

# **Multiple Issue Action and Market Algorithms for the World Wide Web**

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## **Multiple Issue Auction and Market Algorithms for the World Wide Web**

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## **Abstract**

The Internet is quickly changing the way business-to-consumer and business-to-business commerce is conducted in the world. The Electronic Revolution has also spawned a trend of price wars and, in some instances, chaos because of the zero-sum nature of the electronic channel. The technology has created an opportunity to get beyond the lose-lose nature of single issue price wars by determining sellers' and buyers' preferences across multiple issues and encouraging negotiations, thereby creating possible joint gains for all parties. We develop simple multiple issue algorithms and heuristics that could be used in electronic auctions and electronic markets, to match businesses to businesses and consumers based on dovetailing underlying interests and preferences. We provide arguments that such dovetailed matches should help stabilize markets and make them more efficient.

**Keywords:** Decision Support; Negotiation Modeling; World-Wide-Web; Electronic Commerce, Computer-mediated Communication, Multiple Criteria Decision Making, Intelligent Agents, Bots.

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# Multiple Issue Auction and Market Algorithms for the World Wide Web

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## 1. Introduction

In order to put our study into perspective, we present a market framework adapted from Guttman and Maes (1998b) (Figure 1). The market framework consists of one or many buyers and one or many sellers. One buyer and one seller, assuming the current tendency towards non-fixed prices, defines a traditional negotiation. The negotiation may take place face-to-face or electronically. One seller and many buyers defines an auction which may be live or on-line. Many sellers and one buyer defines a reverse auction, an example being a government auction. Many sellers and many buyers defines a market or a double auction, which may be live or electronic. Live and electronic versions of stock exchanges exist. Guttman and Maes (1998b) also differentiate between traditional classified ad markets and traditional stock markets, the difference being that in traditional stock markets there is a centralized multilateral exchange compared to classified ad markets, where trading is ad hoc and bilateral. Centralized multilateral exchange markets may become classified ad markets and vice versa due to revolutionary changes made possible through the Internet and the expansion of the world-wide-web for electronic commerce. An example of a previously classified ad market, which is becoming an exchange market, is the US home mortgage market. Traditional retail markets may offer an opposite example. Such markets are becoming more one-to-one, exhibiting features similar to classified ad markets (e.g. Egghead and Dell Computer). Some traditional markets and auctions offer the possibility of trading both electronically or live. Even the traditional large stock exchanges are moving in the direction of partially electronic markets.

Most existing live and electronic auctions and markets mainly focus on a single issue, namely price of the merchandise or stock (Guttman and Maes, 1998b). There exists extensive literature demonstrating the detrimental effects of single issue, distributive (zero-sum) negotiations. Price wars are a concrete example, leading to a volatile market (Burdett and Coles, 1997; Kephart et al., 1998; Guttman, Moukas, and Maes, 1998; Anders, 1998). Exclusive focus on price will also do a disservice to buyers and sellers alike by hiding important value attributes from consideration. Following Guttman and



Maes (1998a), an explicit consideration of such multiple value attributes holds the promise of converting distributive negotiations into integrative negotiations. See also Kersten and Szpakowicz (1998).

|                |      | <b>BUYERS</b>   |         |
|----------------|------|-----------------|---------|
|                |      | ONE             | MANY    |
| <b>SELLERS</b> | ONE  | NEGOTIATION     | AUCTION |
|                | MANY | REVERSE AUCTION | MARKETS |

Figure 1: Market Framework

In this paper we develop simple, heuristic algorithms for multiple issue electronic markets and auctions, to match businesses with businesses and consumers based on dovetailing buyers' and sellers' underlying interests and preferences. We argue that such dovetailed matches should help stabilize markets and make them more efficient. To the best of our knowledge, there is very little literature on multiple issue markets and auctions whether electronic or live. See, for example, Kagel and Roth (1995, pp. 416-421). When discussing auctions, we focus on auctioning a single good having multiple negotiable attributes/issues or multiple quantities of homogeneous goods, such as stocks. When discussing markets, we focus on the case of multiple diametrically opposed issues as well as the case where quantity is a negotiable issue. A web site that runs the algorithms acts as the exchange mechanism.

The organization of the rest of the paper is as follows. In section 2 we review the relevant literature. In section 3 we discuss some theory of multiple issue markets and auctions; in section 4, auction algorithms and in section 5 market algorithms. Section 6 concludes the paper.

## 2. Literature Overview

We classify the literature into Electronic Negotiation Models, Automated Agents, Auctions, and Markets. The literature is vast and draws upon Economics, Finance, Information Sciences, Marketing, and Negotiation Science, among others. We focus on the recent electronic applications of auctions and markets and also provide some representative web site URLs. In many instances, the developments have been so rapid that the academic journals are lagging behind and many of the applications and publications can only be found on the WWW.

Different aspects of Electronic Commerce have been relatively well documented, although the research in the area is very active (Choi, Staahl and Whinston, 1997; Peterson et al., 1997; Baty and Lee, 1995; Lee and Clark, 1996; Elofson and Robinson, 1998; Barua et al., 1997). FastParts and GE's TPN provide good examples of business to business Electronic Commerce websites. See the reference list for URLs. The dramatic expansion and ease of Electronic Commerce will make electronic markets and auctions common. Obviously it is not unimportant how such electronic markets and auctions are conducted.

### *Electronic Negotiation Models*

Computer aided negotiation support models have been developed to provide analytical aid to negotiators, both as individuals and groups (see, e.g., Teich et al. 1995, 1996). Research in the area of negotiation modeling has been quite active within the last two decades. For a review of the literature see Teich, Wallenius and Wallenius (1994, 1998). Kersten's site is representative of the current state of the art in electronic negotiations. Kersten's site is among the first web-based negotiation support sites and includes the possibility for asynchronous negotiations (Kersten and Noronha, 1997). Segev's site provides an extensive list of references and links.

### *Automated Agents*

In recent years Artificial Intelligence researchers have created several software agents that aid in web browsing, searching, shopping, negotiating and other internet related tasks (see, e.g., Doorenbos, Etzioni, Weld, 1997; Sandholm and Lesser, 1995; Barbuceanu and Fox, 1997; Moukas and Zacharia, 1997; Oliver, 1997; Khoo et al., 1998; BotSpot). The web portal Excite's Jango (developed by Doorenbos, Etzioni and Weld), a shopping agent, is probably the biggest commercial success thus far of agent technology. This agent, and others, however concentrate on finding the lowest price among merchants offering a specific product or service, as discussed in the introduction. MIT Media Lab's forthcoming site T@T promises to be an exception, offering agent technology and Distributed Constraint Satisfaction Problem protocols to aid the multiple issue/attribute choice problem (Moukas, Guttman and Maes, 1998). Their earlier generation Kasbah site offered automated negotiation utilizing different negotiation strategies over a single issue, price. The first generation shopping agents are already being used in some of the auction sites (e.g. Bid4It and AuctionBot).

