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REAL-TIME CONTROL OF WATER QUALITY AND QUANTITY

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PREFACE

IIASA has for some time been associated with studies on real-time forecasting and control of water resources systems. The Institute itself has organized one workshop on the topic and largely as a result of that first workshop the Institute of Hydrology (UK) held a second, extended workshop in July, 1977. This report has been prepared as a contribution to the proceedings of the second workshop.

The report addresses the problem of combined operational management of stream discharge and water quality in a river basin. Emphasis is placed upon water quality aspects of the management problem and in this sense the report illustrates the type of work which may later be undertaken within the scope of the Resources and Environment Area Task 2--*Models for Environmental Quality Control and Management.* At present Task 2 is concerned with the development and application of water quality models. This report discusses the use of models for real-time forecasting of pollutant movement in a river; it examines features of river basin management within the framework of multivariable control theory; and it offers a speculative investigation of the application of fuzzy control techniques.

ABSTRACT

The paper considers the application of estimation, forecasting, and control techniques to the problem of combined real-time control of stream discharge and water quality in a river basin. A simple recursive estimation procedure is presented for the on-line estimation of pollutant movement and dispersion in a reach of river. Some important features of the linear multivariable control system design problem are then considered in the context of controlling downstream discharge and quality given an upstream effluent discharge and surface storage facility as input control variables. Because of the very basic difficulties of visualizing water quality regulation according to most conventional control engineering approaches, a final section of the paper offers a speculative examination of the possibilities for fuzzy control applications in operational river basin management.

Real-Time Control of Water Quality and Quantity*

M.B. Beck

1. Introduction

This paper deals primarily with some problems, and possible solutions, of *combined* real-time control of river water quantity and quality. Hitherto, in so much as real-time operations in water resources systems have been discussed at all, the literature shows a strong tendency to distinguish the subject of stream-flow forecasting/control from the subject of in-stream water quality control, see Szöllösi-Nagy and Wood (1977). Most studies of in-stream quality regulation have focused on a *single* index of water quality, usually dissolved oxygen (DO) concentration, and the associated problem of effluent discharge control. Thus before considering the combined quantity/quality problem it is appropriate first to return to the single subject of quality control, and to place it in the broader context of river basin management.

There are a number of reasons why few, if any, cases of real-time control of river basin water quality have been demonstrated in practice. Among the most important of these reasons are:

i) a lack of suitable instrumentation and monitoring facilities for all the indices of water quality that may from time to time be of interest;

ii) the invalidity of the assumption that wastewater discharge characteristics are freely manipulatable for control purposes;

iii) a lack of clearly defined objectives for the management of water quality in a river basin and a lack of precisely specified standards of stream quality to be maintained by the application of real-time control.

With reference to item i) it has been said that [Briggs (1975)] reliable sensors exist for measurement of temperature, dissolved oxygen (DO), pH value, conductivity, and suspended matter. More recently, for instance Briggs et al (1977), ammonia, nitrate, and organic matter have been added to the list of readily measurable indices, although the reliability of these latter is perhaps still a matter for debate. On the subject of real-time

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waste discharge control several articles are available which give an exhaustive treatment of current problems and (limited) capabilities [Olsson (1977) , Olsson and Hansson (1976), Andrews and Stenstrom (1977), Ranta et al (1977)].

The third item, operational objectives and stream quality standards, poses two particularly awkward questions; a consideration of these questions will form the central part of this paper. We do not, however, intend to address the particularly controversial issue of choosing a set of desired values for stream quality standards, although it may be noted that such a choice is probably best cast within a statistical framework [Taylor (1977)]. The first major, potential problem is that the combination of water quantity and quality control, even for the case of a single index of quality, changes the principal features of the control system design from a single variable to a *multivariable* control situation. The second major problem, which arises partly from this multivariable situation and partly from the lack of clearly defined operational goals, concerns the implementation of control (regulatory) actions in the face of *imprecise and conflicting objectives* for quantity/quality control. By "conflicting objectives" we mean here that the attainment of two (or more) individual desired levels of variables are not mutually inclusive events. Hence there may need to be some coded hierarchy of real-time control objectives. For example, is it more important to avoid low DO regimes than high nitrate levels? By "imprecise objectives" we are questioning whether these "low" DO regimes and "high" nitrate levels can be sensibly interpreted as some exact point value(s) or should they be more appropriately represented as loosely defined ranges of values?

The first part of this paper (section 2), however, deals with a more straightforward aspect of river water quality forecasting, namely the on-line estimation of pollutant dispersion along a river. Section 3 presents an example of the multivariable control system design problem. The final section, and this is the section of the paper upon which primary emphasis is placed, attempts to introduce a framework for organizing a systematic approach to the problems of real-time river basin management. In particular, the concept of fuzzy control [Zadeh (1965, 1973)] is introduced, although here more as an analytical tool than as an immediately practicable element of control system technology. The attraction of this concept stems from its ability to deal both with imprecisely defined operational objectives and with linguistic, empirical statements about river basin management.

2. On-Line Estimation of Pollutant Dispersion in a River

The purpose of this section is to illustrate how a compact estimation algorithm might be employed for the on-line estimation of pollutant dispersion in a river. This problem is the water quality analog, as it were, of rainfall-runoff/river flow prediction. A similar simple recursive least squares estimator is also used to solve the parameter estimation problem which is implicit in the self-tuning controller discussed by Ganendra (1977). The essence of the approach suggested here is that a measure of dispersion can be interpreted from the impulse response function, which itself is computed from an input (upstream)/output (downstream) time-series model of a reach of river. An important modification of the estimation algorithm is that it is required to track time-varying parameter values; indeed, the estimator would be substantially deficient in this particular application if it did not possess such a capability.

Consider then a river system in which there is an upstream discharge of effluent and a downstream abstraction of public potable water supply. The downstream abstractor would almost certainly require protection of his intake in the form of advanced warning of accidental toxic spillages and of storm discharges from the upstream sewer network and treatment plant. The most important information is the time taken for the peak loading to reach the abstractor and the degree of attenuation in dissolved material concentrations effected by dispersion mechanisms.

Since, as we have said, suitable instrumentation does not exist for frequent measurement of the vast majority of water pollutants it is assumed that sampled observations of the upstream specific conductivity, $u(t_k)$, and downstream specific conductivity, $y(t_k)$, are available at time t_k for computation (see Figure 1). Further assumptions are that the toxic and undesirable substances of interest behave as conservative substances in the river and duplicate the same patterns of distribution as conductivity. There is, of course, the not inconsiderable requirement that the accidental spillage can be detected, its principal chemical composition identified and some estimate of its initial (upstream, in-plant) strength provided.

A) The Model

The input/ouput (black box) relationship between $u(t_k)$ and $y(t_k)$ can be represented by,

$$y(t_k) = A(q^{-1})y(t_k) + q^{-\delta}B(q^{-1})u(t_k) + \xi(t_k) \ , \qquad (1)$$

in which q^{-1} is the backward shift operator,

$$q^{-1}\{y(t_k)\} = y(t_{k-1}) \quad ,$$

and $\xi(t_k)$ is a noise sequence with rational spectral density which accounts for the lumped effects of measurement error and stochastic disturbances.

B) Polynomial Definitions

Before defining the form of the A and B polynomials in equation (1) let us suppose that the pure time delay T_d (see Figure 1) before any response is observed downstream be bounded as

$$\delta \leq T_d \leq \Delta$$

Here δ is an expected minimum value and Δ is an expected maximum value for T_d; both δ and Δ are expressed as integer multiples of the sampling interval $(t_k - t_{k-1})$. The primary feature of the model, equation (1), must be its ability to accommodate and detect a time-variable time delay $T_d(t_k)$ so that,

$$\left.\begin{aligned} B(q^{-1}) &= b_1q^{-1} + b_2q^{-2}+\ldots+b_{\Delta-\delta+n}q^{-\Delta+\delta-n} \quad , \\ &\text{and} \\ A(q^{-1}) &= a_1q^{-1} + a_2q^{-2}+\ldots+a_nq^{-n} \quad . \end{aligned}\right\} \quad (2)$$

In general, the order n of the polynomial A is a priori unknown; we shall discuss a suitable choice of n below.

