HYDROGEN: MECHANISMS AND STRATEGIES OF MARKET PENETRATION

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1. Introduction and Summary

This conference provides clear evidence of the growing interest in hydrogen as an energy vector and of the increasing variety of efforts to devise water-splitting processes based on non-fossil forms of primary energy. The time seems appropriate for assessing the economic potential of hydrogen in the energy game and for estimating the discounted value of this potential. We need quantitative estimates of the time lags, probabilities of success, and the costs of R. & D. in order to provide guidelines for the allocation of the substantial sums of money that will be needed for a successful and timely development program.

In this paper, we shall describe two successive models-one for quantifying the benefits and the other for optimizing the level and the structure of the research effort. Our aim has been to devise sufficiently simple analyses so as to keep intuition on the track. These models require numerical values for certain parameters, and in each case we have attempted to work with prudent estimates. Because of the inherently subjective nature of these parameters, we have run

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a series of sensitivity analyses. In all cases--even with the most pessimistic assumptions concerning a non-growing, slow-learning society--the prospective benefits appear high.

Compared with these benefits, the costs of exploratory research are so low that it would make good sense for the U.S. alone to support 50-100 parallel projects during the next five years. These would include laboratory and benchscale experiments and then unit operations tests. By the end of the 1970's, it should be possible to determine which projects are the most promising candidates for pilot plant construction. Demonstration plants would be built during the middle 1980's, and these would be followed by largescale commercial facilities during the 1990's. This is the scenario for which we shall attempt to estimate the costs and benefits.

2. Hydrogen and the Energy Market

Most presentations of the "Hydrogen Economy" emphasize the use of hydrogen as an energy vector with superior properties: clean-burning, cheaply transportable, and readily storable. Once we start looking at the size and structure of the energy market, we soon see that it will take many years before hydrogen is extensively used as a fuel. From the very beginning, however, water-splitting will help to economize on fossil resources. The new technology can first be used to replace those quantities of oil and natural gas that are now used in the manufacture of chemical hydrogen.

-2-

This application will come first because it commands a high price per BTU and because demands are concentrated in large units, e.g. ammonia plants and oil refineries. Concentration means that a water-splitting plant could use the output of a large high-temperature nuclear reactor. The process heat source could be identical to that used for electricity generation. A large and proved reactor type will provide the cheapest source of nuclear process heat. In this way, large water-splitting plants could precede the construction of a distribution net for hydrogen.

For orientation on the numerical magnitudes, see Table 1 and Figure 1, reproduced from Meadows and De Carlo [4]. Note that there are wide ranges of uncertainty in these longterm forecasts of hydrogen demand, but that ammonia and petroleum refining continue to be the principal customers for hydrogen through the year 2000.

In the following section, our calculation of benefits will be extrapolated from the U.S. "low adjusted" figure of 15.5 trillion SCF of hydrogen for the year 2000. This is $4 \ 10^{15}$ BTU, equivalent to 2.3% of that year's aggregate demand for primary energy (see Associated Universities, AET-8 [1, p.15]). Despite this small percentage, hydrogen will be an enormous industry. Assuming a price of \$6 per million BTU, the annual sales of hydrogen would amount to \$24 billions for the U.S. plus an even greater amount for the rest of the world.

-3-

	Esti- mated lemand 1968	U.S. forecast base 2000	Demand in year 2000			
c End use			United	States	Rest of the world 1	
			Low	High	Low	High
Anhydrous						
ammonia . Petroleum	. 872	3,060	2,460	4,490	7,200	12,700
refining	. 775	4.580	2.340	32.640	6.000	36.000
Other uses 2 .	413	1,450	1,450	24,660	2,000	25,000
Total. Adjusted	2,060		6,250	61,790	15,200	73,700
range			15,500	52,530	24,950	63,950
			(Median	34,015)	(Median	44,450)

TABLE 1.-Contingency forecasts of demand for hydrogen by end use, year 2000 (Billion standard cubic feet)

¹ Estimated 1968 hydrogen demand in the rest of the world was

2,995 billion cubic feet. ² Includes hydrogen used in chemicals and allied products, for hydrogasification of coal and oil shale, in iron ore reduction, and for miscellaneous purposes except plant fuel.

