THE ECONOMICS OF RISKS TO LIFE

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ABSTRACT

This paper examines the economic welfare implications of mortality change within a framework that both recognizes general equilibrium effects and incorporates full age-specific accounting. Two formal results are derived. Under a life-cycle welfare criterion, changes in the age-pattern of mortality, caused say by a medical breakthrough, should be assessed on the utility of additional life-years, production, and reproduction, less expected additional social costs of support. Loss of life at a specific age should be assessed on the opportunity costs of expected lost years of living and lost production and reproduction, less expected social support costs. From these results it is seen that current methods, in general, leave out an important social transfer term, that the valuation of life-risks is highly age-dependent, and that the degree of diminishing returns to consumption plays an important part in calculations of the economic cost of risks.

ACKNOWLEDGMENTS

One of the more difficult questions the economist faces is how to assess activities – engineering projects, safety procedures, medical advances – that raise or lower risks to human life. It is clear that in most situations proper safety should be a matter of degree: engineering constructions should neither be infinitely solid nor built too close to their limits of strength. But how safe is safe enough? What meaning can be given to phrases such as "the value of life" or "the cost of hazards to life?" And what are the economic consequences of the fact that mortality risks are gradually falling – that life is lengthening?

One method for evaluating mortality risks, in widespread use by government agencies and engineers, is the *human capital* approach. It has been proposed in various forms,[1] but the basic method sums earnings forgone by individuals lost through death or incapacitation, and sets these against the net economic benefits of the activity. (Whether these earnings should be net of the individual's consumption or not has been the subject of some contention.) Useful as this method is in giving precise dollar values, from a welfare theory viewpoint it is founded on thin logic. As Schelling argued in 1968, by concentrating purely on wage or GNP loss it ignores the individual's own desire to live. Under human capital, a medical breakthrough that prolonged life from 70 to 80 years, for example, would have no particular social justification – it would not raise GNP.

The *willingness-to-pay* method, proposed by Mishan (1971) does recognize the natural desire to live longer. Under this method a scheme

[1] See, for example, Weisbrod (1961), Fromm (1962), and Rottenberg (1967). For surveys of the general literature on the evaluation of mortality risk, see Acton (1976) and Linnerooth (1978).

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that increased life from 70 to 80 would be socially justified if those who benefited were willing, in theory at least, to pay more for their extra years than the cost of the scheme. Wider social benefits, to close relatives for example, would be included by assessing willingness to pay for the increased life of loved ones. This method, based on welfare utilities and not on dollar earnings, has obvious difficulties of quantification. Recently, Conley (1976), Usher (1973), and Jones-Lee (1974) have proposed separate methods to put the criterion on a quantitative footing, by modeling the rational person's willingness to buy extra life-years and valuing it in consumption terms.

The two methods, human capital and willingness to pay, are worth comparing for a moment. The new willingness-to-pay literature, unlike the human-capital writings, is grounded solidly on welfare theory logic. It starts from commonly accepted assumptions and proceeds deductively to its findings. On the other hand, human capital has the appealing property that it is *actuarial*: it uses full age-specific accounting to evaluate changes in mortality. Thus it can discriminate between lives lost at different ages and also between activities with equal risk but with different age-patterns of incidence. The new willingness-to-pay literature loses this property. It bases its analyses on the implications of a change in probability p of death at some arbitrary single future age or time. It would be difficult in this framework to evaluate a cure for cancer that causes a continuous pattern of changes over the entire mortality age schedule.

Both methods, whether actuarial or based on welfare theory or not, suffer a common major deficiency. They are fundamentally partial-equilibrium approaches. They ignore the chain of wider economic transfers set up through society when life is lengthened. To return to the earlier example, willingness-to-pay, as currently interpreted, would approve an advance in life from 70 to 80 years if those affected and their kin were willing to pay the cost of the increase. Forgotten, however, is that prolongation of life is not costless to wider society: those who live longer, consume longer, and this extra consumption must be financed by transfers from people at younger productive ages. Proper accounting we would suspect should include intergenerational transfer costs, felt in this case as a heavier social security burden on the young.

This paper sets out to deduce the economic welfare implications of mortality change from a simple set of assumptions in a framework that both recognizes general equilibrium effects and incorporates full age-specific accounting. Two formal results are derived. These show that, under the chosen life-cycle welfare criterion, alterations in the

mortality schedule, caused say by a medical breakthrough, should be assessed on the utility of expected additional life-years, production, and reproduction, less expected additional social costs of support. Loss of life at a specific age, due to an accident say, should be assessed on the opportunity costs of expected lost years of living, lost production and reproduction, less expected social support costs. From these results it is seen that current methods, in general, leave out an important social transfer term, that the valuation of life-risks is highly age-dependent, and that the degree of diminishing returns to consumption plays an important part in calculations of the economic costs of risks.

1 THE ECONOMICS OF CHANGES IN MORTALITY RISK

To set the context for the analysis, I first set up a neoclassical, age-specific model of the economy and population. Within this model, the effect of a change in the mortality pattern on life-cycle well-being is then derived. Both population and economy are assumed to be in steady-state growth, individuals to be alike in tastes and behavior, and production to show constant returns. Later, I shall discuss whether the results hold up when these assumptions are relaxed.