C) *Least Squares Estimation*

If the following vectors are defined,

$$\left.\begin{aligned} \underline{z}^T(t_k) &= [y(t_{k-1}),\ldots,y(t_{k-n}),\ u(t_{k-\delta-1}),\ldots u(t_{k-\Delta+\delta-n})] \quad , \\ \underline{a} &= [a_1,\ldots,a_n,b_1,\ldots,b_{\Delta-\delta+n}]^T \quad , \end{aligned}\right\} \quad (3)$$

then equation (1) becomes,

$$y(t_k) = \underline{z}^T(t_k)\underline{a} + \xi(t_k) \quad , \qquad (4)$$

and the well-known recursive least squares algorithms for the estimates $\hat{\underline{a}}$ of $\underline{a}$ are given by [e.g. Young (1969),Young (1974a)],

$$\left.\begin{aligned} \hat{\underline{a}}(t_k) &= \hat{\underline{a}}(t_{k-1}) - P(t_{k-1})\underline{z}(t_k)[1 + \underline{z}^T(t_k)P(t_{k-1})\underline{z}(t_k)]^{-1} \bullet \\ &\qquad [\underline{z}^T(t_k)\hat{\underline{a}}(t_{k-1}) - y(t_k)] \quad , \\ P(t_k) &= P(t_{k-1}) - P(t_{k-1})\underline{z}(t_k)[1 + \underline{z}^T(t_k)P(t_{k-1})\underline{z}(t_k)]^{-1} \bullet \\ &\qquad \underline{z}(t_k)P(t_{k-1}) \quad , \end{aligned}\right\} \quad (5)$$

in which

$$P(t_k) = [\sum_{j=1}^{k} \underline{z}(t_j)\underline{z}^T(t_j)]^{-1} \quad . \qquad (6)$$

D) *The Problem of Bias*

For most practical situations of interest $\xi(t_k)$ is not a white, Gaussian sequence and the estimates $\underline{\hat{a}}$ from equation (5) will be biased. In this event the problem of bias can be overcome by posing the estimation in terms of, say, a recursive instrumental variable [Young (1974a)] or recursive generalized least squares [Hastings-James and Sage (1969)] formulation. The essential components of all three algorithms are similar and, therefore, the simple least squares form is retained for the purposes of illustration.

E) *Dynamic Least Squares Estimation (variable parameters)*

It emerges from section *B)* above that a primary function of the estimation algorithms is to track variations in the parameter values, and especially so for the B polynomial coefficients. For example, under low flow conditions in the river we might expect the parameters b_1, b_2 to be zero (since $T_d \rightarrow \Delta$) but they would clearly be non-zero under storm conditions ($T_d \rightarrow \delta$). A straightforward extension of equation (5) permitting the estimation of parameters which vary in a random walk fashion, i.e.

$$\underline{a}(t_k) = \underline{a}(t_{k-1}) + \underline{v}(t_{k-1}) \quad , \tag{7}$$

where $\underline{v}$ is a vector of white, Gaussian $(\underline{0},D)$, stochastic disturbances, is given by [Young (1974a)],

$$\left.\begin{aligned}
\underline{\hat{a}}(t_k) &= \underline{\hat{a}}(t_{k-1}) - [P(t_{k-1}) + D]\underline{z}(t_k)[1 + \underline{z}^T(t_k)(P(t_{k-1}) \\
&\quad + D)\underline{z}(t_k)]^{-1} \cdot [\underline{z}^T(t_k)\underline{\hat{a}}(t_{k-1}) - y(t_k)] \quad , \\
P(t_k) &= P(t_{k-1}) + D - [P(t_{k-1}) + D]\underline{z}(t_k)[1 + \underline{z}^T(t_k)(P(t_{k-1}) \\
&\quad + D)\underline{z}(t_k)]^{-1} \cdot [\underline{z}^T(t_k)P(t_{k-1}) + D] \quad .
\end{aligned}\right\} \tag{8}$$

The choice of D, which should reflect the expected rates of variation in the individual parameters, is an obvious problem. Most likely this matrix should be diagonal with elements denoting the *relatively* stationary properties of the a coefficients in $A(q^{-1})$ but with correspondingly larger entries for the *relatively* quickly changing values of the lower order (b_1, b_2) coefficients of $B(q^{-1})$.

F) *Impulse (pulse) Response Function Calculation*

At time t_k the impulse (pulse) response function is easily computed recursively from the model, equation (4), with the substitution of the current estimates $\underline{\hat{a}}(t_k)$ for $\underline{a}$ (and with $\xi(t_k) = 0$ since this is a deterministic computation). The response could be displayed graphically and from it, or otherwise, the time of travel, T_p, a measure of dispersion, γ, and the expected attenuation in the magnitude of the pulse input of toxic agent can all be readily determined (see Figure 1 for interpretations of T_p and γ).

G) *Computational Effort*

Recursive estimation algorithms have been chosen deliberately because of their advantages of minimal computational requirements and their ability to track time-varying parameters. Other factors which strongly influence the amount of computational effort include the orders of the A and B polynomials. It seems prudent to choose $n = 2$; a choice which would be consistent with the probable approximation of the impulse response to that of an overdamped second-order system.

Further examples of the application of similar techniques are given in Figure 1. We might also note in passing the potential for the application of adaptive (self-tuning) predictors and state reconstruction (Kalman filtering) methods in wastewater treatment plant control [Beck (1977a, 1977b)].

3. The Multivariable Control Problem: An Example

In an earlier paper [Beck (1977b)] some basic principles of control theory, e.g. feedforward, feedback, proportional-integral-derivative (PID) controllers, and the regulator and servomechanism problems, are discussed in relation to wastewater treatment and river water quality control. (It should be added, however, that that paper restricts itself essentially to a consideration of the possibilities for real-time regulation of wastewater discharge characteristics.) The said principles derive primarily from what is now called the *classical* phase of control theory development. From about the late 1950's and continuing on through the 1960's much effort was directed at the development of a *modern* control theory constructed, as it is, around the *state-space* approach to dynamic systems analysis, see for example Szöllösi-Nagy (1976) and Duong et al (1975). Perhaps because it is "modern" and perhaps because the application of control techniques to hydrological processes is itself a recent development, modern, usually "optimal", control has received a much better publicity than classical control in this applied research area, even though it has several notable limitations and disadvantages [see e.g. Young (1974b)]. Two of the most actively pursued avenues of present-day control theory development are *stochastic adaptive* control methods [Wittenmark (1975)], from which comes the self-tuning controller, and (linear) *multivariable* control system synthesis procedures, for example, MacFarlane (1972), MacFarlane and Kouvaritakis (1977). We have no intention of considering the latter in any detail here; rather, some initial comments on the multivariable nature of water quality/quantity control are offered as cautionary messages.

Let us consider again the hypothetical river system of section 2 with one effluent discharger and one potable water abstractor. In this case, however, we have upstream of the effluent discharge a measured lateral inflow to the river system from a gauged tributary and yet further upstream a regulating reservoir (Figure 2). Superficially, at least, the system does not appear to be particularly complicated. The control objectives are to maintain, say, a specified flow and DO concentration* at the downstream abstraction point.

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*It is recognized that in practice DO alone would not be a particularly useful index for determining the suitability of river water for potable abstraction.

For control system design purposes the following categories of variables can be distinguished:

a) Measurable input disturbances, e.g. gauged lateral inflow, d_1, and effluent flow-rate, d_2;

b) (Input) disturbances that are not measurable, or not measurable with sufficient speed, e.g. effluent quality (as BOD), n_1;

c) Regulating (input) control variables, e.g. reservoir discharge, u_1;

d) Controlled output variables, e.g. downstream river water flow, y_1, and water quality, y_2;

e) Desired values for the controlled output variables, r_1, and r_2.

A possible configuration for the controller is given in Figure 3.