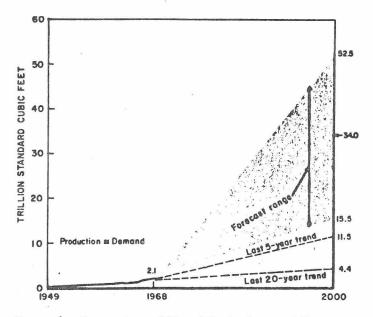


FIGURE 1 .-- Comparison of Trend Projections and Forecasts for Hydrogen Demand.

Source: Meadows and DeCarlo (1970).

Why might it be reasonable to project a price of \$6 per million BTU for hydrogen from fossil fuels? With today's mature technology for steam reforming, it takes roughly 2 BTU of oil or gas primary energy input per BTU of hydrogen output. To cover non-fuel operating costs plus a return on capital, the price of hydrogen is approximately three times the price per BTU of oil or gas. Implicitly, then, we are projecting an oil price of \$2 per million BTU or \$12 per barrel for the year 2000.

Until water-splitting captures most of the hydrogen market, it seems likely that hydrogen prices will be determined, not by the costs of water-splitting but rather by the costs of steam reforming and similar processes tased upon fossil fuels. This might put large profits into the pockets of the innovating enterprises--sufficient profits to more than offset their initial teething troubles and R. & D. expenses.

Once water-splitting has captured the entire market, hydrogen prices will be dominated by the evolution of costs for this new technology. These costs will be lowered successively by economies of scale for individual plants and by the cumulative learning experience acquired by the watersplitting industry. We shall focus upon the latter component because it is more easily correlated with the size and dynamics of the market.

It is convenient to summarize these dynamics with the learning parameter λ , defined as the percentage reduction in

-5-

manufacturing costs for every 1% increase in the industry's cumulative production. That is, let Q_{τ} denote the industry's output in year $\tau \leq t$. Then the average costs and the price in year t+1 are given by

$$P_{t+1} = k \left(\sum_{\tau = -\infty}^{t} Q_{\tau}\right)^{\lambda} .$$
 (1)

The price history of the chemical industry suggests that, with a well supported R. & D. program and a fast expanding market, manufacturing costs may be reduced by roughly 20% with every doubling of the cumulative production. This would imply that the learning parameter $\lambda = -.3$. In the following calculations, to be on the conservative side, we have supposed that $\lambda = -.2$, and that a doubling of the cumulative production will reduce costs by only 13%. This would put water-splitting technology in a sleepier league than methanol or PVC. This is not very reasonable in view of the enormous interest--economic, intellectual and political-linked to an already launched hydrogen economy. On the other side, nuclear reactors and associated chemical plants will be affected by the low metabolic rate characteristic of large animals, and this will tax their rate of evolution.

In addition to the learning parameter λ , equation (1) contains a constant of proportionality k. We have estimated this parameter by supposing that a constant amount of new capacity will be added during each of the 10 years preceding

year 0, the date of capture of the entire chemical hydrogen market. The cumulative production during these preceding years will therefore be 4.5 times the production in year 0. Hence, $k = P_0/(4.5Q_0)^{-.2}$.

3. The Demand curve for Hydrogen; Market Simulation

Even before water-splitting captures the entire chemical market, hydrogen will begin to be used for steel making and for air and road transport. For these applications, hydrogen has intrinsic advantages which will more than compensate for its high price. In the case of air transportation, this is due to hydrogen's high heating value per unit weight. Because it increases the productivity of an airplane, hydrogen would be preferable to conventional jet fuel even if its price per BTU were three times higher. Similarly, hydrogen should command a premium price per BTU for steel making and for road transport in areas where the air is heavily polluted. During the 1990's, it is likely that these applications will represent only a small percentage of the hydrogen market. Nontheless, they will prepare the way for the period of large-scale expansion beginning, say, in the year 2000.

