



Interim Report

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Ecological Network Model and Analysis for Rawal Lake, Pakistan

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Abstract

Ecosystems are complex dynamic systems that form highly structured biotic and abiotic interactions. One common approach for studying ecosystems is to consider the structure based on thermodynamics and to use energy transfers between components to investigate the resultant network in terms of its throughflow, storage, and organizational properties. Energy is continually input into an ecosystem in the form of light energy, and some energy is lost with each transfer to a higher trophic level. There are many physical, chemical, and biological limitations to the effectiveness and energy transfer between components of an ecosystem in nature. A water reservoir called Rawal Lake in sub-tropical forest ecosystem in the Margalla Hills of Islamabad forms the domain of this study. The main objectives of this study are: 1) to identify the species present in the Rawal Lake ecological network, 2) to summarize what is known about the energy flows on the basis of the food consumption, and 3) to build a model that represents its dynamics. Ecosystems are impacted by growing anthropogenic interventions, and water resources in particular face many potential threats from climatic events like flooding, droughts, etc., which can result in structural and functional changes in different living and nonliving interactions. This study catalogues the potential threats/ risks to these systems due to climate change or other disturbances. A linear donor controlled ecological network model has been developed to analyze the resilience of the aquatic ecosystem due to these external threats.

Keywords: aquatic ecosystem; mathematical ecology; ecological network model; energy flow; anthropogenic threats/risks; donor controlled model.

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Muhammad Amjad is serving as a Scientific Officer at the Global Change Impact Studies Centre (GCISC), Islamabad, Pakistan. The results achieved in this research were carried out with the Dynamic Systems (DYN) Program (now known as “Advanced Systems Analysis”) within the framework of IIASA’s Young Scientists Summer Program in 2010.

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Ecological Network Model and Analysis for Rawal Lake, Pakistan

1. Introduction

Solar radiation is the basis for all ecological processes driving the flow of energy among trophic levels on earth. Sunlight is necessary for the lifecycle of plants both on land as well as in water. The energy they capture is transferred through a rich diversity of heterotrophic consumers. Understanding a food web is largely an exercise in identifying these energy pathways. While most of pathways involve active feeding relations, passive energy flows of respiration, excretion, and death also link the system together in a web of interconnections. In the process of finding the trophic connections among the biotic and abiotic populations of an ecosystem, the emergent relationship become a mishmash, sometimes referred to as a "bird's nest" or "spaghetti" diagram. However, within such delineation valuable clues exist to understand and interpret ecosystem function. Ecological network analysis and simulation models are useful tools to gain insight into the functioning of a food web.

In this study, we examine a lake ecosystem in Islamabad called Rawal Lake as a part of Margallah Hills National Park (MHNP). The lake is an important part of the local ecosystem and contributes many factors socio-ecological benefits, i.e., drinking water supply, recreational activities, fishing, and habitat for 400 local and migratory bird species.

The mathematical methodology to develop an ecological network model to study system and boundary flows (environmental) and interactions is an emerging field in ecosystems studies (Higashi and Burns, 1991). The idea of using ecological networks was introduced by Hannon (1973), while applying input-output analysis (Leontief, 1951, 1966) to study the flows in ecosystems. By developing a network model he studied the interdependence of different compartments based on direct and indirect flows among them. A great deal of scientific literature has been written since then on ecological network analysis

(see e.g., Hannon, 1979, 1986; Higashi and Burns, 1991; Higashi and Patten, 1989; Matis and Patten, 1981; Patten, 1985, 1991; Ulanowicz, 1986, Fath and Patten, 1999, Fath 2004, 2007). Based on this, Fath et al. (2007) proposed a methodology of developing ecological network models and data requirements. The connections among different compartments in ecological network system are usually based on the flow of conservative units such as energy, carbon, nitrogen flow between the compartments and across the system boundary (Fath and Borrett, 2006). The conceptual and mathematical understanding of such connections could be very useful to envisage the past, present, and future changes in the system and its environment. It is really important to study the dynamics of different partitions of the environment to foresee both the exclusive and exhaustive nature of the ecosystem. Uncovering the flow patterns and influence of different flows in a system's perspective using mathematical simulations can be more helpful to understand the different scenarios of ecological changes and stability as a part of larger web of interactions.

In this study, we have developed an ecological network model on the basis of the aquatic ecosystem structure from the existing literature on Lake Rawal. Flow values were generated from linear donor controlled equations and values of system and boundary flow coefficients were generated randomly using Monte Carlo simulations. Walter (1980) stated that compartmental models with donor controlled flows are stable in the sense of Margalef if the system starts off in a steady state. It is seen that stability varies with the variation in the magnitude of the boundary flows. The variation in the initial conditions does not play very important role on the mathematical stability of the model but non-homogeneous terms in the equations.

2. Margallah Hills National Park

Margalla Hills National Park (MHNP), situated in the foothills of the Himalayas at the border of Punjab and Khaber-Phaktoonkhawa provinces, ranges in elevation from 500 to 1600 meters. Figure 1 shows the general plan of the park, its relation to Islamabad, and the location of Rawal Lake. According to a study

conducted by Capital Development Authority (CDA) on “Environmental Study and Best Utilization of land Margallah Hills National Park and Surrounding Areas” in 2007 (CDA, 2007), the total park area spread over approximately 12,604 hectares containing grass farms (3,481 hectares), reserved forest (4,698 hectares) and acquired areas (4,425 hectares) along with cultural and sports complex, picnic resorts and Rawal Lake.

Flora: The park has rich biodiversity in flora. Mainly trees, shrubs and vines are represented by 104 families, 465 genera and 616 species. The common vegetation includes cheel (*Pinus longifolia*), kao (*Wild olive*), phulai (*Acacia modesta*), sienetha (*Dodona-bura manniana*), shisham (*Dalbergia sisso*), toot (*Morus-alba*) and paper mulberry. The flowering trees/shrubs include kachnar (*Bauhinia purpurea*), wild pomegranate (*granatum*), black berries, raspberries, cranberries and wild pears etc. For the most part, the hills are covered with subtropical flora consisting of dry, semi-evergreen vegetation and pines.

Fauna: The vast park area is habitat for many species of mammals (38 species), reptiles (13 taxa) and birds (266 species). The main mammal species include Common leopard (*Panthera pardus*), rhesus monkey (*Macacca mulata*), gray goral (*Nemorhaedus goral*), barking deer (*Muntiacus muntjak*), fox (*Vulpes bengalensis*), wild boar (*Sus scrofa*), porcupine (*Hystrix indica*) and fruit bat (*Pteropus giganteus*).

The common bird's species like himalayan griffon (*Gypus himalayensis*), spotted owlet (*Athene brama*), rose-ringed parakeet (*Pisttacula krameri*), golden oriole (*Oriolus oriolus*), common babbler (*Turdoides caudatus*) and ringed dove (*Streptopelia decaocta*) and few migratory birds like water-fowl include: marsh harrier (*Circus aeruginosus*), white- backed kingfisher (*Halcyon smynrensis*), pintail (*Anas acuta*), little egret (*Egretta grazatta*) and mallard (*Anas platyrhynchos*) lives in park and adjacent areas.

The wetland part of the park is also popular for fish. doula (*Channa channa*), rahu (*Labeo rohita*), thaila (*Catla catla*), mori (*Cirrhinus mrigala*), carp fish (*Cyprinus carpio*) and talapia (*Tilapia mossambica*) are common fish species in the Park.