Neoclassical Model

Begin with the economy. Output is produced by combining capital K with labor L in a constant-returns production function F.[2] The economy stores no consumption goods. Output is split into consumption and investment in capital growth. Thus

$$F(K(t), L(t)) = C(t) + \dot{K}(t), \qquad C(t) \geq 0. \tag{1}$$

For the population we need a fair degree of age-specific detail. The population grows according to the Lotka dynamics

$$B(t) = \int_0^\omega B(t - x)p(t, x)m(t, x)dx, \tag{2}$$

where B is births per unit time, $p(t, x)$ is the proportion of those born at time $t - x$ who survive to age x, and m is the proportion reproducing at age x, time t; ω is an upper bound on the length of life, and the initial birth sequence is assumed given. This year's flow of births, in other words,

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[2] F is assumed concave, first-degree homogeneous, and continuously differentiable; for simplicity, capital depreciation is ignored.

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is produced by those who were born x years ago and have survived to reproduce.

Assume the population is *stable*,[3] and is growing exponentially at rate g. In this case equation (2) has the solution

$$B(t) = B(0)e^{gt}, \tag{3}$$

where the growth rate g is connected to mortality p and fertility m by substituting equation (3) in equation (2) and canceling B to yield

$$1 = \int_0^\omega e^{-gx} p(x) m(x) dx. \tag{4}$$

If $\lambda(x)$ is the age schedule of labor participation, the labor force L and total population N are given by

$$L(t) = \int_0^\omega B(t-x) p(x) \lambda(x) dx = B(t) \int_0^\omega e^{-gx} p(x) \lambda(x) dx \tag{5}$$

$$N(t) = \int_0^\omega B(t-x) p(x) dx = B(t) \int_0^\omega e^{-gx} p(x) dx. \tag{6}$$

The labor/population ratio L/N and the birth rate B/N will be denoted by $h(g)$ and $b(g)$, respectively.

Individual consumption varies with age, as do the mortality, fertility, and labor participation rates above. (How it varies is determined below.) Putting population and economic variables together, we can express total consumption C as the sum of individual age-related consumption $c(t,x)$ by

$$C(t) = \int_0^\omega B(t-x) p(x) c(t,x) dx. \tag{7}$$

Later, we shall need three parameters: the average ages of producing A_L, consuming A_c, and reproducing A_m, in the population, defined by

$$A_L = \int_0^\omega x e^{-gx} p(x) \lambda(x) dx \bigg/ \int_0^\omega e^{-gx} p(x) \lambda(x) dx$$

$$A_c = \int_0^\omega x e^{-gx} p(x) c(t,x) dx \bigg/ \int_0^\omega e^{-gx} p(x) c(t,x) dx$$

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[3] That is, its age-specific rates of fertility and mortality and its normalized age-distribution are all constant over time; g is assumed positive.

$$A_m = \int_0^\omega xe^{-gx}p(x)m(x)dx \Big/ \int_0^\omega e^{-gx}p(x)m(x)dx.$$

Assuming the economy has reached a Solow-type steady state, where the growth rate of the economy equals that of population and per capita variables are constant, and assuming investment is chosen to maximize total consumption, then

$$\dot{K}/K = g; \qquad c(t,x) = c(x); \qquad \text{and} \quad F_K = g. \tag{8}$$

One central fact in society is that consumption, which takes place at all ages, must be supported by production, which takes place only at labor-participative ages. The economy, in other words, functions at all times under the budget identity

$$C \equiv F(K,L) - gK, \tag{9}$$

that is,

$$\int_0^\omega B(t-x)p(x)c(x)dx \equiv (F/L - gK/L)\int_0^\omega B(t-x)p(x)\lambda(x)dx.$$

Using equation (3) and dividing through by $B(t)$, with usual per-unit labor notation this societal budget constraint becomes

$$\int_0^\omega e^{-gx}p(x)c(x)dx \equiv (f(k) - gk)\int_0^\omega e^{-gx}p(x)\lambda(x)dx. \tag{10}$$

Thus, intergenerational transfers are introduced by the inescapable requirement that, when growth, labor-participation rates, and the capital–labor ratio remain unchanged, any increase in consumption for one age group must be matched by decreases for other age groups.

To complete the model, it remains to determine the life-cycle pattern of consumption. Let $U[c, x]$ be the utility rate of being alive at age x, given consumption rate c. It is assumed that people individually allocate their consumption to maximize their expected lifetime welfare W, where

$$W = \int_0^\omega U[c(x), x]p(x)dx. \tag{11}$$

In aggregate, of course, they must do this in such a way that the societal budget constraint continues to hold at all times. Standard consumption-loan mechanisms (Samuelson 1958) ensure that this happens: a market interest rate and social insurance arrangements appear that encourage people to distribute their consumption over their life cycle to maximize W in such a way that the societal budget constraint is always met. The

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exact mechanisms of this need not concern us; it is sufficient to say that the individual spreads his consumption so that life-cycle welfare is maximized subject to (10) being met. Finding the life-cycle consumption pattern is thus a simple constrained variational problem, the solution of which yields

$$\partial U/\partial c(x) = \partial U/\partial c(0)e^{-gx}. \tag{12}$$

Thus life-cycle consumption is patterned according to age-related need, so that its marginal usefulness is the same at all ages, modified only by the ability to invest at an interest rate g, which equals the rate of population growth. Condition (12) therefore is the continuous-age generalization of Samuelson's "biological interest rate" condition.

