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Market Substitution Models and Economic Parameters

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PREFACE

Market penetration by new technologies is an established fact. The curves of penetration obey simple mathematical rules and fit past experience very well. However, it has not been possible to argue rigorously that future market penetrations would follow the same rules, because a theoretical basis for these rules was lacking.

In a 1977 IIASA report (RR-77-22), Peterka proposed such a basis for centrally planned economies; it followed from detailed consideration of their investment practices. Thus, there remained a need for a model that would be heuristically reasonable for market economies. This report explores two such models.

The work reported here provides a basis for including market penetration considerations in the research activities of the IIASA Energy Systems Program. In particular, it has been used in constructing two reference scenarios for 1975–2030, called "High" and "Low," which are important ingredients in the global energy analysis described in detail in the forthcoming book *Energy in a Finite World*.

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SUMMARY

Peterka (1977) has proposed a theoretical economic framework from which the logistic model for market penetration may be derived. His basic equation is consistent with the use of capital charge rates equal to amortization rate plus industry growth rate to determine total costs of a technology and with the use of a price that exactly recovers these costs on an industrywide basis. This formalism is consistent with the practice of centrally planned economies, which use the charge and price rules just set forth.

In addition, the Peterka model can also be interpreted as a strategic principle. Using the principle that the attractiveness of investment is proportional to the degree to which a technology is in use, and also to a figure of economic merit, this paper explores companion models for market economies. The most attractive one, which is called the price model, is derived from a strategic principle that rates the economic attractiveness of a technology in proportion to the inverse of the price that would have to be charged for its product. This judgment of attractiveness of the model is based on synthetic problems simulating market substitution in the electric utility industry.

The Peterka model and the market model can be expressed in identical mathematical form, so their qualitative features must be similar.

The mathematical form of the combined model is

$$\dot{f}_i/f_i = \gamma_i \sum_j W_j(d_j - d_i)$$

$$W_i = (f_i\gamma_i)/\sum_j f_j\gamma_j$$

where f_i is the market share of a particular technology, d_i is the total production cost, including capital charges and amortization, and γ_i is a constant of the particular technology. In the Peterka model,

$$\gamma_i = 1/\alpha_i$$

where α_i is the specific capital investment per unit of production capacity of technology i. For the price model,

$$\gamma_i = \rho / d_i$$

where ρ is the logarithmic expansion rate of the industry.

Both models are pseudo steady-state models, but all the parameters may be expressed as functions of time without violating the principles of the heuristics on which they are based.

CONTENTS

1 THE PETERKA MODEL

The fact of market substitution is well established, as is the generally logistic curve shape of the process [see, for example, Fisher and Pry (1970), Marchetti and Nakicenovic (1979), Nakicenovic (1979), or Fleck (in preparation)]. However, the theoretical basis of logistic substitution has only been suggested. One attempt is that of Peterka (1977), who provides a model in which investment in a technology is made at a rate such that new facilities are financed by the marginal income from existing facilities of the same type. Mathematically, this is expressed as

$$\alpha_i \dot{P}_i = P_i(p - c_i) \quad (1)$$

where P_i is capacity of plants exhibiting technology i, α_i is investment required for unit increase of that capacity, p is price of the commodity, and c_i is operating cost per unit commodity. For example, in electrical generation, P_i might be kilowatts; α_i, dollars/kW, and p and c_i dollars/kW-yr; with $\dot{P}_i$ then being yearly capacity addition rate in kW/yr. The operating cost is defined, according to Peterka, so as to include fixed charges against capital such as those for amortization and taxes, but not charges for profit or for accumulation of new capital by the enterprise. These latter items are, rather, taken up in the term $p - c_i$.

The Peterka model has qualitative features that lead to the logistic curves that are observed for market substitution. It is, therefore, an appropriate model to examine for validation or generalization.

1.1 A More Detailed Statement of the Peterka Model

Better insight into the Peterka model can be gained by a more detailed statement of its fundamental principle. This is that the rate of investment in new construction of facilities of a given type is governed by the cash flow generated by existing facilities. The rate of investment in new construction is not entirely due to expansion but also arises from the need to replace existing plant as it is retired. Thus, this rate of investment is $\alpha_i(\dot{P}_i + a_i P_i)$, where a_i is the amortization rate. The amortization rate is not, of course, necessarily a constant. It could be very small for a technology that is just beginning to penetrate and that therefore consists primarily of new plants. This point can be of importance, as we shall discuss later. However, for the time being we assume that a_i is constant in time. An approximate justification for this can be made by considering that $a_i P_i$ is an allowance for amortization that is applied to replacement construction as required; then, we interpret the term as a required addition to a sinking fund, which, however, neither pays interest when it goes negative nor receives interest when it is positive, and which averages over the long term to zero value.

The cash flow generated by existing facilities is the difference between income pP_i and costs. These costs consist of:

- Operating costs for labor, materials, fuels, services, and other items purchased in proportion to production rate. The unit operating costs are defined as b_i, and the operating costs are therefore $b_i P_i$.
- Value-added taxes, which can be expressed as a fraction β of operating costs, or $\beta b_i P_i$.
- Regularly assessed capital charges, such as those for dividends and interest in market economies, property taxes, insurance, and maintenance. We lump these under a fixed-capital-charge rate δ, and the costs are $\delta \alpha_i P_i$. Note here that we have assumed that the rate δ is invariant among competitive technologies. This is generally the case as a first approximation, but a detailed treatment would show some variation among δ_i defined for different technologies.

