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A PROBLEM OF FAIR DIVISION:
APPORTIONING THE EUROPEAN PARLIAMENT

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Preface

The problem of how to make a "*fair division*" of resources between competing interests arises in many areas of application at IIASA. One of the tasks in the System and Decision Sciences Area is the systematic investigation of different criteria of fairness and the formulation of allocation procedures satisfying them.

A particular problem of fair division having wide application in governmental decision-making is the so-called apportionment problem. An application arises in the current debate over how many seats in the European Parliament to allocate to the different member countries. Current discussions swirl around particular numbers, over which agreement is difficult to achieve. A systematic approach that seeks to establish agreement on the principles of fair division involved should stand a better chance of acceptance in that it represents a scientific or system analytic approach to the problem.

Abstract

In this paper certain natural and desirable properties for apportionment are formulated, and it is shown that they lead to a unique procedure for determining solutions. This procedure is then applied to the case of the European Parliament.

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INTRODUCTION

The mass media and, ipso facto, the declarations of political leaders of several of the nine countries of the EEC point to a major current problem, apportionment: how are the seats in the European Parliament to be distributed among the member nations? Several plans have been advanced. The "*Draft*" plan¹⁾ as revised apportions 359 seats, an alternate *French proposal*²⁾ is based on 291 seats, and another *Irish plan*²⁾ suggests 388 seats. The discussion, if abstracted from its universal suffrage and direct election aspects, swirls around numbers, the need to assure the small nations a voice which can be heard, and yet the fundamental desire that the Parliament should be genuinely representative of the demographic distribution which exists today.

To students of the history of the United States the preoccupations and arguments have a decidedly familiar ring. A considerable investment of time and thought was given to the apportionment question at the Constitutional Convention in Philadelphia in 1787. Attended by the political and intellectual leaders of the American colonies, the philosophical children of the European 18th Century enlightenment, this Convention sought to give legal definitions to the ideals of individual freedoms and equality among men. The

¹⁾ Elections to the European Parliament by Direct Universal Suffrage, Draft Convention with Explanatory Statement, Special Issue based on PATIJN report (Doc. 368/74), (Resolution adopted 14 January 1975).

²⁾ As reported in the press.

commonly held philosophical principle that, as James Madison stated it,³⁾ the states "ought to vote in the same proportion in which their citizens would do if the people of all the states were collectively met," was checked only by the fear of the small states "solicitous to guard ... against an improper consolidation" of a league of large states. From this emerged, on the one hand, the language of Article I, Section 2 of the U.S. Constitution, "Representatives and direct taxes shall be apportioned among the several states which may be included within this Union, according to their respective numbers;" and, on the other hand, a Senate, with each state accorded two seats.

But, exactly how the proportional representation was to be made was left open. This led to controversies every ten years (i.e., at each new census) about how the phrase "according to their respective numbers" should be interpreted. Endless discussions ensued in which rival regional and party factions suggested multitudes of alternate ways of apportioning (naturally, each faction suggesting plans to its peculiar advantage). In the end such discussions were doomed to lead to ad hoc results and more jockeying over numbers while insufficient attention was being paid to the basic principles of fair division. Thomas Jefferson,⁴⁾ in recommending that George Washington veto one of these apportionment bills, clearly saw the issue: "[The bill] seems to have avoided establishing ... a rule, lest it might not suit on another occasion. Perhaps it may be found next time more convenient to distribute them among the smaller states; at another time among the larger states; at other times according to any other crochets which ingenuity may invent and the combinations of the day give strength to carry."

³⁾ Writings of James Madison, vol. III, Ed. Gaillard Hunt, G.P. Putnam, New York, 1902, p. 385.

⁴⁾ The Works of Thomas Jefferson, vol. VI, Ed. Paul L. Ford, G.P. Putnam & Sons, New York, 1904, p. 469.

