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Abstract

We discuss some issues involved in modeling of complex systems composed of dynamically interacting agents. We describe a prototype of simulation environment INFOGEN created for modeling of such systems with the aim of evaluating strategies of enterprizes in the information economy, but applicable to general multiagent systems. The case study is presented along with the mathematical description of the multi-agent systems.

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1 Introduction

In this paper we describe an approach for modeling complex systems composed from independent entities called *agents* dynamically interacting between each other. The original aim was to create a simulation tool for evaluation of strategies of enterprises in the new emerging information industry. This industry is developing now as a merger of telecommunications, computer industry and content provision. In this context the modelled system is information economy and agents are the enterprizes and business units involved in creation, production and distribution of information products, network providers, consumers of information products, government and regulation agencies. Such agents make decisions about consumption, transformation and exchange of information and other *resources*, expand their production facilities, formulate their strategies in order to achieve specific aims. These decisions are taken in asynchronous and distributed manner. Agents may combine different *roles* within economy, like content provision and delivery of information service. If carried far enough this project could involve creation of virtual information economy. At this point we have created the prototype of agent-based simulation system INFOGEN and the methodological framework for its further development. This paper summarizes the work done so far and indicates some directions for further research. Our contribution goes beyond our original aim of modeling information economy. In fact, INFOGEN can be used potentially for simulation of wide range of complex systems with distributed decision making operating in changing and/or uncertain environment, for example financial markets.

Modeling of information industry and, more generally, economic system undergoing rapid technological and structural change, poses the challenges which are not yet fully addressed by traditional economic modeling. Some of these modeling challenges consist in finding adequate approaches for treating nonstationarity and uncertainty of economic environment, bounded rationality of economic agents, rich variety and complexity of dynamic interrelations between different agents.

Nonstationarity. The large part of traditional economic theory and modeling is centered around perfect markets in the state of equilibrium. In such systems the operation of market forces smooth out disturbances introduced by uncertainty and random events and leads the system to ergodic state of equilibrium. In case of rapid technological change this

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is no longer the case because relatively small disturbances and decisions with small immediate impact can have self-magnifying properties due to the positive feedbacks present in the system (technologies with increasing returns) [4]. This leads to nonergodicity of the system which requires from modeller the shift of the emphasis from stationary to transient behavior.

Uncertainty. The lack of ergodicity increases importance of adequate treatment of uncertainty present in the system. There are two levels of uncertainty present in the system. There is external uncertainty represented by demand patterns, technological change and differend kinds of random perturbations. Internal uncertainty is due to the fact that each agent takes decisions without full knowledge about states and actions of other agents. Thus, both models of uncertainty and behavior of economic agents under uncertainty should be included in the system.

Complexity. Traditional economic modeling deal with systems composed from fairly homogeneous agents with similar behavior patterns. We needed instead the capabilities to model rich variety of relations where the same agents can compete in one field and collaborate in another overlapping field, assume different combinations of industry roles, possess different knowledge about the state of the whole system. This complexity leads to the multitude of positive and negative feedbacks in the system which under different values of system parameters can lead to different equilibriums, and even chaotic behavior. Even without chaos the presence of multiple equilibria leads to catastrophic behavior, i.e. in certain points the system abruptly switches between different equilibria with arbitrarily small change of system parameters. Important objective here would be to define regions of stability in the space of the system parameters.

Bounded rationality. Traditional economic theory assumes that economic agents are perfectly rational and their behavior is governed by maximization of certain utility function. Nonstationarity, uncertainty and complexity makes this assumption too restrictive in many cases. Besides these traditional models we included in our system some more realistic models of agent behavior which assume the bouded rationality of agents, i.e. that their decision actions result from the set of heuristics which vary according to changing of information patterns, environment and goals [3]. Such heuristics are constantly being evaluated against obtained results and new heuristics are generated.

Our objective was to create a system capable of modeling these features of information economy. In order to achieve this we draw upon recent advances in methodology of operations research, simulation, computer science and economic modeling. In particular, simulation models of asynchronous systems were developed in the theory of Discrete Event Dynamic Systems (DEDS) together with interplay between simulation and optimization [2, 11, 15, 16, 17, 19, 24, 25, 26]. Decision making principles in the presence of uncertainty were considered in the field of stochastic programming [6, 8, 12, 14, 10, 18, 20, 22]. Dynamic behavior of systems composed from interacting agents was studied within the framework of evolutionary approach [1, 5, 4, 9, 21, 23]. Related work in computational economy and market-oriented programming resulted in creation of several tools for distributed resource allocation in financial and other fields [13, 27, 28, 29].

The rest of the paper is organized as follows. Architecture of the system INFOGEN is described in section 2. Section 3 is dedicated to a case study of competition between the producers of an innovative product. Section 4 contains the mathematical description of multiagent system which underlies INFOGEN.

2 Architecture of the system INFOGEN

INFOGEN stands for INFOrmation economy modeling through aGENt programming. In this section we provide an informal description of INFOGEN architecture. It is a discrete event simulator which consists of three main components: *resources*, *agents* and *market*. It simulates the evolution of the economy as distributed transformation of resources by agents and exchange of resources through market. The mathematical description of the general multiagent system which underlies INFOGEN can be found in section 4.

2.1 Resources

These are the elementary entities from which the system is composed. Or, they can be viewed as an alphabet in which the system is described. In our terminology we consider ´resources` any commodity or entity which is exchanged, satisfied, manufactured or in any other way changed by economic agents relevant to modeling purposes. Thus, besides resources in the economic sense of the word other examples of resources are money, all kinds of products, services and needs. In our system resources are divided in five types: *money*, *input respource*, *output resource*, *internal resource* and *final demand*.

Money. This is obligatory resource which is always present in the system and which flow is treated separately from flow of other resources. This is due to its economic function of exchange and because performance of agents is often measured in money terms.

Input resources. These are the resources which are used by agents for creation of products and services and satisfaction of needs. For example, in the case of the agent representing an Internet provider one of the input resources may be the lines which he leases from a telephone company. In case of the agent representing an Internet user some of the input resources are fixed local phone service and the Internet connection. Input resources are bought by an agent at the market and may be stored.

Output resources. These are the products and services into which agents transform input resources and which are offered to the market. For example, for Internet provider an output resource is the capacity to provide an Internet connection of given quality, while for telephone company the output resource is the capacity to provide a phone connection. From these examples it is clear that the output resources for one agent are the input resources for some other agents. Output resources can constitute the *offers* to the market and they can be stored.

Internal resources. These are resources which are possessed by agents and are necessary for transformation of the input resources into output resources. Examples of such resources are qualified manpower or production capacities. For example for Internet provider his Internet node would be his internal resource, for Internet user it would be his personal computer and specialized software, for a phone company it is her network. Input resources can be expanded and otherwise developed and they should be subjected to maintenance. Money and input resources are needed for both maintenance and development.

Final demand and needs . These are the final resources which drive the economic activity of the system. They are not transformed or exchanged in the sytem and constitute needs and demands of the end user. What are the final resources very much depend on the purpose of the modeling. Suppose, for example, that we model the penetration of the new telecommunication voice service, like voice over Internet. Then the final resource may be just "demand for voice over Internet" represented by some expert prediction. On the other hand we might be interested in looking closer how this demand is formed according to some hypotheses about behavior of customers and price and quality structure of competing voice services. In this case the final resource would be "the need for voice communication" measured, for instance, by distribution of time per day for various types of customers. Input resources in this case may be "fixed phone connection", "mobile phone connection", "Internet connection" and "other means".

This resource structure is very flexible and can be easily modified by reassigning resources to different types and aggregation/disaggregation according to modeling needs.

2.2 Agents

Agents transform and exchange resources described previously. We developed generic agent structure which can be specialized in the rich collection of agents by specifying agent parameters for particular purposes. This structure permits to model a variety of economic actors from enterprizes to individual users. Such flexibility is important because we needed the capabilities to model agents which combine the multitude of industry *roles*.

Roles. In the rapidly evolving information economy one of the most important issues for newly emerging company as well as for established industry leader is which industry roles to assume. Should established fixed network provider go into providing Internet service, or form a strategic alliance with provider of cable television? Thorough analysis of information industry roles can be found in [7]. After preliminary analysis we understood that all industry roles except regulatory roles can be represented in the alphabet of resources described above, i.e. as transformation of specific set of resources into another set of resources and their exchange. From this resulted that the agents themselves can be represented in terms of this alphabet.