With these qualifications, we may write Eq. (1) in more detailed form as

$$\alpha_i(\dot{P}_i + a_i P_i) = P_i[p - (1 + \beta)b_i - \delta\alpha_i] \tag{2}$$

Transposing the term $\alpha_i a_i P_i$ gives

$$\alpha_i \dot{P}_i = P_i[p - (1 + \beta)b_i - (\delta + a_i)\alpha_i] \tag{3}$$

Equation (3) is identical with Eq. (1) provided that we define

$$c_i \equiv (1 + \beta)b_i + (\delta + a_i)\alpha_i \tag{4}$$

Indeed, Eq. (4) provides a more precise interpretation of "cost."

1.2 Mathematical Inferences from the Peterka Model

If we divide both sides of Eq. (1) by α_i and then sum over i, we can derive

$$\dot{P} = \sum_i [P_i(p - c_i)/\alpha_i] \tag{5}$$

Defining logarithmic expansion rate ρ as

$$\rho \equiv \dot{P}/P \tag{6}$$

and market share f_i of technology i as

$$f_i \equiv P_i/P \tag{7}$$

where P is total production capacity of the industry, we can then get

$$\rho = \sum_i [f_i(p - c_i)/\alpha_i] \tag{8}$$

or solve for the price p as

$$p = [\rho + \sum_i (f_i c_i/\alpha_i)] / \sum_j (f_j/\alpha_j) \tag{9}$$

Thus, the price is fixed within the model by the market shares.

Equation (1) can be expressed in market shares by

$$\begin{aligned} \alpha_i \dot{f}_i &= f_i(p - c_i - \alpha_i \rho) \\ &= f_i \left[\frac{\rho}{\Sigma_j (f_j/\alpha_j)} + \frac{\Sigma_j (f_j c_j/\alpha_j)}{\Sigma_j (f_j/\alpha_j)} - c_i - \alpha_i \rho \right] \\ &= \frac{f_i \, \Sigma_j (f_j/\alpha_j)(c_j + \rho\alpha_j - c_i - \rho\alpha_i)}{\Sigma_j (f_j/\alpha_j)} \end{aligned} \tag{10}$$

where, in deriving Eq. (10), we have used the fact that the sum of the f_i is unity.

The term $(c_j + \rho\alpha_j)$ has the character of an augmented cost, the true cost plus a "profit" required to maintain system expansion. We define this as

$$d_j \equiv c_j + \rho\alpha_j \tag{11}$$

The price can be expressed in terms of d_j as

$$p = \frac{\Sigma_j (f_j d_j/\alpha_j)}{\Sigma_j (f_j/\alpha_j)} \tag{12}$$

and market shares change as

$$\dot{f}_i = \frac{(f_i/\alpha_i)\, \Sigma_j (f_j/\alpha_j)(d_j - d_i)}{\Sigma_k (f_k/\alpha_k)} \tag{13a}$$

$$= (f_i/\alpha_i)(p - d_i) \tag{13b}$$

1.3 Economic Implications of the Peterka Model

The statement of the principle of self-financing of an industry's expansion is implicit in Eq. (1), as a consequence of the detailed balancing of each component technology. As is seen in Eq. (9), there is also an implicit price-setting in the Peterka model, which results in there being no excess "profit" beyond what is needed to finance expansion. Thus, no external funds flow into the industry, nor do funds leave the industry for application to other sectors.

This set of conditions describes in an idealized way the principles of price determination in centrally planned economies, often referred to as Libermanism. The industrial expansion rate replaces the investment charge rate of market economies, and plays the same role as a cost factor. Because flows of capital to and from other parts of the economy are not considered, there is no room for external or for distributed profit. [Peterka does, in fact, exhibit a formalism where extra investment, as is necessary to introduce a technology in the first place, is explicitly included. However, this formalism is not developed; the Peterka model is usually stated as Eq. (1).]

Equation (1) suggests that there is a figure of economic merit by which technologies may be ranked. Those technologies for which $(p - c_i)/\alpha_i$ are greatest grow fastest. This makes intuitive sense. It states that one emphasizes those technologies for which the ratio of cash accumulation to investment is greatest. The principle is plausible for both centrally planned and market economies. However, market economies have a price-setting mechanism different from Eq. (9); further, as we shall see later, the economic assumptions of classical market theory suggest that different figures of merit should be used.

1.4 The Peterka Model as a Strategic Principle

The concept of the ratio $(p - c_i)/\alpha_i$ as a figure of merit invites extension. There must be some one of the technologies for which this figure of merit is a maximum. Why do we not concentrate all new construction on this "best" technology? Fleck (in preparation) has analyzed the decision process as one that involves psychological components and that is essentially stochastic. Simplifying these arguments, one can say that the probability of adopting a particular choice has two components. One of these is a figure of merit, and this has just been noted for the Peterka model. The other is a measure of confidence in the specific choice. There are always "opportunity-conscious" and "risk-averse" decision makers. The most opportunity-conscious decision maker will always choose the option with the highest figure of merit. The most risk-averse decision maker will, on the other hand, always choose the option that is most common at the time of decision – the tried and true, so to speak.