Once water-splitting captures the premium-price chemical market, the industry's further expansion will depend upon its ability to lower costs and prices. Each time the fabrication cost of hydrogen can be reduced, a new set of customers will be attracted. As a shortcut summary of price responsiveness, it is convenient to define the elasticity n. This parameter

-7-

indicates the percentage expansion of the hydrogen market associated with each 1% reduction in the current price. For the reference case, it has been supposed that the elasticity $\eta = -2$. This seems like an underestimate of the elasticity of demand for hydrogen in view of its small share of the energy market and its significant advantages for steel making, air and road transport. The demand for hydrogen is surely more elastic than that for electricity, a well-established energy vector. In the case of electricity, it has been estimated that $\eta \approx -1$ (see Doctor and Anderson [2, pp. 37-40]).

For projecting demands, we shall suppose that future growth may be factored into two components: one that is dependent upon the hydrogen price and one that is independent. The first of these effects is summarized through the elasticity parameter n, and the second through the growth parameter y. The growth parameter allows for those long-term trends in hydrogen demand that are related to the growth of population, per capita income, per capita use of energy, and the rate of learning how to utilize hydrogen in place of conventional fossil fuels. It is supposed that at constant prices, the demand for hydrogen would grow at the constant annual rate of 5% after the year 2000. This trend factor lies well below the above 10% growth rates experienced during the 1960's, but recall that this was a period during which prices (in constant dollars) declined at the rate of 2.5% per year. The trend factor y refers only to the rate at which

-8-

hydrogen demand would grow if its price were to remain constant.

It will be convenient to represent prices and quantities as index numbers relative to their values in year 0. We may then write the market demand curve as

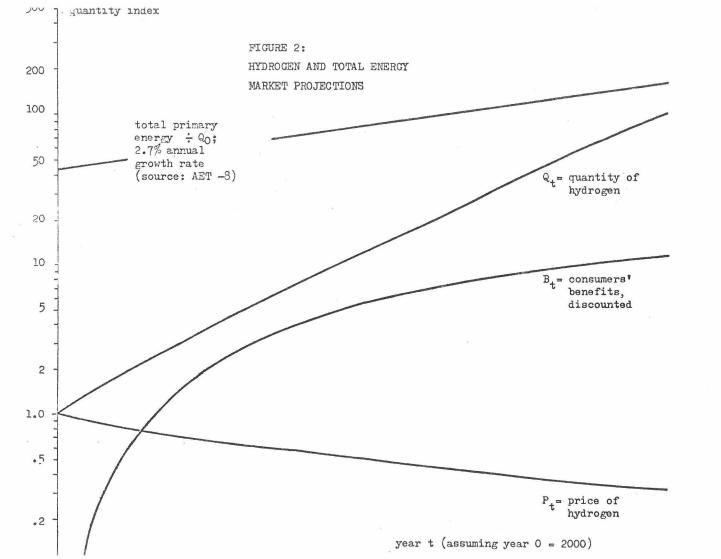
$$\begin{bmatrix} quantity \\ demanded \\ in year t \end{bmatrix} = \begin{bmatrix} long-term \\ growth factor \\ at constant \\ hydrogen prices \end{bmatrix} \begin{bmatrix} price \\ elasticity \\ factor \end{bmatrix}$$

$$Q_t = \begin{bmatrix} \gamma^t \\ \gamma^t \end{bmatrix} \begin{bmatrix} P_t^n \\ P_t^t \end{bmatrix}$$

$$= \begin{bmatrix} 1.05^t \\ 1.05^t \end{bmatrix} \begin{bmatrix} P_t^{-2} \\ P_t^{-2} \end{bmatrix}.$$
(2)

Having specified numerical values for the parameters appearing in the dynamic equations (1) and (2), it is straightforward to trace the evolution of the hydrogen market over time (see Figure 2). It turns out, for example, that P_{10} = .725, and that Q_{10} = 3.099. Expressed at annual rates, this means that prices decline at the rate of 3%, and that demand increases at the rate of 12% during the decade beginning in 2000.¹ These growth rates slow down a bit during subsequent years. Intrepidly extrapolating to the year 2050, we note that the hydrogen demands would still lie well below the total primary energy demands even if these were to grow at the annual rate of only 2.7%. These projections leave ample scope for the continuing employment of our colleagues in the

As a rough check note that (v-1) + () (-03).