Thus, the *generic agent structure* in our system consists of *resource sets*, *transformation functions* and *strategies.*

Resource sets. There is the total set of resources for all system. Each agent is characterized by four subsets of this set, i.e. set of input resources, set of output resources, set of internal resources and set of needs. Input resources are all resources which are transformed by this particular agent into internal and output resources and in need satisfaction. For partucular agents generated from the general structure some of these sets may be empty. At each time moment the state of an agent is characterized by available money and internal resources and by stocks of input and output resources.

Transformation functions. There are four sets of such functions in the general agent structure: *production functions*, *development functions*, *maintenance functions* and *satisfaction functions*. *Production functions* tell how much of money, internal and input resources are needed for production of the given quantity of the output resource. They have the following structure:

$$v_i = \psi(a, v_o) \tag{1}$$

where v_i is the volume of specific input or internal resource or money, v_o is the volume of the output resource and a are production parameters. In the simplest case these functions could be linear, however we are specifically interested in case of increased returns and economies of scale. In such case $\psi(a, \cdot)$ is concave function which may asymptoticaly tend to linear with increasing argument. The simplest case of such function is the following:

$$v_i = a_1 v_o \frac{1 + a_2 v_o}{1 + a_3 v_o}$$

where the case when $a_2 > a_3$ describes increasing returns and $a_2 < a_3$ corresponds to diminishing returns.

All other types of transformation functions have the same structure (1) as production functions. *Development functions* describe amounts of input resources and money necessary for expanding production capacities for given amount. *Maintenance functions* define amount of money and input resources necessary for maintenance of internal resources and stocks of input and output resources. *Satisfaction functions* define amount of money and input resources necessary for satisfaction of a need.

Strategies. Strategies are actions which agents undertake in order to achieve specific aims. Strategies depend on amount of money and other resources available to an agent and on information available on the states and strategies of other agents. The general agent structure includes three types of interrelated strategies: *pricing strategies*, *development strategies* and *purchasing strategies*. All these strategies in some cases may be derived by solving dynamic optimization problems (see section 4). In more complex cases such strategies can be based on principles of adaptivity and bounded rationality.

Pricing strategies define the price which an agent offers for its output resources (products and services). In one our case study we implemented the principles of bounded rationality as follows. Each agent had a set of several strategies: keep the market price, increase the price or decrease the price based on previous history. On each step an agent could choose from one of such strategies according to probabilities which were updated according to their performance in terms of income and revenue, similar to the theory of learning automata.

Development strategies. If demand exceeded production capacities an agent can choose between increasing the price or expanding production capacities. Development strategies govern such expansion taking into account that newly added capacities are becoming operational after some delay.

Purchasing strategies. These strategies are employed by the agents for selection of offers for required input resources present in the market. The simplest strategy is, of course, to choose the offer with smallest price. We take into account, however, that for real economic agents the price considerations are not necessarily unique and allow customers to migrate between offers with different price with some dynamics dependent on other attributes of an offer.

At this moment we implemented some basic set of strategies which is in the process of expansion.

Specific agents are generated from this general agent structure by specifying its elements. Here are some types of the agents with which we experimented.

Production agent. This agent puts on the market products and services producing them on production capacities using input resources bought on the market, but do not have final needs to satisfy. These agents further differ by their set of strategies.

End user agent. This agent satisfies the final needs by purchasing products and services on the market. This agent is further characterized by capability to substitute different products to satisfy the same need. For example, the need for voice communication can be satisfied by fixed phone, mobile phone or voice over Internet.

Pure supplier. This agent has only output resource which supplies for the price derived from the expert estimates. This agent is useful to model supply of important product which flow we do not want to describe in much detail due to modeling purposes. One example is the regulatory commission which distributes frequences for broadcast transmission.

2.3 Market

Market is the environment in which agents operate. Each period of time agents which produce output resources put their *offers* on the market. Each offer consists of quantity and price of specific resource. Agents which are customers for the input resources go to the market and choose between offers. For the case when demand exceeds supply the system has the set of rules which distribute available supply between customers. Producing agents may then decide to increase the price for the next period and/or to expand production capacities. There is the set of balancing mechanisms which are needed because unsatisfied demand of one agent may result in decreasing of its offer to the market which in turn may result in diminishing satisfaction of demand of other agent. One possibility is to use the generalizations of Walras tatonnement process [29].

3 Case study: competition of producers of an innovative product

In order to illustrate our modeling approach we present in this section a simple but illuminating case study. Consider a market which caters for satisfaction of some need of pool of end users. At some point as the result of technological innovation appears some other product, or several products which can satisfy the same need, but in some new way or some new aspect of it. Some companies, old or new, start to develop these products and offer them to the market. Doing so they can adopt different development and market strategies. It is important to understand which parameters of such strategies are crucial for success.

There are many examples of such situation in the present technological reality. Consider, for example, the recent developments in provision of voice telecommunications. The need of the end users to be satisfied is the need of remote communication by voice. There was established market with traditional product which is fixed telephony. Then appeared new services like mobile telephony and very recently the possibility of voice through Internet. Another example is video on demand versus more traditional film distribution like cinema and video rentals. Still another example is the competition between different Internet providers which compete between themselves and against traditional means of provision of information services.

In what follows we present a simplified description how this situation can be modeled with the help of INFOGEN and some results of experimentation. Let us follow the description of INFOGEN architecture presented in the previous section.

Resources. In the simplest case we have five resources in the system: money, end user need, traditional product, innovative product (may be more then one), production capability of innovative product.

Agents. There are three types of agents present in the system: end users, producers of innovative product, suppliers of traditional product.

End users. They operate on four of the resources defined above: end user need, two input resources (traditional product and innovative product) and money. Their production function describes how fixed amounts of traditional and innovative products satisfy their need, taking into account the possibility to substitute one product by another. Their strategy consists of two components: maximization of need satisfaction given amount of money available each period and purchasing strategy according to which they select between offers of producers of innovative product.

Producers of innovative product. In the simplest case they operate on three resources: money, production capacity as internal resource and innovative product as output resource. In this case the only resource which is used for expansion of production capacity is money. For more detailed modeling production capacities may be disaggregated and input resources may be added for both production and capacity expansion. Production, development and maintenance functions of these agents describe how much money and production capacity is needed for production of the given quantity of innovative product and how much money is needed for production capacity expansion and maintenance. These agents have the pricing strategy and production expansion strategy. We modeled several types of these agents which differ by their strategy sets.

Suppliers of traditional product. In the simplest case this agent operate only one resource: traditional product which is the output resource for him. It has infinite stock of this resource which he supplies to the market for given price which may vary between time periods. In more involved modeling we might be interested in possible strategies of this type of agents for countering the invasion of the new product, in this case it is necessary to specify the structure of this agent in more detail.

We used INFOGEN for different experiments with this model. We have found that the behavior of the system very much depends on the set of strategies adopted by differend producers of innovative product and on parameters of such strategies. System may tend to equilibrium in which some producers will conquer certain market shares, while others will perish. There are multiple equilibria which are characterized by different sets of survived agents and the system may switch between different such equilibria in discontinuous fashion, i.e. with arbitrarily small change of strategy parameters. In some cases the system can exhibit chaotic behavior with different producers having their market shares oscillate widely.

Two sets of experiments are presented here in order to illustrate these points. The first set is comprised by Figures 1-4 (see the end of the paper). In this case there are two identical producers of innovative product which differ by their market strategies. The first agent sest its price independently selecting on each step from several adaptive strategies according to their past performance with respect to revenue maximization. The second agent have the information about the price adopted by the first agent and varies his price around the price of the first agent again according to adaptive strategies. On these figures the upper straight line represents the maximal possible value of the market for the innovative product, i.e. in the case when all traditional product is substituted. The lower horisontal line represents the half of the maximal market value, this is the reasonable market share for each agent since they are identical. The thick upper curve represents the total market for the innovative product and two lower intertwined curves represent revenues of individual agents.

Figures 1-4 represent various patterns of market evolution depending on the parameters of the market strategies of the agents. On Figure 1 we have the total collapse of the market for the innovative product followed by near collapse with rebound on Figure 2 and the victory of the innovative product on Figure 3. Figure 4 represent the case of chaotic behavior with survival of both traditional and innovative products and oscillations of market shares both between new and traditional products and between manufacturers of the new product.