One could also justify the factor P_i on the right-hand side of Eq. (1) in a related, but slightly less psychologically oriented, way. At the time of decision, there is always some uncertainty about achieving the economic performance predicted as the figure of merit. The more experience that exists, the less the uncertainty will be. The reciprocal of uncertainty then measures the confidence that one has in the figure of merit, and this positive attribute increases with P_i.

Thus we can say that the expansion rate of technology i, $\dot{P}_i$, can be considered to be a function of two parameters: economic attractiveness, described by a figure of merit E_i, and confidence in the technology, described by a figure of merit C_i. Most generally,

$$\dot{P}_i = \dot{P}_i(E_i, C_i) \tag{14}$$

where the functional dependence is such that $\dot{P}_i$ increases with E_i, C_i within the domain of realizable systems.

Equation (14) can be explored through examination of a variety of functional relations: additive laws, multiplicative laws, and additions and multiplications of powers of E_i and C_i. Fleck's analysis offers justification for the mathematically tractable simple multiplication law

$$\dot{P}_i = kE_i \cdot C_i \tag{15}$$

and Peterka's model is an expression of Eq. (15) for which E_i is identified with $(1/\alpha_i)[\Sigma_j\,(f_j d_j/\alpha_j)/\,\Sigma_k\,(f_k/\alpha_k) - c_i]$, C_i is identified with P_i, and k solves to be unity.

Equation (15) is the *strategic principle* adopted throughout this paper, and the identification of C_i with P_i is likewise robust. We shall later be examining other figures of merit, believed to be more descriptive of market economies.

Considering Peterka's model to be a strategic principle removes one heuristic objection to it. As pointed out, whatever the price-determination mechanism is, the figure of merit $(p - c_i)/\alpha_i$ is a plausible one. Maximizing the ratio of earnings to investment is in the investor's interest, be the investor a public body or a private one. Logically, this leads to the model set:

$$\begin{aligned} P_i &= \dot{P} \qquad (i = k) \\ &= 0 \qquad (i \neq k) \end{aligned} \tag{16}$$

where k is that technology for which $(p - c_k)/\alpha_k$ is a maximum. The model of Eq. (16) is optimal, but there is considerable evidence that it is incorrect; there are many instances of favorable technologies that were never deployed extensively because they remained "unfashionable" up to the time that the industry to which they pertained declined. Equation (16) also predicts that small changes in d_i over time cause sudden activities of technology, whereas social systems do not easily accommodate to such "bang–bang" control.

2 MARKET ECONOMY MODELS

We have noted that Peterka's model implicitly incorporates a price-setting mechanism that corresponds to a standard practice of centrally planned economies. This arises from the absence of capital flows into or out of the particular industry. In market economies, such capital flows exist and are (ideally) controlled by the market for capital. Thus, we first look for models that differ from Peterka's only by permitting such capital flows.

2.1 Fixed-Price Models

The most direct extension of the Peterka strategy is to retain the figure of merit but to let the price be fixed arbitrarily. That is, as in the Peterka model,

$$\dot{P}_i = kP_i[(p - c_i)/\alpha_i] \tag{17}$$

It is important to note that the term c_i includes, for market economies, interest and fixed dividends to investors, as well as capital taxes. The inclusion of the constant k, from Eq. (16), as an arbitrary normalizer, permits the price, p, to be an extrinsic parameter. We can solve for k by summing both sides of (17) over all i and noting that $\Sigma_i \dot{P}_i = \rho P$. The result is

$$k = \frac{\rho}{\Sigma_i\ [f_i(p - c_i)/\alpha_i]} \tag{18}$$

and leads to

$$\dot{P}_i = \frac{\rho P_i(p - c_i)/\alpha_i}{\Sigma_i\ [f_i(p - c_i)/\alpha_i]} \tag{19a}$$

or, after some manipulation,

$$\dot{f}_i = \rho f_i\ \frac{\Sigma_j f_j[(p - c_i)/\alpha_i - (p - c_j)/\alpha_j]}{\Sigma_j\ [f_j(p - c_j)/\alpha_j]} \tag{19b}$$

The similarity to the Peterka model is emphasized if, given an arbitrary price p, we define a parameter

$$\lambda \equiv (1/\rho) \sum_j\ [f_j(p - c_j)/\alpha_j] \tag{20}$$

Then, price is expressible as

$$p = \frac{\Sigma_j\ (f_j/\alpha_j)(c_j + \lambda\rho\alpha_j)}{\Sigma_j\ (f_j/\alpha_j)} \tag{21}$$

Instead of "excess profit" $\rho\alpha_j$, this excess profit is $\lambda\rho\alpha_j$. If we consider total charges d_i as incorporating excess profit, we can for arbitrary prices define

$$d_i \equiv c_i + \lambda\rho\alpha_i \tag{22}$$

Algebraic manipulation then leads to

$$\dot{f}_i = \frac{(f_i/\lambda\alpha_i)\ \Sigma_j\ (f_j/\alpha_j)(d_j - d_i)}{\Sigma_k\ (f_k/\alpha_k)} \tag{23}$$

which differs from (13a) only in having the extra divisor λ on the right-hand side. The system behaves exactly as if each specific investment α_i had been arbitrarily renormalized by the factor λ. Notice, however, that the analogue of Eq. (1) is

$$\alpha_i \dot{P}_i = P_i(p - c_i)/\lambda \tag{24}$$

so that if λ is different from unity, we can tell whether the actual cash flow is into the industry ($\lambda > 1$), or out of it ($\lambda < 1$). This situation permits us to define λ as an investment flow parameter.