Since it may not be obvious at first sight why it is desirable to distinguish between categories a) and b) variables, we shall consider first some simple properties of feedforward and feedback controllers before discussing multivariable control problems. Taken in isolation, the feedforward controller is designed to cancel out the effects on the ouputs of the *measured* disturbances, such as gauged lateral inflow. In other words, information about the disturbance is relayed to the controller (often in the form of a prediction) which in turn initiates control actions designed to nullify the effects of these disturbances before they "reach" the outputs. Many of the disturbances of process behaviour, however, are immeasurable and their effects on the outputs can only be detected as an error between desired and actual values. The principle of the feedback controller is thus one of manipulating the control input(s) so as to reduce the effects of *immeasurable* disturbances.

Out attention can now be turned to the following observations*:

i) that quantitative (flow) disturbances of the system affect *both* the downstream flow and quality conditions;

ii) that the feedback portion of the controller will respond to an error between set-point and actual values in either flow or quality; and hence

iii) that the control input variable simultaneously brings about a change in both output variables.

In control engineering terms these properties of the controlled system are called *interaction*. For an assessment of the implications of interaction with respect to control system design we may consider a simple, hypothetical operational situation. Suppose in Figures 4(a) and 4(b) that at time t_o the controller is maintaining discharge and quality at their respective *steady* values of $r_1(t_o)$ and $r_2(t_o)$ such that the *spatial* profile of DO along the lower part of the river is given by the continuous line in Figure 4(c).

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*In fact, as it is posed in Figure 3 this (initial) illustrative example of multivariable control is a rather special case in which only one input variable is available for control, i.e. reservoir release; the true multivariable problem arises when there is more than one controlled output and two, or more, inputs available to effect this control.

At time $t = t_1$ the desired flow at the abstraction point is reduced in a step change fashion to $r_1(t_1)$. The controller responds and manipulates the reservoir release whereby, after a period of time (the continuous line response in Figure 4(a)), the new desired downstream discharge is being achieved. The accompanying reduction in effluent dilution, however, disturbs the downstream DO concentration away from its set-point value (Figure 4(b)) until a new steady-state situation is reached which might be represented by the dashed line spatial profile in Figure 4(c). The subsequent persistent error between desired and actual performance in Figure 4(b) is referred to as *steady-state offset*. The net result of interaction in this system, therefore, is that while one control objective can be achieved, that is the desired downstream discharge, it appears that the physical constraints of the system are such that the other objective, desired DO concentration, is unlikely ever to be achieved for the given discharge conditions.

Since it is quite possible that the kind of interaction we have described is not evident in the process model used for control system design, considerable care must be exercised in any attempts to improve controlled DO performance. For instance, it is customary to incorporate an "integral action" characteristic into a feedback controller in order to counteract steady-state offset, where integral action means that part of the controller's function is to take corrective action which is proportional to the integral of the error between desired and actual output. Thus because of the physical constraints of mutually exclusive objectives, the inclusion of integral action on the error signal $(y_2 - r_2)$ would be an incorrect design and would cause excessive control effort to be expended to no achievable purpose.

Intuitively, it seems desirable to have an extra "degree of freedom" with which to implement control of both downstream discharge and DO concentration. The extra degree of freedom can be interpreted as an additional regulating input variable, for which there is only one candidate in Figure 3, namely the flow-rate of effluent discharge to the river.* We are now confronted with the truly multivariable control system design problem and here *stability* considerations are of primary importance. Let us suppose that the feedback controller consists in fact of two independent loops:

i) u_1, reservoir release, is manipulated according to the error, $y_1 - r_1$, in downstream discharge; and

ii) u_2, formerly d_2, the effluent discharge rate, is altered as a function of variations in the error, $y_2 - r_2$.

It might further be assumed that when only one of the control loops is operative at any one time it achieves successful performance in its respective task (quantity or quality). However, if both control loops are required to operate simultaneously, it may happen that instability is induced whereby, for example, a decrease in reservoir release is followed by an increase in effluent discharge which in turn is accompanied by a further decrease in reservoir release, and so on. In other words, for this controller,

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*The practicality of this assumption is open to debate, as has been mentioned in the introductory section; nevertheless, manipulation of effluent flow-rates instead of effluent quality characteristics would seem to be a more immediately feasible technical solution to the problem of regulating, say, the total BOD load discharged to the river [Young and Beck (1974), Beck (1977b)].

and given the presence of strong interaction, a small disturbance of downstream quality away from its set-point may be followed by a sequence of control actions which rapidly worsen the situation.

Once stability of the controlled behaviour is assured, one can then examine whether the manner in which the error signals are connected across the feedback controller to the input control signals is such that the distribution of control effort between the two inputs does not encourage any tendency for one regulating input to cancel the effectiveness of the other. And thus the design would continue--by reference to the model of process dynamics--with the satisfaction of less important criteria such as the elimination of steady-state offset (see above), and appropriate *transient response* characteristics. For completeness, therefore, Figure 5 shows three examples of transient responses of the controlled performance when there is a step increase in desired downstream discharge. Response (1) is rather sluggish in nature, which might possibly be a consequence of the integral action used to avoid offset, whereas response (2), although much faster, has the undesirable features of excessive overshoot and oscillations. The compromise solution, response (3), is relatively fast, settles quickly at the desired value, and for most purposes would be deemed a perfectly adequate design transient performance.

4.. Real-Time Control and River Basin Water Quality Management Objectives

There is a tacit assumption in the preceding section that it is realistic to talk about *precise* set-points for water quality in the context of operational (as opposed to legal) river basin management. A further assumption is that the relevant communications and data collection networks are available for fully automatic closed-loop control without any element of human intervention or decision-making. And there are also assumptions made for the purposes of analysis and design, e.g. that *accurate* mathematical models are available and that these models might be constrained within the boundaries of *linear* control system synthesis techniques. Many would disagree with the validity of such assumptions, and in particular there would be disagreement over the benefits of maintaining dissolved oxygen concentration, say, at some precise value.

The nature of river basin water quantity/quality control would seem to be profoundly different from the nature of control in, for example, the petrochemcials industry where it is necessary to have accurate control of precisely determined temperature and pressure profiles in a distillation column. Indeed, the terminology which might be employed for conceptualizing real-time operational control in river basins is somewhat imprecise and informal. For instance, if asked to formulate a set of operating rules for river basin water quantity/quality management it seems natural to start thinking in terms of statements like:

i) "If river discharge increasing rapidly then decrease reservoir release by a lot";

ii) "If DO concentration much less than $4 \mathrm{gm}^{-3}$ then achieve high effluent dilution ratio".

The difficulties of quantifying a "DO concentration much less than $4gm^{-3}$" or of implementing the control action "decrease reservoir release by a lot" are immediately recognizable. On the other hand, if it were possible to obtain a complete list of such rules, then it might also be possible to use them as a support service in the day-to-day decisions which have to be made for river basin management. What is really required, and this is what it is hoped the introduction of fuzzy set theory notions will achieve, is both a framework for evolving a consensus of opinion on appropriate operating rules, and a calculus for manipulation of these rules. This section, then, is a first attempt at considering the application of fuzzy control in river basin management.

4.1 The Concept of Fuzzy Control

The idea of using fuzzy variables as a means of describing *qualitative* relationships of the type illustrated above is due to Zadeh (1965). Let us consider first, however, what is meant by the following fuzzy set,

$$A = \{\text{DO concentration much less than } 4gm^{-3}\}.$$

It is possible to define, as in Figure 6, a *membership function* $\mu(A)$ which expresses the degree of membership of any given DO concentration in the fuzzy set of "DO concentrations much less than $4gm^{-3}$". For $\mu(A) = 1.0$ the corresponding DO concentration is clearly considered to be much less than $4gm^{-3}$, while for $\mu(A) = 0.0$ any corresponding DO concentration is not thought of as belonging to the fuzzy set A. Similarly the fuzzy sets B and C can be defined (see also Figure 6) as alternative characterizations of the river DO regime, where B and C are,

$$B = \{\text{satisfactory DO concentrations}\},$$

$$C = \{\text{high DO concentrations}\}.$$

and so on for flow conditions and any other quality determinants which are of interest to the operational management. Notice, however, that certain ranges of DO concentration, e.g. about $3gm^{-3}$ and about $10gm^{-3}$, are somewhat indeterminately placed with partial membership of more than one fuzzy set.