Example of discontinuous switch between different equilibrium points is presented on Figures 5-8. Here again we have two identical agents this time defining their market strategies independently. They differ, however, by their initial market share and by the capability of rapidly, or decisively react to observed revenue fluctuations measured by one of the strategy parameters which we call reaction parameter. In reality such greater flexibility may be caused by larger innovation capacity. The larger reaction parameter the larger variations can introduce the agent in its prices. In all four cases the first agent has larger initial market share and the value of reaction parameter fixed to 0.05. On the Figure 5 the second agent has the same value of reaction parameter as the first agent and the larger initial market share proves to be decisive: the first agent quickly conquers the whole market. While the second agent increases its reaction parameter it survives longer and longer and finally between values 0.066657 and 0.066658 the system changes equilibrium in a discontinuous (catastrofic) fashion: the second player starts to win the whole market share (Figures 6 and 7) which continues with the further increase of the reaction parameter (Figure 8).

4 Mathematical description of underlying multiagent system

This section contains more precise mathematical description of multiagent system which underlies INFOGEN described in section 2. It should be read keeping in mind informal explanations of this section.

4.1 Definition of resources

We consider the set W of n resources ν_i, $i = 1 : n$. Each of these resources can be characterized by its quantity z_i, and the vector of other attributes $v_i = (v_{i1}, ..., v_{iK_i})$. Some of the components of these vectors can be continuous, while others can be discrete. For example, among these attributes can be quality for information resources, or packet loss for connections in data network. The values of these quantities belong to specified sets:

$$z_i \in Z_i, \quad v_i \in V_i,$$

Often there are additional constraints on the possible values of quantities and attributes which can be expressed in the following way:

$$(z_i, v_i) \in Y_{ik}, \quad k = 1 : M_i, \quad Y_{ik} \subseteq Z_i \times V_i \tag{2}$$

We treat money here as special resource $\nu_0 \in W$ which has empty attribute, thus money is characterized by couple $(z_0, \emptyset)$ where z_0 is the amount of money.

4.2 Structure of resource space

The space of resources can be associated with oriented *resource graph* (W, A). The set W of vertices of this graph coincide with the set of resources while the set of oriented arcs $A \subseteq W \times W$ defines resource transformations. Let us explain this in more detail.

For each $\nu_i \in W$ let us denote by W_i^+ the set of all vertices from which oriented arcs point to ν_i and by W_i^- the set of all vertices to which oriented arcs point from ν_i:

$$W_i^+ = \{\nu_j : (\nu_j, \nu_i) \in A\} \quad W_i^- = \{\nu_j : (\nu_i, \nu_j) \in A\}$$

The set W_i^+ is called *consumption set* for resource ν_i because it is constituted from all resources which are consumed in the process of "production" or "transformation" of resource ν_i and the set W_i^- is constituted from all resources for which transformation the resource ν_i is needed.

The resources ν_i for which $W_i^+ = \emptyset$ are called *primary resources.* They are not transformed within the system and are taken from outside. As far as modeling of information industry is concerned, the examples of such resource are land or manpower.

The resources ν_i for which $W_i^- = \emptyset$ are called *final resources.* Such resources are not used in further transformations within the system. In our case examples of such resources are needs of the end users and the government.

Resources ν_i for which $W_i^+ \neq \emptyset$ are produced or transformed within the system. With each such resource is associated the *production function*:

$$(z_i, v_i) = \psi_i((z_j, v_j), \nu_j \in W_i^+) \tag{3}$$

which defines the quantity z_i and attributes v_i of resource ν_i which can be obtained from resources belonging to consumption set taken in quantities z_j with attributes v_j. Sometimes production function can be approximated by a linear function with respect to quantities:

$$z_i = \sum_{j:\nu_j \in W_i^+} c(v_i, v_j) z_j$$

where $c(v_i, v_j)$ are some coefficients which depend on the attributes of respective resources. More often this is monotonously increasing concave function of quantities which reflects *economy of scale.*

With each production function (3) is associated *expenditure function* which defines the amount of money u_i necessary to purchase the resources $(z_j, v_j)_{\nu_j \in W_i^+}$

$$u_i = \phi_i((z_j, v_j), \nu_j \in W_i^+) \tag{4}$$

Often this is a linear function of resource quantities.

4.3 Role layer

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Roles define relations and structures on the production graph defined above and are associated with different operations which can be applied to resources. We consider here only transformation roles. Such roles are responsible for transformation of one resources into others. Many structural and infrastructural roles of information industry fit into this category, among them information and information service production, provision and brokerage roles.

Each such role is associated with some subset B of vertices of production graph, $B \subset W$. Different resources belonging to B have different functions within this subset. Some of them are principal resources, others depend on principal resources. Let us consider, for example, information production role, like production of movies. In this case there is principal resource which is movie and secondary resource which is promotion material about movie. According to different functions of resources belonging to the subset B of a particular role this role can be subdivided into subroles, some subroles are dedicated to production proper, while other subroles are dedicated to maintenance of relations with customers/suppliers.

More formally, let us consider the set R of transformation roles,

$$R = \{r_1, ..., r_K\}$$

We assume that there is a set $\bar{W} \subseteq W$ of *principal resources* and collection of subsets W_k, $k = 1, ..., K$. Each transformation role is associated with a subset W_k. Suppose that $\bar{W}_k$ is the set of principal resources among W_k:

$$\bar{W}_k = W_k \cap \bar{W}$$

We assume that the collection of sets $\bar{W}_k$ constitutes a partition of set $\bar{W}$:

$$\bar{W} = \cup_{k=1}^{K} \bar{W}_k, \quad \bar{W}_k \cap \bar{W}_l = \emptyset, \quad k \neq l$$

4.4 Agent layer

Agents constitute independent entities which combine one or more transformation roles and form supplier/consumer relationships with each other.

Let us define this notion formally. By P we denote the set of agents,

$$P = \{p_1, ..., p_M\}$$

Each agent can be associated with some subset of transformation roles $R_m \subseteq R$. Recall that for each role r_i correspond some set of resources W_i.

4.5 Supplier/consumer graph

Resource graph and mapping between roles and agents permit to define supplier/consumer graph. This graph plays fundamental role in multiagent system. It is oriented graph (P, D) with vertices which coincide with the set of agents P and set of oriented arcs $D \subseteq P \times P$. For each vertice p_i of this graph let us define the set of all resources Ω_i produced by corresponding agent:

$$\Omega_i = \bigcup_{j:r_j \in R_i} W_j$$

and by Ω_i^+ the set of all consumed resources for this agent:

$$\Omega_i^+ = \bigcup_{j,k:\nu_k \in W_j, r_j \in R_i} W_k^+$$

Similar to resource graph for each vertice p_i let us define the sets of predecessors P_i^+ and children P_i^-:

$$P_i^+ = \{p_j : (p_j, p_i) \in D\} \quad P_i^- = \{p_j : (p_i, p_j) \in D\}$$

The meaning of this structure is the following. The vertices correspond to production and/or transformation of different resource sets. Oriented arcs point from suppliers to producers/consumers. Resources flow in the direction of these arcs and money flows in the opposite direction. Thus, the set P_i^+ is the set of all agents which supply resources needed for production and transformation of resources from Ω_i and the set P_i^- is the set of all agents which consume resources from Ω_i. There should be the following relation between this graph and resource graph:

$$W_k^+ \subseteq \bigcup_{j:p_j \in P_i^+} \Omega_j, \quad \forall k : \nu_k \in \Omega_i$$

which means simply that for all resources produced by a agent p_i the set of resources needed for its production is among the set of all resources produced by suppliers of this agent.

4.6 Market layer

This layer describes relations between agents as they are unfolded in time. They are perceived as flow of resources along the supplier/consumer graph.

4.6.1 Time structure

The system is evolving in discrete time $t = 0, 1, \ldots$. The units of this time can correspond to different units of real time, from months to years. The consumer/supplier graph described in the previous section depends on time. This is done in order to describe such phenomena as emergence of new agents, union of different agents into one agent, change of consumer/supplier relations described by oriented arcs of this graph, change of roles played by different agents:

$$(P, D) = (P(t), D(t)), \quad P(t) = \{p_1(t), ..., p_{M(t)}(t)\},$$

$$p_m(t) = (R_m(t), C_m(t), O_m(t)), \quad P_i^+ = P_i^+(t), \quad P_i^- = P_i^-(t)$$

4.6.2 State of the system

The state of the system $S(t)$ at time t consists of the states of all agents:

$$S(t) = (S_1(t), ..., S_M(t))$$

The state of a agent p_i consists of the following components:

$$S_i(t) = \{w_i(t), d_i(t), \Psi_i(t), \Pi_i^+(t), \Pi_i^-(t)\}$$

where
$w_i(t)$ - current available resources;
$d_i(t)$ - current money supply;
$\Psi_i(t)$ - current production/investment strategy;
$\Pi_i^+(t)$ - current consumer contracts;
$\Pi_i^-(t)$ - current supplier contracts;
In following subsections we describe in more detail each of these components.