2.2 Investment Opportunity Models

The standard description of investment planning in market economies, and particularly in industrial sectors, is *not* one in which cash flow per unit investment is to be maximized. Instead, it is assumed that there exists a "fee" for the use of money, and that any amount of capital is available if that fee is paid. Such "fees" are included in the c_i of market economies. For an investment in a new industry, the fee is the going interest rate, augmented by a (market-determined) rate to accommodate the factor of risk.* The objective is then to maximize earnings over and above that fee (see Riggs 1968 and Massé 1962).

The topic of market penetration assumes an existing industry, so only this case will be treated further.

The existing rate of return, to be denoted by r, is simply the cash flow rate divided by the total investment. Then,

$$r = \sum_j\ [(p - c_j)f_j] / \sum_j d_j f_j \tag{25}$$

We may derive the price from Eq. (25) as

$$p = \sum_j f_j(c_j + r\alpha_j) \tag{26}$$

*This statement is a condition that the *enterprise* does not have the opportunity to invest in other, more profitable industries. One might say that this is the decision of the owners (shareholders). If the opportunity exists for them, capital will flow out of one and into the other until (risk-adjusted) rates of return are balanced – at least under ideal conditions. In a dynamic economy, of course, this balance is hardly ever achieved.

Now suppose the industrial expansion target is taken as some fixed δP. If that δP is constructed using technology i, we will make money at a rate $\delta P(p - c_i)$.

δP is a constant and can be absorbed into the constant k of Eq. (15). "Excess earnings" as a figure of merit then leads to

$$(p - c_i) = E_i \tag{27}$$

Assuming as usual that C_i is given by P_i, Eq. (15) becomes

$$\dot{P}_i = kP_i(p - c_i) \tag{28}$$

Summing both sides and expressing the result in terms of the f_i gives

$$\rho = k(p - \sum_i f_i c_i) = kr \, \Sigma \, f_i \alpha_i \tag{29}$$

This development leads to

$$k = \rho/(r \, \Sigma_i \, f_i \alpha_i) \tag{30}$$

and to

$$\dot{f}_i = \frac{\rho f_i (p - c_i)}{r \, \Sigma_i \, f_i \alpha_i} - \rho f_i \tag{31}$$

which in turn reduces to

$$\dot{f}_i = \frac{\rho f_i \, \Sigma_j \, f_j (c_j - c_i)}{r \, \Sigma_j \, f_j \alpha_j} \tag{32}$$

Equation (32) is a close analogue of the Peterka model. The economic differences are the following. First, the availability of capital, in large amounts at a standard rate, has the effect of averaging specific investment as an inhibiting factor; α_i is replaced by $\bar{\alpha}$. Second, the c_i already includes interest and normal dividends on capital investment, and in this sense is analogous to the Peterka model's d_i. Third, the ratio of expansion rate to excess rate of return is a specific accelerator for market substitution. These differences all seem heuristically plausible and make Eq. (32) a candidate for the desired market-economy analogue of the Peterka model.

2.3 A Price-Suggested Model

Equation (32), while plausible, is not quite satisfactory, because the multiplier $1/\bar{\alpha} = 1/\Sigma_j \, f_j \alpha_j$ is on the right-hand side. If we argue that the availability of capital is not a basic problem in market economies (that only the *cost* of capital must be considered), this term, which has the force of an accelerator of technological change, is not heuristically consistent.

Therefore, we look further for a new model. We define

$$d_i \equiv c_i + r\alpha_i \tag{33}$$

We note the identity

$$\sum_i P_i p = Pp = \sum_i P_i d_i \tag{34}$$

If we differentiate (34) with respect to time, we get

$$\sum_i \dot{P}_i d_i = p\dot{P} + P\dot{p} \tag{35}$$

By using the theory of price–demand coefficients and the definition of ρ, we could express the right-hand side of Eq. (35) as a constant multiplied by pP; but that is unnecessary. The important point to note is that the right-hand side is not a function of i.

Now, we note that the Peterka model derives its basic weighting from the appearance of a term $\Sigma_i\ \alpha_i \dot{P}_i$ in the equation describing investment rate balancing. Applying the same reasoning as that model, we get

$$E_i = 1/d_i \tag{36}$$

when we consider income balancing. The figure of merit is the reciprocal of the cost, computed at the rate of return of capital for the industry. There is no need for any other factor in E_i, since any constants from the right-hand side of Eq. (35) can be absorbed into the k of our strategic model, Eq. (15).

