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PREFACE

Interest in water resources systems has been a critical part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resources management must be developed further. This in turn requires an increase in the degree of detail and sophistication of the analysis, including economic, social and environmental evaluation of water resources development alternatives aided by application of mathematical modelling techniques, to generate inputs for planning, design and operational decisions.

In 1976 and 1977 IIASA initiated a concentrated research effort focusing on modelling and forecasting of water demands. Our interest in water demands derived from the generally accepted realization that these fundamental aspects of water resources management have not been given due consideration in the past.

This paper, the sixth in the IIASA water demand series, reports on a price coordination method proposed for the solution of a complex demand-supply problem in water resources management. It is assumed that mathematical models are available for the description of each "supply" and "demand" unit in the region. The method presented in this paper allows one to determine optimum levels of development for both supply and demand units, such as to maximize total net benefits from water use in a given region. Although the complexity of the problem under consideration necessitated some simplifying assumptions, practical applicability of the method is demonstrated and recommendations are made as to how it could be extended further.

Janusz Kindler
Task Leader

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Summary

A scheme is proposed for the coordination by prices of water supplies and demands in a region. The objective is to maximize the total regional net benefit from water use and it is achieved when the marginal benefit at each demand point is equal to the marginal cost of delivering water to that point. The class of problems to which the scheme can be applied is determined from the graph of the network connecting supplies and demands. An example is presented in which the scheme is applied to analyze possible interbasin water transfers in the Northwest Water Plan in Mexico.

Supply-Demand Price Coordination in Water Resources Management

1. INTRODUCTION

In water resources management, the demands for the use of water traditionally have been treated as fixed "requirements". These requirements are based on coefficients (e.g. so many liters per person per day, so many liters per ton of steel) which have been assumed to be constant over time. Future water requirements have been forecast by extending past trends.

In many industrialized countries, this approach to the management of natural resources is proving inadequate. Demands are not growing as forecast; factories and power plants are switching to recycling; the number of water-using appliances in homes is leveling off; irrigation is becoming more efficient. Moreover, it is being increasingly recognized that instead of investing more money on the supply side of the supply-demand picture to bring extra water from further away, it may be more effective to invest this money on the demand side to ensure more efficient use of the water currently available.

To cope with this rapidly changing situation new management approaches are being developed and applied. Water demands are being forecast using the alternative futures concept in which various scenarios of future changes in demand are developed. The factors affecting the various types of water demands are being modelled by mathematical programming and statistical techniques.

Now that better models for water demands are becoming available, it is necessary to seek ways in which they can be combined with the models currently existing for water supply (e.g. models for reservoir releases) in order to solve problems of regional water management. The objective of this paper is to develop an algorithm for the coordination of supplies and demands in a region assuming that each significant unit is represented by such a model, and that there is no economic linkage among demand and supply units.

Two general approaches to this type of analysis may be noted: aggregated and disaggregated. In the aggregated approach, a large mathematical model is formulated to represent the supplies and demands over the whole region and solved to yield the optimal solution, which maximizes the total net benefit from water use. When the supplies and demands are connected in a complex network under centralized management, the aggregated approach may be the most appropriate one to use, but it is often difficult to synthesize all the information into one aggregated model. Computer time and memory requirements may be prohibitive; the models which currently exist for various types of demands may not be compatible (e.g. agricultural water demands are commonly modelled by linear programming while statistical regression is usually used to estimate municipal water demands); because of insufficient data, it may not be feasible to construct any satisfactory "mathematical" model for some of the demands.

A disaggregated approach attempts to avoid these difficulties by treating supplies and demands as independent entities which must be coordinated sequentially by a supervisor. Such a unit needs information only on the structure of the system, namely upon the interconnections existing among the different supplies and demands. Using this information, the supervisor can guide a kind of game consisting of a sequence of questions and answers between himself and the agencies responsible for the different supplies and demands. These questions and answers are repeated in a well specified order until convergence to an optimal balance, or equilibrium, between supplies and demands is obtained.

The objective of the coordination algorithm is to maximize the total regional net benefit but instead of approaching this by accounting for total benefits and costs, as is usually the case, the algorithm works with marginal benefits and costs. The key idea in determining the optimal solution is that if a certain flow is to be transferred from a supply to a demand, the cost of delivering the final unit of water (which is the marginal cost of this flow) must be equal to the benefit generated by this final unit, or marginal benefit. This is analogous to determining the equilibrium price in a market, so "price" is often substituted for marginal cost or benefit in the discussion which follows.

The snag in applying a market equilibrium approach to regional water management is that water supplies and demands are not independent of one another as is usually assumed in market equilibrium analysis. For example, upstream users affect downstream users; groundwater withdrawals deplete surface waters. Although some of these complexities are treated in the present paper, a number of simplifying assumptions still had to be made including the following: water quality is not explicitly considered; all flows are made available at the same time; all supplies have the same reliability.

The paper is structured in the following way: Section 2 provides background information on the basic concepts which are used in Section 3 for the analysis of a typical water resources management problem involving the coordination of two supplies and two demands; Section 4 expands this analysis into a general scheme which is applied in Section 5 to determine the optimal interbasin water transfers in the Northwest Water Plan in Mexico; Section 6 presents concluding remarks.

2. SOME BACKGROUND INFORMATION

Since the main tools of the method presented in this paper are the so-called demand and supply models we now shortly review what they are.

The benefit B of a demand unit (a firm, a city, an agricultural area, an entire region, ...) is, in general, a function of the amount Q of water consumed or used by the unit, i.e.

$$B = B(Q) \quad .$$

Naturally, in real cases, the benefits depend on the timing and reliability of the flow delivered. For example, in irrigation planning it is important to know the flow able to be delivered with high reliability (e.g. 95%, 99%) during the critical weeks of the growing season. However, most irrigation demand models are formulated to yield the total volume needed in the whole growing season rather than some critical peak flow, so the assumption in the present analysis of a constant average flow is consistent with the output of these models. The time period over which the flow is being delivered should be the same for all demands.