All preliminaries are now completed. Population and economic growth are well-defined [equations (3), (4), and (8)], as is the pattern of life-cycle consumption (12). And the societal budget identity (10) connects the demography of consumption with that of production.

Change in Age-Specific Risks

We now introduce a particular, but small age-specific change in age-specific risks, so that the mortality schedule $p(x)$ becomes $p(x) + \delta p(x)$, and derive the implications of our chosen criterion — the representative person's expected lifetime welfare, W.[4]
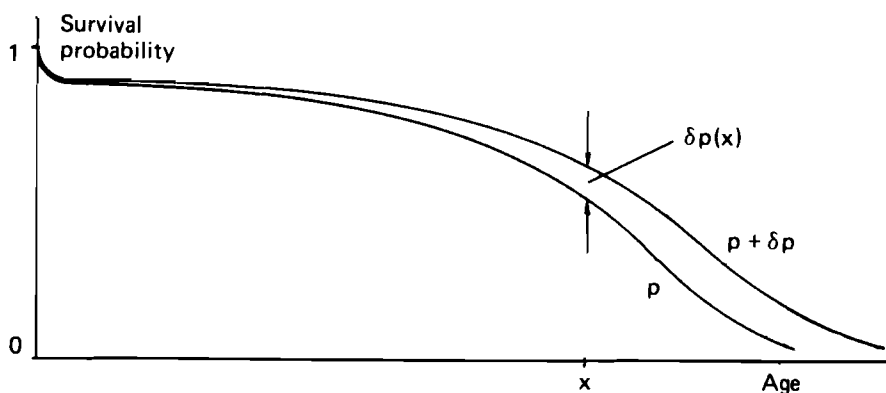

FIGURE 1   Change in age-specific mortality risks.

[a] A word on the choice of expected lifetime utility as the social criterion. It is quite legitimate to ask what the consequences are of risk-change for *any* arbitrary criterion. Suitability of a particular criterion depends on how well it "represents" social interests and on the "reasonableness" of the implications, both judgmental matters. Assuming risk-neutral individuals with identical tastes who fulfill the von Neumann–Morgenstern choice axioms, W is arguably representative. Reasonableness of implications will be judged later.

Figure 1 illustrates the shape of the function δp – the variation in the mortality curve – for a decrease in the incidence of cancer (scale of δp exaggerated slightly). For convenience, I shall assume in this section that the mortality variation lengthens life; for shortened life, the argument is symmetrical.

When the mortality schedule changes, several variables are forced to change with it: the growth rate g, the consumption pattern $c(x)$, life-cycle welfare W, and others. I shall write $\delta g[\delta p]$ as the differential change[5] in growth due to the particular age-specific mortality variation δp. Where the variation δp is understood, I shall simply write δg. Similar practice will be followed with other variables.

At this point some new notation will be useful. Let

$$U_{ex} = \int_0^{\omega} U[c(x), x]\,\delta p(x)\mathrm{d}x$$

$$c_{ex} = \int_0^{\omega} e^{-gx}c(x)\delta p(x)\mathrm{d}x$$

$$\tag{13}$$

$$L_{ex} = \int_0^{\omega} e^{-gx}\lambda(x)\delta p(x)\mathrm{d}x$$

$$v_{ex} = \int_0^{\omega} e^{-gx}m(x)\delta p(x)\mathrm{d}x$$

The first three can be viewed as the expectations of extra utility from lengthened life, of extra lifetime consumption, and of extra man-years of production resulting from the particular variation δp. The fourth, v_{ex}, is in demographic terms the change in reproductive value at birth – loosely speaking, it is the expected additional children per person due to the mortality variation. (The last three are discounted because future consumption utilities are later valued to date of birth.)

To derive $\delta g[\delta p]$, the change in the intrinsic growth rate due to the mortality variation, recall equation (4):

$$1 = \int_0^{\omega} e^{-gx}m(x)p(x)\mathrm{d}x.$$

Using the appropriate chain rule

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[5] Technically, $\delta g[\delta p(x)]$ is a Fréchet differential – a differential whose argument is a function and not a single-valued variable.

$$0 = \int_0^\omega e^{-gx} m(x)\delta p(x)\,\mathrm{d}x - \delta g \int_0^\omega x e^{-gx} m(x) p(x)\,\mathrm{d}x,$$

whence

$$\delta g\,[\delta p] = \frac{\displaystyle\int_0^\omega e^{-gx} m(x)\delta p(x)\,\mathrm{d}x}{\displaystyle\int_0^\omega x e^{-gx} m(x) p(x)\,\mathrm{d}x} = v_{ex}/A_m . \qquad (14)$$

The altered mortality pattern affects the growth rate by the change in reproductive value at birth divided by the average age of motherhood (average length between generations). Note that if the mortality variation affects only postreproductive ages, v_{ex} is zero, so that no change in the growth rate occurs.

