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Interactive Fuzzy Programming for Stochastic Two-level Linear Programming Problems through Probability Maximization

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Foreword

In this paper, we focus on stochastic two-level linear programming problems involving random variable coefficients both in objective functions and constraints. Using the concept of chance constraints, stochastic constraints are transformed into deterministic ones. Following the probability maximization model, the minimization of each stochastic objective function is replaced with the maximization of the probability that each objective function is less than or equal to a certain value. Under some appropriate assumptions for distribution functions, the formulated stochastic two-level linear programming problems are transformed into deterministic ones. Taking into account vagueness of judgments of the decision makers, we present interactive fuzzy programming. In the proposed interactive method, after determining the fuzzy goals of the decision makers at both levels, a satisfactory solution is derived efficiently by updating the satisfactory degree of the decision maker at the upper level with considerations of overall satisfactory balance among both levels. It should be emphasized here that the transformed deterministic problems for deriving an overall satisfactory solution can be easily solved through the combined use of the bisection method and the phase one of the simplex method. An illustrative numerical example is provided to demonstrate the feasibility of the proposed method.

Abstract

This paper considers stochastic two-level linear programming problems. Using the concept of chance constraints and probability maximization, original problems are transformed into deterministic ones. An interactive fuzzy programming method is presented for deriving a satisfactory solution efficiently with considerations of overall satisfactory balance.

Keywords: two-level linear programming problems, random variables, chance constraints, probability maximization, interactive decision making

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About the Authors

Masatoshi Sakawa joined the Integrated Modeling Environment in April 2009. His research and teaching activities are in the area of systems engineering, especially mathematical optimization, multiobjective decision making, fuzzy mathematical programming and game theory. In addition to over 300 articles in national and international journals, he is an author and coauthor of 5 books in English and 14 books in Japanese. At present Dr. Sakawa is a Professor at Hiroshima University, Japan and is working with the Department of Artificial Complex Systems Engineering. Dr. Sakawa received BEng, MEng, and DEng degrees in applied mathematics and physics at Kyoto University, in 1970, 1972, and 1975 respectively. From 1975 he was with Kobe University, where from 1981 he was an Associate Professor in the Department of Systems Engineering. From 1987 to 1990 he was Professor of the Department of Computer Science at Iwate University and from March to December 1991 he was an Honorary Visiting Professor at the University of Manchester Institute of Science and Technology (UMIST), Computation Department, sponsored by the Japan Society for the Promotion of Science (JSPS). He was also a Visiting Professor of the Institute of Economic Research, Kyoto University from April 1991 to March 1992. In 2002 Dr. Sakawa received the Georg Cantor Award of the International Society on Multiple Criteria Decision Making.

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1 Introduction

Decision making problems in decentralized organizations are often formulated as two-level programming problems with a DM at the upper level (DM1) and another DM at the lower level (DM2) [28]. Under the assumption that these DMs do not have motivation to cooperate mutually, the Stackelberg solution [39, 3, 37, 17] is adopted as a reasonable solution for the situation. On the other hand, in the case of a project selection problem in the administrative office of a company and its autonomous divisions, the situation that these DMs can cooperate with each other seems to be natural rather than the noncooperative situation. Lai [11] and Shih et al. [38] proposed solution concepts for two-level linear programming problems or multi-level ones such that decisions of DMs in all levels are sequential and all of the DMs essentially cooperate with each other. In their methods, the DMs identify membership functions of the fuzzy goals for their objective functions, and in particular, the DM at the upper level also specifies those of the fuzzy goals for the decision variables. The DM at the lower level solves a fuzzy programming problem with a constraint with respect to a satisfactory degree of the DM at the upper level. Unfortunately, there is a possibility that their method leads a final solution to an undesirable one because of inconsistency between the fuzzy goals of the objective function and those of the decision variables. In order to overcome the problem in their methods, by eliminating the fuzzy goals for the decision variables, Sakawa et al. have proposed interactive fuzzy programming for two-level or multi-level linear programming problems to obtain a satisfactory solution for DMs [29, 30]. The subsequent works on two-level or multi-level programming have been developing [14, 26, 27, 31, 32, 40, 18, 1, 19, 28]. In actual decision making situations, however, we must often make a decision on the basis of vague information or uncertain data. For such decision making problems involving uncertainty, there exist two typical approaches: probability-theoretic approach and fuzzy-theoretic one. Stochastic programming, as an optimization method based on the probability theory, have been developing in various ways [45, 4], including two stage problems considered by Dantzig [8] and chance constrained programming proposed by Charnes et al. [5]. Especially, for multiobjective stochastic linear programming problems, Stancu-Minasian [44]

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considered the minimum risk approach, while Leclercq [13] and Teghem Jr. et al. [43] proposed interactive methods.

Fuzzy mathematical programming representing the vagueness in decision making situations by fuzzy concepts have been studied by many researchers [20, 21]. Fuzzy multiobjective linear programming, first proposed by Zimmermann [47], have been also developed by numerous researchers, and an increasing number of successful applications has been appearing [36, 16, 48, 42, 12, 21, 41, 22].

As a hybrid of the stochastic approach and the fuzzy one, Wang et al. considered mathematical programming problems with fuzzy random variables [46], Liu et al. [15] discussed chance constrained programming involving fuzzy parameters. In particular, Hulsurkar et al. [9] applied fuzzy programming to multiobjective stochastic linear programming problems. Unfortunately, however, in their method, since membership functions for the objective functions are supposed to be aggregated by a minimum operator or a product operator, optimal solutions which sufficiently reflect the DM's preference may not be obtained. To cope with the problem, after reformulating multiobjective stochastic linear programming problems using several models for chance constrained programming, Sakawa et al. [24, 23, 25] presented an interactive fuzzy satisficing method to derive a satisficing solution for the DM as a generalization of their previous results [33, 36, 34, 35, 21].

Under these circumstances, in this paper, we deal with two-level linear programming problems with random variable coefficients in both objective functions and constraints. Using the concept of chance constraints, stochastic constraints are transformed into deterministic ones. Following the probability maximization model, the minimization of each stochastic objective function is replaced with the maximization of the probability that each objective function is less than or equal to a certain value. Under some appropriate assumptions for distribution functions, the formulated stochastic two-level linear programming problems are transformed into deterministic ones. By considering the fuzziness of human judgments, we present an interactive fuzzy programming method for deriving a satisfactory solution for the DMs by updating the satisfactory degree of the DM at the upper level with considerations of overall satisfactory balance among both levels.

2 Stochastic two-level linear programming problems

Consider two-level linear programming problems with random variable coefficients formulated as:

$$\left.\begin{array}{ll} \underset{\text{for DM1}}{\text{minimize}} & \bar{z}_1(\boldsymbol{x}_1,\boldsymbol{x}_2) = \bar{\boldsymbol{c}}_{11}\boldsymbol{x}_1 + \bar{\boldsymbol{c}}_{12}\boldsymbol{x}_2 + \bar{\alpha}_1 \\ \underset{\text{for DM2}}{\text{minimize}} & \bar{z}_2(\boldsymbol{x}_1,\boldsymbol{x}_2) = \bar{\boldsymbol{c}}_{21}\boldsymbol{x}_1 + \bar{\boldsymbol{c}}_{22}\boldsymbol{x}_2 + \bar{\alpha}_2 \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \le \bar{\boldsymbol{b}} \\ & \boldsymbol{x}_1 \ge \boldsymbol{0},\ \ \boldsymbol{x}_2 \ge \boldsymbol{0} \end{array}\right\} \tag{1}$$

where $\boldsymbol{x}_1$ is an n_1 dimensional decision variable column vector for the DM at the upper level (DM1), $\boldsymbol{x}_2$ is an n_2 dimensional decision variable column vector for the DM at the lower level (DM2), $\bar{\boldsymbol{c}}_{lj}$, $l = 1, 2$, $j = 1, 2$ are n_j dimensional random variable row vectors expressed as $\bar{\boldsymbol{c}}_{lj} = \boldsymbol{c}^1_{lj} + \bar{t}_l\boldsymbol{c}^2_{lj}$ where $\bar{t}_l$, $l = 1, 2$ are mutually independent random variables with mean M_l and their distribution functions $T_l(\cdot)$, $l = 1, 2$ are assumed to be nondecreasing, and $\bar{\alpha}_l$, $l = 1, 2$ are random variables expressed as $\bar{\alpha}_l = \alpha^1_l + \bar{t}_l\alpha^2_l$. In addition, $\bar{b}_i$, $i = 1, 2, \ldots, m$ are mutually independent random variables whose distribution function are also assumed to be nondecreasing.