If the water is paid for at a price p the profit of the unit is [B(Q) - pQ] so that a particular amount of water will be demanded for each given price. Under the assumption that the unit is profit maximizing, this amount of water can easily be determined by solving the following optimization problem

$$\max_{Q}[B(Q) - pQ] \quad . \qquad (1)$$

If problem (1) is solved for all values of the parameter p, a function, called the demand function

$$Q = Q^D(p) \qquad (2)$$

is obtained which gives the amount of water demanded by the unit as a function of the price of the water. If there are no explicit inequality constraints added to problem (1) the necessary conditions for optimality implies that

$$\frac{d}{dQ} B(Q) = p$$

so that the demand function (2) can be interpreted as the marginal benefit of the unit (actually the inverse of this function).

The same considerations can be applied to a supply unit (a reservoir, a desalination plant, a pumping station, a region, ...) which at cost C(Q) supplies an amount Q of water and sells it at a price p. In this case the optimization problem solved by the unit is

$$\max_{Q}[pQ - C(Q)] \quad ,$$

and the corresponding solution gives a supply function

$$Q = Q^{S}(p) \quad , \tag{3}$$

which is nothing but the inverse of the marginal cost of supply.

The benefit and cost functions B(Q) and C(Q) and the demand and supply functions $Q^D(p)$ and $Q^S(p)$ may be themselves the result of complex optimization procedures (e.g. optimal design of the plant, determination of the optimal size of the reservoir, ...) and for this reason they may not be explicitly known, i.e. their functional form may not be available. What is available instead is a procedure that, given a price, allows the determination of the water demanded or supplied at that price. In many cases this procedure may be a complex mathematical programming model (see, for example, [1,2] for industrial demand, [3,4] for agricultural demand, and [5,6] for municipal demand). In other cases the procedure may be a sequence of operations based on more or less empirical observations eventually integrated by some simulation study. This is the way typically followed by consulting agencies when designing supply units such as reservoirs, interbasin transfers, and pumping stations. Although, strictly speaking, these procedures are not mathematical models, they can be regarded as models for the purpose of developing the algorithms.

In the following we assume a model is available for the description of each supply, demand, and transfer unit of the system. In particular, the demand and supply units will be described by models of the kind Q(p), namely procedures that can allow the determination of the flows given the prices, while the transfer operations will be described by models of the kind p(Q). Moreover, we will assume that the models are such that the corresponding demand and supply functions are differentiable and strictly monotonic as shown in Figure 1, and that the flows and the prices involved in the system are not explicitly constrained. These assumptions are needed in order to justify the algorithm presented in the paper and are not very severe in practical situations. In particular, the property that the flows are unconstrained, which seems to be quite limiting at first glance, can often be obtained by means of a suitable formulation of the models. For example, the fact that the flow Q supplied by an artificial reservoir still to be constructed cannot be greater than a value $\bar{Q}$, is automatically taken into account if the model describing this supply unit has a supply function which goes to $\bar{Q}$ for p going to infinity.

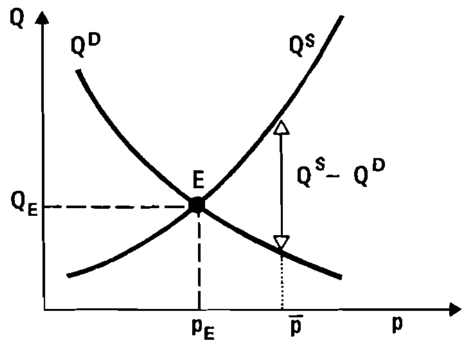

Figure 1. Demand (Q^D) and supply (Q^S) functions and equilibrium solution E.

When a supply and a demand unit are connected, the value of water exchanged can simply be obtained from the plot of Figure 1. The point E in the plot, called equilibrium point, is characterized by

$$Q^S(p_E) = Q^D(p_E) \quad ,$$

and gives the value Q_E of the flow that must be exchanged between the two units in order to maximize the total net benefit of the system, i.e.

$$\max_Q [B(Q) - C(Q)] \quad .$$

Consequently, the equilibrium price p_E is the price that leads the supply and demand units not only to spontaneously exchange the same amount of water but also to select that particular value (namely Q_E) which maximizes the total net benefit of the system.

Of course, if the two functions $Q^D(p)$ and $Q^S(p)$ are explicitly given the equilibrium solution is immediately obtained. If, on the contrary, only the models for computing Q^D and Q^S are available it becomes important to obtain the equilibrium solution without using the models too many times. One way of doing this consists of fixing a price $\overline{p}$ and computing the corresponding imbalance $Q^S - Q^D$ (see Figure 1) and then iterating on the price until the imbalance is zero. This is the essence of the classical price coordination method that is applied in this paper to complex management problems characterized by the presence of many demand and supply units. (See [7,8] for interesting reviews and applications of the classical price coordination method. Price coordination in water resource systems is very well surveyed in [9].)

3. ANALYSIS OF A TYPICAL PROBLEM

The aim of this section is to present an ideal but typical water management problem characterized by many supply and demand units and to outline our price coordination scheme for the solution of such problems.

The system is shown in Figure 2 and comprises two supply and two demand units. It is actually a part of the problem considered later in the application example. The groundwater extracted by the pumping station S1 is transferred through an artificial open channel (the dimension of which is to be determined) to an irrigation area D1. The water supplied by the reservoir S2 is first transferred to point A through a natural channel (no cost of transfer) and then diverted to the two irrigation areas D1 and D2 through two artificial channels. In all of these channels a specified fraction, defined as a, (e.g. 5%) of the inflow is lost through seepage.