We can now derive the change in expected lifetime welfare, $\delta W[\delta p]$. From equation (11)

$$\delta W = \int_0^\omega U[c(x), x]\,\delta p(x)\,\mathrm{d}x + \int_0^\omega \partial U/\partial c(x) \cdot \delta c(x) p(x)\,\mathrm{d}x$$

$$= \int_0^\omega U[c(x), x]\,\delta p(x)\,\mathrm{d}x + \partial U/\partial c(0) \int_0^\omega e^{-gx}\,\delta c(x) p(x)\,\mathrm{d}x. \quad (15)$$

Life-cycle welfare is changed directly by extra years and indirectly by the alteration in the consumption pattern needed to accommodate these extra years. The latter can be evaluated by taking differentials across the societal budget identity (10). This yields, on collecting terms,

$$0 = \int_0^\omega e^{-gx} c(x)\delta p(x)\,\mathrm{d}x + \int_0^\omega e^{-gx}\,\delta c(x) p(x)\,\mathrm{d}x$$

$$-(f(k)-gk)\int_0^\omega e^{-gx}\lambda(x)\delta p(x)\,\mathrm{d}x - \delta k(f'-g)\int_0^\omega e^{-gx}\lambda(x) p(x)\,\mathrm{d}x - \beta\delta g$$

where (16)

$$\beta = \int_0^\omega x e^{-gx} c(x) p(x)\,\mathrm{d}x - (f(k)-gk)\int_0^\omega x e^{-gx}\lambda(x) p(x)\,\mathrm{d}x$$

$$- k \int_0^\omega e^{-gx}\lambda(x) p(x)\,\mathrm{d}x.$$

From the savings rule $f' = g$, the fourth term in equation (16) disappears. Where $\bar{c}$ is per capita consumption, β, the life-cycle value of a marginal

increase in the growth rate, can be expressed as

$$\beta = \frac{1}{b}[\bar{c}(A_c - A_L) - kh].$$ (17)

Finally, using equation (16) to substitute for the second term in equation (15), and noting that for constant returns $f - kg$ is F_L, we obtain

$$\delta W = \int_0^\omega U[c(x), x]\, \delta p(x) \mathrm{d}x$$

$$+ \frac{\partial U}{\partial c(0)} \left\{ F_L \int_0^\omega \mathrm{e}^{-gx} \lambda(x) \delta p(x) \mathrm{d}x - \int_0^\omega \mathrm{e}^{-gx} c(x) \delta p(x) \mathrm{d}x + \beta \delta g \right\}.$$

Reexpressed in more convenient notation, this becomes our first main result. The net life-cycle utility value of a particular age-specific change in mortality risk is given by

$$\delta W = U_{ex} + \frac{\partial U}{\partial c(0)} \left\{ F_L L_{ex} - c_{ex} + v_{ex} \beta / A_m \right\}.$$

| Life-cycle welfare increase | = | Utility of extra life-years | + | Value of extra labor-years | − | Social cost of consumption upkeep | + | Value of additional children | (18) |

The net increase in individual life-cycle welfare thus consists of four components. When mortality is improved, the individual is blessed with extra years of life, extra years of productive work if preretirement years are affected, and extra children if reproductive years are affected. On the other hand, extra years must somehow be supported. The third term shows the total amount of consumption support needed – a burden on social security, or a burden on private savings earlier in life, or a burden on one's children, depending on the particular social insurance arrangement that ensures support for the elderly.

These welfare changes occur at different periods in the life cycle. Those in the younger productive age-groups carry the consumption cost; only in later life do they reap the utility of extra years, the costs now turned over to a new generation. To the extent that population is growing, younger age-groups are larger than older ones and transfers toward later ages are easier on the individual; this is why the analysis discounts costs at rate g over the life cycle in the above terms.

2 VALUE OF LIFE

Until now I have viewed activities that put life under hazard in rather inconvenient terms as causing variations in the mortality age-profile. Is

it possible to proceed more directly and value actual lives lost or saved? In the literature, most writers prefer to deal with marginal changes in risk rather than with direct loss of life, feeling possibly that increase of risk is more approachable somehow, less awesome, than loss of life. From an actuarial viewpoint, however, risk and death cannot be separated. For any sizable population, an increase in age-specific risk means, in life-table terms, an increase in numbers of deaths at specific ages. We might therefore expect valuation of risk and valuation of lives lost to be closely connected.

Let us approach the valuation of lives lost by asking a specific question. Suppose in the community an unspecified activity were to take one life at random at age a, how much welfare would the community as a whole be prepared to give up to rid itself of the increased risk? The result will be called the Social Welfare Equivalent (SWE) of life at age a.

To answer this question, go back to the life table – to how $p(x)$ is constructed. A life-table is calculated by taking a base number of births, $\bar{B}$, (for example 10,000) and observing the year-by-year decrements in survivorship. Assume now that every $\bar{B}$ people born undergo one additional death at age a. Until age a there is no difference in survivorship; at age a there are $\bar{B}p(a) - 1$ survivors instead of $\bar{B}p(a)$; at age $x > a$ there are $(\bar{B}p(a) - 1)(p(x)/p(a))$ survivors instead of $\bar{B}p(x)$. The additional death therefore causes a variation in the mortality schedule (see Figure 2) equal to the difference in numbers surviving divided by the base:

$$\delta p(x) = \begin{cases} 0 & 0 \leqslant x \leqslant a \\ p(x)/p(a)\bar{B} & a < x \leqslant \omega \end{cases} \qquad (19)$$