Discussions should not revolve about the rival merits of one numerical solution over another. The issue is rather to decide upon a rule or method which is "fair," i.e., whose qualitative properties satisfy principles acceptable to citizen and politician alike, and which provides quantitative solutions for any desired size of Parliament. Statesmen should reach agreement on principles; the principles must then, through computation, determine apportionments for whatever size elective body is sought. This procedure is possible scientifically. Moreover it may be recommended on political grounds because agreements in principle are usually more readily reached than agreements on numbers. The "Congressional Record" of the U.S. bears witness to the interminable discussions which followed each census and the heated tempers over the numbers to be apportioned in the House of Representatives. Europe today bears witness to the difficulties of numbers as proposed in at least three "*plans*." Numbers are pernicious: they invite, or at least allow, endless manipulation and horse trading; whereas properties of methods or principles of fairness merge the divergences on numbers into agreement on method. It is as a U.S. Representative said in 1900:⁵⁾ "The apportionment of Representatives to the population is a mathematical problem. Then why not use a method that will stand the test ...?"

⁵⁾ Representative E.W. Gibson, Congressional Record, vol. 70, 70th Congress, 2nd Session, 1929, p. 1500.

BASIC PRINCIPLES AND THE EEC PLAN

In the "Draft Convention with Explanatory Statement" entitled Elections to the European Parliament by Direct Universal Suffrage cited above (hereafter referred to as the EEC Draft), certain criteria are presented which it is desired that an apportionment method should satisfy. The first and most significant of these criteria is the following general "fairness" principle:

- (i) "the highest degree of proportionality should be achieved between the number of inhabitants of a State and the number of its representatives in the European Parliament."

This fundamental principle underlies what is in fact meant by the very notion of apportionment -- namely, an assignment of representatives to the several states proportionately according to their size. According to Webster, "To apportion is to distribute by right measure, to set off in just parts, to assign in due and proper proportion."⁶) The same principle underlies the United States Constitutional mandate on apportionment, "representatives shall be apportioned ... according to their respective numbers."

Having adopted this basic proportionality principle, however, the EEC Draft goes on to enunciate two other criteria that have a more distinctly political character, and which (it will be seen) are at odds with the above-mentioned principle. Thus the second desired criterion for an apportionment is

- (ii) "all the important political forces of a State should be represented in the European Parliament."

This criterion in effect imposes a minimum required number of representatives for each member country depending on the number of its political parties -- a number which of

⁶) The Writings and Speeches of Daniel Webster, vol. VI, National Edition, Little, Brown & Co., Boston, Mass., 1903, p. 107.

course may change substantially from time to time. The third criterion mentioned in the EEC Draft is of similar nature,

(iii) "the new distribution of seats should not lead to a reduction in the present number of any State's representatives."

In fact, the present apportionment gives enough seats to every country so that the third requirement alone implies that the second will in practice almost certainly be satisfied. However, unless the number of delegates is to be vastly enlarged, it is evident that the third requirement is quite incompatible with the first. Indeed, Luxembourg, with a present population of 357,000, has less than 2/10 of one percent of the total population of the nine EEC countries, and it also has six seats. Thus if in a new apportionment Luxembourg is to receive at least six seats and have a number of representatives proportional to its population, it will be seen that the total number of seats would have to be upwards of 3,000 -- not an altogether reasonable number. Even granting that Luxembourg's position is somewhat special, Ireland presently has ten seats and only 1.2 percent of the population, so that one would have to have over 750 seats for Ireland to be represented proportionately with the other countries.

Thus it must be recognized that, barring unreasonably large sizes of the Parliament, strict proportionality of representation cannot be achieved in the presence of criteria (ii) and (iii). Precisely the same issue arose in the debates at the United States Constitutional Convention in 1787. Governor Morris of New York noted that if representatives were apportioned proportionally to population, "it would exclude some states altogether ... who would not have a sufficient number to entitle them to a single representative."⁷⁾

⁷⁾ Writings of James Madison, Ed. Gaillard Hunt, G.P. Putnam, New York, 1902.