In this manner a description of the operational quantity/quality *state* of the river system can be obtained. With a suitable discretization of the continuum of DO concentrations the membership functions of Figure 6 can be approximated according to Table 1.

Table 1: Fuzzy Set Definitions for DO Concentrations

DO concn(gm^{-3})	<2.0	2.0-3.0	3.0-4.0	4.0-6.0	6.0-8.0	8.0-10.0	10.0-12.0	>12.0
$\mu(A)$	1.0	0.7	0.2	0	0	0	0	0
$\mu(B)$	0	0.3	0.6	1.0	1.0	0.7	0.1	0
$\mu(C)$	0	0	0	0	0.2	0.8	0.9	1.0

A measured DO concentration of $3.75gm^{-3}$ thus has a membership $\mu(A) = 0.2$ of the fuzzy set A, and a membership $\mu(B) = 0.6$ of the fuzzy set B. So in terms of the overall fuzzy control problem shown in Figure 7 we now have a mechanism through which all the available measurements (or forecasts), as real numbers, can be translated into a framework suitable for manipulation by the fuzzy control algorithms, see part (1) of Figure 7.

The principal feature of the controller algorithms, part (2) in Figure 7, is the list of logical statements about desirable river basin management practice. Suppose that for the river system of Figure 2 operational control decisions can be implemented through the setting of reservoir releases and through the rate of release of treated sewage from a detention lagoon, i.e. effluent discharge manipulation. A suitable set of fuzzy control statements (algorithms) might then be:

1) "IF" (Discharge is approaching flood conditions) "THEN" (Set reservoir release to very low);

2) "IF" (Discharge is low) "AND" "IF" (DO much less than $4gm^{-3}$) "THEN" (Set effluent discharge rate to low);

3) "IF" (Reservoir storage is high) "AND" "IF" (Nitrate concentration is much greater than $15gm^{-3}$) "THEN" (Set reservoir release to medium);

4) "IF" (Reservoir storage is low) "AND" "IF" (DO concentration is satisfactory) "AND" "IF" (Nitrate concentration is low) "THEN" (set reservoir release to low);

5) "IF" (DO concentration is high) "THEN" (Set effluent discharge rate to high).

These five rules, together with the fuzzy set operations of union, intersection, etc., permit the computation of a fuzzy control decision, or action, as output from the algorithms, given the input information on the system's (fuzzy) operational state. It is helpful to visualize the controller as a kind of look-up table: the particular combination of operational conditons determines the entry in the look-up table, and for each entry there will be an associated combination of control actions. Thus we have a real-time, operational analog of the idea of a water quality state map previously discussed by Newsome (1972) in the planning context of river basin management.

We are now in a position to consider part (3) of Figure 7. As with the controller input variables so too can the output variables be defined in fuzzy terms. Figure 8 gives example definitions of three fuzzy sets for reservoir releases. The computations of the controller algorithms lead to an output membership function, say Figure 9, which then has to be interpreted as a unique choice of reservoir release. For the control action of Figure 9 it seems intuitively reasonable that the reservoir release should be set to a value of $2m^3s^{-1}$, at which point $\mu_c = 1.0$. The reason why the computed membership function of Figure 9 does not exactly match any of the set definitions in Figure 8 is because the fuzzy set calculus allows the rules for control to be used to interpolate for situations which they do not nominally cover*. However, difficulties can arise when either there is inherent conflict in the rules, or a combination of operating conditions exists for which no obvious control action has been stated. In such situations the computed membership function might be respresented by Figure 10 where there are two peaks, neither of which has a value $\mu_c = 1.0$.

Clearly in a subject area which is relatively young--the original paper of Zadeh dates only from 1965--it has to be admitted that there are many problems yet to be resolved [Tong (1976, 1977)], one of which we have just illustrated. A second major obstacle to the implementation of fuzzy control algorithms in practice is the derivation of the operating rules and the associated fuzzy set definitions. The synthesis of a "controller" based on the empirical experience of a manager, rather than on the analytical properties of a set of equations, is certainly an appealing concept. But there is still no guarantee of agreement by managers on how to control the quantity/quality resources of a river basin. In particular, the purpose of the following section is to emphasize the point that further discussion of the priorities for management decisions is required.

4.2 A Scenario for River Basin Management

Figure 11(a) shows the first period of real time management of the hypothetical river basin. During this period the three variables of interest, stream discharge, DO concentration, and nitrate concentration remain naturally within their imprecisely defined boundaries of maximum and/or minimum permissible values. Thus no managerial decisions are required. In Figure 11(b), the second operational period, we notice that the DO concentration of the river is progressively decreasing and a violation of stream quality standards seems probable. This probable violation is subsequently confirmed during the third period of operation, Figure 11(c), so that it becomes necessary to take some form of control action. Here we might consider what precisely ought to be the basic policy for that control action. It seems reasonable that the control response to the situation of Figure 11(c) should be one of taking action which returns the offending variable to an acceptable range of operation without degrading the acceptable states of all other variables. Hopefully this kind of policy is already embedded in the logic of the supporting service provided by the fuzzy controller algorithms.** Or alternatively, and in addition, the consequences of different operating decisions could be evaluated for the short-term future, e.g. several days ahead of the current time, by reference to an accompanying simulation model of the river system. The assumption of Figure 11(c) is that successful control action is implemented and the DO regime of the river improves satisfactorily. Later in this third operational period nitrate conditions deteriorate; yet although they make an excursion into the border zone of acceptable/unacceptable, the management's decision is that, say, the costs of control action exceed the benefits and thus control is not implemented.

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*For example, a nitrate concentration which is not "low" but not "much greater than $15 gm^{-3}$".

**The algorithms are easily programmed and usually require little computational time and storage.

The final period of operation, Figure 11(d), shows that again both DO and nitrate concentrations are approaching unsatisfactory levels. In this event the simple control policy outlined above is inadequate since it does not resolve the crucial issues of:

i) whether there exists any control action which would simultaneously return both variables to suitable values; and

ii) if condition i) cannot be met, is it desirable first to deal with the unacceptable nitrate conditions, and then to correct for the violation of DO standards, or vice-versa?

These are questions, then, that should be considered for the future, if and when real-time control becomes realistic and desirable.

5. Conclusions

The problems of real-time control of water quality/quantity in river basin management may well demand a rather unusual approach for their solution. What this paper has attempted to achieve is to isolate the reasons why an unconventional approach is required, e.g. because of imprecise and conflicting objectives for quality control, and to offer something of a speculation on what form such an approach might take, i.e. the introduction of fuzzy control notions. On the other hand, if one is both less realistic and more optimistic about the potential for conventional control applications, section 3 of the paper demonstrates that for combined quantity/quality control there exist non-trivial problems of analysis and control system synthesis. In general, control engineering techniques of the forecasting and state estimation type are more likely to find their way into operational river basin management practice.

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LIST OF FIGURE CAPTIONS:

Figure 1: A Scheme for on-line estimation of pollutant dispersion and time of travel between two spatial locations in a river system.

Figure 2: A hypothetical river system with variables defined for the analysis of the multivariable control problem.

Figure 3: Conceptual controller design; the dashed line indicates that effluent flow may be treated either as a disturbance or as a controlling input variable.

Figure 4: Transient and steady-state responses under controlled conditions: (a) downstream discharge; (b) downstream dissolved oxygen concentration; (c) spatial, steady-state profiles of dissolved oxygen concentration under initial(continuous line 1) and final conditions (dashed line 2).

Figure 5: Example design transient responses of controller performance given a step-change increase in desired downstream discharge.

Figure 6: Membership functions for three fuzzy sets of in-stream dissolved oxygen concentration: A = (DO concentration much less then $4gm^{-3}$); B = (satisfactory DO concentrations); C = (high DO concentrations).

Figure 7: The fuzzy control system synthesis problem.