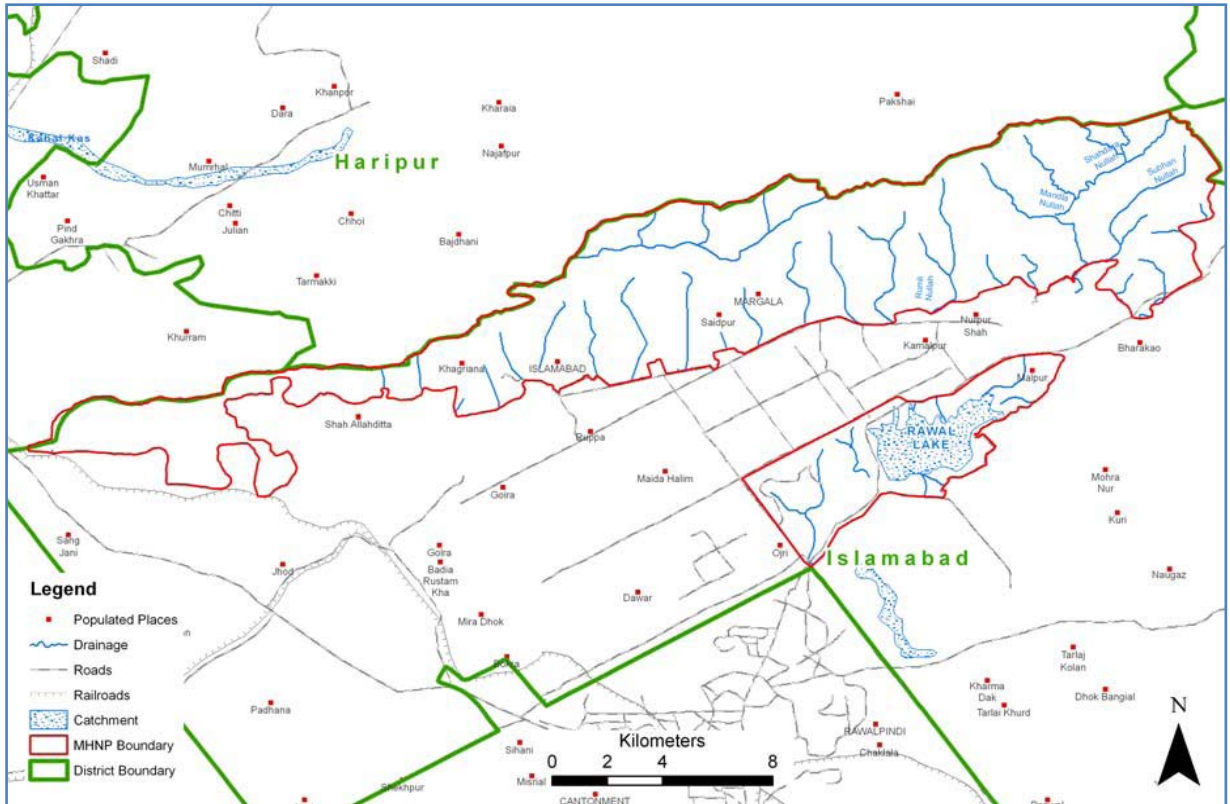


Figure 1: Topographic map of Margalla Hills National Park, Islamabad, Pakistan
Source: WWF - Pakistan

2.1 Rawal Lake

Rawal Lake, located at 33°42'N 73°07'E, is an artificial reservoir that was built in 1960 across Korang River to fulfill the water needs for the cities of Rawalpindi and Islamabad. Its average depth is 33.54 m, 213 m long and covers 8.8 km² of area in an isolated section of Margalla Hills National Park (MHNP) in Islamabad. It has storage capacity of 58.6 million cubic meters and a catchment area spread over 275.2 km². On an average rainfall year, it gathers 3,659,040,000 (3.6×10⁹) square feet (3.4×10⁸ square meters) of water with the contribution from 4 major and 43 small streams coming from the Margalla Hills also contribute to Rawal Lake.

3. Main Threats and Control Parameters

Anthropogenic activities on earth are promoting fragmentation in diversified natural ecosystems. Climate change and its consequences have compounded such challenges like excess exploitation and pollution for ecosystems. Such impacts disturb the functioning of the entire ecosystem, and they diminish the ecosystem's ability to provide services such as food, medicines, and resources for humans.

Wetlands have an important role in human subsistence and ecological functioning on earth. Aquatic ecosystems are under stress due to losses and threats from many reasons like runoff from municipal and agricultural areas, water diversion activities such as irrigation systems, dams, upstream water use, water pollution, conversion to agricultural land, reclamation for urban expansion and the invasion of exotic species. The risk of extinction of aquatic species is higher than for mammals and birds (EPA, 2011). The catchment area of Rawal Lake is also under influence of similar risks due to human settlements i.e., Bhara Kahu, Malpur, Bani Gala and Noorpur Shahan. The indiscriminate construction of residential housing for the last two decades near the lake without knowing the adverse contributes to water contamination due to sewerage and solid waste.

Deforestation, erosion and sedimentation are also increasing threats to the ecological balance of these resources. Some people rely on wood stoves, fireplaces, or fireplace inserts as the primary home heating source. The construction of residential and commercial buildings and roads causes the removal of natural vegetation. This deforestation destroys the habitat of migratory and local birds of this region as well as encourages the erosion of soil which increases siltation and turbidity levels in the lake.

A few economic activities like cultivation of crops, such as wheat and corn on small patches and stone crushing are occurring around and inside the forest area. The pesticides and herbicides used in the agricultural activities are a source of toxic pollution as they wash away and enter into Rawal Lake (Ahad et

al. 2006). The stone crushing causes forest fires in the catchment area. The disposal arrangements of scratch for approximately 170 poultry farms and 370 poultry sheds in the catchment area are also not up to date which further damage the water quality.

The fertilizer use and wastes from livestock are the major sources of nutrient input. The excess or shortage of nutrients input (Nitrogen and Phosphorus) from the land to fresh water changes the primary productivity of the phytoplankton which leads to eutrophication of the aquatic ecosystem. Therefore, a balanced supply of nutrients in Rawal Lake ecosystem could have potentially positive impacts on higher trophic levels.

Fishing and recreational activities near the lake are emerging threats for the sustainable management of ecological system. Chattar Park lies adjacent to the Korang River and Lake View Park on Rawal Lake attracts many visitors. The tourists leave waste, which ends up in the lake and causes of pollution. Excessive boating by visitors increases the turbidity which limits the growth of phytoplanktons. The activities of tourists have also limited the movement of birds and ducks in the lake.

The changing patterns of precipitation and temperature due to climate change could alter the ecological balance of rivers and lakes and threaten the distribution of different species, which respond to the climate conditions. There could be mistiming of the arrival of the migratory birds due to shifting in seasons in context of climate change which could alter the breeding season with food availability to wading birds (Göteborg Award, 2010). Climatically induced extreme events are also testing the ecological resilience of the available resources on earth. Water storage and management of water reservoirs/lakes is gaining importance in view of water scarcity, ecology and food security issues both in the developed and developing world. Therefore, better understanding of the functioning, penetration and propagation of the environmental problems is required. Water bodies are not only important for food security, water supply, and economic activity; but also, they play a very important role in preserving the local and regional ecosystem. Human induced climate perturbations potentially

have serious threats like flooding, soil erosion and drought conditions to the dynamics of aquatic organisms and ecosystem processes.

Increasing population, road construction, over-grazing and wastes from the poultry farms are degrading the catchment area of Rawal Lake. Urbanization, industrialization, technological development, over-fishing, water sports, and recreational activities need to be reduced to conserve the ecosystem and water quality for domestic supply for Rawalpindi and Islamabad.

4. Data

Data for the different species of Rawal Lake are acquired from different sources including scientific literature, World Wide Fund (WWF) for Nature, Lahore, Pakistan and Capital Development Authority (CDA) and summarized in Table 1. Mostly, the functional studies that give metabolic information for specific organisms are available for Rawal Lake. To the authors' knowledge, there are no existing studies of the structural relationship among the Lake species before this effort.