### *Auctions*

The most common types of auctions are the open English auction (ascending price), the open Dutch auction (descending price), the closed Sealed bid auctions (1st or 2nd price, the latter also known as Vickery auction). Other more complicated auctions are the Double auction (multiple buyers and sellers), Multi-unit auctions, such as the open English clock auction, the Combinatorial auction, which permits bids on groups of assets, and the Reverse or Procurement auction. Some of these auctions may be conducted as closed or as open auctions, simultaneous or sequential. There is extensive literature which discusses and tests these auctions using multiple performance measures, including revenue equivalence, the extent of price discrimination, and the efficiency of

auctions (Kagel and Roth, 1995; Smith, 1991; Rothkopf et al., 1995; McAfee and McMillan, 1987 and 1996; Milgrom, 1989; Rothkopf and Harstad, 1994; Ashenfelter, 1989; Feldman and Mehra, 1993a, 1993b; Hausch, 1986; Rothkopf et al., 1986; McCabe, Rassenti, Smith, 1990 and 1991a, 1991b; Cox, Smith and Walker, 1984; Engelbrecht-Wiggans, 1988; Tenorio, 1993; Roth's site includes links to literature). The auctions listed above are single-issue (price) auctions, with the exceptions of Multi-unit auctions (price and quantity) and Combinatorial auctions (groupings of assets).

Web-based electronic auctions have recently become very popular. For a review, see Wurman, Walsh, and Wellman (1998) and Wellman and Wurman (1998). As discussed by Schwartz (1998), hundreds of different types of electronic auctions exist. Design features include whether or not sellers specify reservation prices, whether there is automatic bidding (for example AuctionBot, Kasbah, Bid4it), whether bidders rate each other via blacklists, when the auctions close and rules regarding the closing, the type of merchandise (new, used) and the quantities offered (one or multiple), whether we have a regular or reverse/procurement auction (see Freemarkets and Priceline). Priceline is a type of reverse auction for unused capacity on airlines -- being extended for automobiles and home mortgages, where bidders (with a credit card commitment) "name their price" (Walker, Schneier, Jorasch, 1996).

### *Markets*

Our market framework differentiates between auctions and markets. Yet, in practice there is an overlapping area, specifically referred to in the auction literature as double auctions. Such double auctions serve many buyers and sellers. In fact, the continuous double auction is probably the oldest practiced type of market exchange of goods and stocks, where buyers and sellers post their bids/asks continuously and transactions occur when they overlap, resulting in price discrimination throughout the trading day. A popular competing market is known as the Call market (AKA single price auction), where the auctioneer/marketer balances the supply and the demand and determines a single price at which all goods are exchanged at that uniform price during the trading day. Matching is an important aspect to some markets, especially the more complicated ones (Roth and Oliveira, 1990; Mongell and Roth, 1991; Foner, 1997; Kallio and Salo, 1993).

Dissatisfaction with the high costs of using intermediaries when trading on organized exchanges has contributed to the development of electronic markets, either web-based or non web-based. These costs include direct costs in terms of commissions paid to brokers and indirect costs, such as market impact, where high volume trades result in higher/lower trading prices. In particular, large institutional investors are highly motivated to avoid the indirect costs of market impact. See, for example, Angel et al. (1997), Lupien and Rickard (1997), Schwartz (1991). This has led to the development of what are known as electronic fourth markets, such as Instinet and POSIT (Portfolio System for Institutional Trading), and the more recent web-based electronic markets. As examples of web-based electronic markets, see the Arizona Stock Exchange (a call market), band-X (a classified ad market for the exchange of bandwidth), FastParts (a market for electronic components), and GE's TPN (a market for components).

### Case of Optimark

Similar to Instinet and POSIT, Optimark is an electronic stock exchange market, developed for institutional traders. Even though the market is not web-based nor currently in operation (Optimark is cooperating with both the Pacific Exchange, set to begin Fall, 1998 and NASDAQ scheduled for 1999), we will discuss it more in-depth because it is the only multiple (2) issue market we are aware that exists. Those two issues being price of stock and quantity traded. Again, the motivation is to reduce the market impact of large institutional trades by encouraging traders to state preferences across ranges of price and size. These preferences are stated anonymously, hence eliminating the market impact phenomenon, yet still matching buyers and sellers. Such large traders may be willing to accept a higher or lower price than the current market price for such large volumes of trades. See Lupian and Rickard (1997) (IBM provides the patent at their site) and Optimark's site. In economic terms, they try to eliminate the shifts of demand or supply curves when new buyers/sellers enter or exit the market. Whether Optimark will be successful in eliminating the market impact, stabilizing markets and reducing price volatility remains to be seen.

We have reproduced three figures from Optimark's web site and explain their electronic market system based on those. See Figures 2, 3, 4. In Figure 2, one buyer's satisfaction density profile (based on the buyer's underlying value function) is exhibited.

Lupian and Rickard (1997) use the following notation. There are  $M$  buyers and  $N$  sellers. The size/price combinations are assigned a satisfaction value ranging from 0 to 1; a value of 0 indicates unwillingness to trade at that price/size combination; a value of 1 indicates the highest level of satisfaction. Only discrete levels of type .1, .2, .3 etc. are allowed, higher values indicating higher levels of preference for that price/size trade. Every trader is required to indicate his/her satisfaction profiles for each stock he/she wants to trade, using the 0 to 1 scale, however the actual detail and accuracy will vary from trader to trader. The satisfaction density profiles are defined as  $B_i(p,s)$  for buyer  $i$  and  $S_k(p,s)$  for seller  $k$  where  $p$  is price and  $s$  is quantity of stock.

Optimark then matches buyers and sellers based on a two-stage system. Figure 3 explains the aggregation procedure, which is the first stage. Starting with the size/price cells containing a value "1" (for both buyer and seller), the algorithm attempts to match traders by combining/aggregating smaller quantity traders to larger quantity traders at a single price. In the second stage, in Figure 4, for remaining buyer-seller combinations, a "mutual satisfaction density profile value" is calculated by multiplying the individual satisfaction density profiles. They define the mutual satisfaction density profile between the  $i$ th buyer and the  $k$ th seller to be

$$J_{ik}(p,s) = B_i(p,s)S_k(p,s), \quad i = 1, \dots, M; \quad k = 1, \dots, N.$$

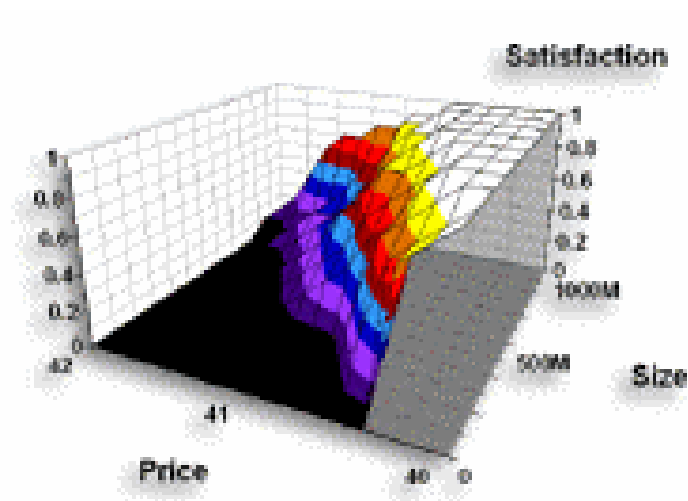


Figure 2 Optimark: Buyer's Satisfaction Density Profile

Source: [www.optimark.com](http://www.optimark.com)

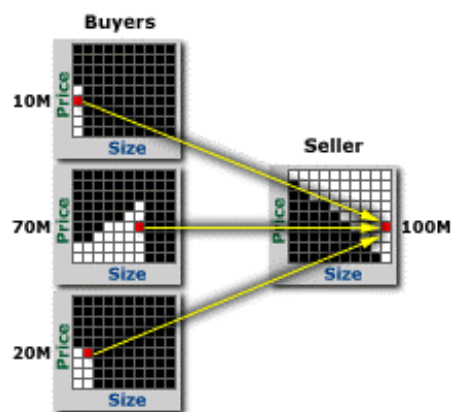


Figure 3 Optimark Aggregation Stage

Source: [www.optimark.com](http://www.optimark.com)

