DYNAMICS OF RESERVOIR OPERATION UNDER POWER PRODUCTION

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One persistant problem within mathematical simulation modelling of river basins has been the operation of reservoirs--especially reservoirs for power production. These reservoirs consistantly end up empty in their attempt to meet power targets. The reason for this behavior arises from the combined effects of stochastic inflows and the prescribed operating rule. Most simulation models follow the well known "normal" operating policy which tries to fulfill the targets (either for power production, irrigation demands, etc.) as long as there is water in the reservoir. No thought is given for the next period's demand. Such a policy is especially poor in power production because power (P) is generated as a combination of outflow (Q) and hydraulic head (H),

$$(1) \qquad P = \alpha QH \quad ,$$

so that the worse the state of the reservoir (near empty) the worse it becomes.

This small paper looks at some of the dynamics of reservoir operation and suggests some areas of continued work. Furthermore, new operating rules could be formulated that would consider the expected value of future power.

Dynamics of Power Production

Power production is a function of both the discharge and the hydraulic head;

(2) $P = \alpha QH$

where

P = power (k-watts/hour)

Q = discharge (m /s)

H = hydraulic head (m)

α = conversion constant (2.73×10^{-6}) .

Figure 1 shows lines of constant power production. One interesting question is: starting at an initial head, H_o, where will the state of the reservoir be at time t_1? This state will depend upon the operating policy. Assume that the operating policy is to maintain a constant level of power production, P_o. Then starting from H_o, Q_o the reservoir will follow the P_o power curve to H_1, Q_1. Another operating rule may be to hold Q_o constant. Then starting from H_o, Q_o, the amount of power produced will decrease and after a time t_1 the reservoir will be at a different set of head and discharge H_1', Q_1'. Figure 2 illustrates these two operating rules. The location of H_1, Q_1 and H_1', Q_1' will now be found analytically.

Case 1: Constant Power Production, P_o. The assumption that the inflow for the season will occur instantaneously at $t = 0$ will be made. The storage after the inflow will be S_o with the hydraulic head H_o. The power at any time t will be