Constructing an ecological network model requires deep subject knowledge and understanding of the system. The first step is to identify the major components in the system, which will comprise the network nodes, and the currency of exchange which links them together. For Lake Rawal an aggregation of 11 functional groups has been identified, linked together by the energy flow among the different living and non living groups. For these aggregated groups, data are available for most of them, but recorded at different times. Ideally, one would have detailed information about the standing biomass for each compartment as well as the exchange rates between compartments, but this is rarely the case. The availability of data to calculate the biomass of each group was a problem but still we were able to roughly calculate the biomass for each compartment the following formula has been used:

$$Total\ Biomass = \frac{Weight\ for\ one\ organism\ (g)}{1000} \times Total\ number\ of\ counts$$

Mostly, the data are available in the form of relative abundance (e.g., rare, available, present, or absent, etc.). Nonetheless, building on a general understanding of lake ecosystems, the available data were sufficient to produce a crude, first-approximation network model of Lake Rawal. Below is a synopsis of the functional groups and the available data for each one.

4.1 Phytoplankton

Phytoplankton are the ecological base of the reservoir water body. Their growth depends on carbon dioxide, nutrients, and availability of sunlight in the water. The other factors which can influence phytoplankton growth are water temperature, salinity, water depth, wind and predators grazing them. In the water column, near Rawal dam a total of 296 algal species belonging to 114 genera of 11 phyla were observed (Leghari et al. 2004a). Among the 296 species, 133 are listed as very rare (48.72%), 27 are rare (9.89%), 50 species are common (18.32%) and 63 species are very common (23.07%) as reported in Leghari et al. (2004a).

4.2 Water Plants and Macro Algae

There is a rich diversity of water plants and algae species in Rawal Lake. Out of 273 total algae species observed by Leghari et al. (2004a), 63 species of algae are listed as very common, 50 species are common, 27 and 133 species are rare and very rare respectively. Green algae were the most dominant group among different species. Leghari et al. (2004a) collected data on the relative abundance of water plants as well as physical parameters (pH, DO, etc.). They identified 23 different species of macrofauna in the lake during the period June 2000 to May 2002.

4.3 Zooplankton

Zooplankton are tiny animals that move passively in the water column and play an important role in any aquatic ecosystem. The data for zooplankton's composition in Rawal Lake were taken from Baloch et al. (2005), on the basis of

their study in April 2004. The zooplankton community in the lake is dominated by rotifer, having 10 species. The data in Table 2 of Baloch et al. (2005) are given in the form relative abundance instead of exact biomass in the lake.

4.4 Fish Fauna

Data for fish fauna in the Rawal Lake are mainly taken from the World Wildlife Fund (WWF) records and Afzal et al. (1995). WWF recorded 28 species of fish in Rawal Lake, and a study by Afzal et al. (1995) identified 15 species belong to 11 genera. The concentration of carnivores (2 species) was found to be very low as compared to foragers (9 species). But the desired value of biomass for fish fauna for this study is not available in the WWF Pakistan records. The biomass approximation in Afzal et al. (1995) is available on the basis of desirable and undesirable species in the local market.

4.5 Amphibian and Reptilian Fauna

The data for amphibian and reptile fauna were taken from the WWF data record. The data were noted for 9 species of frogs, 10 species of lizards and 7 species of snakes in the form of relative abundance (common or rare) respectively.

4.6 Avian Fauna

4.6.1 Ducks

The data for ducks are taken from the study by Hussain et al. (2002). This study was conducted during 1999, 2000 and 2001 to record the number of ducks/birds and to identify any trends in annual abundance. Rawal Lake is the winter resort for migratory ducks due to its existence in subtropical, subhumid ecozone. The major migratory and resident species were identified including 9 species in 1999 and 23 species in 2000.

4.6.2 Birds

Due to Rawal Lake's location in the migratory zone, the presence of bird species is high and important. There are approximately 266 bird species found in the lake area. A few species are more abundant than others but almost all are present in winter. The data for model's bird compartment for Rawal Lake are taken from the WWF Pakistan's data records (2009).

4.7 Benthic Fauna

The productivity of benthic fauna in Rawal Lake reservoir is generally low but there is a gradual increase in February. The data source for benthic fauna is the annual technical report of Pakistan Science Foundation (PSF) project titled "Hydrobiological Studies of the Lakes of Punjab" of 1980/81 (PSF, 1980/81). There are 9 different organisms included in the benthic fauna. Three different surveys were conducted in July, October, December, February and April of 1980/81. The total number of organisms and total weight of each was recorded in g/m² units.

4.8 Detritus/Total Suspended Solid

The value of total suspended solid is taken from a study by Leghari et al. (2004b). The recorded value in the above mentioned study is 2.1 mg/L.

4.9 Sediment Bacteria

Data for sediments bacteria are not available.

Table 1: Details of the available data to develop an ecological network model for Rawal Lake, either in the form of relative abundance or list of counts. The blanks show where the information is lacking.

| S. No. | Compartment Name | Time period for data collection | Total Species | Data type | Reference | Estimated Biomass ² (k.cal/m ² /day) | Flows |
|--------|-----------------------------|---------------------------------|---------------|--------------------|---------------------------------|--|---------|
| 1 | Phytoplankton | June 2000–May 2002 | 296 | Relative abundance | Laghari et al. 2004a | missing | Missing |
| 2 | Zooplankton | April 2004 | - | Relative abundance | Baloch et. al. 2005 | missing | |
| 3 | Small Fish | 1986 | 13 | Relative abundance | WWF (2009) Afzal et al. 1995 | 1771 | |
| 4 | Large Fish | - do- | -do- | -do- | -do- | | |
| 5 | Water Plants and Macroalgae | June 2000–May 2002 | 273 | - | Leghari et al. 2004a | missing | |
| 6 | Detritus/TSS | June 2000–May 2002 | - | - | Leghari et al. 2004b | 2.4-2.6 mg/L | |
| 7 | Benthic Fauna | 1980-1981 | 7 | Total weight/month | PSF (1980/81) | 37.7 | |
| 8 | Amphibian and Reptiles | - | 29 | Relative abundance | WWF (2009) | 494.8 | |
| 9 | Ducks | 1999-2001 | - | List of counts | Hussain et al. 2002 | 16 | |
| 10 | Birds | - | 266 | List of counts | WWF (2009) | 2504.5 | |
| 11 | Sediment Bacteria | - | - | - | - | missing | |

5. MODEL

5.1 Conceptual Model

Technological development and climate perturbations are posing serious impacts to the environment, which leads to major quantitative variations of certain species of ecological importance. These changes at a local scale could potentially alter the entire ecological structure in the region. Predicting the possible environmental impacts on a complex ecosystem is a daunting task. To deal with such environmental problems, it is extremely useful to develop a model, which expose the system interactions, simulate the system dynamics, and help identify the data gaps for understanding ecosystem functioning.

By creating a partition of the Rawal Lake we constructed a conceptual model consisting of 11 compartments. On the basis of general biological knowledge, the Rawal Lake ecosystem was fragmented into who consumes whom and by how much in the aquatic food web. The food web ecosystem in the lake has been conceived to describe both anthropogenically perturbed and unperturbed dynamics to get insight into the impact in the lake ecosystem. Researchers have studied effects of perturbations on input-output systems (Sherman and Morrison, 1950; Christ, 1955) in economics but here we have applied similar knowledge in assessing the perturbation effects on the Rawal Lake ecosystem. External threats to Rawal Lake have been identified such as variation in nutrients and run-off due to extreme events like flooding or droughts. Other potential threats which could be controlled are fishing and migration of birds, ducks, and amphibians. The aim here is to model the biomass dynamics to: (i) Study the interrelation of all biotic and abiotic partitions in a lake ecosystem, (ii) Probe into the source in these interrelations as a reaction to small but actual perturbations, (iii) Test the model properties (stability, sensitivity and control parameters) to explore the usefulness of the linear systems analysis, and (iv) Identify the data gaps to develop an improved network model for the Rawal Lake ecosystem in the future.