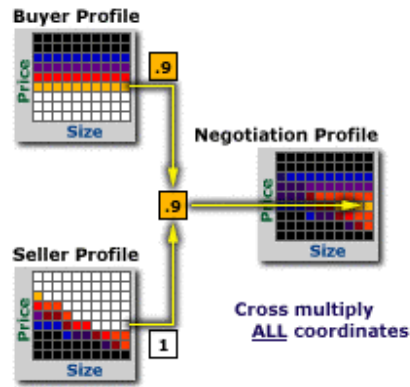


Figure 4 Optimark Cross Products in Stage 2

Source: [www.optimark.com](http://www.optimark.com)

The matching is based on the ranked list of MN cross products for all price/size cells, the maximum of which is basically a Nash Bargaining Solution. Optimark is planning to match buyers and sellers every 90 seconds. In case of ties, five rules exist to break them.<sup>1</sup>

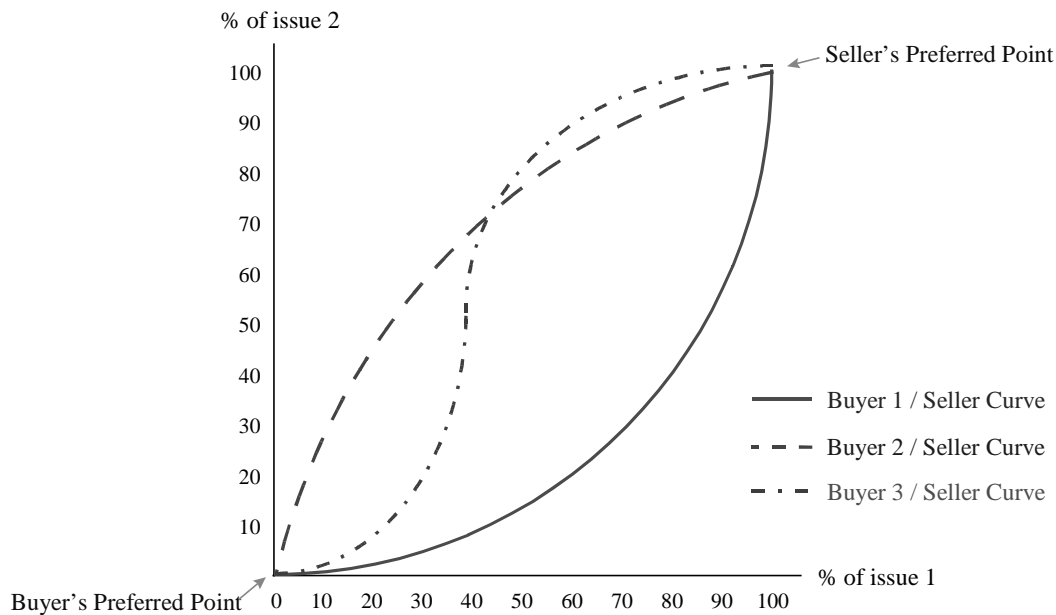
### 3. Multiple Issue Markets and Auctions: Some Theory

#### *Quantity is not an issue*

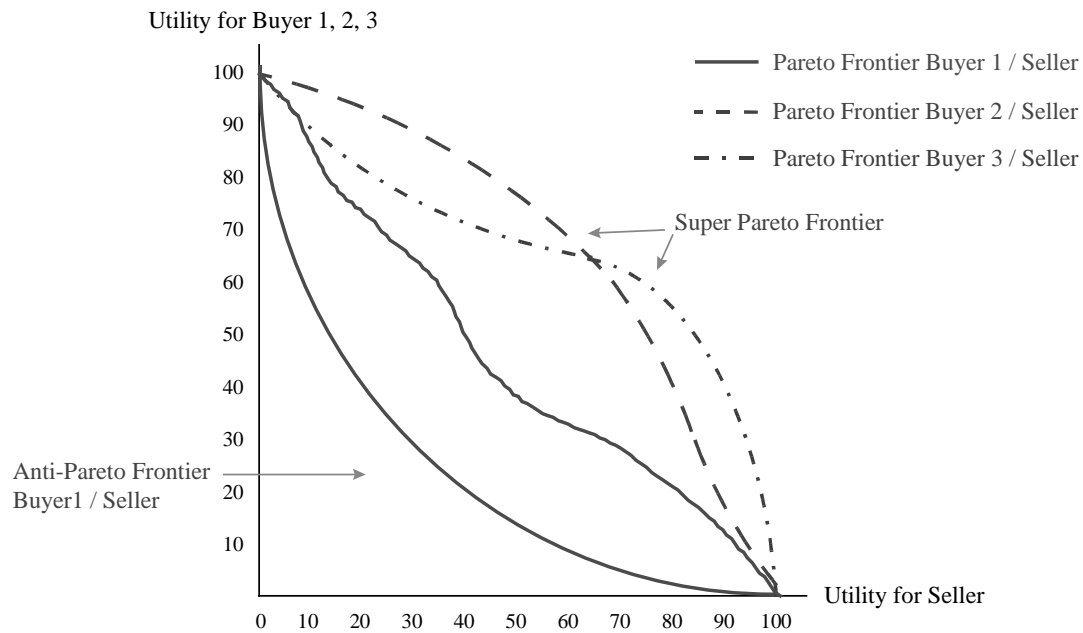
Figure 5a presents contract curves for a two-issue market/auction example, where there exists one seller and three potential buyers. When dealing with multiple issues in auctions and markets, in a diametrically opposed issue space, some matches of buyers and sellers make more sense than others because of dovetailing underlying values. If we map the contract curves from Figure 5a to the utility space, we obtain Figure 5b. The result is the three Pareto frontiers from which no joint gains are possible for that individual buyer/seller pair. The Anti-Pareto frontier is defined as the lower bound of the feasible region in the utility space. One such frontier for buyer1/seller, from which no joint losses are possible, is represented in the figure. Of course, Anti-Pareto frontiers exist for the other buyer/seller combinations as well, but are not depicted in the figure. Assuming all individual “value points” derived from the buyers’ and seller’s value functions were known as would be the case in experimental settings (computer simulations or human experiments), we could make the following argument. From the

<sup>1</sup> Ties are broken by a series of rules that correspond to conventional notions of fairness in trading. The priority for allocations is basically in the following (nested) order: 1) mutual preference value; 2) “standing” status of the profile pair; 3) “class” status of the profile pair; 4) times of entry of the profile pair; 5) trade size within the same pair of profiles; and 6) price, in favor of the earlier submitted profile, in the event all of the above are tied. “Standing” refers to the willingness of a trader to accept any partial fill out to some maximum amount, at a particular price. “Class” refers to the regulatory status of the trader (we use four classes in the U.S. system, i.e., book, agency, principal, and away market quote). Time of entry, size, and price are self-explanatory. (Source: Private communication from Optimark President, Dr. T. Rickard.)

