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Significance of Life Table Estimates for Small Populations: Simulation-Based Study of Estimation Errors

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Contents

Abstract

We study bias, standard errors and distributions of characteristics of life tables for small populations. Theoretical considerations and simulations show that statistical efficiency of different methods is, above all, affected by the population size. Yet it is also significantly affected by the life table construction method and by a population's age composition. Study results are presented in the form of ready-to-use tables and relations, which may be useful in assessing the significance of estimates and differences in life expectancy across time and space for the territories with a small population size, when standard errors of life expectancy estimates may be high.

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1 Introduction: Data and Methods

Life expectancy is a key characteristic of human longevity and development; world-wide policies aim to increase it. Effective policies may be based on informative monitoring systems. This sets a high priority for estimating and comparing life expectancy for small populations.

Using the Monte-Carlo simulation approach, Silcocks et al. (2001), Toson et al. (2003), and Eayres and Williams (2004) have evaluated methodologies for small-area life expectancy estimation in the UK context. They showed that expectancy at birth is distributed normally, while estimates of its standard error are shown to be distributed with a significant skew for the small population size. They also demonstrated that traditional life table methodology without special corrections for age bands with zero deaths in a small population performs rather well, and that the choice of the minimal age of the open age interval and modeling the mortality in that interval are important for estimating life expectancy and its standard error. Based on simulated dependency of standard errors on population size, a minimum population years-at-risk size of 5,000 for estimating life expectancy at birth was recommend in the UK context.

Our work extends this previous research in several directions. First, we confirm some of the findings in the literature in a wider context of mortality schedules and population structures. We conduct simulations based on all available male and female life tables for Austria, Italy, Japan, Russia, Spain, Sweden and the UK. We use data from the *Human Mortality Database* (2010). For each life table scrutinized, we consider five stable population age compositions corresponding to -2 percent, -1 percent, 0 percent, 1 percent and 2 percent annual population growth rates. Based on those mortality and population schedules, we consider six population sizes of 1,000, 5,000, 10,000, 25,000, 50,000, 100,000, 250,000 and 1 million persons (altogether, 47,680 populations).

Second, we present empirical relations between the estimation error of life expectancy indicators and the corresponding life table and population characteristics (life expectancy, life table standard deviation of age at death, and population growth rate). Third, we evaluate estimation errors for both the life expectancy at birth and for life expectancy at age 60, the measurement of which is demanded in the context of policies oriented on population ageing and pension systems. Fourth, we provide more in-depth analysis of the normality of life expectancy estimates for small populations, and illustrate that age composition may play a crucial role affecting the normality of estimates (which is important for establishing confidence limits and significance of the observed variation for the life expectancy).

Unlike the previous works, we consider indicators of unabridged life tables. We have also studied the estimates for abridged life table calculations based on the age groups 0, 1, 5, 10, ..., 85+ years. However, both the previous works and our own study (not reported here) indicate that usage of abridged life tables as opposed to unabridged ones has only a small effect on estimation accuracy as compared to the choice of procedure for the open age interval. At the same time, we found that the estimates for abridged life tables tend to be systematically biased when age composition deviates from the stationary population, irrespective of the population size (these distortions are caused by a deviation from the stationary age composition within individual age intervals). Therefore, it might be recommended to avoid using abridged life tables unless the population age composition is fairly close to stationary.

In the three works cited above, the open age interval was chosen to start at 85, 90, or 95 years; in the event that no deaths occur in the open age interval, the corresponding mortality rate was taken from a known life table and not from the simulated population. We use a different approach, adjusting the open age interval in such a way that there is at least one death in it and, hence, we do not use (unavailable in practice) rates from the theoretical life table in order to conclude the life table for the simulated population.

In this paper we describe how the simulations are designed; then we continue with a discussion of life table calculations, followed by the presentation of results. We conclude by presenting illustrative case studies and general recommendations. The paper is supplemented by an Appendix with table material.

2 Simulations Design

Simulations are carried out in a multi-step procedure. For each of the life tables used, for each population growth rate (-2 percent, -1 percent, 0 percent, 1 percent, 2 percent per annum), and for each of the studied population sizes, we generate a stable population with respective characteristics (the population is generated in single-year age groups and concluded by open age intervals starting at age 100). All generated populations consist of integer numbers of people at each age. In the event that rounded population numbers at individual age groups do not sum up exactly to the desired population size, we randomly add or withdraw people from the population (with probabilities determined by the stable population age structure). Once the stable population of a given size is prepared, we run 25,000 simulations of the number of deaths by age group imputing the mortality rates from the life table. At each age group, the number of deaths is generated according to the binomial distribution (we use R package version 2.11.1). Once death counts are generated, we compute a life table for the simulated situation and store the results for life expectancy and other life table characteristics.

The number of simulations mentioned above (25,000) is considerably higher than those used in our above-cited literature (2,000 by Silcocks et al. and Toson et al.; 10,000 by Eares and Williams). Such a high number was chosen in order to reduce statistical errors of the simulations' outcome to an acceptable minimum as described next. The standard error of normal sample standard deviation S is given as $\sigma_S \approx \sigma \frac{1}{\sqrt{2(n-1)}}$, where σ is unknown standard deviation estimated by S, and n is the sample size (Ahn and Fessler 2003). At $n = 2000$, the standard error amounts to about 1.6 percent of the standard deviation, which, being relatively small, may nonetheless considerably affect the outcome of the estimation (especially in view of necessity to study normality of the estimates and their confidence limits). We increased the number of simulations to 25,000 so that the relative standard error of the standard deviation falls below 0.5 percent.

3 Life Table Procedures

Small population size creates specific problems when constructing a life table in the usual way. In particular, the absence of deaths at certain age intervals brings the death rate to zero and may distort the life table. Comparing life tables computed with zero death rates for such age groups or, alternatively, with artificially-imputed small death rates indicates that the former, more naive method performs better (Toson et al. 2003; Eares and Williams 2004) (we also came to a similar conclusion based on simulations in the case of Russia, not presented here). This does not apply, however, to the open age interval, where applying zero mortality would result in assuming immortality with profound implications for life expectancy estimates. Toson et al. (2003) and Eares and Williams (2004) proposed to impute an externally determined mortality (e.g., from the national life table) for the open age interval with no deaths observed. We have examined this method and, indeed, extra knowledge about mortality at open age intervals improves the life expectancy estimates considerably. However, in many practical cases there could be no basis to assume that old-age mortality in a certain small population will be exactly the same as observed elsewhere or nation-wide. Often, the very purpose of estimating life expectancy for small areas would be to reveal the differences; for this purpose imputing standard mortality at open age intervals may not be sufficient. Therefore, we present here an alternative approach, where the boundary of the open age interval is lowered to such a level (from the original level of 110 years) that it comprises at least one observation of death. As rough as it may be, this method performed better in our simulations than alternatives with a minimum of 2, 3, …, 7 observations in the open age interval (we do not present the results for those alternatives here). Except for very small and growing populations, standard deviations of life expectancy estimates produced by this method were comparable to standard deviations of estimates produced by imputing the theoretical mortality from the original life table for the open age interval. For a stationary population of 1,000 people, the former standard deviation is about 20 percent higher than the latter; for 2,000 people it is 5 percent higher; 2 percent for 5,000 people; and 1 percent for a stationary population of 25,000 people.

4 Results

4.1 General overview

In this section, we outline the general variation of estimation biases and standard errors according to population size, stable growth rates, and mortality levels. A more detailed analysis of the factors of estimation errors follows in the sections below.

Although, there are distinguishable differences in the results for males and females, the differences are by far smaller as compared to the estimation errors themselves. Therefore, we pool all results together, irrespective of the gender of the population.

In Appendix Tables A1 to A5, we present simulation results for populations with different growth rates. Each table is split into two parts, corresponding to lower and higher mortality (with life expectancy at birth exceeding and falling below 55 years, respectively). Our results confirm that estimates of life expectancy at birth become more and more biased (upwards) as population size decreases. As indicated by the columns denoted by bias_e0 and bias_e60 in Appendix Tables A1 to A5, there are notable upward biases in life expectancy estimates for all population sizes up to 10,000 people inclusively. For stationary and shrinking populations, the biases amounted to about one year for populations as small as 1,000 people, and 0.2 years for populations of 5,000 people. For growing populations, these estimates must be doubled.

The biases were significantly smaller as compared to the standard errors of the life expectancy estimates, which are presented in the Appendix Tables in several alternative ways. The fourth and fifth columns contain the standard errors SD_0 and SD_{60}, while the subsequent columns contain the standard errors rescaled to a hypothetical population of 1,000 people in total:

$$s_0 = SD_0 \sqrt{N/1000} \tag{1}$$

$$s'_{60} = SD_{60} \sqrt{N/1000} \tag{2}$$

and in the case of life expectancy at 60, to a population with 1,000 people at age 60 and above:

$$s_{60} = SD_{60} \sqrt{N_{60+}/1000} \tag{3}$$

(N and N_{60+} are the total population size and population at age 60 and above).

Theoretically, if there were no problems associated with zero death counts at some age groups, especially at the open age interval, the rescaled standard errors should be similar at all population sizes, because the standard errors would be reversed square-root functions of population size. However, with extremely small population sizes, the problem of the ‘zero count’ persists and pushes both the biases and the standard errors of the estimates upwards. This problem is particularly strong for growing populations.

4.2 Detailed view: Biases

Simulated associations between estimation biases and standard deviations (the determinants of the latter will be considered in the next section) for different population sizes and age compositions are presented in Figure 1. For population sizes exceeding 10,000, estimation biases may be neglected. For smaller populations, the bias is considerably higher and strongly depends on population and mortality age patterns. For populations as small as 1,000 people, it may vary between 0.5 and 3.5 years depending primarily on the age composition. Given the strong dependency of the bias on age structure, we recommend considering individual corrections in each specific case depending on the actual age composition of the population.