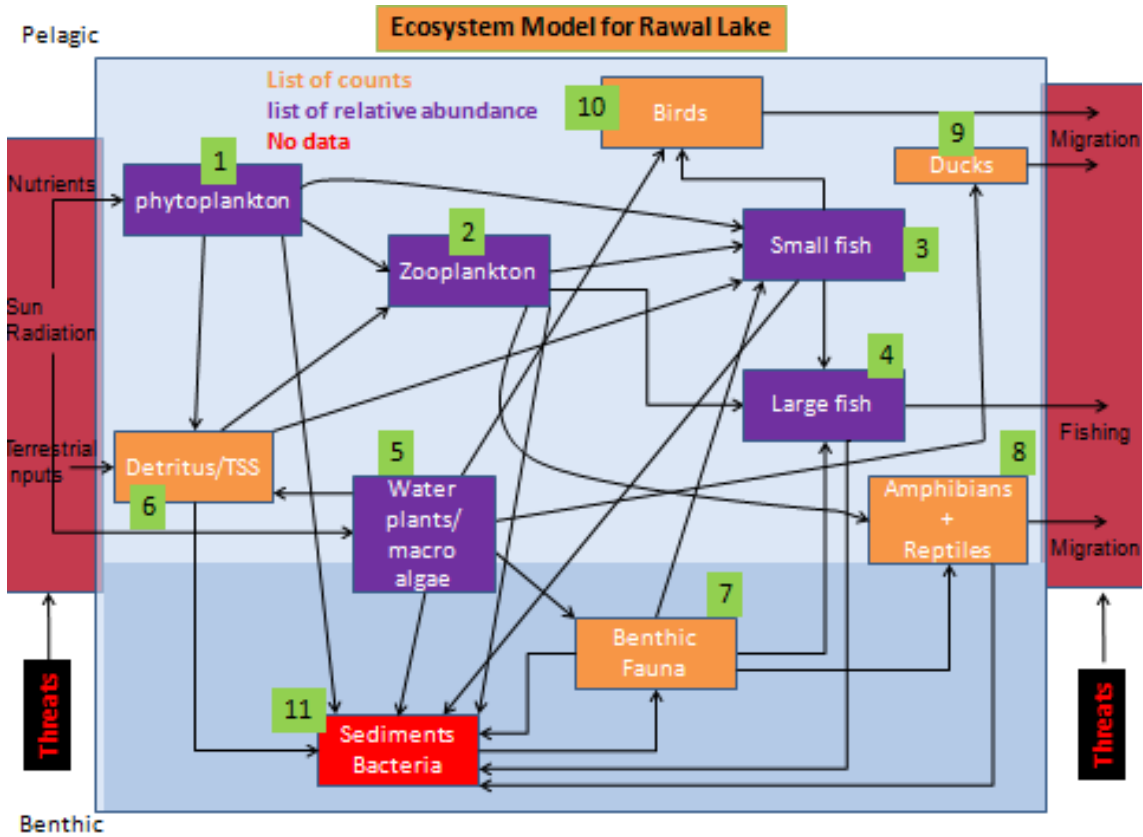


Figure 2: Block diagram of conceptual Rawal Lake ecosystem network model, the different colors represent availability of biomass data for each compartment, arrow lines show the flow of energy in the lake ecosystem

Figure 2 shows the different partitions of Rawal Lake and the conceptual biological model on the basis of energy flow. The ecosystem, and its interaction with the environment, has been envisaged to form first order linear differential equations that describe the state variables. The passage from external environmental threats into the lake ecosystem has been studied by varying the inflow and outflow rates of biomass. This conceptual development remained very useful in identifying the availability (and gaps) of data to develop and analysis of the ecological network model. The different colors represent the different forms of data available for the available partitions.

The adjacency matrix in Table 2 for the Rawal Lake ecosystem model (Figure 2) illustrates the connectedness of different compartments. A zero corresponds to the absence of flow (from columns to rows) of energy and a one indicates a

flow connection between the two compartments. There are 25 connections between different compartments yielding a system connectivity of 20%.

Table 2: Adjacency matrix represents the flow of energy as in Figure 2, the binary number 0 indicates no interaction and 1 corresponds to flow connection between two compartments from columns to rows.

| | Phyto-plankton | Zoo-plankton | Small Fish | Large Fish | Water plants/ macro algae | Detritus /TSS | Benthic Fauna | Amphibian + Reptiles | Ducks | Birds | Sediment Bacteria |
|-----------------------------|----------------|--------------|------------|------------|------------------------------|---------------|---------------|----------------------|-------|-------|-------------------|
| Phytoplankton | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Zooplankton | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Small Fish | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| Large Fish | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| Water plants/ Macroalgae | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Detritus/ TSS | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Benthic Fauna | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| Amphibian+ Reptiles | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| Ducks | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Birds | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Sediment Bacteria | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |

The state variables are listed vertically and horizontally in the adjacency matrix and the process of discerning the adjacency matrix value is helpful to gain the system overview and relevant links in modeling. All the diagonal entries in the matrix are zero, which show that species are not self consuming.

6. Modeling Methodology

Several analyses and modeling techniques on the complex relationships in ecosystem have been developed for compartmental models of energy/ matter storage and dynamics. There are three commonly used model types: 1) donor controlled, 2) recipient controlled, and 3) donor-recipient controlled models (Kazancı, 2007)

$$F_{ij} = f_{ij} x_j \quad (\text{Example of donor controlled flow})$$

$$F_{ij} = f_{ji} x_i \quad (\text{Example of recipient controlled flow})$$

$$F_{ij} = f_{ij} x_j x_i \quad (\text{Example of donor-recipient controlled flow})$$

where F_{ij} , f_{ij} , is the total flow and flow coefficient from compartment j to i . The initial biomasses of donor and recipient compartments are represented by x_j and x_i respectively.

The donor and recipient controlled models are linear, but the donor-recipient controlled models are nonlinear. The linear models have single steady state while the nonlinear models have multiple steady states. The steady states in nonlinear models may have much difference in magnitudes in each steady state. The linear models could be stable or unstable but those that describe the ecosystem are always stable. In the ecological literature (Hairston et al. 1960), conclusions have been made that natural ecological processes at the macro level work as energy flow resource limited. The source of limitations is expressed by “donor controlled” in linear compartmental models. The flows in each compartment have been modeled by taking into account the sum of inflows, minus the sum of outflows using analytical approaches in system theory to study the biomass dynamics. Two flow types have been introduced known as system flows and environmental flows.