Stochastic two-level linear programming problems formulated as (1) are often seen in actual decision making situations, e.g., a supply chain planning [19] where the distribution center (DM1) and the production part (DM2) hope to minimize the distribution cost and the production cost respectively under constraints about inventory levels and production levels. Since coefficients of these objective functions and those of the right-hand side of constraints like product demands are often affected by the economic conditions varying at random, they can be regarded as random variables and the supply chain planning is formulated as (1).

Since (1) contains random variable coefficients, solution methods for ordinary deterministic two-level linear programming problems cannot be directly applied. Consequently, in this paper, we consider the constraints involving random variable coefficients in (1) as chance constraints [5] which mean the probability that each constraint is fulfilled must be greater than or equal to a certain probability (satisficing level). Namely, replacing constraints in (1) by chance constraints with satisficing levels $\beta_i \in (0, 1)$, $i = 1, 2, \ldots, m$, problem (1) can be transformed as:

$$\left.\begin{array}{ll} \underset{\text{for DM1}}{\text{minimize}} & \bar{z}_1(\boldsymbol{x}_1, \boldsymbol{x}_2) = \bar{\boldsymbol{c}}_{11}\boldsymbol{x}_1 + \bar{\boldsymbol{c}}_{12}\boldsymbol{x}_2 + \bar{\alpha}_1 \\ \underset{\text{for DM2}}{\text{minimize}} & \bar{z}_2(\boldsymbol{x}_1, \boldsymbol{x}_2) = \bar{\boldsymbol{c}}_{21}\boldsymbol{x}_1 + \bar{\boldsymbol{c}}_{22}\boldsymbol{x}_2 + \bar{\alpha}_2 \\ \text{subject to} & \Pr\{\boldsymbol{a}_{i1}\boldsymbol{x}_1 + \boldsymbol{a}_{i2}\boldsymbol{x}_2 \le \bar{b}_i\} \ge \beta_i, \ \ i = 1, 2, \ldots, m \\ & \boldsymbol{x}_1 \ge \boldsymbol{0}, \ \ \boldsymbol{x}_2 \ge \boldsymbol{0} \end{array}\right\} \tag{2}$$

where $\boldsymbol{a}_{i1}$ and $\boldsymbol{a}_{i2}$ is the i th row vector of A_1 and A_2, and $\bar{b}_i$ is the i th element of $\bar{\boldsymbol{b}}$.

Since the distribution function $F_i(r) = \Pr\{\bar{b}_i \le r\}$ of each random variable $\bar{b}_i$ is nondecreasing, the i th constraint in (2) can be rewritten as:

$$\begin{array}{rcl} \Pr\{\boldsymbol{a}_{i1}\boldsymbol{x}_1 + \boldsymbol{a}_{i2}\boldsymbol{x}_2 \le \bar{b}_i\} \ge \beta_i & \Leftrightarrow & 1 - \Pr\{\boldsymbol{a}_{i1}\boldsymbol{x}_1 + \boldsymbol{a}_{i2}\boldsymbol{x}_2 \ge \bar{b}_i\} \ge \beta_i \\ & \Leftrightarrow & 1 - F_i(\boldsymbol{a}_{i1}\boldsymbol{x}_1 + \boldsymbol{a}_{i2}\boldsymbol{x}_2) \ge \beta_i \\ & \Leftrightarrow & F_i(\boldsymbol{a}_{i1}\boldsymbol{x}_1 + \boldsymbol{a}_{i2}\boldsymbol{x}_2) \le 1 - \beta_i \\ & \Leftrightarrow & \boldsymbol{a}_{i1}\boldsymbol{x}_1 + \boldsymbol{a}_{i2}\boldsymbol{x}_2 \le F_i^*(1 - \beta_i) \end{array}$$

where $F_i^*(\cdot)$ is a pseudo-inverse function of $F_i(\cdot)$ defined by $F_i^*(r) = \inf\{y \mid F_i(y) \ge r\}$.

Letting $\hat{b}_i = F_i^*(1 - \beta_i)$, problem (2) can be rewritten as:

$$\left.\begin{array}{ll} \underset{\text{for DM1}}{\text{minimize}} & \bar{z}_1(\boldsymbol{x}_1, \boldsymbol{x}_2) = \bar{\boldsymbol{c}}_{11}\boldsymbol{x}_1 + \bar{\boldsymbol{c}}_{12}\boldsymbol{x}_2 + \bar{\alpha}_1 \\ \underset{\text{for DM2}}{\text{minimize}} & \bar{z}_2(\boldsymbol{x}_1, \boldsymbol{x}_2) = \bar{\boldsymbol{c}}_{21}\boldsymbol{x}_1 + \bar{\boldsymbol{c}}_{22}\boldsymbol{x}_2 + \bar{\alpha}_2 \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \le \hat{\boldsymbol{b}} \\ & \boldsymbol{x}_1 \ge \boldsymbol{0}, \ \ \boldsymbol{x}_2 \ge \boldsymbol{0} \end{array}\right\} \tag{3}$$

where $\hat{\boldsymbol{b}} = (\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_m)^T$.

In addition to the chance constraints, it is now appropriate to consider objective functions with randomness on the basis of some decision making model. As such decision making models, expectation optimization, variance minimization, probability maximization and fractile criterion optimization are typical. For instance, let the objective function represent a profit. If the DM wishes to simply maximize the expected profit without caring about the fluctuation of the profit, the expectation optimization model [7] to optimize the expectation of the objective function is appropriate. On the other hand, if the DM hopes to decrease the fluctuation of the profit as little as possible from the viewpoint of the stability of the profit, the variance minimization model [7] to minimize the variance of the objective function is useful. In contrast to these two types of optimizing approaches, as satisficing approaches, the probability maximization model [7] and the fractile criterion optimization model or Kataoka's model [10] have been proposed. When the DM wants to maximize the probability that the profit is greater than or equal to a certain permissible level, probability maximization model [7] is recommended. In contrast, when the DM wishes to optimize such a permissible level as the probability that the profit is greater than or equal to the permissible level is greater than or equal to a certain threshold, the fractile criterion optimization model will be appropriate. In this paper, assuming that the DM wants to maximize the probability that the profit is greater than or equal to a certain permissible level for safe management, we adopt the probability maximization model as a decision making model.