Let us now imagine that a model describing each supply, demand, and transfer unit of the problem is available. In particular, let us assume that supply and demand models of the kind Q(p) are available for the pumping station, the reservoir, and the two irrigation areas, and that models of the kind p(Q) are available for the determination of the marginal transfer costs $T_{S1,D1}$, $T_{A,D1}$, $T_{A,D2}$ for any possible amount of transferred water. In the case of demand units that have more than one source of supply as is the case of the irrigation area D1, we assume that the economy of the unit is only sensitive to the sum of the inflows. This is equivalent to saying that the same price p must be associated with all inflows and that the model gives the total inflow demanded for each given price. This assumption is justified only if the flows have the same reliability, timing, and quality. The same is true of supply units whose outflow goes to more than one demand.

The problem consists in finding the overall "equilibrium" solution, i.e. the flows and the prices associated to them that

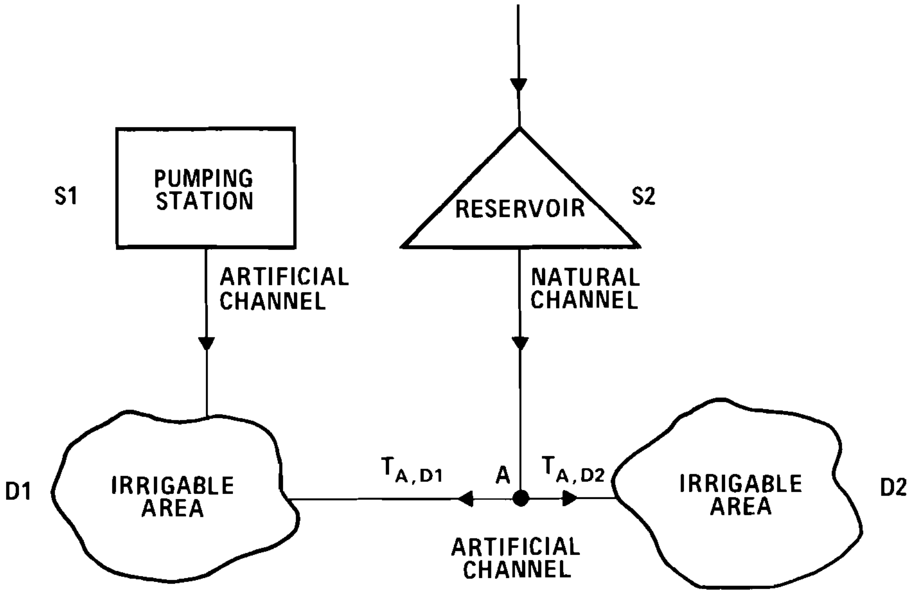

Figure 2. A typical example.

maximize the total net benefit of the region. This problem can be solved in one shot if the models describing the different units of the system (supplies, demands, and channels) can be aggregated. For example, if all units are described by linear programming models, then the aggregation is straightforward and the overall problem is still a linear one although the problem becomes sometimes far too big to be handled. Alternatively, the problem can be solved in a disaggregated way by means of the classical price coordination method [9], which essentially consists in associating a price to each independent flow (three in the case of Figure 2), and then searching for the optimum in the space of these prices. Because of their particular structure, the problems we are dealing with in this paper can be solved by more efficient price coordination schemes. For example, for the problem described in Figure 2 the following one-dimensional searching scheme can be used:

1. Given a price p_{S1} determine, by means of the model describing the pumping station S1, the amount Q_{S1} of water supplied by that unit.

2. Compute the water losses a Q_{S1} incurred in the transfer from S1 to D1, the corresponding amount of water supplied to the irrigation area D1 ($Q_{D1} = (1 - a)Q_{S1}$), and the

marginal costs $T_{S1,D1}$ of this operation (the total cost of transfer is $T_{S1,D1}Q_{S1}$). Finally, determine the price $p_{D1} = (p_{S1} + T_{S1,D1})/(1 - a)$. (This relationship comes from the following balance equation: cost of water at point S1 + cost of transfer from S1 to D1 = cost of water at point D1; i.e. $p_{S1}Q_{S1} + T_{S1,D1}Q_{S1} = p_{D1}Q_{D1}$.)

3. Determine by means of the model describing the irrigation area D1 the total amount of water demanded by this area at the price p_{D1} and, consequently, the flow that the channel (A,D1) must supply in addition to the flow coming from S1.

4. Compute the flow entering the channel (A,D1) by taking the water losses into account, the corresponding marginal cost $T_{A,D1}$, and the new price $p_A = p_{D1}(1 - a) - T_{A,D1}$ associated with point A.

5. Determine the water supplied by the reservoir S2 at price $p_{S2} = p_A(1 - a)$ and the corresponding losses in the natural channel connecting S2 with A. Then, by means of the mass balance equation in point A determine the amount of water entering the channel (A,D2).

6. Compute the losses in channel (A,D2), the amount Q^S of water supplied to the irrigation area D2, the marginal cost $T_{A,D2}$, and the price $p_{D2} = (p_A + T_{A,D2})/(1 - a)$.

7. Determine the amount Q^D of water demanded by D2 at the price p_{D2}.

If the demand Q^D equals the supply Q^S, the flows and prices computed in the above seven steps are the optimal ones. In fact, by construction, at each point in the system marginal benefits equal marginal costs and hence the maximum total net benefit has been obtained. If, on the contrary, Q^D differs from Q^S the price p_{S1} must be suitably updated and the operations repeated until $Q^S - Q^D = 0$. This corresponds to finding the value of p_{S1} for which the imbalance of flow $Q^I = Q^S - Q^D$ given by the seven proceeding operations is equal to zero.

Therefore, one must first determine a pair of prices (p_{S1}^1, p_{S1}^2) such that the corresponding imbalances are of opposite sign and then progressively reduce the interval of uncertainty $[p_{S1}^1, p_{S1}^2]$ by following a suitable scheme, such as the bisection procedure or the Fibonacci search. This procedure will certainly converge to the optimal solution under the assumption that the optimal price p_{S1} is the only value for which the imbalance in the initial interval of uncertainty is zero. This assumption is not at all a restrictive one since in almost all practical situations good lower and upper bounds of the optimal price are a priori known. As far as the speed of convergence of the method is concerned we can expect that just a few iterations are needed. For example, if a bisection procedure is used after ten iterations the interval of uncertainty for the price will be reduced more than a thousand times.