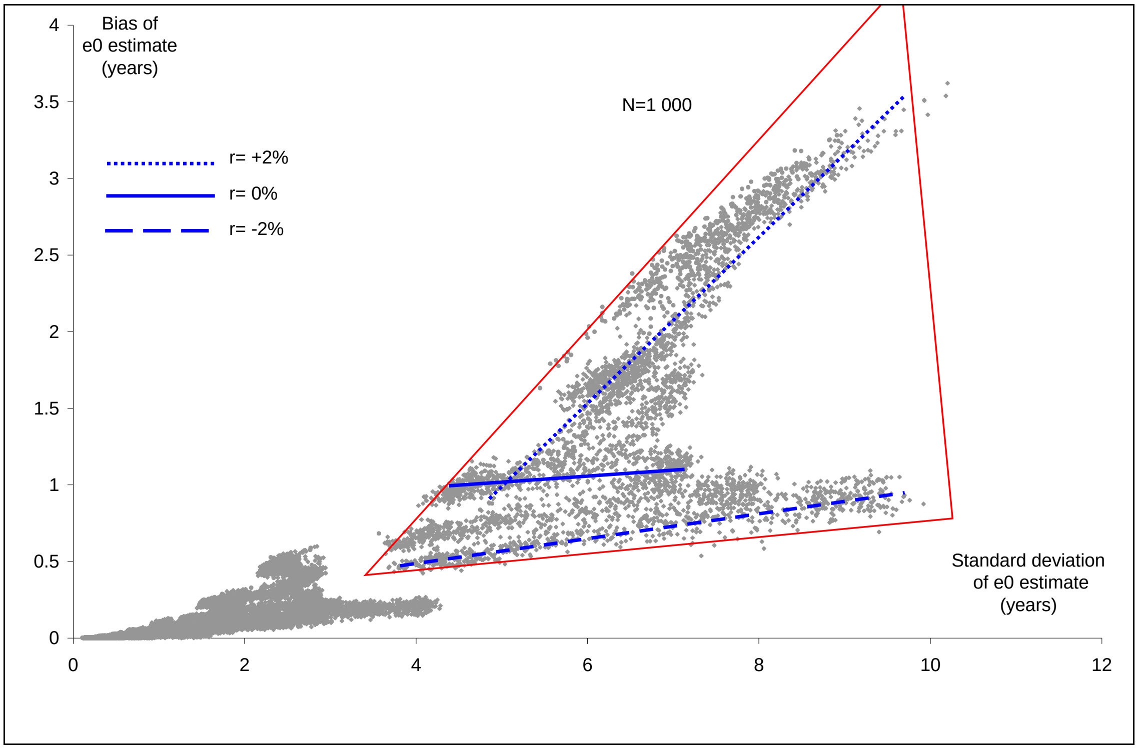

Figure 1. Estimation biases of life expectancy at birth for selected populations as a function of the standard deviation of the estimate, stable population size, and growth rate. Note: Each dot in the chart represents the average of 25,000 simulations. The results corresponding to population sizes of 1,000 people are within the triangle; the lines within the triangle mark the results corresponding to three selected stable population age structures (growth rates -2 percent, 0 percent, and +2 percent per annum).

For the sample set of stable populations examined, we found the following regression relation which may be used to assess the estimation bias for life expectancy at birth:

$$Bias_0 = 0.10 \cdot SD_0 + 0.015 \cdot SD_0{}^2 + 0.050 \cdot SD_0 \cdot r + err \qquad (4)$$

where $\sigma_{err} = 0.1$ years (r is the stable population growth rate in percentage per annum: $r = 1$ for 1 percent growth rate, etc.). For large populations, we recommend using the averages presented in the Appendix Tables.

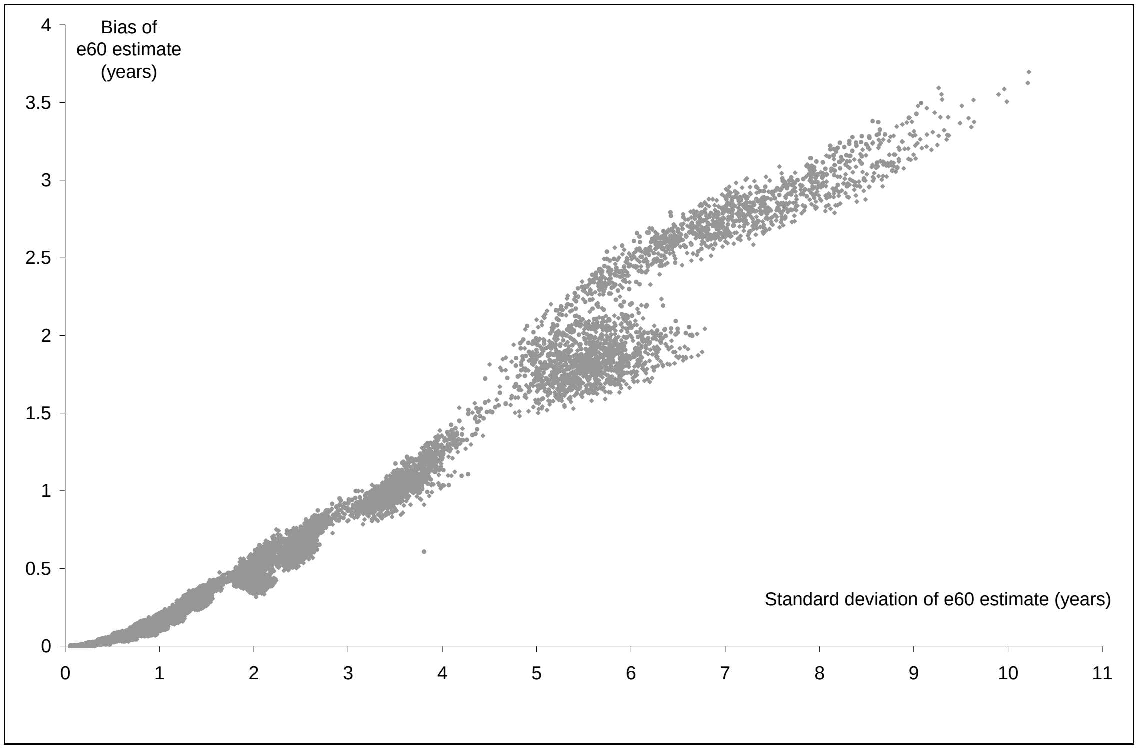

Figure 2. Estimation biases (in years) of life expectancy at 60 for selected populations as a function of the standard deviation of estimates. Note: Each dot in the chart represents the average of 25,000 simulations. The outlier below the main set of dots corresponds to an exceptional case, where the 'zero count' problem was especially severe at the open age interval (Italy, males, 1918, population size 1,000, growth rate 2 percent per annum).

Estimates of life expectancy at 60 for small populations are also considerably biased. Unlike the case of e_0, the estimation bias of e_{60} is well correlated with the standard deviation of the estimation, yet other factors (e.g., the mortality level) play an important role (see Figure 2). For small populations where the estimates from the Appendix Tables are not accurate enough, the following regression relation may be used:

$$Bias_{60} = 0.366 \cdot SD_{60} + 0.0265 \cdot SD_{60}^{\ 2} - 0.0094 \cdot SD_{60} \cdot r + err \qquad (5)$$

with $\sigma_{err} = 0.04$ years.

4.3 Detailed view: Standard deviation of the estimates

As population size decreases, standard deviations of life expectancy estimates increase approximately as an inverse square-root of population size. Yet at a smaller population size, the square-root-approximation underestimates the standard errors, as seen in the results presented in the Appendix Tables (because of the higher prevalence of cases where some age groups do not contain deaths or even the exposed population).

Age composition and the underlying mortality schedule also contribute to standard errors. To study those effects, we first eliminate the population size effect by considering the standard errors s_0, s_{60} rescaled for a hypothetical population of 1,000 persons (Eqs. 1 and 3). We then average the rescaled standard errors for large populations, where the distortions caused by zero death counts at individual age groups are not pronounced. The minimum population size, from which we started averaging the rescaled standard errors, was different for different population growth rates. We averaged the rescaled errors from a population size of 1,000 for declining populations, from a population size of 5,000 for stationary or moderately (1 percent) growing populations, and from a population size of 10,000 for those growing at 2 percent per annum. Coefficients, which could be used to correct those averages to obtain actual rescaled standard errors for different population sizes and growth rates, are presented in Table 1 (in most applications, these corrections may be neglected).

Table 1. Correction coefficients for rescaled standard errors at different population sizes and growth rates.

Population size	Corrections for s_0					Corrections for s_{60}				
	-2%	-1%	0%	1%	2%	-2%	-1%	0%	1%	2%
1000	0.998	1.000	1.011	1.037	1.066	1.009	1.021	1.062	1.124	1.157
5000	0.999	0.999	1.001	1.003	1.015	1.000	1.000	1.008	1.017	1.048
10000	0.999	0.999	1.000	1.001	1.006	0.999	0.997	1.002	1.005	1.018
25000	1.000	1.000	1.000	1.000	1.001	0.998	0.996	0.999	0.999	1.004
50000	1.000	1.001	1.000	1.000	1.000	0.998	0.996	0.998	0.997	0.999
100000	1.001	1.001	1.000	0.999	0.999	0.998	0.996	0.998	0.995	0.995
250000	1.001	1.000	0.999	0.998	0.998	0.998	0.996	0.997	0.994	0.993
1000000	1.002	1.000	0.999	0.998	0.997	0.999	0.996	0.997	0.993	0.991

Overall association between estimated parameters s_0 and the underlying true life expectancy at birth is presented in Figure 3. In the figure, we present only stationary populations with zero growth rates; more general results follow next. Despite the evident overall association between the life expectancy at birth and its estimation error, the particularities of mortality age patterns may strongly affect the estimation errors (note the case of Russian males highlighted in the figure).