The solution was to make an exception to the proportionality principle in this one case; thus the U.S. Constitutional requirement "but each State shall have at least one Representative."

Granted that some degree of representation must be afforded to every member country, the objective must always be to obtain an apportionment that is proportional to population as closely as is possible, subject to the requirement of minimum representation. In the U.S. case this point of view was aptly summed up by Webster in 1832 as follows, "Of representation there can be nothing less than one representative ... it is quite obvious, therefore, that the apportionment of representative power can never be precise and perfect ... the Constitution, therefore, must be understood ... as requiring of Congress to make the apportionment of representatives among the several states according to their respective numbers as near as may be." ⁸⁾

It seems evident from even a casual inspection of the apportionment proposed in the EEC Draft that this principle of allocating seats to states proportionally to population as near as may be, has not been adhered to. Indeed, one may compute the precise proportion of the total population that each country has, and hence the precise number of representatives (including fractions) that each country would ideally be entitled to. This number is called the exact quota for that country. The populations, exact quotas, and currently proposed EEC apportionment are shown in *Table 1*. (Population figures are the numbers used in the revised EEC Draft.)

⁸⁾ Ibid, Webster, pp. 107-108.

	FRG	UK	Italy	France	Neth.	Belg.	DK	IR	Lux.	Total
Pop. (000's)	62,041	56,056	55,361	53,780	13,450	9,772	5,052	3,086	357	258,955
Exact Quota	86.010	77.713	76.749	74.557	18.646	13.547	7.004	4.278	0.495	359
EEC Appment	72	68	67	66	27	23	17	13	6	359

T A B L E 1

ROUNDING

In a perfect apportionment, every country would receive its exact quota, and indeed should receive its exact quota if (in an unlikely event) this turns out to be a whole number. But we cannot expect that the exact quota will ever, in practice, turn out to be integer; hence, as fractional representatives are not to be countenanced, it is necessary to "round" the exact quotas either up or down in such a way that the total number of representatives apportioned is the prescribed number. But it will be seen from *Table 1* that the proposed apportionments are nowhere near the exact quotas; even taking into account the desire for certain minimum levels of representation, this could not be said to be an apportionment to population as near as may be. In particular, therefore, the proposed apportionment is not at all compatible with the first and foremost principle of proportionality enunciated by the EEC Parliament under (i) above.

To appreciate the inequities that result from an apportionment that deviates from the proportionality or quota principle, one need only consider the implied disparities between countries in the number of persons

that each delegate has to represent. For, as the delegates in the newly apportioned Parliament are to be directly elected by specific constituencies, it is relevant to ask how large these constituencies will be. Put differently, to what extent will citizens of different countries have unequal shares in their representatives? In Luxembourg, with a population of 357,000 there would be six seats, hence on the average each delegate would have to represent 59,500 persons, while in Ireland each delegate would have to represent on the average 237,385 persons (about four times as many), in the Netherlands each would have to represent on the average 498,148 (over eight times as many) and in the F.R.G. each would have to represent on the average 861,681 persons (over fourteen times as many as in Luxembourg).

The fundamental fairness principle that any acceptable apportionment method must be based on is the principle of proportionality. The EEC apportionment is quite evidently not based on proportionality and hence, although it is found by a procedure, the procedure lacks justification. Let us now consider what the implications of the proportionality principle are for an acceptable apportionment method.