4. THE COORDINATION SCHEME

Particular coordination schemes, like the one described in the preceding section, can be formally derived from the general price-coordination method [9] when one assumes that the economy of each unit only depends upon the sum of all inputs or outputs. One can also obtain these schemes by directly imposing the necessary conditions for optimality, namely by setting to zero the first order derivatives of the total net benefit with respect to all the flows present in the system. By interpreting these conditions in terms of relationships between marginal transfer costs and supply and demand functions and by reading them in a suitable sequential order one exactly obtains a scheme of the kind used in the preceding section. Since both these ways of deriving the coordination scheme are abstract and difficult to follow in complex cases, we prefer to suggest a heuristic method which is based only upon the characteristics of the graph describing the interactions among the units of the system. By inspecting and interpreting the case presented in the preceding section we will identify the class of systems for which a one-dimensional coordination scheme can be devised in order to solve the problem. Moreover, we will point out how the sequence of operations of the scheme can be obtained.

Let us first analyze the properties of the graph shown in Figure 3 which describes the system considered in Section 2. A model of the kind $Q_i(p)$ is associated to each node i of this graph representing supply and demand units, while no model is associated to the diversion point A which represents only the mass balance equation. On the other hand, each arc (i,j) of the graph is characterized by a pair (Q_i, Q_j) of flows representing the upstream and downstream flows of the channel. Thus, a relationship between Q_i and Q_j describing the water losses is associated to the arc (i,j) together with a model of the kind $p_{ij}(Q_i)$ giving the marginal transfer cost. In other situations some supply (demand) nodes can have input (output) arcs as in the case of reservoirs in cascade, or recycling. In such cases the model associated with the node enables the computation of the net amount of water supplied or demanded for any given price. Moreover, the arcs of the graph can also represent instream uses of the water and, therefore, the model $p_{ij}(Q_i)$ gives in such situation the marginal benefit of the use. Of course, in the limit case of no consumptive use on an arc (i,j) the relationship between Q_i and Q_j is the identity function.

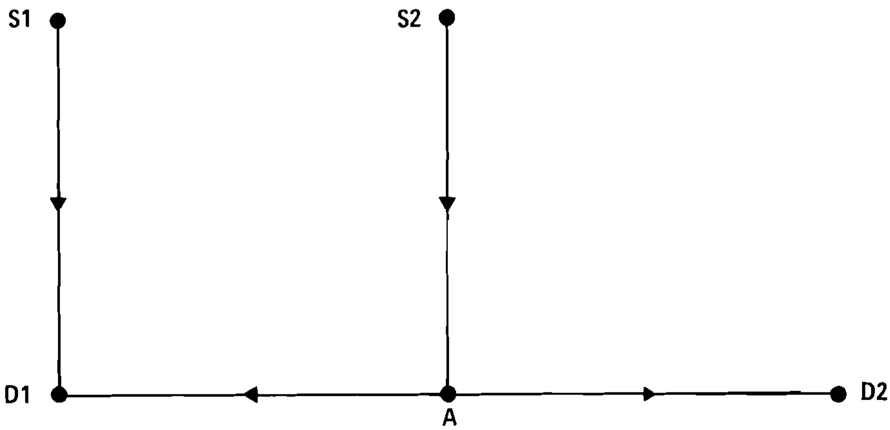

Figure 3. The interaction graph.

Let us now analyze, on the graph shown in Figure 3, the seven steps of the one-dimensional algorithm described in the preceding section. The first operation consists in associating a price p_{S1} with the node S1 of the graph and in computing the corresponding flow Q_{S1}. Since node S1 is a terminal one (i.e. there is only one arc connected to it) the flow Q_{S1} is uniquely associated with arc (S1,D1), so that the flow Q_{D1}, the marginal transfer cost $T_{S1,D1}(Q_{S1})$, and the price p_{D1} can be computed (step 2 of the algorithm). At this point one can eliminate node S1 and arc (S1,D1) from the graph of Figure 3 and consider the reduced graph shown in Figure 4 in which node D1 is characterized by the new demand function $Q^*_{D1}(p) = Q_{D1}(p) - Q_{D1}$. Thus, we are in the same situation we were at the beginning of our analysis since we can associate the price p_{D1} to the terminal node D_1 and proceed by eliminating node D1 and arc (A,D1) (see steps 3 and 4 of the algorithm). Unfortunately, after these two operations we are not in the same condition as with the previous node since node A is not a terminal node. Nevertheless, we can turn our attention to the terminal node S2 since the price p_{S2} must be equal to $p_A(1 - a)$ because of the absence of any transportation cost on the arc (S2,A).

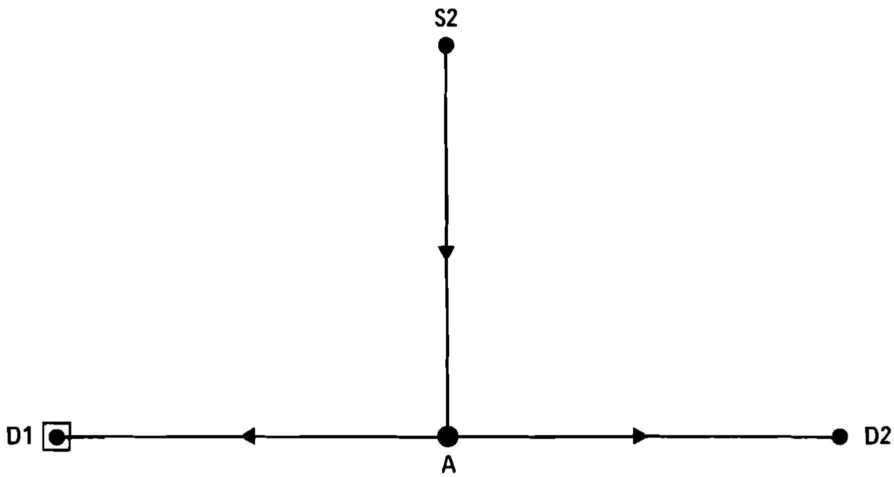

Figure 4. The reduced graph after the first two operations with price associated to D1.