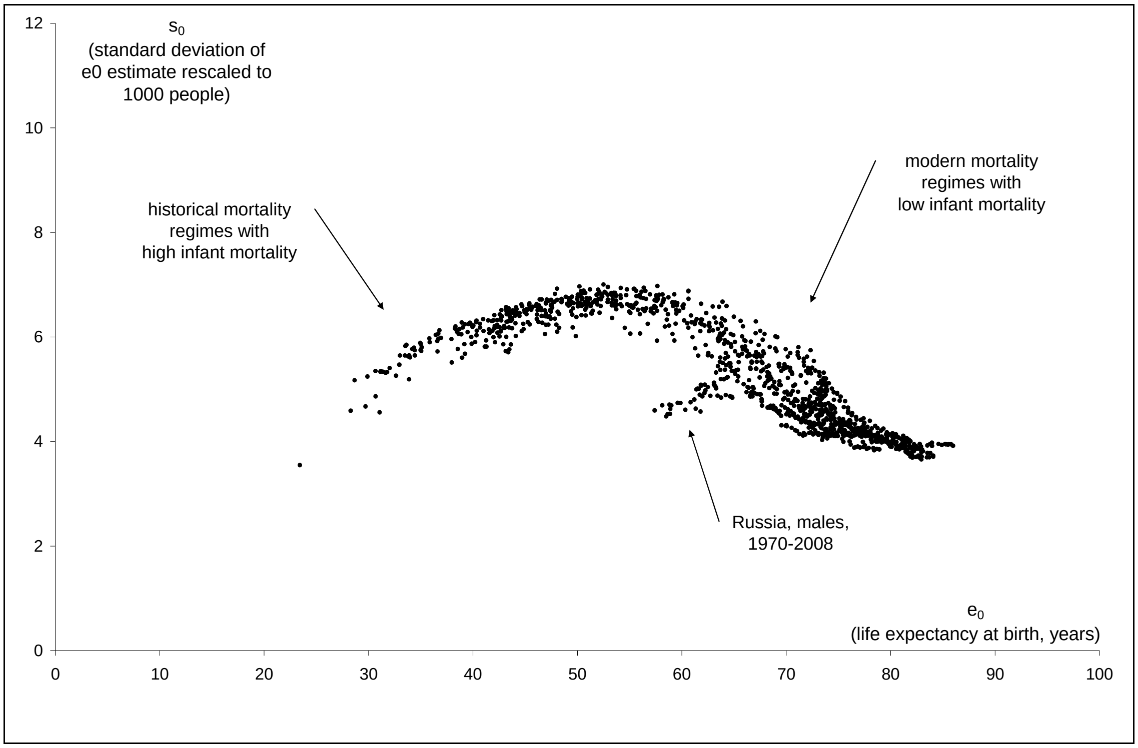

Figure 3. Association between standard deviations of estimates and underlying theoretical values of life expectancy at birth for those stationary populations analyzed.

There is a stronger association between the standard deviation of the estimated life expectancy at birth and the life table standard deviation of the age at death (see Figure 4 for stationary populations). We calculate the life table standard deviation of the age at death by the following formula:

$$SDAD_0 = \sqrt{\frac{\sum_{x=0}^{X} d_x \left(x + 0.5 - e_0\right)^2}{\sum_{x=0}^{X} d_x}}, \qquad (6)$$

Here e_0 is life expectancy at birth, X=100 is the maximum life table age, and d_x is the life table number dying at age x (in practical calculations, this may be assessed, e.g., from the national life table). Despite pooling together results for males and females, for different mortality regimes, $SDAD_0$ seems to be a good predictor of the estimation standard errors. Yet for historical mortality schedules with high infant mortality, the life expectancy at birth may still be a better predictor.

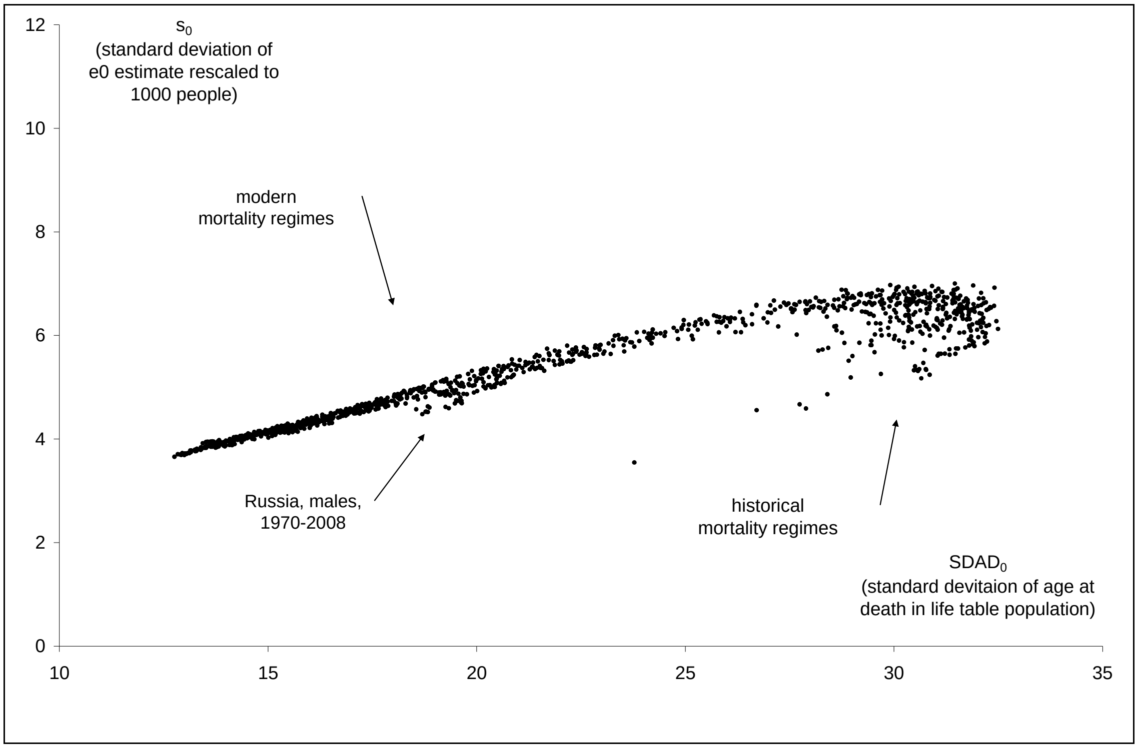

Figure 4. Association between standard deviations of estimates of life expectancy at birth and standard deviations of life table distributions of deaths for those stationary populations analyzed.

For practical applications, the results of our simulations may be summarized in the following regression relation:

$$s_0 = \alpha_0 + \alpha_1 \cdot SDAD_0 + \alpha_2 \cdot SDAD_0 \cdot e_0 + err\,, \qquad (7)$$

where $\sigma_{err} = 0.08$ years and parameters depend on the population age composition modeled in our simulations by the growth rate r (expressed in percent per annum: $r = 0$ for stationary population, $r = 1$ for population age composition formed by 1 percent annual growth of births, etc.):

$$\alpha_0 = -0.03 + 1.15 \cdot r\,,$$

$$\alpha_1 = 0.108 - 0.039 \cdot r\,,$$

$$\alpha_2 = 0.0022 - 0.00047 \cdot r + 0.00013 \cdot r^2\,. \qquad (7a)$$

Standard deviations of estimates of life expectancy at 60, not affected by the specific influence of infant mortality, follow a more consistent association with life expectancy at 60 (see Figure 5 for stationary populations). The wide variety of simulated cases may be described by the following regression:

$$s_{60} = 0.58 - 0.038 \cdot e_{60} + 0.0081 \cdot e_{60} \cdot SDAD_{60} + err\,, \qquad (8)$$

with $\sigma_{err} = 0.01$ years. If the life table standard deviation $SDAD_{60}$ cannot be established, another regression might be used:

$$s_{60} = 0.082 \cdot e_{60} - 0.0010 \cdot e_{60}^{\ 2} + \delta' , \qquad (9)$$

with a standard error of 0.04 years.

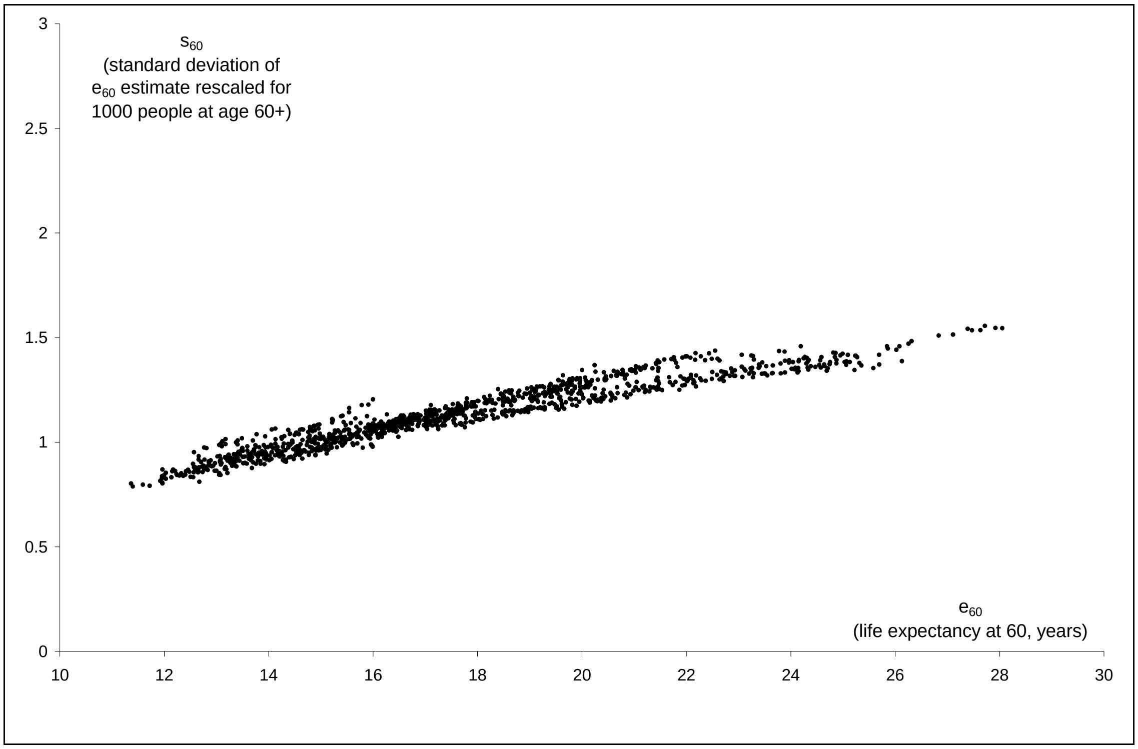

Figure 5. Association between standard deviations of estimates and underlying theoretical values of life expectancy at 60 for those stationary populations analyzed.