I shall write $p(x)/p(a)$ as $p_a(x)$, the probability of survival to age x given survival already to age a.
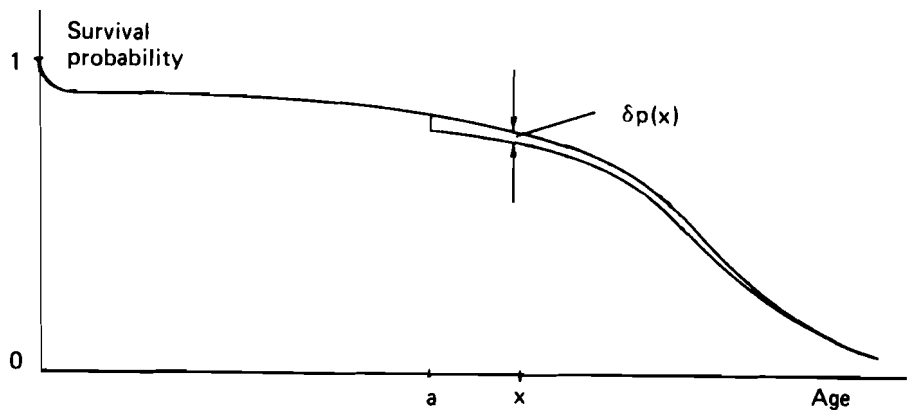

FIGURE 2   Mortality variation caused by an additional single death at age a.

We have now translated the value-of-life problem into one of valuing changes or variations in the mortality schedule; hence we can use the machinery of the previous section. Substituting the variation (19) into equation (18), the additional death imposes a risk that lowers the expected life-cycle welfare of each representative individual by an amount

$$
\mathrm{WE} = \int_a^\omega U[c(x)]\frac{p_a(x)}{\bar{B}}\,dx + \frac{\partial U}{\partial c(0)}\left\{ F_L \int_a^\omega e^{-gx}\lambda(x)\frac{p_a(x)}{\bar{B}}\,dx \right.
$$

$$
\left. - \int_a^\omega e^{-gx}c(x)\frac{p_a(x)}{\bar{B}}\,dx + \frac{\beta}{A_m}\int_a^\omega e^{-gx}m(x)\frac{p_a(x)}{\bar{B}}\,dx \right\}. \qquad (20)
$$

This expression tells us how much additional life-cycle welfare would compensate the representative person for taking on this small additional risk. It would therefore take $\bar{B}$ times this amount to compensate the total number of persons at risk, $\bar{B}$. Hence we multiply equation (20) by $\bar{B}$ to arrive at the social welfare equivalent, SWE, that would compensate for the increased risk corresponding to loss of one life at age a. This yields our second main result — a result that has an obvious actuarial interpretation

$$
\mathrm{SWE} \quad = \quad \int_a^\omega U[c(x)]\,p_a(x)\,dx \quad +
$$

Welfare equivalent of loss of life at age a $=$ Value of remaining years of life at age a $+$

$$
\frac{\partial U}{\partial c(0)}\int_a^\omega e^{-gx}\left\{ F_L\lambda(x)p_a(x) - c(x)p_a(x) + \frac{\beta}{A_m}m(x)p_a(x) \right\}dx.
$$

Value of remaining labor years at age a — Cost of remaining consumption upkeep at age a + Value of remaining reproduction at age a

$$(21)$$

Where the utility and consumption rates are roughly constant at $U(a)$ and $c(a)$ over the remaining years; where $w(=F_L)$ is the wage rate; where e_x is the expected value of remaining survival years at age x; and where $\tilde{e}_x$, $\tilde{e}_{lx}$, $\tilde{e}_{mx}$ are the discounted expected values of remaining survival-years, labor-years, and net fertility at age x, we can write equation (21) in the useful form

$$
\mathrm{SWE}(a) = U(a)e_a + \frac{\partial U}{\partial c(0)}\left\{ w\tilde{e}_{la} - c(a)\tilde{e}_a + \frac{\beta}{A_m}\tilde{e}_{ma} \right\}. \qquad (22)
$$

The result tells us that a marginal life lost is valued in terms of opportunity lost — opportunity to enjoy further life, to produce further output,

to have additional children, less, of course, consumption support costs no longer necessary.

Thus far we have assessed the value of a single life lost at a particular age. The analysis can be extended fairly simply to the case of numbers of lives lost at various ages. Consider an activity R (air travel say) that costs De^{gt} lives in yeat t, where the numbers of deaths are small relative to total deaths and are growing at the same rate as the population. Assume these deaths are distributed as $d(a)e^{gt}$ at age a, so that the probability that a life lost to this activity is aged a is $\phi_R(a) = d(a)/D$. In our analysis the cost of lives lost is imputed to this year's cohort, which stands to lose $d(a)e^{g(t+a)}$ lives at age a in year $t + a$. The value-of-life argument above is additive over lives lost; therefore for this activity in year t, total (welfare-equivalent) losses are

$$\text{Total SWE} = \sum_a d(a)e^{gt}e^{ga} \text{SWE}(a).$$

Finally, multiplying above and below by D gives the needed result

$$\text{Total SWE} = De^{gt} \sum_a \phi_R(a)e^{ga} \text{SWE}(a). \tag{23}$$

Cost of lives lost, in other words, is the number of deaths per year times the expected cost of a death in the activity in question.[6]