(3) $P = \alpha Q(t)H(t)$

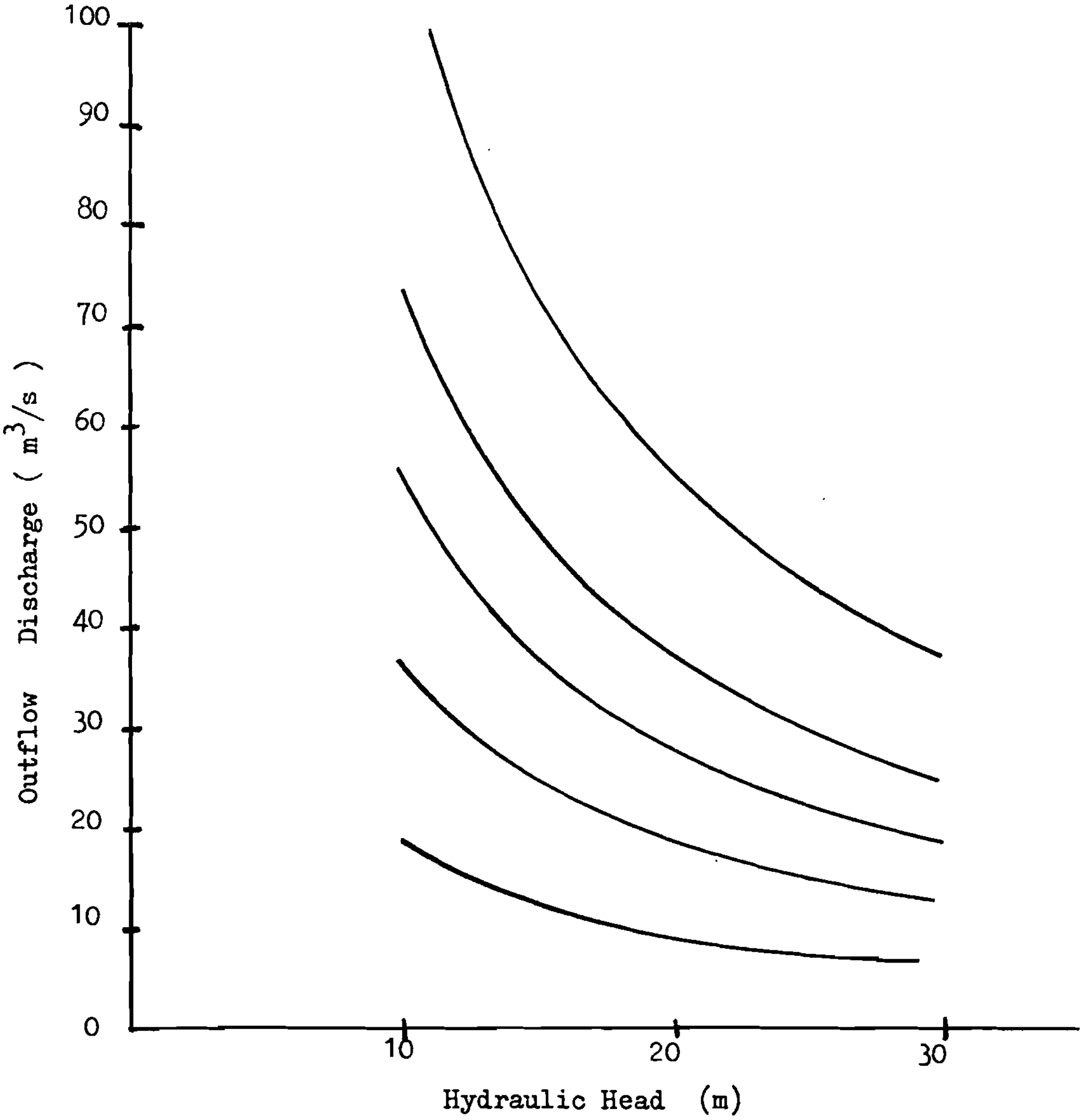

100
90
80
70
60
50
40
30
20
10
0
Outflow Discharge (m^3/s)
10
20
30
Hydraulic Head (m)

Figure 1.

