Figure 17 shows changes in the working population and adjusted population for human capital in Poland. It is assumed that the average individual human capital in 2002 equals 1. The working population increases slightly to 2012 and then it decreases gradually to the level of 18 million people. The pace of increase of the adjusted population is higher than the working population because of the progress of education. After 2012 human capital decreases, but the pace is slower than the changes in the working population. The results show that aggregated human capital in Poland will decrease in the next 50 years.

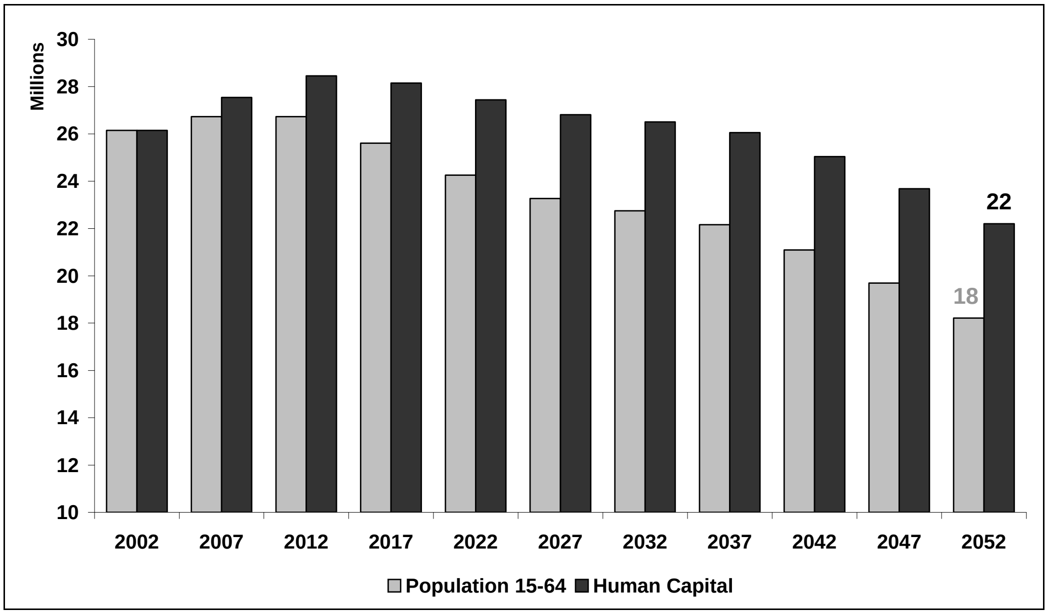


Figure 17. Working population (aged 15-64) and human capital (aged 15-64) in Poland in 2002-2052. Source: Author's own calculations.

From an individual human capital formation point of view, the future for Poland will be very optimistic. The average human capital per capita will increase about 22 percent (see Figure 18). The convex part of the line indicates those years in which well-educated people with large cohorts reach the age where individual human capital is the highest.

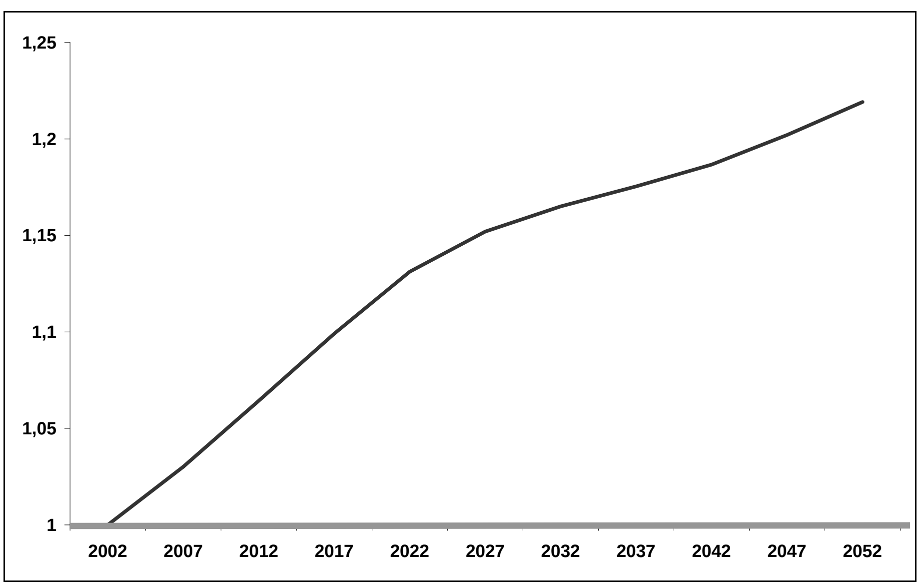


Figure 18. Average human capital per capita in Poland in 2002-2052. Source: Author's own calculations.