4.6.3 Current available resources and money flow

The vector of all resources owned by agent at the beginning of period t is described by its quantity and attributes:

$$w_i(t) = \{(z_{ij}(t), v_{ij}(t)), j : \nu_j \in \Omega_i\}$$

Let us recall that money is included in the set of resources and have index 0, i.e. ν_0 denotes money. We could manage it in our model as any other resource. However, due to specific economic function of money we prefer to treat it separately and denote the current amount of money available to agent p_i as $d_i(t)$. In case if $\nu_0 \in \Omega_i$ we have

$$d_i(t) = z_{i0}(t)$$

Usually there are some constraints on the possible values of resources, for example in the most cases they should be nonnegative. Generally, admissible sets U_j are given such that

$$(w_i(t), d_i(t)) \in U_j, \quad j = 1 : J \tag{5}$$

4.6.4 Current production/investment strategy

Let us consider again production functions from (3). We want to generalize their definition to include delay phenomena. For example, it takes some time to train human resources and acquire necessary technological capabilities. Generally, some resources are produced within the same time period, while other resources become operational after some time after initial investment in terms of money and other resources is made. Similarly, some types of needs of end users may be satisfied for some time after acquisition of certain resources is made. We describe this phenomena by allowing the amount and attributes of resources produced at time t to depend on resources from consumption set at current and previous periods of time:

$$(z_k(t), v_k(t)) = \psi_k^i((z_j(\tau), v_j(\tau)), j : \nu_j \in W_k^+,\ t - \Delta_k^1 \le \tau \le t) \tag{6}$$

where Δ_k^1 is the depth of the memory of the system which can differ for different resources. Here we allow production function depend explicitly on agent p_i.

The production/investment strategy $\Psi_i(t)$ has *duration* which lasts from time t to time $t + t_\Psi$ which can be different for different strategies. At each time $\tau :\ t \le \tau \le t + t_\Psi$ it involves decision $\Psi_i(t, \tau)$:

$$\Psi_i(t) = \{\Psi_i(t, \tau),\ t \le \tau \le t + t_\Psi\} \tag{7}$$

Each such decision consists of decisions $\Psi_{ik}(t, \tau)$ for all resources which belong to production set Ω_i of the agent p_i:

$$\Psi_{ik}(t, \tau) = \{\Psi_{ik}(t, \tau),\ k : \nu_k \in \Omega_i\}$$

Each resource strategy $\Psi_{ik}(t, \tau)$ consists of two parts: production part $\Psi_{ik}^0(t, \tau)$ and consumption part $\Psi_{ik}^+(t, \tau)$. Production part is simply quantity and attributes of resource ν_k which is planned at time t to produce at time τ:

$$\Psi_{ik}^0(t, \tau) = (z_{ik}^p(\tau), v_{ik}^p(\tau))$$

while consumption part consists of all resources necessary to consume at time τ in order to produce resources defined by production strategy:

$$\Psi_{ik}^+(t, \tau) = \{(z_j^c(\tau), v_j^c(\tau)),\ j : \nu_j \in W_k^+\}$$

Obviously, production strategy $\Psi_{ik}^0(t, \tau)$ and consumption strategy $\Psi_{ik}^+(t, \tau)$, $t \le \tau \le t_\Psi$ are connected with production functions (6).

From definition of production strategy follows that the total amount z_j^c of resource ν_j with attributes v_j needed to be consumed in order to execute the production strategy at time t can be expressed as follows:

$$z_j^c = \sum_{k:\ \nu_k \in \Omega_i, (z_j^c(t), v_j^c(t)) \in \Psi_{ik}^+(t,t), v_j^c(t) = v_j} z_j^c(t) \tag{8}$$

In case of money z_0^c which has empty attribute this expression becomes:

$$z_0^c = \sum_{k:\ \nu_k \in \Omega_i, (z_0^c(t), \emptyset) \in \Psi_{ik}^+(t,t)} z_0^c(t) \tag{9}$$

4.6.5 Current consumer contracts

This describes relations between agent p_i and its suppliers from the set $P_i^+(t)$ which is the set of its predecessors on consumer/supplier graph. A *contract* q is defined by the following quantities:

$$q = \{i_q; j_q; k_q; t_q^i; t_q^f; (z_q(\tau), v_q(\tau), \pi_q(\tau)), t_q^i \leq \tau \leq t_q^f\} \tag{10}$$

where

i_q - the number of consumer agent;

j_q - the number of supplier agent;

k_q - the number of supplied resource;

t_q^i - the time of the contract beginning;

t_q^f - the time of the contract ending;

$(z_q(\tau), v_q(\tau), \pi_q(\tau))$ - respectively quantity, attributes and unit price of resource ν_{k_q} which agent p_{j_q} supplies to agent p_{i_q} at time τ. In case if contract involves borrowing money, i.e. $k_q = 0$ then $v_q(\tau) = \emptyset$ and $\pi_q(\tau)$ means the amount of money to return at time period τ.

Thus, the results of execution of contract q at time τ are the following:

- agent p_{i_q} receives from agent p_{j_q} resource ν_{k_q} with quantity and attributes $(z_q(\tau), v_q(\tau))$;

- agent p_{j_q} receives from agent p_{i_q} amount of money $\pi_q(\tau)z_q(\tau)$.

The set of all contracts present in the system at time t is denoted by $Q(t)$.

The flow of resources follows direction of arcs in consumer/supplier graph, while the flow of money is opposite to this direction. In order for contract $q \in Q(t)$ to be admissible it should satisfy a set of constraints, for example

$$\nu_{k_q} \in \Omega_{j_q}$$

i.e. resource ν_{k_q} is among resources produced by agent p_{j_q}. The set of consumer contracts for agent p_i is then defined as the set of all contracts which have p_i as consumer agent and are active at time t:

$$\Pi_i^+(t) = \{q : \; i_q = i, t_q^i \leq t \leq t_q^f\} \tag{11}$$

This set defines the set of arcs in consumer/supplier graph:

$$(p_j, p_i) \in D(t) \Leftrightarrow \exists q \in \Pi_i^+(t) : j = j_q$$

The following expression defines the total amount z_k of resource ν_k with attributes v_k received by agent p_i from execution of contracts:

$$z_k = \sum_{q:\; q \in \Pi_i^+(t), k_q = k, v_q(t) = v_k} z_q(t) \tag{12}$$

Some contracts involve borrowing money. The total amount z_0^b of money borrowed during time period t equals:

$$z_0^b = \sum_{q:\; q \in \Pi_i^+(t), k_q = 0} z_q(t) \tag{13}$$

The amount of money z_0^s spent on debt servicing equals

$$z_0^s = \sum_{q:\; q \in \Pi_i^+(t), k_q = 0} \pi_q(t) \tag{14}$$

and the total amount of money z_0^c spent by agent p_i on contracts not involving borrowing equals

$$z_0^c = \sum_{q:\; q \in \Pi_i^+(t), k_q \neq 0} \pi_q(t) z_q(t) \tag{15}$$

4.6.6 Current supplier contracts

Following definition (10) of contract from the previous section, the set of current supplier contracts $\Pi_i^-(t)$ for agent p_i includes all contracts $q \in Q(t)$ which are active at time t and contain agent p_i as supplier agent:

$$\Pi_i^-(t) = \{q : j_q = i, t_q^i \leq t \leq t_q^f\} \tag{16}$$

The following expression defines the total amount z_k of resource ν_k with attributes v_k supplied by agent p_i in accordance with contracts:

$$z_k = \sum_{q:\, q \in \Pi_i^-(t), k_q = k, v_q(t) = v_k} z_q(t) \tag{17}$$

We are interested here in agents which do not lend money. The total amount of money z_0^r received by such agent from fulfilling supplier contracts equals

$$z_0^r = \sum_{q:\, q \in \Pi_i^-(t)} \pi_q(t) z_q(t) \tag{18}$$

4.7 Flow of resources

Flow of resources is governed by supplier/consumer contracts and production strategies. Admissible contracts should conform with production strategies in such a way that for each agent and each resource amount of supplied resource plus amount of consumed resource should not exceed amount of received resource plus amount of produced resource and current available resource. Taking into account (8),(12),(17) we obtain the following equality for every agent p_i:

$$z_{ik}(t+1) = z_{ik}(t) + z_{ik}^p(t) + \sum_{q:\, q \in \Pi_i^+(t), k_q = k, v_q(t) = v_k} z_q(t) -$$

$$\sum_{l:\, \nu_l \in \Omega_i, (z_j^c(t), v_j^c(t)) \in \Psi_{il}^+(t,t), v_j^c(t) = v_j} z_j^c(t) - \sum_{q:\, q \in \Pi_i^-(t), k_q = k, v_q(t) = v_k} z_q(t) \tag{19}$$

This equation holds for all resources ν_k which belong to the union of sets Ω_i and Ω_i^+. If some resource belongs to Ω_i^+, but does not belong to Ω_i then the second and the last terms from the right hand side of (19) disappear.