4.4 Normality of life expectancy estimate's distribution

Previous research (Silcocks et al. 2001; Eares and Williams 2004) suggested that the distribution of estimates may be considered to be approximately normal, which might simplify practical use of standard errors of estimates (in applications such as the construction of confidence intervals, hypotheses testing, examining the significance of temporal or geographical variation of life expectancy, etc.) However, our study, based on a wider set of population structures, indicates that the estimates' normality may be assumed only under certain conditions, which we discuss below.

We examined the normality of simulated distributions for stable populations and mortality schedules corresponding to the Japanese female life table in 2007 (see Figures 6 and 7 for selected histograms with superimposed normal distributions; for this particular exercise, we used 10,000 simulations). Even at a remarkably large population size, the distributions of estimates for life expectancy at birth may significantly deviate from normal distributions where population age structure is non-stationary (see the case of the stable population declining at 2 percent per annum in Figure 6; even at a population size of 100,000, not shown in the figure, the test for normality of life expectancy at birth estimates fails for such stable populations). For stationary or moderately growing stable populations, however, the distribution of life expectancy at birth estimates is closer to normality beginning with a population size of 10,000 (all distributions of estimates are strongly skewed for a population of 1,000 persons). Distributions of estimates of life expectancy at age 60 do not deviate significantly (at the 90 percent confidence level, Pearson's criterion, 10,000 simulations) from the normal distribution for stationary populations at a population size of 5,000. However, they become skewed for growing populations.

Although deviations from normality may be statistically significant at some population sizes and age compositions, these are certain distribution percentiles and not the normality of distributions as such, which are important for most applications. Estimates of selected percentiles derived from simulated distributions and from the corresponding normal distributions are presented in Appendix Tables A6 and A7. Percentiles obtained assuming the normality of estimates of life expectancy at birth or at age 60 are fairly close to those obtained directly from simulated distributions at a population size of 50,000 or more. Assuming normality for a population of 5,000 or less might be discouraged, unless the tested difference in life expectancies falls far beyond the confidence limits. When studying populations of an intermediate size between 5,000 and 50,000, one must be aware of the possible effects of deviation of the population age composition from the stationary age composition.

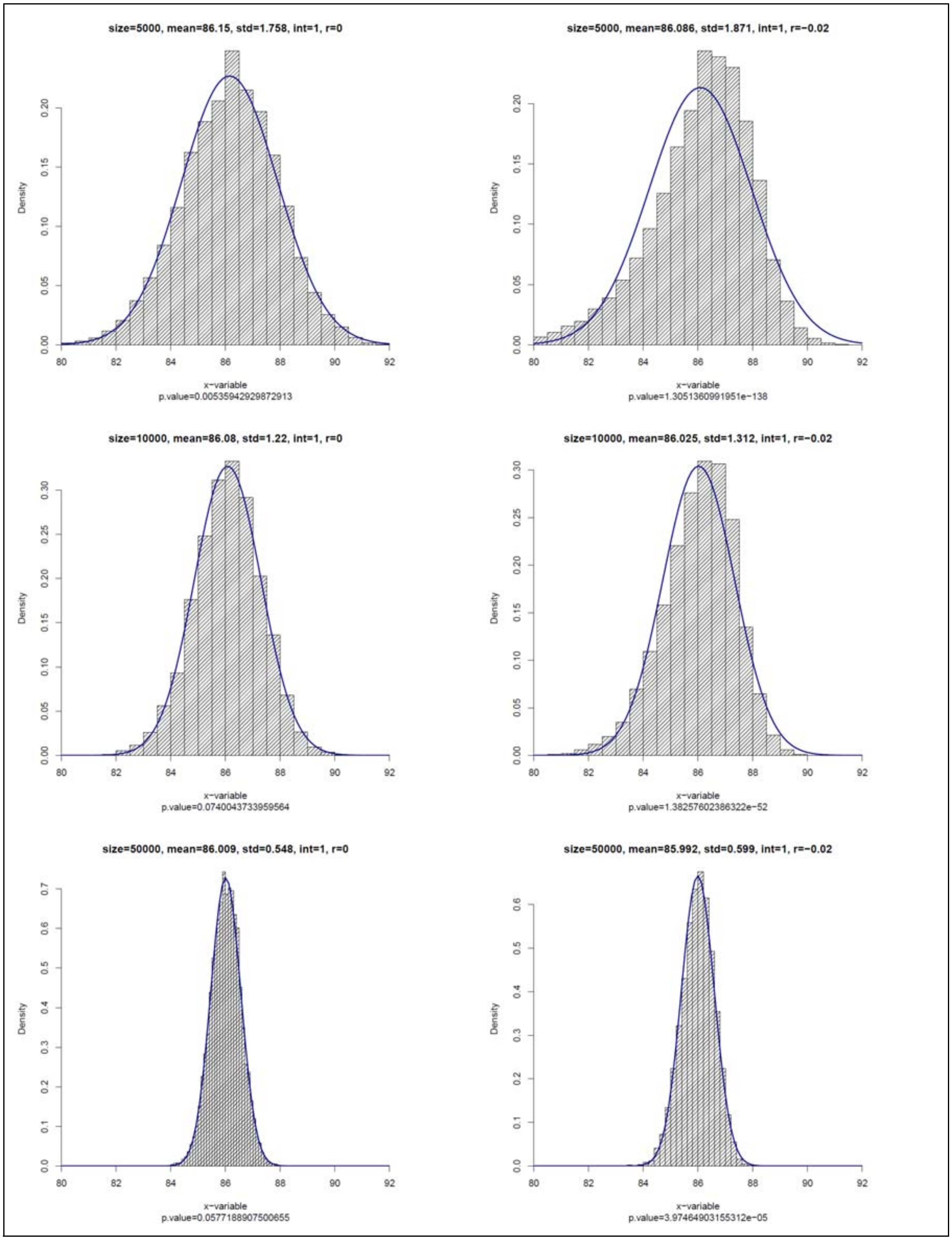

Figure 6. Selected distributions of the estimates of life expectancy at birth (10,000 simulations based on stable populations and mortality schedules of the life table of the Japanese female population in 2007).

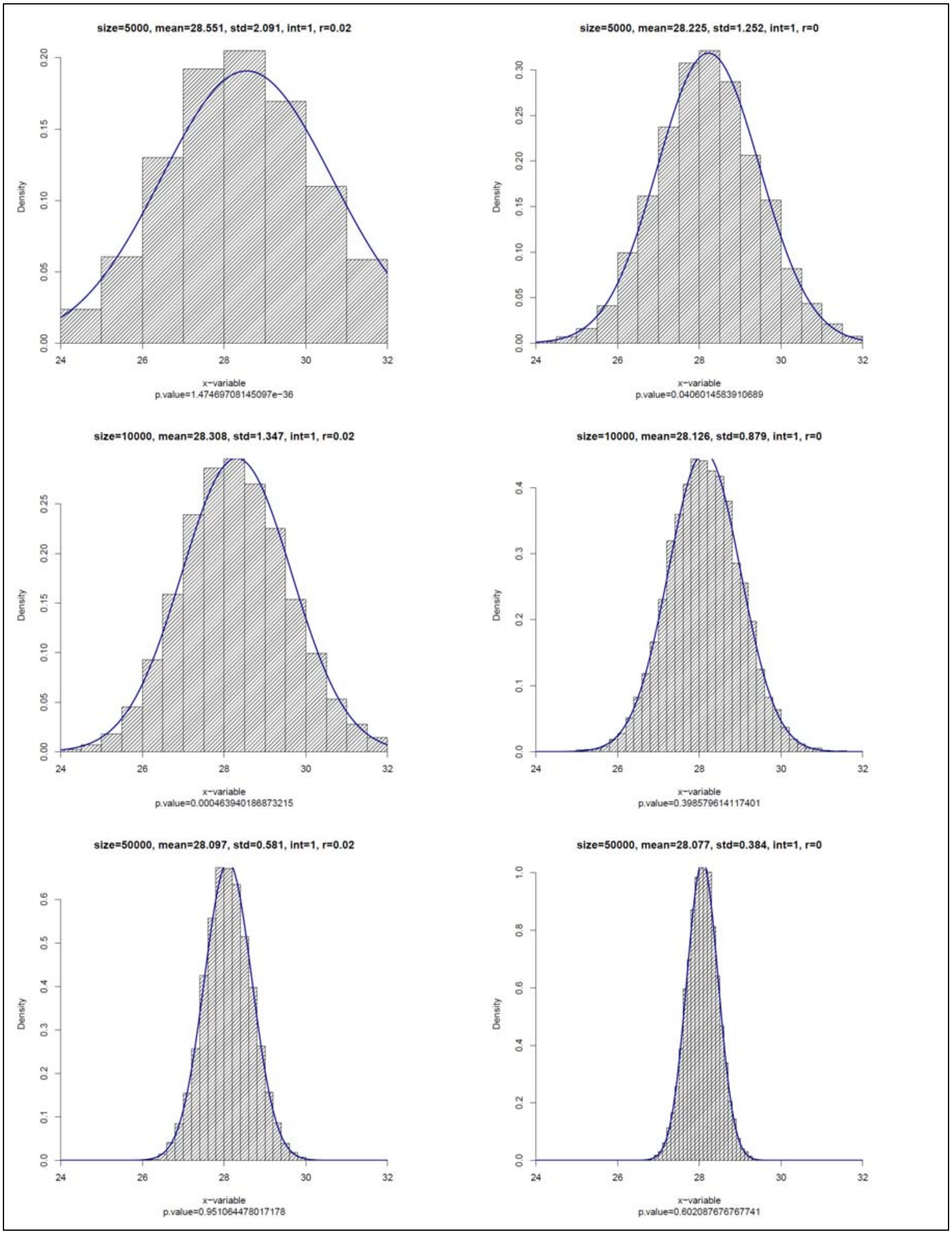

Figure 7. Selected distributions of the estimates of life expectancy at age 60 (10,000 simulations based on stable populations and mortality schedules of the life table of the Japanese female population in 2007).