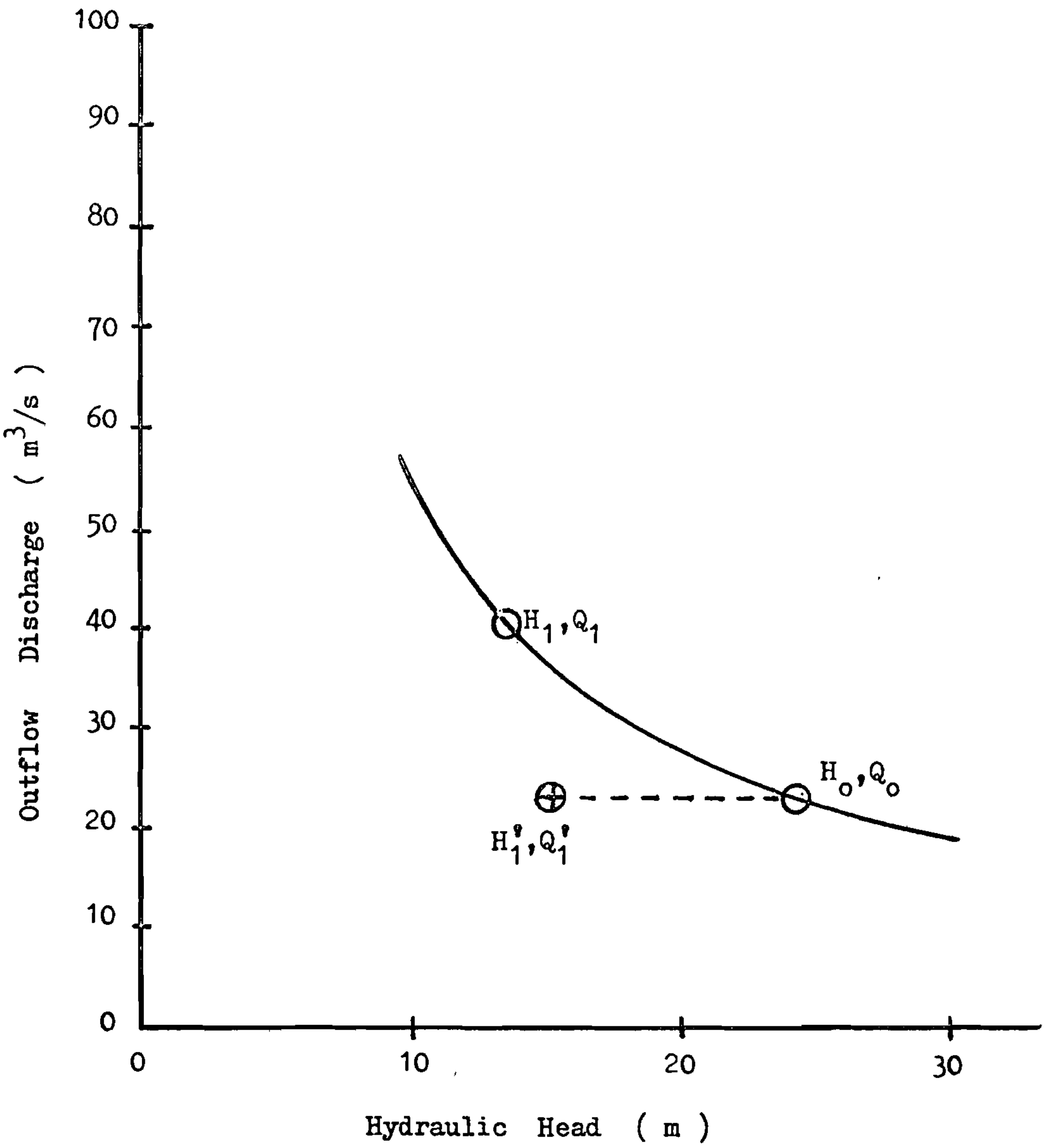

100
90
80
70
60
50
40
30
20
10
0
Outflow Discharge (m3/s)
H1,Q1
Ho,Qo
H1',Q1'
0
10
20
30
Hydraulic Head (m)

Figure 2.

and the energy generated up to time t = T is

$$(4) \qquad E(t) = \int_0^T P dt = \int_0^T \alpha Q(t) H(t) dt \quad .$$

Since P is constant at P_o, the energy developed is $P_o T$. To find the state of the reservoir the right hand integral must be solved. From the dynamics of reservoir storage the following holds true

$$(5) \qquad Q(t) dt = -dS(t)$$

where dS(t) is the change in storage at time t.

A genéral relationship between hydraulic head and storage is

$$(6) \qquad H = (\gamma S)^{\beta}$$

where

β is about .5

γ is a constant

$$(7a) \qquad \frac{dS}{dH} = \frac{1}{\beta\gamma} H^{\frac{1-\beta}{\beta}}$$

and for = .5

$$(7b) \qquad \frac{dS}{dH} = \frac{2}{\gamma} H$$

Equation (4) can be rewritten, using equations (5) and (7a) as

(8) $$P_oT = \int_{H_o}^{H_1} -\frac{\alpha}{\beta\gamma} H^{1/\beta}\, dH = \frac{\alpha}{\gamma(1+\beta)} \left. H^{\frac{1+\beta}{\beta}} \right]_{H_1}^{H_o}$$

Thus H_1, the final head level after time $t = T$ is just

(9) $$H_1 = \left[H_o^{\frac{1+\beta}{\beta}} - \frac{(1+\beta)\gamma}{\alpha} P_oT \right]^{\frac{\beta}{1+\beta}}$$

and for $\beta = .5$

(10) $$H_1 = \left[H_o^3 - \frac{1.5\gamma}{\alpha} P_oT \right]^{1/3}$$

Case 2. Outflow Discharge Is Constant At Q_o. For this analysis, we still assume that the inflow for the season occurs instantaneously at $t = 0$, resulting in an initial storage S_o. The outflow discharge, Q_o, is constant which results in a decreasing power output. If the initial storage level was S_o, then the storage level after $t = T$ would be $S_o - Q_oT$. The total energy output, $E(t)$, can be found by solving

(11) $$E(t) = \int_{t=0}^{T} \alpha\, Q\, H(t)dt = \alpha\gamma\beta \int_{S_o}^{S_o-Q_oT} - S^\beta dS$$

$$= \frac{\alpha\gamma\beta}{\beta+1} [S_o^{\beta+1} - (S_o - Q_oT)^{\beta+1}]$$

The second operating rule produces less energy but since the final hydraulic head is larger the <u>potential</u> for generating power in the next time period is higher. This may be important if the probability of a low inflow is quite high.