It is very important to notice that the price p_{S2} could also be computed from p_A if the arc (S2,A) were characterized by a constant marginal cost for transfer $\overline{T}_{S2,A}$, since we would obviously have

$$p_{S2} = p_A(1 - a) - \overline{T}_{S2,A} \quad .$$

On the contrary, if the transfer cost and/or the seepage losses are not linearly related to the flow (e.g. when there are economies of scale) there is no possibility of extending the analysis to a new terminal node. In conclusion, once the price of a non-terminal node i has been computed it is possible to determine the price of a new terminal node j only if the arcs between i and j have constant marginal transfer costs and are characterized by a fixed fraction of water lost through seepage. For example, in the case of Figure 5 where the dashed arcs are assumed to have constant marginal costs and linear seepage losses one can start from the terminal node 7 and eliminate node 7 and arc (7,8), thus finding the price p_8, and then continue to reduce the graph by jumping on the terminal node 1 which is connected to node 8 through the arcs (8,4), (2,4) and (1,2). The price p_1 is given by

$$p_1 = \left[p_8 + \frac{\overline{T}_{84}}{1 - a_{84}}(1 - a_{24}) - \overline{T}_{24}\right](1 - a_{12}) - \overline{T}_{12} \quad .$$

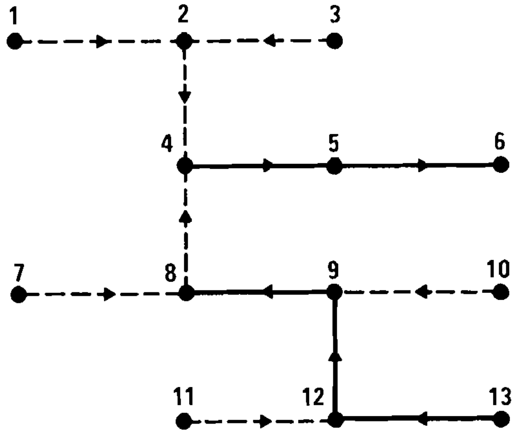

Figure 5. Interaction graph (dashed arcs have linear transfer costs and losses).

Coming back to our example we are now reduced to the graph constituted by the three nodes (S2,A,D2) and by the two arcs (S2,A) and (A,D2). A price is associated to the terminal node S2 so that we can eliminate it (and, consequently, the arc (S2,A)) as indicated in step 5 of the coordination scheme, thus reducing the graph to a pair of nodes connected by an arc. By means of the mass balance equation in node A we compute the flow Q_A in arc (A,D2) and then we can eliminate arc (A,D2) as indicated in step 6, thus reducing the graph to the only node D2. Since a price p_{D2} is associated with this node the final operation (step 7) consists in computing the amount of water Q^D demanded by the unit. Thus, a comparison between Q^D and the flow Q_{D2} associated with arc (A,D2) is possible.

It is important to notice that even when the problem is solvable by means of a one-dimensional coordination scheme it is not possible in general to start the procedure from any terminal node. For example, the case described in Figure 5 can be solved by alternatively eliminating nodes and arcs in the following order: 6,(5,6), 5,(4,5), 1,(1,2), 3,(3,2), 2,(2,4), 4,(8,4), 7,(7,8), 8,(9,8), 10,(10,9), 9,(12,9), 11,(11,12), 12,(13,12), 13. On the contrary, if one starts from node 7 the following sequence is obtained: 7,(7,8), 1,(1,2), 3,(3,2), 2,(2,4) which leads to the reduced graph shown in Figure 6 where a price is associated to the nonterminal node 4. Since this node is not connected with any other terminal node through a dashed path the graph cannot be reduced any more. It is therefore of interest to first identify the class of the problems solvable by means of a one-dimensional coordination scheme and then to give a rule for constructing at least one sequence of operations that solves the problem in this way.

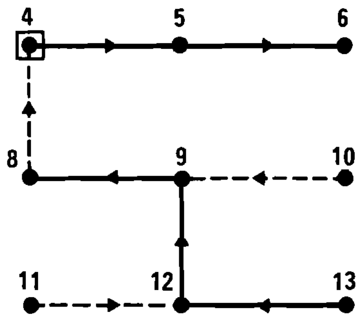

Figure 6. The graph that cannot be further reduced, with a price associated with node 4.

As far as the first question is concerned, it is possible to prove that "solvable" problems are characterized by a graph that satisfies the following two conditions:

(i) the graph is a tree (i.e. interpreted as an undirected graph, is acyclic and connected);

(ii) for any node of degree $k > 2$ at least $(k - 2)$ of the disconnected subgraphs obtained by eliminating this node from the graph must have constant marginal costs and linear losses on all arcs (the degree of a node is the number of arcs connected with it).

For example, in the case of Figure 5 there are five nodes of degree 3 (nodes 2, 4, 8, 9, 12). Since for each one of these nodes condition (ii) is satisfied (easy to check), and the graph is a tree, one can a priori conclude that the problem can be solved by means of a one-dimensional coordination scheme.

In order to answer the second question (determination of the sequence of operations) we must first introduce the notion of critical nodes as follows: a node of degree $k > 2$ is said to be critical if two of the disconnected subgraphs obtained by eliminating it from the graph contain arcs with nonconstant marginal cost and/or nonlinear losses. These are termed critical subgraphs in the following. For example, the critical nodes of the graph shown in Figure 5 are the nodes 4, 8, 9, and 12, while the node 2, which is of degree 3, is not critical since only one of its disconnected subgraphs contains nonconstant marginal costs and/or nonlinear losses. At this point we can state the following for the selection of the initial node of the sequence in solvable problems:

(a) If there are no critical nodes the first node to be considered can be any terminal node.