Figure 8: Membership functions for three fuzzy sets of reservoir release: X = (very low); Y = (low); Z = (medium).

Figure 9: Example computed membership function for reservoir release which is broadly unambiguous.

Figure 10; Example computed membership function for reservoir release which is ambiguous.

Figure 11: Real-time management of a hypothetical river basin - the hatched areas indicate approximate fuzzy boundaries of acceptable river flow/quality conditions.

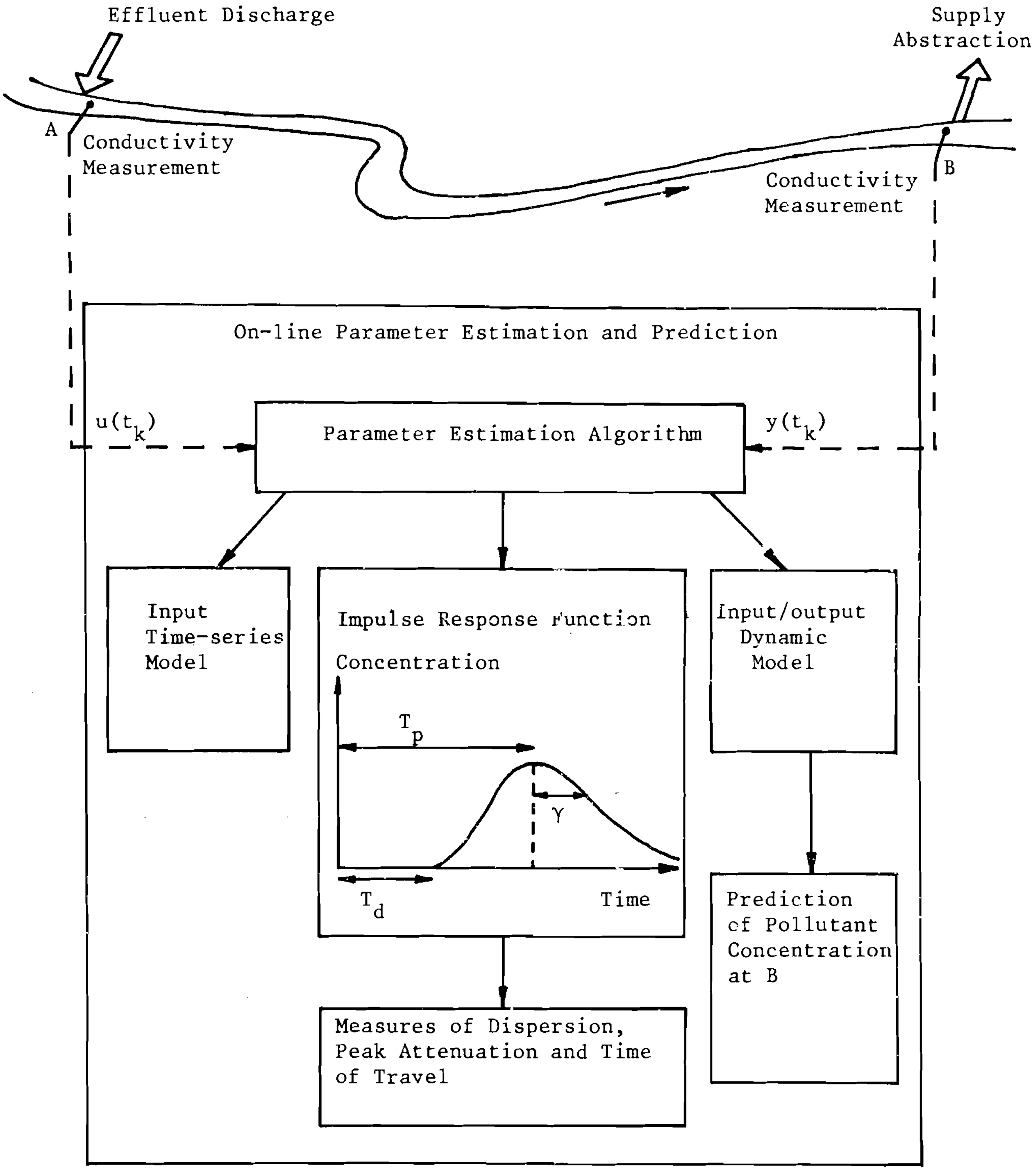
Effluent Discharge
Supply Abstraction
A
Conductivity Measurement
B
Conductivity Measurement
On-line Parameter Estimation and Prediction
$u(t_k)$
Parameter Estimation Algorithm
$y(t_k)$
Input Time-series Model
Impulse Response Function
Concentration
T_p
γ
T_d
Time
Input/output Dynamic Model
Prediction of Pollutant Concentration at B
Measures of Dispersion, Peak Attenuation and Time of Travel

Fig. 1

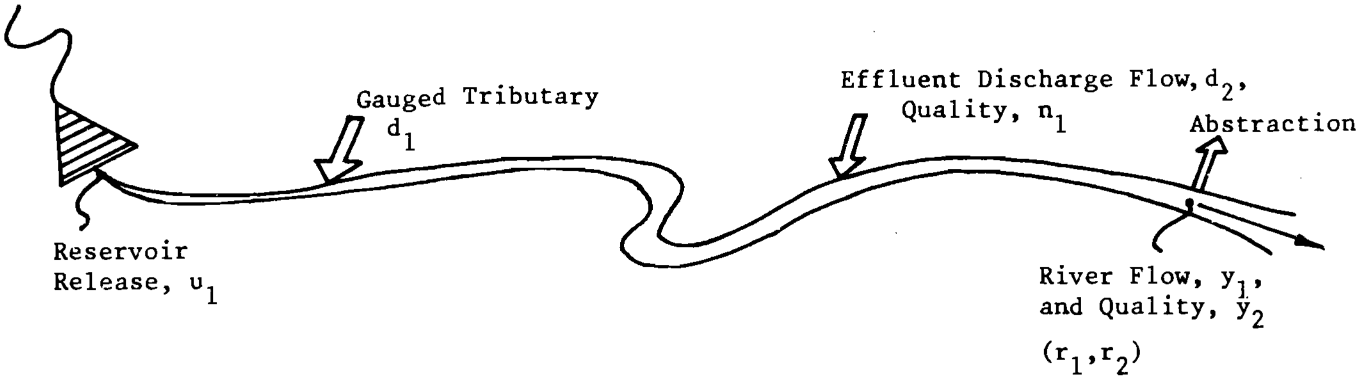
Gauged Tributary
d_1
Effluent Discharge Flow, d_2,
Quality, n_1
Abstraction
Reservoir
Release, u_1
River Flow, y_1,
and Quality, y_2
(r_1, r_2)

