RM-73-2

INTERNATIONAL INSTITUTE FOR IIASA APPLIED SYSTEMS ANALYSIS

RESEARCH MEMORANDUM

"RISK" AND ENERGY SYSTEMS: DETERMINISTIC VERSUS PROBABILISTIC MODELS

Robert L. Winkler

September 1973

SCHLOSS LAXENBURG
2361 Laxenburg
AUSTRIA

"RISK" AND ENERGY SYSTEMS: DETERMINISTIC VERSUS PROBABILISTIC MODELS

Robert L. Winkler

September 1973

"Risk" and Energy Systems: Deterministic Versus Probabilistic Models *

Robert L. Winkler

Introduction

At the conference on "Energy Systems" that was held in Baden July 17-20, 1973, the discussion regarding models and model-building was, for the most part, limited to deterministic models. Only at the end of the conference, in the discussion of "risk and reliability problems," was the question of deterministic models versus probabilistic models brought up. The brief discussion that ensued indicated that there exists some hesitancy on the part of model builders in the energy systems area to include probabilities in their models. Such model builders recognize the presence of uncertainty in the situations they are modeling, but they appear to feel uncomfortable about formally representing this uncertainty in terms of probabilities. This uncomfortable feeling may be due to several factors, including a lack of familiarity with probabilistic models, a question about the source of probabilities for probabilistic models, a feeling that deterministic models are perfectly adequate, and a concern that probabilistic models regarding energy systems would be too complex and difficult to handle.

Unfortunately, because the question of deterministic models versus probabilistic models arose so late in the conference, adequate time was not available for a full discussion of the question. Of course, a full discussion would require several days with many papers and presentations. Since that is not immediately feasible, this paper represents an attempt to present an overview of some of the issues that are involved in the question of deterministic models versus probabilistic models.

The contents of this paper may be summarized as follows. In Section 2 an argument is made for the use of probabilistic models in situations in which there is uncertainty about some of the variables of interest. The next two sections consider questions of implementation: the question of moving from deterministic models to probabilistic models and the question of "determining" probabilities to use as inputs to probabilistic models. In Section 5 an area related to energy systems for which the notion of probability has been used, the area of "risk and reliability," is considered briefly, and the distinction between uncertainty concerning events, or variables, and preferences concerning outcomes, or

---

*This paper was presented at the IIASA Planning Conference on Energy Systems, Baden, Austria, July 17-20, 1973, and appeared in the Conference Proceedings (IIASA-PC-3).

consequences, is discussed. Section 6 contains a short summary emphasizing the implications of this paper for future research in the area of energy systems.

Why Use Probabilistic Models?

Model-building activities in the area of energy systems have focused almost exclusively on deterministic models. These models are deterministic in the sense that they assume that the variables of interest are known to take on certain fixed values. That is, the model builder acts as though the variables are known and fixed, even if it is clear that they are not. For example, an important variable in the study of energy systems is future demand for energy. A variable such as the total demand for energy in the world in the year 1980 might be included in a model of energy systems. Of course, it is recognized that a variable such as this is not observable several years in advance. The usual procedure is to arrive at an estimate of the demand for energy in the year 1980 and to treat this estimate as a certainty equivalent. The term "certainty equivalent," which comes from the area of statistical decision theory, indicates that even though one is not certain about the value that the variable will assume, one acts as though one were certain, treating the estimate as if it were an actual value that had already been observed.

Unfortunately (from the point of view of ease of analysis), the world is not deterministic in nature. Certain variables, such as variables relating to current technology, current demand for energy, etc., are known or can be estimated precisely enough so that they can be assumed known for most purposes. For many other variables, however, particularly variables involving future points in time (e.g., future demand for energy, future advances in technology), there may be a considerable amount of uncertainty. This uncertainty can be represented formally in terms of probability and can thus be incorporated into models of energy systems. The use of probabilistic models gives the model builder an opportunity to represent the "current state of knowledge" much more accurately than is possible with deterministic models.