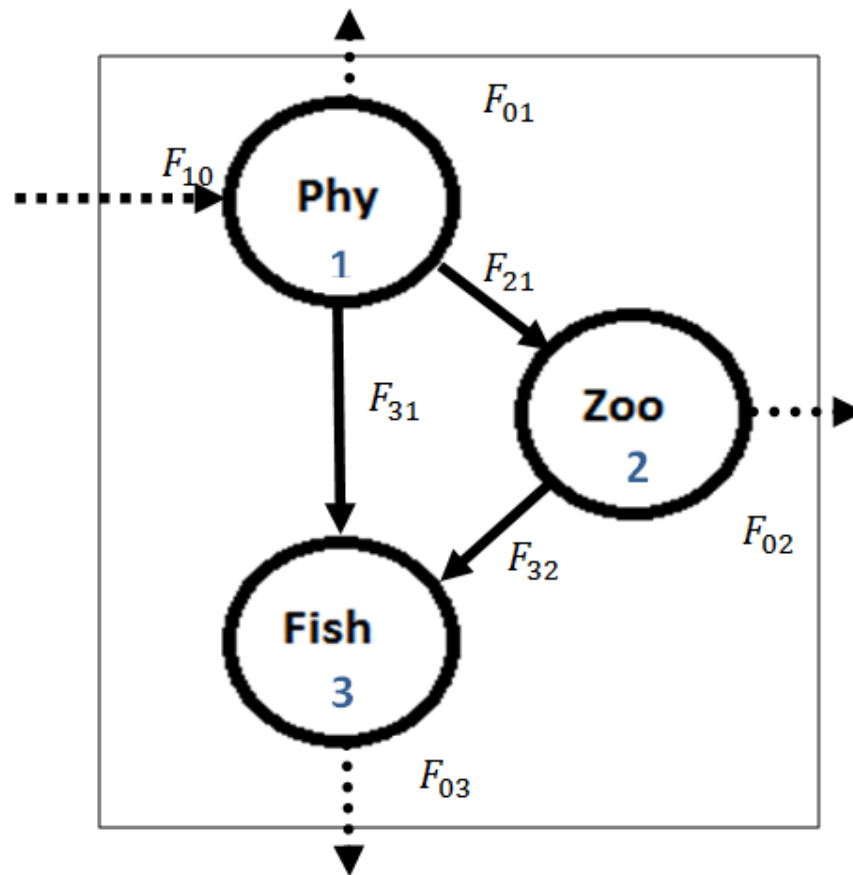


Figure 3: A simple ecological network model with three compartments

To demonstrate this approach, a simplified food web model is described below. Figure 3 exemplifies a food web structure of simplified ecological network consisting of only three compartments and its interaction with the external environment. The first compartment represents phytoplankton, the second compartment is for zooplankton, and third compartment is for fish, while 0 represents the environment. It has been depicted in the model that the first compartment receives energy from the environment, it losses some energy during respiration process to the environment and passes fractions of energy to the second and third compartments. The second compartment gets energy in the food web from the phytoplankton compartment and losses energy to environment and also passes some to the fish compartment. Similarly, the fish compartment consumes both phytoplankton and zooplankton and releases energy to environment during respiration or through fishing processes. The flows mentioned along with different compartments in Figure 3 can be modeled using the flows (both system and boundary) among different compartments, the

initial stock and speed of the flow. The mathematical description to model the biomass dynamics of each compartment above using donor controlled flow technique can be written as follows:

$$\frac{d(\text{phy})}{dt} = \frac{dx_1}{dt} = F_{10} - F_{21} - F_{31} - F_{01} = (\mathfrak{n} + f_{10}x_1) - f_{21}x_1 - f_{31}x_1 - \lambda_{01}x_1$$

$$\frac{d(\text{zoo})}{dt} = \frac{dx_2}{dt} = F_{21} - F_{32} - F_{02} = f_{21}x_1 - f_{32}x_2 - \lambda_{02}x_2$$

$$\frac{d(\text{fish})}{dt} = \frac{dx_3}{dt} = F_{31} + F_{32} - F_{03} = f_{31}x_1 + f_{32}x_2 - (\mathfrak{n} + f_{03}x_3)$$

Where \mathfrak{n} and $\lambda_{0i}x_i$ are the constant terms with those compartments interacting with environment and energy losses to environment during respiration process for each compartment respectively. A similar systems oriented framework has been designed to study the impacts of climate change and extreme events on Rawal Lake.

7. The Basic Model

In this section, we will briefly derive the mathematical model described in Figure 2 that we shall study in this paper. An attempt has been made to include all basic aspects on the basis of our general biological knowledge. The energy flow dynamics have been modeled using a system of linear differential equations or difference equations that corresponds to the system response to the external factors. The steady state represents a state when time derivatives are zero. Each compartment is considered as the transient place for the energy flow when input energy is equal to the output. The energy flow analysis formulations in ecological network modeling starts with the flow matrix, \mathbf{F} ; F_{ij} which gives a the amount of matter/ energy flowing from compartment j to i . To determine the rate of flow, we introduce flow coefficients f_{ij} with each system flow, which represents the flow from the donor compartment j to recipient compartment i is the flow coefficient which determine the flow speed where $i, j = 0,1,2,3 \dots 11$. The system flows represent the interaction of the different species within the lake and the connections of different species to environments are recognized as

boundary flows. In order to represent the macro scale ecological processes, the flow of energy among different compartments in the model has been modeled using linear donor controlled equations (Walter 1980). The flows from the environment to phytoplankton, detritus/TSS and water plants/macro algae compartments have been modeled using a constant term and recipient/donor compartment term ($\eta_0 + \mu_{i,0}x_i$). Similarly the large fish compartment has a constant term and compartment outflow for fishing ($-\eta_4 - \alpha_{0,4}x_4$). The loss of energy to the environment during metabolism in each compartment can be modeled by $\lambda_{0,i}x_i$ term in each difference equation. The term $\alpha_{0,i}x_i$ is for migration (or fishing along with a constant term η_4 for fishing) of species like amphibians, birds and ducks.

$$\dot{x}_1 = \frac{dx_1}{dt} = \eta_0 + \mu_{1,0}x_1 - f_{6,1}x_1 - f_{11,1}x_1 - f_{2,1}x_1 - f_{3,1}x_1 - \lambda_{0,1}x_1 \quad (1)$$

$$\dot{x}_2 = \frac{dx_2}{dt} = f_{2,1}x_1 + f_{2,6}x_6 - f_{3,2}x_2 - f_{4,2}x_2 - f_{8,2}x_2 - f_{11,2}x_2 - \lambda_{0,2}x_2 \quad (2)$$

$$\dot{x}_3 = \frac{dx_3}{dt} = f_{3,1}x_1 + f_{3,2}x_2 - f_{3,6}x_6 - f_{3,7}x_7 - f_{4,3}x_3 - f_{10,3}x_3 - f_{11,3}x_3 - \lambda_{0,3}x_3 \quad (3)$$

$$\dot{x}_4 = \frac{dx_4}{dt} = f_{4,3}x_3 + f_{4,2}x_2 + f_{4,7}x_7 - f_{11,4}x_4 - \alpha_{0,4}x_4 - \eta_4 - \lambda_{0,4}x_4 \quad (4)$$

$$\dot{x}_5 = \frac{dx_5}{dt} = \eta_0 + \mu_{5,0}x_5 - f_{7,5}x_5 + f_{9,5}x_5 - f_{10,5}x_5 - f_{6,5}x_5 - f_{11,5}x_5 - \lambda_{0,5}x_5 \quad (5)$$

$$\dot{x}_6 = \frac{dx_6}{dt} = \eta_0 + \mu_{6,0}x_6 + f_{6,1}x_1 + f_{6,5}x_5 - f_{2,6}x_6 - f_{3,6}x_6 - f_{11,6}x_6 \quad (6)$$

$$\dot{x}_7 = \frac{dx_7}{dt} = f_{7,5}x_5 + f_{7,11}x_{11} - f_{3,7}x_7 - f_{4,7}x_7 - f_{8,7}x_7 - f_{11,7}x_7 - \lambda_{0,7}x_7 \quad (7)$$

$$\dot{x}_8 = \frac{dx_8}{dt} = f_{8,10}x_{10} + f_{8,7}x_7 - f_{11,8}x_8 - \alpha_{0,8}x_8 - \lambda_{0,8}x_8 \quad (8)$$

$$\dot{x}_9 = \frac{dx_9}{dt} = f_{9,5}x_5 - \alpha_{0,9}x_9 - \lambda_{0,9}x_9 \quad (9)$$

$$\dot{x}_{10} = \frac{dx_{10}}{dt} = f_{10,3}x_3 + f_{10,5}x_5 - \alpha_{0,10}x_{10} - \lambda_{0,10}x_{10} \quad (10)$$

$$\dot{x}_{11} = \frac{dx_{11}}{dt} = f_{11,3}x_3 + f_{11,1}x_1 + f_{11,2}x_2 - f_{11,7}x_7 - f_{7,11}x_{11} - \lambda_{0,11}x_{11} \quad (11)$$

Where $X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}]$ represents the stock vector with eleven compartments, $\mu_{i,0}x_i$ is the flow and $\mu_{i,0} (= \eta_0 + \xi_{i,0})$ is coefficient for input from the environment to the i^{th} compartment in the form of sunlight (η_0) and nutrients ($\xi_{i,0}$); $f_{i,j}$ represents the flow coefficients from compartment j to compartment i ; $\alpha_{0,i} \in [0,1]$ denotes the migration from compartment i to the environment η_0 , the value of this parameter should be kept under control

because the migration cannot be larger than the available biomass; $\lambda_{0,i}$ denotes the energy consumed by respiration and metabolism for each living species. η_j represents the constant term in those compartments which are interacting with environment; their flow has been modeled using one constant term and one dependent on the interacting compartment in the lake. It is also worth mentioning here that all parameters used in the deterministic differential equations are assumed to be positive. A unique solution of the linear differential equations (1 to 11) can be investigated if value of all parameters and initial conditions are specified for each of the state variables x_i at time $t = 0$, e.g., $x_i(0) = c$, where c is constant. In our model, the flow parameter values were generated randomly between 0 and 1 while the initial conditions are given between 1 and 10, to see different variations. The steady state solution does not depend on the initial conditions but the flow rate coefficients.