Case 3. Constant Power P_o, Constant Inflow Q_I. A more realistic analysis would consider the condition when a constant inflow Q_I occurs throughout the season. This analysis is much more complicated because the constant inflow contributes to a smaller and smaller portion of the energy target as the hydraulic head (and storage) decreases. The remaining portion of the power target is met by taking water from storage. Figure 3 illustrates this. The dotted lines are curves of constant power, like those illustrated in Figure 1. The solid lines illustrate the power component generated from water released from storage when Q_I is as specified. The curves correspond to a total power target of 3×10^{-3} m-w/hr. When the curves indicate $Q_{st} = 0$, that is, no water being released from storage, then the reservoir is either filling or remaining stationary. For a power target P, the curves follow the relationship:

$$(12) \qquad P = \alpha[Q_I + Q_{st}(t)] \cdot H(t)$$

Let H_o be the intial hydraulic head and $Q_{st}(0)$ be the initial outflow from storage, i.e. $Q_{st}(t)$ at time $t = 0$. Then it can be shown that:

$$(13) \qquad H(t) = H_o \left[\frac{Q_{st}(0) + Q_I}{Q_{st}(t) + Q_I}\right] \quad .$$

Using the head-storage relationship of equation (7b), i.e. $\beta = .5$ and the outflow storage relatioship of equation (5), then the following relatioships hold

$$(14a) \qquad dS(t) = -Q_{st}(t)dt$$

$$(14b) \qquad \qquad = \frac{2}{\gamma} H(t)\ dH(t)$$

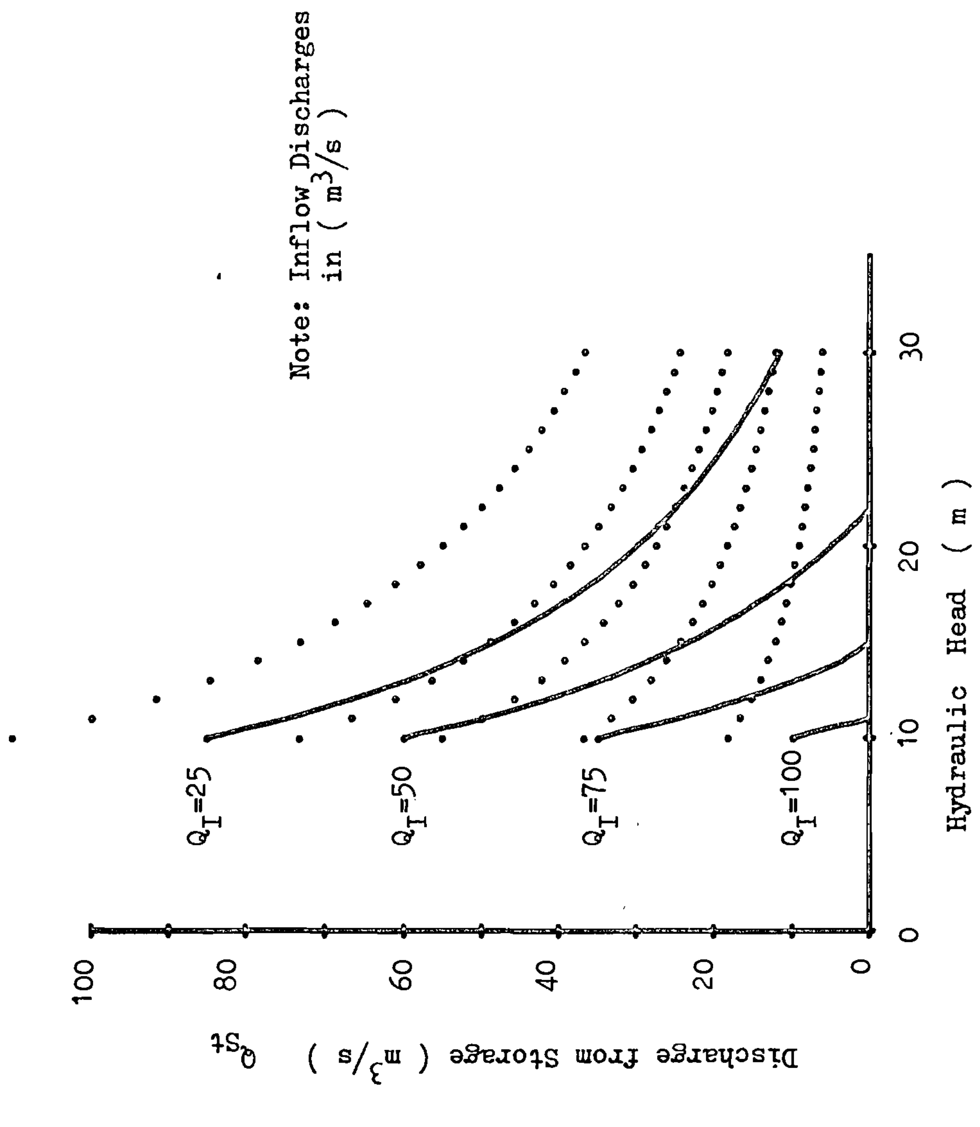

Note: Inflow Discharges
in (m³/s)
Q_I=25
Q_I=50
Q_I=75
Q_I=100
Hydraulic Head (m)
0
10
20
30
Discharge from Storage (m³/s) Q_St
0
20
40
60
80
100