seller's point of view it would make sense to match him with buyer 2 for lower levels of Issue 1 and Issue 2 and with buyer 3 for higher levels of Issue 1 and Issue 2. This is what we call the "Super Pareto Frontier" consisting of the most northeasterly segments of the combined contract curves. In reality, we would not know the value points and we need algorithms to match buyers and sellers based on underlying dovetailed interests. Even without the value points, and considering the problem of interpersonal comparison of utilities, the inward or outward bulging shape of the utility curves could facilitate the identification of good matches and the concession making process in the issue space. The reason being that for outward bulging utility curves, a large concession in issue space may result in a small decrement of utility. For inward bulging utility curves the opposite is true (see Mumpower, 1991). We are not advocating the automatic matching of Seller with Buyers on the Super Pareto frontier. We expect the seller to be individually rational and negotiate with the buyer with whom he/she can receive the highest utility, and that could be Buyer 1 in Figure 5b and hence a Super Pareto solution would not be attained. Therefore the concept of "good matches" and "dovetailing interests" may not always be useful in real situations because it depends on the trader's BATNAs (Best Alternative To a Negotiated Agreement) and power, among others



**Figure 5a. Contract Curves for a Multiple Issue Market Example:  
One Seller and Three Potential Buyers  
in a Diametrically Opposed Issue Space**

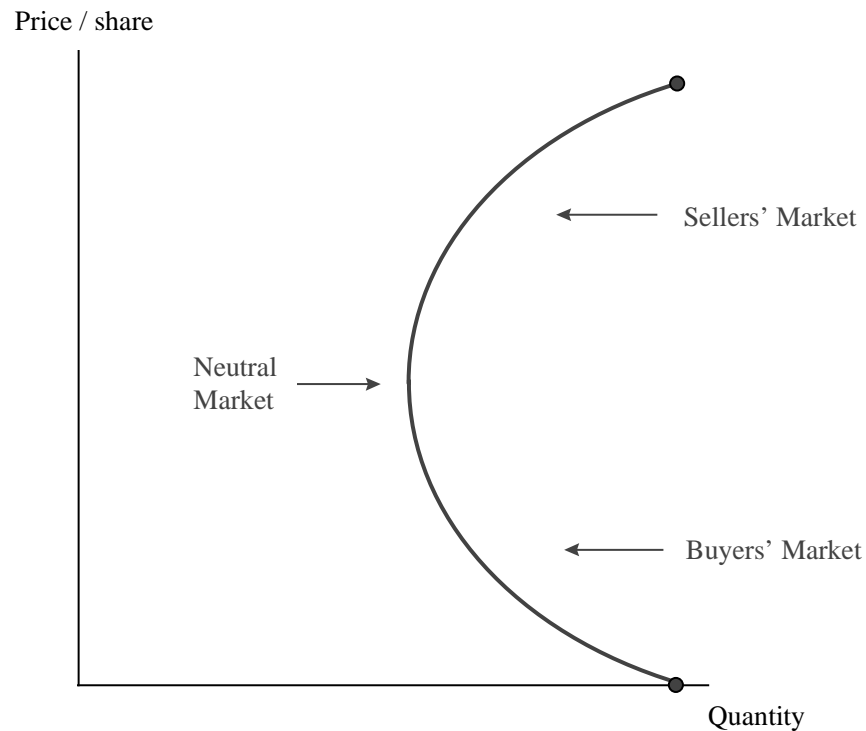


**Figure 5b. Super Pareto Frontier in Utility Space for Figure 5a Example**

### *Quantity is an issue*

In a multiple issue market, when quantity is an issue, value is a function of both quantity AND price, among others. The value functions of groups of sellers and buyers result in indifference curves in the multiple issue space. These can then be viewed as a type of Edgeworth Box forming a contract curve at the tangency points of the indifference curves. This concept is demonstrated in a two issue (price/quantity) example in Figure 6. A single point on the contract curve will be implicitly “negotiated” by market participants. Exactly where this negotiated point is on the curve is indeterminate, but will be converged upon (at least with an efficient market mechanism) by market forces. In a seller’s market, a point in the upper right-hand-side of the curve will be converged upon, in a buyer’s market, a point in the lower right. In a simulated market with defined value functions, a researcher could determine whether the resulting trade agreement lies on this contract curve (which would be Pareto Optimal) or off it.





**Figure 6. Contract Curve for Groups of Sellers and Buyers**

## 4. Multiple Issue Auction Algorithms

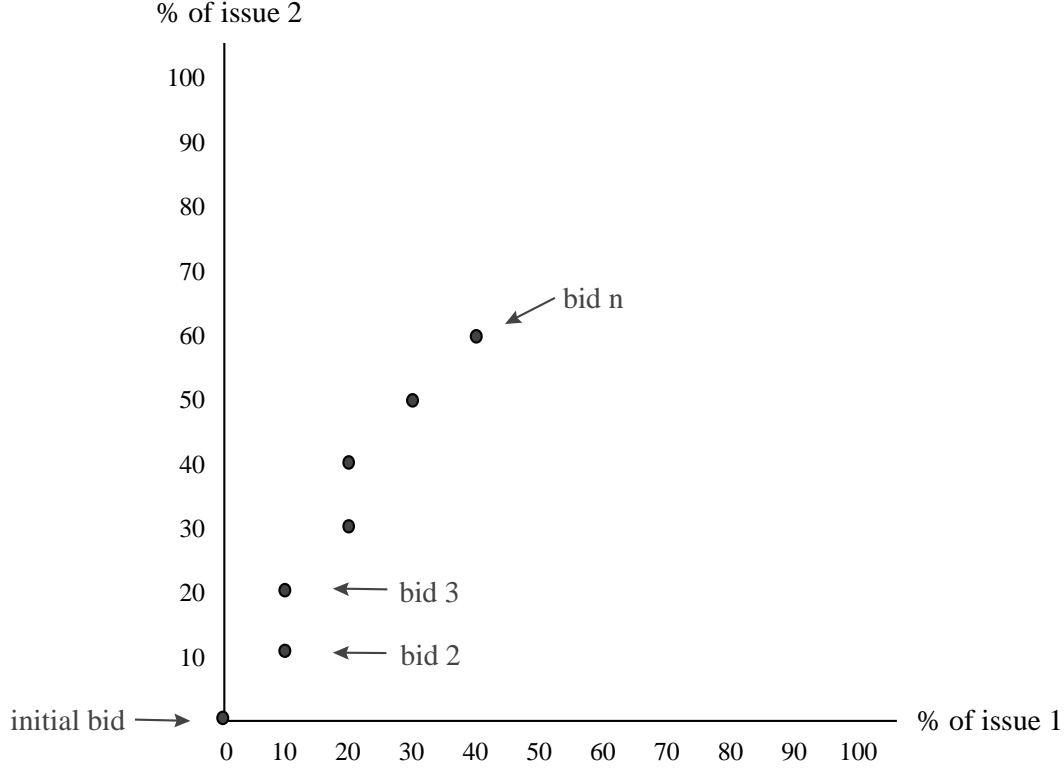
### *Quantity is not an issue*

Figure 7 illustrates the Leap Frog Method where bidders determine the path of bids. This is a natural extension of a typical single issue English auction where bidders shout out their bids. Each bid must be an improvement over the previous bid in at least one of the issues and no worse in any of the issues. We cannot advocate this method because the path is somewhat arbitrarily determined and it does not consider the preference of the auction maker at all.