The situation seems to be rather pessimistic when we consider the aggregated human capital for population. Figure 19 shows the relative changes in human capital with reference to the year 2002 (HC=1). Human capital peaks in 2012 with an increase of 10 percent. After 2012 a decrease is observed. The reason for this decrease is the projected development of education in Poland, as well as a decrease in the size of the workforce. In 2037 (after 35 years) the level of human capital is the same as it was at the beginning of the period under consideration! Then human capital decreases even faster and in 2052 reaches a level that is 15 percent lower than in 2002. Thus, the potential of human capital that can be used in creating economic growth will shrink within the next 50 years. The consequences could be very negative for the Polish economy, which is trying to catch up to the level of development of the "old EU" countries.

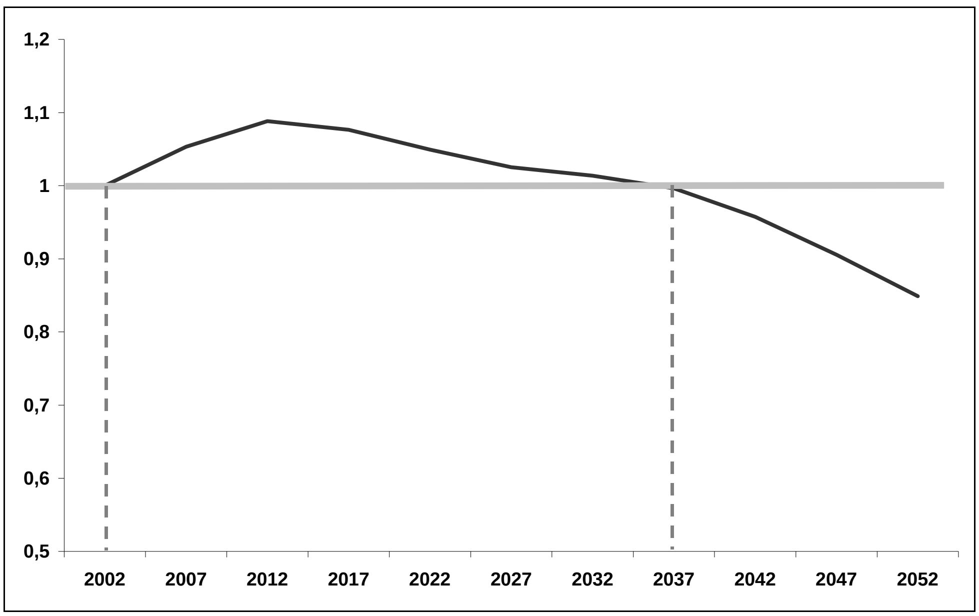


Figure 19. Aggregated human capital (workforce) in Poland in 2002-2052. Source: Author's own calculations.

Conclusions

We present a modified method of human capital measurement. In contrast to other education-based methods, rather than considering only one factor, ours takes into account the following factors affecting human capital formation: education, acquiring knowledge and experience, obsolescence of and forgetting knowledge, and the impact of health. The complexity of the human capital formation process requires simplifications and assumptions which could be disputable, but the method presented in this paper seems to be suitable for following through this formation in the situation of workforce ageing.

The results suggest that individual human capital follows an inverted U-shaped profile where the significant decreases of the stock are found after the age of 45. It is independent of an education level, but there are differences between the profiles – the better the education, the steeper the curve. The very flat profile is found for people with primary not-completed education. The steeper profile is observed for those with a tertiary education. The peak of human capital stock is achieved in the age group 45-55.

The processes of population ageing and de-population will influence more and more societies and economies. In the labor market it will cause a shrinkage of the workforce and changes in workforce structure. An increase in a population's level of education can slow down the negative consequences of this demographic process, but it cannot stop them.

Consequences of workforce ageing can be considered in the short and long term. In the short term, accumulated human capital could increase if well-educated workers from large cohorts reach the age at which their human capital is the highest. However, in the long term, a decrease in aggregate human capital is expected because the working-age population shrinks. This could also affect economic growth levels in the future.

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Appendix 1

Calculations of acquisition rates - α_{as}

We use Eq. (5) [notations of equations are the same as in the text]:

$$h_{as}^{*} = h_s + \sum_{i=1}^{E_{as}} \Delta C_{is} \tag{5}$$

a) $i \in \langle 1, E_{ls} \rangle$,

$$\alpha_{as} = 1 + \frac{E_{as}\Delta C_{1s}}{h_s} \tag{6}$$

b) $i \in \langle E_{ls}+1, E_{hs} \rangle$

$$h_{as}^{*} = h_s + \sum_{i=1}^{E_{ls}} \Delta C_{is} + \sum_{i=E_{ls}+1}^{E_{as}} \left[\frac{\Delta C_{1s} - \Delta C_{zs}}{(E_{ls}+1) - E_{hs}} i + \left(\Delta C_{zs} - \frac{(\Delta C_{1s} - \Delta C_{zs})E_{hs}}{(E_{ls}+1) - E_{hs}} \right) \right]$$