5 Illustrative Examples

Simulation results illustrate that estimations of life expectancy for small populations may be associated with considerably high standard errors and biases. Those must be taken into account both in designing the system of statistical observations and in interpreting geographical, temporal and other variations of longevity obtained from small populations. Further down we present several illustrations of this kind.

Case 1. Establishing confidence limits for life expectancy. Let life expectancy at birth be estimated at 86 years in a population of 20,000 people. What, roughly, would be the confidence limits for the actual life expectancy at the 95 percent confidence level assuming stationary age composition? From Appendix Table A1, we may assess $s_0 \approx 4.8$ (years per 1,000 persons). Hence, the standard error calculated for the actual population size would be $SD_0 = 4.8 \Big/ \sqrt{\frac{20000}{1000}} = 1.07$ (years). Assuming normality, this yields $e_0 = 86 \pm 2.1$ years at a 95 percent confidence level.

Case 2. Examining the significance of life expectancy variation. Consider the hypothetical case of comparing life expectancy in two small populations, say A and B. These populations may either represent two geographically or otherwise defined subpopulations of the total or the same population at two points in time. In the first case, we examine the significance of spatial or social variation in life expectancy, while in the second, we examine the significance of temporal variation. Suppose the two populations are characterized by the following indicators:

	Population A	Population B
Total population, persons	20,000	50,000
Life expectancy at birth	86.0	83.5
Life table standard deviation of age at death	14.0	16.0
Population at age 60 or more	5,930	15,629
Life expectancy at age 60	25.5	26.1

Then assume that the age composition of both populations is near stationary. Is the difference in life expectancy between the two populations significant (say, at the 5 percent significance level)?

To investigate the question above, we estimate standard errors of the estimates of life expectancy for the two populations. In both populations, life expectancy exceeds 55 years. From Appendix Table A1 we may assess $s_0 \approx 4.8$ (years per 1,000 persons) for both populations. More accurate estimates based on Eq. (7) yield $s_0^{(1)} \approx 4.1$ for the first population and $s_0^{(2)} \approx 4.6$ for the second population. Hence, standard errors calculated for the actual population sizes would be:

$$SD_0^{(1)} = 4.8 \Big/ \sqrt{\frac{20000}{1000}} = 1.07 \text{ and } SD_0^{(2)} = 4.8 \Big/ \sqrt{\frac{50000}{1000}} = 0.68 \text{ (years)}$$

More accurate estimates based on Eq. (7) yield 0.92 and 0.66 years, respectively.

Assuming the independence of the estimates for the two populations, we may compute the standard error of the difference between the estimates of life expectancy:

$$SD_0^{(1)-(2)} = \sqrt{\left(SD_0^{(1)}\right)^2 + \left(SD_0^{(2)}\right)^2} = 1.27 \text{ (years).}$$

More accurate calculations based on Eq. (7) yield 1.13 years.

Given the standard deviation and assuming normal distribution, the observed difference of 86.0-83.5=2.5 years yields *p*-value 4.9 percent (double-sided alternative), i.e., *the difference is significant* at the 5 percent significance level. The two populations are different with respect to life expectancy at birth at the 95 percent confidence level. Based on more accurate estimates presented above, the *p*-value may be estimated at a lower level: 2.7 percent.

Let us examine the significance of the difference in life expectancy at age 60. From Appendix Table A1 we obtain $s_{60} \approx 1.16$ for each of the populations analyzed (years per 1,000 persons of age 60 or more). Hence, standard errors estimated for the actual population sizes would be

$$SD_{60}^{(1)} = 1.16\Big/\sqrt{\tfrac{5930}{1000}} = 0.48 \text{ and } SD_{60}^{(2)} = 1.16\Big/\sqrt{\tfrac{15629}{1000}} = 0.29 \text{ (years).}$$

Assuming the independence of the estimates for the two populations, we may compute the standard error of the difference between the estimates of life expectancy:

$$SD_{60}^{(1)-(2)} = \sqrt{\left(SD_{60}^{(1)}\right)^2 + \left(SD_{60}^{(2)}\right)^2} = 0.56 \text{ (years).}$$

Given the standard deviation and assuming normal distribution, the observed difference of 26.1-25.5=0.6 years yields *p*-value 38.2 percent (double-sided alternative), i.e., *the difference may not be considered significant* at the 5 percent significance level. The two populations do not differ significantly with respect to life expectancy at age 60. The result remains under more accurate calculations: Estimates based on Eq. (8) produce *p*-value of 28.4 percent. Estimates based on Eq. (9) produce *p*-value 39.0 percent.

Case 3. Minimal population size meeting the required level of estimation accuracy. Consider a situation where life expectancy at age 60 is estimated to be about 25 years, the proportion of the population aged 60 and more is 30 percent, and the age composition is stationary. Then suppose that the policy maker demands measurements of life expectancy at age 60 to be made at the regional level with errors not exceeding 0.75 years at a 95 percent confidence level. What would the recommendation be about minimal population size for estimating the life expectancy at age 60 with the required accuracy? A difference of 0.75 years would not be statistically significant at the 95 percent confidence level at a standard deviation higher than $\frac{0.75}{1.96} = 0.38$ years (assuming normal distribution, double-sided hypothesis). For a stationary population with $e_{60} = 25$, Eq. (9) implies $s_{60} = 0.082 \cdot 25 - 0.001 \cdot 25^2 \approx 1.43$ years, i.e., the critical threshold 0.38 of standard deviation may be reached at population size $N_{60+} = \left(\frac{1.43}{0.38}\right)^2 = 14$ (thousands) at age 60 or higher, i.e., at total population size

$N = \frac{14}{0.3} \approx 46$ (thousands). Hence, estimation of life expectancy at 60 may be recommended for areas with at least 46,000 people.

6 General Recommendations

We have shown that both the standard errors and the estimation bias become very high at a population size of around 5,000 or less. Additionally, the distributions of estimation errors deviate strongly from normality at such population sizes, which precludes building confidence limits and conducting other statistical analyses. Therefore, estimating life expectancies for such populations must be discouraged.

Based on rough estimates from Appendix Tables A1-A5 and assuming that the standard error of the estimates of the life expectancy at birth is about 1 year or less, we may conclude that population exposure years should be about 25,000 people or more for a low-mortality population. To estimate life expectancy at 60 with a standard error of about 0.25 years, the population size should be about 100,000 or more for stationary populations, 50,000 for populations declining at 2 percent, and 200,000 for populations growing at 2 percent per annum. These rough estimates only outline how strict the requirements on population size could be in order to secure relatively accurate estimations.

Our study indicates that more precise assessments of estimation errors and of minimal population size may vary considerably depending on actual population age composition and mortality schedules. Even the requirements for estimation errors may vary from population to population, depending, e.g., on observed spatial and social variation of mortality as well as on policy demands. In a country with high spatial diversity in life expectancy (e.g., Russia), even a low-precision estimate of life expectancy at the municipal level may reveal important regional differences, while for a country with more homogeneous regional mortality variation, like many western European countries, estimates must be conducted with higher precision, so that they reveal informative variations of mortality levels and not the random sample-size effects.

In most applications of estimation errors, it is convenient to assume a normal distribution of the estimates. However, our simulations indicate that such assumptions may safely be used only starting from a population size of 50,000. For populations of 5,000 or less, such assumptions are not acceptable. In intermediate situations, normality assumptions may only be used as a rough approximation. More precise assessments, if necessary, may demand a detailed analysis and perhaps additional simulations tailored to the particular situation.

We do not find any advantages in using abridged life tables instead of unabridged ones even for a small population with many age groups containing no death observations. Even more, abridged life table calculations may lead to strong biases when the population age composition deviates from the stationary composition. Hence, it might be advised to use the unabridged life tables rather than the abridged ones when the population is not stationary.

Our simulation results show that procedures for the open age interval are crucial for the efficiency of life expectancy estimation. Although we found efficiency in our simple approach based on adjusting the open age interval in such a way that there is at least one death observed, more research on procedures for the open age interval might be important.

7 References

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Appendix. Supplementary Tables

Table A1. Standard errors and estimation biases for life expectancy estimates for selected population sizes at a population growth rate of zero percent.