In Figure 8, we describe an Auction Maker Controlled Bid Mechanism, which could be used in either a regular auction (single seller) or a reverse auction (single buyer). In Figure 8 we represent a preference path for a seller in a situation where the seller and the buyers are diametrically opposed in a two issue space. The preference path is determined as follows: the seller rank orders his/her most important jumps from his/her nadir (worst) point for issues that are "discretized". By discretized issues we mean

continuous issues that have been given a set of discrete levels. If an issue already has a set of discrete levels, we could use those. The ranking process continues until all levels of each issue have an associated rank. The ranks determine the preference path for the

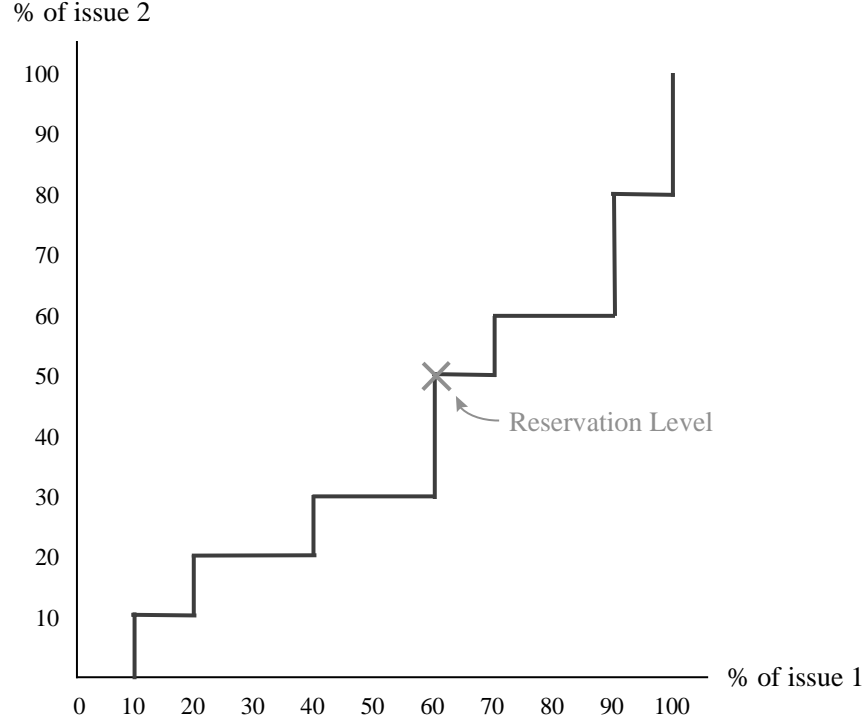
seller and the path then in which the bidders would follow. If desired, the auction maker could provide the auctioneer (or computer) with a reservation level below which a bid is not accepted. We anticipate this method will be preferred by the sellers, and the result will be more efficient than the result with the leap frog method.



**Figure 7. Leap Frog Method:  
Bidders Determine Path**

#### *Quantity is an issue*

We propose a discriminative auction for the multi-unit case in a computerized web environment. The advantage to the seller is that revenue should be maximized (although this needs to be verified), and the advantage to the buyer is that the bid required to enter the “action” is posted while keeping the actual bids sealed. We speculate bidders will appreciate the additional information. By allowing some degree of price discrimination, but less than in a typical discriminative auction, the “winners curse” effect can be reduced, thus encouraging active bidding.



**Figure 8. Auction Maker Specified Path**

Let us next introduce some terminology and notation. An **active bid** is one, which would be accepted at that price and quantity, if the auction were to close at that point in time. An **inactive bid** is one that has expired because it was outbid. A **semi-active bid** is one in which the bidder will only receive a partial quantity if the auction closed at that point in time. Our algorithm assumes that bidders will accept partial quantities, if their bid is semi-active.

$S_i = (p_i, q_i)$ , is the bid of bidder  $i$ , where  $p_i$  = per unit price for bidder  $i$ , and  $q_i$  = quantity desired by bidder  $i$ , ( $i = 1, \dots, n$ );

$n$  = number of bidders;

$D$  = number of units for sale;

$t$  = iteration counter;

$sp_t$  = suggested price at iteration  $t$ .

The steps of the multiple unit discriminative auction algorithm are as follows:

Step 1. Auction owner specifies quantity for sale  $D$ , reservation prices, the closing time of the auction, and the minimum increment in bids (epsilon).

Step 2. Bidder  $i$  enters auction by specifying a desired quantity  $q_i$ .

- Step 3. Bidders request a price. The suggested price  $sp_t$  is either at the reservation level if supply has not yet depleted, or at the previous level, or is calculated an epsilon amount above the previous price. The determination is based on whether total demand at that price can be met by the supply. If so, the price remains the same as previously; if not, it is increased by an epsilon amount.
- Step 4. Bidders submit their bid  $S_i$  either at the level suggested by the algorithm, or above that level, or they drop out. Bids below the suggested level are not accepted.
- Step 5. Bidders whose status changes are informed and requested to make a decision. Go to step 2. Repeat until auction closes.

The revenue, increasing at every iteration, is calculated as follows:

$$\text{Total Revenue} = \sum_{i=j+1}^n p_i q_i + q_{jr} p_j, \text{ where } j \text{ refers to the semi-active bidder; } j+1, \dots, n$$

are the active bidders. The residual quantity for semi-active bidder  $j$  is :

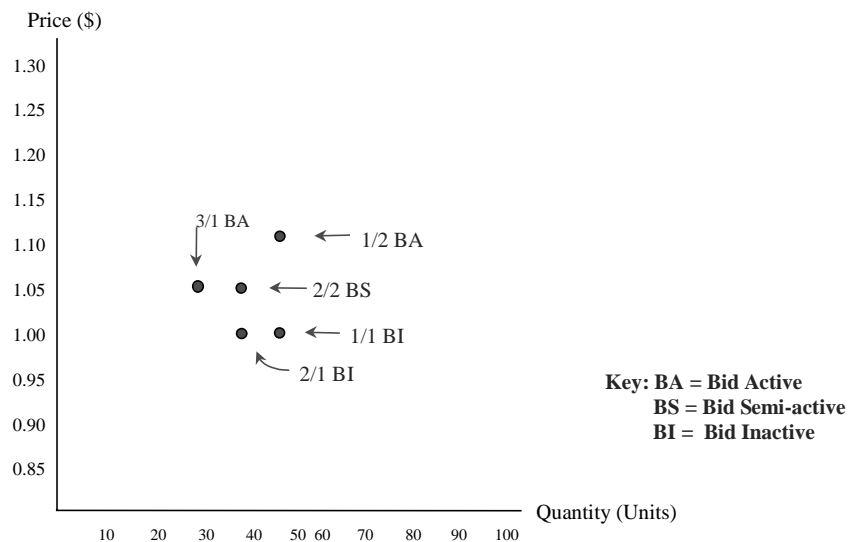
$$q_{jr} = D - \sum_{i=j+1}^n q_i \text{ iff } \sum_{i=j+1}^n q_i + q_j > D, \text{ otherwise } q_{jr} = 0.$$

As an example, assume a seller has 100 units of a homogenous good to auction, with a reservation price of \$1 per unit, and epsilon is 5%. At time 1, Bidder 1 enters the auction, specifies a quantity of 50 and requests a price from the auction mechanism. Since there are no other bids, the reservation price is suggested to the bidder. He makes his bid (1, 50) and it becomes active in status. At time 2, Bidder 2 enters, specifies a quantity of 40 and requests a price. Again the reservation price of \$1 is suggested, because the supply has not yet been depleted. Bidder 2 makes his bid (1, 40) and it becomes active in status. At time 3, bidder 3 enters the auction, and specifies a quantity of 30. At this point the supply is depleted and a new price must be calculated. The auction mechanism calculates a price an epsilon percentage (5%) above the latest price, and a price of \$1.05 is suggested to the bidder. He then makes his bid (1.05, 30). Bidder two then is outbid and thus becomes semi-active with a quantity of 20 units, because he was the last one to bid at the price of \$1. Bidder one remains active. At time 4, bidder two has three options. He can withdraw from the auction completely, stay semi-active in status, or re-bid. Assume he decides to re-bid at the same quantity of 40 units, and requests a price. The price \$1.05 is suggested by the mechanism because at that price the quantities of bidders 2 and 3 would be met by the supply. Bidder 2 then makes his bid (1.05, 40). Bidder 3 remains active in status and bidder 1 becomes semi-active with a quantity of 30 units. At time 5, bidder 1 has, again, three options, i.e. withdraw, remain semi-active or re-bid. Assume he decides to re-bid and requests a price. The mechanism then returns a price of \$1.1025 because at \$1.05 the demand is greater than the supply. Therefore the new price must be calculated at 5% above the most recent price suggested (of \$1.05). If he makes this bid (1.1025, 50), he will become active in status, bidder 3 will remain active, and bidder 2 will become semi-active with a quantity of 20. The process repeats until the auction closes. See Figure 9 for status and positions at the close and the following table for the sequence of the bids.