In the probability maximization model, the minimization of each objective function $\bar{z}_l(\boldsymbol{x}_1, \boldsymbol{x}_2)$ in (3) is substituted with the maximization of the probability that $\bar{z}_l(\boldsymbol{x}_1, \boldsymbol{x}_2)$ is less than or equal to a certain permissible level h_l under the chance constraints. Through probability maximization, problem (3) can be rewritten as:

$$\left.\begin{array}{ll} \underset{\text{for DM1}}{\text{maximize}} & \Pr\{\bar{z}_1(\boldsymbol{x}_1, \boldsymbol{x}_2) \leq h_1\} \\ \underset{\text{for DM2}}{\text{maximize}} & \Pr\{\bar{z}_2(\boldsymbol{x}_1, \boldsymbol{x}_2) \leq h_2\} \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \leq \hat{\boldsymbol{b}} \\ & \boldsymbol{x}_1 \geq \boldsymbol{0},\ \ \boldsymbol{x}_2 \geq \boldsymbol{0} \end{array}\right\}. \tag{4}$$

Supposing that $\boldsymbol{c}_{l1}^2\boldsymbol{x}_1 + \boldsymbol{c}_{l2}^2\boldsymbol{x}_2 + \alpha_l^2 > 0,\ l = 1, 2, \ldots, k$ for any feasible solution $(\boldsymbol{x}_1, \boldsymbol{x}_2)$ to (4), from the assumption on the distribution function $T_l(\cdot)$ of each random variable $\bar{t}_l$, we can rewrite objective functions in (4) as follows.

$$\begin{aligned} &\Pr\{\bar{z}_l(\boldsymbol{x}_1, \boldsymbol{x}_2) \leq h_l\} \\ &= \Pr\left\{(\boldsymbol{c}_{l1}^1 + \bar{t}_l\boldsymbol{c}_{l1}^2)\boldsymbol{x}_1 + (\boldsymbol{c}_{l2}^1 + \bar{t}_l\boldsymbol{c}_{l2}^2)\boldsymbol{x}_2 + (\alpha_l^1 + \bar{t}_l\alpha_l^2) \leq h_l\right\} \\ &= \Pr\left\{(\boldsymbol{c}_{l1}^2\boldsymbol{x}_1 + \boldsymbol{c}_{l2}^2\boldsymbol{x}_2 + \alpha_l^2)\bar{t}_l + (\boldsymbol{c}_{l1}^1\boldsymbol{x}_1 + \boldsymbol{c}_{l2}^1\boldsymbol{x}_2 + \alpha_l^1) \leq h_l\right\} \\ &= \Pr\left\{\bar{t}_l \leq \frac{h_l - (\boldsymbol{c}_{l1}^1\boldsymbol{x}_1 + \boldsymbol{c}_{l2}^1\boldsymbol{x}_2 + \alpha_l^1)}{(\boldsymbol{c}_{l1}^2\boldsymbol{x}_1 + \boldsymbol{c}_{l2}^2\boldsymbol{x}_2 + \alpha_l^2)}\right\} \\ &= T_l\left(\frac{h_l - \boldsymbol{c}_{l1}^1\boldsymbol{x}_1 - \boldsymbol{c}_{l2}^1\boldsymbol{x}_2 - \alpha_l^1}{\boldsymbol{c}_{l1}^2\boldsymbol{x}_1 + \boldsymbol{c}_{l2}^2\boldsymbol{x}_2 + \alpha_l^2}\right) \end{aligned}$$

Hence, (4) can be equivalently transformed into the following deterministic two-level programming problem.

$$\left.\begin{array}{ll} \underset{\text{for DM1}}{\text{maximize}} & p_1(\boldsymbol{x}_1, \boldsymbol{x}_2) = T_1\left(\dfrac{h_1 - \boldsymbol{c}_{11}^1\boldsymbol{x}_1 - \boldsymbol{c}_{12}^1\boldsymbol{x}_2 - \alpha_1^1}{\boldsymbol{c}_{11}^2\boldsymbol{x}_1 + \boldsymbol{c}_{12}^2\boldsymbol{x}_2 + \alpha_1^2}\right) \\ \underset{\text{for DM2}}{\text{maximize}} & p_2(\boldsymbol{x}_1, \boldsymbol{x}_2) = T_2\left(\dfrac{h_2 - \boldsymbol{c}_{21}^1\boldsymbol{x}_1 - \boldsymbol{c}_{22}^1\boldsymbol{x}_2 - \alpha_2^1}{\boldsymbol{c}_{21}^2\boldsymbol{x}_1 + \boldsymbol{c}_{22}^2\boldsymbol{x}_2 + \alpha_2^2}\right) \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \leq \hat{\boldsymbol{b}} \\ & \boldsymbol{x}_1 \geq \boldsymbol{0},\ \ \boldsymbol{x}_2 \geq \boldsymbol{0} \end{array}\right\} \tag{5}$$

3 Interactive fuzzy programming

In general, it seems natural that the DMs have fuzzy goals for their objective functions when they take fuzziness of human judgments into consideration. For each of the objective functions $p_l(\boldsymbol{x}_1, \boldsymbol{x}_2)$, $l = 1, 2$ in (5), assume that the DMs have fuzzy goals such as "$p_l(\boldsymbol{x}_1, \boldsymbol{x}_2)$ should be substantially greater than or equal to some specific value." Then, (5) can be rewritten as:

$$\left.\begin{array}{ll} \underset{\text{for DM1}}{\text{maximize}} & \mu_1(p_1(\boldsymbol{x}_1, \boldsymbol{x}_2)) \\ \underset{\text{for DM2}}{\text{maximize}} & \mu_2(p_2(\boldsymbol{x}_1, \boldsymbol{x}_2)) \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \leq \hat{\boldsymbol{b}} \\ & \boldsymbol{x}_1 \geq \boldsymbol{0},\ \ \boldsymbol{x}_2 \geq \boldsymbol{0} \end{array}\right\} \tag{6}$$

where $\mu_l(\cdot)$ is a membership function to quantify a fuzzy goal for the l th objective function in (5) and it is assumed to be nondecreasing.

Although the membership function does not always need to be linear, for the sake of simplicity, we adopt a linear membership function. To be more specific, if the DM feels that $p_l(\boldsymbol{x}_1, \boldsymbol{x}_2)$ should be greater than or equal to at least $p_{l,0}$ and $p_l(\boldsymbol{x}_1, \boldsymbol{x}_2) \geq p_{l,1} (> p_{l,0})$ is satisfactory, the linear membership function $\mu_l(p_l(\boldsymbol{x}_1, \boldsymbol{x}_2))$ is defined as:

$$\mu_l(p_l(\boldsymbol{x}_1, \boldsymbol{x}_2)) = \begin{cases} 0 & , \quad p_l(\boldsymbol{x}_1, \boldsymbol{x}_2) < p_{l,0} \\ \dfrac{p_l(\boldsymbol{x}_1, \boldsymbol{x}_2) - p_{l,0}}{p_{l,1} - p_{l,0}} & , \quad p_{l,0} \leq p_l(\boldsymbol{x}_1, \boldsymbol{x}_2) \leq p_{l,1} \\ 1 & , \quad p_l(\boldsymbol{x}_1, \boldsymbol{x}_2) > p_{l,1} \end{cases} \tag{7}$$

and it is depicted in Fig. 1.

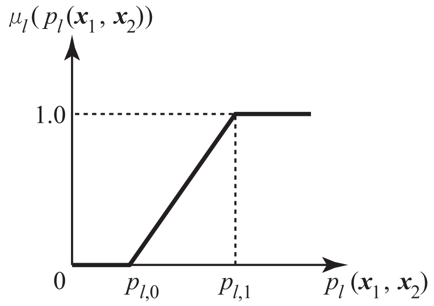

Figure 1: Linear membership function

Zimmermann [47] suggested a method for assessing the parameter values of the linear membership function. In his method, the parameter values $p_{l,1}$, $l = 1, 2$ are determined as

$$\begin{aligned} p_{1,1} &= p_{1,\max} = p_1(\boldsymbol{x}^1_{1,\max}, \boldsymbol{x}^1_{2,\max}) = \max_{(\boldsymbol{x}_1^T, \boldsymbol{x}_2^T)^T \in X} p_1(\boldsymbol{x}_1, \boldsymbol{x}_2) \\ p_{2,1} &= p_{2,\max} = p_2(\boldsymbol{x}^2_{1,\max}, \boldsymbol{x}^2_{2,\max}) = \max_{(\boldsymbol{x}_1^T, \boldsymbol{x}_2^T)^T \in X} p_2(\boldsymbol{x}_1, \boldsymbol{x}_2) \end{aligned}$$

and the parameter values $p_{l,0}$, $l = 1, 2$ are specified as

$$p_{1,0} = p_1(\boldsymbol{x}^2_{1,\max}, \boldsymbol{x}^2_{2,\max})$$
$$p_{2,0} = p_2(\boldsymbol{x}^1_{1,\max}, \boldsymbol{x}^1_{2,\max})$$

where $(\boldsymbol{x}^l_{1,\min}, \boldsymbol{x}^l_{2,\min})$ is an optimal solution to the following problem

$$\left.\begin{array}{ll}\text{maximize} & p_l(\boldsymbol{x}_1, \boldsymbol{x}_2) = T_l\left(\dfrac{h_l - \boldsymbol{c}^1_{l1}\boldsymbol{x}_1 - \boldsymbol{c}^1_{l2}\boldsymbol{x}_2 - \alpha^1_l}{\boldsymbol{c}^2_{l1}\boldsymbol{x}_1 + \boldsymbol{c}^2_{l2}\boldsymbol{x}_2 + \alpha^2_l}\right) \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \le \hat{\boldsymbol{b}} \\ & \boldsymbol{x}_1 \ge \boldsymbol{0},\ \ \boldsymbol{x}_2 \ge \boldsymbol{0}\end{array}\right\}. \tag{8}$$