(b) If there are critical nodes, determine for each one of them the set (called terminal critical set) of the terminal nodes of its two critical subgraphs and then determine the nodes that are in common to all the terminal critical sets. Any one of these nodes (which can be proved to exist) can be considered as initial nodes of the sequence.

For example, for the case of Figure 5 the terminal critical sets are shown on the rows of Table 1, so that the possible initial nodes are the nodes 6 and 13 (recall that a sequence starting from node 6 and solving the problem has already been indicated). As for the determination of the rest of the sequence, the problem is straightforward since the application of the criteria outlined above naturally leads to the complete reduction of the graph.

Table 1. The terminal critical sets for the graph of Figure 5.

CRITICAL NODES \ NODES	1	2	3	4	5	6	7	8	9	10	11	12	13
4						*	*			*	*		*
8	*		*			*				*	*		*
9	*		*			*	*				*		*
12	*		*			*	*			*			*

It is now worthwhile to interpret the algorithm in terms of a two-level recursive decision-making process. For this let us consider Figure 7 where the central block represents the supervisor, while the external blocks represent the seven steps of the scheme discussed in Section 3. Their order corresponds to a sequence of operations that solves the problem and has been predetermined by the supervisor by applying rules (a) and (b). The algorithm can therefore be interpreted as a recursive sequence of questions and answers between the supervisor and the single components. The question of the supervisor in general depends upon some of the preceding answers, while the answer of each component is totally independent from the preceding ones. In this way the global problem of maximizing the total net benefit of the system is solved without requiring the information on the economy of all components to be centralized. Each subproblem relates with only one component and is solved by means of the corresponding model. A final interesting remark is that the sequence of questions and answers between the supervisor and one component of the system develops like a common recursive negotiation until the equilibrium is reached.

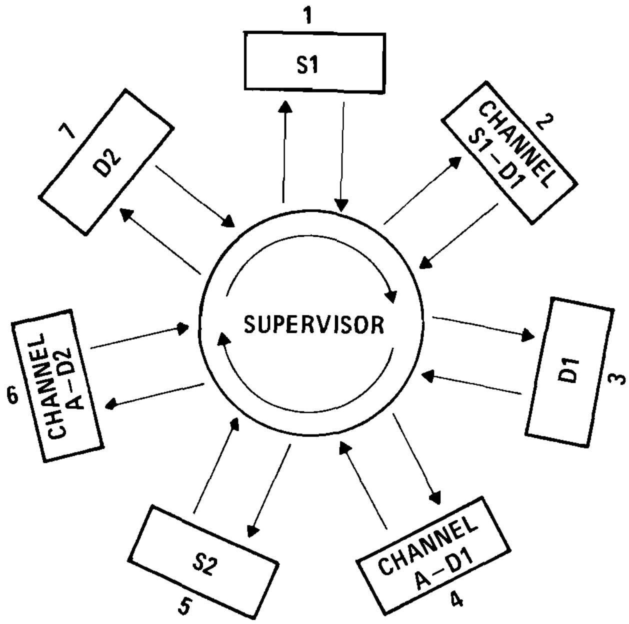

Figure 7. The two-level recursive decision-making process.

5. APPLICATION EXAMPLE

In this section the algorithm previously presented is applied to the proposed Northwest Water Plan in Mexico. This project is part of the Plan Hidraulico del Noroeste which involves the transfer of water from south to north along the Mexican coast of the Gulf of California. The motivation for the project is that the groundwater level in the aquifer of the Costa de Hermosillo at the northern end of the region is continually declining due to excessive pumping for irrigation, leading to seawater intrusion of the aquifer at the rate of about 1 km per year (see Figure 8). The rate of recharge of the aquifer is estimated as 350 million cubic meters per year, and the current rate of pumping is more than twice this rate.

It is proposed to slow down this seawater intrusion by bringing water about 480 km north to the Costa de Hermosillo from the Fuerte River through a series of canals but this water could also be used in other irrigation areas located closer to

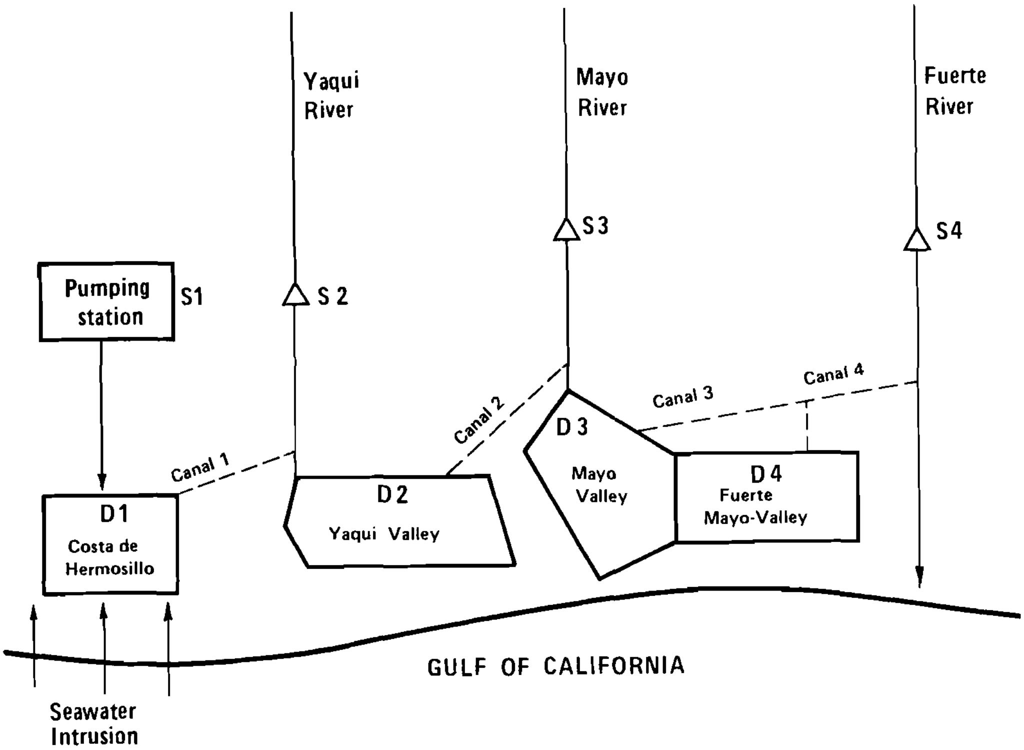

Figure 8. The Northwest Water Plan in Mexico.

the Fuerte River. The algorithm is applied to determine the optimal balance between supplies and demands in the region and the associated pattern of interbasin water transfers. All data and models used in this example are taken from the comprehensive economic analysis of the region given in [10].