Figure 3.

which results in

$$\text{(15a)} \qquad H(t)dt = \frac{-2H\ (t)dH(t)}{\gamma Q(t)}$$

$$\text{(15b)} \qquad = \frac{2H^3(t)\ dH(t)}{H_o[Q_{st}(o) + Q_I] - H(t) \cdot Q_{st}(o)}$$

The energy generated in time T can be found by integrating equation (12) with respect to time:

$$\text{(16)} \qquad E = P \cdot T = \alpha \int_{t=0}^{t=T} [Q_I + Q_{st}(t)] \cdot H(t)dt \quad .$$

Equation (16) can be divided into two components, one component accounts for the energy developed by the water leaving storage. The value of this energy is

$$\text{(17)} \qquad E_1 = \int_{t=0}^{t=T} \alpha\, Q_{st}(t)\ H(t)dt$$

which is similar to case 1 when no inflow occurred. The solution of equation (17) is

$$\text{(18)} \qquad E_1 = \frac{2\alpha}{3\gamma} [H_o^3 - H_T^3] \quad .$$

The other component is from the energy developed by the inflow and can be calculated by solving:

$$\text{(19)} \qquad E_2 = Q_I \int_{t=0}^{t=T} H(t)dt$$

Case 4. Constant Inflow, Constant Outflow. The fourth operating condition that can be analyzed is when there is a constant ourflow target and a constant inflow discharge. Let

Q_o represent the outflow and Q_I the inflow. Then the discharge into or out of storage is just

(20) $$Q_{st} = Q_I - Q_o \quad .$$

The final storage, after a time T, given S_o being the initial storage, is

(21) $$S_t = S_o + Q_{st} \cdot T \quad .$$

Assuming that the head-storage relationship of equation (6) is still valid, then the final hydraulic head, H_T is

(22) $$H_T = \gamma^{\beta} \cdot (S_o + Q_{st}T)^{\beta} \quad .$$

The amount of energy produced in time T is

(23) $$E = \alpha Q_I \int_{t=0}^{t=T} H(t)dt - \alpha \int_{t=0}^{t=T} H\, Q_{st}\, dt$$

The first part can be solved utilizing equation (22) and is

(24) $$\frac{\alpha\gamma^{\beta} Q_I}{(\beta + 1)} \{S_o + Q_{st}T)^{1+\beta} - S_o^{\,1+\beta}\}$$

The amount of energy produced in time T depends upon whether Q_{st} is 0 (inflow equals outflow), > 0 (inflow greater than outflow) or < 0 (inflow less than outflow). These three cases can be easily analyzed

a) Inflow equals Outflow

(23) $$E = Q_I \int_{t=0}^{t=T} H\, dt = \alpha Q_I \gamma^{\beta} S_o^{\,\beta}\, T \quad ;$$

b) Inflow greater than Outflow

(24) $$E = Q_I \alpha \int_{t=0}^{t=T} H(t)dt - \alpha \int H(t)\ Q_{st}dt$$

The first part can be solved utilizing equation (22) and is

(25) $$\frac{\alpha\gamma^{\beta}}{(\beta+1)} Q_I \{(S_o + Q_{st}\ T)^{1+\beta} - S_o^{(1+\beta)}\}$$

The second part of equation (21) can be solved by utilizing equations (5) and (6). The solution to the second part is

(25) $$\frac{\alpha\gamma^{\beta}}{(1+\beta)} Q_{st}(S_o + Q_{st}\ T)^{1+\beta} - S_o^{1+\beta}$$

So the complete solution to (24) is

(26) $$E = \frac{\alpha\gamma^{\beta}}{(1+\beta)} \Big\{Q_I[(S_o + Q_{st}\ T)^{1+\beta} - S_o^{1+\beta}] + (S_o + Q_{st} \cdot T)^{1+\beta} - S_o^{1+\beta}\Big\}$$

c) Inflow less than Outflow

(25) $$E = \alpha Q_I \int_{t=0}^{T} H(t)dt + \alpha \int_{t=0}^{t=T} H(t)\ Q_{st}\ dt$$