The stability of the ecological network model and corresponding Rawal Lake ecosystem under specific flow to each compartment can be determined by investing the eigenvalues of the Jacobian matrix (J) of the system of equations (1 to 11).

$$J = \begin{bmatrix} \frac{dx_1}{dx_1} & \dots & \frac{dx_1}{dx_{11}} \\ \vdots & \ddots & \vdots \\ \frac{dx_{11}}{dx_1} & \dots & \frac{dx_{11}}{dx_{11}} \end{bmatrix} \quad (12)$$

The characteristic equation of the Jacobian matrix is used to calculate the eigenvalues to test the stability of the system.

$$\det(J - \rho_i I) = 0 \quad (13)$$

Where I is the identity matrix and ρ_i are the eigenvalues of the system. If the real part of the all eigenvalues is less than zero, then system is defined to be stable in every simulation. If the eigenvalues all have a negative real part, then the system is stable in the certain values of system and boundary flows; and, if any eigenvalue has a positive real part, then the system is defined to be unstable (Hearon, 1963).

8. Results

Due to limitations both in quantity and quality of the observed database, the developed model is simple and rests largely on general ecosystem principles and previous approaches for modelling such systems. A demanding effort is required both to calibrate and validate the model and to simulate external threats and their consequences to the Rawal Lake ecosystem. We presume that external threats can enter into the lake in the form of energy through nutrients and terrestrial input via run off or a variation in solar radiation, population dynamics (migration) of migratory and local birds, ducks, and in fishing. Since we have included all these threats in the model equations, their impact could be seen through the solution and stability of the system. The stable ecological condition will correspond to the stable mathematical model under the corresponding inflows and outflows. However, if the biomass of a few compartments is converging to zero or unbounded in others, then this corresponds to an unstable model.

To check the stability of the donor controlled ecological network model the eigenvalues of the Jacobian matrix have been calculated from equations 1 to 11. To cope with data scarcity issue with Rawal Lake ecosystem, Monte-Carlo simulations have been performed for 10,000 experiments by generating the random numbers in MATLAB ranging between 0 and 1 for system flow coefficients. To check the maximum and minimum stability under different flow values in the network model, the boundary flow coefficient values have been varied from 0 to 6 by varying one and fixing the values of other three boundary flow coefficients. The percentage of stable cases was plotted for all boundary flow coefficients. It was noted that the stability is higher when the values of boundary and system flows are comparable. However, the increase or decrease in the fishing flow coefficient does not affect the stability of the model.

Generally the stability of a mathematical model assumed that the value of biomass at steady state will always converge to a number, it could be positive or negative to fulfill the convergence criterion. The convergence to a positive

number for biomass meets the ecological criterion of stability, but biomass weight could not be negative; therefore, convergence to a negative number does not satisfy the ecological criteria of stability. If the exogenous input is removed, then all the compartments biomass will decay down to zero. To emphasize this fact, the following results will show the percentage of the positive biomass from 10,000 experiments under the Monte Carlo sample of inflow coefficients in each experiment. The four cases with positive stock values for each compartment are given by the following pictures. The y-axis in the following figures (4-10) shows the percentage of models exhibiting stable behavior.

The phytoplankton compartment receives energy from solar radiation and nutrient input from the adjoining areas and streams which drain in the lake. The phytoplankton compartment, which is the base of the Lake Rawal food web, is largely influenced by the variation in solar radiation and nutrient input from the surrounding area. The more input to this compartment the more rapid the biomass increase, which can increase the turbidity of water and decrease the dissolved oxygen. Therefore, moderate supply to this compartment will be helpful for ecological balance. We have studied how the variation in input to the phytoplankton compartment from the environment could possibly change the stability of the ecosystem. We have also added ecological constraints such as that the biomass values must be non-negative. The results (Figure 4) show that ecosystem stability (with positive biomass) strongly depends on the phytoplankton fluctuations in that an increase in solar radiation and nutrients will decrease the percentage of stable situations. It can be clearly seen in Figure 4 that increasing the inflow energy in the boundary flow coefficient to the phytoplankton compartment makes the ecosystem more and more unstable with positive biomass values decreasing rapidly.

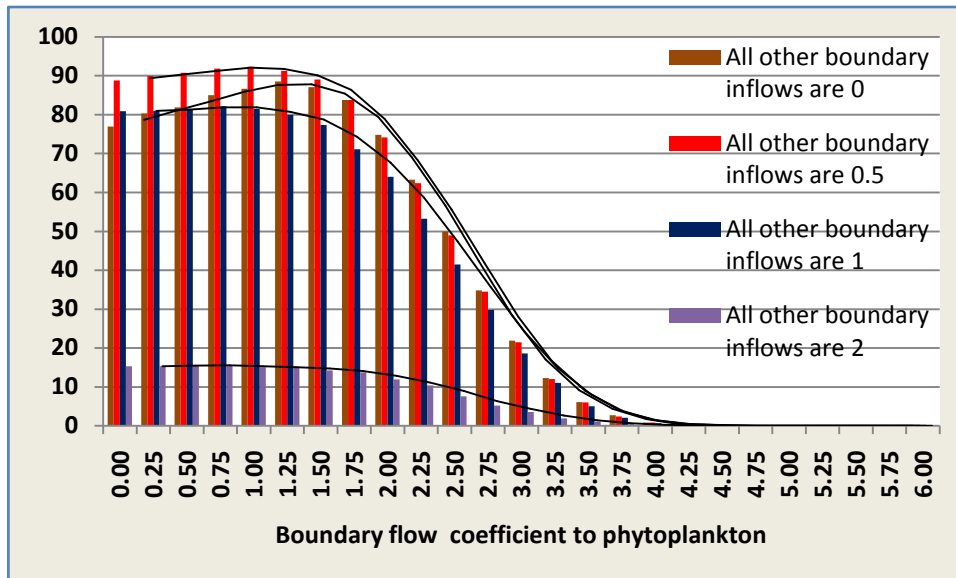


Figure 4: The percentage of stable results with positive values of biomass of ecological network model by varying the boundary flow coefficient to phytoplankton compartment by fixing the other boundary flows in Monte-Carlo simulations

The Water Plants/ Macro Algae compartment also contributes importantly to the importation of energy into the ecosystem. This lake compartment also intermingles with the environment as well as other species like birds, ducks and sediments in the lake ecosystem. The inconsistency in biomass inflow to Water plants/Macroalgae compartment from the environment could play an important role in quest of stability and positivity of biomass in model simulations. The external threats through water plants/ macroalage compartment penetrate slower than the phytoplankton compartment in the lake ecosystem such that the response is not seen until the input loading reaches a higher level. This compartment receives energy from the environment and provides habitat and food to fish and other living organisms in the lake. Their presence in the lake limits floating material, boating and water turbidity.

