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INTERNATIONAL INSTITUTE FOR **IIASA** APPLIED SYSTEMS ANALYSIS
RESEARCH MEMORANDUM

A MAXIMIZATION PROBLEM ASSOCIATED WITH
DREW'S INSTITUTIONALIZED DIVVY ECONOMY

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Define variables as in Dantzig [1]. Let $U_k(C_{ik})$ be the utility derived by a member of the k th group, and let

$$U = \sum_k \mu_k U_k(C_{ik}) \quad (A)$$

be an aggregate utility function. Equation numbers refer to Dantzig's paper; letters to this paper.

The economy is described by Dantzig's equations (2) - (4)--(2) being subdivided here as (2a) and (2b). The equations can be written in full, with explicit subscripts, as follows:

$$\sum_j L_{ij} - x_j = \sum_k C_{ik} \mu_k \quad (2a)$$

$$\sum_i y_i L_{ij} = \sum_\ell \lambda_\ell R_{\ell j} \quad (2b)$$

$$\sum_j \lambda_\ell R_{\ell j} x_j = \ell \gamma_\ell \quad (3)$$

$$\sum_i y_i C_{ik} \mu_k = \ell \delta_k . \quad (4)$$

Equations (2a) and (2b) can be combined to determine $\underline{x}$ and $\underline{y}$:

$$x_i > \sum_j \{L\}_{ij}^{-1} \sum_k C_{jk} \mu_k \quad (2a')$$

$$y_j > \sum_i \{L\}_{ij}^{-1} \sum_\ell \lambda_\ell R_{\ell i} . \quad (2b')$$

We can substitute for $\underline{x}$ and $\underline{y}$ in (3) and (4), but now write them as equations in λ_ℓ , μ_k and C_{ik} : equations (5) and (6) then become

$$\sum_{ik} \lambda_\ell M_{\ell i} \mu_k \cdot C_{ik} = \ell \gamma_\ell \quad (B)$$

$$\sum_{i\ell} \lambda_\ell M_{\ell i} \mu_k C_{ik} = \ell \delta_k \quad (C)$$

where

$$M_{\ell i} = \sum_j R_{\ell j} \{L\}_{ji}^{-1} . \quad (D)$$

The proposed model is

Max U in equation (A)

subject to (B) and (C).

$\underline{x}$ and $\underline{y}$ are given by (2a'), (2b').

Possible Uses of the Model

1) In an existing equilibrium situation, we can assume that $\underline{C}$, $\underline{\lambda}$ and $\underline{\mu}$ exist such that equations (C), (D), (2a'), (2b') are satisfied and U in (A) is a max.

We can then explore the use of the model to investigate the consequences of various types of change.

2) Change in tastes: $U_k \rightarrow U'_k$. Then since there are fewer constraints (B) and (C) than there are C_{ik} 's, there should exist a set of C_{ik} 's which give the new equilibrium.

3) It may be, however, that imposing conditions (B) and (C) is too strong. An alternative would be to calculate $\underline{C}$ to maximize (A) subject to (C) (which is like a budget constraint), for initially assumed $\underline{\lambda}$; and then to compute $\underline{x}$ from (2a'), $\underline{\lambda}$ from (B), $\underline{y}$ from (2b') and to iterate until a new equilibrium is found.

4) Alternative schemes on the lines of (3) could be investigated for other initial changes--e.g. in $\underline{\lambda}$.

5) So far, we have assumed that the μ_k 's remain fixed. A further outer loop could be added to the iteration which solved the LP problem of max U in (A) as a function of μ_k subject to (B) and (C). Alternatively, find μ_k to equalize consumer surplus per capita.

6) A further alternative would be to produce entropy maximizing versions of these models by

$$\text{Max } S = - \sum \log C_{ik} ! \quad (E)$$

subject to, say (B) and (C) and

$$\sum u_k U_k(C_{ik}) = \bar{U} \quad (A')$$

where $\bar{U}$ is assumed given at some suboptimal value.

References

- [1] Dantzig, G.B. "Drew's Institutionalized Divvy Economy,"
Report 73-7, revised, Dept. of Operations Research,
Stanford University, 1973.