One reason that the model builder should be concerned with using probabilistic models instead of deterministic models, then, is that probabilistic models enable the formal consideration of uncertainty. Important problems such as the study of energy systems involve a considerable degree of uncertainty with respect to many of the variables of interest. For many variables, some information is available, but the variables are not known for certain. In the case of variables involving the future, it would be nice to possess clairvoyance and to be able to foretell the values that will be assumed by these variables; unfortunately, this is not possible. To represent such variables in deterministic models by using estimates as certainty equivalents is to ignore the uncertainty concerning the variables. In acting as though one had more information than one actually has, one is, in fact, ignoring information. For example, in acting as though the demand for energy in 1980 were known for certain in 1973, a model builder is ignoring information that indicates that the actual demand in 1980 might be considerably above or below the value that is being used as a certainty equivalent.

Of course, the fact that deterministic models ignore uncertainty about variables is not sufficient to justify the use of probabilistic models in place of deterministic models. The most important aspect of the model for decision-making purposes is the output of the model, not the model itself. For instance, if deterministic models always yielded results identical to those of probabilistic models, then the simplification of not formally representing the uncertainty in the world (i.e., the use of deterministic models) would provide perfectly adequate results and would have the advantage of improved tractability (in comparison with probabilistic models). From statistical decision theory, it is known that under certain conditions, an entire probability distribution may be replaced by a single certainty equivalent, such as the mean of the distribution, without affecting the results of the model. For example, if the "payoff functions" for the various actions in a decision-making problem can be represented as linear functions of a particular uncertain quantity (random variable), then knowledge of the mean of the probability distribution of that uncertain quantity is sufficient for decision-making purposes. In this situation, the mean provides as much information (for the specific decision-making problem of interest) as does the entire probability distribution, so the mean can be used as a certainty equivalent.

There are situations, then, in which the use of certainty equivalents leads to perfectly acceptable results. Even if these situations, care is needed in the choice of a certainty equivalent, as this choice should depend on the structure of the decision-making problem at hand. In some instances, such as the example in the preceding paragraph, the mean should be used as a certainty equivalent. In other instances, the use of the median or some other fractile is indicated. For example, the extreme tails of a distribution may be very important, in which case the .001 fractile or .999 fractile of a distribution is much more useful than a value from the "center" of the distribution. It appears that such considerations have been ignored in the determination of estimates to be used as inputs to deterministic models of energy systems.

Although it should not be ignored, the question of the determination of appropriate certainty equivalents in situations where certainty equivalents are adequate representations of entire probability distributions is much less important than the question of whether certainty equivalents are adequate representations of entire probability distributions. In complex models that include many variables, it is generally invalid to replace probability distributions of the variables with certainty equivalents. That is, the replacement of the probabilistic model by a deterministic model that uses certainty equivalents leads to different inferences and decisions. This is particularly true when uncertainties exist about many variables and when the variables are not independent. In complex situations such as energy systems, the number of variables is large and there are obvious dependencies among variables. For instance, the demand for energy is clearly not independent of the price of energy, and the demand for energy in, say, 1985 is not independent of the demand for energy in 1980. As a result, a probabilistic model that takes into account such stochastic dependence will generally yield results different from a deterministic model using estimates as certainty equivalents. (This point will be discussed further in Section 3.) In the case of complex situations such as energy systems, the differences may be quite substantial.

Probabilistic models, then, have the advantage of formally representing the model builder's uncertainty about variables of interest, including stochastic dependency among variables. Such models are thus more realistic than deterministic models in that they do not ignore the uncertainty inherent in most real-world situations. Furthermore, probabilistic models have the advantage of being adaptive with respect to new information. As new information becomes available about the variables of interest, the probability distributions used in the model can be updated, so that at any point in time, these probability distributions represent the current state of information. For example, the information that a particular new technological advance has been discovered may cause revision of probability distributions regarding future costs of providing certain types of energy, regarding future technological advances, and so on.