Flow of money is considered separately, although we could manage it similarly to other resources utilizing framework of contracts. The profit $d_i^p(t)$ of agent p_i is expressed as the difference between amount of money received from supplier contracts and amount spent on production and consumption (9),(15),(18):

$$d_i^p(t) = \sum_{q:\, q \in \Pi_i^-(t)} \pi_q(t) z_q(t) - \sum_{k:\, \nu_k \in \Omega_i, (z_0^c(t), \emptyset) \in \Psi_{ik}^+(t,t)} z_0^c(t) - \sum_{q:\, q \in \Pi_i^+(t), k_q \neq 0} \pi_q(t) z_q(t) \tag{20}$$

If α is the tax level on profit and β the money return from agent activities outside the model then considering relations (13),(14) for money borrowing the money flow becomes:

$$d_i(t+1) = (1+\beta) d_i(t) + (1-\alpha) \max\{0, d_i^p(t)\} + \min\{0, d_i^p(t)\} +$$

$$\sum_{q:\, q \in \Pi_i^+(t), k_q = 0} z_q(t) - \sum_{q:\, q \in \Pi_i^+(t), k_q = 0} \pi_q(t) \tag{21}$$

Note that there are constraints (5) on the admissible values of available resources and money, most notably nonnegativity constraints. It may happen that some of these constraints are not satisfied by pair $(w_i(t+1), d_i(t+1))$ defined by equations (19),(21), i.e.

$$(w_i(t), d_i(t)) \bar{\in} U_j$$

for some j. This means that current production strategy of agent p_i is not compatible with current contract set. New contracts should be made and/or old contracts changed according to rules described in the following sections.

4.8 Strategies of agents

Each time period t the agents decide which decisions to take. These decisions involve production/investment plan, consumer and supplier contracts and offers to other agents.

In our notations *offers* $G_i(t)$ of the agent p_i can be described by the set of prospective contracts

$$G_i(t) = \{(z_{jk}(\tau), v_{jk}(\tau), \pi_{jk}(\tau)), t \le \tau \le t + t_G\} \tag{22}$$

which define the amount $z_{jk}(\tau)$ of resource ν_k with attributes $v_{jk}(\tau)$ and unit price $\pi_{jk}(\tau)$ which agent p_i offers to agent p_j at time τ. The agent p_j may accept this offer, with maybe lesser quantity, and then the offer becomes the contract.

Thus, the strategy $y_i(t)$ of agent p_i at time t consists of the following components:

$$y_i(t) = \{\Psi_{i0}(t), \Pi^+_{i0}(t), \Pi^-_{i0}(t), G_i(t)\} \tag{23}$$

where

$\Psi_{i0}(t)$ - change to production/investment strategy;
$\Pi^+_{i0}(t)$ - new consumer contracts;
$\Pi^-_{i0}(t)$ - new supplier contracts;
$G_i(t)$ - current offer to other agents.

The quantities $\Psi_{i0}(t), \Pi^+_{i0}(t), \Pi^-_{i0}(t)$ have the structure similar to respective quantities $\Psi_i(t), \Pi^+_i(t), \Pi^-_i(t)$ which make part of the state $S_i(t)$ of agent p_i and are described previously.

The strategy $y_i(t)$ depends on information $F_i(t)$ available to agent p_i at time t. Generally this includes partial knowledge about the state of the system and the strategies of other agents:

$$F_i(t) \subseteq \{S(t), y_j(t), j = 1:M\}, \quad y_i(t) = y_i(t, F_i(t))$$

The strategy $y_i(t, F_i(t))$ is selected according to some decision principle. Some of such decision principles are discussed in the following section.

4.9 Performance measures and selection of strategies

Performance measures formalize such notions as mission of business unit and its performance, need satisfaction for end users. Each agent p_i may have more than one performance measure $\theta_{ir}(\cdot)$ which are functions of the agent state and, maybe, the states of other agents, like in the case of performance measure which formalize market penetration:

$$\theta_{ir}(\cdot) = \theta_{ir}(S(t)), \quad r = 1, n_r \tag{24}$$

where $S(t)$ is the state of the system at time t. Through the states performance measures depend on the strategies $y_j(t)$ of different agents. Note that even if performance measure of agent p_i depends explicitly only on its own state $S_i(t)$ it still implicitly depends on the strategies of other agents trough current contracts $\Pi_i^+(t)$, $\Pi_i^-(t)$.

Among performance measures there would be measures of constraint satisfaction (5), like not go in debt below certain level, or measures of contract fulfillment.

A agent should select strategies $y_i(t)$ in order to obtain "good" values of its performance measures. Since some of them could be conflicting, there could be different notions of tradeoffs between different performance measures. Here are some examples:

1. *Staying within desirable sets.* Select strategies such that

$$\theta_{ir}(S(t)) \in \Theta_{ir}(t), \quad t = 0, ... \tag{25}$$

where $\Theta_{ir}(t)$ are some desirable sets which can change with time.

2. *Maximizing selected criterion with constraints on others.* Suppose that there is one "the most important" performance measure $\theta_{i0}(\cdot)$, like, for example, net profit. Then the objective of the agent p_i may be to maximize the value of this measure with constraints on all others. In case when current strategy affects only current period ("myopic" case) the problem becomes:

$$\max_{y_i(t)} \theta_{i0}(S(t)) \tag{26}$$

$$\theta_{ir}(S(t)) \in \Theta_{ir}(t), \quad r = 1 : n_r \tag{27}$$

Much more often, however, the current strategy affects the future states. In this case the problem of strategy selection may become

$$\max_{y_i(\tau), t \le \tau \le t+T} \sum_{\tau=t}^{t+T} \theta_{i0}(S(\tau)) \tag{28}$$

$$\theta_{ir}(S(\tau)) \in \Theta_{ir}(\tau), \quad r = 1 : n_r, \quad t \le \tau \le t + T \tag{29}$$

there may be different dynamic formulations, like maximization of selected criterion at the end of the planning period of specified length.

3. Maximizing integrated criterion. This is the case when there is no criterion of paramount importance, but all criterions have their relative weights λ_r. In myopic case such tradeoff strategies can be selected by solving the problem

$$\max_{y_i(t)} \sum_{r=1}^{n_r} \lambda_r \theta_{ir}(S(t)) \tag{30}$$

4. Principles of game theory. The principles (25)- (30) are sufficient for situations when the state of a agent p_i does not depend considerably on the strategies of other agents, for example the agent p_i holds a monopoly on some important product/service. Generally, however, the performance measures of different agents depend considerably on the strategies of other agents. Game theory developed different notions of strategy selection in such conditions.

For example, in fiercely competitive environment *the worst case* or *minimax* strategy may give better results then (25)-(30). It select the best strategy for agent p_i in then case when the strategies of all other agents are aversive. In the case (28),(29) this leads to solution of the following problem:

$$\max_{y_i(\tau),t\leq\tau\leq t+T} \quad \min_{y_j(\tau),t\leq\tau\leq t+T,j\neq i} \sum_{\tau=t}^{t+T} \theta_{i0}(S(\tau)) \tag{31}$$

$$\theta_{ir}(S(\tau)) \in \Theta_{ir}(\tau), \quad r = 1 : n_r, \quad t \leq \tau \leq t+T \tag{32}$$

In the case when environment is a mixture of competition and cooperation other strategies may prove to be more advantageous, like *Pareto strategies* and *coalition strategies.*

4.10 Coalitions of agents

Agents may make a coalition in order to improve their performance measures. Generally, there may be more than one coalition among agents. Coalitions E_k, $k = 1 : K_c$ are subsets of the set of agents P and make a partition of this set:

$$\bigcup_{k=1}^{K_c} E_k = P, \quad E_j \cap E_k = \emptyset, \ j \neq k$$

Agents belonging to the same coalition select their strategies according to common decision principles and exchange information. There may be different types of coalitions, some of them being outlawed by regulating authority. Generally, coalitions are described by type of information on which their common strategies depend and by decision principles. Let us describe one such type of coalition.