The necessary algebra then gives us

$$k = \rho / \sum_j (f_j/d_j) \tag{37}$$

and

$$\dot{f}_i = \frac{(\rho f_i/d_i)\ \Sigma_j\ (f_j/d_j)(d_j - d_i)}{\Sigma_k\ (f_k/d_k)} \tag{38}$$

Equation (38) cannot be proven to be the best analogue of the Peterka model for a market economy, but it seems to be free of the heuristic objections raised against Eq. (32).

3 COMPARING THE MODELS

We display again the models that are favored, in their market share form:

Peterka, Planned Economy, Eq. (13a)

$$\dot{f}_i = \frac{(f_i/\alpha_i)\ \Sigma_j\ (f_j/\alpha_j)(d_j - d_i)}{\Sigma_k\ (f_k/\alpha_k)}$$

Cost Model, Market Economy, Eq. (32)

$$\dot{f}_i = \frac{\rho f_i \, \Sigma_j f_j(c_j - c_i)}{r \, \Sigma_k f_k \alpha_k}$$

Price Model, Market Economy, Eq. (38)

$$\dot{f}_i = \frac{(\rho f_i / d_i) \, \Sigma_j \, (f_j/d_j)(d_j - d_i)}{\Sigma_k \, (f_k / d_k)}$$

They all can be expressed in a common form:

$$\dot{f}_i = \gamma_i f_i \sum_j W_j (e_j - e_i) \tag{39a}$$

where W_i are weighting factors defined by

$$W_i = \gamma_i f_i / \sum_k \gamma_k f_k \tag{39b}$$

The parameters are different for the three cases, however.

In the Peterka model,

$$\gamma_i = 1/\alpha_i \; ; \qquad e_i = d_i = c_i + \rho\alpha_i \tag{40a}$$

In the cost model,

$$\gamma_i = \rho/(r \, \Sigma_j \, f_j \alpha_j) = \rho/(r\bar{\alpha}) \; ; \qquad e_i = c_i \tag{40b}$$

In the price model,

$$\gamma_i = \rho/d_i \; ; \qquad e_i = d_i = c_i + r\alpha_i \tag{40c}$$

The analogy between the e_i in the Peterka model and the price model is notable, and we have already observed that this makes the price model a more desirable market economy analogue of the Peterka model than the cost model would be.

3.1 Comparative Behavior of the Models

We can get considerable insight into the comparative behavior of the models simply by examining the γ_i. This parameter is essentially an acceleration parameter for technological substitution: it is a factor in the weights W_j and a separate factor in the equation for $\dot{f}_i$.

In the Peterka model, $\gamma_i = 1/\alpha_i$, or, as I prefer to write it, $\gamma_i = \rho/(\rho\alpha_i)$. Regardless, it is clear that, in this model, technologies of high capital cost are inhibited.

In the cost model, $\gamma_i = \rho/(r\bar{\alpha})$. There is no longer any specific inhibition of technologies of high capital cost, but, because of the factor $1/\bar{\alpha}$, capital-intensive industries are inhibited in their rates of technological change. In addition, the factor ρ/r suggests that industries expanding faster than their rate of return will exhibit more rapid market replacement than those for which the converse is true.

In the price model, $\gamma_i = \rho/d_i$. We note that on the average $d_i > r\bar{\alpha}$, so that market replacement will be slower in the price model than in the cost model.

3.2 Amortization Revisited

Both market economy models predict that a no-growth industry will be technologically stagnant. That is, the market share of competing technologies will not change with time. The Peterka model predicts very rapid penetration of low-operating-cost technologies under these conditions. Heuristically, we expect changes to occur, even in a no-growth industry. For this to be within the scope of models, we must now examine amortization more carefully in the market models. (The treatment presented in the Peterka model requires, however, no elaboration.)

There are actually two separate effects of amortization in a market economy. One is to impose an amortization charge on the existing capital plant, to take into account the (financial) decrease of plant value over its lifetime. The other is to require new construction as old plant is retired. The financial amortization charge $a_i\alpha_i$ is included in the c_i in market economies. However, the rate of new construction is altered from $\dot{P}$ to $\dot{P} + \Sigma_i a_i P_i$ as well. This has the effect of changing our strategic model (15) to

$$\dot{P}_i + a_i P_i = kE_i C_i \tag{41}$$

Without following through the details, the cost model (32) then becomes

$$\dot{f}_i = \frac{f_i}{r\,\Sigma_j\,\alpha_j f_j}\left[(\rho + \sum_k f_k a_k)\sum_j f_j(c_j - c_i) + r\sum_k \alpha_k f_k \sum_j f_j(a_j - a_i)\right] \tag{42}$$

For the standard case, in which all the a_i are the same, this reduces to

$$\dot{f}_i = \frac{(\rho + a)f_i\,\Sigma_j\,f_j(c_j - c_i)}{r\,\Sigma_j\,\alpha_j f_j} \tag{43}$$

where a is the (common) value of all the a_i.