The adaptive nature of probabilistic models has very important implications for decision making. It enables the model builder to treat the decision-making process as a dynamic process. This means that it is possible, within the framework of an uncertain world, to formally consider the interrelationships among decisions that are made at different times. The effect of today's decisions on tomorrow's alternatives, the possibility of delaying a decision until further information is available, the anticipation of future decisions, and so on, can all be formally considered within the framework of probabilistic models. A considerable amount of theoretical work in this area has been conducted in the past decade, and the theory of adaptive probabilistic models and of dynamic decision-making models is quite well developed.

In summary, the world we live in is an uncertain world, it is a changing world where new information continually becomes available, and it is a world where decisions made at one time may have strong effects on alternatives available at other times. Deterministic models simply fail to include some of the salient aspects of this world, and this failure casts doubt upon the results of such models, both in terms of inferences about the future and in terms of decisions that are based on the models. In comparison, probabilistic models are capable of providing a more realistic view of the world that faces the model builder. Probabilistic models allow for uncertainty by the inclusion of probability distributions to represent the uncertainty; they allow for new information by being adaptive and revising probability distributions as new information is obtained; and they allow for interrelationships among decisions at different times by being dynamic and formally considering such interrelationships. In theory, at least, the argument for probabilistic models to deal with complex systems such as energy systems seems compelling. The next two sections of the paper consider the question of implementation.

Moving from Deterministic Models to Probabilistic Models

In the previous section, an argument was made for the use of probabilistic models instead of deterministic models in situations in which there is uncertainty about some of the variables of interest. This argument may have given the impression that the position of this paper is that deterministic models are of little value. On the contrary, the building of deterministic models can be viewed as a very important first step in the development of probabilistic models. Building deterministic models is by no means an easy task, particularly in the case of complex situations such as energy systems. While a deterministic model that involves the use of estimates as certainty equivalents does not take into account the uncertainty present in the real-world situation, it does provide a result for a particular scenario. This scenario is simply the situation in which all of the variables take on exactly the values given by their certainty equivalents.

If the particular scenario represented by a deterministic model were sure to occur, then the deterministic model would provide an accurate representation of reality. Of course, because of the uncertainty about the real world, everyone recognizes that any single scenario is highly unlikely to occur exactly. At the Baden conference on energy systems, this was exemplified by disagreements concerning the appropriate values to use as estimates in some cases. One participant commented with respect to a particular model that one should not place too much reliance on the results of the model because the model incorporated an estimate of the demand for energy in the year 2000 and there is considerable uncertainty as to what that demand will be. In effect, the comment implies that one should not place too much reliance on the results generated from a single scenario.

The first step in moving from deterministic models to probabilistic models is to consider several scenarios instead of just one. In other words, try different sets of values for the variables of interest and see how the results vary as the inputs are varied. This approach is called sensitivity analysis. In this manner, it may be possible to identify some variables for which the uncertainty is not crucial (with respect to the results of the model) and other variables for which the uncertainty _is_ crucial. That is, large variations in the estimate of a particular variable may not lead to changes in the essential nature of the results of the model. The model is then said to be insensitive to changes in the value of that variable. On the other hand, very small variations in a second variable may lead to substantial changes in the results. The model is then said to be highly sensitive to changes in the value of the second variable. This sort of analysis helps the model builder identify variables for which a probabilistic analysis would be most valuable.

In a sense, a sensitivity analysis indicates how adequate a deterministic model is. If many scenarios are considered, and the results of the model (whether in terms of inferences or in terms of decisions) do not change much, then the deterministic model provides a good approximation to a probabilistic model. In this case, unless a great deal of precision is desired, it may not be worth the time and effort required to develop a probabilistic model. If the different scenarios lead to different results, on the other hand, then the deterministic model is highly suspect and a probabilistic model would be quite valuable.