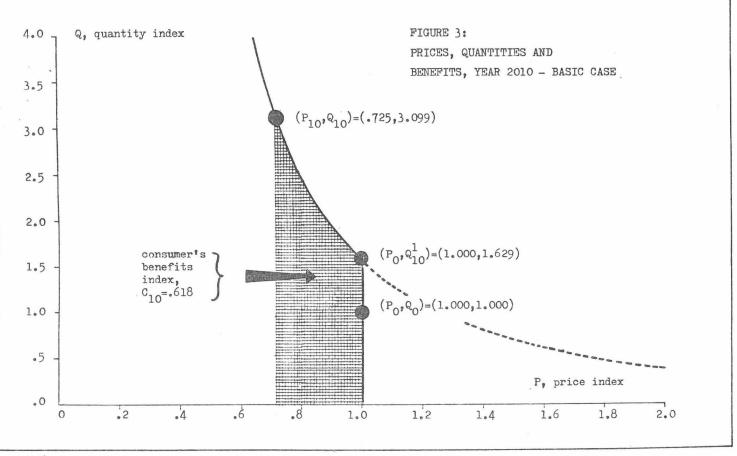
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electricity industry, but probably not for those in oil, gas, and coal.

4. Evaluation of Benefits

In itself, this market simulation does not permit us to evaluate the benefits of water-splitting. We do so through the "consumers' surplus" measure illustrated in Figure 3 for year t = 10. It can be seen that if the hydrogen price remained constant at its initial level Po = 1, demands would grow at the constant rate of only 5%, and that the value $Q'_{10} = 1.05^{10} = 1.629$. We would then observe that the consumers' surplus from water-splitting was zero, for this means that the new technology would provide no price reduction to consumers. In our basic case, however, there are substantial price reductions, and $P_{10} = .725$. Accordingly, there are Q'_{10} consumers each of whom have enjoyed the price reduction of $(P_0 - P_{10})$. In addition, there are other consumers who have been attracted to using hydrogen by the price reduction, but who would have been unwilling to pay Po. Altogether, the consumers' benefits in year 10 are measured by the shaded area C10 shown in Figure 3. Similar calculations may be performed for each year t = 0, 1, 2, ... 50. With an annual discount rate of 10% before taxes, the present value of these benefits in year O is²

²Year 0 has been defined here as the date at which watersplitting has captured the entire hydrogen market--roughly the year 2000. Recall that this technology will already have been incorporated in commercial-scale plants during the entire preceeding decade. In evaluating the present value of the benefits in equation (3), we have taken no credit for consumers'



$$B_{t} = \sum_{\tau=0}^{t} \left[\frac{1}{1 \cdot 1} \right]^{t} C_{t} \quad .$$
 (3)

According to Table 2, the benefits index $B_{20} = 4.319$. To convert this into the <u>dollar</u> value of benefits in the year 2000, we must recall that P_0 corresponds to \$6 per million BTU, that $Q_0 = 4 \ 10^{15}$ BTU, and that $P_0Q_0 = 24 billions. Accordingly, the value of water-splitting discounted to the year 2000 is (\$24 billions)(4.319) \approx \$100 billions. Discounting to 1975 at the annual rate of 10%, the present value of consumers' benefits from water-splitting would be of the order of \$10 billions.

For those who wish to test the effects of other numerical parameter values, we have run a series of progressively more pessimistic calculations than the basic case. For example, if consumers are "unresponsive" to the price of hydrogen, the elasticity $\eta = -1.5$. This would reduce the discounted benefit index B_{20} by a relatively small amount--from 4.319 to 3.685. With slow learning (the "low I.Q." column with $\lambda = -.1$), there would be a slow rate of price decline, and the benefits index $B_{20} = 1.743$. With a "no growth" society, $\gamma = 1.00$, and the benefits $B_{20} = 2.026$. Combining these pessimistic assumptions, we arrive at the rightmost column , a "living fossil" society. Even in this case the benefits index would be .819 (\$24 billions) \approx \$20 billions discounted to the year 2000 \approx \$1.8 billions discounted to 1975.

