International Institute for
Applied Systems Analysis
IIASA www.iiasa.ac.at

Search Techniques for Multi-Objective Optimization of Mixed-Variable Systems Having Stochastic Responses

Walston, J.G.

IIASA Interim Report
May 2007

International Institute for
Applied Systems Analysis
Schlossplatz 1
A-2361 Laxenburg, Austria

Tel: +43 2236 807 342
Fax: +43 2236 71313
E-mail: publications@iiasa.ac.at
Web: www.iiasa.ac.at

Interim Report IR-07-014

Search Techniques for Multi-Objective Optimization of Mixed-Variable Systems Having Stochastic Responses

Jennifer G. Walston (jennifer.walston@afit.edu)

Approved by

Marek Makowski (marek@iiasa.ac.at)
Leader, Integrated Modeling Environment Project

May 2007

Foreword

The optimization techniques is a key tool for analyzing complex socio-economic, environmental and engineering systems. Increasing complexity of practical applications arise various complications in using standard optimization methods. As a consequence, new practical problems as a rule require appropriate adjustments and new developments of existing methods.

This report deals with a rather difficult stochastic optimization models of engineering design involving multiple competing objectives which can be estimated only by using Monte Carlo type simulations. This situation is typical for various integrated assessment models although the main goal of the report is to consider only the problem of conceptual aircraft design. Accordingly, references and methods of this report reflect only approaches primarily adopted in this field.

The essential feature of stochastic engineering design problems is their relatively small dimensionality in contrast to general problems of stochastic optimization arising in integrated socio-economic and environmental assessments. Using this fact, the author develops a novel approach combining specific global stochastic optimization procedures with multicriteria analysis.

This report documents the research the author advanced when she joined the Integrated Modeling Environment (IME) Project during the Young Scientists Summer Program (YSSP) 2006.

Yuri Ermoliev
Marek Makowski

Abstract

A method is proposed for solving stochastic multi-objective optimization problems. Such problems are typically encountered when one desires to optimize systems with multiple, often competing, objectives that do not have a closed form representation and must be estimated via simulation. A two-stage method is proposed that combines generalized pattern search/ranking and selection (GPS/R&S) and Mesh Adaptive Direct Search (MADS) developed for single-objective stochastic problems with three multi-objective methods: interactive techniques for the specification of aspiration/reservation levels, scalarization functions, and multi-objective ranking and selection. This combination is devised specifically so as to keep the desirable convergence properties of GPS/R&S and MADS while extending application to the multi-objective case.

Acknowledgments

I would to acknowledge the contributions of others to my research. I would like to thank the United States National Science Foundation and National Academy of Science for sponsoring my research and their suport of the YSSP program. Additionally, thanks to all the wonderful people at the International Institute for Applied Systems Analysis for their help and support this summer. The insights I gained and friends I made at YSSP will last a lifetime. Also, many thanks my advisor, Dr. James Chrissis, the United States Air Force and the Air Force Institute of Technology for allowing me to participate in YSSP. And last but not at all least, thanks and love to my husband and sons for being such cooperative and happy travelers.

About the Author

Jennifer Walston graduated in 1994 from the California Polytechnic State University, San Luis Obispo, California with a Bachelor of Science degree in Aeronautical Engineering and then accepted a commission in the United States Air Force. She returned to school and graduated in 1999 from the Air Force Institute of Technology (AFIT), Wright-Patterson Air Force Base, Ohio with a Master of Science degree in Operations Research specializing in Simulation. After again returning to AFIT, she is currently a third year Ph.D. student and will graduate in September 2007. The title of her dissertation is *Search Techniques for Multi-Objective Optimization of Mixed-Variable Systems Having Stochastic Responses.*

Jennifer currently holds the rank of Major and has served as a space systems suitability manager, enlisted personnel analyst, instructor, and academic evaluations division chief. Her main fields of scientific interest include optimization of systems with multiple objectives, particularly those for which no closed form objective functions exist and thus must be estimated via simulation. Following graduation, she will be serving as a logistics analyst.

Contents

1 Introduction **1**

1.1 Requirements Analysis . . . 1
1.2 Modeling Uncertainty . . . 2
1.3 Optimizing Multiple Objectives . . . 3
1.4 Optimizing Over a Non-Continuous Decision Space . . . 3
1.5 Problem Formulation . . . 4
1.6 Overview . . . 4

2 Existing Methods **5**

2.1 Stochastic Optimization Methods . . . 5
2.2 Multi-Objective Optimization Methods . . . 5
2.3 Stochastic Multi-Objective Optimization Methods . . . 6

3 Research Methodology **6**

3.1 Method Integration and Extension . . . 6
3.1.1 GPS/R&S . . . 6
3.1.2 Mesh Adaptive Direct Search . . . 7
3.1.3 Interactive Specification of Aspiration/Reservation Levels and Scalarization Functions . . . 9
3.1.4 Multi-Objective Ranking and Selection . . . 10
3.2 Considerations . . . 11
3.2.1 Convergence of Subproblems using GPS/R&S . . . 11
3.2.2 Convergence of Subproblems using MADS . . . 12
3.2.3 Quality of the Pareto Set Approximation . . . 13

4 Proposed Method **13**

4.1 Stage One . . . 13
4.1.1 Aspiration and Reservation Level Analysis . . . 13
4.1.2 GPS/R&S for Problems with Linear Constraints . . . 15
4.1.3 MADS for Problems with Non-Linear Constraints . . . 15
4.1.4 Adding Points to the Efficient Set . . . 15
4.1.5 Tests for Quality of the Pareto Set . . . 15
4.2 Stage Two . . . 15
4.2.1 Creation of the Mesh . . . 16
4.2.2 Multi-Objective Ranking and Selection . . . 16

5 Implementation of the Solution Methodology **16**

5.1 Testing . . . 17
5.1.1 Test Results . . . 17

6 Future Work **18**
6.1 Integrated Software 18
6.2 Automated Decision Agent 19
6.3 Extensive Testing 19
6.4 Engineering Design Optimization Application 19
6.5 Algorithm Termination Criteria 19

7 Conclusion **20**

List of Figures

1 The Mixed-variable GPS Ranking and Selection (MGPS-RS) algorithm Algorithm [10] 8
2 A General MADS Algorithm [49] 9
3 Graphical illustrations of functions used for analysis of Pareto optimal solutions 10
4 Stochastic Multi-objective Pattern Search (SMOPS) 14
5 Notional Approximated Pareto Set and Mesh for a Two-Objective Problem with Two Design Variables 16
6 Comparison of Initial Test Results to Published Solution 18

Search Techniques for Multi-Objective Optimization of Mixed-Variable Systems Having Stochastic Responses

*Jennifer G. Walston (jennifer.walston@afit.edu)** ***

1 Introduction

With the advent of advanced numerical techniques for analyzing complex engineering problems, engineers are seeking to integrate them into smart design processes. Thus, optimization techniques have become increasingly important in engineering design problems, specifically in the area of conceptual aircraft design [1]. However, complications arise when applying traditional optimization techniques to aircraft design. Such design problems typically contain multiple, often competing, objectives [2, 3, 4, 5]. Additionally, these objectives are often subject to measurement error or must be estimated with simulations [6, 7, 8, 2]. Thus, multi-objective and stochastic optimization techniques should be used.

1.1 Requirements Analysis

Because engineering design optimization and many other practical optimization applications are generally multi-objective and contain both stochastic elements and mixed variables, an optimization method capable of handling the following characteristics is desired.

1. **Stochastic.** Stochastic optimization algorithms able to determine the aforementioned Pareto frontier approximation for problems in which function evaluations are known to contain measurement error or must be estimated via simulation. Further, such an algorithm should be convergent to Pareto solutions thus guaranteeing a representation of the frontier in the region of interest can be found. However, as stated by Sriver [9], convergence for stochastic methods is usually given in terms of probability *e.g.*, with probability 1.

2. **Multi-Objective.** Algorithms capable of finding a reasonably accurate approximation of the Pareto frontier are desired for problems for which no preference information is explicitly known or even exists. However, in many engineering design problems, some information about desired performance goals, as well as minimum

*Operations Research Doctoral Candidate, Air Force Institute of Technology, Department of Operational Sciences (AFIT/ENS), Wright-Patterson Air Force Base, Ohio, USA.

**Disclaimer: The views expressed in this article are those of the author and do not reflect the official policy of the United States Air Force, Department of Defense, or the U.S. Government.

acceptable performance, does in fact exist. These so-called aspiration and reservation levels respectively can be used to determine a region of interest. Thus, in this research, this type of preference information is assumed to exist.

3. **General Purpose.** An algorithm applicable to a wide range of problems, with any combination of variable types, is desired. Additionally, such algorithms should be indifferent or robust to the source of the function evaluations, *i.e.* the algorithm is able to treat function evaluations as a "black-box."

4. **Efficient.** To be practical and useful for real-world design problems, an algorithm should perform well with respect to the number of function evaluations required. In many design applications, such function evaluations will be obtained via costly simulation runs and should therefore be used as parsimoniously as possible.