3 DISCUSSION AND ILLUSTRATIONS

Any risk-evaluation method must unavoidably compare two very different things: the enjoyment of additional living [U_{ex} in equation (18)] and the enjoyment of additional consumption [the terms within the braces in equation (18)]. We can simplify further discussion greatly by expressing all terms in consistent units. To do this we apply the results to the special case where the form of the utility function U does not vary with age, and U has constant elasticity of consumption ϵ, given in the usual way by

$$\epsilon = \frac{dU}{dc}\frac{c}{U(c)}.$$

In this special case, with some further algebra it can be shown that equation (18) reduces to

$$\delta W = \frac{\partial U}{\partial c(0)}\left\{\left(\frac{1}{\epsilon}-1\right)c_{ex} + wL_{ex} + \frac{\beta}{A_m}v_{ex}\right\}, \tag{24}$$

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[6] The e^{ga} factor enters to preserve consistency: the cost-of-loss-of-life argument was developed on a cohort (life-cycle) basis, whereas deaths are introduced on a period (current-year) basis.

where $w(=F_L)$ is the wage rate. Utility of additional years now reduces to c_{ex}/ϵ when translated into consumption terms. Finally, dropping the $\partial U/\partial c(0)$ factor, we may express the value of the mortality change to the individual directly as marginal consumption equivalent (CE) to

$$\text{CE}\,[\delta p] \;=\; \left(\frac{1}{\epsilon} - 1\right) c_{ex} + wL_{ex} + (\beta/A_m)v_{ex}. \qquad (25)$$

This equation shows that a crucial, but arbitrary, element in the evaluation of mortality change is the degree of diminishing returns to consumption — the degree to which pure enjoyment of additional years is offset by its consumption cost. In our well-off society we could expect additions to longevity to outweigh consumption considerations (ϵ is low), but in poorer societies (ϵ is high) utility of additional living might be offset by the additional burden of support; in certain nomadic tribes, for example, older members, if no longer productive, are expected to separate themselves from the tribe and die.[7]

One often hears two different ethical arguments where activities that put life at risk are under discussion: "life is infinitely valuable" versus "social product is what counts." In our schema these follow from different positions on returns to consumption. When ϵ tends to zero, equation (25) shows that additional life-years outweigh any consumption considerations: activities should be judged only on whether they preserve and prolong life. When ϵ is one, "utility is consumption," and extensions to life are perfectly offset by their consumption cost: only social product considerations remain. Normally, where returns to consumption are in the usual range, ϵ between zero and one, equation (25) retains elements of both ethical positions.

We can use equation (25) to comment on the two methods in present use. *Willingness-to-pay*, as usually interpreted, ignores the negative social burden term. In the usual case where the reproductive term is negligible, it will therefore overstate the value of mortality reduction and unduly bias against risky projects. *Human capital* tends to understate this value and therefore to bias toward risky projects. Only in the special case where (a) altered risks do not affect childbearing ages, (b) population growth is vanishingly small, and (c) utility shows constant returns to consumption ($\epsilon = 1$), would the (gross) human capital method be justifiable and correct. In this case additional life-years would be exactly offset by their

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[7] Even in Western society, life could not be extended much beyond 100 years unless retirement age were also increased. See Boulding (1965) for an entertaining essay on the economic menace of extreme longevity.

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consumption cost, so that equation (25) would reduce to the human capital measure:

$$CE = wL_{ex}. \tag{26}$$

An Example: Cardiovascular Diseases

To illustrate equation (25), let us assess the worth to the individual of elimination of cardiovascular diseases in the United States. Using the cause-deleted life tables of Preston *et al.* (1972), Table A1 in the Appendix shows the age-specific mortality variation that would result. Under 1975 U.S. data (again see the Appendix) and the definitions in (13), complete elimination of cardiovascular diseases yields the differentials

Extra years	c_{ex}($)	L_{ex}(years)	v_{ex}
7.69	42,670	0.692	0.00135

Cardiovascular diseases attack for the most part postproductive and post-reproductive age-groups. Hence, though longevity increases significantly, expected working-life and expected number of children increase only a little.

Where $\epsilon = 1.0, 0.6$, and 0.4, from equation (25) we obtain

$$CE = \begin{matrix} (1.0 & -1)\,42{,}670 \\ (1.667 & -1)\,42{,}670 \\ (2.5 & -1)\,42{,}670 \end{matrix} + (13{,}749)\,0.692 + (-68{,}125)\,0.00135$$

$$= \begin{cases} \$\ 9{,}400 \\ \$37{,}800 \\ \$73{,}400 \end{cases}$$

This of course does not imply the United States should spend corresponding amounts per person on the elimination of cardiovascular diseases. A flood of research dollars would by no means guarantee such a breakthrough. The illustration, however, gives an idea of the potential returns to the individual.

Value of Life

Having expressed the value of mortality change in consumption terms, we can do the same with the cost of a life lost for the special constant-elasticity case treated above. Expression (21) may then be reexpressed as a social consumption equivalent (SCE) of a life at age a:

TABLE 1   Expected additional life-years, labor-years, and reproduction, and illustrative cost of loss of life at age a.[a]

	Age a								
	0	10	20	30	40	50	60	70	80
$\tilde{e}_a$	70.3	62.5	52.9	43.5	34.3	25.6	18.0	11.7	6.7
$\tilde{e}_{la}$	31.6	32.5	31.4	24.7	17.6	10.8	4.4	0.3	—
$\tilde{e}_{ma}$	0.921	0.949	0.882	0.339	0.038	—	—	—	—
SCE ($1000) $\epsilon = 1.0$	371	382	371	316	239	148	61	4	—
$\epsilon = 0.6$	668	664	619	520	399	265	139	54	31
$\epsilon = 0.4$	1,055	1,031	942	783	605	417	241	119	72