| time | bidder/bid# | q  | p      |
|------|-------------|----|--------|
| 1    | 1/1         | 50 | 1      |
| 2    | 2/1         | 40 | 1      |
| 3    | 3/1         | 30 | 1.05   |
| 4    | 2/2         | 40 | 1.05   |
| 5    | 1/2         | 50 | 1.1025 |

Prior to the auction, the seller has the right to set a reservation price. This reservation price could refer to the total minimum revenue generated from the auction, or he/she could use multiple reservation prices for different quantities. For example, he/she could specify that for quantities between 1 and 10 units the reservation price is, say, \$100 per unit, and for quantities above 10 the reservation price per unit is \$90.

In our algorithm, the price discrimination is reduced to an epsilon difference if the bidders accept the suggested bid. If, however they bid above the suggested bid, then, the price discrimination level could be higher. Why would a bidder be willing to pay above the suggested price level? This could happen if the bidder wants to decrease the probability of being outbid. If an automatic bidding mechanism is used, then the bidder could specify the top price to bid at his quantity, and the mechanism would automatically re-bid on his behalf up to that point. Beyond that point, he could specify a reduced quantity up to another level and so on.



**Figure 9. Price/Quantity Auction Example**

## 5. Multiple Issue Market Algorithms

### *Quantity is an issue*

Optimark's two issue market algorithm is novel and is gaining momentum among practitioners. Optimark's training institute has taught over 2000 traders to use their forthcoming system. It does have several appealing features: anonymity, possible elimination of market impact, preference elicitation over two issues, and the aggregation of small trades matched with larger quantities. However, we do have some criticisms towards the algorithm, which we wish to discuss. We also provide some suggestions for improving the preference elicitation and the matching.

Optimark allows price discrimination, that is trading same stocks at the same time at different prices (possibly opening arbitrage opportunities). Optimark accepts price discrimination, because it allows greater quantities to be traded -- a feature Optimark and apparently their customers see desirable. Since their system is 'black box', the level of price discrimination will go unreported. Hence the level of price discrimination could be larger with Optimark contrasted with traditional stock exchange trading. "Winners curse" is avoided because no one knows they have been "cursed". Ignorance may be bliss in this case.

We are also critical of the 0, .1, .2, ... , .9, 1 preference elicitation scale. Optimark explains that a 0 means unwillingness to trade and a 1 means total satisfaction with the trade. Scores in between differentiate between levels of preference. This seems to be too complicated to be done every 90 seconds, even though Optimark counters that traders can input preference scores as quick/dirty as they desire and yet still obtain desirable trades. We question why anyone would be willing to furnish preference scores between 0 and 1 with their system and are concerned that traders do not specify a unique single best '1' cell. Furthermore, some traders may be more skillful than others in strategically manipulating scores to gain advantage (arbitrage?) over other traders. Obviously values/preference scores cannot be compared across people. As discussed below, our suggestion is to use a cruder scale.

Even if traders shared the same preference scale and accurately portrayed it, ranking the cells based on the product of preference scores, is arbitrary. See Table 1, where we have reproduced the possible Optimark cross products. For example, the product of .6 and .6 is the same as .4 and .9. Nash and Optimark treat the cells as equally good. Subjectively, we would argue that .6 times .6 would be a better match (assuming that .6 means the same for both traders). Similarly, Optimark's matching algorithm would prefer .5 times 1 to .7 times .7. Again, we would argue that the latter would be a better match. For a criticism of the use of the Nash bargaining solution in negotiation literature, see Raiffa (1982).

In Tables 2 and 3 we have calculated cross products associated with two simple preference elicitation schemes. Table 2 is based on a 0-4 scheme and Table 3 on an even simpler 0-2 scheme. In the 0-4 scheme a trader will only specify a single '4' value, as well as at most a single '2' value.

### Table 1: Optimark Cross Product Scores

|                |      |      |      |      |      |      |      |      |      |      |      |      |
|----------------|------|------|------|------|------|------|------|------|------|------|------|------|
| Buyer Profile  | 1.00 | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 |
|                | 0.90 | 0.00 | 0.09 | 0.18 | 0.27 | 0.36 | 0.45 | 0.54 | 0.63 | 0.72 | 0.81 | 0.90 |
|                | 0.80 | 0.00 | 0.08 | 0.16 | 0.24 | 0.32 | 0.40 | 0.48 | 0.56 | 0.64 | 0.72 | 0.80 |
|                | 0.70 | 0.00 | 0.07 | 0.14 | 0.21 | 0.28 | 0.35 | 0.42 | 0.49 | 0.56 | 0.63 | 0.70 |
|                | 0.60 | 0.00 | 0.06 | 0.12 | 0.18 | 0.24 | 0.30 | 0.36 | 0.42 | 0.48 | 0.54 | 0.60 |
|                | 0.50 | 0.00 | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
|                | 0.40 | 0.00 | 0.04 | 0.08 | 0.12 | 0.16 | 0.20 | 0.24 | 0.28 | 0.32 | 0.36 | 0.40 |
|                | 0.30 | 0.00 | 0.03 | 0.06 | 0.09 | 0.12 | 0.15 | 0.18 | 0.21 | 0.24 | 0.27 | 0.30 |
|                | 0.20 | 0.00 | 0.02 | 0.04 | 0.06 | 0.08 | 0.10 | 0.12 | 0.14 | 0.16 | 0.18 | 0.20 |
|                | 0.10 | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 |
| 0.00           | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |      |
|                | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 |      |
| Seller Profile |      |      |      |      |      |      |      |      |      |      |      |      |

Likewise, in the 0-2 scheme a trader would specify only a single ‘2’ value. The 0-4 and the 0-2 scales are quick and dirty and fairly easy to specify. In both of these scales, the maximum of the minimum scores is the same as the maximum of the cross products, which is untrue in the original Optimark scheme, as demonstrated above. Raiffa (1996) argues that the max-min rule may be more fair than the max cross product rule. This perceived benefit comes with the additional cost of an increased number of ties, which must be resolved one way or another.