Therefore, the purpose of this research is to develop a general-purpose class of methods for solving multi-objective, stochastic optimization problems that apply to the mixed-variable case and are indifferent to the source of function evaluations. Such methods should be provably convergent to Pareto optimal solutions in the region of interest. Each of these desired properties presents specific challenges (discussed in sections 1.2, 1.3, and 1.4). In response to these unique challenges, a method is proposed that would extend the applicability of generalized pattern search with ranking and selection—developed by Torczon and later extended by Audet, Dennis, Abramson, and Sriver [10]—and Mesh Adaptive Direct Search—developed by Abramson, Audet, and Dennis [11]—to multi-objective problems through the use of interactive specification of aspiration/reservation levels [12], scalarization functions [13], and multi-objective ranking and selection methods [14].

1.2 Modeling Uncertainty

Given a classical optimization problem, a myriad of classic solution methods are available to the analyst (linear programming, steepest descent, etc.). But if parameters of the optimization problem are random, what changes to classical optimization must the analyst make in order to smartly optimize this random system? The answer to this question is class of solution methods called stochastic optimization and, as the name implies, deals with systems in which one or more of the problem components—objective function(s) and/or constraints—contain a random element. Similar to classic statistical modeling in that one seeks to model a function by repeated sampling of a population; however, in this case, the function is dependent not only on a random element, but also on controllable design variables.

Such a problem can be formulated as shown in equations (1a)-(1d) where x and ω represent the controllable design variables and random environment-determining variables, respectively. Thus, the task becomes finding x so that equation (1a) is minimized in a certain sense over all feasible x and all possible values of ω. Notions of feasibility and optimality for stochastic systems are highly dependent on the specific problem under study and must be precisely defined [15].

In this research, it is assumed that all constraints are deterministic and that the systems under study are those in which the objective function value cannot be explicitly evaluated and must be estimated through some kind of simulation. Here *simulation* refers to a generic numerical method by which input (control) variables are used to produce an output measure of interest (response) [16, 10]. Therefore, in this *simulation-based optimization*, the observed system response $F(x, \omega)$ is a function of both the design variables and the random error associated with the simulation model. The problem is to minimize, in a certain sense [15]

$$F(x, \omega) \tag{1a}$$

subject to

$$g_i(x, \omega) \leq 0, \quad i \in \{1, \ldots, M\} \tag{1b}$$

$$x \in (\Re^{n_1}) \tag{1c}$$

$$\omega \in (\Re^{n_2}) \tag{1d}$$

For example, for simulation-based optimization, it is typical to replace the general form of the stochastic objective and constraint functions given in (1) with their mathematical expectations [15]. With this convention the observed response can be represented by $F(x, \omega) = f(x) + \varepsilon(x)$ where f is the deterministic, "true" objective function and $\varepsilon(x)$ is the random error function associated with the simulation where $E[\varepsilon(x)] = 0$.

1.3 Optimizing Multiple Objectives

In addition to the complexity of design optimization problems due to the stochastic element, often there exists no single criterion for choosing the *best* solution. In fact, even the notion of "best" is not defined when multiple objectives are present because improvement to one objective actually degrades the performance of another. Such objectives are called competing objectives and are the motivation for the study of multi-objective optimization; if the objectives were independent, they could be collapsed into a single objective and classic solution techniques would apply [17]. The multi-objective optimization problem requires an order relation (or dominance relation) between potential solutions [18]. Though many different dominance relations have been proposed [18, 19], consider the notion of *Pareto dominance* or *Pareto optimality*. To find a Pareto optimal solution is to find a solution corresponding to an objective function vector, to which no other solution is superior in all objectives [20, 21]. There are several equivalent definitions of Pareto optimal solutions—also called non-dominated or efficient solutions—in the literature. For the purpose of this research, consider *Definition 1.1* [19].

Definition 1.1. *A solution to a multi-objective optimization problem of the form* $\min_{x \in \Theta} F(x)$, $F(x) : (\Re^{n^c} \times \mathbb{Z}^{n^d}) \to \Re^J$ *is said to Pareto Optimal at the point* $\hat{x}$ *if there is no* $x \in \Theta$ *such that* $F_k(x) \leq F_k(\hat{x})$ *for* $k = 1, \ldots, J$ *and* $F_i(x) < F_i(\hat{x})$ *for some* $i \in \{1, \ldots, J\}$ *[19].*

1.4 Optimizing Over a Non-Continuous Decision Space

The complexity of these problems is further increased when the decision space is non-continuous, *i.e.* there exist decision variables that are either discrete (e.g. integer valued) or categorical. Categorical variables are those which can only take on predetermined values that do not necessarily have an ordinal relationship to each other. However, categorical variables can be mapped to discrete-numeric values, thus these two types of variables are grouped and considered as a single variable type.

This characteristic of the decision space as non-continuous is common in engineering design problems. For example, in the design of aircraft, certain design variables are non-continuous. The number of engines is integer valued and the type (turboprop, turbofan, etc.) and placement (wing, aft, or combination) of the engine are categorical. Other non-continuous variables include airfoil type, wing configuration, and cabin layout. The class of optimization problems that contains continuous, discrete-numeric, and categorical variables is known as *mixed variable programming* (MVP) problems [10, 22].

In this research, the mixed variables are included as follows. The decision space is partitioned into continuous and discrete variables, Ω^c and Ω^d respectively, where the discrete variables may include categorical variables as described previously. By further mapping the discrete values to the integers, the discrete part of the decision space can be represented as a subset of the integers, *i.e.* $\Omega^d \subseteq \mathbb{Z}^{n^d}$ where n^d is the dimension of the discrete partition. A solution $x \in \Omega$ is denoted as $x = (x^c, x^d)$ where $x^c \in \Re^{n^c}$ and $x^d \in \mathbb{Z}^{n^d}$ and $n = n^c + n^d$ is the dimension of the decision space [10][22].

1.5 Problem Formulation

Thus, with the inclusion of stochastic and multi-objective elements to the classic optimization problem formulation, the problem can be formulated as:

$$\min E[F(x,\omega)] = E[f(x) + \varepsilon(x)] \tag{2a}$$

subject to

$$g_i(x) \leq 0, \quad i \in \{1, \dots, M\} \tag{2b}$$

$$x \in (\Re^{n^c} \times \mathbb{Z}^{n^d}) \tag{2c}$$

$$f(x) : (\Re^{n^c} \times \mathbb{Z}^{n^d}) \to \Re^J \tag{2d}$$

1.6 Overview

The remainder of this paper is organized as follows. Section 2 briefly reviews several existing solution methods for both stochastic and multi-objective problems, as well as the few that exist for problems with both characteristics. Section 3 presents specific elements of the proposed methodology before the actual methodology is outlined in section 4. Results of a prototype algorithmic implementation of the method tested against a multi-objective test problem with known results are presented in section 5. Finally, section 6 suggests areas for further research.

2 Existing Methods

Though much research has been conducted in the individual areas of stochastic, multi-objective, and mixed-variable programming, no single method encapsulates all of the characteristics listed in section 1.5. Typically, solution methods are either applicable to stochastic optimization or multi-objective optimization.

2.1 Stochastic Optimization Methods

Various solution techniques exist for stochastic optimization [15]. When a closed form of the objective function is known, the optimization problem can be solved via standard non-linear programming techniques like steepest descent, gradient projection methods, and linearization methods. But if the closed form is not known, or it is difficult to obtain values for the pdf, other methods must be used. Types of solution methods that estimate the value of the objective function using simulation are aptly called simulation-based optimization. Simulation, though not an optimization technique per se, can be used in conjunction with numerical methods of optimization to solve difficult optimization problems [15, 23]. Typical methods to solve simulation-based optimization problems include response surface methodology [24, 10, 23]; the gradient-based finite difference stochastic approximation [25, 26, 27, 28], stochastic quasi-gradient methods [15], and simultaneous perturbation [29, 23]; gradient-free methods (also called direct search methods [9]) like the pattern search method of Hooke and Jeeves [30], the Nelder-Mead method (also called the downhill simplex method and flexible polygon search), [23, 31, 32, 33, 34], and generalized pattern search (GPS) [35]; and discrete optimization techniques like ranking and selection techniques [36, 37, 23], meta-heuristics like simulated annealing [38, 39, 29], genetic algorithms [38, 29, 39], and tabu search [40, 41, 39].