$$h_{as}^{*} = h_s + E_{ls}\Delta C_{1s} + (E_{as} - E_{ls}) \left[\Delta C_{zs} - \frac{(\Delta C_{1s} - \Delta C_{zs})E_{hs}}{(E_{ls}+1) - E_{hs}} \right] + \frac{(\Delta C_{1s} - \Delta C_{zs})}{(E_{ls}+1) - E_{hs}} \sum_{i=E_{ls}+1}^{E_{as}} i$$

$$h_{as}^{*} = h_s + E_{ls}\Delta C_{1s} + (E_{as} - E_{ls}) \left[\Delta C_{zs} - \frac{(\Delta C_{1s} - \Delta C_{zs})E_{hs}}{(E_{ls}+1) - E_{hs}} \right] + \frac{(\Delta C_{1s} - \Delta C_{zs})}{(E_{ls}+1) - E_{hs}} \cdot \frac{(E_{ls}+1+E_{as})(E_{as} - E_{ls})}{2}$$

$$= h_s \left[1 + \frac{2E_{ls}\Delta C_{1s} + (E_{as} - E_{ls}) \left[2\left(\Delta C_{zs} - \frac{(\Delta C_{1s} - \Delta C_{zs})E_{hs}}{(E_{ls}+1) - E_{hs}} \right) + \frac{(\Delta C_{1s} - \Delta C_{zs})(E_{ls}+1+E_{as})}{(E_{ls}+1) - E_{hs}} \right]}{2h_s} \right]$$

$$= h_s \left[1 + \frac{2E_{ls}\Delta C_{1s} + (E_{as} - E_{ls}) \left[2\Delta C_{zs} - \frac{(\Delta C_{1s} - \Delta C_{zs})(2E_{hs} - (E_{ls}+1+E_{as}))}{(E_{ls}+1) - E_{hs}} \right]}{2h_s} \right]$$

$$\alpha_{as} = 1 + \frac{2E_{ls}\Delta C_{1s} + (E_{as} - E_{ls}) \left[2\Delta C_{zs} - \frac{(\Delta C_{1s} - \Delta C_{zs})(2E_{hs} - (E_{ls}+1+E_{as}))}{(E_{ls}+1) - E_{hs}} \right]}{2h_s} \tag{7}$$

c) $i \in \langle E_{hs}+1, \infty \rangle$

$$h_{as}^{*} = h_s + \sum_{i=1}^{E_{ls}} \Delta C_{is} + \sum_{i=E_{ls}+1}^{E_{hs}} \left[\frac{\Delta C_{1s} - \Delta C_{zs}}{(E_{ls}+1) - E_{hs}} i + \left(\Delta C_{zs} - \frac{(\Delta C_{1s} - \Delta C_{zs}) E_{hs}}{(E_{ls}+1) - E_{hs}} \right) \right] + \sum_{i=E_{hs}+1}^{E_{as}} \Delta C_{zs}$$

$$\alpha_{as} = 1 + \frac{2E_{ls}\Delta C_{1s} + (E_{hs} - E_{ls}) \left[2\Delta C_{zs} + \frac{(\Delta C_{1s} - \Delta C_{zs})(E_{ls} + 1 - E_{hs})}{(E_{ls}+1) - E_{hs}} \right] + 2(E_{as} - E_{hs})\Delta C_{zs}}{2h_s}$$

$$\alpha_{as} = 1 + \frac{E_{ls}\Delta C_{1s} + 0.5(E_{hs} - E_{ls})(\Delta C_{1s} + \Delta C_{zs}) + (E_{as} - E_{hs})\Delta C_{zs}}{h_s} \qquad (8)$$

Appendix 2

Calculation of depreciation rates - β_{as}

We use the Eq. (10) from the text:

$$h_{as}^{*} = 8 + \sum_{l=9}^{h_s} \left[1 - E_{as} \cdot 0.000625(l-8)\right] \quad (10)$$

$$j = l - 8$$

$$h_{as}^{*} = 8 + (h_s - 8) - E_{as} \cdot 0.000625 \sum_{j=1}^{h_s - 8} j$$

$$h_{as}^{*} = h_s - 0.000625 \cdot E_{as} \frac{(h_s - 7)(h_s - 8)}{2}$$

$$h_{as}^{*} = h_s \left[1 - 0.000625 \cdot E_{as} \frac{(h_s - 7)(h_s - 8)}{2h_s}\right]$$

$$\beta_{as} = 1 - 0.000625 \cdot E_{as} \frac{(h_s - 7)(h_s - 8)}{2h_s} \quad (11)$$