**TABLE 2. Modified 0-4 Point Scheme: Cross Products**

|                |          |          |          |          |          |          |
|----------------|----------|----------|----------|----------|----------|----------|
| Buyer Profile  | <b>4</b> | 0        | 4        | 8        | 12       | 16       |
|                | <b>3</b> | 0        | 3        | 6        | 9        | 12       |
|                | <b>2</b> | 0        | 2        | 4        | 6        | 8        |
|                | <b>1</b> | 0        | 1        | 2        | 3        | 4        |
|                | <b>0</b> | 0        | 0        | 0        | 0        | 0        |
|                |          | <b>0</b> | <b>1</b> | <b>2</b> | <b>3</b> | <b>4</b> |
| Seller Profile |          |          |          |          |          |          |

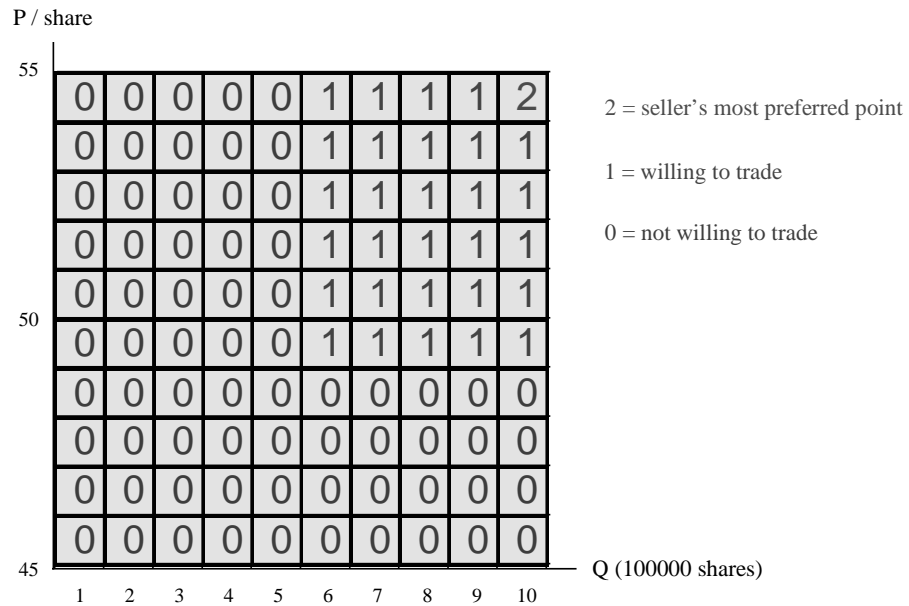
Key: 4 best of threes, 3 happy with trade, 2 best of ones, 1 willing to trade, 0 not willing to trade.

**TABLE 3. Modified 0-2 point scheme: Cross Products**

|               |                                       |          |          |          |
|---------------|---------------------------------------|----------|----------|----------|
|               | <b>2</b>                              | 0        | 2        | 4        |
| Buyer Profile | <b>1</b>                              | 0        | 1        | 2        |
|               | <b>0</b>                              | 0        | 0        | 0        |
|               |                                       | <b>0</b> | <b>1</b> | <b>2</b> |
|               | Seller Profile                        |          |          |          |
|               | Key: 2 Best, 1 will trade, 0 no trade |          |          |          |

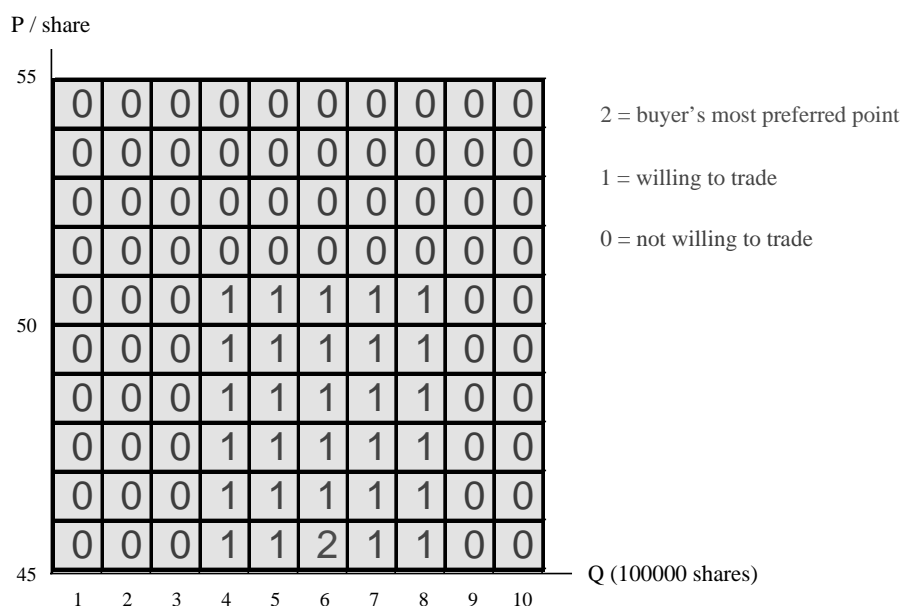
In both the 0-4 scheme and the 0-2 scheme, there are two phases in the matching process:

- phase 1: Find the cross products (the same as in OptiMark) and match the ranked list.
- phase 2: There will be ties, especially on the “one” cross products. We describe below three possible tie breaker rules. Our description is specifically tailored for the 0-2 scheme (see, for example, Figure 10a and 10b), however with slight modification they could also be applied to the 0-4 scheme.