From the monotonicity of the distribution function $T_l(\cdot)$, problem (8) is equivalent to:

$$\left.\begin{array}{ll}\text{maximize} & \dfrac{h_l - \boldsymbol{c}^1_{l1}\boldsymbol{x}_1 - \boldsymbol{c}^1_{l2}\boldsymbol{x}_2 - \alpha^1_l}{\boldsymbol{c}^2_{l1}\boldsymbol{x}_1 + \boldsymbol{c}^2_{l2}\boldsymbol{x}_2 + \alpha^2_l} \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \le \hat{\boldsymbol{b}} \\ & \boldsymbol{x}_1 \ge \boldsymbol{0},\ \ \boldsymbol{x}_2 \ge \boldsymbol{0}\end{array}\right\}. \tag{9}$$

Using the variable transformation method by Charnes and Cooper [6]: $s^l = 1/(\boldsymbol{c}^2_{l1}\boldsymbol{x}_1 + \boldsymbol{c}^2_{l2}\boldsymbol{x}_2 + \alpha^2_1)$, $\boldsymbol{y}_j = s^l \cdot \boldsymbol{x}_j$, $s^l > 0, l = 1, 2, j = 1, 2$, problem (9) is equivalently transformed as:

$$\left.\begin{array}{ll}\text{maximize} & -\boldsymbol{c}^1_{l1}\boldsymbol{y}_1 - \boldsymbol{c}^1_{l2}\boldsymbol{y}_2 - (\alpha^1_l - h_l) \cdot s^l \\ \text{subject to} & A_1\boldsymbol{y}_1 + A_2\boldsymbol{y}_2 - \hat{\boldsymbol{b}} \cdot s^l \le \boldsymbol{0} \\ & \boldsymbol{c}^2_{l1}\boldsymbol{y}_1 + \boldsymbol{c}^2_{l2}\boldsymbol{y}_2 + \alpha^2_l \cdot s^l = 1 \\ & \boldsymbol{y}_1 \ge \boldsymbol{0},\ \ \boldsymbol{y}_2 \ge \boldsymbol{0},\ \ s^l > 0\end{array}\right\}. \tag{10}$$

Since (10) is a linear programming problem, it can be easily solved by the simplex method of linear programming.

To derive an overall satisfactory solution to the membership function maximization problem (6), we first find the maximizing decision of the fuzzy decision proposed by Bellman and Zadeh [2]. Namely, the following problem is solved for obtaining a solution which maximizes the smaller degree of satisfaction between those of the two DMs:

$$\left.\begin{array}{ll}\text{maximize} & \min\limits_{l=1,2}\{\mu_l(p_l(\boldsymbol{x}_1, \boldsymbol{x}_2))\} \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \le \hat{\boldsymbol{b}} \\ & \boldsymbol{x}_1 \ge \boldsymbol{0},\ \ \boldsymbol{x}_2 \ge \boldsymbol{0}\end{array}\right\}, \tag{11}$$

or equivalently,

$$\left.\begin{array}{ll}\text{maximize} & v \\ \text{subject to} & \mu_1(p_1(\boldsymbol{x}_1, \boldsymbol{x}_2)) \ge v \\ & \mu_2(p_2(\boldsymbol{x}_1, \boldsymbol{x}_2)) \ge v \\ & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \le \hat{\boldsymbol{b}} \\ & \boldsymbol{x}_1 \ge \boldsymbol{0},\ \ \boldsymbol{x}_2 \ge \boldsymbol{0}\end{array}\right\}. \tag{12}$$

Since $\mu_l(\cdot)$, $l = 1, 2$ are nondecreasing, (12) can be converted as:

$$\left.\begin{array}{ll}\text{maximize} & v \\ \text{subject to} & p_1(\boldsymbol{x}_1, \boldsymbol{x}_2) \ge \mu^*_1(v) \\ & p_2(\boldsymbol{x}_1, \boldsymbol{x}_2) \ge \mu^*_2(v) \\ & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \le \hat{\boldsymbol{b}} \\ & \boldsymbol{x}_1 \ge \boldsymbol{0},\ \ \boldsymbol{x}_2 \ge \boldsymbol{0}\end{array}\right\} \tag{13}$$

where $\mu_l^*(\cdot)$ is a pseudo-inverse function of $\mu_l(\cdot)$ defined by $\mu_l^*(r) = \inf\{y \mid \mu_l(y) \geq r\}$. Since

$$p_l(\boldsymbol{x}_1, \boldsymbol{x}_2) = T_l\left(\frac{h_l - \boldsymbol{c}_{l1}^1\boldsymbol{x}_1 - \boldsymbol{c}_{l2}^1\boldsymbol{x}_2 - \alpha_l^1}{\boldsymbol{c}_{l1}^2\boldsymbol{x}_1 + \boldsymbol{c}_{l2}^2\boldsymbol{x}_2 + \alpha_l^2}\right)$$

and distribution functions $T_l(\cdot)$ are assumed to be nondecreasing, problem (13) is equivalently transformed as:

$$\left.\begin{array}{ll}\text{maximize} & v \\ \text{subject to} & \dfrac{h_1 - \boldsymbol{c}_{11}^1\boldsymbol{x}_1 - \boldsymbol{c}_{12}^1\boldsymbol{x}_2 - \alpha_1^1}{\boldsymbol{c}_{11}^2\boldsymbol{x}_1 + \boldsymbol{c}_{12}^2\boldsymbol{x}_2 + \alpha_1^2} \geq T_1^*(\mu_1^*(v)) \\ & \dfrac{h_2 - \boldsymbol{c}_{21}^1\boldsymbol{x}_1 - \boldsymbol{c}_{22}^1\boldsymbol{x}_2 - \alpha_2^1}{\boldsymbol{c}_{21}^2\boldsymbol{x}_1 + \boldsymbol{c}_{22}^2\boldsymbol{x}_2 + \alpha_2^2} \geq T_2^*(\mu_2^*(v)) \\ & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \leq \hat{\boldsymbol{b}} \\ & \boldsymbol{x}_1 \geq \boldsymbol{0},\ \ \boldsymbol{x}_2 \geq \boldsymbol{0}\end{array}\right\}, \tag{14}$$

where $T_l^*(\cdot)$ is a pseudo-inverse function of $T_l(\cdot)$ defined by $T_l^*(r) = \inf\{y \mid T_l(y) \geq r\}$.

Obtaining the optimal value of v to (14) is equivalent to finding the maximum of v so that the set of feasible solutions to (14) is not empty. Noting that the constraints of (14) are linear when v is fixed, we can easily find the maximum of v through the combined use of the bisection method and the phase one of the simplex method.