The price model reduces similarly to

$$\dot{f}_i = f_i\left[\frac{\rho + \Sigma_j\,f_j a_j}{d_i\,\Sigma_j\,(f_i/d_i)} - \rho - a_i\right] \tag{44}$$

and, for constant a, to

$$\dot{f}_i = \frac{(\rho + a)f_i\,\Sigma_j\,(f_j/d_j)(d_j - d_i)}{d_i\,\Sigma_j\,(f_j/d_j)} \tag{45}$$

In other words, the existence of amortization has the effect of permitting penetration of a technology into any industry where new construction is justified. Only when the industry is declining at the amortization rate or faster is market substitution entirely inhibited.

3.3 Comparative Calculations

For comparing the three models, a set of calculations was run on a synthetic case suggested by the structure of the investor-owned electrical utility system of the United States. Three competing technologies were examined simultaneously:

1. A (relatively) low-capital-cost, high-operating-cost technology
2. A higher-capital-cost, lower-operating-cost technology
3. A very-high-capital-cost, very-low-operating-cost technology

These may be thought of, qualitatively, as resembling fossil-fueled generation, nuclear generation, and solar power, respectively. Indeed, an estimate of actual costs of these types of generation (Spinrad 1980) was used to derive initial values, but the numbers were altered considerably in order to examine cases that had variable penetration.

The costs are given in Table 1 for the cases considered. They correspond in terms of charge rates to inflation-free conditions in the United States. However, an excise tax has been added, which is not common in the United States. It corresponds to mild encouragement to conserve energy.

All calculations reported here were made on market economy assumptions. That is, the term in δ, capital charge rate, and so on, in Eq. (4) explicitly includes dividends and interest.

Actual costs and prices, given in terms of dollars per kilowatt-year of electricity, are presented in Table 2 for the three technologies under the economic assumptions of Table 1. It can be seen that Technology 2 is the cheapest in terms of cost, but that as extra capital charges are added, it gives way in terms of price to Technology 1. Technology 3 is also cheaper than Technology 1 in terms of cost, but its price escalates even faster than that of Technology 2 as additional capital charges are assessed. These additional capital charges are ρ, the industrial growth rate, in the Peterka model, and r, the excess return on capital, in the other models.

The various formulae were approximated by year-by-year difference equations. No problems were encountered in the forward integration as long as round-off errors were not allowed to initiate mathematical instabilities in the solutions. This was avoided by renormalizing the sum of the market shares to unity after each integration step.

The case $\rho = 0.025$ corresponds to a stagnant industry (since amortization was not explicitly incorporated into the equations except as a financial charge –

TABLE 1 Economic assumptions used to test market penetration models.

Parameters	Annual rate (%)	Technology No. 1	2	3
Property taxes and insurance	2			
Amortization	2.5			
Dividends and interest	3.5			
Excise taxes on sales	20			
Capital cost[a] [$/kW(e)]		925	1,500	2,000
Operating cost [$/kW-yr]		100	35	6

[a]At design capacity factor. That is, capital cost per unit rating is total capital cost per unit nameplate rating, divided by annual average design capacity factor.

TABLE 2 Costs and prices of power under varying parameter values.

Model and parameter value	Cost or price of power [$/kW-yr] from Technology No. 1	2	3
Cost	174	155	166
Price *Peterka model*[a]			
$\rho = 0.025$	236.55	231	259.20
0.05	264.30	276	319.20
0.075	292.05	321	379.20
Price *Other models*[b]			
$r = 0.01$	219.90	204	223.20
0.02	231.00	222	247.20
0.04	253.20	258	295.20

[a]ρ = Growth rate of industry, fraction per year.
[b]r = Expected excess return on capital, fraction per year.

see Section 3.2). For this case, market shares of Technologies 2 and 3 are listed in Table 3 for the various models and excess capital charge rates used. The initial condition is the set of market shares listed for year 0.

In the Peterka model for this case, there is a slow growth of Technology 2 at the expense of Technology 3, which has a higher-priced product than Technology 1 or 2. Technology 1, which commands a slightly higher price than Technology 2, retains an almost static market share. It is being displaced by Technology 2 at a very slow rate at the end of a 100-year period.

The cost model shows penetration of both Technologies 2 and 3 into the market, at faster rates than Technology 2 penetrated in the Peterka model.

TABLE 3 Comparison of market shares[a] for industrial growth rate $\rho = 0.025$, Technologies 2 and 3.[b]