Fig. 2

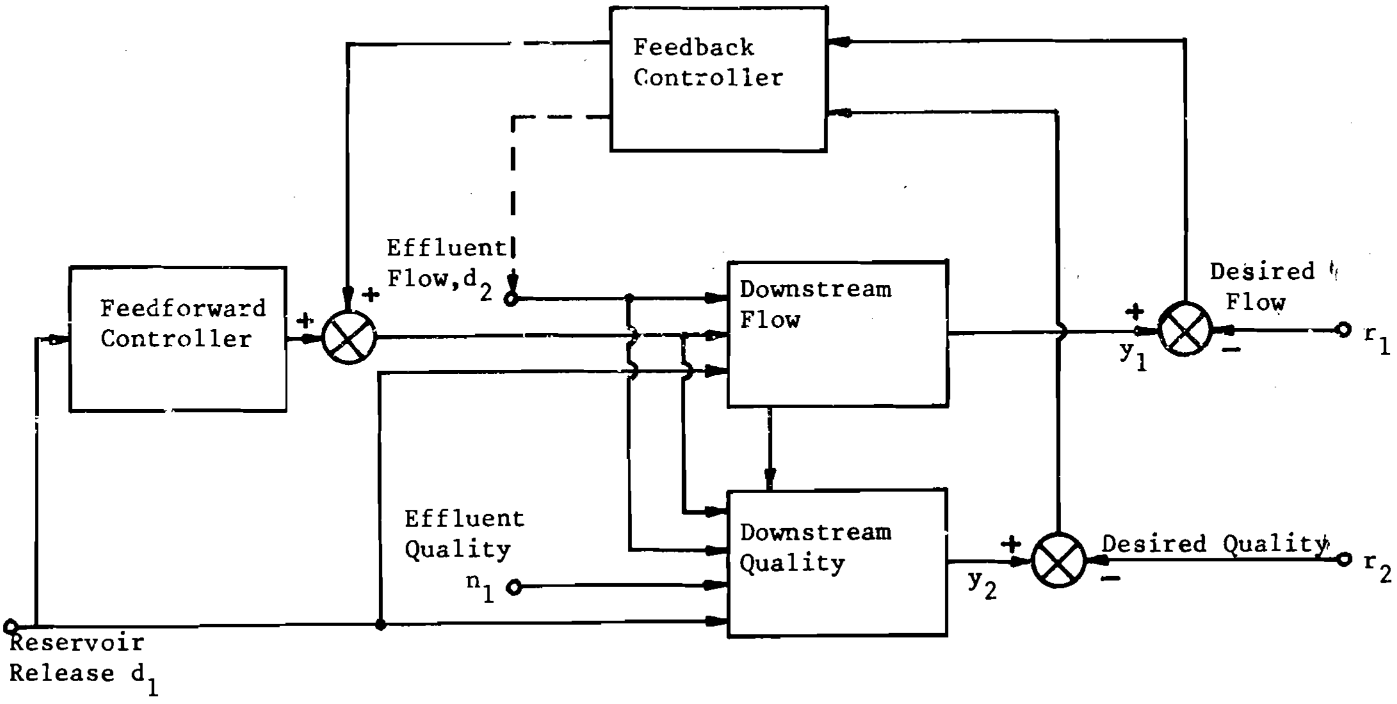
Feedback
Controller
Feedforward
Controller
+
+
Effluent
Flow, d_2
Downstream
Flow
Downstream
Quality
Effluent
Quality
n_1
Reservoir
Release d_1
+
y_1
Desired
Flow
−
r_1
+
y_2
Desired Quality
−
r_2

Fig. 3

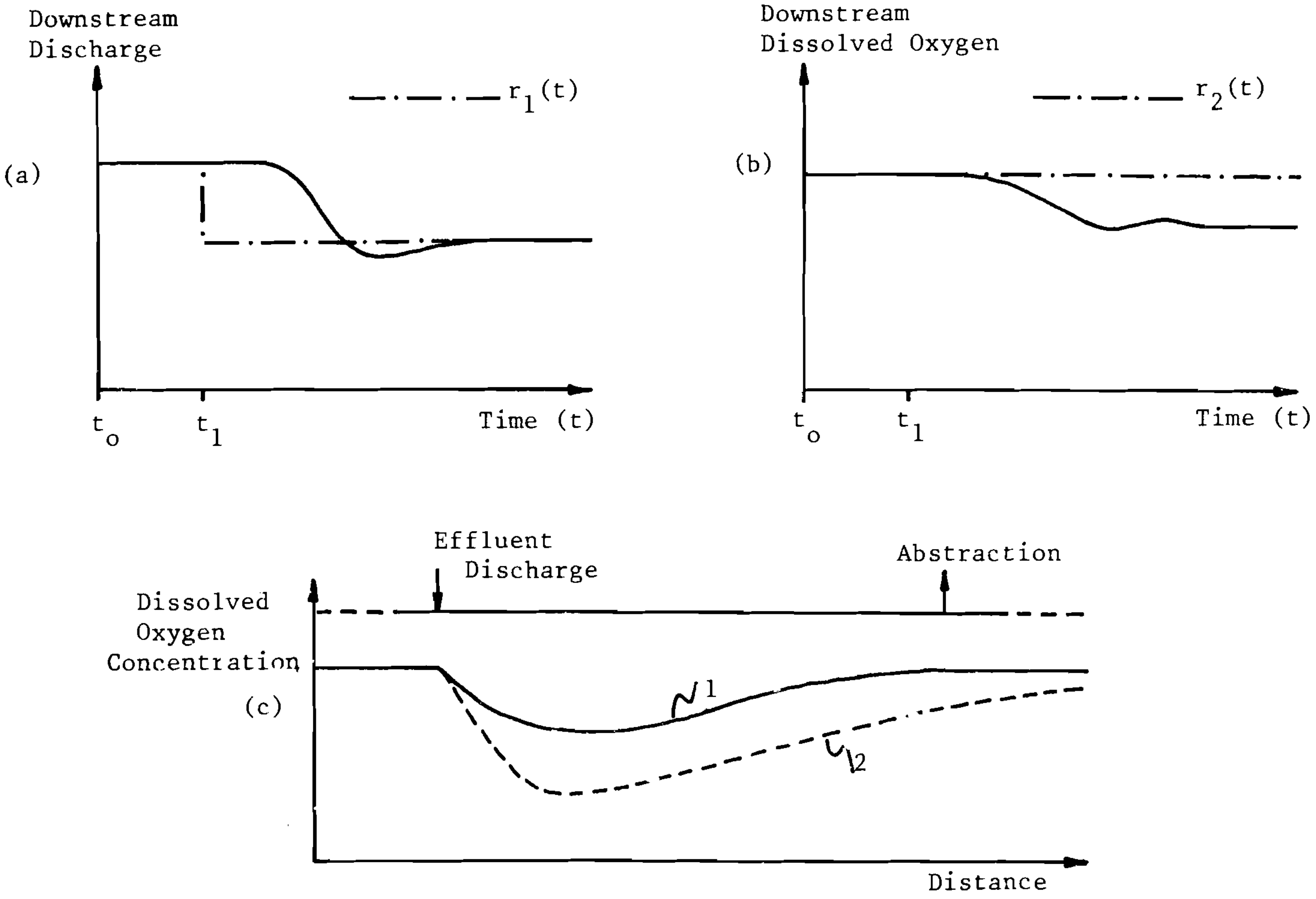
Downstream
Discharge
$r_1(t)$
(a)
t_o
t_1
Time (t)
Downstream
Dissolved Oxygen
$r_2(t)$
(b)
t_o
t_1
Time (t)
Effluent
Discharge
Abstraction
Dissolved
Oxygen
Concentration
(c)
1
2
Distance

Fig. 4

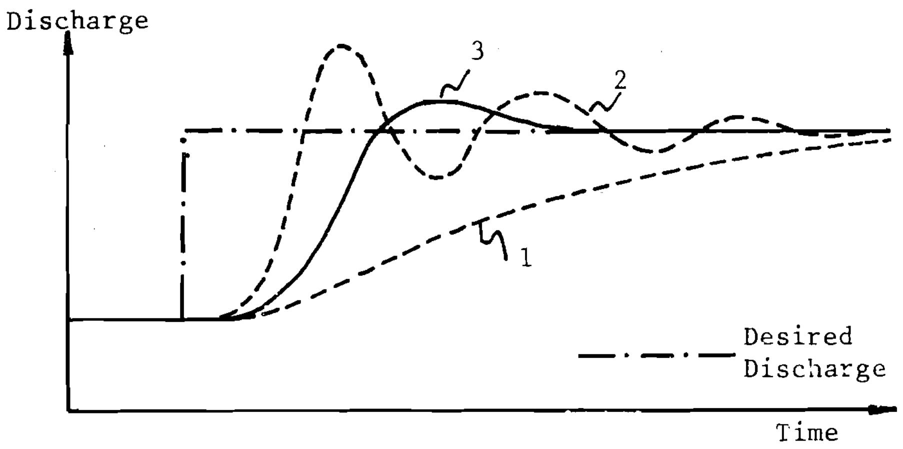
Downstream
Discharge
3
2
1
Desired
Discharge
Time