The combined use of the bisection method and the phase one of the simplex method

Step 1: Set $r := 0$ and $v := 0$. Test whether the set of feasible solutions to (14) for $v = 0$ is empty or not using the phase one of the simplex method. Let $v_{\text{feasible}} := v$ and go to step 2.

Step 2: Set $v := 1$. Test whether the set of feasible solutions to (14) for $v = 1$ is empty or not using the phase one of the simplex method. If it is not empty, $v = 1$ is the optimal value v^* to (14) and the algorithm is terminated. Otherwise, the maximum of v so that the set of feasible solutions to (14) is not empty exists between 0 and 1. Let $v_{\text{infeasible}} := v$ and go to step 3.

Step 3: Set $v := (v_{\text{feasible}} + v_{\text{infeasible}})/2$, $r := r + 1$ and go to step 4.

Step 4: Test whether the set of feasible solutions to (14) for v determined in step 3 is empty or not using the phase one of the simplex method. It should be noted that we can use the sensitivity analysis technique when we carry out the above test. If it is not empty and $(1/2)^r \leq \varepsilon$, the current value of v is regarded as the optimal value v^* to (14) and the algorithm is terminated. If it is not empty and $(1/2)^r > \varepsilon$, let $v_{\text{feasible}} := v$ and go to step 3. On the other hand, if it is empty, let $v_{\text{infeasible}} := v$ and go to step 3.

For the optimal value v^* obtained in this way, we can determine the corresponding optimal solution $\boldsymbol{x}^*$ by solving the following linear programming problem.

$$\left.\begin{array}{ll}\text{maximize} & \dfrac{h_1 - \boldsymbol{c}_{11}^1\boldsymbol{x}_1 - \boldsymbol{c}_{12}^1\boldsymbol{x}_2 - \alpha_1^1}{\boldsymbol{c}_{11}^2\boldsymbol{x}_1 + \boldsymbol{c}_{12}^2\boldsymbol{x}_2 + \alpha_1^2} \\ \text{subject to} & \dfrac{h_2 - \boldsymbol{c}_{21}^1\boldsymbol{x}_1 - \boldsymbol{c}_{22}^1\boldsymbol{x}_2 - \alpha_2^1}{\boldsymbol{c}_{21}^2\boldsymbol{x}_1 + \boldsymbol{c}_{22}^2\boldsymbol{x}_2 + \alpha_2^2} \geq T_2^*(\mu_2^*(v^*)) \\ & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \leq \hat{\boldsymbol{b}} \\ & \boldsymbol{x}_1 \geq \boldsymbol{0},\ \ \boldsymbol{x}_2 \geq \boldsymbol{0}\end{array}\right\} \tag{15}$$

Letting $\tau = T_2^*(\mu_2^*(v^*))$ and using the variable transformation method by Charnes and Cooper [6], problem (15) can be transformed into the following linear programming problem:

$$\left.\begin{array}{ll} \text{maximize} & -\boldsymbol{c}_{11}^1\boldsymbol{y}_1 - \boldsymbol{c}_{12}^1\boldsymbol{y}_2 - (\alpha_1^1 - h_1)\cdot s \\ \text{subject to} & \tau\cdot(\boldsymbol{c}_{21}^2\boldsymbol{y}_1 + \boldsymbol{c}_{22}^2\boldsymbol{y}_2 + \alpha_2^2\cdot s) \\ & \qquad +\boldsymbol{c}_{21}^1\boldsymbol{y}_1 + \boldsymbol{c}_{22}^1\boldsymbol{y}_2 + (\alpha_2^1 - h_2)\cdot s \le 0 \\ & A_1\boldsymbol{y}_1 + A_2\boldsymbol{y}_2 - \hat{\boldsymbol{b}}\cdot s \le \boldsymbol{0} \\ & \boldsymbol{c}_{11}^2\boldsymbol{y}_1 + \boldsymbol{c}_{12}^2\boldsymbol{y}_2 + \alpha_1^2\cdot s = 1 \\ & \boldsymbol{y}_1 \ge \boldsymbol{0},\ \ \boldsymbol{y}_2 \ge \boldsymbol{0},\ \ s > 0 \end{array}\right\}. \tag{16}$$

From the optimal solution $(\boldsymbol{y}_1^*, \boldsymbol{y}_2^*, s^*)$ to (16), we can obtain the optimal solution $(\boldsymbol{x}_1^*, \boldsymbol{x}_2^*)$ to (11) which maximizes the smaller satisfactory degree between those of both DMs.

If DM1 is satisfied with the optimal solution $(\boldsymbol{x}_1^*, \boldsymbol{x}_2^*)$ to (11), it follows that the optimal solution $(\boldsymbol{x}_1^*, \boldsymbol{x}_2^*)$ becomes a satisfactory solution; however, DM1 is not always satisfied with the solution $(\boldsymbol{x}_1^*, \boldsymbol{x}_2^*)$. It is quite natural to assume that DM1 specifies the minimal satisfactory level $\hat{\delta} \in (0,1)$ for the membership function $\mu_1(p_1(\boldsymbol{x}_1, \boldsymbol{x}_2))$ subjectively.

Consequently, if DM1 is not satisfied with the solution $(\boldsymbol{x}_1^*, \boldsymbol{x}_2^*)$ to problem (11), the following problem is formulated:

$$\left.\begin{array}{ll} \text{maximize} & \mu_2(p_2(\boldsymbol{x}_1, \boldsymbol{x}_2)) \\ \text{subject to} & \mu_1(p_1(\boldsymbol{x}_1, \boldsymbol{x}_2)) \ge \hat{\delta} \\ & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \le \hat{\boldsymbol{b}} \\ & \boldsymbol{x}_1 \ge \boldsymbol{0},\ \ \boldsymbol{x}_2 \ge \boldsymbol{0} \end{array}\right\} \tag{17}$$

equivalently,

$$\left.\begin{array}{ll} \text{maximize} & \dfrac{h_2 - \boldsymbol{c}_{21}^1\boldsymbol{x}_1 - \boldsymbol{c}_{22}^1\boldsymbol{x}_2 - \alpha_2^1}{\boldsymbol{c}_{21}^2\boldsymbol{x}_1 + \boldsymbol{c}_{22}^2\boldsymbol{x}_2 + \alpha_2^2} \\ \text{subject to} & \dfrac{h_1 - \boldsymbol{c}_{11}^1\boldsymbol{x}_1 - \boldsymbol{c}_{12}^1\boldsymbol{x}_2 - \alpha_1^1}{\boldsymbol{c}_{11}^2\boldsymbol{x}_1 + \boldsymbol{c}_{12}^2\boldsymbol{x}_2 + \alpha_1^2} \ge T_1^*(\mu_1^*(\hat{\delta})) \\ & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \le \hat{\boldsymbol{b}} \\ & \boldsymbol{x}_1 \ge \boldsymbol{0},\ \ \boldsymbol{x}_2 \ge \boldsymbol{0} \end{array}\right\}. \tag{18}$$

where DM2's membership function $\mu_2(p_2(\boldsymbol{x}_1, \boldsymbol{x}_2))$ is maximized under the condition that DM1's membership function $\mu_1(p_1(\boldsymbol{x}_1, \boldsymbol{x}_2))$ is larger than or equal to the minimal satisfactory level $\hat{\delta}$ specified by DM1.