Population size:	e0	e60	SD_0	SD_{60}	s_0	s'_{60}	s_{60}	bias_e 0	bias_e 60	r
Stationary populations with low mortality (e0 more than 55 years)										
1000	71 (7)	19 (3)	5.3 (0.8)	3.5 (0.3)	5.3 (0.8)	3.5 (0.3)	1.63 (0.15)	1.07 (0.09)	1.01 (0.12)	0%
5000	71 (7)	19 (3)	2.2 (0.4)	1.16 (0.05)	4.9 (0.9)	2.59 (0.11)	1.2 (0.12)	0.2 (0.02)	0.18 (0.02)	0%
10000	71 (7)	19 (3)	1.5 (0.3)	0.8 (0.03)	4.9 (0.9)	2.54 (0.11)	1.17 (0.13)	0.098 (0.012)	0.088 (0.012)	0%
25000	71 (7)	19 (3)	0.97 (0.17)	0.5 (0.02)	4.8 (0.9)	2.51 (0.11)	1.16 (0.13)	0.042 (0.007)	0.037 (0.006)	0%
50000	71 (7)	19 (3)	0.69 (0.12)	0.355 (0.015)	4.8 (0.9)	2.51 (0.11)	1.16 (0.13)	0.022 (0.004)	0.019 (0.003)	0%
100000	71 (7)	19 (3)	0.49 (0.09)	0.25 (0.011)	4.9 (0.9)	2.5 (0.11)	1.16 (0.13)	0.01 (0.003)	0.009 (0.002)	0%
250000	71 (7)	19 (3)	0.31 (0.05)	0.158 (0.007)	4.8 (0.9)	2.5 (0.11)	1.16 (0.13)	0.004 (0.002)	0.003 (0.001)	0%
1000000	71 (7)	19 (3)	0.15 (0.03)	0.079 (0.003)	4.8 (0.9)	2.5 (0.11)	1.16 (0.13)	0.001 (0.001)	0.001 (0.001)	0%
Stationary populations with high mortality (e0 less than 55 years)										
1000	45 (6)	14.1 (1.2)	6.6 (0.5)	3.8 (0.3)	6.6 (0.5)	3.8 (0.3)	1.42 (0.11)	0.99 (0.12)	1.2 (0.13)	0%
5000	45 (6)	14.1 (1.2)	2.8 (0.2)	1.16 (0.06)	6.4 (0.5)	2.59 (0.13)	0.97 (0.07)	0.19 (0.03)	0.22 (0.03)	0%
10000	45 (6)	14.1 (1.2)	2 (0.15)	0.79 (0.04)	6.3 (0.5)	2.51 (0.11)	0.94 (0.07)	0.102 (0.015)	0.11 (0.016)	0%
25000	45 (6)	14.1 (1.2)	1.26 (0.09)	0.49 (0.02)	6.3 (0.5)	2.46 (0.11)	0.92 (0.07)	0.042 (0.009)	0.045 (0.006)	0%
50000	45 (6)	14.1 (1.2)	0.89 (0.07)	0.346 (0.015)	6.3 (0.5)	2.45 (0.11)	0.92 (0.08)	0.018 (0.006)	0.022 (0.004)	0%
100000	45 (6)	14.1 (1.2)	0.63 (0.05)	0.244 (0.011)	6.3 (0.5)	2.44 (0.11)	0.91 (0.08)	0.01 (0.004)	0.011 (0.002)	0%
250000	45 (6)	14.1 (1.2)	0.4 (0.03)	0.154 (0.007)	6.3 (0.5)	2.43 (0.11)	0.91 (0.08)	0.004 (0.002)	0.005 (0.001)	0%
1000000	45 (6)	14.1 (1.2)	0.2 (0.015)	0.077 (0.003)	6.3 (0.5)	2.44 (0.11)	0.91 (0.08)	0.001 (0.001)	0.001 (0.001)	0%

* Numbers in parentheses represent the standard deviation of the indicators (there were 884 populations with low mortality and 308 populations with high mortality analyzed for each combination of population size and stable growth rate).

Table A2. Standard errors and estimation biases for life expectancy estimates for selected population sizes at a population growth rate of -1 percent.

Population size:	**e0**	**e60**	SD_0	SD_{60}	s_0	s'_{60}	s_{60}	**bias_e 0**	**bias_e 60**	**r**
Declining populations with low mortality (e0 more than 55 years)										
1000	71 (7)	19 (3)	5.2 (1.1)	2.48 (0.14)	5.2 (1.1)	2.48 (0.14)	1.34 (0.11)	0.79 (0.11)	0.62 (0.07)	-1%
5000	71 (7)	19 (3)	2.3 (0.5)	0.98 (0.04)	5.2 (1.1)	2.2 (0.1)	1.18 (0.12)	0.16 (0.03)	0.12 (0.014)	-1%
10000	71 (7)	19 (3)	1.6 (0.4)	0.69 (0.03)	5.2 (1.1)	2.17 (0.09)	1.17 (0.13)	0.075 (0.014)	0.056 (0.007)	-1%
25000	71 (7)	19 (3)	1 (0.2)	0.432 (0.019)	5.2 (1.1)	2.16 (0.1)	1.17 (0.13)	0.025 (0.007)	0.021 (0.003)	-1%
50000	71 (7)	19 (3)	0.74 (0.16)	0.306 (0.013)	5.2 (1.1)	2.16 (0.1)	1.17 (0.13)	0.013 (0.005)	0.01 (0.003)	-1%
100000	71 (7)	19 (3)	0.52 (0.11)	0.216 (0.01)	5.2 (1.1)	2.16 (0.1)	1.17 (0.13)	0.007 (0.003)	0.006 (0.002)	-1%
250000	71 (7)	19 (3)	0.33 (0.07)	0.137 (0.006)	5.2 (1.1)	2.16 (0.09)	1.17 (0.13)	0.002 (0.002)	0.002 (0.001)	-1%
1000000	71 (7)	19 (3)	0.16 (0.04)	0.068 (0.003)	5.2 (1.1)	2.16 (0.09)	1.17 (0.13)	0.001 (0.001)	0.001 (0)	-1%
Declining populations with high mortality (e0 less than 55 years)										
1000	45 (6)	14.1 (1.2)	7.3 (0.5)	2.6 (0.18)	7.3 (0.5)	2.6 (0.18)	1.15 (0.07)	0.86 (0.11)	0.74 (0.09)	-1%
5000	45 (6)	14.1 (1.2)	3.2 (0.2)	0.96 (0.04)	7.2 (0.6)	2.14 (0.09)	0.95 (0.07)	0.18 (0.03)	0.141 (0.016)	-1%
10000	45 (6)	14.1 (1.2)	2.26 (0.18)	0.66 (0.03)	7.1 (0.6)	2.09 (0.09)	0.93 (0.08)	0.082 (0.014)	0.068 (0.009)	-1%
25000	45 (6)	14.1 (1.2)	1.43 (0.11)	0.415 (0.017)	7.1 (0.6)	2.08 (0.09)	0.92 (0.08)	0.023 (0.009)	0.024 (0.004)	-1%
50000	45 (6)	14.1 (1.2)	1.01 (0.08)	0.294 (0.012)	7.2 (0.6)	2.08 (0.09)	0.92 (0.08)	0.018 (0.006)	0.014 (0.003)	-1%
100000	45 (6)	14.1 (1.2)	0.72 (0.06)	0.208 (0.009)	7.2 (0.6)	2.08 (0.09)	0.92 (0.08)	0.008 (0.005)	0.007 (0.002)	-1%
250000	45 (6)	14.1 (1.2)	0.45 (0.04)	0.131 (0.005)	7.2 (0.6)	2.07 (0.09)	0.92 (0.08)	0.003 (0.003)	0.003 (0.001)	-1%
1000000	45 (6)	14.1 (1.2)	0.226 (0.018)	0.066 (0.003)	7.2 (0.6)	2.07 (0.09)	0.92 (0.08)	0.001 (0.001)	0.001 (0)	-1%

* Numbers in parentheses represent the standard deviation of the indicators (there were 884 populations with low mortality and 308 populations with high mortality analyzed for each combination of population size and stable growth rate).

Table A3. Standard errors and estimation biases for life expectancy estimates for selected population sizes at a population growth rate of 1 percent.

Population size:	e0	e60	SD_0	SD_{60}	s_0	s'_{60}	s_{60}	bias_e 0	bias_e 60	r
Growing populations with low mortality (e0 more than 55 years)										
1000	71 (7)	19 (3)	6.4 (0.4)	5.7 (0.4)	6.4 (0.4)	5.7 (0.4)	2.2 (0.3)	1.71 (0.1)	1.82 (0.15)	1%
5000	71 (7)	19 (3)	2.2 (0.3)	1.44 (0.07)	4.9 (0.6)	3.22 (0.16)	1.25 (0.12)	0.28 (0.02)	0.29 (0.03)	1%
10000	71 (7)	19 (3)	1.5 (0.2)	0.97 (0.05)	4.8 (0.6)	3.07 (0.15)	1.19 (0.13)	0.141 (0.012)	0.142 (0.018)	1%
25000	71 (7)	19 (3)	0.96 (0.13)	0.6 (0.03)	4.8 (0.6)	3.02 (0.13)	1.17 (0.13)	0.052 (0.008)	0.055 (0.009)	1%
50000	71 (7)	19 (3)	0.68 (0.09)	0.424 (0.019)	4.8 (0.6)	3 (0.13)	1.16 (0.13)	0.028 (0.004)	0.028 (0.004)	1%
100000	71 (7)	19 (3)	0.48 (0.06)	0.299 (0.013)	4.8 (0.6)	2.99 (0.13)	1.16 (0.13)	0.014 (0.003)	0.015 (0.003)	1%
250000	71 (7)	19 (3)	0.3 (0.04)	0.189 (0.008)	4.8 (0.6)	2.98 (0.13)	1.16 (0.13)	0.006 (0.002)	0.006 (0.001)	1%
1000000	71 (7)	19 (3)	0.15 (0.02)	0.094 (0.004)	4.8 (0.6)	2.98 (0.13)	1.16 (0.13)	0.001 (0.001)	0.001 (0.001)	1%
Growing populations with high mortality (e0 less than 55 years)										
1000	45 (6)	14.1 (1.2)	6.4 (0.5)	5.3 (0.3)	6.4 (0.5)	5.3 (0.3)	1.65 (0.19)	1.32 (0.2)	1.92 (0.14)	1%
5000	45 (6)	14.1 (1.2)	2.63 (0.18)	1.49 (0.1)	5.9 (0.4)	3.3 (0.2)	1.03 (0.07)	0.24 (0.03)	0.35 (0.05)	1%
10000	45 (6)	14.1 (1.2)	1.84 (0.13)	0.98 (0.05)	5.8 (0.4)	3.11 (0.17)	0.96 (0.07)	0.121 (0.017)	0.18 (0.03)	1%
25000	45 (6)	14.1 (1.2)	1.16 (0.08)	0.6 (0.03)	5.8 (0.4)	3 (0.15)	0.93 (0.07)	0.05 (0.009)	0.071 (0.011)	1%
50000	45 (6)	14.1 (1.2)	0.82 (0.06)	0.42 (0.02)	5.8 (0.4)	2.97 (0.14)	0.92 (0.08)	0.026 (0.006)	0.036 (0.007)	1%
100000	45 (6)	14.1 (1.2)	0.58 (0.04)	0.295 (0.014)	5.8 (0.4)	2.95 (0.14)	0.91 (0.08)	0.013 (0.004)	0.018 (0.004)	1%
250000	45 (6)	14.1 (1.2)	0.37 (0.03)	0.186 (0.009)	5.8 (0.4)	2.94 (0.14)	0.91 (0.08)	0.005 (0.002)	0.007 (0.002)	1%
1000000	45 (6)	14.1 (1.2)	0.183 (0.013)	0.093 (0.004)	5.8 (0.4)	2.93 (0.14)	0.91 (0.08)	0.001 (0.001)	0.002 (0.001)	1%

* Numbers in parentheses represent the standard deviation of the indicators (there were 884 populations with low mortality and 308 populations with high mortality analyzed for each combination of population size and stable growth rate).