It must be emphasized that in conducting a sensitivity analysis, it is important to vary the values of different variables simultaneously. Because of dependencies among variables, it is not sufficient to adjust just one variable at a time. This implies that a thorough sensitivity analysis requires a large number of scenarios, with all sorts of combinations of values of the variables investigated. As the number of scenarios increases, of course, the time and effort required increase and the results of the sensitivity analysis become more difficult to interpret. The larger the number of variables and the greater the degree of dependence among variables, the greater the difficulties are.

An even more serious problem involving sensitivity analysis is that although the sensitivity analysis may give the model builder some idea of the potential variations in the results, it does not do so in a probabilistic manner. For example, suppose that a deterministic model, using certainty equivalents, is built to predict the demand for energy in the year 2000, and the point prediction turns out to be 2 Q/yr., where 1 Q = 10^{18} BTU. Suppose further that the model includes many variables (values of the demand for energy at intermediate times, technological advances, etc.) and that a sensitivity analysis is conducted, using a large number of scenarios. The sensitivity analysis indicates that the demand for energy in the year 2000 might be as small as 1 Q/yr. or as large as 4 Q/yr. This is a very large range of values, but it still does not provide any information about the probability distribution of the demand for energy in the year 2000. It may be that the distribution is relatively tight, with a probability of, say, .90 that the demand will be between 1.9 Q/yr. and 2.2 Q/yr.; in which case more extreme values are possible but not too likely. On the other hand, it may be that the distribution has a large dispersion and that the probability is .90 that the demand will be between 1.2 Q/yr. and 3.5 Q/yr., in which case the "extreme" values are not so unlikely. From the sensitivity analysis it is not possible to tell how likely the various scenarios are and thus how likely the various results are.

Therefore, although a sensitivity analysis may give the model builder some idea of how sensitive the results of a deterministic model are to variations in the inputs, it is only a first step beyond the deterministic model. The next step is to build a reasonably simple probabilistic model. Such a model can be constructed by considering a few "representative" values of each variable of interest and assessing a probability distribution over all possible combinations of values. Note that it is not sufficient to consider just marginal distributions of the variables; in order to include the interrelationships among the variables, a joint distribution is needed. In practice this joint distribution is usually broken down into a marginal distribution and a series of conditional distributions. For example, it can be expressed as a product of the marginal distribution of the first variable, the conditional distribution of the second variable given the first variable, the conditional distribution of the third variable given the second variable, and so on. Usually this is expressed schematically in terms of a tree diagram, with the initial "fork" containing "branches" representing values of the first variable, each of which is followed by a second fork with branches representing values of the second variable, and so on. In a sense, this tree diagram can be thought of as a procedure for considering various scenarios, with probabilities assigned to the scenarios. The tree diagram makes it relatively simple to see the logical relations among the variables and to understand the stochastic nature of the model. Note, by the way, that the question of the source of the probability distribution is being avoided here and will be discussed in the next section.

The type of probabilistic model described in the preceding paragraph provides a representation of the uncertainty in the situation of interest. With complex systems involving many random variables, tree diagrams quickly become cumbersome. By utilizing results from the theory of probability, the theory of stochastic processes, and so on, it is possible to generate models that are somewhat easier to work with. Conceptually, however, the idea is the same in the simple probabilistic model as it is in fancier probabilistic models, and the degree of sophistication used in the development of a probabilistic model depends on factors such as desired "closeness" of approximation, computational ease, and so on.

In summary, a deterministic model provides a first step in the analysis of a problem involving uncertainty. A sensitivity analysis can then be used to indicate how variations in the inputs affect the results. Care must be taken in the interpretation of the sensitivity analysis, however, since such an analysis is not probabilistic. In rare instances, the results of the sensitivity analysis may be completely unequivocal (e.g., when virtually all scenarios considered lead to the same result), but in most cases the sensitivity analysis is limited to indicating where a probabilistic analysis would prove most fruitful. The next step is to build actual probabilistic models, with the degree of sophistication of the probabilistic inputs depending on the situation at hand. In general, the process of constructing a probabilistic model is a sequential process, with the model-building activities at each step depending to some extent on the results of the previous step. The aim, of course, is to balance off the realism of the model with such factors as the cost of building and solving the model.