Using the variable transformation method by Charnes and Cooper [6], problem (18) can be easily reduced to the following linear programming problem:

$$\left.\begin{array}{ll} \text{maximize} & -\boldsymbol{c}_{21}^1\boldsymbol{y}_1 - \boldsymbol{c}_{22}^1\boldsymbol{y}_2 - (\alpha_2^1 - h_2)\cdot s \\ \text{subject to} & \lambda\cdot(\boldsymbol{c}_{11}^2\boldsymbol{y}_1 + \boldsymbol{c}_{12}^2\boldsymbol{y}_2 + \alpha_1^2\cdot s) \\ & \qquad +\boldsymbol{c}_{11}^1\boldsymbol{y}_1 + \boldsymbol{c}_{12}^1\boldsymbol{y}_2 + (\alpha_1^1 - h_1)\cdot s \le 0 \\ & A_1\boldsymbol{y}_1 + A_2\boldsymbol{y}_2 - \hat{\boldsymbol{b}}\cdot s \le \boldsymbol{0} \\ & \boldsymbol{c}_{21}^2\boldsymbol{y}_1 + \boldsymbol{c}_{22}^2\boldsymbol{y}_2 + \alpha_2^2\cdot s = 1 \\ & \boldsymbol{y}_1 \ge \boldsymbol{0},\ \ \boldsymbol{y}_2 \ge \boldsymbol{0},\ \ s > 0 \end{array}\right\} \tag{19}$$

where $\lambda = T_1^*(\mu_1^*(\hat{\delta}))$.

If there exists an optimal solution $(\boldsymbol{x}_1^*, \boldsymbol{x}_2^*)$ to problem (17), it follows that DM1 obtains a satisfactory solution having a satisfactory degree larger than or equal to the minimal satisfactory level specified by DM1's self. However, the larger the minimal satisfactory level $\hat{\delta}$ is assessed, the smaller the DM2's satisfactory degree becomes when the membership functions of DM1 and DM2 conflict with each other. Consequently, a relative difference between the satisfactory degrees of DM1 and DM2 becomes larger, and it follows that the overall satisfactory balance between both DMs is not appropriate.

In order to take account of the overall satisfactory balance between both DMs, DM1 needs to compromise with DM2 on DM1's own minimal satisfactory level. To do so, the following ratio of the satisfactory degree of DM2 to that of DM1 is helpful:

$$\Delta = \frac{\mu_2(p_2(\boldsymbol{x}_1, \boldsymbol{x}_2))}{\mu_1(p_1(\boldsymbol{x}_1, \boldsymbol{x}_2))}$$

which is originally introduced by Lai [11].

DM1 is guaranteed to have a satisfactory degree larger than or equal to the minimal satisfactory level for the fuzzy goal because the corresponding constraint is involved in problem (17). To take into account the overall satisfactory balance between both DMs, DM1 specifies the lower bound $\Delta_{\min}$ and the upper bound $\Delta_{\max}$ of the ratio Δ, and Δ is evaluated by verifying whether or not it is in the interval $[\Delta_{\min}, \Delta_{\max}]$. The condition that the overall satisfactory balance is appropriate is represented by

$$\Delta \in [\Delta_{\min}, \Delta_{\max}].$$

At the iteration k, let $(\boldsymbol{x}_1^k, \boldsymbol{x}_2^k)$, $p_l^k = p_l(\boldsymbol{x}_1^k, \boldsymbol{x}_2^k)$, $\mu_l(p_l^k)$ and $\Delta^k = \mu_2(p_2^k)/\mu_1(p_1^k)$ denote the current solution, DMl's objective function value, DMl's satisfactory degree and the ratio of satisfactory degrees of the two DMs, respectively. The interactive process terminates if the following two conditions are satisfied and DM1 concludes the solution as an overall satisfactory solution.

[Termination conditions of the interactive process]

Condition 1 DM1's satisfactory degree is larger than or equal to the minimal satisfactory level $\hat{\delta}$ specified by DM1's self, i.e., $\mu_1(p_1^k) \geq \hat{\delta}$.

Condition 2 The ratio Δ^k of satisfactory degrees lies in the closed interval between the lower and the upper bounds specified by DM1, i.e., $\Delta^k \in [\Delta_{\min}, \Delta_{\max}]$.

Condition 1 ensures the minimal satisfaction to DM1 in the sense of the attainment of the fuzzy goal, and condition 2 is provided in order to keep overall satisfactory balance between both DMs. If these two conditions are not satisfied simultaneously, DM1 needs to update the minimal satisfactory level $\hat{\delta}$. The updating procedures are summarized as follows.

[Procedure for updating the minimal satisfactory level $\hat{\delta}$]

Case 1 If condition 1 is not satisfied, then DM1 decreases the minimal satisfactory level $\hat{\delta}$.

Case 2 If the ratio Δ^k exceeds its upper bound, then DM1 increases the minimal satisfactory level $\hat{\delta}$. Conversely, if the ratio Δ^k is below its lower bound, then DM1 decreases the minimal satisfactory level $\hat{\delta}$.

Case 3 Although conditions 1 and 2 are satisfied, if DM1 is not satisfied with the obtained solution and judges that it is desirable to increase the satisfactory degree of DM1 at the expense of the satisfactory degree of DM2, then DM1 increases the minimal satisfactory level $\hat{\delta}$. Conversely, if DM1 judges that it is desirable to increase the satisfactory degree of DM2 at the expense of the satisfactory degree of DM1, then DM1 decreases the minimal satisfactory level $\hat{\delta}$.

In particular, if condition 1 is not satisfied, there does not exist any feasible solution for problem (17), and therefore DM1 has to moderate the minimal satisfactory level.

Now we are ready to propose interactive fuzzy programming for deriving a satisfactory solution by updating the satisfactory degree of the DM at the upper level with considerations of overall satisfactory balance among all the levels.

Computational procedure of interactive fuzzy programming

Step 1: Ask the DM at the upper level, DM1, to subjectively determine satisficing levels $\beta_i \in (0, 1)$, $i = 1, 2, \ldots, m$ for constraints in (2). Go to step 2.

Step 2: In order to determine permissible levels h_l, $l = 1, 2$, the following problems are solved to find the minimum and maximum of $\mathrm{E}\{\bar{z}_l(\boldsymbol{x}_1, \boldsymbol{x}_2)\} = (\boldsymbol{c}^1_{l1} + M_l \boldsymbol{c}^2_{l1})\boldsymbol{x}_1 + (\boldsymbol{c}^1_{l2} + M_l \boldsymbol{c}^2_{l2})\boldsymbol{x}_2 + (\alpha^1_l + M_l \alpha^2_l)$ for each objective function under the chance constraints with satisficing levels β_i, $i = 1, 2, \ldots, m$.

$$\left.\begin{array}{ll} \text{minimize} & (\boldsymbol{c}^1_{l1} + M_l \boldsymbol{c}^2_{l1})\boldsymbol{x}_1 + (\boldsymbol{c}^1_{l2} + M_l \boldsymbol{c}^2_{l2})\boldsymbol{x}_2 + (\alpha^1_l + M_l \alpha^2_l) \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \leq \hat{\boldsymbol{b}} \\ & \boldsymbol{x}_1 \geq \boldsymbol{0},\ \ \boldsymbol{x}_2 \geq \boldsymbol{0} \end{array}\right\} \tag{20}$$

$$\left.\begin{array}{ll} \text{maximize} & (\boldsymbol{c}^1_{l1} + M_l \boldsymbol{c}^2_{l1})\boldsymbol{x}_1 + (\boldsymbol{c}^1_{l2} + M_l \boldsymbol{c}^2_{l2})\boldsymbol{x}_2 + (\alpha^1_l + M_l \alpha^2_l) \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \leq \hat{\boldsymbol{b}} \\ & \boldsymbol{x}_1 \geq \boldsymbol{0},\ \ \boldsymbol{x}_2 \geq \boldsymbol{0} \end{array}\right\} \tag{21}$$

If the set of feasible solutions to these problems is empty, the satisficing levels β_i, $i = 1, 2, \ldots, m$ must be reassessed and return to step 1. Otherwise, let $z^E_{l,\min}$ and $z^E_{l,\max}$ be optimal objective function values to (20) and (21). Since (20) and (21) are linear programming problems, they can be easily solved by the simplex method. Ask DM1 to determine permissible levels h_l, $l = 1, 2$ for objective functions in consideration of $z^E_{l,\min}$ and $z^E_{l,\max}$. Go to step 3.