Fig. 5

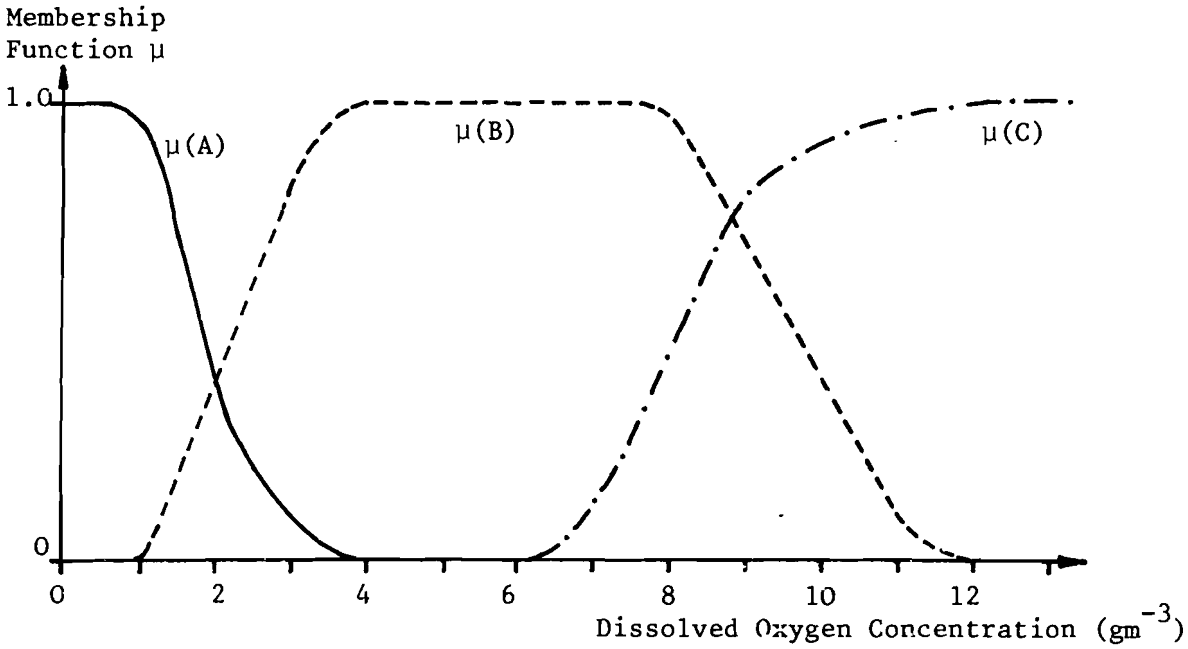
Membership
Function μ
1.0
0
μ(A)
μ(B)
μ(C)
0
2
4
6
8
10
12
Dissolved Oxygen Concentration (gm^-3)

Fig. 6

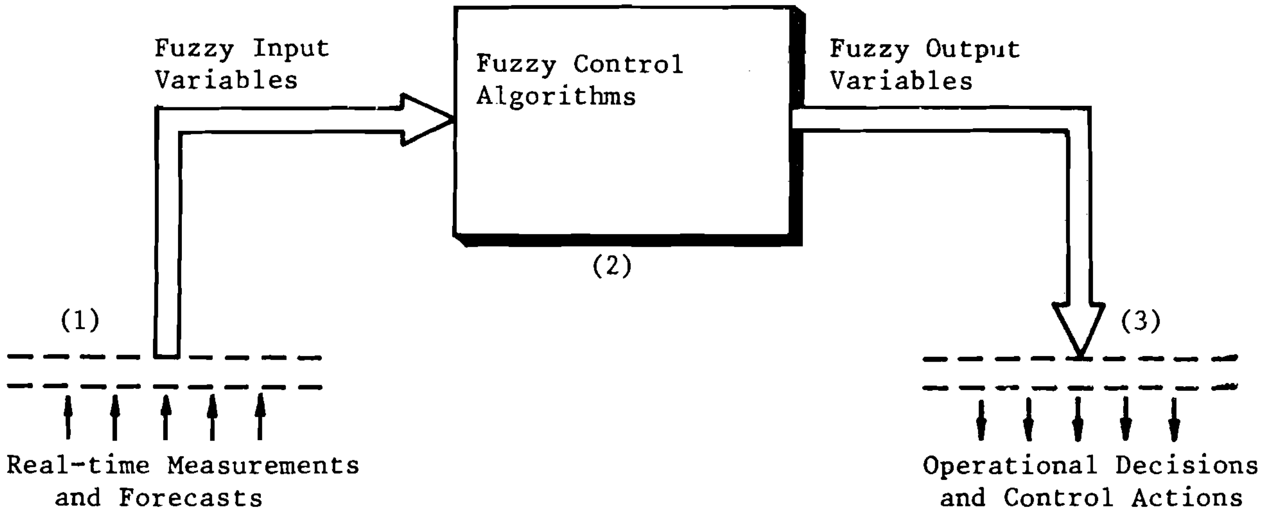
Fuzzy Input
Variables
Fuzzy Control
Algorithms
Fuzzy Output
Variables
(1)
(2)
(3)
Real-time Measurements
and Forecasts
Operational Decisions
and Control Actions