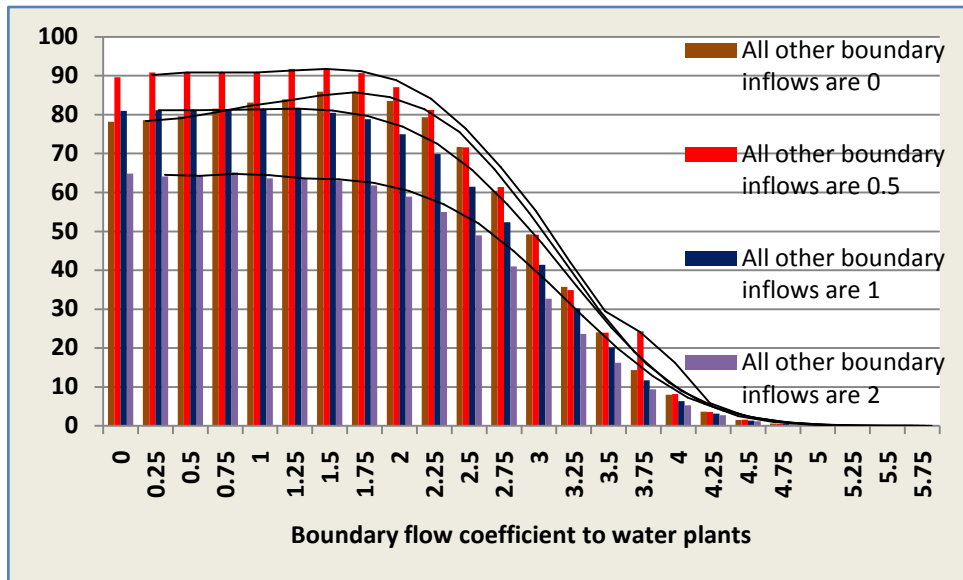


Figure 5: The percentage of stable results with positive values of biomass of ecological network model by varying the boundary flow coefficient to water plants and macroalgae compartment by fixing the other boundary flows in Monte-Carlo simulations

The variation of inflow to Detritus/TSS compartment links with the terrestrial input from the environment in the lake ecosystem. More waste produced in the associated misused areas will aggregate the detritus or total suspended solid in the lake. This compartment feeds Zooplankton and Small Fish compartments in the ecosystem network. The stability of the ecosystem model is highly dependent on the energy interactions in the Detritus/TSS compartment. This compartment provides a good example of biotic and abiotic interactions in different components of the ecosystem. Input into this compartment larger than system flow values (Figure 6) rapidly decreases with the Monte Carlo sampling of system flow coefficients to simulate stability or collapse (intuitively) of the model. The nutrients are also consumed by the water plants and macroalgae and their food can affect the fluctuation in biomass of this compartment. The stability of the simulation model due to variation in the Detritus/TSS compartment changes with the accumulation in boundary inflow. The percentage of the stable results having negative real part eigenvalues and with positive biomass out of 10,000 experiments are as shown in Figure 6.

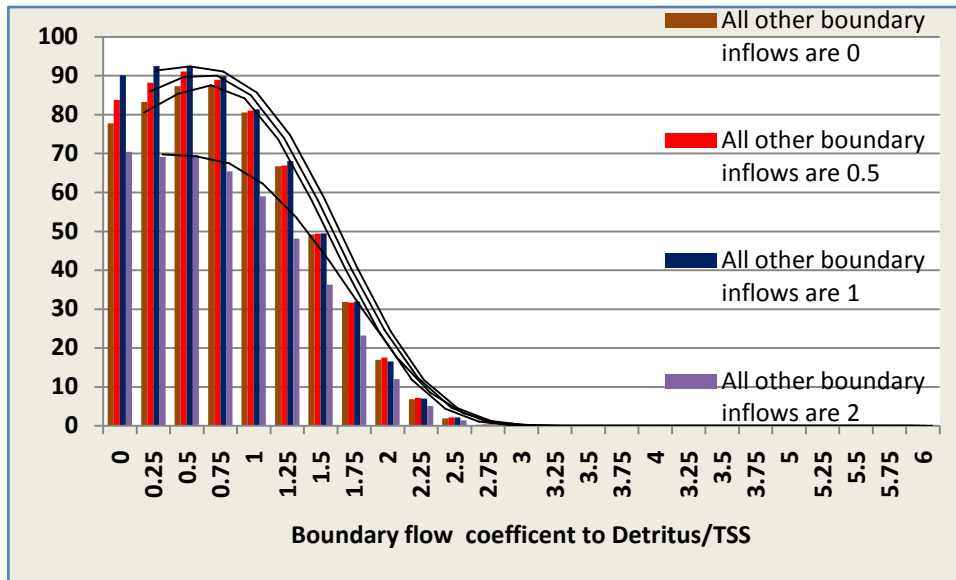


Figure 6: The percentage of stable results with positive values of biomass of ecological network model by varying the boundary flow coefficient to detritus/TSS compartment by fixing the other boundary flows in Monte-Carlo simulations

If the top predator like large fish biomass leaves the compartment through fishing, then the model remains stable which could be seen in Figure 7. The variation in the fishing component from the Large Fish compartment does not directly influence the stability of the model, as all model runs were stable. However, when all other boundary inflows given four different values the stability changes, showing that stability of fishing component does not have any influence on stability of the system (Figure 7).

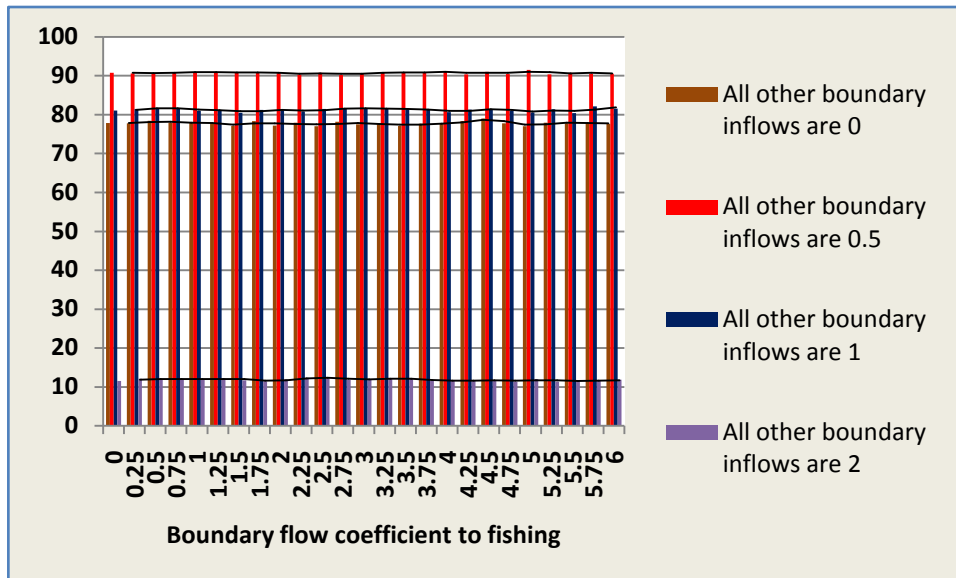


Figure 7: The percentage of stable results with positive values of biomass of ecological network model by varying the boundary flow coefficient to large fish compartment by fixing the other boundary flows in Monte-Carlo simulations

The model simulations can be used to estimate the values of different ecological parameters like biomass, system and boundary flows. The importance of direct and indirect connectedness among different species or compartments can also be measured approximately. The steady state situation in the model results means that “input = output” for all compartments. If the exogenous input is kept zero, then the biomass in each compartment will also decay down rapidly, hence will cause the instability in the ecosystem. The following figures (Figure 8 & 9) demonstrate the physical look of amplified, attenuated and dead time of different compartments in the network. The sensitivity of the model for stable and unstable dynamics helps making procedures to lucidly adjust the results with observed data and its behavior.