Table A4. Standard errors and estimation biases for life expectancy estimates for selected population sizes at a population growth rate of -2 percent.

Population size:	e0	e60	SD_0	SD_{60}	s_0	s'_{60}	s_{60}	bias_e 0	bias_e 60	r
Stationary populations with low mortality (e0 more than 55 years)										
1000	71 (7)	19 (3)	5.8 (1.5)	2.04 (0.09)	5.8 (1.5)	2.04 (0.09)	1.25 (0.12)	0.66 (0.14)	0.42 (0.05)	-2%
5000	71 (7)	19 (3)	2.6 (0.7)	0.87 (0.04)	5.9 (1.5)	1.94 (0.09)	1.19 (0.13)	0.14 (0.04)	0.081 (0.009)	-2%
10000	71 (7)	19 (3)	1.9 (0.5)	0.61 (0.03)	5.9 (1.5)	1.93 (0.09)	1.18 (0.13)	0.08 (0.02)	0.042 (0.005)	-2%
25000	71 (7)	19 (3)	1.2 (0.3)	0.386 (0.018)	5.9 (1.5)	1.93 (0.09)	1.18 (0.13)	0.039 (0.011)	0.019 (0.003)	-2%
50000	71 (7)	19 (3)	0.8 (0.2)	0.272 (0.013)	5.9 (1.5)	1.93 (0.09)	1.18 (0.13)	0.016 (0.006)	0.008 (0.002)	-2%
100000	71 (7)	19 (3)	0.59 (0.15)	0.192 (0.009)	5.9 (1.5)	1.92 (0.09)	1.18 (0.13)	0.01 (0.004)	0.004 (0.001)	-2%
250000	71 (7)	19 (3)	0.38 (0.09)	0.122 (0.006)	5.9 (1.5)	1.92 (0.09)	1.18 (0.13)	0.004 (0.002)	0.002 (0.001)	-2%
1000000	71 (7)	19 (3)	0.19 (0.05)	0.061 (0.003)	5.9 (1.5)	1.93 (0.09)	1.18 (0.13)	0.001 (0.001)	0 (0)	-2%
Stationary populations with high mortality (e0 less than 55 years)										
1000	45 (6)	14.1 (1.2)	8.7 (0.7)	2.02 (0.1)	8.7 (0.7)	2.02 (0.1)	1.04 (0.07)	0.83 (0.12)	0.49 (0.05)	-2%
5000	45 (6)	14.1 (1.2)	3.8 (0.3)	0.82 (0.03)	8.4 (0.7)	1.84 (0.08)	0.94 (0.08)	0.18 (0.03)	0.095 (0.012)	-2%
10000	45 (6)	14.1 (1.2)	2.7 (0.2)	0.58 (0.02)	8.4 (0.7)	1.82 (0.07)	0.93 (0.08)	0.105 (0.02)	0.048 (0.007)	-2%
25000	45 (6)	14.1 (1.2)	1.68 (0.14)	0.363 (0.015)	8.4 (0.7)	1.81 (0.07)	0.93 (0.08)	0.051 (0.012)	0.021 (0.003)	-2%
50000	45 (6)	14.1 (1.2)	1.19 (0.1)	0.256 (0.011)	8.4 (0.7)	1.81 (0.07)	0.93 (0.08)	0.02 (0.008)	0.009 (0.002)	-2%
100000	45 (6)	14.1 (1.2)	0.84 (0.07)	0.181 (0.007)	8.4 (0.7)	1.81 (0.07)	0.93 (0.08)	0.01 (0.006)	0.004 (0.001)	-2%
250000	45 (6)	14.1 (1.2)	0.53 (0.05)	0.114 (0.005)	8.4 (0.7)	1.81 (0.07)	0.93 (0.08)	0.005 (0.003)	0.002 (0.001)	-2%
1000000	45 (6)	14.1 (1.2)	0.27 (0.02)	0.057 (0.002)	8.4 (0.7)	1.81 (0.08)	0.93 (0.08)	0.001 (0.002)	0 (0)	-2%

* Numbers in parentheses represent the standard deviation of the indicators (there were 884 populations with low mortality and 308 populations with high mortality analyzed for each combination of population size and stable growth rate).

Table A5. Standard errors and estimation biases for life expectancy estimates for selected population sizes at a population growth rate of 2 percent.

Population size:	e0	e60	SD_0	SD_{60}	s_0	s'_{60}	s_{60}	bias_e 0	bias_e 60	r
Stationary populations with low mortality (e0 more than 55 years)										
1000	71 (7)	19 (3)	7.7 (0.7)	7.5 (0.8)	7.7 (0.7)	7.5 (0.8)	2.4 (0.5)	2.6 (0.3)	2.9 (0.2)	2%
5000	71 (7)	19 (3)	2.44 (0.17)	1.98 (0.14)	5.5 (0.4)	4.4 (0.3)	1.4 (0.14)	0.47 (0.04)	0.51 (0.06)	2%
10000	71 (7)	19 (3)	1.65 (0.13)	1.25 (0.07)	5.2 (0.4)	4 (0.2)	1.26 (0.13)	0.225 (0.018)	0.25 (0.03)	2%
25000	71 (7)	19 (3)	1.02 (0.09)	0.75 (0.04)	5.1 (0.4)	3.75 (0.18)	1.19 (0.13)	0.086 (0.01)	0.096 (0.014)	2%
50000	71 (7)	19 (3)	0.72 (0.06)	0.52 (0.02)	5.1 (0.4)	3.7 (0.17)	1.18 (0.13)	0.045 (0.005)	0.049 (0.007)	2%
100000	71 (7)	19 (3)	0.51 (0.04)	0.368 (0.017)	5.1 (0.4)	3.68 (0.17)	1.17 (0.13)	0.021 (0.003)	0.024 (0.004)	2%
250000	71 (7)	19 (3)	0.32 (0.03)	0.231 (0.011)	5 (0.4)	3.66 (0.17)	1.16 (0.13)	0.009 (0.002)	0.01 (0.002)	2%
1000000	71 (7)	19 (3)	0.159 (0.014)	0.115 (0.005)	5 (0.4)	3.65 (0.17)	1.16 (0.13)	0.002 (0.001)	0.003 (0.001)	2%
Stationary populations with high mortality (e0 less than 55 years)										
1000	45 (6)	14.1 (1.2)	6.2 (0.6)	5.8 (0.6)	6.2 (0.6)	5.8 (0.6)	1.5 (0.3)	1.5 (0.3)	2.3 (0.3)	2%
5000	45 (6)	14.1 (1.2)	2.58 (0.19)	2.1 (0.2)	5.8 (0.4)	4.8 (0.5)	1.19 (0.09)	0.36 (0.04)	0.61 (0.09)	2%
10000	45 (6)	14.1 (1.2)	1.78 (0.13)	1.3 (0.1)	5.6 (0.4)	4.1 (0.3)	1.03 (0.07)	0.18 (0.02)	0.3 (0.04)	2%
25000	45 (6)	14.1 (1.2)	1.11 (0.08)	0.76 (0.04)	5.6 (0.4)	3.8 (0.2)	0.95 (0.07)	0.07 (0.01)	0.121 (0.019)	2%
50000	45 (6)	14.1 (1.2)	0.79 (0.06)	0.52 (0.03)	5.6 (0.4)	3.7 (0.19)	0.93 (0.08)	0.036 (0.007)	0.062 (0.012)	2%
100000	45 (6)	14.1 (1.2)	0.55 (0.04)	0.366 (0.019)	5.5 (0.4)	3.66 (0.19)	0.92 (0.08)	0.019 (0.004)	0.032 (0.006)	2%
250000	45 (6)	14.1 (1.2)	0.35 (0.03)	0.23 (0.012)	5.5 (0.4)	3.63 (0.18)	0.91 (0.08)	0.008 (0.002)	0.013 (0.003)	2%
1000000	45 (6)	14.1 (1.2)	0.175 (0.013)	0.115 (0.006)	5.5 (0.4)	3.62 (0.18)	0.91 (0.08)	0.002 (0.001)	0.003 (0.001)	2%

* Numbers in parentheses represent the standard deviation of the indicators (there were 884 populations with low mortality and 308 populations with high mortality analyzed for each combination of population size and stable growth rate).

Table A6. Percentiles of distribution of estimates for life expectancy at birth, derived from simulated distribution and from corresponding normal distribution (simulations are based on stable populations and mortality schedules corresponding to the life table of the Japanese female population in 2007).