Using

(26) $$H_T = \gamma^{\beta} \cdot (S_o - Q_{st}\ T)^{\beta}$$

the first integral equals

(27) $$\frac{\alpha Q_I \gamma^{\beta}}{(1+\beta)} [S_o^{1+\beta} - (S_o - Q_{st}\ T)^{1+\beta}]$$

Using Equations (5) and (6) the second integral equals

$$(28) \qquad \frac{\alpha\gamma^{\beta}}{(1+\beta)} [S_o^{1+\beta} - (s_o - Q_{st}\ T)^{1+\beta}]$$

So that the complete solution of Equation (28) is

$$(29) \qquad E = \frac{\alpha\gamma^{\beta}}{1+\beta} \{ Q_I\ [S_o^{1+\beta} - (S_o - Q_{st}\ T)^{1+\beta}] + S_o^{1+\beta} - Q_{st}\ T)^{1+\beta} \}$$

Some Results

It is interesting to look at some results of various operating rules. The most realistic cases are when inflow arrives throughout the season, rather than as a 'lump' at the beginning of the season. Consider Figure 4 which simulated the generation of a constant power target. The inflow level was 50 m^3/sec for both time periods. Line A considered the inflow to be uniformly distributed over time while line B assumed that the inflow came instantaneously at the beginning of the time period. While reservoir case A went 'dry' in the middle of period 2, case B ended period 2 with a hydraulic head of 16.25 feet.

Figure 5 illustrates some reservoir behavior under various inflow patters for constant power production. Case A was for Q_1 = 50 m^3/s, Q_2 = 50 m^3/s; Case B was for Q_1 = 50 m^3/s, Q_2 = 75 m^3/s, Q_3 = 75/100 m^3/s; and Case C was for Q_1 = 50 m^3/s, Q_2 = 100 m^3/s, and Q_3 = 50 m^3/s. In studying these curves there appears to be two decision