[a] Based on U.S. 1975 data (see Appendix). Values in 1975 U.S. dollars.

$$\text{SCE}(a) = \left(\frac{1}{\epsilon} - 1\right) \int_a^\omega e^{-gx} c(x) p_a(x) dx + w \int_a^\omega e^{-gx} \lambda(x) p_a(x) dx$$

$$+ \frac{\beta}{A_m} \int_a^\omega e^{-gx} m(x) p_a(x) dx. \tag{27}$$

which can be written, when consumption is at constant level $c(a)$ over the remaining years, as

$$\text{SCE}(a) = \left(\frac{1}{\epsilon} - 1\right) c(a)\tilde{e}_a + w\tilde{e}_{la} + \frac{\beta}{A_m} \tilde{e}_{ma}. \tag{28}$$

Table 1 gives an idea of the magnitude of the SCE at different ages and different returns to consumption. We can see from this illustration[8] that the cost of a life lost, under the chosen criterion of expected lifetime well-being, is highly age-dependent. Saving a life that otherwise might be lost in a maternity ward might therefore be quite different from saving a life that might otherwise be lost to cancer.

This last point can be illustrated by comparing the social gain from saving (restoring to normal survival probabilities) a life chosen randomly, otherwise lost to motor-vehicle-accident death, maternal death, or cancer death. Table 2 gives probability distributions over age, $\phi(a)$, for deaths due to these causes. The expected gain in saving one life at random in

[8] A couple of caveats are necessary here. These figures do not include any cost to kin of the loss of life of their loved one. Secondly, SCE at age 0 would not be a suitable way to measure the desirability of introducing an additional birth: the analysis calculates how much those *already* born would give up to avoid certain types of risk.

TABLE 2   Age patterns of incidence for three causes of death.[a]

Cause of death	Age								
	0	10	20	30	40	50	60	70	80
Motor vehicle	0.027	0.045	0.224	0.152	0.119	0.117	0.119	0.116	0.081
Maternal	–	–	0.25	0.452	0.299	–	–	–	–
Neoplasms	0.003	0.005	0.006	0.014	0.041	0.116	0.229	0.319	0.267

[a] From Preston et al. (1972), data for United States 1964.

TABLE 3   Comparison of preventing death from three alternative causes.

Cause of death	Expected additional			SCE $1,000 (1975)		
	Survival years	Labor years	Net reproduction	$\epsilon = 1.0$	$\epsilon = 0.6$	$\epsilon = 0.4$
Motor vehicle	34.3	17.0	0.32	212	369	574
Maternal	43.1	24.3	0.39	307	509	770
Neoplasms	15.5	3.9	0.02	52	121	221

year 0 is obtained from equation (23) as

$$\text{SCE} = \sum_a \phi_R(a)e^{ga}\text{SCE}(a). \tag{29}$$

From this expression, and the above tables, we obtain the results shown in Table 3. It should be noted that the effort or cost required to prevent loss of life may be quite different for each of these causes and is not considered here.

Extensions to the Analysis

Various other factors could have been included in the analysis of sections 1 and 2 if we cared to complicate the model. Two are mentioned here. First, when life of loved ones is valued, person i's utility rate U^i might include the extra enjoyment α_j^i that loved ones j (with age differences a_j) are alive:

$$U^{i*} = U^i + \sum_j \alpha_j^i p(x + a_j),$$

whence life-cycle welfare becomes for person i

$$W^i = \int_0^\omega U^i p(x)dx + \sum_j \int_0^\omega \alpha_j^i p(x + a_j)p(x)dx.$$

Under this criterion the social value expression (18) would contain an extra kith-and-kin term:

$$\sum_j \alpha_j^i \int_0^\omega (\delta p(x + a_j)p(x) + p(x + a_j)\delta p(x))\mathrm{d}x.$$

Lessened mortality risk, in other words, is twice valuable — for any person it increases both the chance that his parents and grandparents will survive to be enjoyed and the chance he will survive to enjoy his children and grandchildren. The value-of-life expression, (21), would be modified in a similar fashion.

Second, a change in length of life may induce a change in the age of retirement or in the age-specific labor participation schedule. For this case, analysis shows that the expected working-years terms in equations (18) and (21) should be expanded to reflect extra labor years due to increased participation, as well as increased survival.

Robustness

How robust are the results of sections 1 and 2 when the assumptions of the model are replaced by more realistic ones? Recall that we assumed economic and demographic steady-state growth, constant returns in production, perfect life-cycle financial markets, and similar individuals who face similar mortality schedules.

Note first that the most important factors are scarcely changed under increased realism. When risks to life fall for the population or a life is saved, (a) the individual does enjoy extra years, extra working life, and perhaps extra reproduction, and (b) whatever the support mechanism for old age, be it gifts to tribal elders, Robinson Crusoe stockpiling, or a government social-security system, consumption must still be set aside for lengthened life (although the amount may now depend on the transfer mechanism). With nonconstant returns in production and imperfect life-cycle markets, the valuation of these factors would change, however. The marginal value of consumption may well vary more widely than in equation (12), labor would not necessarily be paid its marginal product, and the value of growth, β, would be altered. With nonoptimal investment, an extra capital–labor ratio adjustment term would enter. These changes are relatively minor. More important is the case where altered mortality risks strike the population unevenly, or the mortality change comes suddenly, or demographic and economic growth vary widely from steady-state. In this case, some people may reap the benefits of increased life and production, while others bear the consumption

costs. For example, a sudden mortality improvement can be a windfall to the elderly — they enjoy extra years while escaping the corresponding extra support of the generation that went before.