For given populations of countries and a given number of seats to be apportioned, we begin by computing the exact quota for each country, that is, the precise number of representatives out of the total to which each country is entitled. Any reasonable apportionment method must begin with the exact quotas; in the absence of any requirements concerning the minimum allowable number of seats, the problem then reduces to deciding which countries are to have their exact quota rounded up, and which are to be rounded down. Any apportionment method that operates in this way will be said to satisfy quota. But this rounding cannot, in general, be achieved by the "usual" method of rounding

fractions of less than .5 down, and .5 or more up. The reason is seen from the example of *Table 1*: the "usual" rounding of the exact quotas would yield the apportionment:

FRG	UK	Italy	France	Neth.	Belg.	DK	IR	Lux.
86	78	77	75	19	14	7	4	0

T A B L E 2
The Usual Rounding

But the sum of these numbers is 360 whereas we are required to apportion 359 seats.

HAMILTON'S METHOD AND THE ALABAMA PARADOX

A second reasonable approach that suggests itself is the following. Given the exact quotas, first give each country as many whole number seats as it is entitled to; this leaves a certain number of seats still to be distributed. These remaining seats are given, one each, to those countries having the largest decimal fractions in their exact quota. This method was first proposed by Alexander Hamilton,⁹⁾ first Secretary of the U.S. Treasury, in 1792, and it is consequently known as Hamilton's method. In the European literature this same method is known as "la repartition au plus fort reste."

The solution by Hamilton's method for 359 seats is shown in *Table 3*.

⁹⁾ The Papers of Alexander Hamilton, vol. XI (Feb. 1792 - June 1792), Ed. Harold C. Syrett, Columbia University Press, New York, 1966, pp. 228-230.

FRG	UK	Italy	France	Neth.	Belg.	DK	IR	Lux.
86	78	77	75	19	13	7	4	0

T A B L E 3
Hamilton's Solution for 359 Seats

Hamilton's method (or variants of it) was used in the U.S. from 1850 to 1900. But in 1881 a most serious difficulty with this method came to light. C.W. Seaton, Chief Clerk of the United States Census Office related that while making calculations by Hamilton's method for various sizes of Congress (and fixed populations), he discovered that whereas in a house of 299 members Alabama was entitled to eight seats, when the house was increased to 300 members Alabama was entitled to only seven.¹⁰⁾ This absurd phenomenon was dubbed by Seaton the "Alabama paradox." It was immediately recognized as being inherently unfair and absurd as a property of any apportionment method, and from 1901 on the Hamilton method was abandoned. That the Alabama paradox was no mere quirk of the Hamilton method, but is frequently encountered may be seen from EEC example: for a Parliament of 358 members, the Hamilton method gives Luxembourg one seat (see *Table 4*) whereas in a Parliament of 359 members it would get none (see *Table 3*).

	FRG	UK	Italy	France	Neth.	Belg.	DK	IR	Lux.
Exact Quota	85.770	77.496	76.535	74.350	18.594	13.510	6.984	4.266	0.494
Hamilton Solution	86	77	77	74	19	14	7	4	0

T A B L E 4
Hamilton's Solution for 358 Seats

¹⁰⁾ Apportionment Among the Several States, House of Representatives, 56th Congress, 2nd Session, Report 2130, 20 December 1900, p. 18.

Thus the Hamilton method also turns out to be unacceptable, and we are left with the problem of both satisfying quota and avoiding the Alabama paradox.

THE QUOTA METHOD: PRINCIPLES

Three principles emerge which are essential to any method of apportionment of representation in a union of sovereign States.

(1) A method must be universally applicable, that is, must provide solutions for all sizes of Parliament sufficiently large to satisfy minimum representational requirements, and all possible demographic distributions and numbers of countries.

(2) A method must satisfy quota, insofar as this is possible in view of assuring small countries the minimum required representation.

(3) A method must avoid the Alabama paradox.

In the presence of minimum representational requirements the notion of quota must be extended. Consider, for example, the case where each country of the EEC were required to have at least six seats but that sixty seats were available, then, in particular, the F.R.G. would have an "exact quota" of 14.37 but they can clearly never receive as many as fourteen seats. The applicable exact quota concept in such cases is easily obtained. If, for a certain size of Parliament, the exact quota of some country is less than the minimum mandated required representation, then this country should receive its minimum number of seats and then be considered as eliminated from the problem of apportionment. This leaves fewer countries and fewer seats to be distributed among them and one can now define the essential property of satisfying quota for this altered situation in the natural way. So, in the presence of minimum requirements, satisfying quota has this meaning.