The Assessment of Probabilities

Suppose that the premise of Section 2, that it is advantageous to use probabilistic models instead of deterministic models in situations where uncertainty is present, is accepted. Then, using an iterative approach such as that described in Section 3, certain variables are identified as important random variables, or uncertain quantities, and the model builder decides to treat these variables probabilistically. How can this be done (i.e., how can a probability distribution for the variables of interest be arrived at by the model builder)?

The objective in assessing a probability distribution for a variable or a set of variables is to represent all of the information available concerning the variable(s). Instead of trying to summarize all of this information in terms of a single estimate, as in the deterministic models, the model builder wants to summarize the information in terms of a probability distribution. Experts provide a very important source of information. With regard to energy systems, experts on the demand and supply of energy, experts regarding technological developments, experts regarding social and political considerations, etc., might be consulted. These experts could be asked to assess probability distributions for the variables of interest. In the past decade a considerable degree of work has been done in the area of probability assessment. This work, which is continuing, provides procedures that can be used to elicit probability distributions from experts.

Probability distributions obtained from experts are, of course, subjective probability distributions, and as such they may differ from person to person. When subjective opinions differ a great deal it is often desirable to probe the differences in an attempt to find out the root causes of the differences. For example, two potential causes are different interpretations of terms and experience with different sets of background data. In order to somehow "pool" different opinions, it may be useful to consider probability distributions obtained from a group of experts rather than a single expert. Questions such as the consensus of experts' probability distributions and the consideration of group assessments of probability distributions have received an increasing amount of attention recently. In situations such as energy systems, where experts are available and a great deal of the available information is of a subjective nature, the subjective probabilities of experts provide a key input to models of the situations.

Another source of information is past data. In terms of energy systems, past data regarding variables such as demand and supply of energy costs of power plants, inflation rates, and so on, can be obtained. Attempts can then be made to fit stochastic models to the data and to use these models to generate probability distributions for the variables of interest. Sophisticated results from stochastic processes, time series analysis, and other areas relating to statistics and probability may prove valuable in attempting to analyze the past data and to make predictions concerning future variables.

Most probabilistic models of complicated systems involve uncertainties of both kinds: uncertainties that require subjective assessments and uncertainties that can be investigated in terms of objective data. In fact, when objective data are available but sparse with regard to a particular variable, it is generally desirable to consider both subjective assessments and objective data for that variable, and this sort of situation may be the rule rather than the exception. Indeed, it often may be that the only way to exploit objective data about one set of variables is to incorporate subjective assessments about the same variables or about another set of variables. The refusal to include subjective probabilities in a model may force the model out of the probabilistic mode into the deterministic mode, and as a result objective data as well as subjective assessments wind up being ignored. Sometimes an attempt is made to incorporate objective data in a model while ignoring subjective assessments, but this just amounts to throwing away information, particularly in view of the fact that the model-building process is basically subjective anyway (e.g., elements of the model-building process such as the choice of variables, the representation of relationships among the variables, and even the way in which objective data are used in the model are ultimately subjective in nature).

Ultimately, the model builder must decide upon a probability distribution to use as an input to the model. In doing so, he may utilize probability distributions based on analyses of past data, and any other information that may be available. If, for example, the model builder feels that a particular expert has been overly optimistic with respect to a particular variable in the past, it might be decided to adjust that expert's probability distribution somewhat to correct for the optimism. If the assumptions underlying a statistical analysis are somewhat in doubt, it may be decided to adjust the probability distribution that is based on that analysis. Moreover, just as a sensitivity analysis can be conducted with a deterministic model, a sensitivity analysis can be conducted with a probabilistic model to investigate the sensitivity of the results to variations in the probability distributions. This may provide the model builder with some idea of what aspects of the probability distribution need particular care and what aspects are not so crucial. The overall objective, naturally, is to arrive at a probability distribution that represents the current state of information with regard to the variables of interest.