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		7.208	5.868	2.742	2.589	1.014
discounted to 1975 9.6 8.2 3.9 4.5 1.8 (\$ billions)		9.6	8.2	3.9	4.5	1.8

5. A One-time Decision Model for R. & D. Expenditures

Now that we have made a rough estimate of the potential benefits, we may formulate a model for optimizing the level of research and development expenditures on water-splitting. Given the magnitude of the benefits, there is reason to believe that it pays to investigate several technologies in parallel--electrolytic, thermochemical, and direct thermal dissociation. The primary energy source is likely to be nuclear fission, but it could also be solar, geothermal, or fusion. There are a large number of possible ways to split the water molecule. For example, 16 thermochemical cycles have been identified at just one laboratory, the Ispra Joint Nuclear Research Centre (see EUR 5059e [3, p. 13]). Many additional cycles have been proposed, and are being discussed at other sessions of this conference.

Now suppose that for investigating just one water-splitting technology, it requires 5 years for laboratory and bench-scale experiments and for unit operation tests. Altogether, the present value of the costs for one exploratory investigation will be, say, \$10 millions. It will be convenient to express these costs as a fraction of the potential benefits. Accordingly, if the present value of the potential benefits is \$10 billions, the ratio of costs to gross benefits for a single "experiment" would be c = .001.

Each of these individual investigations would be risky, and there is no assurance of success on any one attempt. By taking a sufficiently large number of such gambles, however, there is a high probability that at least one will be a winner. A "success" might be defined as a water-splitting process for which a commercial-scale plant would be capable of producing hydrogen at a cost of \$6 per million BTU, including a return on capital. This would then be competitive with hydrogen from steam reforming during the 1990's when oil prices might be \$12 per barrel (at today's general price level).

For simplicity, it is supposed that each line of watersplitting research has an identical and independently distributed probability of success. Let p denote the probability of failure. For example, if the probabilities of success are only 1 in 20, the failure probability p = .95. Then the <u>expected</u> benefits minus the costs of a single investigation will be

From the viewpoint of the U.S. economy as a whole, it can be seen that this would be a highly favorable gamble. It can also be seen that there are diminishing returns from parallel R. & D. efforts--especially if we make the fairly realistic assumption that there are no additional benefits from developing <u>more</u> than one successful water-splitting process. To analyze this quantitatively, let x denote the number of parallel investigations. It will be convenient to choose the unit of benefits and costs as 1.0 rather than \$10 billions. Then a one-time decision model for optimizing the level of R. & D. expenditures would be the following unconstrained maximization problem:

$$\begin{bmatrix} expected^{3} \\ net benefits \\ = \begin{bmatrix} payoff from \\ one or more \\ successes \end{bmatrix} \begin{bmatrix} probability of \\ one or more \\ successes \end{bmatrix} - \begin{bmatrix} research and \\ development \\ costs for x \\ parallel in- \\ vestigations \end{bmatrix}$$
$$f(x) = \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} 1 - p^{X} \end{bmatrix} - \begin{bmatrix} cx \end{bmatrix}.$$
(4)

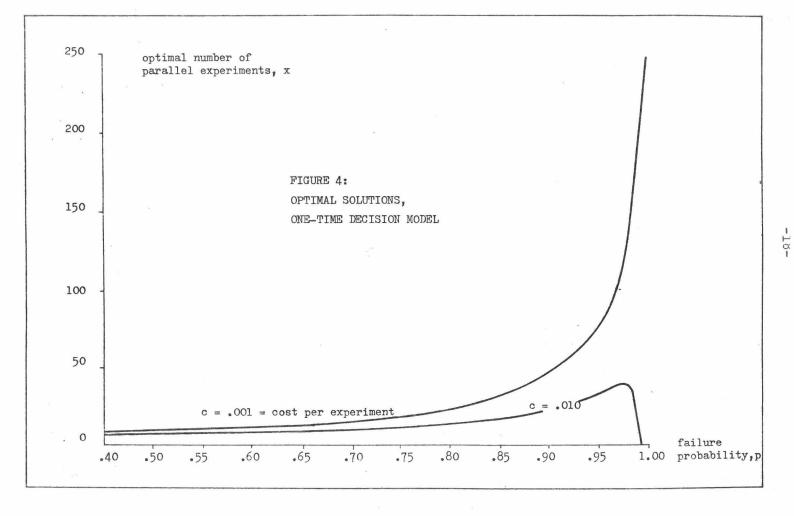
If x is sufficiently large so that we can work with first derivatives rather than first differences, the optimal number of investigations may be calculated by setting f'(x) = 0. Therefore