Coalition of equal trusting agents in adverse environment.

In such coalition E_k each agent knows the strategies, performance measures and states of other agents belonging to coalition plus some information about the state of other agents, but nothing about their performance measures and strategies of agents not involved in the coalition:

$$F_i(t) = \{(S_j(t), y_j(t)), j \in E_k\} \cup \bar{F}_i(t), \quad \bar{F}_i(t) \subseteq \{S_j, j \in P \setminus E_k\}$$

In this case the decision principle can be Pareto optimality within coalition and minimax approach toward outsiders. In case of criterion (28),(29) each agent belonging to coalition solves the following problem.

$$\mathrm{pmax}_{y_l(\tau),t\leq\tau\leq t+T,l\in E_i} \quad \min_{y_j(\tau),t\leq\tau\leq t+T,j\in P\setminus E_k} \sum_{\tau=t}^{t+T} \theta_{i0}(S(\tau)) \tag{33}$$

$$\theta_{ir}(S(\tau)) \in \Theta_{ir}(\tau), \quad r = 1 : n_r, \quad t \leq \tau \leq t+T \tag{34}$$

where by pmax we denoted Pareto maximum.

Coalitions may change their composure due to decisions of particular agents to join other coalitions or play on their own accord if they discover that such action improves their performance measures. Procedures of coalition formation and dissolving should be specified.

4.11 Contract adjustment

This is done when current available resources $(w_i(t+1), d_i(t+1))$ obtained according (19),(21) to do not satisfy constraints (5) and this can not be adjusted by making new consumer contracts according to offers $G_j(t)$ advanced by other agents. This situation may occur, for example, when end users decided to terminate some contracts. Then current contracts and/or production policy should be changed in order to allow the current state of the agent p_i reenter into admissible set. Contract adjustment brings about penalties which are reflected in the money flow.

4.12 Evolution of the system

Now we are ready to describe the time evolution of the simulation model defined in the previous sections.

Initialization. At time $t = 0$ initial states $S_i(0)$ are assigned to all agents and simulation iterval $[0, T]$ is selected.

Generic step. At the beginning of time period t agents are in the states $S_i(t)$ and the current set of agents $P(t)$ is divided in coalitions $E_k(t),\ k = 1 : K_c(t)$. The following actions are performed during the step t:

1. New agents are introduced and, possibly, some old agents are eliminated which changes the set of agents to $P(t+1)$.

2. New coalitions are formed or old coalitions are confirmed which brings the coalition set to $E_k(t+1),\ k = 1 : K_c(t+1)$.

3. By each agent $p_i \in P(t+1)$ the following actions are performed:

- Information $F_i(t)$ is obtained according to participation in coalition;

- Strategy y_i is selected according to one of decision principles discussed above in concordance with other coalition partners. This strategy involves making offer to other agents, selection of new consumer and supplier contracts and adjustment of production/investment plan.

- New state $S_i(t+1)$ is computed according to (19),(21). If it enters in admissible set then go to step $t+1$. If not, try to reenter in admissible set by modifying strategy and/or current contracts. If this is impossible then this agent is eliminated on the next step.

5 Summary

We presented here a general methodology for modeling of complex distributed multiagent systems and presented a prototype of the simulator INFOGEN for simulation of such systems. We had as reference point its application to modeling of information industry, although it is applicable for also for other multiagent systems.

There are still many research issues to be clarified. As we have seen multiagent systems exhibit widely different dynamics under different values of system parameters. It would be highly desirable to develop tools which would permit to identify regions of stability of certain equilibrium points and regions where one strategy is superior with respect to another strategy. Right now we can do this only by trial and error which is very ineffective. One possibility to develop such tools is to extend to the multiagent systems the theory of sensitivity analysis developed for Discrete Event Dynamic Systems [2, 15, 17, 19, 24, 26].

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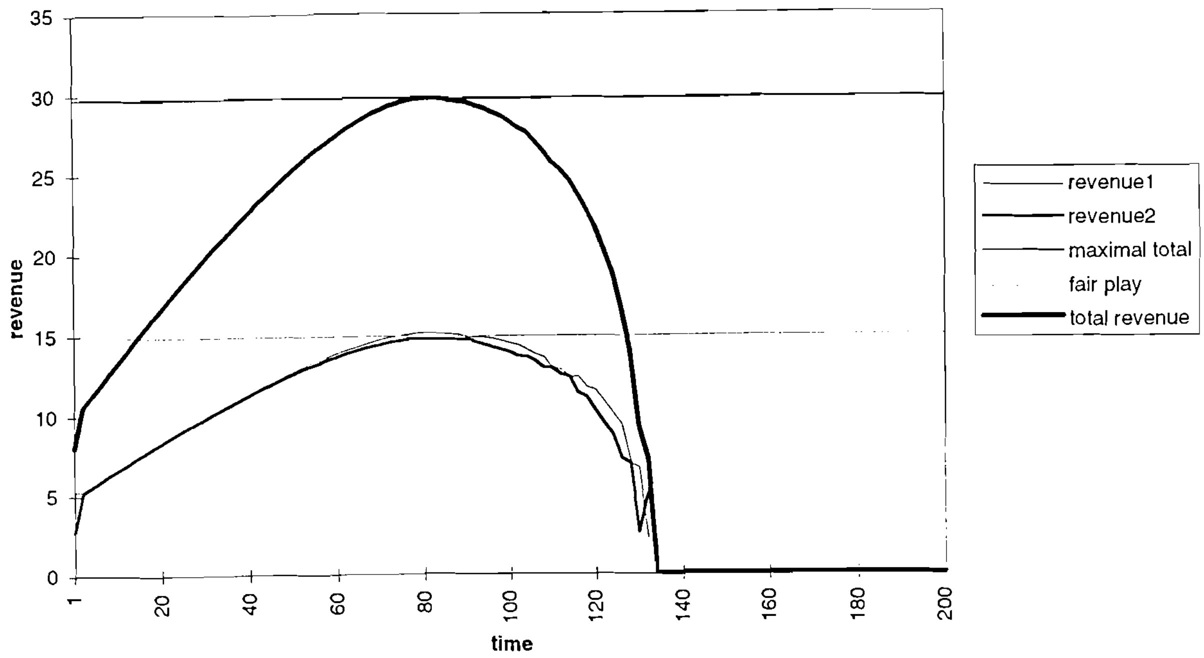