2.2 Multi-Objective Optimization Methods

Similarly, many solution methods also exist for multi-objective optimization. These methods can also be sorted into 3 families: *a priori* methods, progressive methods, and *a posteriori* methods as well as into five sets: scalar methods, interactive methods, fuzzy methods, methods that use a meta-heuristic, and decision aid methods. The scalar methods attempt to transform the multi-objective problem into a single-objective one so that classic optimization techniques can then be used. Many examples exist including the weighted sum of the objective functions method, Keeney-Raiffa method, distance-to-a-reference-point method, and the lexicographic method [42]. Interactive methods, like the surrogate-worth tradeoff method and the aspiration/reservation-based methods [12], belong to the progressive methods family and thus allow the decision maker to tune preferences with regard to tradeoffs as the methods progress [42]. The fuzzy methods, like the Sakawa and Reardon methods, allow the modeler to deal with uncertainty and the imprecision of human knowledge by allowing a progressive transition between states via the membership function [42]. The meta-heuristics used for multi-objective optimization are typically adapted versions of those used for single objective problems. Examples include genetic algorithms [43], simulated annealing [38], and scatter search [44]. Decision aid methods, like the ELECTRE and PROMETHEE methods, are different from the other approaches in that they set up an order relation between a given set of discrete alternatives and thus provide a ranking (order) of solutions with respect to a set of criteria.

2.3 Stochastic Multi-Objective Optimization Methods

The few exceptions that apply to both types are limited in their applicability by simplifying assumptions. Specifically, three methods were proposed by Baba and Morimoto for the solution of multi-objective programming problems subject to noise: learning automata, random optimization, and stochastic approximation [45, 46]. Learning automata is a reinforcement (or feedback) learning scheme where actions by the automaton produce results for which either a reward or punishment result. The feedback then changes the probability of choosing that action. Baba and Morimoto show that an appropriately chosen learning scheme ensures convergence to a ”reasonable solution” for a finite number of candidate solutions [45]. Additionally, they showed that a random optimization algorithm ensures convergence to the Pareto-Optimal solutions. However, it is ensured to converge only under strict assumptions on the decision space, solution space, and error. They suggest further study to find a less restrictive result. Finally, Baba and Morimoto propose a stochastic quasigradient method to solve the stochastic multi-objective optimization problem. Under assumptions of continuity, compactness, and bounded error, they show that the algorithm converges with probability one to the global solution [46].

3 Research Methodology

3.1 Method Integration and Extension

Given the number of previous methods for either multi-objective or stochastic optimization that exist, it is reasonable to hypothesize that an appropriate combination of methods exists to address stochastic multi-objective optimization problems with mixed variables. Particularly, consider the following observations. Generalized Pattern Search with Ranking and Selection (GPS/R&S) has been successfully developed for single objective, stochastic, linearly constrained problems and has been applied to a multi-echelon repair system [47]. Additionally, GPS/R&S has been further extended (or generalized) to include problems that are not linearly constrained. The extended method is called Mesh Adaptive Direct Search. However, GPS/R&S and MADS in their current forms apply to only single objective problems. Alternatively, interactive techniques using aspiration/reservation levels and scalarization functions have been used successfully to find Pareto optimal solutions to deterministic multi-objective problems [12]. Finally, a multi-objective ranking and selection technique called multi-objective optimal computing budget allocation (MOCBA), developed by Lee et al. [14] has been applied to selecting the non-dominated set of inventory policies for aircraft maintenance, a discrete variable problem [48]. A brief description of each method follows.

3.1.1 GPS/R&S

Pattern search algorithms are defined through a finite set of directions used at each iteration. The direction set and a step length parameter are used to construct a discrete set of points, or mesh, around the current iterate. The mesh at iteration k is defined to be

$$M_k = \bigcup_{x \in S_k} \{x + \Delta_k^m Dz : z \in \mathbb{N}^{n_D}\}, \tag{3}$$

where S_k is the set of points where the objective function f has been evaluated by the start of iteration k, Δ_k^m is called the *mesh size parameter*, and D is a positive set that spans $\mathbb{R}^n$. An additional restriction on D is that each direction $d \in D$, $j = 1, 2, \ldots, n_D$, must be the product of some fixed nonsingular generating matrix $G \in \mathbb{R}^{n \times n}$ by an integer vector $z_j \in \mathbb{Z}^n$ [49]. A finite set of trial points called the *poll set* are then chosen from the mesh, evaluated, and compared to the incumbent solution. If improvement is found, the incumbent is replaced and the mesh is retained or coarsened via the mesh size parameter Δ_k^m. If not, the mesh is refined and a new set of trial points is selected. Initially developed by Torczon, GPS was extended by Audet and Dennis to include non-linear constraints and then by Abramson to the mixed-variable case.

The GPS framework, in conjunction with ranking and selection, was used by Sriver to address the random response case. In this case, the poll set at each iteration is given by $P_k(x_k) \bigcup \mathcal{N}(x_k)$ where $\mathcal{N}(x_k)$ is a user-defined set of discrete neighbors around x_k and

$$P_k = \left\{x_k + \Delta_k(d, 0) : d \in D_k^i\right\}, \tag{4}$$

where $(d, 0)$ denotes that continuous variables have been partitioned and that the discrete variables remain unchanged. A generic indifference-zone ranking and selection procedure $RS(P_k, \alpha, \delta)$, with *indifference-zone parameter* δ and *significance level* α, is used to select among points in the poll set for improved solutions, *i.e.* δ-near-best mean. If no improvement can be found, an extended poll step is conducted to search amongst the discrete neighbors of points in the poll set. Sriver showed that this solution algorithm has an iteration subsequence with almost sure convergence to a stationary point "appropriately defined" in the mixed-variable domain [10]. The mixed-variable GPS/R&S Algorithm is shown in figure 1.

3.1.2 Mesh Adaptive Direct Search

Mesh Adaptive Direct Search (MADS) is a class of algorithms developed by Audet and Dennis for minimization of nonsmooth functions of the type $f : \mathbb{R}^n \leftarrow \mathbb{R} \bigcup \{+\infty\}$ under general constraints $x \in \Omega \neq \emptyset \subseteq \mathbb{R}^n$. The feasible region Ω may be defined by blackbox constraints, *e.g.* computer code that returns a yes/no answer to wether a trial point is feasible [49]. Thus, this class of algorithms is applicable to a wider range of problems than GPS/R&S, such as non-linearly constrained problems.

MADS is similar to GPS/R&S in the generation of the Mesh and Poll sets (see equations 3 and 4 in section 3.1.1). However, though similar, the key difference is in the generation of the poll set and the poll step. In MADS a separate *poll size parameter* Δ_p^k is introduced which controls the magnitude of the distance between the incumbent solution and trial points generated for the poll step. In GPS, only one value $\Delta_k = \Delta_k^p = \Delta_k^m$ is used. In the poll step of MADS, the MADS frame (analogous to the poll set in GPS) is defined to be

$$P_k = \{x_k + \Delta_k^m d : d \in D_k\} \subset M_k \tag{5}$$

- INITIALIZATION: Let $X_0 \in \Omega$, $\Delta_0 > 0$, $\xi > 0$, $\alpha_0 \in (0,1)$ and $\delta_0 > 0$. Set the iteration and R&S counters $k = 0$ and $r = 0$ respectively.

- POLL STEP: Set extended poll trigger $\xi_k \geq \xi$. Use R&S procedure $RS(P_k(X_k) \bigcup \mathcal{N}(X_k), \alpha_r, \delta_r)$ to return the estimated best solution $\hat{Y}$. Update $\alpha_{r+1} < \alpha_r$, $\delta_{r+1} < \delta_r$, and $r = r + 1$. If $\hat{Y} \neq X_k$, the step is *successful*, update $X_{k+1} = \hat{Y}$, $\Delta_{k+1} \geq \Delta_k$, and $k = k + 1$ and return to POLL STEP. Otherwise, proceed to EXTENDED POLL STEP.

- EXTENDED POLL STEP: For each discrete neighbor $Y \in \mathcal{N}(X_k)$ that satisfies the extended poll trigger condition $\overline{F}(Y) < \overline{F}(X_k) + \xi_k$, set $j = 1$ and $Y_k^j = Y$ and do the following.

  - Use R&S procedure $RS(P_k(Y_k^j), \alpha_r, \delta_r)$ to return the estimated best solution $\hat{Y}$. Update $\alpha_{r+1} < \alpha_r$, $\delta_{r+1} < \delta_r$, and $r = r + 1$. If $\hat{Y} \neq Y_k^j$, set $Y_k^{j+1} = \hat{Y}$ and $j = j + 1$ and repeat this step. Otherwise, set $Z_k = Y_k^j$ and go to the next step.
  - Use R&S procedure $RS(X_k \bigcup Z_k), \alpha_r, \delta_r)$ to return the estimated best solution $\hat{Y}$. Update $\alpha_{r+1} < \alpha_r$, $\delta_{r+1} < \delta_r$, and $r = r + 1$. If $\hat{Y} = Z_k$, the step is successful, update $X_{k+1} = \hat{Y}$, $\Delta_{k+1} \geq \Delta_k$, and $k = k + 1$ and return to the POLL STEP. Otherwise, repeat the EXTENDED POLL STEP for another discrete neighbor that satisfies the extended poll trigger condition. If no such discrete neighbors remain in $\mathcal{N}(X_k)$, set $X_{k+1} = X_k$, $\Delta_{k+1} < \Delta_k$, and $k = k + 1$ and return to the POLL STEP.