But how should minimum requirements be determined? A natural approach is to say that each country should receive a certain fixed minimum of seats, such as each department in France has at least two députés and each state in the U.S. has at least one Representative. However, perhaps the political reality calls for minima which are different. In this case only one limitation is imposed on the choice of minima: no large country should be overrepresented in comparison with a smaller country at the minima. Or, to put it another way, the assignment of minimum representations must not favor the large countries over the small countries. The device of minima is resorted to as a means of assuring power to small countries beyond what their populations would naturally assign them.

If the three essential properties are to be satisfied, and if the minimum required representations do not favor larger in comparison with smaller countries then it is a scientific fact that there is one mathematically consistent method and only one method which can be used. This method is known as the Quota Method.^{11), 12)} Said in another way, the Quota method is the only mathematically consistent method which can satisfy the three principles. In political terms, if agreement can be reached on these three natural properties then the numerical solutions are determined. This, it must be underlined, is a strong mathematical statement which requires a careful and complex argument. But it allows the political discussion to dispense with the numbers and step up to the choice of principles.

¹¹⁾ M.L. Balinski and H.P. Young, "The Quota Method of Apportionment," American Mathematical Monthly, vol. 82, 1975, pp. 701-730.

¹²⁾ M.L. Balinski and H.P. Young, "A New Method for Congressional Apportionment," Proceedings of the National Academy of Sciences U.S.A., vol. 71, 1974, pp. 4602-4606.

THE QUOTA METHOD: MECHANICS

To obtain solutions the following data must be specified: the population of each country, the size of the European Parliament to be apportioned, and the minimum number of seats to be accorded each country.

The mechanics are relatively straightforward. First, each country is accorded the minimum number of seats which it is required to have. Then, given the apportionment for a Parliament having any given number of seats (e.g., as given by the initial minimum requirement solution) the apportionment for a Parliament having one more seat is determined by singling out that country which should receive the extra seat. This country is the one which would have the largest average district size in the larger Parliament provided that its number of seats does not round its exact quota up by more than one.

The method is natural in its construction for it seeks to assign the extra seat to that country still least represented in the new Parliament. Despite the simplicity of this construction it is both a surprising and a non-obvious mathematical fact that no other construction will satisfy the three basic properties.

THE QUOTA METHOD: SELECTED SOLUTIONS FOR THE EUROPEAN PARLIAMENT

In the current situation the populations are fixed, but minima have not been explicitly stated. Two criteria which concern this decision were stated in the EEC Draft, namely, criterion (ii) which assures each country as many seats as "political forces" within it and criterion (iii) which assures each country as many seats as in the present European Parliament. In the present situation criterion (iii) dominates (ii) and hence would appear to imply that

the minimum number of seats should be

FRG	UK	Italy	France	Neth.	Belg.	DK	IR	Lux.
36	36	36	36	14	14	10	10	6

T A B L E 5
Present Minima

On the other hand, the existence of alternate plans implies the possibility of setting different minima.

A second decision to be made is the total number of seats in the Parliament. The current proposal suggests 355, but this number is not predetermined; rather, it is the result of an ad hoc procedure.

Below, solutions are given for several choices of minimum requirements and several choices for the size of Parliament. These are given as interesting examples and for comparison with the various proposed plans. They also illustrate the numerical solutions which are produced. But, of course, solutions can be found for any choices of minima and Parliament size. The sizes have been taken as 291 (*French Plan*), 359 (*European Parliament Plan*), 388 (*Irish Plan*) and 425 (*a larger alternative*). For each such total size the method has been applied with a common minimum representation of 1, a common minimum of 3, a common minimum of 6, and minima corresponding to the present number of seats (as given in *Table 5*). It should not be forgotten that each quota method solution is the only solution for the given data and choice of minima that can result from the agreement to accept the three principles that have been stated.