Figure 1: Collapse of the market for innovative product

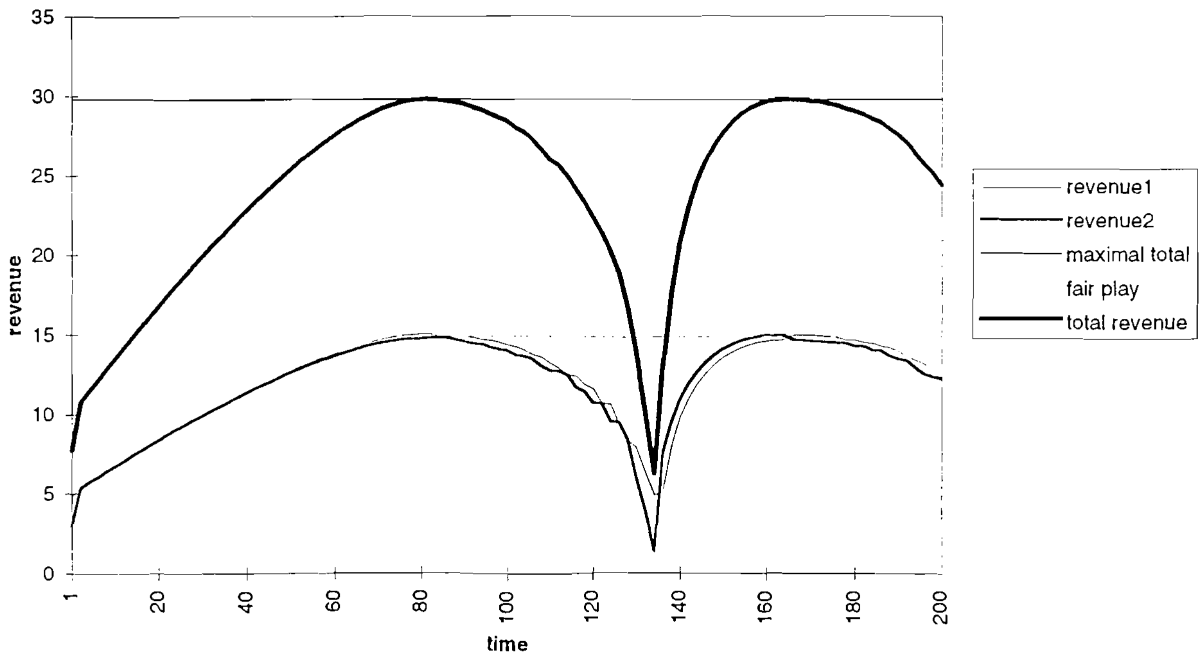

Figure 2: Near collapse of the market for innovative product with rebound

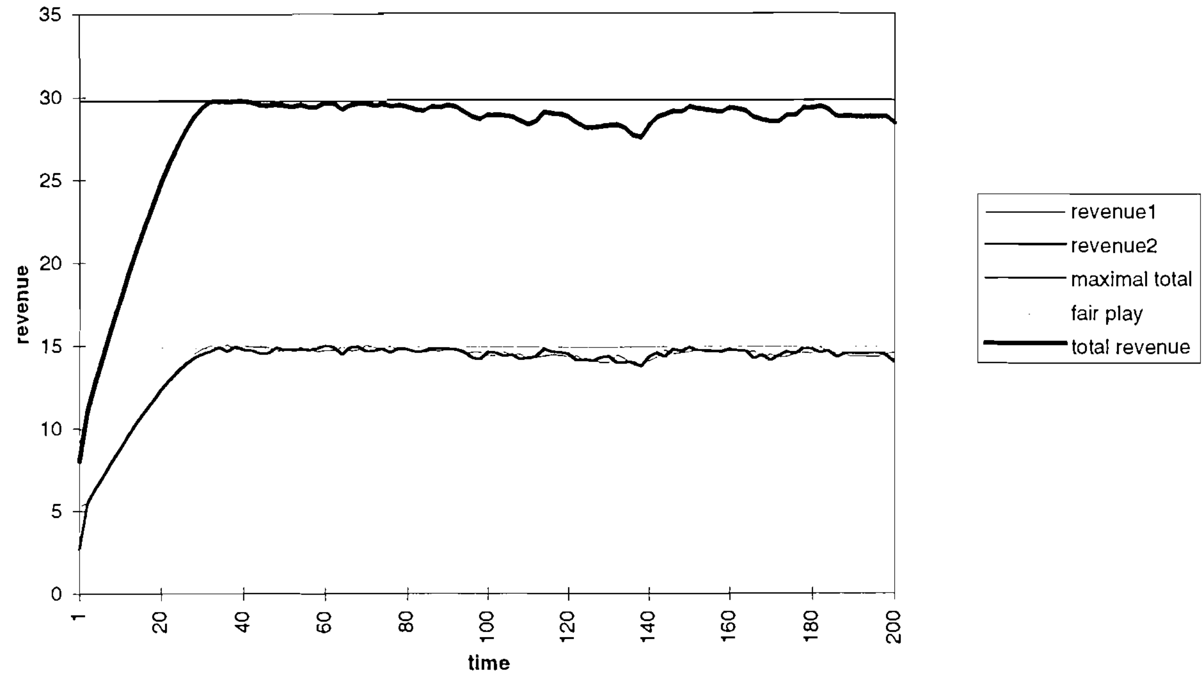

Figure 3: Victory of innovative product

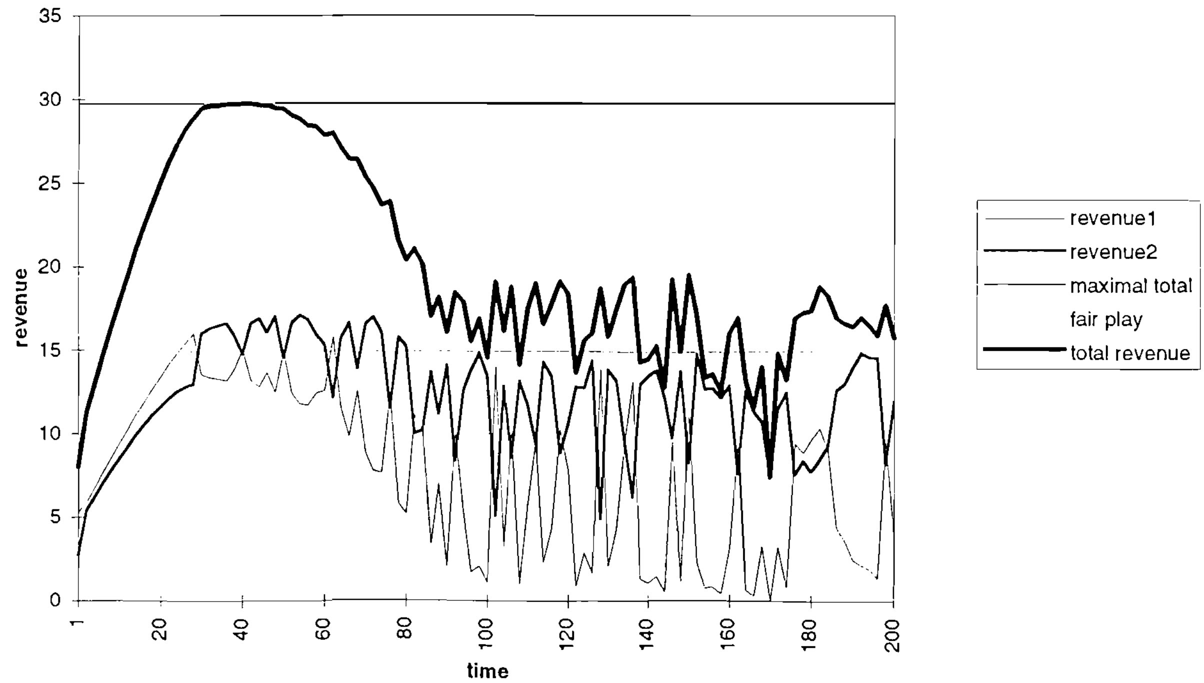

Figure 4: Chaotic market behavior