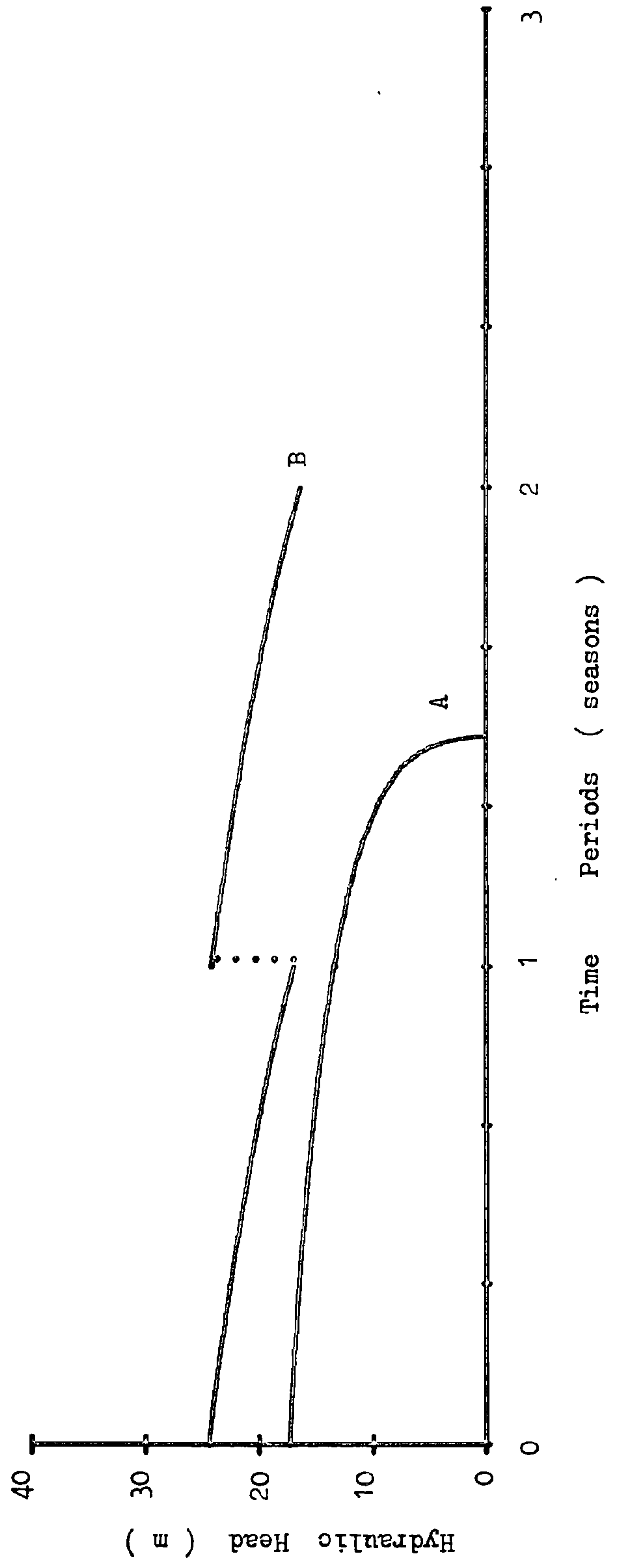


Figure 4.

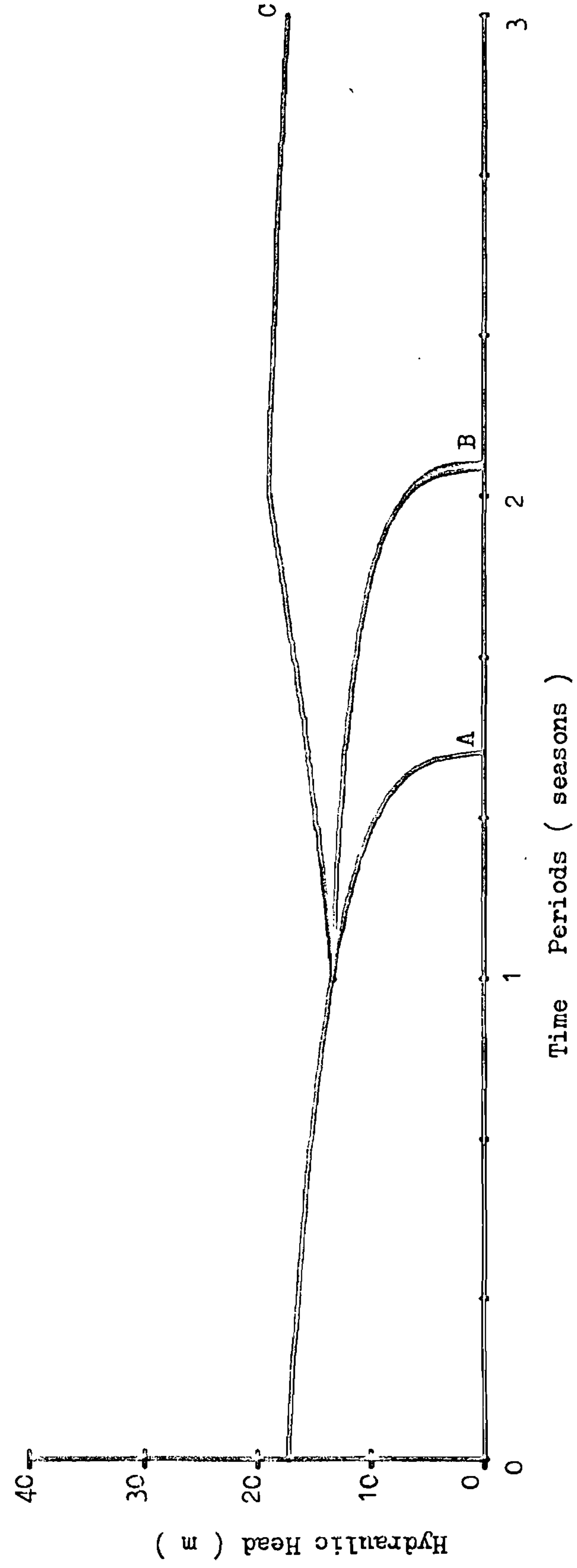
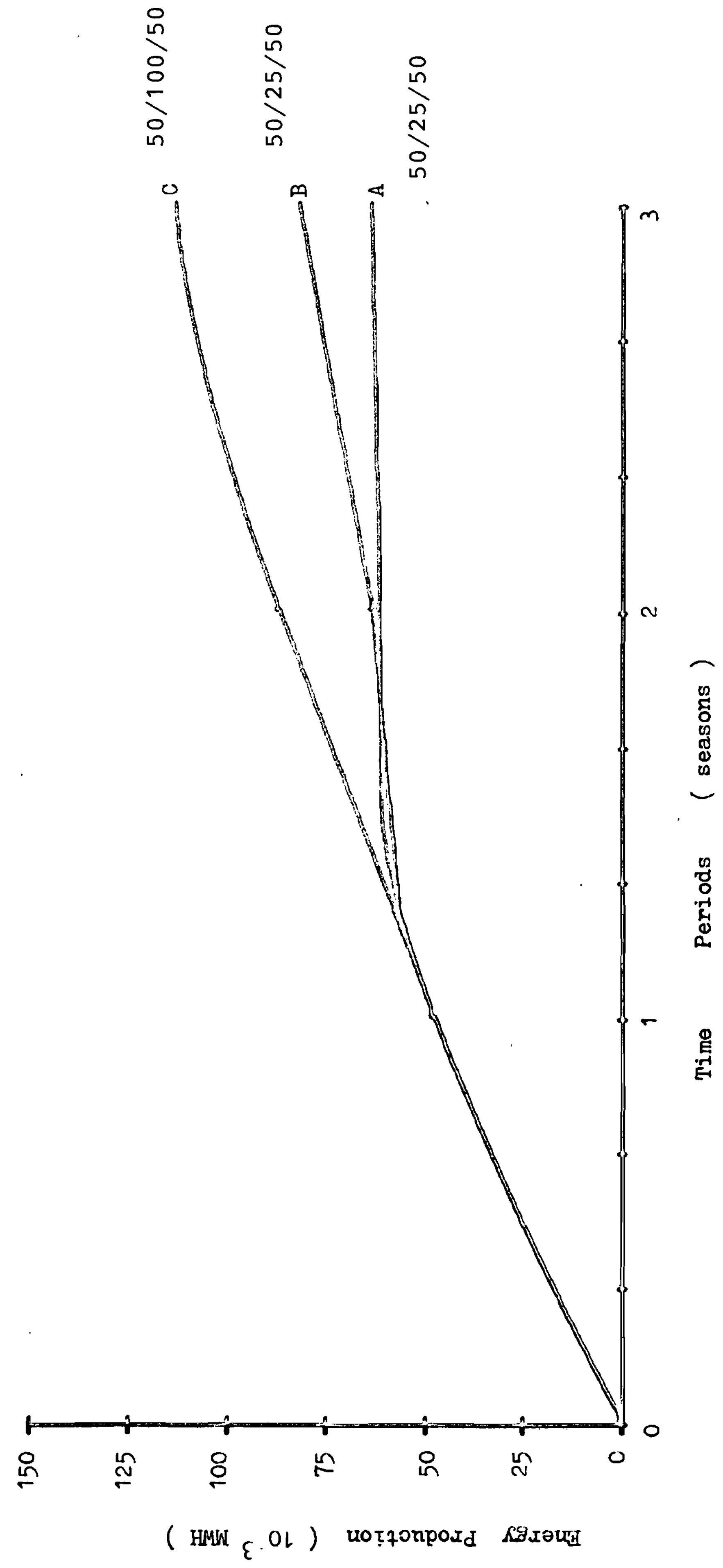