Step 3: Solve (8) for obtaining optimal solutions $(\boldsymbol{x}^l_{1,\max}, \boldsymbol{x}^l_{2,\max})$, $l = 1, 2$ and calculate $p_{l,\max}$. Then, identify the linear membership function $\mu_l(p_l(\boldsymbol{x}_1, \boldsymbol{x}_2))$ of the fuzzy goal for the corresponding objective function. Go to step 4.

Step 4: Set $k := 1$. Solve the maximin problem (11) for obtaining an optimal solution which maximizes the smaller degree of satisfaction between those of the two DMs. For the optimal solution $(\boldsymbol{x}^k_1, \boldsymbol{x}^k_2)$ to (11), calculate $p^k_l = p_l(\boldsymbol{x}^k_1, \boldsymbol{x}^k_2)$, $\mu_l(p^k_l)$, $l = 1, 2$

and $\Delta^k = \mu_2(p_2^k)/\mu_1(p_1^k)$. If DM1 is satisfied with the optimal solution to (11), the optimal solution becomes a satisfactory solution and the interaction procedure is terminated. Otherwise, ask DM1 to subjectively set the minimal satisfactory level $\hat{\delta} \in (0, 1)$ for the membership function $\mu_1(p_1(\boldsymbol{x}_1, \boldsymbol{x}_2))$. Furthermore, ask DM1 to set the upper bound $\Delta_{\max}$ and the lower bound $\Delta_{\min}$ for Δ. Go to step 5.

Step 5: Set $k := k + 1$. Solve problem (17) for finding an optimal solution to maximize DM2's membership function $\mu_2(p_2(\boldsymbol{x}_1, \boldsymbol{x}_2))$ under the condition that DM1's membership function $\mu_1(p_1(\boldsymbol{x}_1, \boldsymbol{x}_2))$ is larger than or equal to the minimal satisfactory level $\hat{\delta}$. For the optimal solution $(\boldsymbol{x}_1^k, \boldsymbol{x}_2^k)$ to (17), calculate $p_l^k = p_l(\boldsymbol{x}_1^k, \boldsymbol{x}_2^k)$, $\mu_l(p_l^k)$, $l = 1, 2$. and $\Delta^k = \mu_2(p_2^k)/\mu_1(p_1^k)$ and go to step 6.

Step 6: If the current solution $(\boldsymbol{x}_1^k, \boldsymbol{x}_2^k)$ satisfies the termination conditions and DM1 accepts it, then the procedure stops and the current solution becomes a satisfactory solution. Otherwise, ask DM1 to update the minimal satisfactory level $\hat{\delta}$, and go to step 5.

It should be noted that all problems (8), (11), (17), (20) and (21) in the interactive fuzzy programming algorithm can be solved by either the simplex method of linear programming or the combined use of the bisection method and the phase one of the simplex method.

4 Numerical Example

To demonstrate the feasibility and efficiency of the proposed method, consider the stochastic two-level linear programming problem formulated as:

$$\left.\begin{array}{ll}\underset{\text{for DM1}}{\text{minimize}} & \bar{z}_1(\boldsymbol{x}_1, \boldsymbol{x}_2) = (\boldsymbol{c}_{11}^1 + \bar{t}_1 \boldsymbol{c}_{11}^2)\boldsymbol{x}_1 + (\boldsymbol{c}_{12}^1 + \bar{t}_1 \boldsymbol{c}_{12}^2)\boldsymbol{x}_2 + (\alpha_1^1 + \bar{t}_1 \alpha_1^2) \\ \underset{\text{for DM2}}{\text{minimize}} & \bar{z}_2(\boldsymbol{x}_1, \boldsymbol{x}_2) = (\boldsymbol{c}_{21}^1 + \bar{t}_2 \boldsymbol{c}_{21}^2)\boldsymbol{x}_1 + (\boldsymbol{c}_{22}^1 + \bar{t}_2 \boldsymbol{c}_{22}^2)\boldsymbol{x}_2 + (\alpha_2^1 + \bar{t}_2 \alpha_2^2) \\ \text{subject to} & \boldsymbol{a}_{i1}\boldsymbol{x}_1 + \boldsymbol{a}_{i2}\boldsymbol{x}_2 \leq \bar{b}_i, \;\; i = 1, 2, \ldots, 7 \\ & \boldsymbol{x}_1 = (x_{11}, x_{12}, x_{13}, x_{14}, x_{15})^T \geq \boldsymbol{0} \\ & \boldsymbol{x}_2 = (x_{21}, x_{22}, x_{23}, x_{24}, x_{25})^T \geq \boldsymbol{0}\end{array}\right\} \quad (22)$$

where $\bar{t}_1$ and $\bar{t}_2$ are Gaussian random variables $N(4, 2^2)$ and $N(3, 3^2)$, and right side coefficients $\bar{b}_i$, $i = 1, 2, \ldots, 7$ are also Gaussian random variables $N(164, 30^2)$, $N(-190, 20^2)$, $N(-184, 15^2)$, $N(99, 22^2)$, $N(-150, 17^2)$, $N(154, 35^2)$, $N(142, 42^2)$. Here $N(p, q^2)$ stands for a Gaussian random variable with mean p and variance q^2. Coefficient values of objective functions and constraints are respectively shown in Table 1 and 2.

In step 1 of the interactive fuzzy programming, DM1 specifies satisficing levels β_i, $i = 1, 2, \ldots, 7$ as:

$$(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7)^T = (0.85, 0.95, 0.80, 0.90, 0.85, 0.80, 0.90)^T.$$

For the specified satisficing levels β_i, $i = 1, 2, \ldots, 7$, in step 2, minimal values $z_{l,\min}^E$ and maximal values $z_{l,\max}^E$ of objective functions $\mathrm{E}\{\bar{z}_l(\boldsymbol{x}_1, \boldsymbol{x}_2)\}$ under the chance constraints are calculated as $z_{1,\min}^E = 1819.513$, $z_{2,\min}^E = 286.583$, $z_{l,\max}^E = 2307.626$ and $z_{2,\max}^E = 758.279$. By considering these values, the DMs subjectively specifies permissible levels as $h_1 = 2150.0$ and $h_2 = 450.0$.

Table 1: Coefficient values of objective functions

$(\boldsymbol{c}_{11}^1, \boldsymbol{c}_{12}^1)$	19	48	21	10	18	35	46	11	24	33	α_1^1	−18
$(\boldsymbol{c}_{11}^2, \boldsymbol{c}_{12}^2)$	3	2	2	1	4	3	1	2	4	2	α_1^2	5
$(\boldsymbol{c}_{21}^1, \boldsymbol{c}_{22}^1)$	12	−46	−23	−38	−33	−48	12	8	19	20	α_2^1	−27
$(\boldsymbol{c}_{21}^2, \boldsymbol{c}_{22}^2)$	1	2	4	2	2	1	2	1	2	1	α_2^2	6

Table 2: Coefficient values of constraints

$(\boldsymbol{a}_{11}, \boldsymbol{a}_{12})$	12	−2	4	−7	13	−1	−6	6	11	−8
$(\boldsymbol{a}_{21}, \boldsymbol{a}_{22})$	−2	5	3	16	6	−12	12	4	−7	−10
$(\boldsymbol{a}_{31}, \boldsymbol{a}_{32})$	3	−16	−4	−8	−8	2	−12	−12	4	−3
$(\boldsymbol{a}_{41}, \boldsymbol{a}_{42})$	−11	6	−5	9	−1	8	−4	6	−9	6
$(\boldsymbol{a}_{51}, \boldsymbol{a}_{52})$	−4	7	−6	−5	13	6	−2	−5	14	−6
$(\boldsymbol{a}_{61}, \boldsymbol{a}_{62})$	5	−3	14	−3	−9	−7	4	−4	−5	9
$(\boldsymbol{a}_{71}, \boldsymbol{a}_{72})$	−3	−4	−6	9	6	18	11	−9	−4	7