An important topic related to the assessment of probabilities is the revision of probabilities on the basis of new information. As noted in Section 2, one important aspect of probabilistic models is the adaptive nature of the models with respect to new information. As new information is obtained, Bayes' theorem provides the formal mathematical mechanism for revising probability distributions. The application of Bayes' theorem requires the assessment of likelihoods that represent the impact of the new information with regard to the variables of interest. These likelihoods are then formally combined with the original probabilities to yield a revised probability distribution. The assessment of likelihoods is similar to the assessment of the original probability distribution; experts may be consulted, certain statistical models may be useful, and so on.

To give a detailed discussion of the assessment of probabilities and the revision of probabilities on the basis of new information would require too much space. The purpose of this section was to cover briefly some of the notions involved in the assessment and revision of probabilities . These notions, together with the discussion in Section 3 of moving from deterministic models to probabilistic models, should provide some indication of how the suggestion of using probabilistic models can be implemented. For more detailed discussions of these notions and of probabilistic models in general, see Raiffa [2] and Winkler [4].

Risk and Reliability

One area related to energy systems for which the notion of probability has been used is the area of "risk and reliability" (e.g., Otway, Lohrding, and Battat, [1], Starr, Greenfield, and Hausknecht, [3]). In this context the term "risk" generally refers to the possibility of effects detrimental to health or, in the extreme, causing death, directly related to installations such as nuclear power plants. These concerns are based on factors such as the potential emission of pollutants (including radioactive pollutants) and the possibility of large-scale "accidents." The "risks" are measured in terms of probabilities that may represent mortality rates, probabilities of various types of accidents, and so on. In turn, these probabilities are related to the "reliability" of the installations in question, hence the term "risk and reliability."

The events of concern in "risk and reliability" studies tend to be relatively rare events, and the probabilities are very small. Such events are difficult to deal with, partially because they occur so seldom that it is difficult to build up any sort of experience with them. In other areas, rare events are considered (in a "risk" context) regularly, and it may be possible to look at such well-established areas to see how rare events are handled and how the concept of risk is considered. A prime example is the area of insurance, where for a certain premium, an insurance company will assume the risk associated with a particular rare event.

Probabilities such as those mentioned in the preceding paragraphs are certainly relevant with respect to models involving power plants, particularly where alternative types of power plants are being considered. With respect to the modeling of energy systems, however, this is but a small portion of what one might call energy systems. As indicated in the previous sections, probabilities should be considered for many different types of variables relating to energy systems, and it would seem that probabilities relating to risk and reliability are no more valuable than probabilities relating to other aspects of energy systems. The probability of adverse health effects or death due to a particular type of installation is important. But what about the probability of a severe energy shortage within the next two decades? What about the probability that the cost of a particular form of energy will increase tenfold over the next decade? What about the probability that technological developments will lead to a new, cheaper form of energy that is not now known? Probabilities such as these all seem very important and very relevant for the modeling of energy systems, but they do not seem to be considered (at least formally) in current models relating to energy systems.

In a sense, the use of probabilities in "risk and reliability" studies entered through the back door, under the category of "risk." Indeed, it appears that in such studies probabilities such as the probability of death are treated as measures of risk. This is in accord with the everyday use of the term "risk" by the layman, but it is an oversimplification from the standpoint of statistical decision theory. In statistical decision theory, a decision maker's attitude toward risk in general is measured by a utility function that represents the decision maker's preferences for various outcomes, or consequences. For any specific decision-making problem, the action chosen by the decision maker should depend on the probability of various consequences <u>and</u> on the preferences for the various consequences. Probability <u>is</u> used to represent the uncertainty concerning the various consequences, but this does not provide any information about the decision maker's preferences.