Population size	Growth rate		Unabridged life tables					Abridged life tables				
		Percentile:	2.5%	5.0%	50.0%	95.0%	97.5%	2.5%	5.0%	50.0%	95.0%	97.5%
5000	-2%	Normal distribution	82.42	83.01	86.09	89.16	89.75	82.02	82.63	85.77	88.92	89.52
		Actual distribution	81.66	82.61	86.33	88.67	89.08	81.31	82.20	85.99	88.48	88.95
	-1%	Normal distribution	82.69	83.24	86.10	88.96	89.50	82.39	82.97	85.97	88.97	89.54
		Actual distribution	82.31	83.03	86.23	88.74	89.19	82.07	82.75	86.09	88.80	89.26
	0%	Normal distribution	82.70	83.26	86.15	89.04	89.60	82.52	83.11	86.17	89.24	89.83
		Actual distribution	82.59	83.17	86.21	88.94	89.49	82.41	83.07	86.19	89.22	89.77
	1%	Normal distribution	82.49	83.10	86.31	89.51	90.12	82.18	82.86	86.45	90.03	90.72
		Actual distribution	82.46	83.14	86.30	89.44	90.03	82.40	83.02	86.39	90.12	90.97
	2%	Normal distribution	81.80	82.55	86.46	90.37	91.12	81.26	82.18	87.00	91.82	92.75
		Actual distribution	82.10	82.81	86.38	90.29	91.22	82.23	82.90	86.74	91.99	93.58
10000	-2%	Normal distribution	83.45	83.87	86.03	88.18	88.60	83.12	83.54	85.74	87.93	88.35
		Actual distribution	83.15	83.70	86.15	87.97	88.25	82.74	83.30	85.87	87.66	87.99
	-1%	Normal distribution	83.65	84.04	86.05	88.06	88.44	83.34	83.74	85.84	87.94	88.34
		Actual distribution	83.51	83.96	86.11	87.92	88.25	83.16	83.64	85.89	87.85	88.20
	0%	Normal distribution	83.69	84.07	86.08	88.09	88.47	83.48	83.90	86.05	88.21	88.62
		Actual distribution	83.58	84.00	86.11	88.04	88.39	83.47	83.87	86.06	88.19	88.63
	1%	Normal distribution	83.49	83.91	86.14	88.36	88.79	83.25	83.73	86.26	88.80	89.28
		Actual distribution	83.40	83.87	86.15	88.34	88.74	83.35	83.79	86.24	88.83	89.41
	2%	Normal distribution	83.14	83.64	86.23	88.83	89.32	82.81	83.41	86.52	89.64	90.24
		Actual distribution	83.24	83.71	86.22	88.88	89.36	83.17	83.66	86.41	89.78	90.59
50000	-2%	Normal distribution	84.82	85.01	85.99	86.98	87.17	84.48	84.67	85.67	86.67	86.86
		Actual distribution	84.77	84.97	86.02	86.93	87.11	84.40	84.63	85.69	86.63	86.79
	-1%	Normal distribution	84.91	85.09	86.01	86.93	87.10	84.66	84.85	85.79	86.74	86.92
		Actual distribution	84.87	85.06	86.02	86.89	87.07	84.63	84.83	85.80	86.72	86.89
	0%	Normal distribution	84.94	85.11	86.01	86.91	87.08	84.80	84.99	85.95	86.91	87.09
		Actual distribution	84.92	85.11	86.02	86.90	87.06	84.82	85.00	85.95	86.92	87.10
	1%	Normal distribution	84.83	85.02	86.00	86.99	87.17	84.84	85.04	86.12	87.20	87.40
		Actual distribution	84.83	85.02	86.01	86.99	87.18	84.82	85.02	86.12	87.18	87.41
	2%	Normal distribution	84.69	84.90	86.03	87.16	87.38	84.75	84.99	86.27	87.56	87.80
		Actual distribution	84.70	84.90	86.02	87.16	87.40	84.80	85.02	86.25	87.60	87.85
100000	-2%	Normal distribution	85.17	85.30	85.99	86.68	86.82	84.81	84.95	85.65	86.36	86.49
		Actual distribution	85.13	85.28	86.01	86.66	86.77	84.78	84.92	85.67	86.34	86.46
	-1%	Normal distribution	85.21	85.33	85.99	86.64	86.76	85.00	85.13	85.80	86.46	86.59
		Actual distribution	85.20	85.32	86.00	86.62	86.74	85.00	85.12	85.80	86.45	86.57
	0%	Normal distribution	85.23	85.35	85.99	86.63	86.75	85.13	85.26	85.94	86.62	86.75
		Actual distribution	85.21	85.34	85.99	86.62	86.73	85.11	85.26	85.94	86.60	86.74
	1%	Normal distribution	85.17	85.30	85.99	86.68	86.82	85.19	85.33	86.09	86.85	86.99
		Actual distribution	85.16	85.30	85.99	86.69	86.82	85.19	85.33	86.08	86.85	86.99
	2%	Normal distribution	85.08	85.23	86.01	86.80	86.95	85.16	85.34	86.26	87.18	87.36
		Actual distribution	85.09	85.23	86.01	86.80	86.97	85.17	85.36	86.25	87.21	87.39

Table A7. Percentiles of distribution of estimates for life expectancy at age 60, derived from simulated distribution and from corresponding normal distribution (simulations are based on stable populations and mortality schedules corresponding to the life table of the Japanese female population in 2007).

Population size	Growth rate	Percentile:	Unabridged life tables 2.5%	5.0%	50.0%	95.0%	97.5%	Abridged life tables 2.5%	5.0%	50.0%	95.0%	97.5%
5000	-2%	Normal distribution	26.23	26.53	28.12	29.71	30.02	25.76	26.09	27.82	29.54	29.87
		Actual distribution	26.24	26.53	28.14	29.70	30.00	25.78	26.10	27.79	29.54	29.96
	-1%	Normal distribution	26.05	26.39	28.15	29.92	30.26	25.71	26.08	28.03	29.97	30.35
		Actual distribution	26.08	26.41	28.14	29.92	30.28	25.76	26.12	28.01	30.01	30.46
	0%	Normal distribution	25.77	26.17	28.23	30.29	30.68	25.41	25.87	28.23	30.60	31.05
		Actual distribution	25.85	26.21	28.20	30.32	30.76	25.59	26.01	28.17	30.68	31.28
	1%	Normal distribution	25.29	25.78	28.35	30.92	31.41	24.84	25.43	28.52	31.60	32.19
		Actual distribution	25.42	25.88	28.31	30.93	31.45	25.23	25.73	28.35	31.79	32.60
	2%	Normal distribution	24.45	25.11	28.55	31.99	32.65	23.70	24.57	29.11	33.64	34.51
		Actual distribution	24.89	25.45	28.41	31.88	32.87	24.98	25.49	28.75	33.87	35.48
10000	-2%	Normal distribution	26.76	26.97	28.08	29.20	29.41	26.33	26.56	27.75	28.95	29.17
		Actual distribution	26.75	26.98	28.08	29.20	29.41	26.35	26.60	27.74	28.96	29.21
	-1%	Normal distribution	26.64	26.87	28.10	29.33	29.57	26.29	26.55	27.90	29.26	29.52
		Actual distribution	26.62	26.85	28.10	29.33	29.55	26.33	26.58	27.89	29.25	29.55
	0%	Normal distribution	26.40	26.68	28.13	29.57	29.85	26.20	26.51	28.13	29.74	30.05
		Actual distribution	26.42	26.68	28.12	29.56	29.86	26.29	26.57	28.08	29.79	30.13
	1%	Normal distribution	26.12	26.45	28.20	29.94	30.27	25.82	26.22	28.34	30.46	30.87
		Actual distribution	26.16	26.48	28.18	29.96	30.31	26.02	26.33	28.28	30.55	31.09
	2%	Normal distribution	25.67	26.09	28.31	30.52	30.95	25.23	25.77	28.61	31.45	32.00
		Actual distribution	25.80	26.19	28.26	30.56	31.06	25.70	26.11	28.47	31.65	32.40
50000	-2%	Normal distribution	27.46	27.56	28.06	28.56	28.66	27.07	27.18	27.72	28.26	28.36
		Actual distribution	27.46	27.56	28.06	28.57	28.67	27.08	27.18	27.72	28.26	28.36
	-1%	Normal distribution	27.40	27.51	28.07	28.62	28.73	27.13	27.25	27.86	28.47	28.59
		Actual distribution	27.41	27.51	28.06	28.62	28.73	27.14	27.25	27.86	28.47	28.58
	0%	Normal distribution	27.33	27.45	28.08	28.71	28.83	27.16	27.30	28.02	28.75	28.89
		Actual distribution	27.31	27.45	28.08	28.71	28.84	27.17	27.31	28.02	28.77	28.91
	1%	Normal distribution	27.16	27.30	28.07	28.84	28.98	27.11	27.29	28.19	29.10	29.27
		Actual distribution	27.17	27.30	28.07	28.84	28.99	27.14	27.30	28.18	29.12	29.30
	2%	Normal distribution	26.96	27.14	28.10	29.05	29.24	26.97	27.20	28.36	29.53	29.75
		Actual distribution	26.97	27.15	28.09	29.07	29.26	27.05	27.25	28.33	29.56	29.82
100000	-2%	Normal distribution	27.64	27.71	28.06	28.41	28.48	27.26	27.34	27.71	28.08	28.15
		Actual distribution	27.63	27.70	28.06	28.41	28.48	27.26	27.33	27.71	28.08	28.15
	-1%	Normal distribution	27.59	27.66	28.06	28.45	28.52	27.35	27.44	27.86	28.29	28.37
		Actual distribution	27.58	27.66	28.06	28.44	28.51	27.36	27.44	27.86	28.29	28.38
	0%	Normal distribution	27.53	27.61	28.06	28.51	28.59	27.40	27.50	28.01	28.52	28.62
		Actual distribution	27.53	27.61	28.06	28.51	28.59	27.41	27.50	28.01	28.53	28.63
	1%	Normal distribution	27.42	27.52	28.06	28.60	28.70	27.41	27.53	28.16	28.79	28.91
		Actual distribution	27.43	27.52	28.06	28.61	28.70	27.44	27.55	28.16	28.81	28.93
	2%	Normal distribution	27.29	27.41	28.08	28.75	28.88	27.36	27.52	28.34	29.17	29.33
		Actual distribution	27.30	27.43	28.08	28.76	28.88	27.40	27.55	28.33	29.19	29.37