$$f'(x) = (-\log p)p^{x} - c = 0$$

$$\therefore \text{ optimal } x = \frac{\log[c/-\log p]}{\log p} \quad . \tag{5}$$

The implications of equation (5) are shown on Figure 4. Somewhat paradoxically, the <u>higher</u> the probability of failure, the greater becomes the optimal number of experiments to be

³One extension of this basic model is being investigated by Jean-Pierre Ponssard at IIASA. Working with an exponential "utility" function, he has shown that for decision makers who are averse to taking risks, the optimal number of investigations is generally <u>larger</u> than for the expected value criterion adopted here.



undertaken in parallel. For example, suppose that there is a \$10 billion payoff from water-splitting, a \$10 million cost of each experiment, and therefore c = .001. If the probability of failure is .5, it is optimal to undertake only 9 experiments. With the less favorable situation in which p = .99, the optimal number becomes 230! Needless to say, this monotone increasing relation cannot be extrapolated indefinitely. It is no longer valid for an unfavorable lottery --that is, for c > 1 - p. Hence x = 0 for c = .01 and p > .99.

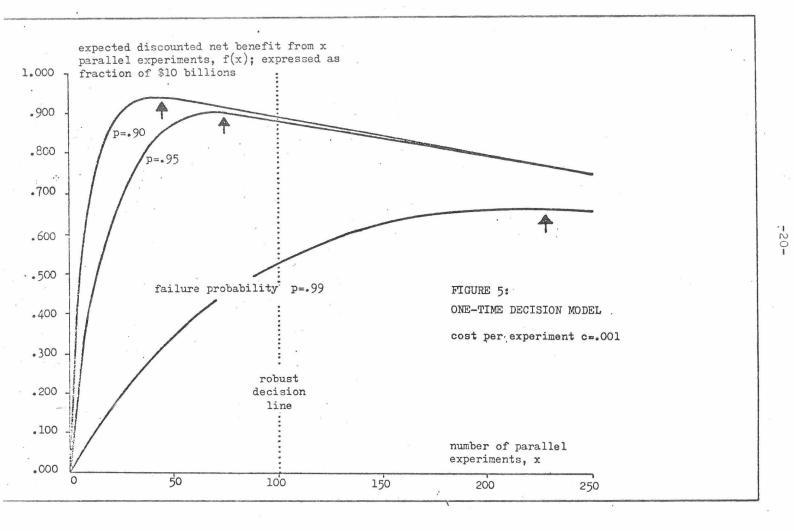
Some additional insights may be obtained from Figure 5. This shows the expected net benefit function f(x) for 3 alternative values of the failure probability p--keeping the cost of experiments fixed at c = .001. The maximum point along each of the 3 curves is indicated by an arrow. It can be seen that these 3 optimal values of x are identical with those on Figure 4.

Figure 5 suggests that if we are uncertain about the value of p, there would be no more than a 20% loss in optimality if we set x = 100. This number of experiments would be "robust" for values of p ranging between the extremes of .90 and .99. With 100 experiments and with p = .95, the probability of discovering one or more successful processes would then be $1 - .95^{100} = .994$.