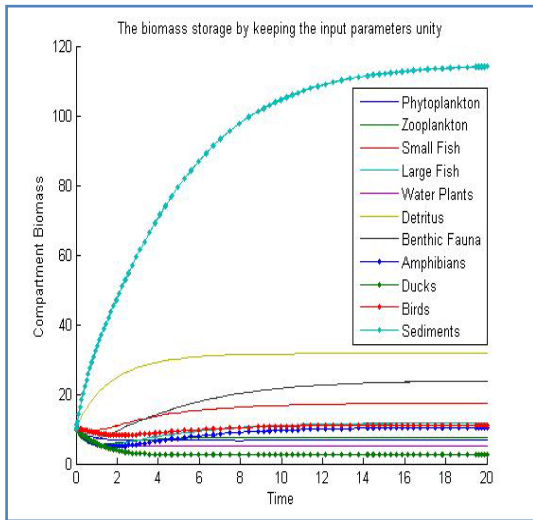


Figure 8: The stable state of compartmental biomass by adjusting the all boundary flows coefficient to unity and system flows coefficient to 0.5

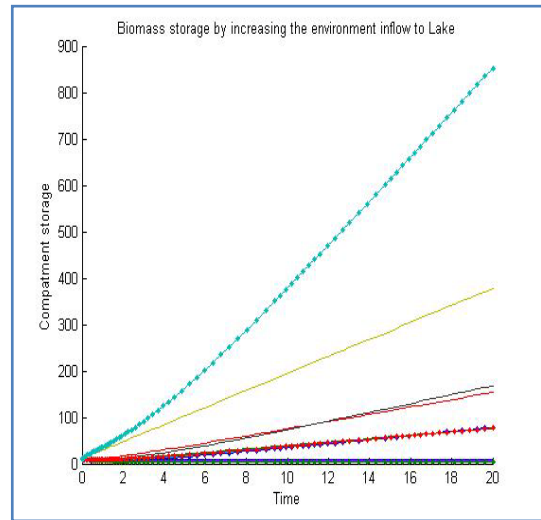
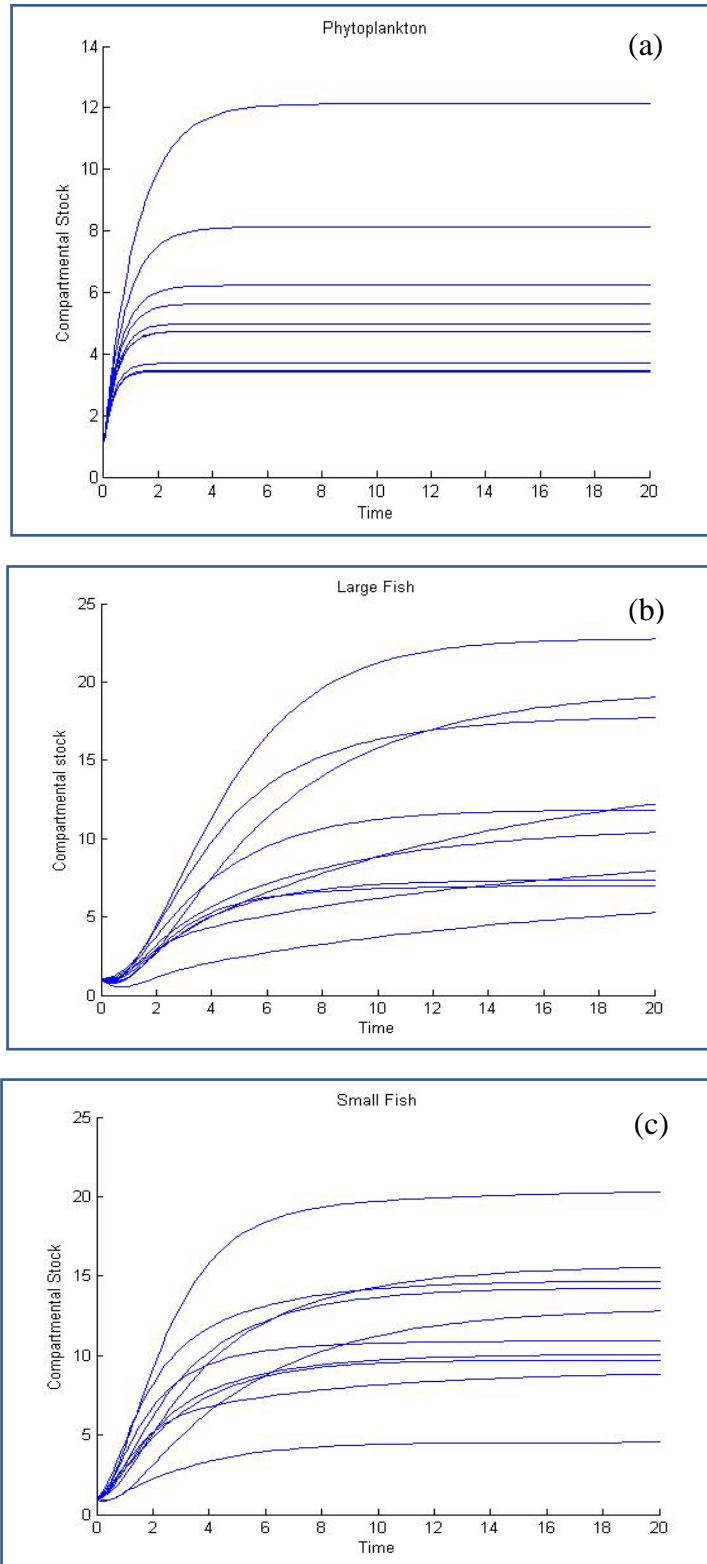


Figure 9: The unstable state of compartmental biomass by adjusting all boundary flows coefficients to 2 and system flow coefficients to 0.5

The above results show that certain perturbations to the ecological system, as represented by the system of equations, can have a negative impact of the system stability. These perturbations could originate from a number of sources including climatic and technological changes to Rawal Lake. It is very important to understand how energy flows through and is stored in those compartment's biomasses which are viable both economically and ecologically. To ascertain these facts, more knowledge on the system compartments and their interactions is required.

A Dynamic Equilibrium is a system's state where conditions are held approximately invariable by negative feedback systems operating within the ecosystem. In stable dynamic equilibrium means, the biomass will converge to a positive biomass. However, the value of the biomass is dependent on the flow rate and stock value. The following Figures 10 (a, b, c) illustrate the resulting biomass in 10 experiments for 20 time steps in Phytoplankton, Large Fish and Small Fish compartments and their convergence to positive biomass depending on different flow rate (using Monte Carlo simulations). By assigning the initial biomass value as 1 and generating random system flow coefficients each time, the compartments reach different steady-state biomass values .



**Figure 10: The process of reaching equilibrium state of two compartments
 (a) Phytoplankton (b) Large Fish (c) Small Fish**

9. Conclusions

The development of linear donor-controlled ecological network model is useful to find the resilience of the lake ecosystem under different boundary flows. The network model of Rawal Lake was developed from the existing literature by considering the major known ecological interactions. The presence of an energy flow between groups within the ecosystem has been identified as described in the adjacency matrix. The initial conditions and boundary flows have been roughly estimated. Due to the unavailability of data for system and boundary flows Monte-Carlo simulations were performed by generating values of flow coefficients between 0 and 1. Simulations have been performed for the different values of boundary flow coefficients. The analysis using real data will increase the significance of the model for management purposes. Due to the unavailability of empirical values of system and boundary flows just show the conceptual behavior of the lake ecology under different scenarios of mainly input boundary flows. The results demonstrate that changes in inflow parameters (environmental) directly affect the stability of the Rawal Lake ecosystem. The data gaps identified in Table 1 invite the need of more studies for the lake. The model is equally useful for other lakes potentially with similar ecology to envisage the behavior under different threats.

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