	Peterka model		Cost model[c]						Price model[c]					
			$r = 0.01$		$r = 0.02$		$r = 0.04$		$r = 0.01$		$r = 0.02$		$r = 0.04$	
Year	No. 2	No. 3	No. 2	No. 3	No. 2	No. 3	No. 2	No. 3	No. 2	No. 3	No. 2	No. 3	No. 2	No. 3
0	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125
5	0.129	0.120	0.152	0.132	0.138	0.129	0.132	0.127	0.126	0.125	0.126	0.124	0.125	0.121
10	0.133	0.115	0.184	0.139	0.153	0.132	0.138	0.129	0.127	0.124	0.126	0.123	0.125	0.117
15	0.136	0.110	0.219	0.145	0.168	0.136	0.146	0.131	0.128	0.124	0.127	0.122	0.125	0.114
20	0.140	0.105	0.258	0.149	0.184	0.139	0.153	0.132	0.129	0.124	0.128	0.121	0.125	0.110
25	0.144	0.101	0.299	0.152	0.201	0.142	0.160	0.134	0.131	0.123	0.128	0.120	0.125	0.107
30	0.149	0.097	0.343	0.153	0.220	0.144	0.168	0.136	0.132	0.123	0.129	0.119	0.125	0.103
35	0.153	0.093	0.389	0.153	0.239	0.147	0.176	0.137	0.133	0.123	0.130	0.118	0.125	0.100
40	0.157	0.089	0.435	0.151	0.258	0.149	0.185	0.139	0.134	0.122	0.131	0.117	0.125	0.097
45	0.161	0.085	0.481	0.148	0.279	0.150	0.193	0.140	0.135	0.122	0.131	0.116	0.125	0.094
50	0.166	0.081	0.527	0.144	0.300	0.151	0.202	0.142	0.136	0.122	0.132	0.116	0.125	0.091
55	0.170	0.078	0.571	0.139	0.322	0.152	0.211	0.143	0.137	0.121	0.133	0.115	0.125	0.088
60	0.175	0.074	0.613	0.133	0.344	0.153	0.220	0.144	0.139	0.121	0.133	0.114	0.125	0.085
65	0.180	0.071	0.653	0.126	0.367	0.153	0.229	0.145	0.140	0.120	0.134	0.113	0.124	0.082
70	0.185	0.068	0.690	0.119	0.390	0.152	0.239	0.147	0.141	0.120	0.135	0.112	0.124	0.080
75	0.190	0.065	0.724	0.111	0.413	0.152	0.249	0.148	0.142	0.120	0.136	0.111	0.124	0.077
80	0.195	0.062	0.755	0.103	0.436	0.151	0.259	0.148	0.143	0.119	0.136	0.110	0.124	0.075
85	0.200	0.059	0.783	0.096	0.460	0.149	0.269	0.149	0.145	0.119	0.137	0.109	0.124	0.072
90	0.205	0.057	0.809	0.089	0.483	0.148	0.280	0.150	0.146	0.119	0.138	0.108	0.124	0.070
95	0.210	0.054	0.832	0.081	0.506	0.145	0.290	0.151	0.147	0.118	0.138	0.108	0.123	0.068
100	0.216	0.052	0.852	0.075	0.528	0.143	0.300	0.151	0.148	0.118	0.139	0.107	0.123	0.066

[a] The body of the table consists of market shares for Technologies 2 and 3, rounded off to three decimal places.
[b] Technology 1 has the remaining market share so that the sum is equal to 1.
[c] The parameter r is the excess profit on investment above and beyond normal dividends and interest, expressed as a fraction per year.

The rate is particularly fast for small r. The price model, on the other hand, shows very sluggish market penetration.

The case $\rho = 0.075$ corresponds to a vigorously growing industry. The actual annual growth rate is closer to 5% than to 7.5% since we have not counted the replacement construction required by amortization. The market share evolution for this case, according to various formulae, is given in Table 4.

The Peterka model shows a decline in market share for both Technologies 2 and 3, for which the price in that model is higher than for Technology 1. The cost model shows a very rapid penetration of Technology 2 – the lowest-cost technology – even under the relatively high excess profit margin $r = 0.04$. The price model shows a relatively sluggish growth of Technology 2 for small r, a sluggish decline for large r, and a decline for the high-cost technology, number 3, in all cases.

4 DISCUSSION AND CONCLUSIONS

In trying to model a phenomenon as complex as market substitution, there is no way of ensuring that any algorithm is correct. Instead, all that can be done is to try models out and see whether the results are reasonable. Since "reasonableness" is subjective, there are always grounds for dispute. Yet, from the examples just exhibited, it seems that some models should be preferred.

Specifically, the cost model, Eq. (32), does not lead to results that are easy to justify heuristically. It leads to very rapid market substitutions, even when intuitively one would think that they would be slow – for example, when the industry is stagnant. Further, it is unstable as the excess rate of return r approaches zero.

The price model, Eq. (38), is free of these defects. It gives a completely definite answer as $r \to 0$. It favors that technology which commands the lowest price under market conditions, including whatever rate of return is appropriate. For the examples tested, it shows rather sluggish market substitution, however.

The Peterka model is simpler, but, as has been pointed out, it is based on an assumption that cannot be justified for market economies. This assumption is that the excess rate of return r can be equated with the industry expansion rate ρ. The Peterka model thus penalizes technologies of high capital cost very heavily when an industry is expanding rapidly; yet this is the circumstance under which capital can usually be attracted easily, and large investments can be tolerated if they lead to production economies.