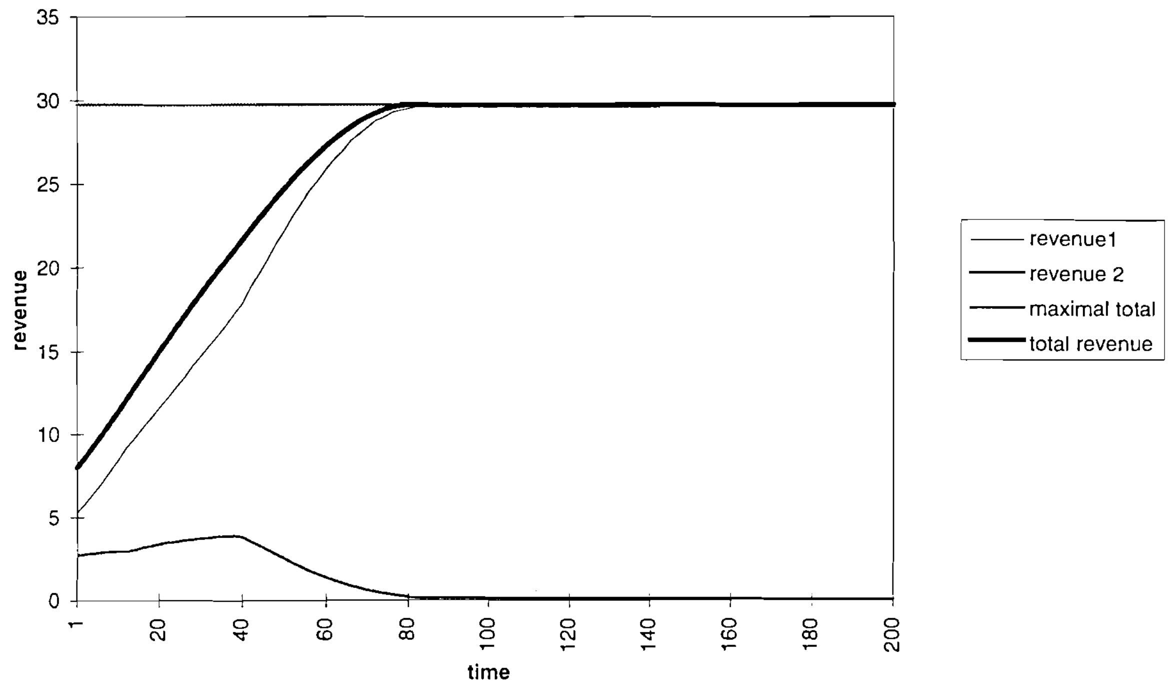

Figure 5: The first producer wins

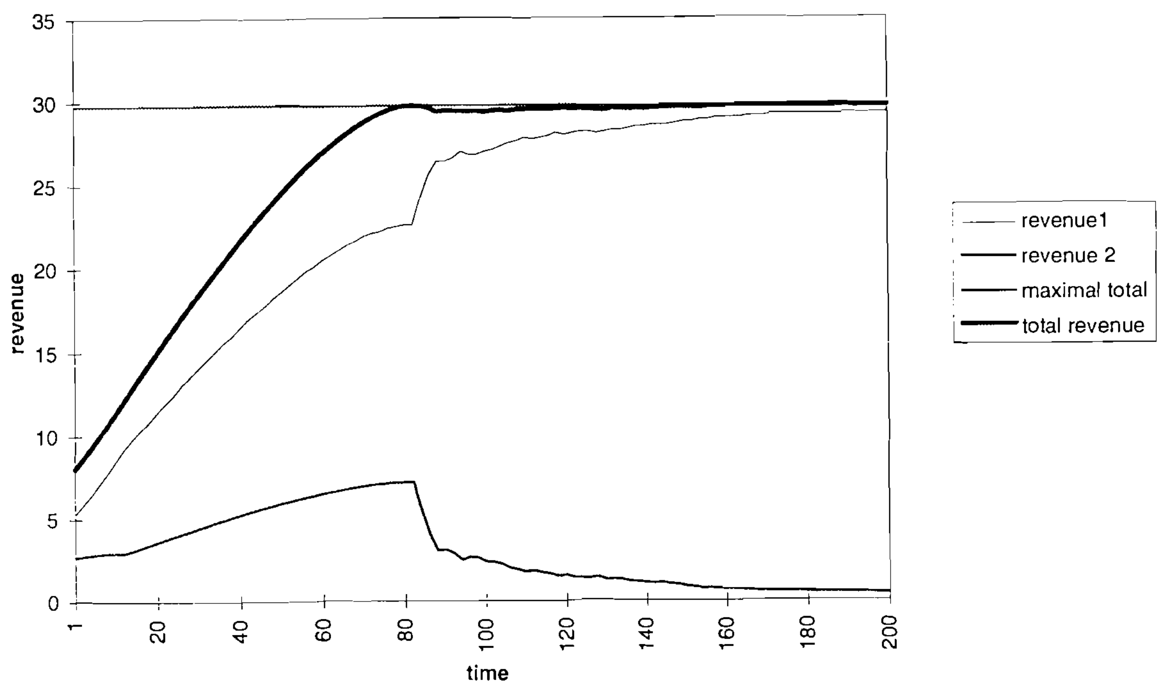

Figure 6: The first producer wins near the catastrophy point

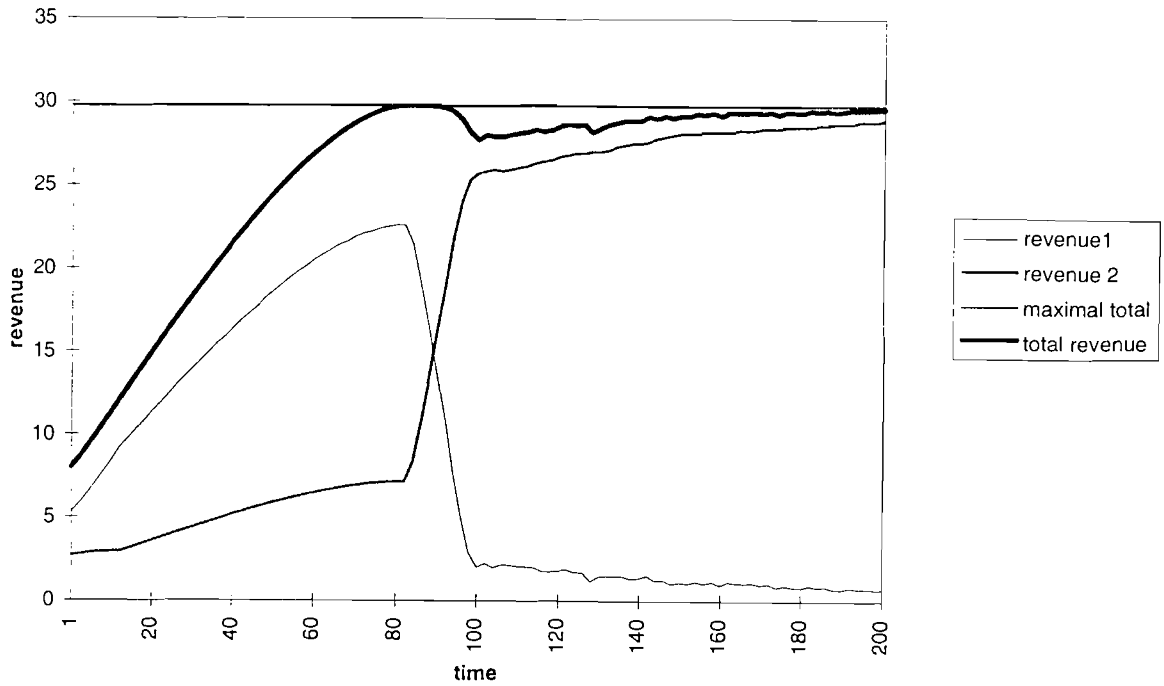

Figure 7: The second producer wins near the catastrophy point

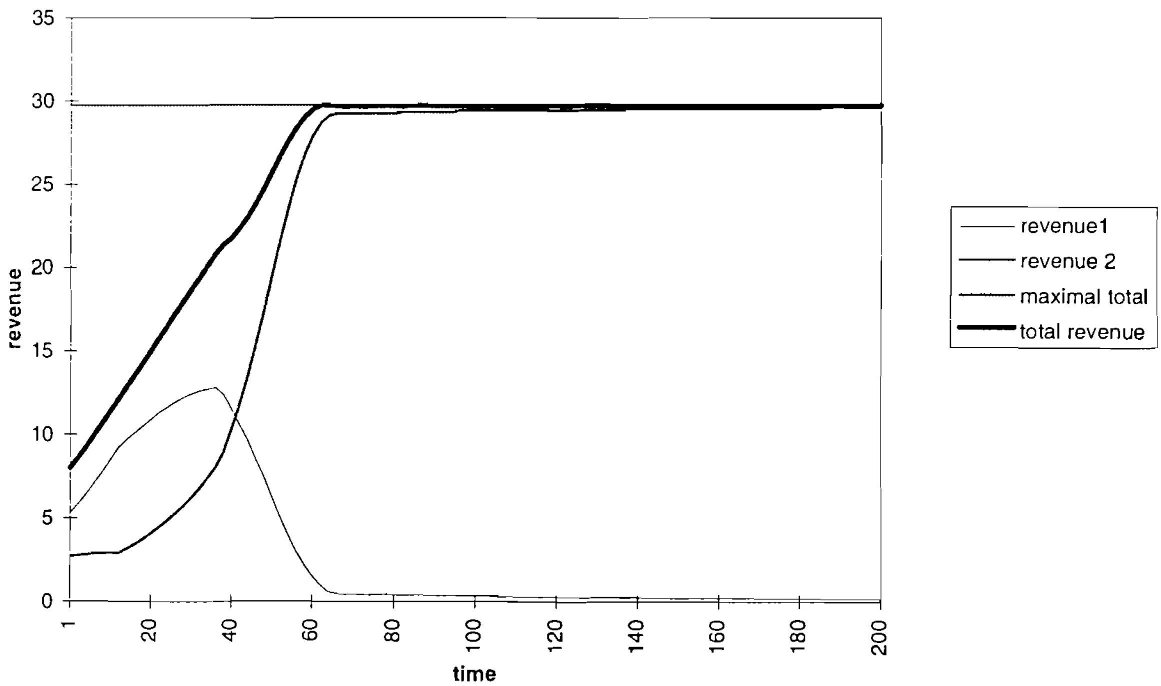

Figure 8: The second producer wins