In step 3, maximal values $p_{l,\max}$ of $p_l(\boldsymbol{x}_1, \boldsymbol{x}_2)$ are calculated as:

$$p_{1,\max} = p_1(\boldsymbol{x}_{1,\max}^1, \boldsymbol{x}_{2,\max}^1) = 0.880, \quad p_{2,\min} = p_2(\boldsymbol{x}_{1,\max}^2, \boldsymbol{x}_{2,\max}^2) = 0.783.$$

Assume that the DMs identify the linear membership function (7) whose parameter values are determined by the Zimmermann method [47]. Then, the parameter values $p_{l,1}$ and $p_{l,0}$, $l = 1, 2$ characterizing membership functions $\mu_l(\cdot)$ are becomes:

$$\begin{aligned} p_{1,1} = &\ p_1(\boldsymbol{x}_{1,\max}^1, \boldsymbol{x}_{2,\max}^1) &= 0.880, \\ p_{1,0} = &\ p_1(\boldsymbol{x}_{1,\max}^2, \boldsymbol{x}_{2,\max}^2) &= 0.598, \\ p_{2,1} = &\ p_2(\boldsymbol{x}_{1,\max}^2, \boldsymbol{x}_{2,\max}^2) &= 0.783, \\ p_{2,0} = &\ p_2(\boldsymbol{x}_{1,\max}^1, \boldsymbol{x}_{2,\max}^1) &= 0.060. \end{aligned}$$

In step 4, let $k := 1$ and the maximin problem is solved. The obtained result is shown at the column labeled "1st" in table 3. For the obtained optimal solution $(\boldsymbol{x}_1^1, \boldsymbol{x}_2^1)$ to the maximin problem, corresponding membership function values are calculated as $\mu_1(p_1(\boldsymbol{x}_1^1, \boldsymbol{x}_2^1)) = 0.551$ and $\mu_2(p_2(\boldsymbol{x}_1^1, \boldsymbol{x}_2^1)) = 0.551$. Then, the ratio of satisfactory degrees Δ^1 is equal to 1.000. Since DM1 is not satisfied with this solution, DM1 sets the minimal satisfactory level $\hat{\delta} \in (0, 1)$ for $\mu_1(p_1(\boldsymbol{x}_1, \boldsymbol{x}_2))$ to 0.600 so that $\mu_1(p_1(\boldsymbol{x}_1, \boldsymbol{x}_2))$ will be improved from its current value 0.551. Furthermore, the upper bound and the lower bound of the ratio of satisfactory degrees Δ are set as $\Delta_{\max} = 0.700$ and $\Delta_{\min} = 0.600$.

In step 5, let $k := 2$ and (17) for $\hat{\delta} = 0.600$ is solved. For the obtained optimal solution $(\boldsymbol{x}_1^2, \boldsymbol{x}_2^2)$ to (17), $\mu_1(p_1(\boldsymbol{x}_1^2, \boldsymbol{x}_2^2)) = 0.600$, $\mu_2(p_2(\boldsymbol{x}_1^2, \boldsymbol{x}_2^2)) = 0.478$. and $\Delta^2 = 0.797$, shown at the column labeled "2nd" in table 3.

In step 6, DM1 is asked whether he is satisfied with the obtained solution. Since the ratio of satisfactory degrees Δ^2 exceeds $\Delta_{\max} = 0.700$, the second condition of termination of the interactive process is not fulfilled. Suppose that DM1 feels that $\mu_1(p_1(\boldsymbol{x}_1, \boldsymbol{x}_2))$

Table 3: Interaction process

Interaction	1st	2nd	3rd	4th
$\hat{\delta}$		0.600	0.700	0.650
x_{11}^k	15.368	15.066	14.423	14.749
x_{12}^k	2.162	1.960	1.532	1.750
x_{13}^k	0.000	0.000	0.000	0.000
x_{14}^k	0.000	0.000	0.000	0.000
x_{15}^k	0.000	0.000	0.000	0.000
x_{21}^k	6.033	5.784	5.255	5.524
x_{22}^k	0.118	0.108	0.086	0.097
x_{23}^k	14.276	14.489	14.953	14.707
x_{24}^k	1.516	1.775	2.325	2.046
x_{25}^k	17.848	17.997	18.315	18.153
$p_1(\boldsymbol{x}_1^k, \boldsymbol{x}_2^k)$	0.734	0.767	0.796	0.781
$p_2(\boldsymbol{x}_1^k, \boldsymbol{x}_2^k)$	0.458	0.406	0.301	0.353
$\mu_1(p_1(\boldsymbol{x}_1^k, \boldsymbol{x}_2^k))$	0.551	0.600	0.700	0.650
$\mu_2(p_2(\boldsymbol{x}_1^k, \boldsymbol{x}_2^k))$	0.551	0.478	0.333	0.405
Δ^k	1.000	0.797	0.475	0.623

should be considerably better than $\mu_2(p_2(\boldsymbol{x}_1, \boldsymbol{x}_2))$, and DM1 updates the minimal satisfactory level $\hat{\delta}$ from 0.600 to 0.700 in order to improve $\mu_1(p_1(\boldsymbol{x}_1, \boldsymbol{x}_2))$. Consequently, in step 5, let $k := 3$ and (17) for $\hat{\delta} = 0.700$ is solved. The obtained result is shown at the column labeled "3rd" in table 3. For the obtained optimal solution $(\boldsymbol{x}_1^3, \boldsymbol{x}_2^3)$ to (17), $\mu_1(p_1(\boldsymbol{x}_1^3, \boldsymbol{x}_2^3)) = 0.700$, $\mu_2(p_2(\boldsymbol{x}_1^3, \boldsymbol{x}_2^3)) = 0.333$ and $\Delta^3 = 0.475$.

In step 6, since the ratio of satisfactory degrees Δ^3 is less than $\Delta_{\min} = 0.600$, the second condition of termination of the interactive process is not fulfilled. Hence, he updates the minimal satisfactory level $\hat{\delta}$ from 0.700 to 0.650 for improving $\mu_2(p_2(\boldsymbol{x}_1, \boldsymbol{x}_2))$ at the sacrifice of $\mu_1(p_1(\boldsymbol{x}_1, \boldsymbol{x}_2))$. As a result, in step 5, let $k := 4$ and (17) for $\hat{\delta} = 0.650$ is solved. For the obtained optimal solution $(\boldsymbol{x}_1^4, \boldsymbol{x}_2^4)$ to (17), corresponding membership function values are calculated as $\mu_1(p_1(\boldsymbol{x}_1^4, \boldsymbol{x}_2^4)) = 0.650$ and $\mu_2(p_2(\boldsymbol{x}_1^4, \boldsymbol{x}_2^4)) = 0.405$ as shown at the column labeled "4th" in table 3. Then, the ratio of satisfactory degrees Δ^4 is equal to 0.623.

In step 6, since the current solution satisfies all termination conditions of the interactive process and DM1 is satisfied with the current solution, the satisfactory solution is obtained and the interaction procedure is terminated.

5 Conclusions

In this paper, we focused on stochastic two-level linear programming problems with random variable coefficients in both objective functions and constraints. Through the use of the probability maximization model in chance constrained programming, the stochastic two-level linear programming problems are transformed into deterministic linear programming ones under some appropriate assumptions for distribution functions. Taking into account vagueness of judgments of the DMs, interactive fuzzy programming has been proposed. In the proposed interactive method, after determining the fuzzy goals of the DMs at both levels, a satisfactory solution is derived efficiently by updating the satisfactory degree of the DM at the upper level with considerations of overall satisfactory balance among both levels. It is significant to note here that the transformed deterministic problems to derive an overall satisfactory solution can be easily solved through the combined use of the bisection method and the phase one of the simplex method. An illustrative numerical example was provided to demonstrate the feasibility of the proposed method. Extensions to other stochastic programming models will be considered elsewhere. Also extensions to two-level linear programming problems involving fuzzy random variable coefficients will be required in the near future.

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