**Figure 10a. Seller's Preference Profile  
in 2-1-0 Method Example**



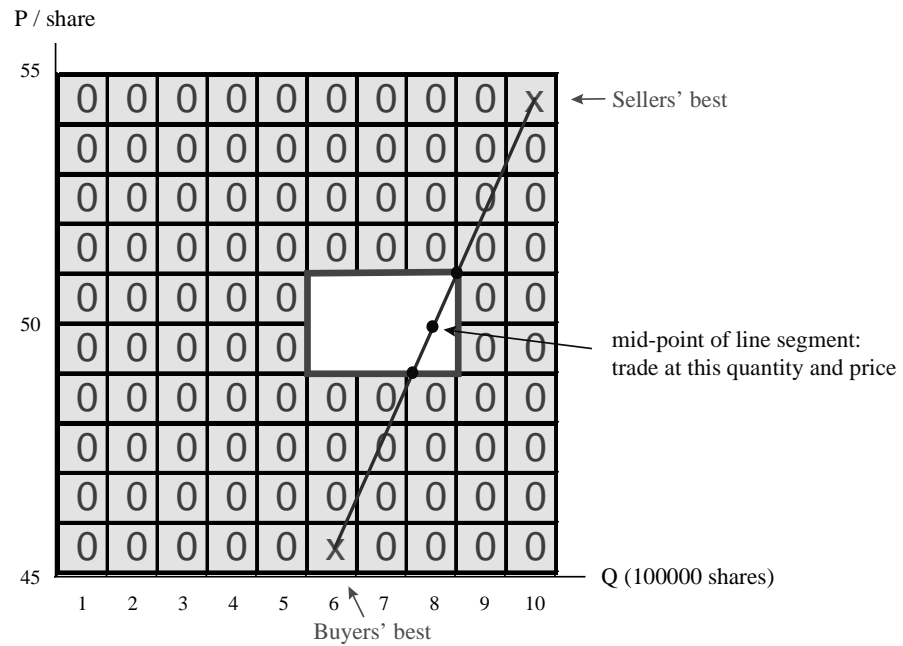


**Figure 10b. Buyer's Preference Profile in 2-1-0 Method Example**

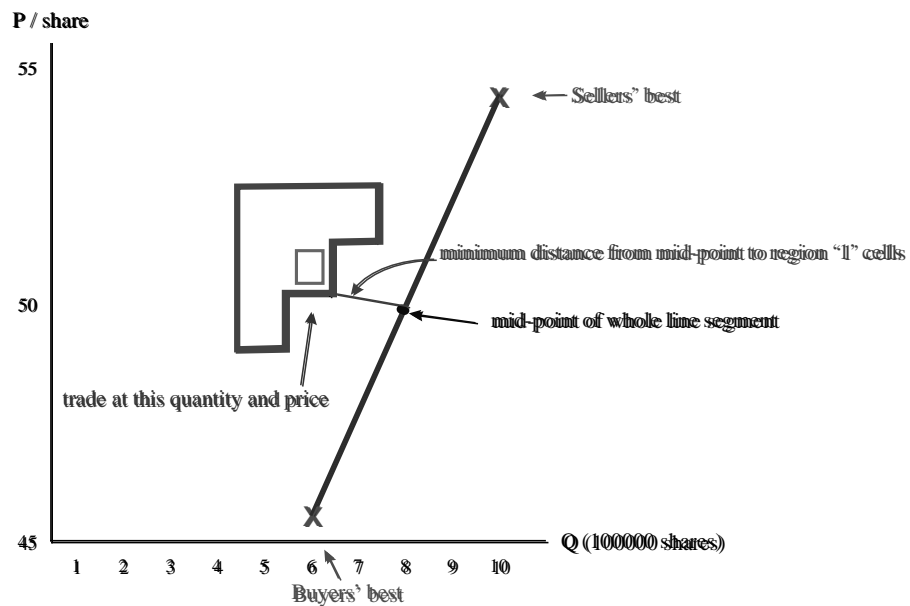
Idea A (B) tie breaker: Count the number of ties for each pair. Start matching based on the LARGEST (SMALLEST) number of ties (1s most likely). For each matching pair, draw a line between the 2s (most preferred points). Mark the tied region. See Figure 10c. If the line passes through the marked region, select the cell where the midpoint of the line segment passing through the marked region is located. If the line does not pass through the marked region, then select the cell that is closest to the overall midpoint of the line connecting the two best cells. See Figure 10d.

Idea C: Force the matches which simply maximize quantity of shares traded. The true maximum quantity would be computationally difficult to calculate with a large number of tied traders (a combinatorial/maximum flow network problem). Therefore, we suggest a greedy heuristic approach to approximate this maximum quantity. For all tied traders, first calculate the maximum quantity for each paired buyer/seller combination. Match the buyer/seller whose quantity is highest (in case of no unique price, split the difference in price), delete that pair and repeat until all feasible pairs have been matched.

Which of these ideas performs best awaits further testing and analysis as well as thorough comparison to the operation of Optimark's algorithm.



**Figure 10c. Cross Products of Buyer/Seller Profiles From Figures 10a and 10b: Line Crosses Region of '1s'**



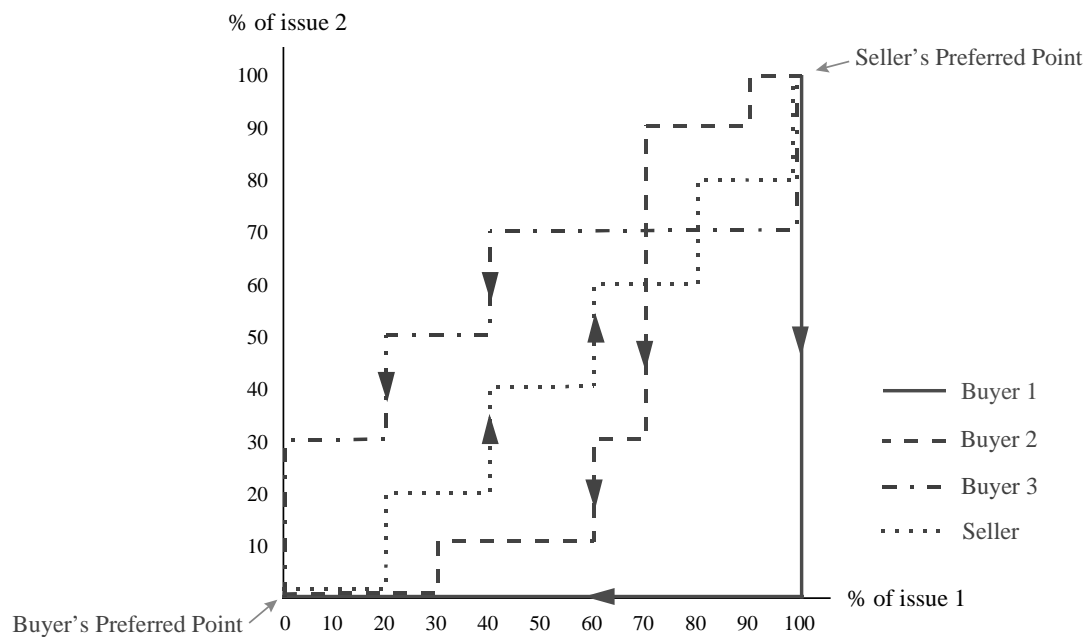
**Figure 10d. Cross Products of Buyer/Seller Profiles: Line Does Not Cross Region of '1s'**

### *Quantity is Not an Issue*

In Figure 11, we present the preference paths for three buyers and one seller who are diametrically opposed in a two issue space (see also Korhonen et al., 1995). As explained in the Auction Section above, the preference paths are determined by having each party rank order the most important jumps from their nadir (worst) point for issues that are "discretized". The ranks determine the preference path for each trader. The traders could specify their reservation levels on their preference path if desired. We match sellers and buyers based on their "closeness" of the path and suggest they negotiate with that party. If they have specified their reservation level on that path, we can check if there is an overlap and a possible agreement zone. If a match occurs, we inform the two parties and they close the deal.

A word of explanation regarding the definition of closeness and why closeness is desirable is in order. If a buyer's and seller's preference paths overlap completely, this implies that there is complete dovetailing interests. In other words, what one party desires most, the other party desires least. On the other extreme, if preference paths are completely divergent, then there is no dovetailing interests, meaning both parties desire the same things. Geometrically a simple measure of closeness of two preference paths is the area between the paths. In higher dimensions this concept is more difficult to operationalize. Instead we recommend that we base the measure of closeness on the distance between the coordinates of the points on the paths.

In case the participating sellers and buyers agree, we could automate the matching based on minimum distance and overlapping reservation levels and select a settlement point. One settlement option could be the midpoint of the overlapping reservation levels. If the sellers and buyers don't agree to automate, we would simply notify the participants of a close match and allow them to negotiate.



**Figure 11 Preference Paths for Four Parties in Figure 5a Example**

The closer the paths, the better the match. Of course, if reservation levels are specified and overlap, then the agreement zone exists. Even if a good match exists, reservation levels may not overlap. The reverse is also possible, in other words, bad matches may have overlapping reservation levels.

## 6. Concluding Discussion

In this paper we have presented and discussed several multiple issue auction and market algorithms. Much literature exists that discusses auctions and markets. However, very few mention multiple issue auctions and markets and few algorithms and procedures exist for such situations. To compare and contrast such algorithms in an experimental setting, a number of performance measures could be utilized. They mostly relate to the quantity or value of goods traded, and stability and efficiency of trades. Quantity and value of goods traded is self explanatory. One common measure of efficiency of markets is the percentage of the maximum possible gains from trade which is realized by the allocation process. It is computed as the sum total of consumer surplus and producer surplus divided by total possible sum. A traditional measure of stability is the Nash Equilibrium. Pareto Optimality of realized trades is of particular interest in multiple issue markets because of the potential of logrolling and generating joint gains.

Our future work includes experimentation with human subjects and computer simulation of various algorithms and their impact on markets and auctions under controlled experimental settings. We are in the process of implementing the algorithms to the web environment. The aim is to improve the performance of auctions and markets by matching consumers and producers based on their underlying preferences and dovetailing interests. Such matching will reduce the likelihood of damaging price wars and increase the satisfaction of the traders.

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