Figure 1: The Mixed-variable GPS Ranking and Selection (MGPS-RS) algorithm Algorithm [10]

- INITIALIZATION: Let $x_0 \in \Omega$, $\Delta_0^m \leq \Delta_0^p$, D, G, τ, w^-, and w^+ satisfy the requirements of a MADS frame set given in Definition 2.2 of [49]. Set the iteration counter $k \leftarrow 0$.
- SEARCH AND POLL STEP: Perform the SEARCH and possibly the POLL steps (or part of them) until an improved mesh point x_{k+1} is found on the mesh M_k (where M_k is defined as for GPS in equation 3 in section 3.1.1).
  - OPTIONAL SEARCH: Evaluate f_Ω on a finite subset of trial points on the mesh M_k.
  - LOCAL POLL: Evaluate f_Ω on the frame P_k (where P_k is as given in equation 4 in section 3.1.1).
- PARAMETER UPDATE: Update Δ_{k+1}^m according to definition 2.1 of [49] and Δ_{k+1}^p so that definition 2.2 of [49] is satisfied. Set $k \leftarrow k+1$ and go back to the SEARCH AND POLL step.

Figure 2: A General MADS Algorithm [49]

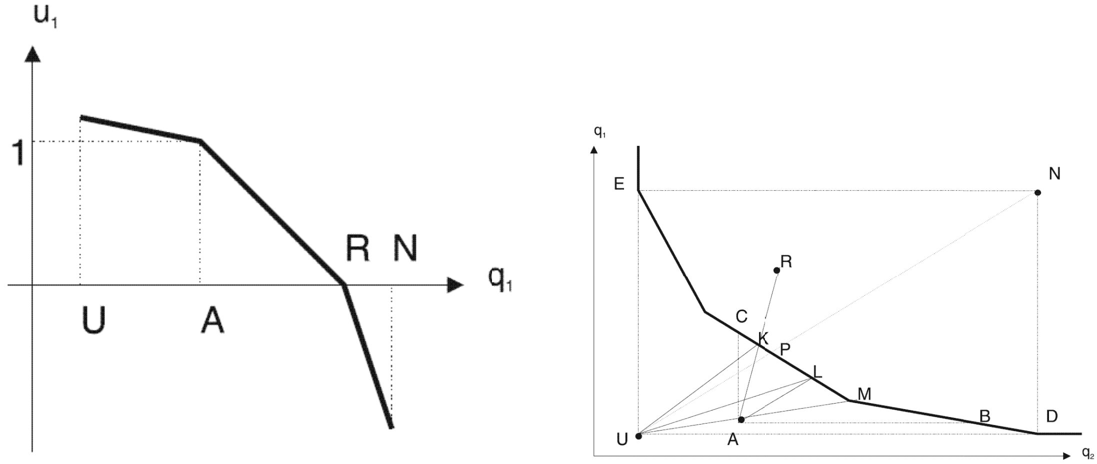
where D_k is a positive spanning set such that $0 \notin D_k$ and for each $d \in D_k$ the following conditions must be met [49]:

- d can be written as a nonnegative integer combination of the directions in $D : d = Du$ for some vector $u \in \mathbb{N}^{n_{D_k}}$ that may depend on the iteration number k,
- the distance from the frame center x_k to a frame point $x_k + \Delta_k^m d \in P_k$ is bounded above by a constant times the poll size parameter: $\Delta_k^m \parallel d \parallel \leq \Delta_k^p \max\{\parallel d\prime \parallel : d\prime \in D\}$,
- limits of the normalized sets $D_k = \left\{\frac{d}{\|d\|} : d \in D_k\right\}$ are positive spanning sets.

The general MADS algorithm, as developed by Audet and Dennis, is shown in figure 2.

3.1.3 Interactive Specification of Aspiration/Reservation Levels and Scalarization Functions

As shown in Figure 3(b), points on the Pareto front can be found by varying the relative importance of the distance to a given point. Using the utopia point **U**, any point between points **D** and **E** can be found. By using aspiration point **A** and varying the slope of the ray emanating from it, points between **B** and **C** can be found. There are many methods for determining which ray to use [51]. This particular method uses the reservation point **R** as the second point in determining the direction of the ray [50]. This technique is based on the assumption that the decision maker has an idea of what is desired for each objective, as well as what minimum, or maximum, values are acceptable. These values are referred to as the aspiration and reservation values, respectively; *i.e.* points **A** and **R** discussed previously and shown in Figure 3(b). These values are then used inside of an achievement scalarization function of the form shown in equation 6. The function u_i is called a component achievement function, *i.e.* a strictly monotone function of the objective vector components q_i. An example of such a function is shown in equation 7 and



(a) Component achievement functions for a minimized criterion (figure 4 in [50]).

(b) Pareto solutions corresponding to different component achievement functions (figure 3 in [50]).

Figure 3: Graphical illustrations of functions used for analysis of Pareto optimal solutions

Figure 3(a). The maximization of $S(\cdot)$ provides proper Pareto optimal solutions nearest to the aspiration level.

$$S(q, \bar{q}, \underline{q}) = \min_{1 \le i \le n} u_i(q_i, \bar{q}_i, \underline{q}_i) + \epsilon \sum_{i=1}^{n} u_i(q_i, \bar{q}_i, \underline{q}_i) \tag{6}$$

$$u_i(q_i, \bar{q}_i, \underline{q}_i) = \begin{array}{ll} \alpha_i w_i(\bar{q}_i - q_i) + 1, & q_i < \bar{q}_i \\ w_i(\bar{q}_i - q_i) + 1, & \bar{q}_i \le q_i \le \underline{q}_i \\ \beta_i w_i(\underline{q}_i - q_i), & \underline{q}_i < q_i \end{array} \tag{7}$$

3.1.4 Multi-Objective Ranking and Selection

Lee et al. propose a performance index to measure the degree that a point is dominated in the Pareto sense when the objective function evaluations are subject to noise. This index can then be used inside of a ranking and selection framework to find the set of non-dominated points rather than a single best point [20]. This performance index was then used to develop the Multi-objective Optimal Computing Budget Allocation algorithm (MOCBA). It has been shown that the observed Pareto set determined by MOCBA approaches the true Pareto set asymptotically with probability 1 [14]. Thus, this method can be substituted for the single objective ranking and selection method inside of the GPS/R&S or MADS algorithms to develop multi-objective versions.

3.2 Considerations

In the determination of the new solution methodology, this research considers the tradeoff between the proven convergence properties of GPS/R&S and relative simplicity of scalarization techniques. Additional considerations include the computational efficiency and required CPU processing time of the methods [9, 52].

3.2.1 Convergence of Subproblems using GPS/R&S

The following are assumed:

1. The problem is of the form $\min_{x \in \Theta} E[F(x,\omega)]$, $\bar{F}(x) := E[F(x,\omega)]$.

2. $\Theta \subseteq (\Re^{n^c} \times \mathbb{Z}^{n^d})$ represents the feasible, mixed-variable domain where the continuous variables are restricted by bound and linear constraints.

3. $\bar{F}(x) : (\Re^{n^c} \times \mathbb{Z}^{n^d}) \to \Re^J$, *i.e.* there exist J multiple objectives $\bar{F}_i(x)$, $i = 1, \ldots, J$. Let $I = \{1, \ldots, J\}$.

Lemma 3.1. *Given a global minimizer of a convex combination of the J objectives, i.e.* $\overline{x}^* = \arg\min_{\overline{x} \in \Theta} \left(\sum_{i=1}^{J} c_i F_i(\overline{x}) \right)$, $c_i \geq 0 \Longrightarrow \overline{x}^*$ *is Pareto optimal.*

Proof. Assume to the contrary that $\overline{x}^*$ is not Pareto optimal.
If $\overline{x}^*$ is not Pareto optimal, by *Definition 1.1*, there exists some $\overline{x} \in \Theta$ such that $F_k(\overline{x}) \leq F_k(\overline{x}^*)$ for $k = 1, \ldots, J$ and $F_i(\overline{x}) < F_i(\overline{x}^*)$ for some $i \in \{1, \ldots, J\}$. Thus, the positive sum, $\sum_{i=1}^{J} c_i F_i(\overline{x}) < \sum_{i=1}^{J} c_i F_i(\overline{x}^*)$. Which contradicts the assumption that $\overline{x}^* = \arg\min_{\overline{x} \in \Theta} \left(\sum_{i=1}^{J} c_i F_i(\overline{x}) \right)$. Therefore, $\overline{x}^*$ is Pareto optimal.