For inferential purposes, probabilities will suffice. For decision-making purposes, some consideration must be given to "values," or preferences for consequences. Furthermore, the consideration of probabilities should be separated from the consideration of values; the formal decision-theoretic framework can be used to take both aspects into consideration in determining an "optimal" decision.

The consideration of values is a difficult question that requires careful investigation. For most problems of interest, and certainly for large-scale problems such as energy, the consequences of concern involve multiple attributes. Decisions regarding energy systems involve considerations such as the costs of alternative systems, the cost of energy to the consumer, the impact on the environment, the impact on the climate, and so on. Some work has been done in recent years regarding multiattribute utility, and hopefully this will prove useful in the modeling procedure. Another point of interest is that large-scale problems invariably involve societal effects as well as individual effects, and the question of aggregating individuals' preferences or talking of "society's preferences" is a complex and difficult one. Nevertheless, issues such as this need to be considered in modeling large-scale systems.

In summary, the area of "risk and reliability" has utilized probabilities to some extent, but there are many more ways in which probabilities would be useful in the study of energy systems, potentially even more useful than in the context of risk and reliability. Moreover, the term "risk" suggests considerations of preferences for various consequences, and such preferences are an important input for decision-making models. It is important to distinguish between uncertainty concerning variables and preferences concerning consequences; these two concepts should be considered separately and brought together by the formal model.

Implications for Future Research on Energy Systems

As noted in Section 2, model-building activities in the area of energy systems have focused almost exclusively on deterministic models. In this paper an argument is presented for the use of probabilistic models. The world we live in is an uncertain world, and probabilistic models enable the model builder to formally include uncertainty in models. Because the world is not deterministic, the results of deterministic models must be viewed with some suspicion; in contrast, probabilistic models have the advantage of being adaptive and allowing decision making to be treated in a dynamic sense.

Deterministic models represent a first step in model-building, and it is an important first step. Sensitivity analysis provides further information and may help to suggest variables for which a probabilistic treatment would be most useful. The probabilistic models themselves can range from very simple decision trees to very complex models that use advanced mathematical results. The probability distributions that represent the model builder's uncertainty may be based on probability distributions assessed by experts, on past data, on forecasts generated by sophisticated statistical procedures. Once the probabilities are assessed, the model can be solved, using analytical techniques if possible and numerical methods otherwise.

One small aspect of energy systems, that of risk and reliability, has received probabilistic treatment, as noted in Section 5. In addition, probabilities should be considered for many different types of variables relating to energy systems.

Furthermore, an aspect other than uncertainty should be considered: preferences for various outcomes, or consequences. This can be thought of as the "value" side of the question. To the extent that models of energy systems have decision-making implications, consideration of values as well as consideration of uncertainties should prove most valuable. The overall objective, of course, is to make the model as realistic as possible, including as much information as possible, within the constraint of keeping it workable.

The major implication of this paper with regard to future research on energy systems is that probabilistic models should be investigated. Initially, this might best be accomplished by starting with a deterministic model that has already been constructed and moving to a simple probabilistic model. To avoid getting bogged down in details with an initial application, the model chosen might be a relatively small-scale model. Hopefully the use of probability could then be extended to more complex models. Two parallel streams of research, one involving continuing work on methodology related to probabilistic models and one involving applications of probabilistic models to energy systems, would complement each other quite well.

References

[1] Otway, H.J., Lohrding, R.K., and Battat, M.E. "A Risk Estimate for an Urban-Sited Reactor," Nuclear Technology, 12 (1971), 173-184.

[2] Raiffa, H. Decision Analysis. Reading, Mass., Addison-Wesley, 1968.

[3] Starr, C., Greenfield, M.A., and Hausknecht, D.F. "A Comparison of Public Health Risks: Nuclear vs. Oil-Fired Power Plants," Nuclear News (October 1972).

[4] Winkler, R.L. An Introduction to Bayesian Inference and Decision. New York, Holt, Rinehart and Winston, 1972.