For these reasons, the price model, Eq. (38), seems to be the most sensible starting point for a market-economy analogue of the Peterka model, which is a valid interpretation of the economic protocols of centrally planned economies. An interesting point of departure for future research would be to see how the market penetration process might vary between market and centrally planned economies. Neither of these models, however, can be adopted as more than a suggestion to try, until their correlation with reality is well checked. The

TABLE 4 Comparison of market shares[a] for industrial growth rate $\rho = 0.075$, Technologies 2 and 3.[b]

	Peterka model		Cost model[c]						Price model[c]					
			$r = 0.01$		$r = 0.02$		$r = 0.04$		$r = 0.01$		$r = 0.02$		$r = 0.04$	
Year	No. 2	No. 3	No. 2	No. 3	No.2	No. 3	No. 2	No. 3	No. 2	No. 3	No. 2	No. 3	No. 2	No. 3
0	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125
5	0.119	0.106	0.216	0.145	0.167	0.136	0.145	0.131	0.128	0.124	0.127	0.122	0.125	0.119
10	0.113	0.090	0.338	0.155	0.218	0.145	0.168	0.136	0.132	0.123	0.129	0.119	0.125	0.114
15	0.108	0.076	0.475	0.152	0.277	0.151	0.193	0.140	0.135	0.122	0.131	0.116	0.125	0.108
20	0.102	0.064	0.608	0.136	0.342	0.154	0.219	0.144	0.139	0.121	0.133	0.114	0.125	0.103
25	0.096	0.054	0.720	0.115	0.410	0.153	0.248	0.148	0.142	0.120	0.136	0.111	0.125	0.099
30	0.091	0.045	0.807	0.091	0.480	0.149	0.278	0.150	0.146	0.119	0.138	0.108	0.125	0.094
35	0.085	0.038	0.870	0.070	0.548	0.142	0.310	0.152	0.150	0.118	0.140	0.106	0.125	0.089
40	0.080	0.032	0.913	0.053	0.612	0.134	0.344	0.153	0.153	0.117	0.142	0.103	0.125	0.085
45	0.076	0.027	0.942	0.039	0.671	0.123	0.378	0.153	0.157	0.115	0.144	0.101	0.124	0.081
50	0.071	0.022	0.961	0.028	0.723	0.112	0.412	0.152	0.161	0.114	0.147	0.098	0.124	0.077
55	0.067	0.019	0.974	0.020	0.769	0.100	0.447	0.150	0.165	0.113	0.149	0.096	0.124	0.073
60	0.063	0.016	0.982	0.015	0.809	0.089	0.482	0.148	0.169	0.112	0.151	0.094	0.124	0.070
65	0.059	0.013	0.988	0.011	0.842	0.079	0.516	0.145	0.174	0.111	0.153	0.091	0.123	0.067
70	0.055	0.011	0.992	0.007	0.870	0.069	0.550	0.141	0.178	0.110	0.156	0.089	0.123	0.063
75	0.052	0.009	0.995	0.005	0.893	0.060	0.582	0.137	0.182	0.109	0.158	0.087	0.123	0.060
80	0.049	0.008	0.996	0.004	0.913	0.052	0.614	0.132	0.187	0.108	0.160	0.085	0.122	0.057
85	0.045	0.006	0.997	0.003	0.928	0.044	0.644	0.127	0.191	0.107	0.163	0.083	0.122	0.054
90	0.043	0.005	0.998	0.002	0.941	0.038	0.672	0.122	0.196	0.106	0.165	0.081	0.121	0.052
95	0.040	0.005	0.999	0.001	0.952	0.033	0.699	0.116	0.201	0.104	0.168	0.079	0.121	0.049
100	0.037	0.004	1.000	0.000	0.960	0.028	0.724	0.111	0.205	0.103	0.170	0.077	0.121	0.047

[a]The body of the table consists of market shares for Technologies 2 and 3, rounded off to three decimal places.
[b]Technology 1 has the remaining market share so that the sum is equal to 1.
[c]The parameter r is the excess profit on investment above and beyond normal dividends and interest, expressed as a fraction per year.

synthetic problems solved in this report are *not* such a check.

Models with extra free parameters could also be tried. One that is suggested by the behavior of the price model, which exhibits market penetrations that are always in the (intuitively) correct direction, but that are slow, would be to multiply the right-hand side of Eq. (38) by a parameter s. To justify such a parameter, however, one would have to invent a new strategic principle.

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APPENDIX: An Exact Solution for a Special Case

Peterka has demonstrated that certain features of the solutions to his equations are quite insensitive to the values of the α_i used. From this observation, one derives some interest in the case where γ_i are replaced by constant values $\bar{\gamma}$. The situation is of even greater interest for the price model, as it is even more likely that the $1/d_i$ values will be close than it is that $1/\alpha_i$ will be close – at least for situations where substitution is slow.

If we replace γ_i by $\bar{\gamma}$, the model equations become

$$\dot{f}_i/f_i = \bar{\gamma} \sum_j f_j(d_j - d_i) \qquad \text{(A1)}$$

This set of equations has a solution in closed form:

$$f_i = \frac{c_i}{\Sigma_j\, c_j \exp[-\bar{\gamma}(d_j - d_i)t]} = \frac{c_i \exp(-\bar{\gamma} d_i t)}{\Sigma_j\, c_j \exp(-\bar{\gamma} d_j t)} \qquad \text{(A2)}$$

Equation (A2) applies for constant $\bar{\gamma}$, d_i, but it is even more generally

$$f_i = \frac{c_i \exp(-\int_0^t \bar{\gamma} d_i \mathrm{d}t')}{\Sigma_j\, c_j \exp(-\int_0^t \bar{\gamma} d_j \mathrm{d}t')} \qquad \text{(A3)}$$

when $\bar{\gamma}$ and the d_i vary with time. The c_i are determined, of course, by conditions at the reference time $t = 0$.

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