Fig. 7

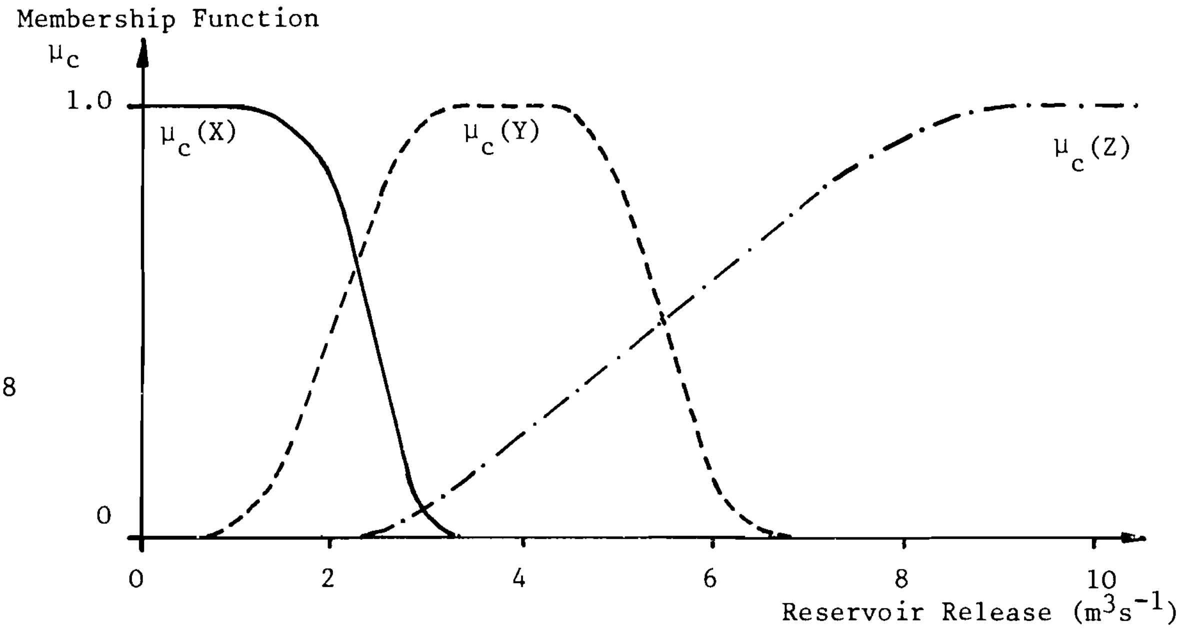

Fig. 8

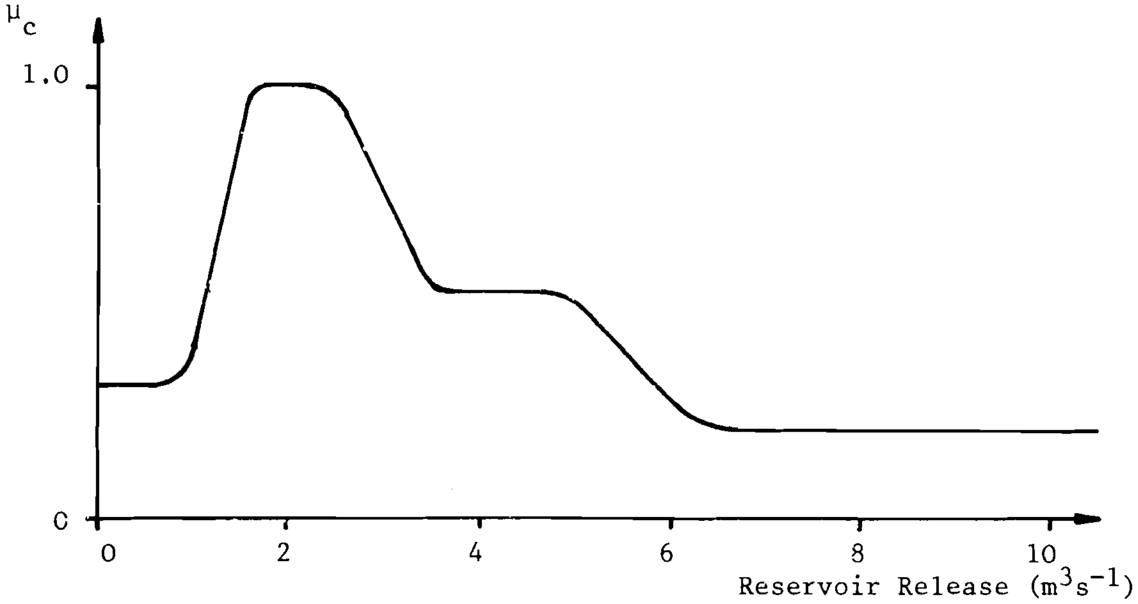

Fig. 9

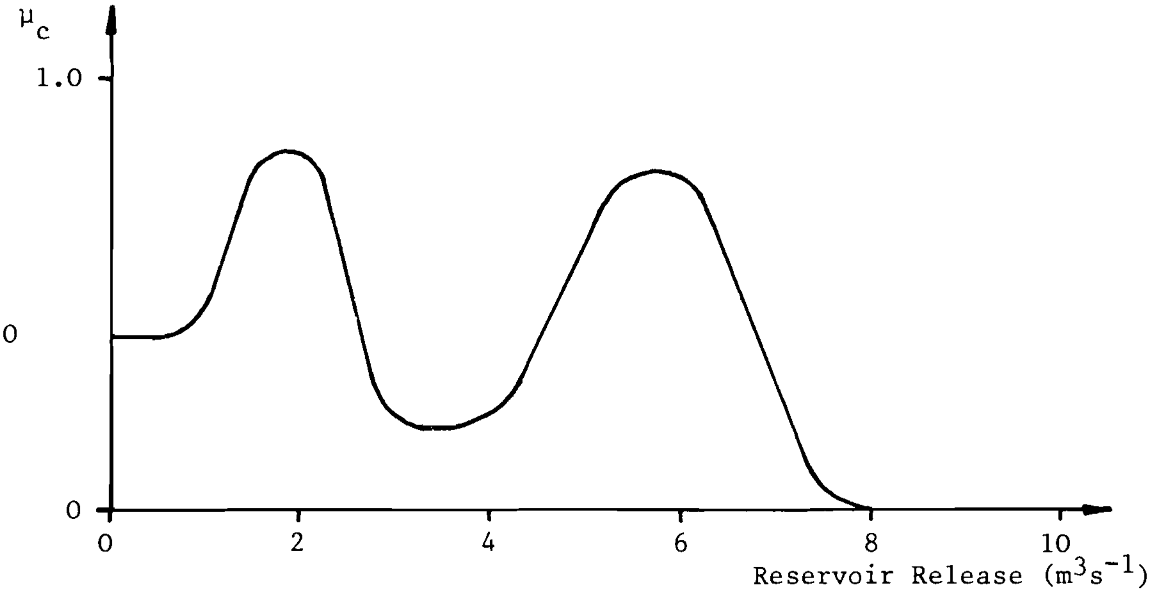

Fig. 10

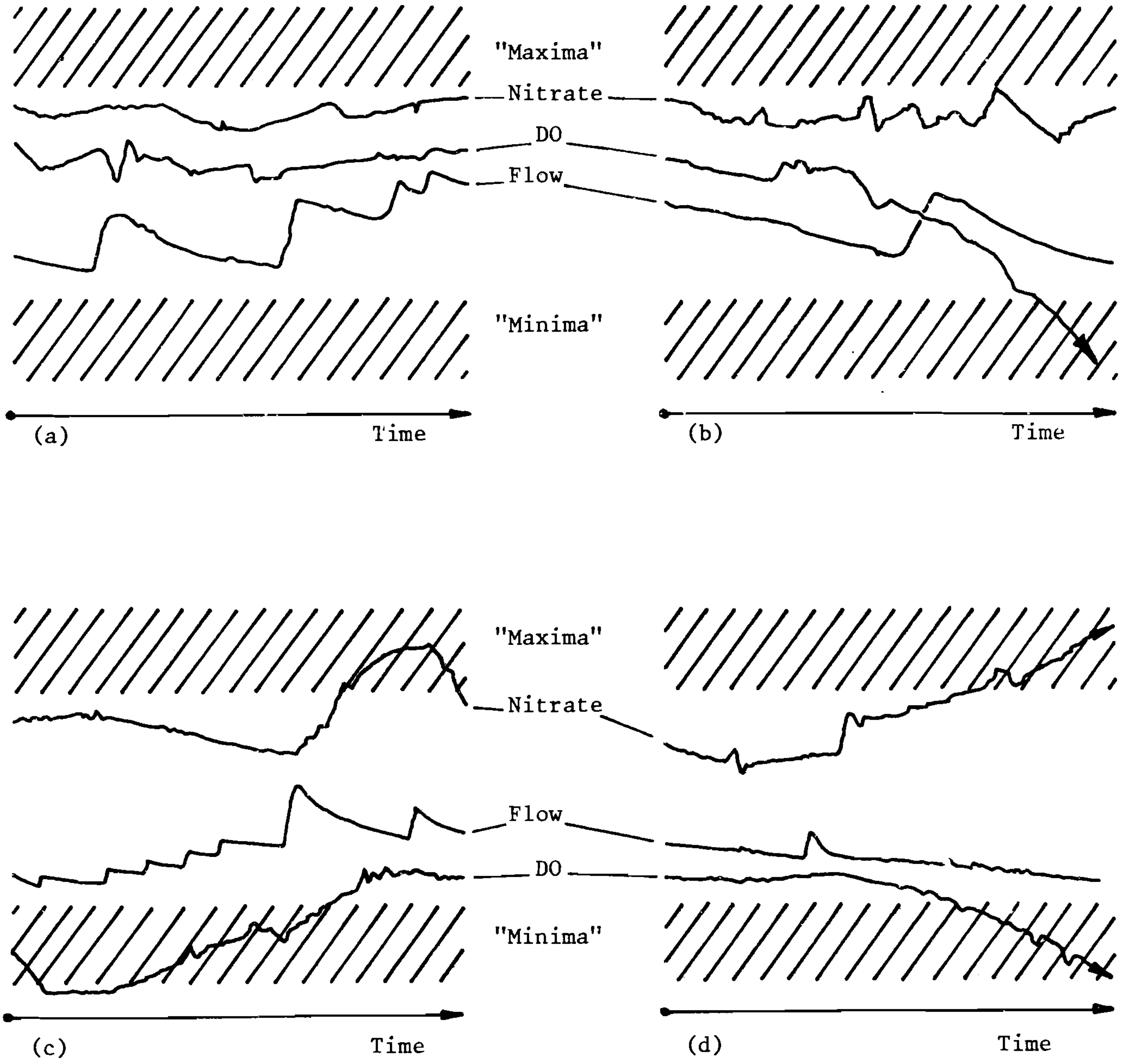
"Maxima"
Nitrate
DO
Flow
"Minima"
(a)
Time
(b)
Time
"Maxima"
Nitrate
Flow
DO
"Minima"
(c)
Time
(d)
Time

Fig. 11