"FRENCH PLAN"

Parliament Size 291

	FRG	UK	Italy	France	Neth.	Belg.	DK	IR	Lux.	Total
French Proposal	66	60	59	57	17	13	9	7	3	291
Exact Quota	69.72	62.99	62.21	60.44	15.11	10.98	5.68	3.47	0.40	291
Quota Solution, min 1	70	63	62	61	15	11	5	3	1	291
Quota Solution, min 3	69	63	62	60	15	11	5	3	3	291
Quota Solution, min 6	68	61	61	59	14	10	6	6	6	291
Quota Solution, present EEC min	65	58	58	56	14	14	10	10	6	291

"EUROPEAN PARLIAMENT PLAN"

Parliament Size 359

	FRG	UK	Italy	France	Neth.	Belg.	DK	IR	Lux.	Total
EEC Proposal	72	68	67	66	27	23	17	13	6	359
Exact Quota	86.01	77.71	76.75	74.56	18.65	13.55	7.00	4.28	0.49	359
Quota Solution, min 1	86	78	77	75	18	13	7	4	1	359
Quota Solution, min 3	86	77	77	74	18	13	7	4	3	359
Quota Solution, min 6	85	76	76	73	18	13	6	6	6	359
Quota Solution, present EEC min	83	74	74	71	17	14	10	10	6	359

"IRISH PLAN"

Parliament Size 388

	FRG	UK	Italy	France	Neth.	Belg.	DK	IR	Lux.	Total
Irish Proposal	75	71	70	69	30	26	20	18	9	388
Exact Quota	92.96	83.99	82.95	80.58	20.15	14.64	7.57	4.62	0.53	388
Quota Solution, min 1	93	84	83	81	20	15	7	4	1	388
Quota Solution, min 3	93	84	83	80	20	14	7	4	3	388
Quota Solution, min 6	92	83	82	79	19	14	7	6	6	388
Quota Solution, present EEC min	90	81	80	78	19	14	10	10	6	388

'ANOTHER ALTERNATIVE'

Parliament Size 425

	FRG	UK	Italy	France	Neth.	Belg.	DK	IR	Lux.	Total
Exact Quota	101.82	92.00	90.86	88.26	22.07	16.04	8.29	5.06	0.59	425
Quota Solution, min 1	102	92	91	88	22	16	8	5	1	425
Quota Solution, min 3	101	92	90	88	22	16	8	5	3	425
Quota Solution, min 6	101	91	90	87	21	15	8	6	6	425
Quota Solution, present EEC min	99	90	88	86	21	15	10	10	6	425

The Quota Method has a tendency of producing solutions which round up the exact quotas of large countries more often than those of small countries. This, as has been said, is unavoidable once the three essential principles are adopted. Moreover, as the minimum requirements give an increasingly large disproportionate representation to the smaller countries, this particular phenomenon tends to disappear.

CONCLUSION

The central idea is that political apportionment must be based on principles of fair division rather than on disputes over numbers. This can be achieved.

Three principles were singled out which are necessary to the case being studied and sufficient to uniquely determine numerical solutions. First, essential to any method, is that it be applicable to all possible situations. Second, any method not satisfying quota seems shocking to common sense, contrary to any reasonable notion of fair division, and risks being unacceptable to an informed electorate. Third, the Alabama paradox has already proven to be politically unacceptable. In the coming years as populations and the number of member nations change, the European Parliament will have to reapportion its seats and it is likely that its size will rise, not fall. Thus, a universally applicable method is necessary which meets the most basic measure of fairness, satisfying quota, and which is free from the defect of the Alabama paradox. The only method which accomplishes this is the Quota Method.