As shown in Figure 8, the system consists of 4 supplies: the groundwater pumping in the Costa de Hermosillo, and the three surface water reservoirs on the Yaqui, Mayo, and Fuerte Rivers. Water is demanded in 4 irrigation areas. The supplies, $\bar{Q}$, considered to be available for allocation from the three surface water reservoirs are 2483, 946, and 1000 million cubic meters per year, respectively, and from the groundwater aquifer, 350 million cubic meters. The reservoir supply functions are assumed to be of the form $Q(p) = \bar{Q}$.

The Fuerte River supply (S4) could be brought by canal 4 to an irrigation area in the Fuerte-Mayo valley (D4) and further by canal 3 to another irrigation area in the Mayo valley (D3). This transfer would release supplies from the Mayo river (S3), formerly used in Mayo valley, to flow through canal 2 to an irrigation area in the Yaqui valley (D2), thus releasing supplies from the Yaqui river (S2) to be pumped to the Costa de Hermosillo along canal 1 to complete the transfer.

The demands for water in the four irrigation areas are described by linear programming models which maximize the total annual net benefits of irrigated farming. The net benefits are given by the value of crop outputs less the costs of production, and allowance is made for the dependence of the nearby urban communities on the farm economy.

Two cost functions for the channels were tried, one linear, and one nonlinear reflecting economies of scale. Since the results obtained from the two analyses did not differ greatly, only the results from the nonlinear cost function are presented. This function is of the general form: marginal cost in dollars per cubic meter = $bV^{-0.4}$ for a transfer of V cubic meters, where b is a coefficient computed for each channel. The values of b are 0.01296, 0.00384, 0.00018, and 0.00612 for channels 1 to 4 respectively. The corresponding seepage losses expressed as a proportion of the inflow are 18%, 2.5%, 5%, and 5%.

The interaction graph for the case at hand is shown in Figure 9. As one can easily verify the problem is solvable with our one dimensional search since the conditions (i) and (ii) of the preceding section are satisfied. To select the sequence of operations we must first choose the starting node. The critical nodes are A1, A2, and A3 and in Table 2 their terminal critical sets are shown, so that one can see that the possible initial nodes are S1 and S4. We choose node S4 as the starting node, and by applying the ideas outlined in the preceding section, the following sequence is obtained: S4,(S4,A4), A4,(A4,A3), D4,(A3,D4), A3,(A3,D3), D3,(A2,D3), S3,(S3,A2), A2,(A2,D2), D2,(A1,D2), S2,(S2,A1), A1,(A1,D1), D1,(S1,D1), S1. The initial range of the price p_{S4}, was the interval 0.01 to 0.25 dollars per cubic meter. Convergence of the algorithm is obtained in 12-15 iterations in this example.

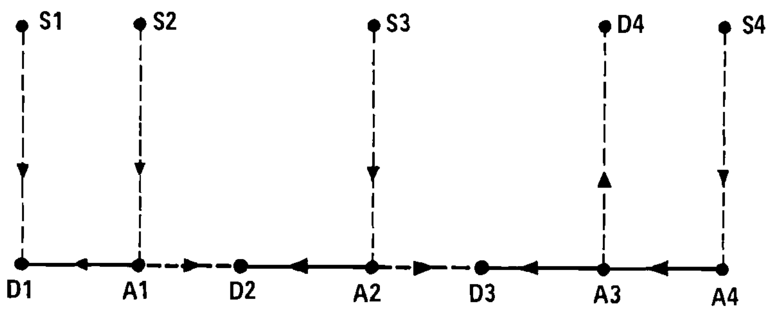

Figure 9. The interaction graph of the Northwest Water Plan.

Table 2. The critical nodes and their terminal critical sets.

critical nodes \ terminal critical set	S1	S2	S3	D4	S4
A1	*		*	*	*
A2	*	*		*	*
A3	*	*	*		*

The results of two cases are presented. In the first, the price coordination algorithm is applied to each supply and demand pair in isolation, i.e. (S1,D1), (S2,D2), ...; in the second, transfers along all canals are feasible. The balance of flows between supplies and demands (Table 3) shows that with each pair considered in isolation the demand is equal to the supply available. It may be noted in Table 3 that D4 receives 5% less flow than S4 supplies because of losses in the transfer channel. When all transfers are possible, D1 (the Costa) draws some water from each of the other demands so that the flow in the channels increases as the Costa is approached.

Table 3. The balance of flows (in millions of cubic meters).

	S1	D1	T21	D2	S2	T32	D3	S3	T43	D4	T44	S4
without transfers	350	350	-	2482	2482	-	946	946	-	950	1000	1000
with transfers	350	779	523	2438	2482	491	748	946	308	642	1000	1000

S = supply, D = demand, T = transfer.

The annual total net benefit in the region increases by 6% from 741.4 million dollars to 784.2 million dollars if the transfer scheme is built with the capacities indicated in Table 3. The annualized construction and maintenance cost is 16.1 million dollars, yielding a benefit-cost ratio of 2.7 for the project.