□

Lemma 3.2. *The sequence of iterates generated by GPS/R&S contains a limit point that satisfies the first-order necessary conditions for optimality, almost surely (a.s.).*

Proof. Follows directly from *Theorem 3.19* and *Theorem 3.24* in the doctoral dissertation of Sriver [9].[1] □

Theorem 3.3. *The sequence of iterates generated by each subproblem of stochastic multi-objective pattern search (SMOPS) (as defined in section 4.1.1) contains a limit point that meets the first-order necessary conditions for Pareto optimality, almost surely (a.s.).*

Proof. The SMOPS algorithm generates each subproblem as a nonnegative combination of the J objectives of the original problem, *i.e.* $Z(x) = \left(\sum_{i=1}^{J} c_i F_i(x) \right)$, $c_i \geq 0$. Each subproblem is then solved using GPS/R&S. Thus, by *Lemma 3.2*, the sequence of iterates

[1] Convergence in pattern search algorithms is dependent on the existence of bounded error in the selection of iterates. In GPS/R&S, ranking and selection is used as a means of error control during the search. As proven by Sriver, with this condition satisfied, GPS/R&S converges almost surely to a stationary point appropriately defined in the mixed-variable domain [10].

produced in the subproblem contains a limit point $\overline{x}^*$ satisfying first-order conditions for optimality *a.s.*

By *Lemma 3.1*, if $\overline{x}^*$ is globally optimal, it is also Pareto optimal. Thus, it follows that the sequence of iterates produced in the subproblem contains a limit point satisfying the first-order necessary conditions for Pareto optimality *a.s.* □

3.2.2 Convergence of Subproblems using MADS

The following are assumed:

1. The problem is of the form $\min_{x \in \Omega} E[F(x, \omega)]$, $\bar{F}(x) := E[F(x, \omega)]$.
2. $\Omega \subseteq (\Re^{n^c} \times \mathbb{Z}^{n^d})$ represents the feasible, mixed-variable domain.
3. $\bar{F}(x) : (\Re^{n^c} \times \mathbb{Z}^{n^d}) \rightarrow \Re^J$, *i.e.* there exist J multiple objectives $\bar{F}_i(x)$, $i = 1, \ldots, J$. Let $I = \{1, \ldots, J\}$.

Lemma 3.4. *Let f be a single objective subproblem of SMOPS (as defined in section 4.1.1). Suppose that the sequence of iterates produced by the subproblem converges to the solution $\hat{x} \in \Omega$. Then the set of refining directions for the entire sequence of iterates is asymptotically dense in $T_\Omega^H(\hat{x})$ a.s. and the following hold.*

- *If f is Lipschitz near $\hat{x}$, then $\hat{x}$ is a Clarke stationary point of f on Ω with respect to the continuous variables.*
- *If f is strictly differentiable at $\hat{x}$ and $T_\Omega^H(\hat{x}) \neq \emptyset$, then $\hat{x}$ is a Clarke KKT stationary point of f over Ω with respect to the continuous variables.*
- *If f is strictly differentiable at $\hat{x}$, Ω is regular at $\hat{x}$, and $T_\Omega^H(\hat{x}) \neq \varnothing$, then $\hat{x}$ is a contingent KKT stationary point of f over Ω with respect to the continuous variables.*

Proof. This lemma follows directly from *Theorem 3.13, Corollary 3.14, Corollary 3.16*, and *Theorem 4.4* in the work of Audet and Dennis [49].[2] □

Theorem 3.5. *Suppose the sequence of iterates generated by a subproblem of SMOPS converges to $\hat{x} \in \Omega$. Then $\hat{x}$ meets the first-order necessary conditions (in the forms listed below) for optimality a.s.:*

- *If f is Lipschitz near $\hat{x}$, then $\hat{x}$ is a Clarke stationary point of f on Ω*
- *If f is strictly differentiable at $\hat{x}$ and $T_\Omega^H(\hat{x}) \neq \emptyset$, then $\hat{x}$ is a Clarke KKT stationary point of f over Ω.*
- *If f is strictly differentiable at $\hat{x}$, Ω is regular at $\hat{x}$, and $T_\Omega^H(\hat{x}) \neq \emptyset$, then $\hat{x}$ is a contingent KKT stationary point of f over Ω.*

[2] Convergence in pattern search algorithms is dependent on the existence of bounded error in the selection of iterates. In GPS/R&S, ranking and selection is used as a means of error control during the search. As proven by Sriver, with this condition satisfied, GPS/R&S converges almost surely to a stationary point appropriately defined in the mixed-variable domain [10].

Further, if $\hat{x}$ is in fact globally optimal, it is also Pareto optimal.

Proof. The SMOPS algorithm generates each subproblem as a nonnegative combination of the J objectives of the original problem, *i.e.* $Z(x) = \left(\sum_{i=1}^{J} c_i F_i(x)\right)$, $c_i \geq 0$. Each subproblem is then solved using MADS. Thus, by *Lemma 3.4*, the limit point $\hat{x}$ satisfies first-order conditions for optimality, *i.e.* is a stationary point, *a.s.*

Therefore, by *Lemma 3.1*, if $\hat{x}$ is optimal, it is also Pareto optimal. □

3.2.3 Quality of the Pareto Set Approximation

Though solving the set of subproblems results in a set of Pareto optimal solutions, such a set is only an approximation of the true, most-likely infinite, set that describes the Pareto frontier. If this frontier is well-behaved, the given approximation most likely will be adequate. However in realistic problems, like engineering design optimization, this may not be the case. As discussed by Collette and Messac et al., under certain conditions, a *distance to a point* method like Aspiration/Reservation level analysis will find most Pareto solutions [42, 53]. However, in general, if the frontier is non-convex or discontinuous, the aforementioned approximation to the Pareto front may still be missing points of potential interest. Thus further investigation is required to determine if other Pareto points exist outside the approximated set.

4 Proposed Method

To extend/integrate these solution methodologies into something that applies to multi-objective, stochastic, and mixed-variable cases, a two-stage method is proposed. In the first stage, a convex combination of objectives, via scalarization functions and aspiration/reservation levels of the decision maker, is used to determine an approximation of the Pareto front in a region of interest. For each single objective sub-problem, GPS/R&S or MADS can be used to determine a Pareto solution. However, since the actual Pareto frontiers of typical design optimization problems are not likely convex [53], some points in the Pareto frontier may not be found from a combination of objectives (see note in *Section 3.2.3*). Thus, a second stage is added to further investigate the region of interest. In this stage, the single-objective ranking and selection routine inside of GPS/R&S is replaced with MOCBA, so that the discrete points in the mesh can be evaluated with respect to multiple objectives. A graphical representation is shown in figure 4 and descriptions of each step follow.

4.1 Stage One

4.1.1 Aspiration and Reservation Level Analysis

As discussed in section 3.1.3, the multiple objectives are combined into a single objective problem of the form shown in equation 6. Each subproblem, or choice of aspiration and reservation levels, produces a point on the Pareto front approximation. There are many ways to produce test points. Historically, in interactive specification of aspiration and

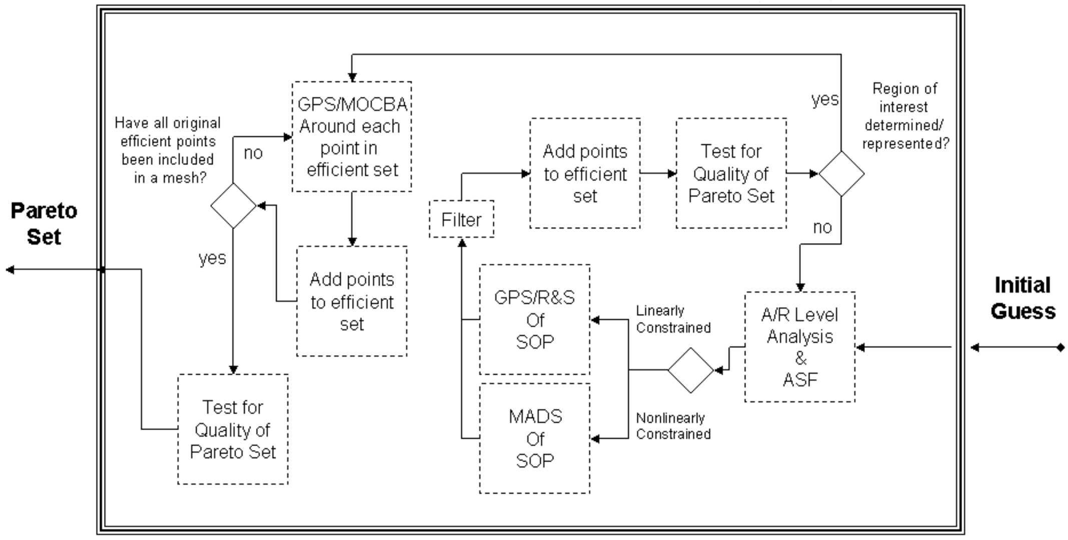


Figure 4: Stochastic Multi-objective Pattern Search (SMOPS)

reservation levels, a decision maker was actively involved in choosing these points [12]. However, if this interaction is not possible or if the decision maker has only specified a range of values for aspiration and reservation levels, some other method must be used. In the case where a range of values has been specified, the problem is that of determining an approximation to the Pareto frontier within a region of interest. Such a problem is similar to that of approximating a response surface with aspiration and reservation levels as the decision variables. Thus, experimental design methods from response surface methodology should apply. Three methods were chosen to include in this method.