Figure 5.



C 50/100/50
B 50/25/50
A 50/25/50
0
1
2
3
Time Periods (seasons)
0
25
50
75
100
125
150
Energy Production (10^3 MWH)

Figure 6.

variables of interest. One is the power target P and the other is a minimum hydraulic head, H_o. The minimum hydraulic head represents the lowest level that the reservoir should reach before power production is cut back. This cutting back of power saves the hydraulic head for power production in the next time period. Figure 6 illustrates this point. In Figure 6 a constant outflow discharge is specified to generate power (75 m^3/s). Case A has a minimum bydraulic head of 0 (empty) and an inflow patter of 50/25/50 m^3/s. Once the reservoir was empty, it was almost impossible to get the reservoir back into a state where a significant amount of energy was produced. Case B had a minimum hydraulic head of 8.5 m. Power production was cut back when the level was reached but energy was still being generated. Case C had an inflow pattern of 50/100/50 m^3/s and no cut back was experienced. The outflow discharge target, Q_o, (or the power target, P) and the minimum hydraulic head, H_o, are two decision variables which should be optimized. In fact, a stochastic dynamic programming algorithm may be quite appropriate for this problem.

Summary

This paper looks at some dynamics of reservoir operation when power production is considered. These dynamics affect the amount of energy that can be produced in the current time period and affect the potential energy that could be produced in future time periods.

Such dynamics could be useful for studying the steady state probability that the reservoir will be in a particular state, given the operating policy. The dynamics are also useful if optimization of reservoir operating rules is to be performed.