The corresponding balances of marginal benefits and costs are shown in Figure 10, where the solid line represents the marginal value of water when all transfers are possible. The flat portions of this line are the equilibria at each demand point and the inclined portions represent the marginal cost of transfer along the channels. As expected, the marginal value rises in the direction of transfer to a maximum in the Costa (D1). The dashed lines in Figure 10 show the price equilibria reached when each supply-demand pair is isolated. When the transfer scheme is introduced, the price falls from 24 to 10 cents per cubic meter in the receiving area (D1) but rises from 4 to 9 cents per cubic meter on average in the donor areas (D2 to D4).

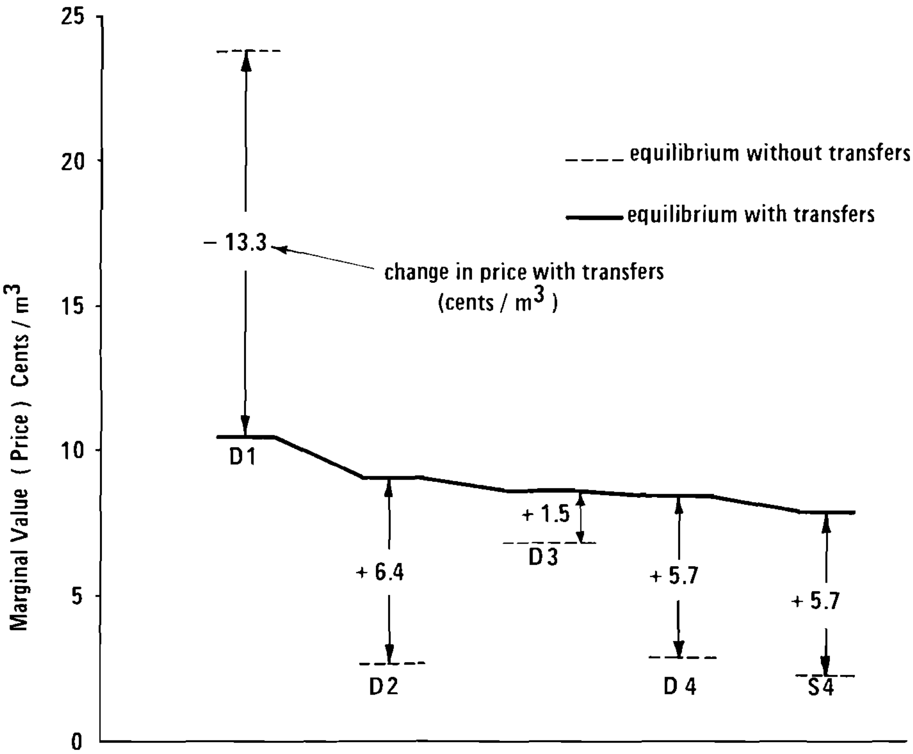

Figure 10. Balance of marginal values.

This change in the prices is an important result because it quantifies the degree to which the farmers in the donor areas are being penalized for the sake of the farmers in the Costa. A common criterion of the viability of such transfer projects is the willingness of the farmers to pay for the water brought to them. If the demand models used in this example properly reflect the real situation, then the prices used in the algorithm are indicative of the marginal value of water to the farmers, and hence what they could afford to pay for the water. Although water is not normally priced as a market commodity, the prices obtained by the algorithm may be useful as reference points against which prices determined by more conventional methods could be compared (such as pricing to cover supply costs).

6. CONCLUSIONS

A particular price coordination scheme for the solution of a complex water management problem has been presented in this paper. The method works in the "marginal domain", thus making use of demand and supply models describing the economy of the single units of the system. The scheme is essentially a one-dimensional search coordinated by a supervisor. The advantages of this scheme with respect to aggregated cost-benefit analysis are the saving of computation time and memory requirements, and the fact that the information structure needed is highly decentralized. Also, the marginal benefits or costs associated with each unit are explicitly obtained as part of the problem solution.

The class of problems to which the method can be applied has been identified by means of a simple topological analysis of the graph describing the interactions among the various units of the system. The classical price coordination scheme applied to the same class of problems would require to search for the optimum in a space of dimensions equal to the number of independent flows to be determined (roughly the number of arcs of the graph). Thus, for example, in the case of the Northwest Water Plan in Mexico analyzed in Section 5, a straightforward application of the classical price coordination method would require an eight-dimensional search, while our one-dimensional scheme has been shown to be well suited for determining the optimal solution.

When the interaction graph is too complex, i.e. when the topological conditions presented in Section 4 are not satisfied, the problem cannot be solved by means of a one-dimensional search. Nevertheless, one can eliminate arcs from the interaction graph until the above-mentioned conditions are satisfied, thus obtaining a subproblem solvable by means of our one-dimensional search. This corresponds to solving the global problem by searching for the optimum values of the flows associated with the arcs eliminated from the original graph and using our one-dimensional searching scheme as a subroutine at each iteration of the multi-dimensional search. In this way one still obtains a great computational advantage with respect to a straightforward application of the classical price coordination method.

The algorithm is applied to the Northwest Water Plan in Mexico, a region involving a groundwater supply, three surface water supplies, and four irrigation areas which could be interconnected by a proposed interbasin water transfer scheme. Convergence is achieved in 12-15 iterations. The results indicate a 6% increase in the total net benefit from crop production in the region if the scheme is constructed but the average price of water would double in the areas from which the water is taken if the water were priced as a market commodity.

The method described in the paper is presented in a quite heuristic way and is not qualified from the mathematical point of view. Some results on this line, related, for example, to the convergence of the method, could certainly be obtained in terms of properties of the demand and supply functions of the various units. Nevertheless, in the spirit of the paper which assumes that the demand and supply functions of the units are not explicitly given, the knowledge of such properties would not be of great interest. On the other hand, the problem of relating these properties to the structural properties of the models seems to be a rather difficult task.

On the contrary, a much more interesting line for further investigation seems to be the extension of the present method to water management problems in which the quality of the water exchanged among the different units, as well as the reliability of the flows exchanged, is taken into account.

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