1. **Full Factorial Design.** The full factorial design has as a design point every possible combination of decision variables (aspiration and reservation levels) and levels of those variables. Though full factorial designs provide information about linear, interaction, and quadratic effects, designs become impractically large for relatively few numbers of design variables and levels. Particularly, in this method, the number of design variables grows twice as fast as the number of objective functions, so the full factorial design is only practical for very small problems.

2. **Central Composite Design.** The central composite design is a variance optimal design used to fit second order models. It is considered quite useful for sequential experimentation. With this model, information about linear, interaction, and quadratic terms of the response model can be determined with relatively few design points [54].

3. **Box-Behnken Design.** The Box-Behnken design was developed as a three level alternative to the central composite design. It is a spherical design that provides good coverage of the design space in general. However, because it is spherical, vice cuboidal, it should not be used if the decision maker is particular concerned with the extreme points of the given range of aspiration and reservation levels [54].

4.1.2 GPS/R&S for Problems with Linear Constraints

This step of SMOPS uses the NOMADm implementation of GPS/R&S [55] to solve each single objective subproblem of the form discussed in section 3.1.3. GPS/R&S is discussed in detail in section 3.1.1 and has been shown to have good convergence properties. (See section 3.2.1 and [9].)

4.1.3 MADS for Problems with Non-Linear Constraints

Similarly, this step of SMOPS uses the NOMADm implementation of MADS [55] to solve each single objective subproblem of the form discussed in section 3.1.3. MADS is discussed in detail in section 3.1.2 and has also been shown to have good convergence properties. (See section 3.2.2 and [49].)

4.1.4 Adding Points to the Efficient Set

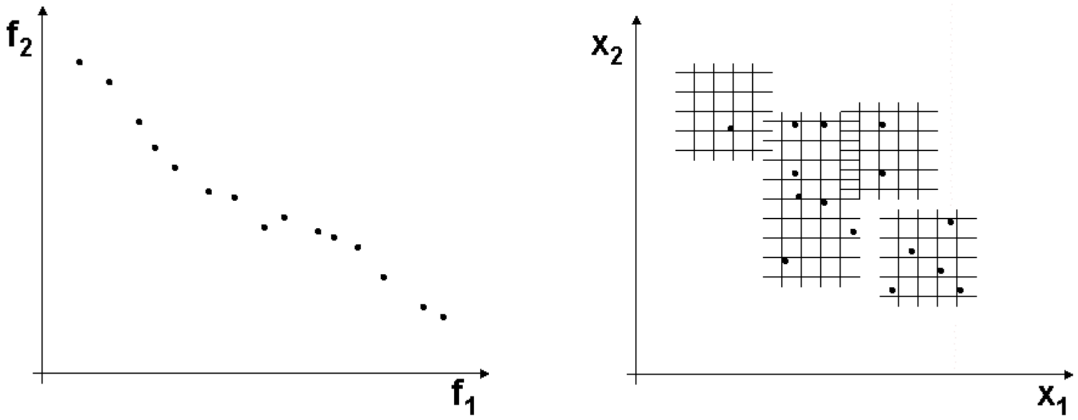
Each subproblem by design, should produce an efficient point. In deterministic problems this is always the case (see *Lemma 3.1*). In stochastic problems, as the number of iterations of the single objective solver is allowed to approach infinity, the solution converges to an efficient point with probability one (see *Theorem 3.2.1*). However, in practice, the number of iterations is finite. Thus, the addition of dominated points is possible. Therefore, in future research, a filter will be added to ensure that a point is non-dominated before it is added to the efficient set. Additionally, the filter will check to see if the new point dominates other points in the current efficient set. Multi-objective ranking and selection [14] will be used to determine if a point is dominated (see sections 3.1.4 and 4.2.2).

4.1.5 Tests for Quality of the Pareto Set

An exact Pareto set may have an infinite number of efficient points. Any multi-objective solver will provide only an approximation of that set. Thus, an item of interest to users of a solver is the quality of its approximation of the Pareto set. Relatively few papers in the literature focus on quality metrics for Pareto set approximations and most make the assumption that the true set is known *a priori*. Because this research is intended for applications like engineering design optimization, assumptions of this type are likely to be invalid. Thus, the quality metrics introduced by Wu and Azarm will be used in future research to assess the quality of the Pareto set because these metrics measure the quality (accuracy, spread, cluster, etc.) of points in the approximated set without any knowledge of the true Pareto set [56].

4.2 Stage Two

As discussed in section 3.2.3, some efficient points may not be found via interactive specification of aspiration/reservation levels and scalarization functions of the type discussed in section 3.1.3. Therefore, a second (optional) stage will be added in future research for those cases in which missing points may pose a particular problem. In this stage, a discrete mesh (similar to that used for GPS/R&S and MADS) around the current efficient points will be determined and then a multi-objective ranking and selection algorithm will



(a) Objective Space

(b) Design Space Showing Mesh

Figure 5: Notional Approximated Pareto Set and Mesh for a Two-Objective Problem with Two Design Variables

be used to check for new efficient points on the mesh. A graphical representation of a notional problem is shown in Figure 5.

4.2.1 Creation of the Mesh

The discrete mesh determined for this step is similar to the frame used by MADS (see section 3.1.2). The mesh is given by

$$M_e = \bigcup_{x \in S_e} \left\{ x + \Delta_e^m D z : z \in \mathbb{N}^{n_D} \right\}, \tag{8}$$

where S_e is the set of efficient points found in stage one, Δ_e^m is the mesh size parameter, and D is a positive set that spans $\mathbb{R}^n$.

4.2.2 Multi-Objective Ranking and Selection

A version of Multi-objective Optimal Computing Budget Allocation algorithm (MOCBA), developed by Lee et al. will be used to check for new efficient points on the mesh. As discussed in section 3.1.4, MOCBA has been used successfully for multi-objective ranking and selection problems [20, 48] and that the observed Pareto set determined by MOCBA approaches the true Pareto set asymptotically with probability 1 [14].

5 Implementation of the Solution Methodology

There exists single objective GPS/R&S and MADS software, called NOMADm, designed to run inside of the MATLAB® computing environment [55]. This software has been verified on several single objective stochastic test problems [9] and has been used in a multi-echelon repair system optimization application [47]. This software will be extended to include the multi-objective case by embedding it as the single-objective subproblem solver as described in section 4. Though interactive decision aid software exists for deterministic optimization problems [12, 50], such code is not written in MATLAB® and would have to be integrated. Thus, existing multi-objective code will not be used in this research and the required code will be written in MATLAB® during future research (see section 6.1). As an initial proof of concept prototype, NOMADm was manually connected (analyst in the loop) with scalarization/Aspiration/Reservation logic to test the performance of stage one of the algorithm. Stage two requires internal modification of NOMADm software and will be developed in further research. Results of the test are given in section 5.1.1.

5.1 Testing

The accuracy of the final version of the algorithm will be verified via a set of test problems having known solutions. These test problems will be modified (*e.g.* the Matlab® RAND function in each objective function evaluation) to introduce random noise into the objective functions to simulate the algorithm's use on a stochastic system. Initially, the prototype was verified by its use on the test problem developed for multi-objective evolutionary algorithms by Viennet given in equation 9(a-c) [57].

$$\begin{aligned} \min\ F_1(X_1,X_2) &= \tfrac{(X_1-2)^2}{2} + \tfrac{(X_2+1)^2}{13} + 3 \\ F_2(X_1,X_2) &= \tfrac{(X_1+X_2-3)^2}{175} + \tfrac{(2X_2-X_1)^2}{17} - 13 \\ F_3(X_1,X_2) &= \tfrac{(3X_1-2X_2+4)^2}{8} + \tfrac{(X_1-X_2+1)^2}{27} + 15 \end{aligned} \tag{9a}$$

subject to

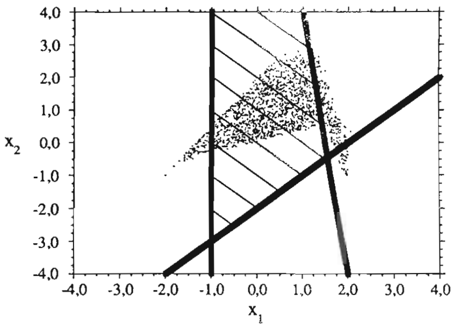
$$\begin{aligned} 4X_1 + X_2 - 4 &< 0 \\ -X_1 - 1 &< 0 \\ X_1 - X_2 - 2 &< 0 \end{aligned} \tag{9b}$$

$$X_1, X_2 \in [-4, +4]^2 \tag{9c}$$

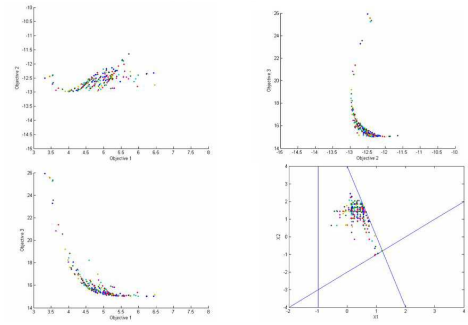
5.1.1 Test Results

The algorithm was tested over a range of aspiration and reservation levels using three different experimental designs: central composite design with 59 design points, Box-Behnken design with 54 design points, and full factorial design with 4,096 design points. Five replications were used at each design point. Each run took less than a minute (with 500 function evaluations) running in Matlab 7.2.0 on a 2.13GHz Pentium(R)M processor with 1GB of RAM.