4 CONCLUSION

This paper derived expressions for the value of activities that alter the mortality schedule and for the cost of premature loss of life, under specific assumptions and a life-cycle welfare criterion. A change in the pattern of the mortality schedule, it was shown, should be assessed by the difference it makes to expected length of life, production, reproduction, and consumption support; loss of life should be assessed by the expected opportunity costs of lost years, production, and reproduction, less support costs.

Full age-specific accounting, where labor participation, consumption, fertility, mortality, and utility depend on age, brings an actuarial precision to the results: the separate implications of mortality change — for length of life, production, consumption, and reproduction — can be assessed quite accurately. It also shows that it is meaningless to talk about a single value of life: the age of the life (or a probability distribution for it) must be specified. Valuation of life in fact depends heavily on age, as the illustrations above show; this follows directly from our choice of a life-cycle criterion. A life lost at age 80 has less opportunity to contribute to this criterion than one lost at age 30, hence the implied value of life decreases with age. This sits comfortably, for the most part, with our intuitive feelings; if we felt, on the other hand, that "a life is a life whatever the age" a life-cycle criterion would be no longer appropriate.

The simple, general-equilibrium framework adopted in this analysis shows that social-support costs figure large in the valuation of risks to life. The degree to which these offset the pure enjoyment of staying alive makes a significant difference to numerical assessments. Where being alive is valued much more highly than pure consumption, additional support costs, like additional wage earning, fade from significance. But where the value of being alive is measured purely by additional consumption — where utility shows constant returns to consumption — the gain from added longevity is canceled completely by the additional consumption support required.

APPENDIX

The illustrations use U.S. data chosen to correspond to year 1975. All data and illustrations are for male and female combined. (In the maternal

TABLE A1 Age-specific survival schedules and variation caused by elimination of cardiovascular diseases.[a]

Survival probability	Age								
	0	10	20	30	40	50	60	70	80
$p_E(x)$	1.000	0.97029	0.96402	0.95260	0.93747	0.91056	0.85807	0.76547	0.62615
$p(x)$	1.000	0.97000	0.96343	0.95091	0.93149	0.88841	0.79228	0.61135	0.34853
$\delta p(x)$	0.000	0.00029	0.00059	0.00169	0.00598	0.02215	0.06579	0.15412	0.27762

[a] From latest available cause-of-death life tables: Preston *et al.* (1972), for United States 1964.

TABLE A2 Labor participation schedule.[a]

	Age									
	–16	16–19	20–24	25–29	30–44	45–49	50–54	55–59	60–64	65+
$\lambda(x)$[a]	—	0.549	0.751	0.755	0.744	0.738	0.704	0.646	0.479	0.131

[a] Source of data: 1LO Year Book 1976; data for United States 1975.

TABLE A3 Fertility schedule.[a]

	Age							
	10–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49
m(x)[a]	0.0006	0.0290	0.0595	0.0566	0.0272	0.0101	0.0024	0.0001

[a] Source of data: Statistical Abstract of the United States 1977; data for 1974.

death illustration, excepting the age-incidence distribution, to preserve consistency the same combined male/female data were used as in the other illustrations.)

The tables above give the survival, labor participation, and fertility schedules used in the illustrations. In Table A1 $p(x)$ is the usual survival table, used throughout the illustrations; $p_E(x)$ would obtain if cardiovascular diseases were eliminated; $\delta p(x)$, the variation caused by eliminating these diseases, is the difference. The 1964 survival probabilities are used throughout the illustrations; mortality in the U.S. has changed but little in the last 15 years. For conciseness, only 10-year intervals are shown above; most calculations, however, were based on 5-year intervals. Preston (1976) contains further details on cause of death.

Other data[9] used in the illustrations are

$$L = 94{,}793{,}000, \quad N = 213{,}137{,}000, \quad \bar{c} = \$6{,}142, \quad K = \$4{,}303 \text{ billion.}$$

In the absence of a usable consumption age-schedule, it is assumed that those 15 and under consume one-half of an adult's standard consumption; those 65 and above, three-quarters. This yields, for consistency with average consumption $\bar{c}$,

	Age		
	0–15	15–65	65 +
$c(x)$	$3,582	$7,164	$5,373

Computations on the above data, smoothed where necessary, yield[10]

$$g = 0.00; \quad b = 0.148; \quad A_c = 38.07; \quad A_L = 38.92; \quad A_m = 25.21$$

$$\beta = -\$1{,}716 \text{ million}; \quad \beta/A_m = -\$68{,}125; \quad w = \$13{,}749.$$

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[9] Demographic data for the United States in 1975 were taken from the ILO Year Book 1976; economic data in 1975 dollars were taken from the 1977 Statistical Abstract of the United States. In the consumption figure, government expenditures were treated as part of consumption. K represents total reproducible assets.

[10] For discussion of β, the value of a marginal increase in g, and why it is negative, see Arthur and McNicoll (1978). For consistency with the life-cycle model here ($g = 0$), w, the wage rate, was computed from equation (12): $\int c(x)p(x)dx \equiv w \int \lambda(x)p(x)dx$.

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