6. A Sequential Decision Model

Now consider the case of sequential decisions, but continue to suppose that the experimental outcomes do not lead

-19-



us to revise our prior estimates of the probability parameter p ("Bygones are bygones."). Today (at time 0), we select x, the number of processes to be investigated during the initial experimental period of, say, 5 years. At the end of this period for bench-scale and unit operations experiments, we learn whether all of these attempts have been failures. If so, there is another opportunity to enter this same type of lottery. If x was an optimal number for the first set of experiments, it will again be optimal for the second set. Similarly, at the end of 10 years--even if all of the preceding experiments were failures--it remains optimal to investigate x more technologies during the third set of experiments. And so on ad infinitum.⁴

This sequential decision process yields a higher value of expected discounted net benefits than f(x) in equation (4). To see this, let β denote the discount factor for each fiveyear period of experimentation. (For example, if the annual discount rate is 10%, $\beta = (1/1.1)^5 = .62.$) Let g(x) denote the expected discounted net benefits from undertaking x projects at each five-year interval--assuming that all previous experiments have ended in failures. It can then be seen

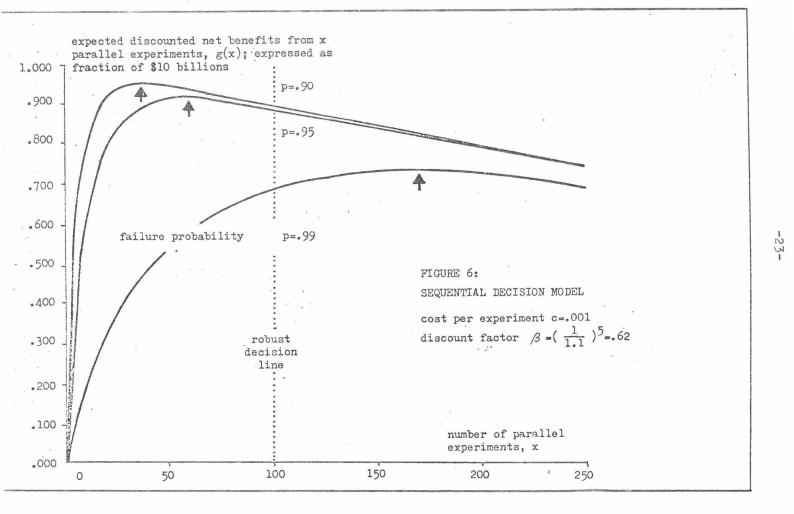
⁴ This sequential decision model has an inherent weakness. There is a small but positive probability that even after a long series of unsuccessful experiments, we will not discontinue the search for water-splitting processes. This logical difficulty may, of course, be overcome by introducing Bayesian revision of the prior probability parameter p.

that

$$\begin{bmatrix} expected net \\ benefits from \\ one five-year \\ period of \\ experiments \end{bmatrix} \begin{bmatrix} discounted sum of \\ probabilities for \\ each possible five- \\ year period of \\ experiments \end{bmatrix}$$
$$g(x) = \begin{bmatrix} 1-p^{x}-cx \end{bmatrix} \qquad [(\beta p^{x})^{0} + (\beta p^{x})^{1} + (\beta p^{x})^{2} + \dots]$$
$$\vdots g(x) = \frac{1-p^{x}-cx}{1-\beta p^{x}} \qquad (6)$$

Figure 6 contains the numerical results for the sequential decision equation (6). As in Figure 5, the cost per experiment c = .001. Again, the net benefit curve is shown for three alternative values of the probability parameter: p = .90, .95 and .99. It will be seen that the maximum value of g(x) is in each case slightly higher than the corresponding value of f(x), and that the optimal value of xis smaller--e.g., for p = .95, the maximum values of f(x)and g(x) are, respectively, .904 and .920 (expressed as fractions of the \$10 billion benefits). The maximizing values of x are 75 and 60 experiments.

For the sequential as well as the one-time model, it remains a robust decision to set the number of initial parallel experiments x = 100. This numerical result makes good common sense. Given an opportunity to enter a favorable lottery, we cannot go far wrong if the size of the initial gamble is 10% of the ultimate prize. If these numbers are



at all realistic, it would not be difficult to justify the expenditure of \$1 billions in the search for economically competitive water-splitting processes.

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