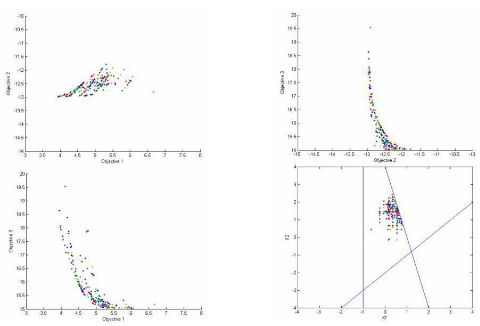
Initial results are quite promising and are shown in Figure 6. The initial runs fall inside the published Pareto set (see Figure 6(a)) implying that stage one of the algorithm is indeed converging to Pareto solutions. The two-dimensional projections of the experimental Pareto front appear to approach a reasonable approximation to the actual Pareto front. However, it is noticeable that the effect of random noise is more prominent in objective two. This results from how the test problem was constructed. The Matlab® RAND function was used in each objective function without scaling. This random noise



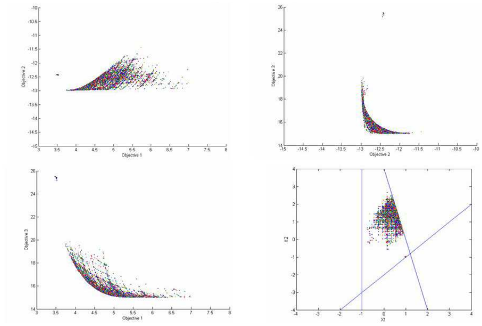
(a) Pareto Set for Deterministic Test Problem, Figure 7 in [57]



(b) Initial Test Results for Test Problem with Added Noise using Central Composite Design



(c) Initial Test Results for Test Problem with Added Noise using Box-Behnken Design



(d) Initial Test Results for Test Problem with Added Noise using Full Factorial Design

Figure 6: Comparison of Initial Test Results to Published Solution

is much larger in comparison to objective two than compared to objectives one and three. In future testing, the noise will be scaled to account for differences in the size of the objective functions. Additionally, because number of iterations of the algorithm is necessarily finite, the filter as described in section 4.1.4 will be added to prevent potentially dominated points from entering the efficient set.

6 Future Work

6.1 Integrated Software

This research provided the algorithmic methodology for a two-stage solution process. However, because the the first stage will converge to an approximation of the Pareto optimal set, its performance was tested on a simple test problem as an initial proof of concept exercise. Future research will integrate a multi-objective ranking and selection algorithm into NOMADm in order to implement the second stage of the algorithm (see sections 3.1.4 and 4.2). Both stages will then be integrated with NOMADm into a single graphical user interface so that interactive specification of aspiration/reservation levels can be accomplished without manual “hand-jamming”of the functions. Additional areas of future research follow.

6.2 Automated Decision Agent

Section 4.1.1 discusses the experimental design built to investigate a range of values of aspiration and reservation levels. If instead, an automated decision agent could be developed, it may provide better insight to the decision maker. In fact, even the decision strategies of a decision maker could be investigated, *e.g.* conservative versus intrepid decision strategies.

6.3 Extensive Testing

After both stages of the algorithm are tested on the simple test problem, the algorithm will be tested on multi-objective test sets as suggested by Deb [58] and Van Veldhuizen [59]. These sets are multi-objective but deterministic. Therefore, random noise will be added to function evaluations to emulate objective function measurement error or the use of simulation.

6.4 Engineering Design Optimization Application

After thorough testing on standard test problems, the algorithm will be applied to a real-world optimization problem. As discussed previously aircraft design problems contain multiple objectives. (Examples of multi-objective aircraft design problems can be found in [2, 3, 4, 5].) Additionally, these objectives are often subject to measurement error or must be estimated with simulations. (Examples of simulation used in aircraft design can be found in [6, 7, 8, 2].) The algorithm should be well suited to solving this type of problem and thus will be applied to an aeronautical engineering design optimization problem.

6.5 Algorithm Termination Criteria

Even if an algorithm is known to converge, the reality of imprecision and roundoff errors make it a necessary to predetermine stopping criteria. Traditionally in pattern search methods, this is accomplished by stopping the algorithm when the step size is less than a threshold value, *i.e.* $\Delta_k \leq \Delta_T$ [30, 9] where Δ_k is defined as in equations 4 and 5. However, in a stochastic environment, termination criteria are typically more complex. Too small a value of Δ_T may increase the required sample size of the ranking and selection portion of the algorithm to an unacceptable level, whereas too large a value may induce premature termination [9]. Thus, an in-depth study of appropriate termination criteria is necessary for practical implementation of the algorithm. Therefore, it is an objective in future efforts of this research to develop heuristic stopping criteria based on based on differences in competing responses compared to variations in the responses and the practically required tolerance of the solution.

7 Conclusion

In this paper, a research approach is suggested that extends the applicability of GPS/R&S and MADS single-objective stochastic optimization algorithms to include problems with multiple objectives via a two-stage algorithm that incorporates the multi-objective optimization methods of interactive specification of aspiration and reservation levels, scalarization functions, and multi-objective ranking and selection. This combination is devised specifically so as to keep the desirable convergence properties of GPS/R&S and MADS while extending application to the multi-objective case. Initial testing of stage one has been conducted on a test problem with known solutions. In further research, stage two will be tested, integrated software for both stages of the algorithm will be developed, thoroughly tested, and then applied to an aircraft design optimization problem.

References

[1] J. Martins and J. Alonso, "Complete configuration aero-structural optimization using a coupled sensitivity analysis method," in *9th Symposium on Multidisciplinary Analysis and Optimization, Atlanta, GA; 4-6 Sept. 2002*, vol. AIAA Paper 2002-5402, 2002.

[2] E. Cramer, V. Du, and J. Gablonsky, "Multi-objective optimization for complex computer simulations," in *44th AIAA Aerospace Sciences Meeting and Exhibit*, 9 - 12 January 2006 2006.

[3] W. Kim, R. V. Grandhi, and M. Haney, "Multi-objective evolutionary optimization method for thermal protection system design," in *46th Structures, Structural Dynamics, and Materials Conference; Austin, TX; 18-21 Apr. 2005*, vol. AIAA Paper 2005-2311, 2005.

[4] Y. Lian and M.-S. Liou, "Multi-objective optimization of a transonic compressor blade using evolutionary algorithm," in *46th Structures, Structural Dynamics, and Materials Conference; Austin, TX; 18-21 Apr. 2005*, vol. AIAA Paper 2005-1816, 2005.

[5] H. Langer, T. Puehlhofer, and H. Baier, "A multi-objective evolutionary algorithm with integrated response surface functionaltities for configuration optimization with discrete variables," in *10th Multidisciplinary Analysis and Optimization Conference; Albany, NY; Aug. 30 - Sep. 1, 2004*, vol. AIAA Paper 2004-4326.

[6] S. Lienard and Y. Lefevre, "Modeling and analysis of the deployment of a rolled inflatable beam using msc-dytran," in *46th Structures, Structural Dynamics, and Materials Conference; Austin, TX; 18-21 Apr. 2005*, vol. AIAA Paper 2005-1968, 2005.

[7] H. Kwon, S. Park, and J. Lee, "Transonic wing flutter simulation using navier-stokes and k-ω turbulent model," in *46th Structures, Structural Dynamics & Materials Conference; Austin, TX; 18 - 21 April 2005*, 2005.

[8] E. J. Cramer, "Using approximate models for engineering design," in *7th Symposium on Multidisciplinary Analysis and Optimization, St. Louis, MO; 2-4 Sept. 1998*, vol. AIAA Paper 1998-4716, 1998.

[9] T. Sriver, *Pattern Search Ranking and Selection Algorithms for Mixed-Variable Optimization of Stochastic Systems*. PhD thesis, Air Force Institute of Technology, September 2004.

[10] T. Sriver and J. Chrissis, "Combined pattern search and ranking and selection for simulation optimization," in *Proceedings of the 2004 Winter Simulation Conference*, 2004.

[11] M. A. Abramson, C. Audet, and J. Dennis, "Filter pattern search algorithms for mixed variable constrained optimization problems." *Pacific Journal of Optimizaton*, to appear. Also appears as Technical Report #TR04-09, Rice University, Department of Computational and Applied Mathematics, 2004.

[12] J. Granat and M. Makowski, "Interactive specification and analysis of aspiration-based preferences," *European Journal of Operational Research*, vol. 122, pp. 469–485, 2000.

[13] K. Miettinen and M. Mäkelä, "On scalarizing functions in multi-objective optimization," *OR Spectrum*, vol. 24, pp. 193–213, 2002.

[14] L. H. Lee, E. P. Chew, S. Teng, and D. Goldsman, "Finding the non-dominated pareto set for multiobjective simulation models." Submitted to IIE Transactions., 2005.

[15] Y. Ermoliev and R. Wets, eds., *Numerical Techniques for Stochastic Optimization*. Berlin, Germany: Springer-Verlag, 1988.

[16] M. Fu, F. Glover, and J. April, "Simulation optimization: A review, new developments, and applications," in *Proceedings of the 2005 Winter Simulation Conference*, 2005.

[17] A. Abraham and J. Lakhmi, *Evolutionary Multiobjective Optimization*. London: Springer-Verlag, 2005.

[18] C. Audet, G. Savard, and W. Zghal, "Multiobjective optimzation through a series of single objective formulations." Technical Paper GERAD G-2007-05, GERAD and Department of Mathematics and Industrial Engineering, École Polytechnique, Montréal, Canada, 2006.

[19] M. Ehrgott, *Multicriteria Optimization*. Berlin, Germany: Springer, second ed., 2005.

[20] L. H. Lee, E. P. Chew, and S. Teng, "Optimal computing budget allocation for multi-objective simulation models," in *Proceedings of the 2004 Winter Simulation Conference* (J. S. S. R .G. Ingalls, M. D. Rossetti and B. A. Peters, eds.), 2004.

[21] Y. Fu and U. Diwekar, “An efficient sampling approach to multiobjective optimization,” *Annals of Operations Research*, vol. 132, pp. 109–134, 2004.

[22] C. Audet and J. Dennis, Jr., “Pattern search algorithms for mixed variable programming,” *SIAM Journal on Optimization*, vol. 11, no. 3, pp. 573–594, 2000.

[23] A. Gosavi, *Simulation-Based Optimization, Parametric Optimization Techniques and Reinforcement Learning*. Boston, MA, USA: Kluwer Academic Publishers, 2003.

[24] R. Jin and et al., “Comparative studies of metamodelling techniques under multiple modelling criteria,” *Structural and Multidisciplinary Design Optimization*, vol. 23, pp. 1–13, 2001.

[25] H. Robbins and S. Munro, “A stochastic approximation method,” *Annals of Mathematical Statistics*, vol. 22, pp. 400–407, 1951.

[26] J. Keifer and J. Wolfowitz, “Stochastic estimation of the maximum of a regression function,” *Annals of Mathematical Statistics*, vol. 23, pp. 462–466, 1952.

[27] J. Blum, “Multidimensional stochastic approximation method,” *Annals of Mathematical Statistics*, vol. 25, pp. 737–744, 1954.

[28] J. Swisher, “A survey of simulation optimization techniques and procedures,” in *Proceedings of the 2000 Winter Simulation Conference* (J. Joines, ed.), 2000.

[29] J. Spall, *Introduction to Stochastic Search and Optimization: Estimation, Simulation, and Control*. Hoboken, NJ, USA: John Wiley and Sons, 1993.

[30] R. Hooke and T. Jeeves, “Direct search solution of numerical and statistical problems,” *Journal of the Association of Computing Machinery*, vol. 8, pp. 212–229, 1961.

[31] J. Tomick and et al., “Sample size selection for improved nelder-mead performance,” in *Proceedings of the 1995 Winter Simulation Conference* (e. C. Alexopoulous, ed.), 1995.

[32] R. Barton and J. Ivey Jr., “Nelder-mead simplex modifications for simulation optimization,” *Management Science*, vol. 42, no. 7, pp. 954–973, 1996.

[33] D. Humphrey and J. Wilson, “A revised simplex search procedurefor stochastic simulation response-surface optimization,” in *Proceedings of the 1998 Winter Simulation Conference* (e. D. J. Medeiros, ed.), 1998.

[34] D. Humphrey and J. Wilson, “A revised simplex search procedurefor stochastic simulation response surface optimization,” *INFORMS Journalon Computing*, vol. 12, no. 4, pp. 272–283, 2000.

[35] V. Torczon and M. Trosset, “On the convergence of pattern search algorithms,” *SIAM Journal on Optimization*, vol. 7, no. 1, pp. 1–25, 1997.

[36] D. Goldsman and B. Nelson, “Ranking, selection, and multiple comparisions in computer simulation,” in *Proceedings of the 1994 Winter Simulation Conference*, 1994.

[37] Y. Hochberg and A. Tamhane, *Multiple comparison Procedures*. Hoboken, NJ, USA: John Wiley and Sons, 1987.

[38] US Department of Energy, “Computational science education project.”

[39] S. Sait and H. Youssef, *Iterative Computer Algorithms with Applications in Engineering: Solving Combinatorial Optimization Problems*. Las Alamitos, CA, USA: IEEE Computer Society, 1999.

[40] Sandia National Laboratories, “Tabu search.”

[41] F. Glover, “Future paths for integer programming and links to artificial intelligence,” *Computers and Operations Research*, vol. 13, no. 5, pp. 533–549, 1986.

[42] Y. Collette and P. Siarry, *Multiobjective Optimization Principles and Case Studies*. Berlin, Germany: Springer-Verlag, 2004.

[43] H. Eskandari, L. Rabelo, and M. Mollaghasemi, “Multiobjective simulation optimization using an enhanced genetic algorithm,” in *Proceedings of the 2005 Winter Simulation Conference*, 2005.

[44] J. Molina, M. Laguna, R. Marti, and R. Caballero, “SSPMO: A scatter search procedure for non-linear multiobjective optimization,” 2005.

[45] N. Baba and A. Morimoto, “Three approaches for solving the stochastic multiobjective programming problem,” in *Proceedings of GAMM/IFIP-Workshop on Stochastic Optimization*, (New York, NY, USA), Springer-Verlag, 1990.

[46] N. Baba and A. Morimoto, “Stochastic approximation method for solving the stochastic multi-objective programming problem,” *International Journal of Systems Science*, vol. 24, no. 4, pp. 789–796, 1993.

[47] D. Tharaldson, “Optimization of a multi-echelon repair system via generalized pattern search with ranking and selection: A computational study,” Master’s thesis, Air Force Institute of Technology, March 2006.

[48] L. H. Lee, S. Teng, E. P. Chew, I. Karimi, K. W. Lye, P. Lendermann, Y. Chen, and C. H. Koh, “Application of multi-objective simulation-optimization techniques to inventory management problems,” in *Proceedings of the 2005 Winter Simulation Conference* (F. B. A. M. E. Kuhl, N. M. Steiger and J. A. Joines, eds.), 2005.

[49] C. Audet and J. Dennis, “Mesh adaptive direct search algorithms for constrained optimization,” *SIAM Journal on Optimization*, vol. 17, no. 1, pp. 188–217, 2006.

[50] M. Makowski, “Methodology and a modular tool for multiple criteria analysis of lp models.” Working Paper 94-102, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1994.

[51] F. Lootsma, T. Athan, and P. Papalambros, “Controlling the search for a compromise solution in multi-objective optimization,” *Engineering Optimization*, vol. 25, no. 1, pp. 65–, 1996.

[52] T. Hanne, “Theory and methodology on the convergence of multiobjective evolutionary algorithms,” *European Journal of Operational Research*, vol. 117, pp. 553–564, 1999.

[53] A. Messac, G. Sundararaj, R. Tappeta, and J. Renaud, “Ability of objective functions to generate points on nonconvex pareto frontiers,” *AIAA Journal*, vol. 38, no. 6, pp. 1084–1091, 2000.

[54] R. H. Myer and D. C. Montgomery, *Response Surface Methodology: Process and Product Optimization Using Designed Experiments*. New York, NY, USA: John Wiley and Sons, Inc., 1995.

[55] M. A. Abramson, “Nomadm version 4.02.” Free Software Foundation, Inc., 59 Temple Place, Suite 330, Boston, MA 02111-1307 USA, 2006.

[56] J. Wu and S. Azarm, “Metrics for quality assessment of a multiobjective design optimization solution set,” *Transactions of the ASME*, vol. 123, no. 1, pp. 18–25, 2001.

[57] R. Viennet and I. Marc, “Multicriteria optimization using a genetic algorithm for determining a pareto set,” *International Journal of Systems Science*, vol. 27, no. 2, pp. 255–260, 1996.

[58] K. Deb, A. Pratap, and T. Meyarivan, “Constrained test problems for multi-objective evolutionary optimization.” KanGAL Report no. 200002, Kanpur Genetic Algorithms Laboratory (KanGAL), Indian Institute of Technology, Kanpur, PIN 208 016, India, 2000.

[59] D. Van Veldhuizen, *Multiobjective Evolutionary Algorithms: Classifications, Analyses, and New Innovations*. PhD thesis, Air Force Institute of Technology, 1999.